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A Generalized Human Development Index*

Satya R. Chakravarty

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LRH : Satya R. Chakravarty

Abstract

The human development index indicates achievement in the living standard of a population in terms of attainment levels of different quality-of-life attributes e.g., educational attainment and life expectancy at birth. This paper axiomatically characterizes a general measure of achievement. This achievement index, which contains the UNDP human development index as a special case, can be regarded as a generalized human development index. The general index enables us to calculate the percentage contributions of individual attributes to overall achievement and hence to identify the attributes that are more/less susceptible to achievement. This kind of breakdown becomes important from a policy point of view. We also provide an empirical illustration of the axiomatically characterized indices.

*Chakravarty : Economic Research Unit, Indian Statistical Institute, 203, B.T. Road, Calcutta 700 035, India, Tel : 91-33-5778085 (Extn. 2622), Fax : 91-33-577-6680, E-mail: satya@isical.ac.in. I wish to thank Sandip Mitra for numerical computations. I express sincere gratitude to a referee for many constructive suggestions on earlier drafts of the paper.

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Address of author : Satya R. Chakravarty, Economic Research Unit, Indian Statistical Institute, 203, B.T. Road, Calcutta 700 035, India, Phone : 91-33-5778085 (Extn. 2622), Fax : 91-33-577-6680, E-mail : satya@isical.ac.in

1 Introduction

Well-being of a population has often been measured in terms of income. (See, for example, Kolm, 1969; Atkinson, 1970; Kakwani, 1980; Shorrocks, 1983; Chakravarty, 1990; Sen, 1997.) However, many researchers have insisted on the inadequacy of income as the sole indicator of welfare and argued that income should be supplemented by other attributes of welfare such as health and literacy. For instance, the basic needs approach advocated by development economists regards development as an improvement in an array of human needs and not just as a growth of income (Streeten, 1981). At the same time people do not isolate different aspects of their lives. Instead, they speak about an overall level of well-being. Therefore, the construction of a composite index of well-being is a worthwhile exercise. (See Kolm, 1977; Atkinson and Bourguignon, 1982; Maasoumi, 1989; Mosler, 1994; Tsui, 1998; Bourguignon and Chakravarty, 1999.) We may also cite major attempts at compilation of multidimensional data, such as the United Nations supported basic needs and physical quality-of-life attributes. (See Ram, 1982, 1992.) Composite indices of well-being have been constructed for the purpose of interpersonal or international comparisons from this kind of data and others derived from micro data sets. For instance, Ram (1982) suggested the principal components of basic needs and physical quality-of-life attributes data. Maasoumi (1989) used the same data set as Ram to illustrate an information theoretic composite indices of well-being; Maasoumi and Nickelsburg (1988) computed similar indices from Michigan Panel data on incomes as well as housing equity and education; and Slottje (1991) employed many more economic and social variables than usual to study alternative indexing techniques based on hedonic regressions, principal components and 'ranked attributes'. Hirschberg, Maasoumi and Slottje (1991) proposed statistical cluster analysis methods to explore different ways and levels for clustering of 23 diverse attributes such as political rights, civil liberties, life expectancy, literacy etc., and computed aggregate measures of welfare for many countries.

Sen (1985, 1987, 1997) defined standard of living in terms of (i) functioning, which indicates attainments of different attributes, and (ii) capability, which is the ability to attain. The capability approach emphasizes what a person can do and not what he can purchase as the ultimate metric of well-being. An example of a functioning achievement index (achievement index, for short) is the human development index (HDI) suggested by UNDP (1990-99). It is given by the arithmetic average of the normalized attainment levels of the three quality-of-life attributes, per capita real GDP, life expectancy and educational attainment rate.

Several studies concerning axiomatic structure of measures of improvement in well-being, which aggregates increments in the attainment levels of different quality-of-life attributes, have been made in both single-dimensional and multidimensional frameworks (see Kakwani, 1993, Majumder and Chakravarty, 1996, Tsui, 1996; Chakravarty and Mukherjee, 1999). But no such exercise has been carried out for achievement measures. In this paper we propose a set of axioms for a measure of achievement and characterize the underlying measure. Thus, our approach deviates from the ad hoc approach to the design of the HDI. Our characterized measure contains the HDI as a special case. Our index can therefore be regarded as a generalized HDI.

Now, 'people do not just want to be alive. They want to be knowledgeable; and they certainly may want a decent life, one that is not considerably undermined by extreme poverty and constant worry about sheer physical survival' (UNDP, 1991, p.88). In other words, all these three components of human life deserve equal weight, and this idea is reflectd in the construction of the HDI[1]. Evidently, the low contributing attributes require attention from policy point of view for improving their levels in order to reach a higher position in achievement. It may be important to note that the HDI (and also our general index) become helpful in calculating the percentage contributions made by individual attributes to overall achievement and hence in isolating the attributes according to their degrees of contribution. This shows an important policy application of our general index. Clearly, according to this notion of policy recommendation an assessment of overall well-being becomes contingent on the implicit valuation of the achievement index. However, an exercise of this type may be useful for two reasons. First, following Sen (1985), the non-welfarist approach to policy analysis is becoming quite popular. Second, in many situations policy is evaluated using specific forms of indices. Therefore, it seems worthwhile to see what type of policy would be implied by the use of a specific achievement index.

