ON A UNIFIED APPROACH TO AVERAGE PHASE SPACE DENSITY IN SOME SULPHUR-INDUCED HIGH ENERGY NUCLEAR COLLISIONS

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The Average Phase Space Density (APSD) of very high energy nuclear collisions at the total "freeze-out" temperature offers an indirect but convenient tool to assess the merit and worth of the pionisation models. We have attempted to apply here a specific multiple production model with a view to estimating the APSD of pions for a set of sulphur-induced ultrarelativistic heavy-ion collisions in a unified manner, to compare them finally with the experiment-based results of NA35 and NA44-groups at CERN and also with the calculational results based on the thermal model. The specific implications of the present approach have also been pointed out in the end.

Keywords: Relativistic heavy-ion collision, two-particle correlation, thermal model, inclusive production, quark–gluon plasma

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1. Introduction

The term "freeze-out" indicates essentially the completion of the process of hadronisation in high energy heavy-ion collisions. This is observationally marked by super-abundance of pions called "pionisation". Recently, the Average Phase Space Density (APSD) of pions at freeze-out has been offered as a reliable and sensitive observable in the studies of heavy-ion collisions.[1,2] Our objective here is to calculate this APSD factor for pionisation in a set of sulphur-induced collisions at high energies with the combination of some approaches which are apparently of non-standard nature. In particular, what we intend to do here is to apply a specific parametrisation of the transverse momentum spectra forwarded by Hagedorn,[3,4] and another conversion parametrisation suggested by Peitzmann,[5,6] to interpret finally in a unified manner the nature of the observed APSD values for production of pions in various nucleus–nucleus reactions (e.g. S+S, S+Cu, S+Au, S+Ag, and S+Pb collisions) at a specific interaction temperature (for $T = 120$ MeV) only, and finally, to test their compatibility with the calculations based on the popular thermal model. The sulphur-induced collisions are of special importance now to us because of the following reasons: (i) Firstly, data on some sulphur-induced collisions are currently available. (ii) More importantly, recently, based on the present approach, we reported[7] results related to the APSD factor for pion production in lead–lead collision alone at CERN–SPS collider with a reasonable degree of success. So, our task here is to check whether the same approach could confront the available APSD data on some selected sulphur-induced collisions at ultrarelativistic energies.

2. APSD: A Brief Outline of the Theory

The average phase space density is normally defined in the standard form[8] by the following expression:

$$\langle f\rangle(\mathbf{p}) = \frac{\int d^3x\, f^2(t > t_f, \mathbf{x}, \mathbf{p})}{\int d^3x\, f(t > t_f, \mathbf{x}, \mathbf{p})}\,, \tag{1}$$

$$\approx P_1(p)\int d^4q\,\delta(q\cdot p)[C(q,p) - 1]\,, \tag{2}$$

where all the symbols and expressions have their contextual significance and are given by

$$E\frac{d^3N}{dp^3} = P_1(p) = \int d^4x\, S(x,p) \tag{3}$$

and

$$C(q,K) - 1 = \exp[-q_o^2R_o^2(K) - q_S^2R_s^2(K) - q_l^2R_l^2(K) - 2q_oq_lR_{ol}^2(K)]\,, \tag{4}$$

[with frequent changes of notation between p and K, i.e. $K \leftrightarrow p$].

In Eq. (4) given above the q_i's are the components of the momentum difference in the out-side-long coordinate system and K stands for the average pair momentum. In terms of the HBT-radii, which reflect the phase space density $S(x,K)$ from a combination of hadronic one-particle and identical two-particle spectra of the abundant hadron species, the integration over q in Eq. (2) leads to the simplified form as given below:

$$\langle f\rangle(\mathbf{p}) = \frac{1}{E}\frac{1}{p_T}\frac{d^3N}{dp_T\,dy\,d\phi}\frac{\pi^{1/2}}{R_s(\mathbf{p})\sqrt{R_o^2(\mathbf{p})R_l^2(\mathbf{p}) - R_{ol}^4(\mathbf{p})}}\,. \tag{5}$$

Elimination of pions from the long-lived resonances which do not contribute to the APSD at freeze-out reduces the working formula to the following expression with a parameter in fraction,[8] $\sqrt{\lambda_{\rm dir}}$

$$\langle f\rangle(\mathbf{p}) = \frac{1}{E}\frac{1}{p_T}\frac{d^3N}{dp_T\,dy\,d\phi}\frac{\pi^{1/2}\sqrt{\lambda_{\rm dir}}}{R_s(\mathbf{p})\sqrt{R_o^2(\mathbf{p})R_l^2(\mathbf{p}) - R_{ol}^4(\mathbf{p})}}\,. \tag{6}$$

3. The Models, Their Applications and the Results

The section comprises the following subsections which offer the necessary conceptual and technical tools to arrive at the final results.

