Hall effects on hydromagnetic falling liquid film

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Abstract

An investigation is made of flow of an electrically conducting falling liquid film over a smooth vertical surface taking Hall effects into account, the liquid being permeated by a transverse magnetic field. Consideration of Hall current into the flow indicates a similarity between the flow of a rotating liquid and that due to the non-rotating system in presence of Hall currents. Discussion has been made for electrically conducting falling film in presence of cross-flow due to hall effect in non-rotating system.

Keywords: Hall effect; Falling film; Cross-flow

1. Introduction

The flow of liquids in thin films is a well known phenomenon in everyday life as well as in numerous technological applications. Although the dynamics of thin film flows of Newtonian liquids has been extensively studied by several authors (Andersson [1]), no attempts have been made to study the gravity-driven film flow in presence of magnetic field. In the present note an investigation is made on the electrically conducting liquid film flowing down along a smooth vertical surface taking Hall currents into account, these effects being important when the magnetic field is very strong (Sato [2], Sherman [3]). This information is fundamental in the gravity-driven flow of liquid in presence of strong magnetic field.

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2. Formulation and discussion

Consider a thin film of thickness d of an electrically conducting liquid flowing down a vertical plate in the presence of a transverse magnetic field and the plate is assumed to be electrically conducting. Let the x-axis be taken along the free surface of the film downwards and y-axis normal to and directed towards the plate, and a strong imposed magnetic field H_0 is along y-axis. At a large distance from the entry section, the flow will be fully developed and in the steady state all the physical variables depend on y only. It is assumed that the free surface of the film is wave free. The solenoidal equation $\nabla \cdot \vec{H} = 0$ gives $H_y = constant = H_0$ everywhere in the flow, where $\vec{H} = (H_x, H_0, H_z)$. The equation of continuity $\nabla \cdot \vec{q} = 0$ and the no-slip condition at the plate give $v = 0$ everywhere, where $\vec{q} = (u, 0, w)$. It is well-known that the introduction of Hall current induces a flow at right angle to the main flow direction and this flow is similar to that in a rotating system (Batchelor [4], Gupta [5]).

Introducing the dimensionless variables $\eta = y/d$, $U = ud/\nu$, $W = wd/\nu$, $h_x = H_x/(\sigma\mu_e H_0 \nu)$, $h_z = H_z/(\sigma\mu_e H_0\nu)$, the x and z components of equations of momentum for fully developed falling film are

$$U'' + M^2 h_x' + G_1 = 0, \tag{1}$$

$$W'' + M^2 h_z' = 0, \tag{2}$$

where $G_1 = gd^3/\nu^2$ is a non-dimensional parameter, $M = \mu_e H_0 d(\sigma/\rho\nu)^{1/2}$ is the Hartmann number.

When the strength of the magnetic field is very large, x and z components of modified magnetic induction equation which include Hall effects [3] are given by

$$mh_z'' - h_x'' = U', \tag{3}$$

$$mh_x'' + h_z'' = -W', \tag{4}$$

where $m = \omega_e\tau_e$ stands for Hall parameter with ω_e and τ_e denoting electron Larmor frequency and electron collision time. The y-component of the equation is identically satisfied. Further it is assumed that $\omega_e\tau_e \sim o(1)$ and $\omega_i\tau_i \ll 1$, where ω_e, ω_i are the cyclotron frequencies of electrons and ions and τ_e and τ_i are the collision times of electrons and ions. In writing the magnetic induction equation, the ion slip effects arising out of imperfect coupling between ions and neutrals as well as the electron pressure gradient are neglected. Introducing the complex quantities $F = U + iW$, $h = h_x + ih_z$, Eqs. (1)–(4) can be written as

$$F'' + M^2 h' + G_1 = 0, \tag{5}$$

$$h'' = -\frac{1}{1 + im} F'. \tag{6}$$

The boundary conditions for the velocity and induced magnetic field are

$$F = 0 \quad at\ \eta = 1, \qquad F' = 0 \quad at\ \eta = 0, \tag{7}$$

$$h' + h/\phi = 0 \quad at\ \eta = 1, \qquad h = 0 \quad at\ \eta = 0, \tag{8}$$

where $\phi = \sigma_1 d_1/\sigma d$ is the dimensionless electrical conductance ratio (Mazumder [6]), with σ_1 *and* d_1 being, respectively, the electrical conductivity and