2 Characterization of Individual Indicators

We assume that there are k attributes of well-being. These attributes may be life expectancy at birth, real GDP per capita, educational attainment, housing, provision of public goods, and so on. Let x_i stand for the attainment level or the value of attribute i for the country under consideration, where $i = 1, 2, \dots, k$. Denote the lower and upper bounds of x_i by m_i and M_i respectively, that is, $x_i \in [m_i, M_i]$, which is a subset of the real line R^1. Clearly, if $m_i = M_i$, then the interval $[m_i, M_i]$ is the singleton set $\{m_i\}$. Therefore, we assume that $m_i < M_i$. That is, the open set (m_i, M_i) is nonempty. (See Morris, 1979; Sen, 1981; Dasgupta, 1993 for discussion on these bounds.)

An indicator for i is a real valued function A that associates a value $A(x_i, m_i, M_i)$ to each $x_i \in [m_i, M_i]$. We assume that this indicator is twice differentiable. Differentiability is just a technical assumption and makes the analysis simple. Further, differentiability of A implies its continuity which ensures that A will not be over sensitive to minor observational errors on attribute i, and its lower and upper bounds. We have also assumed that A is attribute independent, that is, the same functional form is chosen for all attributes. While we can certainly choose different functional forms for different attributes, in empirical applications choice of the same functional form makes computation easy.

We now suggest some properties for an arbitrary index A. The suggested properties are :

Normalization (NM) : $A(x_i, m_i, M_i) = 0$ if $x_i = m_i$,
$= 1$ if $x_i = M_i$.

Monotonicity (MN) : Given m_i and M_i, an increase in x_i increases A.

Translation Invariance (TI) : $A(x_i, m_i, M_i) = A(x_i + c, m_i + c, M_i + c)$, where c is any scalar such that $m_i + c \geq 0$.

Homogeneity (HM) : For any $c > 0, A(x_i, m_i, M_i) = A(cx_i, cm_i, cM_i)$.

Lower Gain in Indicator at Higher levels of Attainment Difference (LI) : Let $x_i \in [m_i, M_i]$ be any attainment level of attribute i. Then for any $\delta > 0$ such that $x_i + \delta \in [m_i, M_i]$, the magnitude of indicator gain $A(x_i + \delta, m_i, M_i) - A(x_i, m_i, M_i)$ is a decreasing function of δ.

Normalization means that indicator levels for attribute i are zero and one in the extreme cases when the attribute takes on its minimum and maximum values respectively. According to monotonicity, given other things, an increase in the attainment level of the attribute increases the indicator. Translation invariance says that if the actual value of the attribute as well as its lower and upper bounds are augmented by the same absolute amount, then there is no change in the value of the indicator. Therefore, the indicator depends on the absolute differences $x_i - m_i$ and $M_i - m_i$. To understand TI more explicitly, let us represent the arguments m_i, x_i and M_i on the straight line $[0, \infty)$. If we regard m_i as the origin for arguments of A, then TI demands that a shift of the origin along with equal shifts of x_i and M_i keeps A unchanged. This is one way of looking at the behavior of A under equal absolute changes in all arguments. In fact, TI can be considered as the absolute counterpart to HM. While TI is concerned with origin independence, HM refers to scale independence. More precisely, HM requires insensitivity of the indicator to the unit of measurement of the attribute. For instance, if attribute i is life expectancy and if we change the unit of measurement from years to months, then the value of indicator should not change. Finally, LI says that an increase in the value of the attribute say, per capita real GDP, represents a greater indicator increase at lower levels than an equivalent increase at higher levels. For instance, an increase in per capital real GDP from 0 to 10 shows a higher degree of indicator gain than an increase from 40 to 50.

It may be noted that these axioms have to be imposed at a time for deriving our indicator, because they are different, and none implies or is implied by a second one. In fact, this is demonstrated more rigorously by our theorem 2, which shows that if one axiom is dropped then our indicator need not be the unique one derived when all the axioms are used at a time. The unique indicator is derived in theorem 1.

To show the relevance of these axioms in the context of the HDI, let us consider the functional form of the HDI for k arbitrary attributes :

$$HDI = \sum_{i=1}^{k} ((x_i - m_i)/(M_i - m_i))/k \ . \tag{1}$$

Now, consider an arbitrary component $(x_i - m_i)/(M_i - m_i)$ of the HDI. We note that this component satisfies NM, MN, TI and HM. But it fails to meet LI. However, its transformed version $((x_i - m_i)/(M_i - m_i))^r, 0 < r < 1$, which we discuss later in the section, satisfies LI.

