ON CONFLUENT HYPERGEOMETRIC SERIES

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Introduction

The function ${}_1F_1(\alpha, \rho, x)$ (with integral or half integral values of the parameter ρ and α) occurs in distributions of many important statistics such as (1) F-Statistic under non-null hypothesis (2) multiple correlation coefficient (for a particular type of parent population) (3) Studentised D^2-Statistic; and in constructing the five or one percent tables of these Statistics we have to use the above function for various values of α, ρ and the argument x In this paper we are giving values of ${}_1F_1(\alpha, \rho, x)$ for $\rho = 1, 2, 3$ and 4; α from 2 to 25; and x from 2 to 13.5. Further values will be published later.

The most general notation of the hypergeometric series was given by Pochhammer, which may be stated as

$${}_pF_q(\alpha_1, \alpha_2, \ldots\ldots \alpha_p, \rho_1, \rho_2, \ldots\ldots \rho_q, x) = \sum_{n=0}^{\infty} \{((\alpha_1)_n (\alpha_2)_n \ldots (\alpha_p)_n\}/\{n!(\rho_1)_n (\rho_2)_n \ldots (\rho_q)_n\}]x^n \qquad (1)$$

where $(\alpha)_n = \alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)$ and $(\alpha)_0 = 1$. ${}_1F_1(\alpha, \rho, x)$ is a particular case of the above series where $p = q = 1$.

Thus

$${}_1F_1(\alpha, \rho, x) = (\rho-1)!/(\alpha-1)! \sum_{n=0}^{\infty} (\alpha+n-1)!/n! \times (\rho+n-1)!\, x^n \qquad .. \quad (2)$$

When $\alpha = \rho$ in the above expression we have

$${}_1F_1(\alpha, \rho, x) = 1 + x + x^2/2! + \ldots\ldots + x^n/n! = e^x \qquad .. \quad (3)$$

We must obviously exclude the case when ρ is a negative integer; if α is a negative integer the series becomes a polynomial in x. The series (2) satisfies the differential equation

$$x\frac{d^2y}{dx^2} - (x-\rho)\frac{dy}{dx} - \alpha y = 0 \qquad .. \quad (4)$$

An independent solution of the above differential equation is $x^{1-\rho}\,{}_1F_1(\alpha-\rho+1, 2-\rho, x)$. For all values of x of finite modulus ${}_1F_1(\alpha, \rho, x) = e^x\,{}_1F_1(\rho-\alpha, \rho, -x)$ when $R(x) < 0$; ${}_1F_1(\alpha, \rho, x)$ admits of the asymptotic expansion which may be written as

$$(\rho-1)!/(\rho-\alpha-1)!\,(-x)^{-\alpha}\,{}_2F_0\left(\alpha, 1-\rho+\alpha, 1-\alpha, -\frac{1}{x}\right) \qquad .. \quad (5)$$

The error which results from stopping at the k-th term of this series is at most of order $x^{-\alpha-k+1}$ when $|x|$ is large and $(-x)^{-\alpha} = \exp\{-\alpha \log(-x)\}$ the logarithm having its principal values whose imaginary part lies between $\pm\pi/2$. The nature of the asymptotic value, when $|x|$ is large and $R(x) > 0$ is

$${}_1F_1(\alpha, \rho, x) = e^x.(\rho-1)!/(\alpha-1)!\,x^{\alpha-\rho}\,{}_2F_0(\rho-\alpha, 1-\alpha, 1/x) \qquad .. \quad (6)$$

where

$$ {}_1F_1(\alpha,\rho,x)=[1+(\alpha/\rho\,1!)\,x+(\alpha(\alpha+1)/\rho(\rho+1)\,2!)\,x^2+\dots] \quad \dots (7)$$