3.1. *The basic outlook*

In proceeding towards calculations for various heavy-ion collisions we are initially guided by a sort of scepticism about the dominant conventional view that the routes to and the models for understanding the physics of heavy-ion collision would be entirely different from what they are for nucleon–nucleon (hadron–hadron) collisions. Obviously, we are not in favor of such total compartmentalisation and exclusivity of them. In fact, we have been driven to undertake the present issue with the prime motive of bridging up this gap as far as possible by providing a unified approach. In other words, we would try to demonstrate here that we could arrive at the results of nucleus–nucleus collisions even at such high energies (with some necessary assumptions, approximations and parametrisations) from a fundamentally very common basis of nucleon–nucleon collisions virtually with no concern for QGP.

3.2. *Model for one-particle transverse spectra*

The one-particle momentum spectrum determined as the space–time integral of the emission function $S(x,p)$ is sensitive to the momentum distribution in $S(x,p)$ and thus allows to constrain essential parts of the collision dynamics. For the heavy nucleus collisions we make the start in our approach by considering the rapidity integral transverse momentum spectrum. We assume first that only the direct "thermally" produced pions are produced and so modelised. Secondly, for the sake of simplifying the calculations, we neglect here the contributions arising out of the resonance decays — both short-lived and long-lived.

The parametrisation for the single-particle spectrum as proposed by the thermal model gives an appropriate fit only in the region $0.8\ \mathrm{GeV/c^2} \le M_T \le 3.0\ \mathrm{GeV/c^2}$. But the various observations over the one-particle spectra reveal that the slopes appear systematically flatter for central collisions than for peripheral ones.[9] The flattening of the distributions with increasing centrality shows that the spectral shapes can be described very poorly by an exponential curve. Actually a concave behaviour of PP data is observed over the whole M_T range for which we make change of choice of the p_T spectra. Hagedorn suggested a parametrisation for inclusive cross-sections in PP collisions in the following form[3,4]:

$$E\frac{d^3\sigma}{dp^3} = C\left(\frac{p_o}{p_T + p_o}\right)^n\,, \tag{7}$$

Table 1.

$C\,(mb)$	p_o (GeV/c)	n
42	4.9	34

where C, p_o, n are free parameters, which are presented in Table 1 and we will make use of this expression in the present work. The longitudinal component of the inclusive cross-section reflected through the rapidity term or variable has been absorbed as nearly a constant in the empirical C term.

3.3. *Conversion of cross-sections: From PP to AB collisions*

Now we try to build up the link between the transition from nucleon–nucleon to nucleus(A)–nucleus(B) collisions. With this purpose in mind, let us introduce a nucleus-dependent empirical term in the form $(A \cdot B)^{\alpha(p_T)}$ in the expression for inclusive cross-section on the physics of factorisation relations and that appears obviously as a product term in the final expression[5] given below:

$$E\frac{d^3\sigma}{dp^3}(A \cdot B) = (A \cdot B)^{\alpha(p_T)} E\frac{d^3\sigma}{dp^3}(PP)\,. \tag{8}$$

The parameter $\alpha(p_T)$ depends weakly on rapidity but strongly on p_T. At relatively low p_T, α increases and this indicates a growing participation of the nuclear volume. In the present work though we take $\alpha(p_T) \sim 1 + (0.1 \pm 0.03)p_T$ for moderate values of $p_T \leq 3$ GeV/c. This is also the limiting behaviour of α for full participation of the target nucleus. Besides, we have also taken into account the probable role of the Cronin effect on the value of α as considered above. Figures 1–3 represent very good

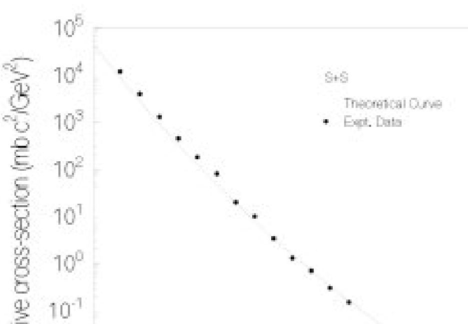

Fig. 1. Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for production of pions in S+S collision. Data points are from Ref. 9. Solid curve is the product of our model-based calculations.