As argued earlier, LI is quite reasonable intuitively. In fact, as shown in section 3, in constructing the gender-related development index, a variant of the HDI, the UNDP has implicitly assumed this postulate.

Theorem 1 : Suppose that the indicator A is twice differentiable. Then it satisfies properties NM, MN, TI, HM and LI if and only if it can be written as

$$A(x_i, m_i, M_i) = f\ ((x_i - m_i)/(M_i - m_i))\ , \tag{2}$$

where $f : [0,1] \to R^1$ is twice differentiable, increasing, strictly concave, and $f(0) = 0$ and $f(1) = 1$.

Proof : See appendix.

The indicator identified in (2) is bounded between zero and one, where the lower and upper bounds are achieved in the extreme cases described in postulate NM. The function $f : [0,1] \to R^1$ satisfying the conditions stated in theorem 1 will be called regular. Given any regular f we have a corresponding indicator. These indices will differ only in the manner how we specify f.

For example, suppose that the indicator is of the form

$$f_r\ ((x_i - m_i)/(M_i - m_i)) = ((x_i - m_i)/(M_i - m_i))^r\ , \quad 0 < r < 1. \tag{3}$$

Since $f_r(0) = 0$ and $f_r(1) = 1$, the index in (3) meets NM. MN follows from increasingness of f_r. Obviously f_r remains unaltered under equal absolute changes in its arguments. Because f_r is homogeneous of degree zero in (x_i, m_i, M_i), HM is satisfied. Finally, LI follows from strict concavity of f_r (guaranteed by $0 < r < 1$). Given x_i, m_i and M_i, an increase in r decreases f_r. For $r = 1, f_r$ becomes $f_1 = (x_i - m_i)/(M_i - m_i)$, a component of the HDI in (1).

As stated earlier, though f_1 meets NM, MN, TI and HM; because of its linearity in x_i, it fails to verify LI. A negative transform of f_1, more precisely, $1 - f_1 = (M_i - x_i)/(M_i - m_i)$, has been considered by UNDP as the deprivation function for attribute i. The deprivation index expresses the shortfall of the attainment level x_i from its maximum attainable value M_i as a fraction of the range $(M_i - m_i)$.

As an alternative to f_r in (3) we may consider the index

$$f_e\left((x_i - m_i)/(M_i - m_i)\right) = (1 - e^{-(x_i - m_i)/(M_i - m_i)})/(1 - e^{-1})\ , \tag{4}$$

where e represents the exponential function. Since $f_e : [0,1] \rightarrow R^1$ is regular, the index in (4) satisfies all the properties considered above.

We conclude this section by demonstrating that the five axioms NM, MN, TI, HM and LI are independent. The demonstration involves construction of an indicator that will satisfy any four of the five axioms but not the remaining one. Thus, independence means that if we drop any one of the five axioms, the characterized index will not be of the from (2).

Theorem 2 : Properties NM, MO, TI, HM and LI are independent.

Proof : See appendix.

3 The Generalized Human Development Index

The indicator given by (2) involves the extent of attainment for one particular attribute of well-being. To get a picture of overall indicator, we have to consider all the attributes simultaneously. That is, we need to construct an achievement index involving all the attributes under consideration. The derived achievement index turns out to be a generalization of the HDI.

We assume at the outset that the achievement index G is a real valued function of single-dimensional indices. This kind of assumption is made in many branches of economics. For instance, in welfare economics social utility is regarded as a function of individual utility levels.

To state this assumption in the current context, let us denote $f((x_i - m_i)/(M_i - m_i))$ by a_i, where $i = 1, \ldots k$. We write $\underline{a}$ for the vector $(a_1, \ldots, a_k)$. Then the relationship can be formally stated as : There exists a function $I : [0,1]^k \rightarrow R^1$ such that for all $(x_1, m_1, M_1), (x_2, m_2, M_2), \ldots$ (x_k, m_k, M_k), the achievement index $G((x_1, m_1, M_1), (x_2, m_2, M_2), \ldots (x_k, m_k, M_k))$ can be written as $I(a_1, a_2, \ldots a_k)$, where $[0,1]^k$ is the k-fold cartesian product of $[0,1]$.

Under the above assumption we now state certain properties for an arbitrary index I.

Normalization (NOM) : For any $z \in [0,1], I(z, \ldots z) = z$.

Consistency in Aggregation (CIA) : For any $\underline{a}, \underline{b} \in [0,1]^k$,

$$I(a_1 + b_1, a_2 + b_2, \ldots, a_k + b_k) = I(a_1, a_2, \ldots, a_k) + I(b_1, b_2, \ldots, b_k). \tag{5}$$

Symmetry (SYM) : For all $\underline{a} \in [0,1]^k, I(\underline{a}) = I(\underline{a}P)$, where P is any $k \times k$ permutation matrix**2**.