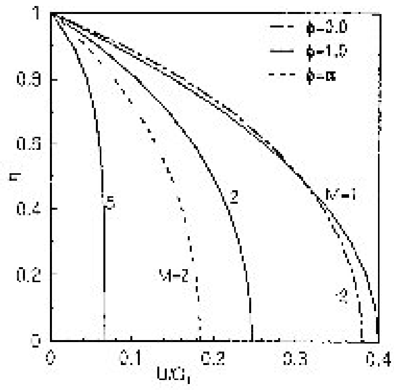

Fig. 1. Velocity profiles for $m = 0.0$.

the thickness of the plate. The first boundary condition of (8) follows from the continuity of the tangential component of the magnetic field at the interface between the vertical plate and the liquid film, while the second boundary condition follows from the fact that the medium adjoining the free surface is assumed to be electrically non-conducting.

The solutions of Eqs. (5) and (6) subject to the boundary conditions (7) and (8) are given by

$$F = \frac{G_1(\phi + 1)(\cosh M_1 - \cosh M_1\eta)}{M_1^2\phi\cosh M_1 + M_1\sinh M_1}, \tag{9}$$

$$h = \frac{G_1}{M^2}\left[\frac{M_1(\phi + 1)\sinh M_1\eta}{M_1^2\phi\cosh M_1 + M_1\sinh M_1} - \eta\right], \tag{10}$$

where

$$M_1 = \frac{M}{\sqrt{1 + im}}.$$

When Hall effects are absent ($m = 0$), there will not be any cross-flow and cross-magnetic field so that $W = h_z = 0$ and the solutions (9) and (10) reduce to the solution of hydromagnetic falling film. Fig. 1 shows the velocity profiles along the hydromagnetic falling film for various values of electrical conductance ϕ and Hartmann number M. It is observed that the velocity gradually decreases with increase in M for fixed ϕ and it becomes flattened near the free surface of the film. From the physical point of view this is due to the retarding influence of the Lorentz force $\mu_e\vec{J}X\vec{H}$ on the flow field. In the absence of imposed magnetic field ($M = 0$), the solution reduces to $U=(G_1/2)(1-\eta^2)$ for laminar falling liquid film (Bird et al. [7]).

Separating Eqs. (9) and (10) into real and imaginary parts for velocity and magnetic fields we have

$$U=\frac{G_1(\phi+1)}{\alpha_1^2+\beta_1^2}(\alpha_1\alpha_2+\beta_1\beta_2), \tag{11a}$$

$$W=\frac{G_1(\phi+1)}{\alpha_1^2+\beta_1^2}(\alpha_1\beta_2-\alpha_2\beta_1), \tag{11b}$$

$$h_x=\frac{G_1}{M^2}\left[\frac{(\phi+1)(\alpha_1\alpha_3+\beta_1\beta_3)}{\alpha_1^2+\beta_1^2}-\eta\right], \tag{12a}$$

$$h_z=\frac{G_1}{M^2}\frac{(\phi+1)(\alpha_1\beta_3-\alpha_3\beta_1)}{\alpha_1^2+\beta_1^2}, \tag{12b}$$

where

$$\alpha_1=\phi(\alpha^2-\beta^2)\cosh\alpha\cos\beta-2\alpha\beta\phi\sinh\alpha\sin\beta$$
$$+\alpha\sinh\alpha\cos\beta-\beta\cosh\alpha\sin\beta$$

$$\beta_1=\phi(\alpha^2-\beta^2)\sinh\alpha\sin\beta+2\alpha\beta\phi\cosh\alpha\cos\beta$$
$$+\alpha\cosh\alpha\sin\beta+\beta\sinh\alpha\cos\beta$$

$$\alpha_2=\cosh\alpha\cos\beta-\cosh\alpha\eta\cos\beta\eta,$$

$$\beta_2=\sinh\alpha\sin\beta-\sinh\alpha\eta\sin\beta\eta$$

$$\alpha_3=\alpha\sinh\alpha\eta\cos\beta\eta-\beta\cosh\alpha\eta\sin\beta\eta,$$

$$\beta_3=\alpha\cosh\alpha\eta\sin\beta\eta+\beta\sinh\alpha\eta\cos\beta\eta$$

$$\alpha,\beta=\pm\frac{M[\pm 1+\sqrt{1+m^2}]^{1/2}}{\sqrt{2(1+m^2)}}.$$

When the vertical wall is perfectly conducting ($\phi\to\infty$) the solutions (9) and (10) reduce to

$$F=\frac{G_1}{M_1^2}\left[1-\frac{\cosh M_1\eta}{\cosh M_1}\right], \tag{13}$$

$$h=\frac{G_1}{M_1^2}\left[\frac{M_1\sinh M_1\eta}{M_1^2\cosh M_1}-\eta\right]. \tag{14}$$