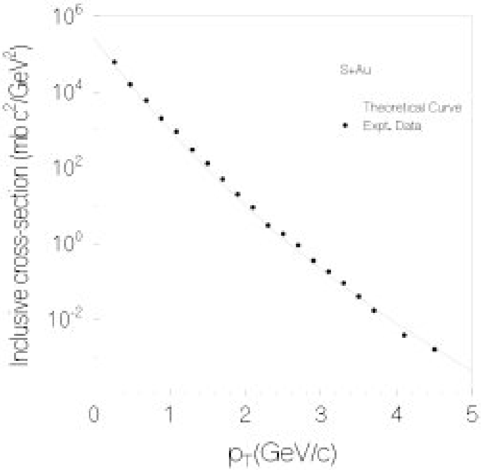

Fig. 2. Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for secondary pions in S+Au collision. Data points are from Ref. 9. Solid curve is drawn here with the help of Eqs. (7) and (8) with a suitable value for the p_T coefficient.

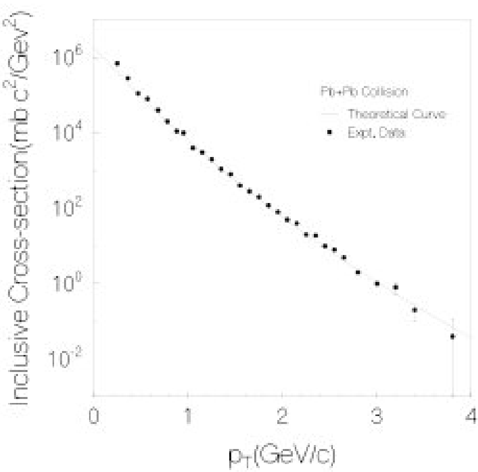

Fig. 3. Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for production of secondary pions in Pb+Pb collision. Data points are from Ref. 6. Solid curve is drawn here with the help of Eqs. (7) and (8) with a suitable value for the p_T coefficient.

fits to the available data on $E\frac{d^3\sigma}{dp^3}$ vs. transverse momentum (p_T) for production of pions in the soft and semi-hard domain of $p_T \leq 3$ GeV/c of various nucleus–nucleus collisions. The data-points for S+S and S+Au collisions are obtained from the results of WA80 collaboration[9] and those for Pb+Pb collision are from Ref. 6. And the modestly satisfactory agreement of the p_T-spectra for a wide range of high energy collisions (from Sulphur+Sulphur to Lead+Lead collisions) might be an index of the strength of the heuristic formula suggested by Peitzmann.[5]

3.4. *The final unification*

The theoretical estimate of the term $\frac{d^3N}{p_T\,dp_T\,dy\,d\phi}$ occurring in the working formula Eq. (5) has to be obtained by dividing Eq. (8) with the total inelastic cross-section $\sigma_{\rm in}$, for $A+B$ collisions. Hence the required expression is

$$\frac{d^3N}{p_T\,dp_T\,dy\,d\phi} = \frac{1}{\sigma_{\rm in}}(A\cdot B)^{\alpha(p_T)}E\frac{d^3\sigma}{dp^3}(PP)\,. \tag{9}$$

Therefore the APSD, denoted by $\langle f_{\rm Hag}\rangle$, using Hagedorn's expression for single particle spectrum, is given by

$$\begin{aligned}\langle f_{\rm Hag}\rangle &= \frac{1}{\sigma_{\rm in}}E\frac{d^3\sigma}{dp^3}(A\cdot B)\frac{\sqrt{\lambda_{\rm dir}}\,\pi^{3/2}}{M_T\cosh Y R_s(p_T)\sqrt{R_o^2(p_T)R_l^2(p_T)-R_{ol}^4(p_T)}}\\ &= \frac{(A\cdot B)}{\sigma_{\rm in}}C\left(\frac{p_o}{p_o+p_T}\right)^n\frac{\sqrt{\lambda_{\rm dir}}\,\pi^{3/2}}{M_T\cosh Y R_s(p_T)\sqrt{R_o^2(p_T)R_l^2(p_T)-R_{ol}^4(p_T)}}\,.\end{aligned} \tag{10}$$

A point must be made here. It is well known that the knowledge of single-particle spectrum and the Bose–Einstein correlations helps us to obtain a reliable estimate of the APSD. Furthermore, one must recognise that the absolutely normalised single-particle spectra carry information about the particle density in momentum space while the width of the distribution in the configuration space can be extracted from the correlation studies. And these extracted values of APSD are, in general, put to use by all, including ourselves, as the experimental measurements. This is surely a limitation of all the approaches whatsoever.

