UNDERSTANDING THE NATURE OF PROTON–AIR CROSS-SECTIONS AT VERY HIGH ENERGIES AND THE IMPLICATIONS

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The purpose of this paper is to focus on the possible effective role of two relatively less-known models in analyzing comprehensively the very up-to-date data on proton–air inelastic cross-sections at high and ultra high energies. The standard versions of all the familiar simulation-based multiparticle production models, which nowadays normally claim front-ranking positions, address on the contrary, only a small part of the cross-section data for a very limited or sectional range of energy values. Against this background, the relevance and impact of the present study have finally been highlighted.

Keywords: Total cross-section; extensive air showers; cosmic ray interaction.

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The cross-section for proton–air collisions at high energies constitutes an important factor for considerations of simultaneous development of the three related fields of physics, viz., particle physics, nuclear physics and cosmic ray physics. Of them we would concentrate here mainly on the impact of the branches of high energy particle and nuclear physics, especially in the field of multiple production dynamics. The physics of both total and inelastic cross-sections for either proton–proton or proton–air collision in the domain of multiparticle production depends on the following important variables: the longitudinal and transverse momenta, the rapidity (pseudo-rapidity) factor and any other scaling variable with limited observance or violation. And all this pertains to the measured observables, like average multiplicity of the

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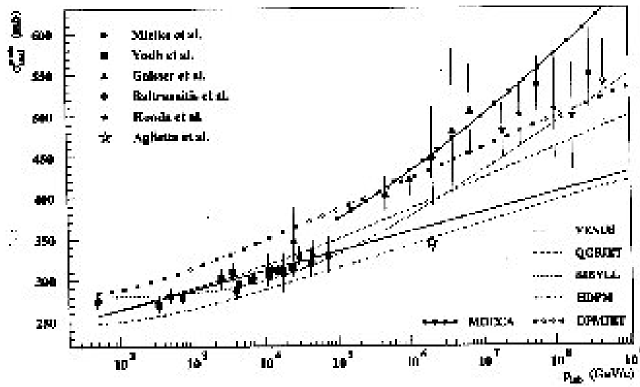

Fig. 1. Inelastic proton–air cross-sections as a function of energy, according to experimental data and according to the models in the CORSIKA and MOCCA Monte–Carlo programs. VENUS, HDPM and DPMJET are the hadronic interaction models at high energies provided in the CORSIKA Monte–Carlo program. All the data and the model references are from Nagano and Watson.[1]

secondary particles, the inclusive cross-section, the total and inelastic cross-section, average transverse momentum, the inelasticity coefficient, the leading particle effect etc. Our objective here is to study the nature of total and inelastic cross-section for proton–air collisions at very high energies on the basis of two chosen models which have hitherto not been applied widely, despite their possession of certain degree of potentiality in explaining the relevant facts.

The adjoining diagram in Fig. 1 of Ref. 1 depicts the assorted data on proton–air inelastic cross-sections for a span of 10^6–10^7 order of magnitude of $P_{\rm Lab}$ (in GeV/c) and the simulation results obtained by various front-ranking multiparticle production models. Indeed, they summarize the performance scenario on proton–air inelastic cross-sections by the near-totality of the main and major phenomenological models[1] in the domain of soft multiparticle production. Quite obviously, none of the models (which are VENUS, QGSJET, SIBYLL, HDPM, DPMJET, MOCCA) could be considered to attain the desired or satisfactory level of success in explaining the measured data at ultrahigh energies on an overall basis. Each of these important and popular models describes spectacularly only a very limited part of the data either on the low or high momentum regime; and none of them addresses comprehensively the entirety of them. Still, the performance on an overall basis, is relatively much better in eye-estimation with SIBYLL, though the fit has degenerated considerably on the higher energy side. The outcome is, on the whole, not very encouraging for any of the models, in so far as the understanding of the comprehensive nature of the data on proton–air inelastic cross-sections is concerned.