Construction of Tables. We have calculated the value of ${}_1F_1(\alpha,\rho,x)$ for $\alpha=2, 3, \dots 25$, $\rho=1, 2, 3$ and 4 and $x=2, 3, 3.8, 5.7, 7, 8.4, 9, 10.5$, and 13.5. For $\alpha=2, 3$, $\rho=1, 2$ and for all values of x we directly calculated the value of the function correct to six significant figures; these we call our basic values. We then used the following recursion formulae for the evaluation of the function for higher values of α and ρ:

$$\alpha\,.\,{}_1F_1(\alpha+1,\rho,x)=(x+2\alpha-\rho)\,{}_1F_1(\alpha,\rho,x)+(\rho-\alpha)\,{}_1F_1(\alpha-1,\rho,x) \quad \dots (8)$$

$$(\rho-\alpha)x\,{}_1F_1(\alpha,\rho+1,x)=\rho(\rho+x-1)\,{}_1F_1(\alpha,\rho,x)+\rho(1-\rho)\,{}_1F_1(\alpha,\rho-1,x) \quad \dots (9)$$

For example for $\rho=1$ we know the values of the function for $\alpha=1$ and 2; substituting these two values in the right hand of (8) we get values for $\alpha=3$. Repeating the process we can get values of the function for any value of α. In the same way we can use (9) in the case of ρ. The following recursion formulae are of general interest.

$$\alpha\rho\,{}_1F_1(\alpha+1,\rho,x)=\rho(\alpha+x)\,{}_1F_1(\alpha,\rho,x)-x(\rho-\alpha)\,{}_1F_1(\alpha,\rho+1,x) \quad \dots (10)$$

$$\alpha\,{}_1F_1(\alpha+1,\rho+1,x)=(\alpha-\rho)\,{}_1F_1(\alpha,\rho+1,x)+\rho\,{}_1F_1(\alpha,\rho,x) \quad \dots (11)$$

One point must be mentioned here. For high values of α and ρ the values of the function become very large; and it is more convenient to use a multiplying factor. Thus to get the actual value of the function we have to multiply the tabulated figure by the common factor e^{15} which, correct to seven significant figures, is 3269017.

Accuracy of the Table. For all values of ρ and x and upto $\alpha=15$, the values of the function are correct up to five significant figures, from $\alpha=16$ to $\alpha=25$ to four significant figures. To test the accuracy of the table we evaluated the function directly for selected values of ρ and x, which are compared with tabulated values in table (1)

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TABLE (1). THE AGREEMENT BETWEEN CALCULATED VALUE BY SERIES AND THE VALUE FROM THE TABLE.

No	α	ρ	x	Series	Table
1	6	1	8·4	3·7674	3·7674
2	9	1	5·7	0·89480	0·89480
3	8	1	9·0	77·298	77·298
4	6	1	9·0	8·7375	8·7375
5	5	2	3·0	$10^{-4}\times 0·68354$	$10^{-4}\times 0·68354$
6	10	2	7·0	1·5427	1·5427
7	11	3	9·0	11·043	11·043
8	15	3	10·5	2745·7	2745·7
9	12	3	10·5	242·24	242·24
10	7	4	3·8	$10^{-4}\times 0·88317$	$10^{-4}\times 0·88317$
11	14	4	10·5	198·99	198·99
12	20	4	13·5	2178 × 10[illegible]	2178 × 10[illegible]
13	22	4	13·5	9450 × 10[illegible]	9450 × 10[illegible]
14	24	4	13·5	3865 × 10[illegible]	3865 × 10[illegible]

