# Multiple bank lending and seniority in claims☆

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**Abstract**

In this paper, we study why multiple banks lend to the same project even though they are not constrained by the availability of funds. The choice of the amount of debt, as well as the hierarchy of debt claim is endogenized. We find that the hierarchy among the creditors is exclusively determined by their monitoring costs. We identify conditions under which group lending is efficient. We also establish a possible rationale behind the observed phenomenon where a system of consortium lending is increasingly being replaced by syndicated lending.

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*Keywords:* Multiple bank lending; Seniority of claims; Endogenous debt

## 1. Introduction

A commonly observed capital structure is one where a firm has more than one lender with unequal priorities. In this paper we study why this may be an optimal outcome when there is the possibility of strategic default on debt by firms.

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Our model adopts the costly state verification framework (Townsend, 1979), with the lenders monitoring in bad states only (Moore, 1993). Here, monitoring is equivalent to verification, or auditing, by the lenders, should the firm default on its debt obligations. In the absence of such auditing, the firm may be encouraged to default even when the actual fund available with it does not warrant such default. Yafeh and Yosha (1996) provide empirical support for monitoring by creditors.

In our model, each creditor has enough funds to finance all the debt requirements of a firm. Since investors are not fund constrained, multiple lending is an endogenous outcome. Thus, our paper is different from those of Winton (1995) and Zender (1991), where multiple lenders are necessary, by assumption, since creditors have insufficient funds to individually finance the firm.

One strand in the literature explains why firms may have different classes of investors and/or securities. The different classes represent either dispersed versus concentrated debt holders or loans with different term structure or maturities. In these models, the optimal number of creditors is determined by the efficiency of liquidation/continuation decisions (Myers, 1977). In Gertner and Scharfstein (1991) and White (1989), bank loans lead to lower renegotiation costs and are therefore optimal for the firm. However, in reality, beside concentrated debt (bank loans or private debt), a sizeable amount of dispersed debt (public debt) is also observed (Seward, 1990). During bankruptcy, banks alone monitor the firm. Thus, monitoring by the bank prevents the manager from absconding with output in the bankruptcy states. In Detragiache (1994), both public and private debt contracts have roles in the optimal capital structure because of their different renegotiation costs. In Bolton and Scharfstein (1995) and Dewatripont and Maskin (1996), the firm has multiple investors because multiple creditors can extract more cash flows from the firm than a single creditor during bankruptcy. Therefore, *strategic defaults* are less likely with multiple investors as renegotiation is likely to break down.[1]

Dewatripont and Tirole (1994) study the role of multiple creditors when the entrepreneur's effort choice is unverifiable. They conclude, if investors hold different claims in different proportions, then the tendency to liquidate the firm when it defaults, increases. Therefore, multiple investors with different claims will induce the manager to put in more effort.

A common strand in all of this literature is the importance of the behavioral role played by different claimants in on-going projects that face the choice of liquidation or continuation. In our paper, on the other hand, the hierarchy of claims is worked out as an optimal outcome in situations where firms are capable of strategic default and banks have costs of auditing.

The literature on priority rules among the creditors, is based upon the various categories of the creditors. The categories are either concentrated versus dispersed debt or short term versus long term debts. There are two ways of looking at optimal hierarchy. It mitigates the underinvestment and overinvestment problems (the *ex ante* criterion). Otherwise it is determined by socially and privately efficient liquidation decisions (the *ex post* criterion). Rajan (1992), White (1980) and Diamond (1991a) analyse the problem in these contexts. These models crucially depend on the information gathering abilities of the banks vis-a-vis the dispersed holders. For instance, while a large lender (say, the bank), can prevent the managers from continuing with a negative net present value (NPV) project, it may often demand liquidation even when continuing with the project would be efficient. In Rajan

(1992), bank lending reduces both—the managerial effort and agency costs as the bank may withdraw financing in the second period. The optimal capital structure may include both monitored loan as well as arm's-length debt. In this model, bank debt must have lower priority. White (1980) and Diamond (1991a) assert that dispersed debt holders must have a higher priority over the concentrated debt holders.