So, now we proceed to analyze the data with the help of two other much less-known models — one forwarded by Y. D. Prokoshkin (YDP)[2] and the other offered by Harry J. Lipkin (HJL).[3] Their models were introduced basically to provide successful fits to the hadronic total cross-sections. It is well known that the total and inelastic cross-sections for proton–proton and proton–air collisions are related with each other through the physics of the Glauber model.[4–6] So we have the strong impression that the models on hadron-involved total cross-sections have surely certain links with the proton–air inelastic cross-sections. With this assumption, we now try to apply the forms of the general expressions for PP total cross-sections as envisaged by the above-mentioned two models to the proton–air collisions at high energies with adjustments of the existing parameters, if and whenever necessary. We are especially interested here to take up the present study on proton–air collisions with the help of the above-stated two models in view of the latest success[7] on the nature of hadronic total cross-sections at the highest energies over the widest range in a previous work.

Some comments on the data sets to be used are in order here. The data used by Akeno (AGASA)[8] and Fly's eye groups[9] are based on some overestimations, as was pointed out by Nikolaev,[10] arising out of the effect of the existence of quasielastic interactions in which the target nucleus breaks up without particle production. Nikolaev showed, on the basis of assumptions of (i) modest energy-dependence of the elasticity factor and (ii) the relatively strong violation of the Feynman scaling, that the data for inelastic cross-sections for PP collisions should be increased on an average by nearly 30 mb. While testing the fits with the two models this correction factor to the data on PP total cross-sections has been introduced in the diagrams depicting PP total cross-section behavior (Figs. 2 and 3). Besides, these corrected

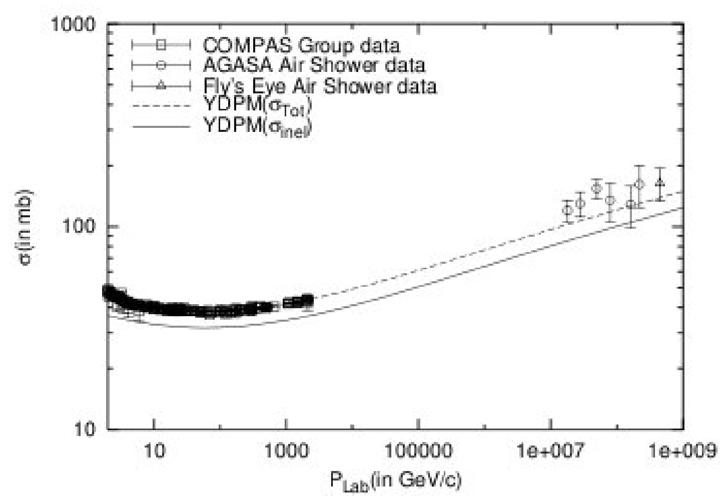

Fig. 2. Plot of cross-section vs. laboratory momentum for PP collisions over the widest possible range. The various experimental points for total cross-section are from Table 1 and Ref. 13. The dashed curve is drawn on the basis of the modified version of the YDP model for total cross-section indicated by "YDPM(σ_{tot})" and the solid one depicts the same for inelastic cross-section indicated by "YDPM(σ_{inel})".