TABLE 2. Value of $e^{-\frac{1}{2}x}\ {}_1F_1(a, 1, x)$

a	Values of x: 2	3	3·8	5·7	7	8·4	9	10·5	13·5
2	$10^{-[illegible]}$ × ·67810	$10^{-[illegible]}$ × ·24577	$10^{-[illegible]}$ × ·65036	$10^{-[illegible]}$ × ·61234	$10^{-[illegible]}$ × ·20837	$10^{-[illegible]}$ × ·12787	$10^{-[illegible]}$ × ·24788	·12775	3·2354
3	$10^{-[illegible]}$ × ·15822	$10^{-[illegible]}$ × ·70658	$10^{-[illegible]}$ × ·21633	$10^{-[illegible]}$ × ·20188	$10^{-[illegible]}$ × ·13251	$10^{-[illegible]}$ × ·72208	·14749	·85878	26·580
4	$10^{-[illegible]}$ × ·32298	$10^{-[illegible]}$ × ·17204	$10^{-[illegible]}$ × ·60080	$10^{-[illegible]}$ × ·89322	$10^{-[illegible]}$ × ·51214	·31401	·67714	4·3415	161·78
5	$10^{-[illegible]}$ × ·61029	$10^{-[illegible]}$ × ·37710	$10^{-[illegible]}$ × ·14320	$10^{-[illegible]}$ × ·20306	·16931	1·1548	2·6763	18·352	809·06
6	$10^{-[illegible]}$ × ·10835	$10^{-[illegible]}$ × ·70741	$10^{-[illegible]}$ × ·31956	$10^{-[illegible]}$ × ·70457	·50062	3·7674	8·7375	68·008	3511·4
7	$10^{-[illegible]}$ × ·18389	$10^{-[illegible]}$ × ·14786	$10^{-[illegible]}$ × ·66884	·17411	1·3614	11·210	26·978	228·73	13683·9
8	$10^{-[illegible]}$ × ·30118	$10^{-[illegible]}$ × ·27168	$10^{-[illegible]}$ × ·13313	·40473	3·4904	31·069	77·299	700·49	48718·8
9	$10^{-[illegible]}$ × ·47911	$10^{-[illegible]}$ × ·48210	$10^{-[illegible]}$ × ·25433	·80489	8·3248	81·059	208·29	2061·4	16160 × $10^{[illegible]}$
10	$10^{-[illegible]}$ × ·74374	$10^{-[illegible]}$ × ·82983	$10^{-[illegible]}$ × ·46946	1·8974	10·124	201·15	533·02	5088·0	50434 × $10^{[illegible]}$
11	$10^{-[illegible]}$ × ·11300	$10^{-[illegible]}$ × ·13917	$10^{-[illegible]}$ × ·84148	3·8811	42·229	478·20	1308·0	14865·	14037 × $10^{[illegible]}$
12	$10^{-[illegible]}$ × ·14880	$10^{-[illegible]}$ × ·22821	·14703	7·6958	90·107	1005·2	3074·5	37418	42202 × $10^{[illegible]}$
13	$10^{-[illegible]}$ × ·24802	$10^{-[illegible]}$ × ·36689	·25124	14·848	186·56	2427·5	7002·4	90826	11466 × $10^{[illegible]}$