An alternate formulation of the hierarchy problem concerns the optimal mix of *short term* and *long term* debts. The short term and long term debts differ on their basis of their maturity structures. Berglöf and Thadden (1994), Gertner and Scharfstein (1991), Hart and Moore (1995), Berkovitch and Kim (1990), Diamond (1992) and (1992) derive the optimal priority structure in this framework. Under different set-ups, they find out that long-term debt should be junior to short-term debt. More recently Park (2000) confirm the above findings in a costly state verification framework. Boot (1995) questions the rationale underlying bondholders having seniority in claims. Guedes and Opler (1996) provide a literature survey on various theoretical models that links optimal maturity to liquidity risk, credit quality, debt agency costs and taxes.

In addition to the above, often the priority structure is based on debt covenants that restrict the firm from issuing any future debt that has the same priority or is senior to the existing debts. Smith and Warner (1979) consider a random sample of 87 public issues registered with Securities and Exchange Commissions (SEC) between January '74 and December '75. They found that more than 90% of bonds contained restrictions on the issuing of additional debt with seniority.

As evident from the literature, most works justify the existence of seniority among different kinds of claims. These claims could differ either across their maturities or their holding concentrations. In our analysis, the priority among the creditors differs due to their different auditing, or monitoring costs. Among the various studies that deal with hierarchy, the papers by Winton (1995) and Park (2000) are closer to ours than any other studies, since they both have costly state verification assumptions. However, in both Park and Winton all lenders have identical monitoring costs. In Park, for instance, lenders monitor with different incentives resulting from their differences in claim structure. Prior to the contracting, all lenders are alike and any one can be chosen as a senior lender. The differential monitoring incentives arise after the contracting. In our paper, because we have different monitoring costs, we can actually solve for who will be the senior lender.

We show that, in a firm with multiple investors, the senior most claimant is the investor with the highest auditing cost. Ascending order of seniority corresponds to an ascending order of monitoring costs to the lenders. These results confirm the universally observed seniority pattern (Barclay and Smith, 1995; Harris and Raviv, 1992).

In the Indian context, it is widely observed that during bankruptcy, while some banks are able to recover their dues before others, some do not recover any money (Anant, Gangopadhyay and Goswami, 1992). There is a sizeable literature on the importance of Priority Rules (Franks and Torous, 1989; Harris and Raviv, 1992; Bulow and Shoven, 1978; White, 1980; etc.). The approach here is to show how deviations from pre-contracted priority rules can affect overall efficiency. Our interest, however, is in how, or why, these priorities get determined in the first place. In this regard, our paper is different from those on priority rules during bankruptcy.

| | | | |
|---|---|---|---|
| $P$ chooses: (i) Number of Lenders (ii) Hierarchy | $L_j$ decides: $D, I(D)$ | $P$ raises $K = X - I$ | $z$ is distributed |

Fig. 1.

We also establish *group lending* to be an efficient outcome, as it reduces the expected auditing costs in the economy. We therefore justify the recommendations of the Narasimham Committee (1993)[2] which suggested that banks should lend collectively to a project. Finally, we answer why a system of *syndicated* lending is more efficient than a system of *consortium* lending.

The rest of the paper is organized as follows. In Section 2, we describe the basic model. In Section 3, we derive the main set of results. In particular, the optimal hierarchy of claims is derived. We also demonstrate the situations where borrowing from multiple lenders would be desirable. Section 4 allows the lenders to communicate with each other. Some important implications of our results are discussed in this section. Section 5 concludes the paper.

## 2. The model

The economy consists of two competitive lending institutions or lenders, $L_i$, $i = 1, 2$ and a promoter (entrepreneur), $P$.[3] The promoter has a project with uncertain return, $z$. It requires a fixed investment $X$, part of which the promoter can raise as debt, $D$ and the remainder, if any, as equity. $X$ is common knowledge.