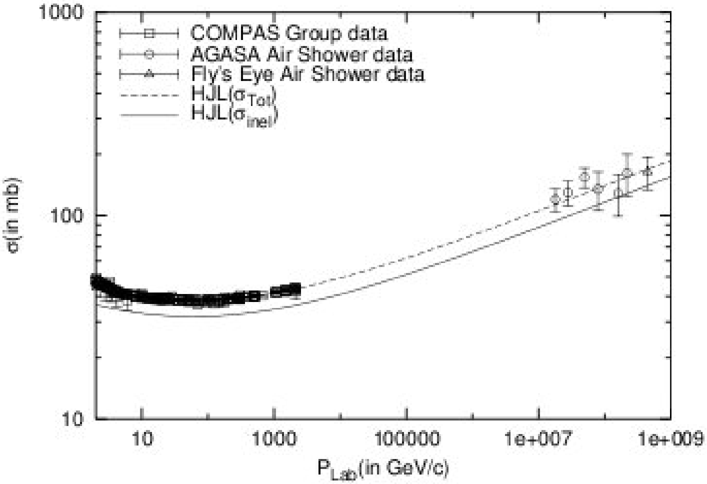

Fig. 3. Plot of cross-section vs. laboratory momentum for PP collisions over the widest possible range. The various experimental points for total cross-section are from Table 1 and Ref. 13. The dashed curve is drawn on the basis of the HJL model for total cross-section indicated by "HJL($\sigma_{\rm tot}$)" and the solid one depicts the same for inelastic cross-section indicated by "HJL($\sigma_{\rm inel}$)".

Table 1. The inelastic and total cross-sections for PP scattering from the AGASA data on the production cross-section for P–air collisions.[10].

$\log_{10} E$ (GeV)	$\sigma_{\rm prod}(P\text{–air})$ (mb)	$\sigma_{\rm in}(PP)$	$\sigma_{\rm tot}(PP)$ (mb)
7.17–7.41	480 ± 33	90 ± 12	120 ± 15
7.41–7.65	500 ± 38	97 ± 14	130 ± 18
7.65–7.89	537 ± 33	111 ± 13	154 ± 17
7.89–8.13	507 ± 61	100 ± 22	135 ± 29
8.13–8.37	498 ± 64	97 ± 24	129 ± 30
8.37–8.61	550 ± 72	117 ± 29	162 ± 38

values for $\sigma_{\rm tot}^{PP}$ and $\sigma_{\rm in}^{PP}$ in Table 1 do also take care of the changes in the K-factor to 1.3 proposed by Block *et al.*[6]

In what follows we present the outlines and some broad features of the two models to be applied here and the necessary working formulas for them.

Model 1. Let us start with the first model. The monotonic nature of rise of total cross-section beyond energies $E \geq 20$ GeV which was discovered around early '70s is now established not only for proton–proton collisions but also for hadron-induced and photon-induced collisions at very high energies, so much so that this observable ($\sigma_{\rm tot}$) has become a strong indicator for the "universal" nature of hadron-initiated reactions at very high energies. But this very observable showed a smooth fall of values for energy domain $E \leq 20$ GeV, according to the dependence predicted by the simple Regge pole model[11]:

$$\sigma_{\text{tot}}^{\text{Regge}}(s) = \sigma_{\infty}^{\text{Regge}} \left(1 + \frac{a}{\sqrt{s}}\right), \tag{1}$$

where $\sqrt{s}$ is the total energy of interaction in c.m. frame.

Prokoshkin[2] mentioned that the observed Serpukhov effect[12] first hinted at the deviation from the dependence given by the above simple Regge formula. The effect could be stated as follows: the experiments conducted at the Serpukhov energies demonstrate convincingly that the total cross-sections pass through the minimum of Serpukhov energies beyond which they increase almost uniformly; and the rate of rise, then, is the only point of dispute which assumes the center-stage regarding studies on both inelastic and total cross-sections. Summing up the observations, Prokoshkin[2] proposed a very well-accepted version of a two-component model as given below. According to the hypothesis of Prokoshkin and that of Serpukhov group, the total cross-section was represented as the sum of the Regge cross-section and the growing "gluonic" piece:

$$\sigma_{\text{tot}}(s) = \sigma_{\text{tot}}^{\text{Regge}}(s) + \sigma_{\text{tot}}^{\text{gluon}}(s). \tag{2}$$