14	$10^{-[illegible]}$ × ·35030	$10^{-[illegible]}$ × ·57956	·42087	27·959	370·04	5225·8	15478	21349 × $10^{[illegible]}$	30114 × $10^{[illegible]}$
15	$10^{-[illegible]}$ × ·51390	$10^{-[illegible]}$ × ·90123	·69262	51·518	740·02	10959	33293	48750 × $10^{[illegible]}$	70450 × $10^{[illegible]}$
16	$10^{-[illegible]}$ × ·7268	·1382	1·122	93·08	1425	2245 × $10^{[illegible]}$	6090 × $10^{[illegible]}$	1065 × $10^{[illegible]}$	1885 × $10^{[illegible]}$
17	$10^{-[illegible]}$ × ·1017	·2091	1·790	165·2	2691	4501 × $10^{[illegible]}$	1435 × $10^{[illegible]}$	2356 × $10^{[illegible]}$	4520 × $10^{[illegible]}$
18	$10^{-[illegible]}$ × ·1410	·3128	2·820	288·5	4990	8618 × $10^{[illegible]}$	2869 × $10^{[illegible]}$	5008 × $10^{[illegible]}$	1061 × $10^{[illegible]}$
19	$10^{-[illegible]}$ × ·1935	·4629	4·388	496·3	9102	1708 × $10^{[illegible]}$	5705 × $10^{[illegible]}$	1043 × $10^{[illegible]}$	2430 × $10^{[illegible]}$
20	$10^{-[illegible]}$ × ·2642	·6781	6·760	842·0	1636 × $10^{[illegible]}$	3243 × $10^{[illegible]}$	1108 × $10^{[illegible]}$	2134 × $10^{[illegible]}$	5454 × $10^{[illegible]}$
21	$10^{-[illegible]}$ × ·3575	·9840	10·28	1410	2998 × $10^{[illegible]}$	6064 × $10^{[illegible]}$	2116 × $10^{[illegible]}$	4290 × $10^{[illegible]}$	1201 × $10^{[illegible]}$
22	$10^{-[illegible]}$ × ·4805	1·417	15·50	2335	5062 × $10^{[illegible]}$	1118 × $10^{[illegible]}$	3984 × $10^{[illegible]}$	8489 × $10^{[illegible]}$	2597 × $10^{[illegible]}$
23	$10^{-[illegible]}$ × ·6415	2·022	23·15	3822	8741 × $10^{[illegible]}$	2033 × $10^{[illegible]}$	7306 × $10^{[illegible]}$	1635 × $10^{[illegible]}$	5524 × $10^{[illegible]}$
24	$10^{-[illegible]}$ × ·8513	2·865	34·31	6191	1492 × $10^{[illegible]}$	3650 × $10^{[illegible]}$	1355 × $10^{[illegible]}$	3181 × $10^{[illegible]}$	1157 × $10^{[illegible]}$
25	·1123	4·033	50·42	9932	2519 × $10^{[illegible]}$	6477 × $10^{[illegible]}$	2454 × $10^{[illegible]}$	6030 × $10^{[illegible]}$	2386 × $10^{[illegible]}$