According to NOM, if the indicator levels for different attributes take on the same value α, then the achievement is also α. The postulate thus shows that achievement is an average of individual indicators. Furthermore, we note that when there is only one attribute, the indicator and achievement are the same. The next property, CIA, can be interpreted as follows. Suppose that attribute i has two components. For instance, if i is educational attainment then its two components can be adult literacy rate, and combined primary, secondary and tertiary enrolment ratios. Let $a_i(b_i)$ be the indicator level of attribute i based on the first (second) component. In case it is not possible to split indicator i, we can attach zero value to a_i (or b_i) so that the second (first) component represents the attribute itself and $b_i(a_i)$ becomes the level of attribute. Then CIA says that sum of the achievements based on the vectors $(a_1, a_2, \ldots, a_k)$ and $(b_1, b_2, \ldots, b_k)$ is same as the achievement based on the added vector $(a_1+b_1, a_2+b_2, \ldots, a_k+b_k)$. The property therefore shows how to calculate achievement when we split the attributes into components. An important implication of CIA is that I is insensitive to the order in which the attributes are broken down. That is, a_i and b_i can be interchanged on the right-hand side of (5), and this can be done for any number of attributes. Finally, symmetry requires insensitivity of I to permutation of its arguments. It means that anything other than the individual indicator levels is completely irrelevant to the measurement of achievement.

Note that CIA forces the overall index to be an additive function of the indicators of k attributes (see equation (6) and the proof of theorem 3 in the appendix). It may be worthwhile to look at the nature of substitutability, as implied by CIA, among different attributes. Assuming that symmetry holds, under CIA, the marginal social rate of substitution between attributes i and j is

$$(f'((x_i - m_i)/(M_i - m_i)))(M_j - m_j)/(f'((x_j - m_j)/(M_j - m_j)))(M_i - m_i) \; ,$$

which is independent of the level of a third attribute n. Under strict concavity of f, this marginal rate of substitution is nonconstant and the well-being contour involving attributes i and j is strictly convex to the origin. As we go down along a contour, more and more units of the attribute plotted along the horizontal axis are required to substitute each additional unit of the other and the substitution becomes increasingly difficult. However, if f is linear, as implicitly assumed in the construction of the HDI, the marginal rate of substitution is constant and one attribute can be perfectly substituted for another. Given simultaneous importance of all the attributes, this may not be desirable. We can similarly analyse the impact of CIA on substitutability among different components of an attribute[3].

Theorem 3 : An achievement index I satisfies NOM, CIA and SYM if and only if it is of the form

$$\begin{aligned} I\,(a_1, a_2, \ldots, a_k) &= \sum_{i=1}^{k} a_i/k \\ &= \sum_{i=1}^{k} \; f\,((x_i - m_i)/(M_i - m_i))/k\,. \end{aligned} \tag{6}$$

Proof : See appendix.

The achievement index given by (6) possesses the following interesting properties :

(i) It is bounded between zero and one, where the lower (upper) bound is obtained in the extreme case $x_i = m_i (x_i = M_i)$ for all i. That is, I takes on the values zero and one when all the attributes achieve their minimum and maximum attainable values respectively.

(ii) It is increasing in individual indicator levels $a_i^{\prime}s$ (hence in $x_i^{\prime}s$).

(iii) It is globally translation invariant - when all the attainment levels as well their lower and upper bounds are augmented/diminished by the same absolute amount, I remains unaltered.

(iv) It is globally homogeneous of degree zero - a multiplication of the values of the attributes along with their lower and upper bounds by a positive scalar does not change the index value.

(v) For any attribute, the achievement difference is greater at lower attainment levels, given that the values of other attributes remain fixed.

(vi) Since the achievement index is simply the arithmetic average of attribute-wise indicators, we can make quantitative assessment of individual attributes. The quantity $T_i = a_i/k$ may be interpreted as the total contribution of attribute i to achievement I, while 100 a_i/I is the percentage contribution of attribute i. Therefore, this kind of breakdown allows us to identify the attributes which are less/more sensitive to the achievement. The less sensitive attributes may require attention from policy point of view for improvement of their contributions so that a higher standard of living can be achieved.

To illustrate the general formula in (6) let us suppose that the indicators are of the form (3). The corresponding achievement index is

$$I_r = \sum_{i=1}^{k} ((x_i - m_i)/(M_i - m_i))^r/k \ , \quad 0 < r < 1. \tag{7}$$

I_r is decreasing in r. For $r = 1, I_r$ becomes $I_1 = \sum_{i=1}^{k}((x_i - m_i)/k(M_i - m_i))$, which is the HDI suggested by UNDP for k attributes. (See equation (1)).