*(A.1:) $z$ has a density function $f(z)$. Its distribution function is $F(z)$, with support $[0,\bar{z}]$, $0<\bar{z}<\infty$ and $F(0)=0$.*

There are four stages. In the first stage, the promoter decides whether to borrow. If the promoter decides not to borrow, then the project will be fully financed through equity. If she decides to borrow, she determines the number of lending institutions to borrow from. If the promoter borrows from multiple lenders, she announces the hierarchy on the basis of which their claims will be met. In the second stage, the lenders announce their debt claims as well as their investment levels (the loan) to the promoter. The total investment provided by the lenders is denoted by $I$. If $I < X$, an amount $K = X - I$ has to be raised through the capital market as equity. The promoter does this in the third stage, with the project being initiated with the announced investment levels. In the fourth stage, the actual realization takes place and $z$ is distributed between the promoter and the lender(s), with debt having senior claim over equity. The complete sequencing is illustrated in Fig. 1.[4]

The lender incurs an auditing cost $\theta$, whenever the promoter defaults on her debt repayments. By incurring $\theta$, the lender can observe the actual realization of the project.[5] In the event that the promoter does not default, the lenders do not audit.

The only uncertainty in our model is about the project realization. The promoter knows the actual realization whereas $L_i$ has to expend resources, $\theta_i$, to learn $z$.

We assume linear costs of raising funds for the promoter, $q$ and the lenders, $r$. In general, $q$ will be higher than the lender's cost of capital, $r$. If, $q \le r_i$, $i = 1, 2$, the promoter will never take a loan.

*(A.2:)* $\theta_1 > \theta_2 > 0$ *and* $0 < r_1 < r_2 < q < X/z^e$, *where* $E(z) = \int_0^{\bar{z}} z\,dF(z) = z^e$.

Lenders specializing in auditing may not be the most efficient fund raisers. Deposit institutions like commercial banks, may raise funds easily, covered as they usually are with deposit insurance. With deposit insurance, they will be less inclined to develop sufficient expertise in monitoring. This could result in higher auditing costs for the commercial banks. Nondepository institutions, on the other hand, with better monitoring expertise will have lower auditing costs. The assumption $q < X/z^e$ ensures that the project is viable, and will be funded by the promoter even if she decides not to borrow. The trade-off between the two costs ensures the functioning of both these kinds of lending institutions in the market.

Thus, given A.2, from now on, whenever we refer to lender $L_1$ we will mean the lender with the higher auditing cost, and a lower cost of raising capital. Similarly, $L_2$ would refer to the lender with a lower auditing cost and a higher capital cost.

The auditing cost is a cost *in addition* to the cost of capital. However, this cost is borne by the bank in the event of a default only. Of course, this affects the bank's payoffs in default states and, hence, the overall profitability of the loan to the bank. This, in turn, affects the bank's investment, given the debt claim. Since the equity market is more costly, lower bank investment hurts the overall profitability of the project. Thus, in principle, the promoter will prefer both capital and auditing costs to be low. In this paper we show that, when banks are differently ranked by their two cost components, and there is credit rationing (banks do not supply all the capital necessary for the project), there exists an optimal hierarchy that is endogenously determined.

We organize the model by dividing it into three parts. In the first part, we deal with a single lender. The next part deals with the case where there is strict hierarchy in claims of lenders. In the third part, we allow for multiple lenders with equal priority in claims.

### *2.1. Lender* j *is the* sole *claimant*

The expected return to $L_j$ is denoted by $R_j$. Therefore,

$$R_j = \int_0^{\delta_j} z\,dF(z) + \int_{\delta_j}^{\bar{z}} \delta_j\,dF(z) - \int_0^{\delta_j} \theta_j\,dF(z)$$

$$= z^e - \int_{\delta_j}^{\bar{z}} (z - \delta_j)\,dF(z) - \theta_j F(\delta_j) \tag{1}$$

where $\delta_j$ is the debt claim of $L_j$ in the project. In the first line of (1), the first term on the right hand side represents the expected return to $L_j$ from the project when the promoter defaults (i.e., $z < \delta_j$). The second term gives the expected return to the lender when her claim of $\delta_j$ is satisfied (i.e., $z \geq \delta_j$). The final term is the expected auditing cost to $L_j$. Recall that a lender audits the firm only when the firm defaults in paying her debt claim and not otherwise.

Earlier, we stated that lending institutions operate in a competitive environment. This essentially means that they make zero profit. If $\eta_j$ is the investment made by $L_j$ in the project, then the zero profit condition is simply $R_j - r_j\eta_j = 0$, $j = 1, 2$. This solves for $\eta_j$ being equal to $R_j/r_j$.