TABLE 3. $e^{-[illegible]}\ {}_1F_1(\alpha, 2, x)$

α	Values of x: 2	3	3·8	5·7	7	8·4	9	10·5	13·5
2	$10^{-[illegible]}\times$ ·22003	$10^{-[illegible]}\times$ ·61442	$10^{-[illegible]}\times$ ·13674	$10^{-[illegible]}\times$ ·81424	$10^{-[illegible]}\times$ ·33546	$10^{-[illegible]}\times$ ·13004	$10^{-[illegible]}\times$ ·24788	$10^{-[illegible]}\times$ ·11109	·22313
3	$10^{-[illegible]}\times$ ·45200	$10^{-[illegible]}\times$ ·16381	$10^{-[illegible]}\times$ ·39655	$10^{-[illegible]}\times$ ·35198	$10^{-[illegible]}\times$ ·15096	$10^{-[illegible]}\times$ ·70739	$10^{-[illegible]}\times$ ·13033	$10^{-[illegible]}\times$ ·69430	1·7293
4	$10^{-[illegible]}\times$ ·82878	$10^{-[illegible]}\times$ ·33703	$10^{-[illegible]}\times$ ·98545	$10^{-[illegible]}\times$ ·11078	$10^{-[illegible]}\times$ ·54233	$10^{-[illegible]}\times$ ·28785	$10^{-[illegible]}\times$ ·59231	·33188	10·013
5	$10^{-[illegible]}\times$ ·14315	$10^{-[illegible]}\times$ ·08354	$10^{-[illegible]}\times$ ·22161	$10^{-[illegible]}\times$ ·30638	$10^{-[illegible]}\times$ ·16871	·10019	·21163	1·3343	47·019
6	$10^{-[illegible]}\times$ ·23658	$10^{-[illegible]}\times$ ·13010	$10^{-[illegible]}\times$ ·40387	$10^{-[illegible]}\times$ ·77301	$10^{-[illegible]}\times$ ·47359	·31102	·68458	4·7377	200·17
7	$10^{-[illegible]}\times$ ·37772	$10^{-[illegible]}\times$ ·23032	$10^{-[illegible]}\times$ ·91915	$10^{-[illegible]}\times$ ·18185	·12294	·88708	2·0207	15·298	752·04
8	$10^{-[illegible]}\times$ ·58646	$10^{-[illegible]}\times$ ·41347	$10^{-[illegible]}\times$ ·17433	$10^{-[illegible]}\times$ ·40400	·20988	2·3031	6·8912	43·787	2596·6
9	$10^{-[illegible]}\times$ ·89963	$10^{-[illegible]}\times$ ·70139	$10^{-[illegible]}\times$ ·31895	$10^{-[illegible]}\times$ ·85093	·69492	5·0512	14·835	128·78	6361·7
10	$10^{-[illegible]}\times$ ·13231	$10^{-[illegible]}\times$ ·11501	$10^{-[illegible]}\times$ ·56811	·17587	1·5427	14·298	36·081	343·44	25388
11	$10^{-[illegible]}\times$ ·19345	$10^{-[illegible]}\times$ ·18730	$10^{-[illegible]}\times$ ·97685	·34802	3·3008	32·982	85·776	873·93	73284
12	$10^{-[illegible]}\times$ ·27805	$10^{-[illegible]}\times$ ·29680	$10^{-[illegible]}\times$ ·16519	·06921	6·8397	73·458	198·01	2147·7	20241 $\times 10^{[illegible]}$