As argued in (v) above, I_r will attach greater weight to achievement differences at lower attainment levels. But the k-attribute HDI I_1 violates this, although it satisfies all other properties of a multidimensional achievement index. This is an advantage of the parametric index I_r over the conventional index I_1.

In (7) if we allow r to take on values in the interval (0, 1], then I_r contains I_1 as a special case. We can therefore say that I_r, for $0 < r \leq 1$, is a generalization of the HDI (I_1) under the restriction that all particular cases of I_r except I_1 satisfy the property mentioned in (v) above.

Next, we wish to compare I_r in (7) with the gender-related development index (GDI) suggested by UNDP. In the case of the GDI the same variables as in the HDI are used, the former only adjusts the average achievement in each variable in accordance with the disparity of achievement between men and women. Let $A_i(g)$ be the indicator for attribute i for the male population, that is, $A_i(g) = (x_i(g) - m_i(g))/(M_i(g) - m_i(g))$, where $x_i(g)$ is the attainment level of attribute i for the male population, $m_i(g)$ and $M_i(g)$ are respectively the lower and upper bounds of $x_i(g)$. We can analogously define $A_i(l)$, the indicator for attribute i for the female. Then the equally distributed attainment index for the ith attribute is

$$\bar{A}_i = \left(p_g\, A_i(g)^{1-t} + p_l\, A_i(l)^{1-t}\right)^{(1-t)^{-1}} \ , \tag{8}$$

where $p_g(p_l)$ = proportion of males (females) in the total population and $t > 0 (t \neq 1)$ is a parameter[4]. Evidently, since $A_i(g)$ and $A_i(l)$ in (8) fulfils NM, MN, HM and TI, $\bar{A}_i$ fulfils them as well. Note that $\bar{A}_i$ is a CES type function and satisfies strict concavity if $t > 0$ (Madden, 1986, pp. 123-24). Therefore, given $t > 0, \bar{A}_i$ meets LI also. The k-attribute GDI suggested by UNDP is then defined by

$$\begin{aligned} GDI &= \sum_{i=1}^{k} \bar{A}_i/k \\ &= \sum_{i=1}^{k} \left(p_g\, A_i^{1-t}(g) + p_l\, A_i^{1-t}(l)\right)^{(1-t)^{-1}} /k \ , \end{aligned} \tag{9}$$

which has the same aggregation rule as the one in (6). The difference between (9) and (6) is that in the former the equally distributed $\bar{A}_i$'s are arguments, whereas in the latter the ordinary

a_i's are arguments. It may be noted that (9) does not have any axiomatic foundation. Clearly, when formulated in terms of $\bar{A}_i$'s, the properties NOM, CIA and SYM will be fulfilled by the GDI[5,6].

The concluding result of this section is concerning independence of NOM, CIA and SYM.

Theorem 4 : The postulates NOM, CIA and SYM are independent.

Proof : See appendix.

4 An Empirical Illustration

The purpose of this section is to illustrate the generalized HDI I_r numerically using UNDP data for 1993 and 1995. The UNDP reports the HDI (I_1) for more than 170 countries. The indicators used by UNDP for calculating I_1 are life expectancy at birth; educational attainment, as measured by a combination of adult literacy (two-thirds weight) and combined primary, secondary and tertiary enrolment ratios (one-third weight); and the real GDP per capita.

It has been found by UNDP that there exist high correlation among these three attributes. This therefore does not seem to justify the choice of these three indicators simultaneously because of possible redundancy in explanatory power. But Principal Component Analysis carried out by UNDP shows that the eigenvector corresponding to the leading eigenvalue (which explains 88% of the total variance) puts virtually equal weight on the three variables. Therefore, omission of a variable here does not seem to be desirable (see UNDP, 1993, p. 119).

For the construction of I_1, fixed minimum and maximum values have been established by UNDP for each of these three indicators :

Life expectancy at birth : 25 years and 85 years (for both 1993 and 1995).

Educational attainment : 0% and 100% (for both 1993 and 1995). (The minima and maxima for adult literacy rate; and combined primary, secondary and tertiary enrolment ratios have been established as 0% and 100%, which in turn show that the minimum and maximum for educational attainment are also 0% and 100% respectively.)

Real GDP per capita (purchasing power parity (PPP) $) : PPP $ 100 and PPP $ 6040 (for 1993); PPP $ 100 and PPP $ 6311 (for 1995).

In this section we calculate I_r for $r = 1$, .5 and .25. We look at the percentage contributions made by the attributes to overall achievement. The countries chosen are USA, Japan, Bulgeria, Peru, India and Uganda. The minima and maxima values we choose for our purpose are the ones established by UNDP.