3.5. *Inputs for calculations*

To develop the present work the analyses are done in the longitudinal comoving system (LCMS) where the longitudinal momentum of the pion-pair vanishes. The HBT radii in LCMS are given by,

$$\begin{aligned}R_l &= \sqrt{\frac{2T_f}{M_T}}\frac{\tau_o}{\cosh(y_k)}\,,\\ R_o &\simeq R_s \simeq 3.5\ {\rm fm}\,,\\ R_{ol}^2 &= \frac{p_T Y}{M_T^2}\frac{T_f}{(\delta\eta)^2}\left[(\delta\tau)^2+\tau_0^2\frac{T}{M_T}\right],\end{aligned}$$

where τ_o is the longitudinal proper freeze-out time, $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$ is the space–time rapidity. In the above expression, $Y = (y_1+y_2)/2$, where y_1 and y_2 are the rapidities of the (observed) particles and $y_K = y - y_0$, when y_0 is the rapidity of the observer and $y = |Y|$ for a system with $y_1 = y_2$. Inserting measured values of R_l and assuming the effective temperature $\sim$ 120 MeV in the above expressions, the decoupling proper time (τ_0) for all systems studied is seen to lie $\sim$ 4 fm/c.[10,11] The freezing-out, for all practical calculations, is reasonably assumed to occur instantaneously, for which $\delta\tau$ in the above expression reduces to zero. $\delta\eta$ is chosen to provide reliably good fits and lies between 1.1 to 1.5. The values of different source parameters used in our calculations are given in Table 2.

Table 2.

T_f (MeV)	τ_o (fm/c)	$\delta\tau$	$\delta\eta$
120	4	0	1.3 ± 0.2

The value of $\lambda_{\rm dir}$ is taken here to be 0.7.

3.6. *Final results*

The results of our calculations based on Hagedorn's model for pionisation are presented in Figs. 4–8. They have also been compared with the NA35 and NA44 experimental (extracted) results indicated by filled boxes and also with the results of thermal model represented here by Bose–Einstein equilibrium distribution[1,8] and depicted in the graphs by the dotted curves or curvilinear lines.

4. Discussion and Concluding Remarks

The results arising out of the combination of our chosen models to analyse data on APSD are depicted in Figs. 4–8. They reveal that the APSD values are modestly in good agreement with the measurements really available at temperature, $T = 120$ MeV, in the neighbourhood of which the physical "freeze-out" is seen to

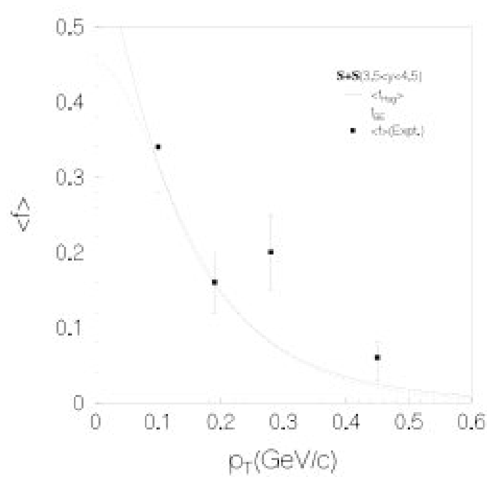

Fig. 4. Comparison of the fit for $\langle f \rangle$, average phase space density in pionisation, based on Hagedorn-model (present work) with NA35 data[1,8] on S+S collision and with the thermal model. Solid curve is our theoretically obtained values and the results of the thermal model are shown by dotted curve.

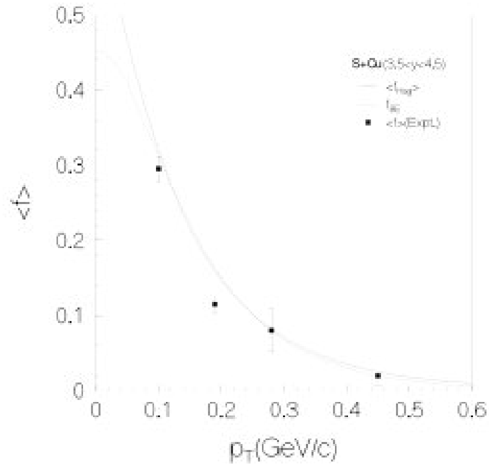

Fig. 5. Comparison of the fit for $\langle f \rangle$, average phase space density in pionisation, based on Hagedorn-model (present work) with NA35 data[1,8] on S+Cu collision and with the thermal model. Solid curve is our theoretically obtained values and the results of the thermal model are shown by dotted curve.