13	$10^{-[illegible]}\times$ ·39800	$10^{-[illegible]}\times$ ·40224	$10^{-[illegible]}\times$ ·27433	1·2347	13·779	158·60	436·44	5080·7	83773 $\times 10^{[illegible]}$
14	$10^{-[illegible]}\times$ ·55040	$10^{-[illegible]}\times$ ·70850	$10^{-[illegible]}\times$ ·41839	2·2003	27·089	333·13	941·52	11682	13798 $\times 10^{[illegible]}$
15	$10^{-[illegible]}\times$ ·77330	$10^{-[illegible]}\times$ ·10722	$10^{-[illegible]}\times$ ·71513	4·1331	51·906	682·61	1979·7	28097	34323 $\times 10^{[illegible]}$
16	$10^{-[illegible]}\times$ ·1004	$10^{-[illegible]}\times$ ·1009	·1129	7·202	97·88	1308	4067	5688 $\times 10^{[illegible]}$	8300 $\times 10^{[illegible]}$
17	$10^{-[illegible]}\times$ ·1452	$10^{-[illegible]}\times$ ·2365	·1700	12·65	180·8	2685	8182	1211 $\times 10^{[illegible]}$	1956 $\times 10^{[illegible]}$
18	$10^{-[illegible]}\times$ ·1965	$10^{-[illegible]}\times$ ·3456	·2709	21·63	328·5	5175	1614 $\times 10^{[illegible]}$	2525 $\times 10^{[illegible]}$	4501 $\times 10^{[illegible]}$
19	$10^{-[illegible]}\times$ ·2639	$10^{-[illegible]}\times$ ·5002	·4125	36·45	587·4	9802	3129 $\times 10^{[illegible]}$	5107 $\times 10^{[illegible]}$	1015 $\times 10^{[illegible]}$
20	$10^{-[illegible]}\times$ ·3521	$10^{-[illegible]}\times$ ·7176	·6218	60·66	1036	1828 $\times 10^{[illegible]}$	5967 $\times 10^{[illegible]}$	1030 $\times 10^{[illegible]}$	2240 $\times 10^{[illegible]}$
21	$10^{-[illegible]}\times$ ·4606	·1021	·9282	99·72	1801	3358 $\times 10^{[illegible]}$	1121 $\times 10^{[illegible]}$	2054 $\times 10^{[illegible]}$	4833 $\times 10^{[illegible]}$
22	$10^{-[illegible]}\times$ ·6146	·1441	1·373	162·1	3093	6086 $\times 10^{[illegible]}$	2075 $\times 10^{[illegible]}$	3999 $\times 10^{[illegible]}$	1034 $\times 10^{[illegible]}$
23	$10^{-[illegible]}\times$ ·8051	·2010	2·015	260·9	5255	1069 $\times 10^{[illegible]}$	3791 $\times 10^{[illegible]}$	7678 $\times 10^{[illegible]}$	2168 $\times 10^{[illegible]}$
24	$10^{-[illegible]}\times$ ·1049	·2811	2·934	415·7	8827	1928 $\times 10^{[illegible]}$	6842 $\times 10^{[illegible]}$	1434 $\times 10^{[illegible]}$	4475 $\times 10^{[illegible]}$
25	$10^{-[illegible]}\times$ ·1360	·3887	4·242	656·4	1408 $\times 10^{[illegible]}$	3366 $\times 10^{[illegible]}$	1220 $\times 10^{[illegible]}$	2719 $\times 10^{[illegible]}$	9108 $\times 10^{[illegible]}$