The relationship between achievement and specific attribute may now be analyzed with the aid of Tables 1-3. The upper part of a table presents all the relevant information for the year 1993 and its lower part gives the same information for 1995. In Table 1, the first column gives the names of the countries for which analysis is done. In columns 2-4 we present, for each country, the levels of indicators for three sources of well-being for $r = 1$. These indicator levels are then averaged to determine the overall achievement I_1 which is shown in column 5. Finally, columns 6-8 provide, for each country, the percentage contributions of indicators for the alternative attributes to overall achievement. Note that the Human Development Reports prepared by UNDP for 1996 and 1998 also present figures of column 5. But the percentage contributions, which play a major role in our analysis, have not been presented and analyzed in the Reports.

Several interesting features emerge from upper part of Table 1. First, the figures in the fifth column indicate arrangement of the countries considered in decreasing order of achievement levels. Next, we note that for the first three countries all the three attributes contribute roughly equally to overall achievement. The standard deviation among the attributes in relation to their contributions to overall achievement is 1.3 for Japan, and for USA and Bulgeria this figure is approximately 2.1. This standard deviation rises to a moderately high amount (6) for Peru, and for India and Uganda the amounts of this kind of dispersion are quite high. We observe that the ranges of the percentage contributions for these two latter countries are 31.35 and 37.83 respectively. In these two countries the major contributors to global achievement are life expectancy at birth and educational attainment. However, we have argued about the need for more or less equal importance of all the attributes of well-being to attain a better living standard. In fact, very low share of real GDP per capita in overall achievement for India and Uganda demonstrates the existence of high poverty in these two countries. An analogous analysis holds for the lower part of the table. An important aspect which comes out from the comparison between the two parts of the table is that for the first five countries there has not been any significant change with respect to the percentage contributions of the attributes between the period 1993-95. But for Uganda while the share of per capital real GDP has gone up by 7.25%, that of life expectancy has gone down by 8.25% . From equal importance point of view this does not appear to be an encouraging picture.

Tables 2 and 3 present similar figures for $r = 0.5$ and 0.25. We can analyze these tables in a manner in which we analyzed table 1. Tables 2-3 show that the index values as well as percentage contributions are sensitive to the value of r. We have pointed out earlier that I_r decreases as r increases. This is confirmed by index values shown in the tables.

5 Conclusion

This paper has developed a generalization of the UNDP human development index through an axiomatic approach. Our index is additive across attributes - it can be written as the simple average of indicators based on individual attributes. These property permits us to determine the percentage contributions made by the attributes to overall achievement. These contributions can in turn be used to separate the attributes according to their degrees of sensitivity to well-being. The less susceptible attributes may need policy attention for improving their contributions since all quality-of-life attributes should carry approximately equal weights to achieve a better standard of living.

Appendix

Proof of theorem 1 : Since $x_i \in [m_i, M_i]$ ensures that $x_i - m_i \geq 0$, by TI we have

$$\begin{aligned} A(x_i, m_i, M_i) &= A(x_i - m_i, 0, M_i - m_i) \\ &= q(x_i - m_i, M_i - m_i) \quad (say). \end{aligned} \tag{10}$$

Now, by HM it follows that

$$\begin{aligned} A(x_i, m_i, M_i) &= q((x_i - m_i)/(M_i - m_i)\ ,\ 1) \\ &= f((x_i - m_i)/(M_i - m_i)) \quad \text{(say)}. \end{aligned} \tag{11}$$

Clearly, the domain of f is $[0, 1]$. MN requires increasingness of f. Because in the extreme cases $x_i = m_i$ and $x_i = M_i$, the index A becomes $f(0)$ and $f(1)$ respectively, for NM to hold we must have $f(0) = 0$ and $f(1) = 1$. Twice differentiability of A implies that of f.

Next, let $h > 0$. Then given $x_i > y_i$, LI requires that

$$f((x_i+h-m_i)/(M_i-m_i))-f((x_i-m_i)/(M_i-m_i)) < f((y_i+h-m_i)/(M_i-m_i))-f((y_i-m_i)/(M_i-m_i))\ . \tag{12}$$

Dividing both sides of (12) by h and letting h tend to zero, we get

$$f'((x_i - m_i)/(M_i - m_i)) < f'((y_i - m_i)/(M_i - m_i)), \tag{13}$$

where f' is the derivative of f. (The argument given above shows that (12) implies (13). To prove the reverse implication, let us take definite integral of the left (right)-hand side of (13) from lower limit $x_i(y_i)$ to upper limit $x_i + h(y_i + h)$ to get

$$\int_{x_i}^{x_i+h} f'((z - m_i)/(M_i - m_i))\, dz < \int_{y_i}^{y_i+h} f'((z - m_i)/(M_i - m_i))\, dz \tag{14}$$

which is (12). Therefore, (12) implies and is implied by (13).) Given $x_i > y_i$, for inequality (13) to hold we require decreasingness of f', that is, strict concavity of f. In other words, we need $f'' < 0$, where f'' the derivative of f'.