To study the effects of wall conductance (ϕ) and the Hall parameter (m) on the hydromagnetic falling liquid film we have presented the dimensionless components of velocity (U,W), induced magnetic field (h_x,h_z) and the current density (J_x^*,J_z^*) against η in Figs. 2–4 for various values of ϕ and m with $M=2.0$. It is observed from Fig. 2a that for fixed M and ϕ the primary velocity (U/G_1) increases progressively with increase in m, while Fig. 2b shows a similar result for the cross-flow (W/G_1). For fixed value of $m=1.5$, maximum of U/G_1 decreases with increase in ϕ, whereas that of cross-flow W/G_1 increases everywhere with increase in ϕ. Further, the flattening effect of the magnetic field on the velocity profiles is also observed as in Fig. 1. It may also be noted that for fixed ϕ, m and M, the primary velocity U/G_1 is as much as three times greater than the induced cross-velocity W/G_1. Fig. 3(a,b) shows the profiles of induced magnetic field h_x/G_1 and h_z/G_1 for various values of m and ϕ with $M=2.0$. It is observed from Fig. 3a that the magnitude of the induced magnetic field decreases with increase in m and increases with ϕ, whereas cross-magnetic field increases with increase in both m and ϕ (Fig. 3b).

The dimensionless components of current density are given by

$$J_x^*/G_1=h_z' \quad \text{and} \quad J_z^*/G_1=-h_x', \tag{15}$$

where $J_x^*=\mathrm{d}J_x/\sigma\mu_e H_0 v$ and $J_z^*=\mathrm{d}J_z/\sigma\mu_e H_0 v$

The components of current density J_x^* and J_z^* are plotted against η for various values of m and ϕ for $M=2.0$ in Figs. 4(a,b). It is observed that the current density J_x^* due to Hall current increases with increase in m, whereas J_z^* decreases with the same. For fixed m both J_x^* and J_z^* increase with ϕ.

The dimensionless shear-stresses at the wall due to the primary and secondary flows are, respectively, given by

$$\tau_x=\frac{1}{G_1}\left[\frac{\mathrm{d}U}{\mathrm{d}\eta}\right]_{\eta=1}=f_1(\phi,M,m), \tag{16}$$

$$\tau_z=\frac{1}{G_1}\left[\frac{\mathrm{d}W}{\mathrm{d}\eta}\right]_{\eta=1}=f_2(\phi,M,m). \tag{17}$$

The values of shear-stress due to primary and secondary flows at the plate are depicted in Table 1 for various values of m and ϕ with $M=5.0$. From the table, it is seen that the magnitude of both τ_x and τ_z at the plate increases with increase

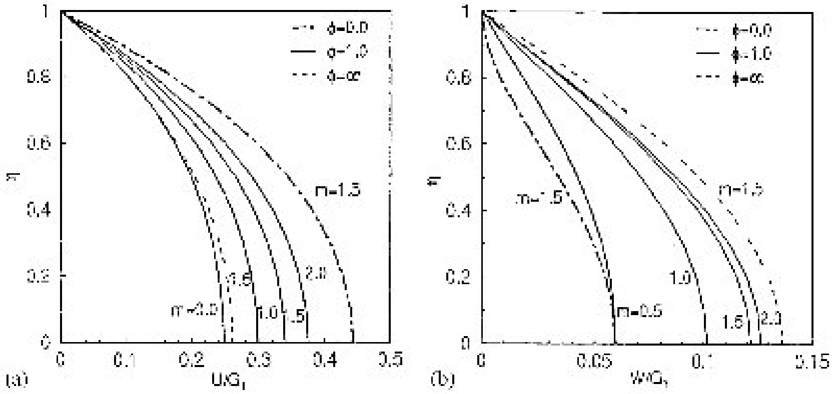

Fig. 2. (a) Primary velocity profiles for $M = 2.0$. (b) Cross-velocity profiles for $M = 2.0$.