TABLE 4. $e^{-13}\,{}_1F_1(a, 3, x)$.

a	Values of x: 2	3	3·8	5·7	7	8·4	9	10·5	13·3
2	10^[illegible] × ·12831	10^[illegible] × ·27987	10^[illegible] × ·53030	10^[illegible] × ·20451	10^[illegible] × ·82167	10^[illegible] × ·28534	10^[illegible] × ·48963	10^[illegible] × ·19145	10^[illegible] × ·30608
3	10^[illegible] × ·22003	10^[illegible] × ·61442	10^[illegible] × ·13874	10^[illegible] × ·91424	10^[illegible] × ·33548	10^[illegible] × ·13604	10^[illegible] × ·24788	10^[illegible] × ·11109	·22313
4	10^[illegible] × ·37672	10^[illegible] × ·12288	10^[illegible] × ·30093	10^[illegible] × ·20513	10^[illegible] × ·11182	10^[illegible] × ·51694	10^[illegible] × ·09150	10^[illegible] × ·49990	1·2272
5	10^[illegible] × ·60275	10^[illegible] × ·23041	10^[illegible] × ·61770	10^[illegible] × ·88037	10^[illegible] × ·32708	10^[illegible] × ·10977	10^[illegible] × ·34083	·19094	5·0201
6	10^[illegible] × ·93426	10^[illegible] × ·41166	10^[illegible] × ·12760	10^[illegible] × ·16373	10^[illegible] × ·87108	10^[illegible] × ·50223	·10510	·64428	22·852
7	10^[illegible] × ·14114	10^[illegible] × ·70812	10^[illegible] × ·23963	10^[illegible] × ·36682	10^[illegible] × ·21504	·13716	·20826	2·0114	81·758
8	10^[illegible] × ·20874	10^[illegible] × ·11810	10^[illegible] × ·43378	10^[illegible] × ·78158	10^{-2} × ·50548	·35142	·70210	5·8076	273·27
9	10^[illegible] × ·30317	10^[illegible] × ·19104	10^[illegible] × ·76117	10^[illegible] × ·18977	·11288	·85433	1·9519	15·802	854·10
10	10^[illegible] × ·43319	10^[illegible] × ·30515	10^[illegible] × ·13008	10^[illegible] × ·31536	·24222	1·9870	4·7836	40·002	2522·5
11	10^[illegible] × ·61142	10^[illegible] × ·47594	10^[illegible] × ·21729	10^[illegible] × ·60403	·50231	4·4189	11·043	101·42	7065·6
12	10^[illegible] × ·85190	10^[illegible] × ·72096	10^[illegible] × ·35577	·11270	1·0111	9·0367	24·431	242·24	19130·
13	10^[illegible] × ·11744	10^[illegible] × ·11030	10^[illegible] × ·57230	·20545	1·0820	20·273	53·294	539·82	49677
14	10^[illegible] × ·16021	10^[illegible] × ·16444	10^[illegible] × ·90614	·36688	3·7973	41·555	112·24	1236·3	12476 × 10^[illegible]
15	10^[illegible] × ·21090	10^[illegible] × ·24222	10^[illegible] × ·14144	·64308	7·1219	83·209	230·71	2745·7	30406 × 10^[illegible]
16	10^[illegible] × ·2911	10^[illegible] × ·3529	10^[illegible] × ·2170	1·108	13·10	163·1	483·9	5859	7212 × 10^[illegible]
17	10^[illegible] × ·3877	10^[illegible] × ·5090	10^[illegible] × ·3318	1·881	23·70	313·7	914·3	1223 × 10¹	1649 × 10^[illegible]
18	10^[illegible] × ·5130	10^[illegible] × ·7273	10^[illegible] × ·4098	3·140	42·18	592·7	1709	2504 × 10¹	3774 × 10^[illegible]
19	10^[illegible] × ·6743	10^[illegible] × ·1031	10^[illegible] × ·7453	5·202	73·09	1102	3300	5032 × 10¹	8358 × 10^[illegible]
20	10^[illegible] × ·8812	10^[illegible] × ·1449	·1101	8·492	128·0	2018	6306	9941 × 10¹	1816 × 10^[illegible]
21	10^[illegible] × ·1145	10^[illegible] × ·2021	·1612	13·71	218·8	3641	1164 × 10^[illegible]	1033 × 10^[illegible]	3874 × 10^[illegible]
22	10^[illegible] × ·1480	10^[illegible] × ·2801	·2343	21·00	340·5	6495	2121 × 10^[illegible]	3705 × 10^[illegible]	8130 × 10^[illegible]
23	10^[illegible] × ·1905	10^[illegible] × ·3856	·3379	34·05	617·2	1144 × 10¹	3814 × 10^[illegible]	7004 × 10^[illegible]	1679 × 10^[illegible]
24	10^[illegible] × ·2439	10^[illegible] × ·5276	·4837	54·32	1021	1991 × 10¹	6779 × 10^[illegible]	1307 × 10^[illegible]	3418 × 10^[illegible]
25	10^[illegible] × ·3110	10^[illegible] × ·7179	·6880	84·44	1671	3430 × 10¹	1192 × 10^[illegible]	2409 × 10^[illegible]	6863 × 10^[illegible]