With a single lender, the investment of the promoter will be the difference between the total investment, $X$, and the investment made by the lender. Thus, the expected profit to the promoter is given by,

$$\pi_j = \int_{\delta_j}^{\bar{z}} (z - \delta_j)\, dF(z) - qK. \tag{2}$$

where $K = X - \eta_j$. While the first term in equation (2) represents the residual claim of the promoter on the project, the second term represents the cost of equity.

Let $\delta_j^*$ maximize $R_j$ given in equation (1).[6] Denote $R_j^* \equiv R_j(\delta_j^*)$. A necessary condition for $0 < \delta_j^* < \bar{z}$ is that at $\delta_j^*$

$$\left.\frac{dR_j}{d\delta_j}\right|_{\delta_j=\delta_j^*} = 0 \Rightarrow [1 - F(\delta_j^*)][1 - \theta_j h(\delta_j^*)] = 0, \tag{3}$$

where,

$$h(.) \equiv \frac{f(.)}{[1 - F(.)]}$$

Note that $h(.)$ is the *hazard rate*. To ensure the second order condition for a solution, we assume the following about the hazard rate.

*(A.3:) The hazard rate is increasing, i.e., $h'(.)>0$*

The returns for $L_j$ initially increase with a rise in the debt claim and then decrease. A higher debt claim implies higher returns during the nondefault states. However, it also increases the probability of default. As a result, the expected auditing cost of the lender increases. Therefore, there exists an (interior) optimal debt claim, $\delta_j^*$, such that the net returns to $L_j$ is maximized. Note that, it may well be the case that the optimal lending by the banks may not cover the amount required for investment.[7]

The net surplus is the sum of the returns accruing to the lender and the promoter. Therefore, the net surplus, $s_j$, is given as $s_j = R_j - r_j\eta_j + \pi_j$. The lenders earn zero profits in equilibrium. This implies that any contract that maximizes the promoter's profit automatically maximizes the net surplus. Therefore, any arrangement that is optimal for the promoter is also efficient. Using (1) and (2), we get

$$s_j = z^e - qX - \theta_j F(\delta_j) + (q - r_j)\eta_j$$

To arrive at the net surplus, we have to deduct three sets of costs: the two opportunity costs of investment—one for the lender and the other for the promoter and the expected dead-weight loss of auditing.[8]

## 2.2. *Lender* j *is the* senior *claimant*

Strict hierarchy implies that, the promoter can pay the junior lender only if the senior lender has been paid in full. Henceforth, we will use the following convention: the subscript $ij$ in variable $x_{ij}$ will imply that $i$ is senior. Thus, $R_{ij}$ will be the return of lender $j$ when $i$ is senior. If, however, there is only one subscript, such as in $I_j$, it will denote the aggregate value of bank investment when $j$ is senior. Then, the expected returns to the lenders are given by

$$R_{jj} = \int_0^{D_{jj}} z dF(z) + \int_{D_{jj}}^{\bar{z}} D_{jj}\, dF(z) - \theta_j F(D_{jj}),$$

$$= z^e - \int_{D_{jj}}^{\bar{z}} (z - D_{jj})\, dF(z) - \theta_j F(D_{jj}). \tag{4}$$

$$R_{ji} = \int_{D_{jj}}^{D_j} (z - D_{jj})\, dF(z) + \int_{D_j}^{\bar{z}} D_{ji} dF(z) - \theta_i F(D_j),$$

$$= \int_{D_{jj}}^{\bar{z}} (z - D_{jj})\, dF(z) - \int_{D_j}^{\bar{z}} (z - D_j)\, dF(z) - \theta_i F(D_j). \tag{5}$$

Note that being the senior claimant is equivalent to being the sole lender. Therefore, $R_{jj}$ is similar to $R_j$.