The latter factor in Eq. (2) here is expected to exhibit flavor-independent behavior which is universal for all hadrons. And this gluonic component has some specific characteristics: (a) It is accepted that the values of $\sigma_{\text{tot}}^{\text{gluon}}$ are nearly identical for particles and antiparticles. (b) Secondly, the dependence of $\sigma_{\text{tot}}^{\text{gluon}}$ on s for different particles differs only in a shift in the $\ln s$ scale. In other words, the cross-section $\sigma_{\text{tot}}^{\text{gluon}}$ as a function of the dimensionless variable s/s_0 is universal, with a critical value for s_0. It is observed that the critical value of $s = s_0$ are larger for nucleon–nucleon collisions than for meson–nucleon interactions. This finding could be an evidence of glueball formation hypothesis since the gluons in the meson are harder than those in the nucleon. The energy dependence of $\sigma_{\text{tot}}^{\text{gluon}}$ was proposed by Prokoshkin[2] in the form given below:

$$\sigma_{\text{tot}}^{\text{gluon}}(s) = \alpha \ln\left(\frac{s}{s_0}\right) + \beta \ln^2\left(\frac{s}{s_0}\right), \tag{3}$$

where $\alpha = 0.46 \pm 0.15$ mb and $\beta = 0.27 \pm 0.10$ mb as prescribed by Prokoshkin[2] for proton–proton total cross-section. But here as we aim at updating the old fit to the latest data we have taken $\alpha = -(1.73 \pm 0.17)$ mb and $\beta = 0.39 \pm 0.01$ mb. Besides, in the entire work we assumed the value of s_0 to be consistently throughout at 10 GeV^2 as the use of this value is just conventional, so no explanation seems to be necessary. In order to test the above model with the total cross-section data on PP collision given by Particle Data Group[12] over a wide energy range, we use the values $\sigma_{\infty}^{\text{Regge}} = 39.8 \pm 0.4$ mb and $a = 0.11 \pm 0.04$ GeV in places of 37.1 mb and[2] 0.32 GeV respectively. The fit result of the modified version, marked as YDPM, is depicted in Fig. 2 with ndf = 140 and $\chi^2/\text{ndf} = 1.503$. The letters "M" and "N" added to the abbreviations represent "modified" and "new" fits respectively.

The final expression of the modified YDP model for PP collision is given below,

$$\sigma_{\mathrm{YDPM}}^{PP} = (39.8 \pm 0.4)\left(1 + \frac{(0.11 \pm 0.04)}{\sqrt{s}} - (1.73 \pm 0.17)\ln\left(\frac{s}{s_0}\right) + (0.39 \pm 0.01)\ln^2\left(\frac{s}{s_0}\right)\right). \tag{4}$$

Model 2. Lipkin[3] used a version of the two-component pomeron model with its basis on the observation that $\sigma_{\mathrm{tot}}^{PP}$ does not have leading Regge exchange contribution describable by duality diagrams. It was assumed to be given by the sum of two Pomeron-like components: (i) the asymptotic component increasing monotonically with energy (σ_∞) expressed in the following form:

$$\sigma_\infty^{PP}(P_{\mathrm{Lab}}) = 19.5\left(\frac{P_{\mathrm{Lab}}}{20}\right)^{0.13} \tag{5}$$

and (ii) the other with monotonically decreasing component (σ_{dec}) describing the "approach to asymptotia" is to be represented as

$$\sigma_{\mathrm{dec}}^{PP}(P_{\mathrm{Lab}}) = 19.8\left(\frac{P_{\mathrm{Lab}}}{20}\right)^{-0.2}. \tag{6}$$

Both the forms and the numerical values in expressions (5) and (6) are the phenomenological choices made by Lipkin himself. So, the total cross-section, according to Lipkin's model, is given by the summation of the above two consecutive expressions:

$$\begin{aligned}\sigma_{\mathrm{tot}}^{PP}(P_{\mathrm{Lab}}) &= \sigma_\infty^{PP}(P_{\mathrm{Lab}}) + \sigma_{\mathrm{dec}}^{PP}(P_{\mathrm{Lab}}) \\ &= 19.5\left(\frac{P_{\mathrm{Lab}}}{20}\right)^{0.13} + 19.8\left(\frac{P_{\mathrm{Lab}}}{20}\right)^{-0.2}.\end{aligned} \tag{7}$$

The main physical considerations behind the form of the Lipkin's final working expression (Eq. (7)) are: (i) the asymptotic component satisfies the naive SU(3) symmetry and some other quark counting rules, (ii) the decreasing part originates from a "double scattering" contribution proportional to the product of the number of non-strange quarks and the total number of quarks. The utility of this model lies on the fact that it offers an asymptotic formula for σ_{tot} from relatively low energy data in meson–nucleon channels to the values of nucleon–nucleon total cross-sections which are then, in the present case, extended to nucleon–nucleus (air) collisions at superhigh energies. Furthermore, the basics are ingrained in a near-universality of the quark-based scatterings without any introduction of the active role of gluons. In what follows we choose to represent σ_{tot} by σ.