TABLE 5. $e^{-x}\,{}_1F_1(n, 4, x)$

n	Values of x: 2	3	3·8	5·7	7	8·4	9	10·5	13·5
2	$10^{-?}\times$ ·91771	$10^{-?}\times$ ·17063	$10^{-?}\times$ ·26013	$10^{-?}\times$ ·10900	$10^{-?}\times$ ·20385	$10^{-?}\times$ ·88135	$10^{-?}\times$ ·14281	$10^{-?}\times$ ·48942	$10^{-?}\times$ ·62576
3	$10^{-?}\times$ ·14858	$10^{-?}\times$ ·33455	$10^{-?}\times$ ·60088	$10^{-?}\times$ ·34196	$10^{-?}\times$ ·10456	$10^{-?}\times$ ·38304	$10^{-?}\times$ ·80304	$10^{-?}\times$ ·20270	$10^{-?}\times$ ·42783
4	$10^{-?}\times$ ·22604	$10^{-?}\times$ ·61443	$10^{-?}\times$ ·13674	$10^{-?}\times$ ·91424	$10^{-?}\times$ ·33348	$10^{-?}\times$ ·13004	$10^{-?}\times$ ·24788	$10^{-?}\times$ ·11109	·22313
5	$10^{-?}\times$ ·33805	$10^{-?}\times$ ·10752	$10^{-?}\times$ ·26045	$10^{-?}\times$ ·22170	$10^{-?}\times$ ·82252	$10^{-?}\times$ ·43171	$10^{-?}\times$ ·80359	$10^{-?}\times$ ·40270	·97619
6	$10^{-?}\times$ ·49728	$10^{-?}\times$ ·18126	$10^{-?}\times$ ·49328	$10^{-?}\times$ ·50050	$10^{-?}\times$ ·22315	$10^{-?}\times$ ·11873	$10^{-?}\times$ ·23672	·13067	3·7625
7	$10^{-?}\times$ ·71578	$10^{-?}\times$ ·29640	$10^{-?}\times$ ·88517	$10^{-?}\times$ ·10489	$10^{-?}\times$ ·55212	$10^{-?}\times$ ·31048	$10^{-?}\times$ ·64398	·39948	13·157
8	$10^{-?}\times$ ·10139	$10^{-?}\times$ ·47288	$10^{-?}\times$ ·15228	$10^{-?}\times$ ·21829	$10^{-?}\times$ ·12409	$10^{-?}\times$ ·76523	·16462	1·0646	42·557
9	$10^{-?}\times$ ·14165	$10^{-?}\times$ ·73443	$10^{-?}\times$ ·25847	$10^{-?}\times$ ·42952	$10^{-?}\times$ ·26712	·17001	·30992	2·8557	129·07
10	$10^{-?}\times$ ·19549	$10^{-?}\times$ ·11321	$10^{-?}\times$ ·42803	$10^{-?}\times$ ·81891	$10^{-?}\times$ ·55133	·40452	·93057	7·1713	370·75
11	$10^{-?}\times$ ·26689	$10^{-?}\times$ ·17079	$10^{-?}\times$ ·88847	$10^{-?}\times$ ·15193	·11147	·87926	2·0465	17·291	1016·3
12	$10^{-?}\times$ ·36080	$10^{-?}\times$ ·25402	$10^{-?}\times$ ·10933	$10^{-?}\times$ ·27523	·21600	1·6528	4·5293	40·235	2674·3
13	$10^{-?}\times$ ·48364	$10^{-?}\times$ ·37300	$10^{-?}\times$ ·17004	$10^{-?}\times$ ·48817	·41033	3·7088	9·5545	90·736	6788·2
14	$10^{-?}\times$ ·64306	$10^{-?}\times$ ·54148	$10^{-?}\times$ ·26256	$10^{-?}\times$ ·84862	·77777	7·6006	19·648	198·99	16086
15	$10^{-?}\times$ ·84879	$10^{-?}\times$ ·77780	$10^{-?}\times$ ·40126	·14657	1·4248	14·876	39·489	428·55	39843
16	$10^{-?}\times$ ·1113	$10^{-?}\times$ ·1107	$10^{-?}\times$ ·6639	·2449	2·564	28·54	77·73	880·6	$9569\times10^{?}$
17	$10^{-?}\times$ ·1450	$10^{-?}\times$ ·1561	$10^{-?}\times$ ·8993	·4066	4·541	53·78	180·1	1821	$2105\times10^{?}$
18	$10^{-?}\times$ ·1878	$10^{-?}\times$ ·2184	$10^{-?}\times$ ·1326	·6670	7·822	99·65	285·0	3659	$4678\times10^{?}$
19	$10^{-?}\times$ ·2420	$10^{-?}\times$ ·3032	$10^{-?}\times$ ·1938	1·081	13·63	181·8	532·4	7223	$1019\times10^{?}$
20	$10^{-?}\times$ ·3103	$10^{-?}\times$ ·4180	$10^{-?}\times$ ·2809	1·731	22·16	327·1	979·9	$1403\times10^{?}$	$2178\times10^{?}$
21	$10^{-?}\times$ ·3959	$10^{-?}\times$ ·5726	$10^{-?}\times$ ·4039	2·745	38·80	580·7	1770	$2084\times10^{?}$	$4575\times10^{?}$
22	$10^{-?}\times$ ·5030	$10^{-?}\times$ ·7708	$10^{-?}\times$ ·5766	4·313	64·69	1019	3188	$5063\times10^{?}$	$9436\times10^{?}$
23	$10^{-?}\times$ ·6362	$10^{-?}\times$ ·1055	$10^{-?}\times$ ·8178	6·719	106·2	1765	5645	$9124\times10^{?}$	$1925\times10^{?}$
24	$10^{-?}\times$ ·8017	$10^{-?}\times$ ·1420	·1153	10·36	172·8	3027	9694	$1733\times10^{?}$	$3463\times10^{?}$
25	$10^{-?}\times$ ·1006	$10^{-?}\times$ ·1002	·1612	15·85	278·8	5137	$1713\times10^{?}$	$3130\times10^{?}$	$7654\times10^{?}$

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