This establishes the necessity part of the theorem. The sufficiency is easy to verify. □

Proof of Theorem 2 : (i) Suppose that the indicator is of the form

$$A_1(x_i, m_i, M_i) = 1 + ((x_i - m_i)/(M_i - m_i))^r, \qquad 0 < r < 1. \tag{15}$$

Note that A_1 is an affine transformation of f_r in (3). Since $A_1(x_i, m_i, M_i) = 1$ if $x_i = m_i$ and $A_1(x_i, m_i, M_i) = 2$ if $x_i = M_i$, the postulate NM is violated. However, A_1 satisfies all the four remaining postulates.

(ii) Consider the following indicator

$$A_2(x_i, m_i, M_i) = (1 - e^{(x_i - m_i)/(M_i - m_i)})/(1 - e)\,. \tag{16}$$

Since A_2 is a decreasing function of x_i, MN is not fulfilled. It is easy to check that A_2 meets the other four axioms.

(iii) The indicator given by

$$A_3(x_i, m_i, M_i) = (1 - e^{-(x_i - m_i)/M_i})/(1 - e^{-(M_i - m_i)/M_i}) \tag{17}$$

violates TI because $e^{-(x_i - m_i)/M_i}$ and $e^{-(M_i - m_i)/M_i}$ do not remain invariant under equal absolute changes in x_i, m_i and M_i. Clearly, A_3 is normalized, monotone, homogeneous of degree zero in its arguments and strictly concave in x_i.

(iv) Since the indicator

$$A_4(x_i, m_i, M_i) = (1 - e^{-(x_i - m_i)})/(1 - e^{-(M_i - m_i)}) \tag{18}$$

is not homogeneous of degree zero in (x_i, m_i, M_i), HM is violated. Nevertheless, A_4 verifies the postulates NM, MN, TI and LI.

(v) We have already noted that the form given by f_1 does not meet LI but meets NM, MN, TI and HM. □

Proof of Theorem 3 : The idea of the proof is taken from Aczél (1966, pp. 239-240). For any $i, 1 \leq i \leq k$, define $I(0, \ldots, 0, a_i, 0, \ldots, 0) = t_i(a_i)$. Now, by CIA

$$\begin{aligned} I(a_1, a_2, 0, \ldots, 0) &= I(a_1, 0, \ldots, 0) + I(0, a_2, 0, \ldots, 0) \\ &= t_1(a_1) + t_2(a_2). \end{aligned}$$

Repeating this procedure we get

$$I(a_1, a_2, \ldots, a_k) = \sum_{i=1}^{k} t_i(a_i).$$

Symmetry of I requires that $t_i's$ are identical, say, $t_i = t$ for all i. Therefore,

$$I(a_1, a_2, \ldots, a_k) = \sum_{i=1}^{k} t(a_i). \tag{19}$$

If $a_i's$ are all identical, say, $a_i = a$ for all i, then

$$I(a, a, \ldots, a) = kt(a). \tag{20}$$

But by NOM, in this extreme case

$$I(a, a, \ldots, a) = a. \tag{21}$$

From (20) and (21) it follows that $t(a) = a/k$. Substituting this explicit form of t in (19) we get

$$\begin{aligned} I(a_1, a_2, \ldots, a_k) &= \sum_{i=1}^{k} a_i/k \\ &= \sum_{i=1}^{k} f((x_i - m_i)/(M_i - m_i))/k\ . \end{aligned}$$

This completes the necessity part of the theorem. The sufficiency can be verified by noting that I in (6) fulfils NOM, CIA and SYM. □

Proof of Theorem 4 : (i) Consider the achievement index

$$G_1(a_1, \ldots, a_k) = \sum_{i=1}^{k} a_i. \tag{22}$$

Since NOM demands that the index should be expressed as an average of attribute-wise indices, G_1 does not satisfy NOM. However, it fulfill CIA and SYM.

(ii) Because the achievement index

$$G_2(a_1, \ldots, a_k) = \left(\sum_{i=1}^{k} a_i^2 / k \right)^{1/2} \tag{23}$$

is nonlinear in $a_i's$, it fails to meet CIA. It is easy to see that G_2 is normalized and symmetric.

(iii) Finally, suppose the achievement index is of the form

$$G_3(a_1, \ldots a_k) = \sum_{i=1}^{k} \alpha_i a_i \tag{24}$$

where $0 < \alpha_i < 1$ and $\sum_{i=1}^{k} \alpha_i = 1$. Since the coefficients $\alpha_i's$ in (24) are not necessarily the same, the functional value of G_3 may not remain unchanged under different permutations of $(a_1, \ldots a_k)$. Hence G_3 does not fulfil SYM, though it meets NOM and CIA. □

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Notes

1. Noorbakhash (1998) defined an alternative human development index in terms of the Euclidean distance of the standardized attribute levels from the respective highest quantities. As in the case of the UNDP index, his modified index also regards all the attributes as equally important.