But, quite significantly, the unaltered version of Lipkin model, which is marked here HJL, is found to describe data even at the highest available energies quite successfully. In other words, no changes in the parameter-values in accommodating the latest data offered by Particle Data Group[13] are warranted. The fit in Fig. 3

is obtained by original Lipkin model with ndf = 140 and $\chi^2/\text{ndf} = 1.374$ by using Eq. (7) and the result is shown by dashed curve.

Recently we reported[7] that these two models (Prokoshkin and Lipkin), after some modifications in the parameter values, produce excellent fit to the data for a set of total cross-sections in hadron–hadron collisions. Being inspired by this success, here we apply these two models to describe theoretically the experimental data on P–air inelastic cross-section over the widest possible laboratory momentum range. Retaining the forms of the expressions to be the same as those of the earlier versions of the models and inserting some changes, whenever necessary, only in the parameter-values therein, we propose the following expressions for describing the proton–air inelastic cross-section data in a phenomenological way:

$$\sigma_{\text{YDPN}}^{P\text{—air}} = (361.0 \pm 29.0) - (21.0 \pm 6.0) \ln\left(\frac{s}{s_0}\right) + (1.6 \pm 0.3) \ln^2\left(\frac{s}{s_0}\right) \tag{8}$$

with ndf = 35 and $\chi^2/\text{ndf} = 2.680$.

$$\sigma_{\text{HJLN}}^{P\text{—air}} = (62.0 \pm 3.0)\left(\frac{P_{\text{Lab}}}{20}\right)^{(0.13\pm0.01)} + (226.0 \pm 9.0)\left(\frac{P_{\text{Lab}}}{20}\right)^{-(0.05\pm0.01)} \tag{9}$$

with ndf = 35 and $\chi^2/\text{ndf} = 2.560$. The curves drawn to show the nature of agreements in both cases are displayed in Figs. 4 and 5. The values of inelastic cross-section (σ_{inel}) for PP collision, which are shown by the solid curve in both figures, can easily be derived by multiplying σ_{tot} with 34/41 as obtained by Bell *et al.*[14]

The models which have been applied here to probe the nature of the inelastic proton–air cross-section measurements do not directly and outwardly manifest any

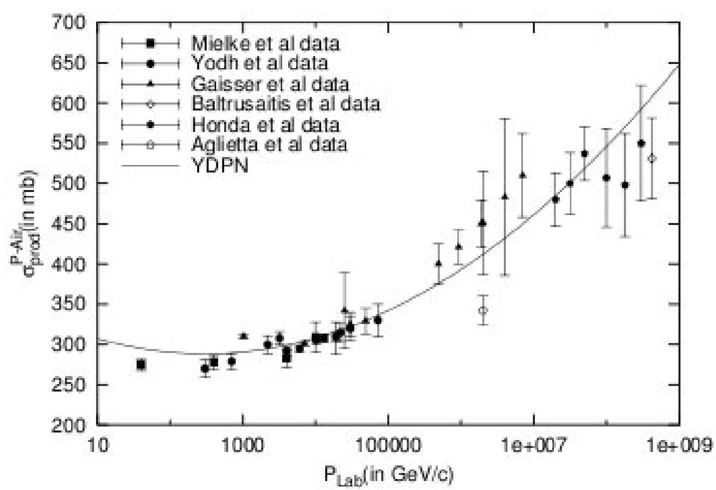

Fig. 4. Plot of inelastic cross-sections vs. laboratory momentum for P–air collisions over the widest possible range. The various experimental points are from Ref. 1. The solid curve represents the theoretical plot on the basis of an extension of the YDP model newly made here by the present authors marked YDPN.