The junior claimant gets paid only after the senior claimant is paid. Therefore, the junior lender does not get paid at all when $z < D_{jj}$. However, for $D_{jj} \leq z < D_j$, the junior lender gets paid $z - D_{jj}$. Thus, the junior lender faces default whenever the realization of $z$ is less than $D_j$. The expected audit cost to the junior lender is, therefore, $\theta_i F(D_j)$.[9] The expected return to $L_i$ when $z \geq D_j$ is $D_{ji}[1 - F(D_j)]$.

As before, we use the zero profit condition for banks to get their levels of investment in the project. The investment by $L_k$, denoted by $I_{jk}$, is given by $I_{jk} = R_{jk}/r_k\ \forall j, k = 1, 2$. The aggregate investment, $I_j$ is given by $I_j = I_{jj} + I_{ji}$.

The expected profit to the promoter is

$$\Pi_j = \int_{D_j}^{\bar{z}} (z - D_j)\, dF(z) - q(X - I_j). \tag{6}$$

The surplus from this set up is given by

$$S_j = z^e - qX - \theta_j F(D_{jj}) - \theta_i F(D_j) + (q - r_j)I_{jj} + (q - r_i)I_{ji}, \qquad i \neq j.$$

### 2.3. Lenders have equal priority in claims

In this case, each unit realization from the project is proportionally distributed among the lenders till their claims are met. It is not possible for any particular lender to have his claim fully satisfied without the claim of the other lender being fully satisfied. This is in contrast to that of strict hierarchy in claims. With equal priority, therefore, both creditors have to audit when $z$ is less than the total claims of the creditors. Here, there is no senior claimant. We will denote this by using 0 in the subscript. Thus, $D_{0j}$ will denote the debt claim of $j$ under equal priority.

Let the share of $L_i$ on the project be denoted as $\alpha_i$. Thus,[10]

$$\alpha_i \equiv \frac{D_{0i}}{D_{01} + D_{02}} = \frac{D_{0i}}{D_0}, \qquad \forall\, i = 1, 2$$

The expected return from the project to $L_i$ is

$$R_{0i} = \int_0^{D_0} \alpha_i z\, dF(z) + \int_{D_0}^{\bar{z}} D_{0i}\, dF(z) - \int_0^{D_0} \theta_i\, dF(z)$$

$$= \alpha_i \left\{ z^e - \int_{D_0}^{\bar{z}} (z - D_0)\, dF(z) \right\} - \theta_i F(D_0) \tag{7}$$

Here, both the lenders have to audit simultaneously in all states where the firm defaults, i.e., $z < D_0$. Similar to the case of strict hierarchy, the aggregate investment in the project by the lenders is given by $I_0$, where $I_0 = I_{0i} = I_{0j} = R_{0i}/r_i + R_{0j}/r_j$.

The expected profit to the promoter, $\Pi_0$, is

$$\Pi_0 = \int_{D_0}^{\bar{z}} (z - D_0)\, dF(z) - q(X - I_0). \tag{8}$$

The total surplus in this case is,

$$S_0 = z^e - qX - \sum_{i=1}^{2} \theta_i F(D_0) + \sum_{i=1}^{2} (q - r_i) I_{0i}.$$

## 3. Results

The promoter has six different options of raising funds. Two of these involve borrowing from only $L_1$ or only $L_2$, respectively. She has another three options, where she borrows from both the lenders—making $L_1$ senior, $L_2$ senior, and giving them equal priority. Finally, the project could be financed entirely through equity. Recall that the superscript '*' denotes the optimal value. Equations (9) and (10) denote the optimal surpluses when $L_1$ and $L_2$ are senior, respectively. Equation (11) denotes the optimal surplus when both the lenders have equal priority in claims. Equations (12) and (13) denote the optimal surpluses when the promoter borrows from $L_1$ and $L_2$ respectively. If we denote the surplus under all equity financing by $s^*$, as in equation (14), we have the following expressions for the different surpluses in the six cases.