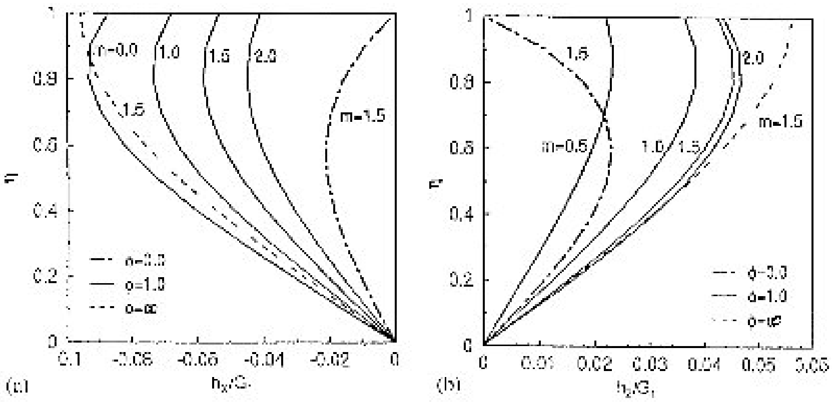

Fig. 3. (a) Profiles of h_x/G_1 for $M = 2.0$. (b) Profiles of h_z/G_1 for $M = 2.0$.

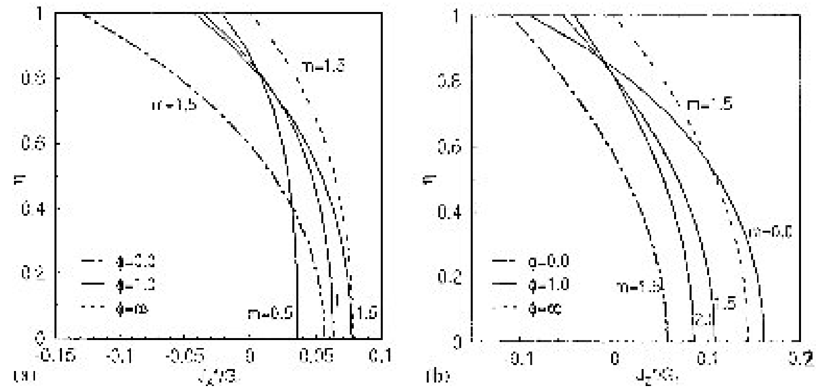

Fig. 4. (a) Profiles of J_x^*/G_1 for $M = 2.0$. (b) Profiles of J_z^*/G_1 for $M = 2.0$.

Table 1
Values of τ_x for $M = 5.0$

$m \setminus \varphi$	0.0	0.5	1.0	1.5	2.0
0.0	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000
0.5	−0.4285	−0.4425	−0.4746	−0.5122	−0.5495
1.0	−0.3333	−0.3441	−0.3696	−0.4004	−0.4322
2.0	−0.2727	−0.2813	−0.3017	−0.3268	−0.3531
∞	−0.1999	−0.2058	−0.2199	−0.2372	−0.2556
Values of τ_z					
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.0000	−0.0728	−0.1298	−0.1706	−0.1998
1.0	0.0000	−0.0667	−0.1218	−0.1646	−0.1984
2.0	0.0000	−0.0598	−0.1107	−0.1521	−0.1864
∞	0.0000	−0.0486	−0.0911	−0.1273	−0.1589

in m and decreases with increase in ϕ. When the plate is non-conducting ($\phi = 0$), Eq. (16) shows that the skin-friction due to primary flow does not vanish, whereas from (17) we see that for $\phi = 0$, the skin-friction due to cross-flow vanishes at the plate. Thus we arrive at an interesting result that in the case of non-conducting plate ($\phi = 0$), the cross flow due to Hall current shows incipient flow reversal although the primary flow does not. On the other hand, from (16) we find that for conducting plate ($\phi \neq 0$) the primary flow at the plate $\eta = 1$ shows incipient flow reversal when $\phi = \phi_{\text{crit}}$.

The present investigation reveals the interesting fact that owing to the introduction of Hall current in the gravity-driven film flow a reduction in falling film velocity takes place due to the cross-flow in a non-rotating system.

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