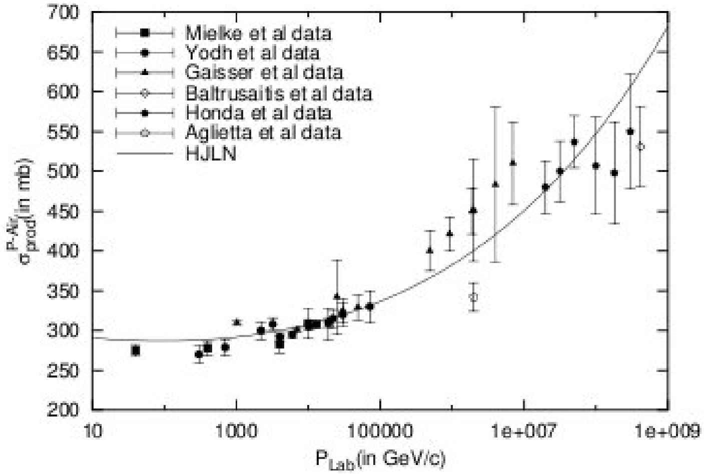

Fig. 5. Plot of inelastic cross-sections vs. laboratory momentum for P–air collisions over the widest possible range. The various experimental points are from Ref. 1. The solid curve represents the theoretical plot on the basis of an extension of the HJL model newly made here by the present authors marked HJLN.

nucleus-dependence factor. Actually the slight alterations of the parameter values valid for proton–proton cross-sections take care of the nuclear effects, as air is just a variety of modestly light nuclei. This will probably not be valid for heavier nuclei. We will now explore how the somewhat hidden nuclear effects even in cases of our calculations containing no clear-cut and direct A-dependent terms could be exposed. In the standard framework, it is quite accepted that (i) the inelastic proton–air collisions can be described as incoherent superposition of the collisions of individual nucleons; (ii) the multiplicity in an event is closely related to the number of participating nucleons, and the fluctuations in the number of participants are large compared to the multiplicity fluctuations from a single collision; (iii) as a result of the above, the produced particle multiplicities are, to a considerable extent, determined by the impact parameter rather than the detailed dynamics, or the structure of the "hadrons" involved in the collisions; and (iv) in actual calculations this impact parameter dependence is reflected through the use of an observable, $\langle\nu\rangle$,[10] the average number of "wounded" or participating nucleons given by

$$\langle\nu\rangle^{P\text{–air}} = \frac{A\sigma_{\text{inel}}^{PP}}{\sigma_{\text{inel}}^{P\text{–air}}}\,. \tag{10}$$

This total inelastic hadron–nucleus cross-section formula follows from the more general form given by $\sigma_{\text{inel}} = \sum \sigma_{\nu}^{pA}$ where ν is the number of wounded (excited) nucleons in a nucleus with mass number A (for air, $A = 14$). The term "wounded" physically means the nucleon which interacts inelastically with a projectile nucleus. Frichter *et al.*[15] made a model-based study on average number of wounded nucleons which we reckon herewith for comparison in Table 2. In the fourth column of this table the $\langle\nu\rangle$-values obtained by Frichter, Gaisser and Stanev (FGS)[15] are shown.

Table 2. Comparison of the model-based $\langle \nu \rangle$ values.

E (GeV)	$\langle \nu \rangle_{\rm YDPM}$	$\langle \nu \rangle_{\rm HJLM}$	$\langle \nu \rangle_{\rm FGS}$
10^3	1.66	1.68	1.69
10^7	2.42	2.78	2.29
10^9	2.65	3.21	2.77
10^{11}	2.80	3.46	3.14

Comparison of the estimations of the average wounded nucleons based on the models chosen here for the present study as given in Table 2 leads to some divergence of values at high energy region from those predicted by Frichter *et al.*[15] The model-based values obtained here indicate an increasing trend in the number of wounded nucleons even with increase in energies by the order of 2 to 4 (10^7–10^{11} GeV, in Table 2). But we cannot ascertain — due to lack of relevant data — whether such a rising nature would be traceable even with the heavy nuclei as the colliding objects. This implies essentially the presence of the effects of cascading and rescattering over a wide band of interaction energy.