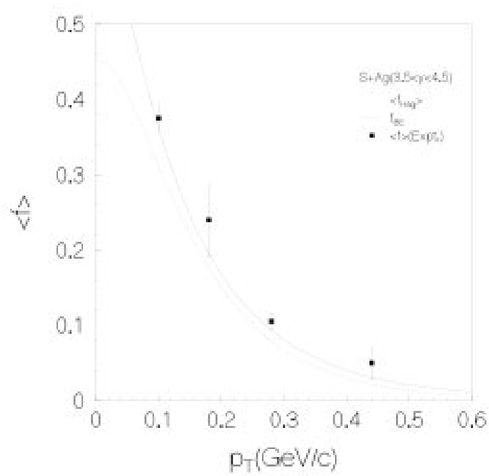

Fig. 6. Comparison of the fit for $\langle f \rangle$, average phase space density in pionisation, based on Hagedorn-model (present work) with NA35 data[1,8] on S+Ag collision and with the thermal model. Solid curve is our theoretically obtained values and the results of the thermal model are shown by dotted curve.

occur. Secondly, the calculated values of $\langle f \rangle$ do not show any strong A-dependence in the studied range of transverse momentum. And this is so even if the mass number (A) of the target heavy nucleus is changed from $A = 32$ (for S) to $A = 208$ (for Pb).

The expression for inclusive cross-section is the main basis of the present work, as it enters into all our calculations. Quite evidently, this accommodates essentially

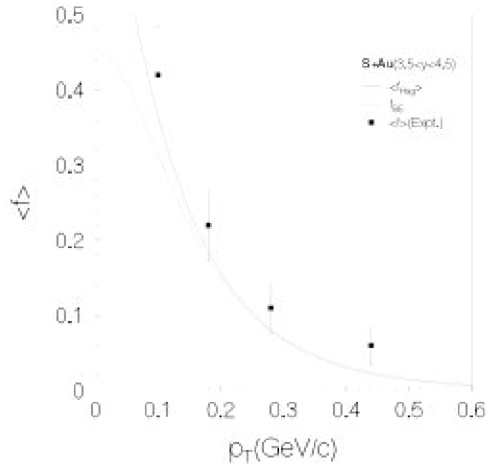

Fig. 7. Comparison of the fit for $\langle f \rangle$, average phase space density in pionisation, based on Hagedorn-model (present work) NA35 data[1,8] on S+Au collision and with the thermal model. Solid curve is our theoretically obtained values and the results of the thermal model are shown by dotted curve.

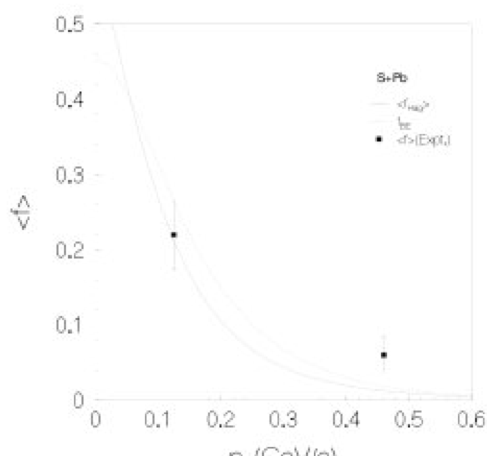

Fig. 8. Comparison of the fit for $\langle f \rangle$, average phase space density in pionisation, based on Hagedorn-model (present work) with NA44 data[1,8] on S+Pb collision and with the thermal model. Solid curve is our theoretically obtained values and the results of the thermal model are shown by dotted curve.

the idea of Feynman Scaling (FS) hypothesis, since there is no energy-dependence of the inclusive cross-section and it has the only dependence on x. But it is well known that FS is valid for the very small range of x ($x \simeq 0$) and p_T-values ($p_T \leq 1$), called the "central" region and "soft" collisions respectively. But the data chosen here for the present study are restricted to the minimum-bias events which might explain the slight departure of the theoretical $\langle f \rangle$ values measured at the relatively higher values of transverse momentum. What we observe here is the fact that Hagedorn's model for nucleon–nucleon collision works quite well even for heavy-ion collisions. In our opinion, this is not just a coincidence; rather it is a matter of great physical importance for future studies in high energy physics.

Finally, the APSD seems to show a striking feature of "universality"[12–15] for over a large value-ranges of the mass number of the target heavy nuclei, while the projectile heavy nucleus remains sulphur though. But this needs further verification by future experiments. The present work provides just the strong hints to support the suspect existence of such effects in future heavy-ion collisions. This apart, the present study is also a pointer to the fact that the insight obtained from the physics of nucleon–nucleon collision could have a high degree of relevance for understanding the physics of nucleus–nucleus collisions.

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