2. A non-negative $k \times k$ matrix is said to be a permutation matrix if each of its rows and columns sums to one.

3. I am grateful to the referee for bringing this to my attention.

4. For $t = 1$, $\bar{A}_i$ becomes the population share weighted geometric average of $A_i(g)$ and $A_i(l)$.

5. For illustrative purpose, UNDP chose $t = 2$. While for calculating indicators for educational attainment and life expectancy at birth, UNDP employed the rule defined in (8), due to nonavailability of some of the required variables, the calculation of per capita real GDP has been modified as follows. Population share weighted equally distributed averaging of appropriately determined male and female proportional income shares has been taken first. This average has been multiplied by the adjusted real GDP per capita to find the actual value of the GDP variable. The indicator has then been determined using the same functional form as employed in the context of a component for HDI (see UNDP, 1995). Under this formulation also, all the properties considered in section 2 are satisfied by the per capita real GDP indicator.

6. UNDP has also suggested a gender empowerment index which aims to measure the relative empowerment of men and women in political and economic spheres of activity and a human poverty index. Since they are related respectively to male-female segregation and poverty, here we do not go for a discussion on them.

Country	Indicator based on			I_1	Percentage contribution based on		
	life expectancy at birth	educational attainment	real GDP per capita		life expectancy at birth	educational attainment	real GDP per capita
				Year : 1993			
USA	.850	.980	.990	.940	30.142	34.752	35.106
Japan	.910	.920	.980	.938	32.338	32.694	34.826
Bulgeria	.770	.840	.710	.773	33.204	36.223	30.617
Peru	.690	.850	.540	.694	33.141	40.826	25.937
India	.600	.520	.190	.436	45.872	39.755	14.526
Uganda	.330	.510	.140	.326	33.742	52.147	14.315
				Year : 1995			
USA	.860	.980	.990	.943	30.399	34.641	34.995
Japan	.910	.920	.990	.940	32.270	32.624	35.106
Bulgeria	.770	.870	.730	.789	32.531	36.755	30.841
Peru	.710	.860	.620	.729	32.465	39.323	28.349
India	.610	.530	.210	.451	45.085	39.172	15.521
Uganda	.260	.540	.220	.340	25.490	52.941	21.569

Table 1 : Attribute-wise Breakdown of the Index I_1 for 1993 and 1995

Country	Indicator based on			$I_{.5}$	Percentage contribution based on		
	life expectancy at birth	educational attainment	real GDP per capita		life expectancy at birth	educational attainment	real GDP per capita
				Year : 1993			
USA	.922	.990	.995	.969	31.716	34.055	34.229
Japan	.954	.959	.990	.968	32.860	33.040	34.100
Bulgeria	.877	.917	.843	.879	33.281	34.761	31.958
Peru	.831	.922	.735	.829	33.394	37.064	29.542
India	.775	.721	.436	.644	40.101	37.332	22.566
Uganda	.574	.714	.374	.554	34.548	42.949	22.503
				Year : 1995			
USA	.927	.990	.995	.971	31.843	33.992	34.165
Japan	.954	.959	.995	.969	32.803	32.983	34.214
Bulgeria	.877	.933	.854	.888	32.931	35.004	32.064
Peru	.843	.927	.787	.852	32.948	36.262	30.789
India	.781	.728	.458	.656	39.700	37.006	23.294
Uganda	.510	.735	.469	.571	29.753	42.878	27.369

Table 2 : Attribute-wise Breakdown of the Index $I_{.5}$ for 1993 and 1995

Country	Indicator based on			$I_{.25}$	Percentage contribution based on		
	life expectancy at birth	educational attainment	real GDP per capita		life expectancy at birth	educational attainment	real GDP per capita
				Year : 1993			
USA	.960	.995	.997	.984	32.520	33.697	33.783
Japan	.977	.979	.995	.984	33.097	33.187	33.716
Bulgeria	.937	.957	.918	.937	33.312	34.045	32.643
Peru	.911	.960	.857	.910	33.399	35.187	31.414
India	.880	.849	.660	.797	36.832	35.538	27.630
Uganda	.758	.845	.612	.738	34.223	38.157	27.620
				Year : 1995			
USA	.963	.995	.997	.985	32.584	33.665	33.751
Japan	.977	.979	.997	.985	33.069	33.159	33.772
Bulgeria	.937	.966	.924	.942	33.137	34.164	32.698
Peru	.918	.963	.887	.923	33.159	34.787	32.054
India	.884	.853	.667	.805	36.611	35.346	28.043
Uganda	.714	.857	.685	.752	31.650	37.995	30.353

Table 3 : Attribute-wise Breakdown of the Index $I_{.25}$ for 1993 and 1995