Most of the standard hadronic interaction models that are conventionally put into use for air shower studies fall under any of the following categories: (i) dual parton model or any of its mutated versions like the most popular quark–gluon string model, (ii) statistical or thermodynamical models producing particles via clusters or fireballs. In the first group multiple production of soft hadrons called minimum bias hadronic interactions proceed by the exchange of strings in various way. Strings, it is believed, do radiate characteristic multiplicity of secondaries. Inelasticity, in such cases, are mainly determined by the momentum distribution of the valence constituents and increases slowly with rising energy as more and more soft strings enter into the exchanges. The second set of models is generally characterized by a more rapid, power-law dependence of multiplicity on invariant mass of the produced clusters. The net effect of this sort of models is to enhance the elasticity factor and the leading particle effect and thus to reduce quantitatively the inelasticity value. Against this, the models applied here, to the best of our knowledge, suffer from incompleteness in the sense that they deal only with total and inelastic cross-sections. Furthermore, unlike the competing models, they are silent on other aspects of the hadron and nuclear physics. Despite this, the YDPN model, with the occurrence of the $\ln^2 s$ term which indicates rising inelastic cross-section indirectly hints at a reducing effect on the leading particle energy and also at increasing the inelasticity coefficients at a very slow pace.

Let us now summarize the important conclusions that we have arrived here by making use of the chosen models.

(i) The models we chose for the present study — with or without modifications — explain very nicely the data on inelastic proton–air cross-section under our consideration here.

(ii) The large error bars in the data-points in proton–air cross-section magnitudes over the wide span of energy-domain explain the deviations of the χ^2/ndf values from unity; the somewhat larger values of χ^2/ndf for proton–air collisions in both models (YDPN and HJLN) are justified in view of this reality.

(iii) Seen in a superficial manner, the transition of physics from nucleon–nucleon collision to nucleon–nucleus collision, in this case, is possible only by change of parameters of the two chosen models with reasonably good χ^2/ndf.

(iv) The measured proton–air production cross-sections are more reliable in the sense that there is as such no hadronic interaction dependence in these measurements, though air-shower experiments, in general, are subject to large systematic errors arising out of two major sources: (a) the conversion of the observed attenuation length to the proton interaction length in air and (b) the deviation of the proton–proton cross-section from the measured proton–air cross-sections.

(v) That the original version of the Lipkin model offers such a good fit brings to focus the grand success of the model over such a wide range of interaction energy. This is surely a noteworthy feature of the model.

(vi) The physics of YDP model is based on the ideas of post-QCD physics whereas that of HJL model is of pre-QCD era. In the latter case the physics of formation of minijets through the radiation of multiple gluons is not taken into account. Still the fit, based on our χ^2/ndf values, is better for HJL model. What it portends to may be an interesting probing point for future studies.

(vii) The logarithmic nature of rise reflects, in general, a modest violation of the Feynman scaling. On the contrary, the power-law behavior normally represents relatively stronger violation of the Feynman scaling. And the observance of scaling is expected if and only if $\langle n \rangle \sim \ln s$ and $\sigma_{\text{inel}} \sim \ln^2 s$. So, the YDP model indicates moderate violation and the HJL model hints at a stronger violation. In any case, the violation of the Feynman scaling — moderate or strong — is the common prediction for hadron physics.

(viii) The sharp rise in proton–air inelastic cross-section at higher energy regime indicates physically and indirectly that inelasticity coefficient may rise very slowly with energy for proton–air collisions because this will have diminutive effect on the energy of the leading particle among the produced secondaries in proton–air collisions.[16–18]

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