$$S_1^* = z^e - qX - \theta_1 F(D_{11}^*) + \theta_2 F(D_1^*) + \sum_{i=1}^{2} (q - r_i) I_{1i}^*. \tag{9}$$

$$S_2^* = z^e - qX - \theta_2 F(D_{22}^*) + \theta_1 F(D_2^*) + \sum_{i=1}^{2} (q - r_i) I_{2i}^*. \tag{10}$$

$$S_0^* = z^e - qX - \sum_{i=1}^{2} \theta_i F(D_0^*) + \sum_{i=1}^{2} (q - r_i) I_{0i}^*. \tag{11}$$

$$s_1^* = z^e - qX - \theta_1 F(\delta_1^*) + (q - r_1)\eta_1^*. \tag{12}$$

$$s_2^* = z^e - qX - \theta_2 F(\delta_2^*) + (q - r_2)\eta_2^*. \tag{13}$$

$$s^* = z^e - qX. \tag{14}$$

The promoter's optimal borrowing decision with the associated hierarchy is simply the surplus with the highest value among all these surpluses. This is sufficient, since we have already argued that the surplus and the promoter's profit are the same in each case.

Before we explore the results involving optimal hierarchy and multiple lending, we enquire as to whether or not the *standard debt contract* (SDC) is the optimal contract in this context. The standard debt contract (Gale and Hellwig, 1985; p. 648) represents " . . . a contract which requires a fixed payment when the firm is solvent, requires the firm to be declared bankrupt if this fixed payment cannot be met and allows the creditor to recoup as much of the debt as possible from the firm's assets."

**Theorem 1.** *The optimal contract is the standard debt contract.*[11]

The intuition underlying the above result is straightforward. As $q > r$, efficiency requires that the entire funding be undertaken by the financial institutions. The question is, how will they collect the returns on their investment. The assumption is that the promoter knows the value of $z$ at no cost, but the institutions have to incur a cost $\theta$ to know $z$. Therefore, if the institutions are shareholders, their payment will be a function of the proportion of the shareholding and, hence, they will have to audit to know how much they should get. This will hold for all values of $z$. The debt contract lowers this audit cost because the institutions incur the audit cost only if there is a default.

Wang and Williamson (1998) derive that the standard debt contract is optimal when screening by the lenders is costly and the borrowers self select. They obtain that the unique (equilibrium) separating contract for good borrowers is a debt contract. Dowd (1992) and Krassa and Villamil (1992) establish the optimality of debt contract when the firm borrows from multiple borrowers.[12]

### 3.1. Optimal hierarchy

We now characterize the optimal hierarchy offered by the promoter to her creditors.

**Proposition 1.** *Let A.1–A.3 hold. If both the lenders invest positive amounts to the project and there is a strict hierarchy of claims, then $L_1$ is the senior claimant.*

The junior claimant always audits in more states than the senior claimant. The senior claimant audits in states where $0 \leq z < D^*_{jj}$, while the junior audits in states where $0 \leq z < D^*_j$. Both banks lend positive amounts with $D^*_{jj} < D^*_j$. Since $L_1$ has the higher auditing cost, making it the senior lender reduces the total expected auditing cost.

**Proposition 2.** *Under A.1–A.3, equal priority in claims is never optimal.*

Propositions 1 and 2 taken together imply that, *in the event of multiple lending, the optimal debt claim structure involves strict hierarchy of claims and the lender with the higher auditing cost is the senior claimant.* This result conforms to the observed pattern of seniority in claims (Harris and Raviv, 1992; Barclay and Smith, 1995). They find that dispersed creditors will be senior to the bank, which in turn will be senior to the promoter. Translated to our model, dispersed creditors are expected to have higher monitoring costs than banks. In India, non-depository investment institutions and the commercial banks often lend long-term capital to the same project. Our model suggests that when both these types of lenders are involved in a project, the depository institutions will be senior to the other type. In the Indian context, Anant, Gangopadhyay and Goswami (1992), empirically corroborate our hypothesis.

**Proposition 3.** *Under A.1 and A.3, if the lenders have the same auditing costs, then the promoter will choose only one bank.*

The proposition implies that, if multiple lenders invest in the same project, they will have different auditing costs. In Winton (1995), multiple lending was obtained with identical auditing and capital costs as the lenders were assumed to be credit constrained. Lenders are not credit constrained in our model and multiple lending will be determined endogenously. We now find out situations where multiple lending is optimal.

### 3.2. Multiple lending

In order to identify the conditions under which multiple lending is optimal, we first derive a feasibility range of the parameters in the model. For the rest of this section, we simplify the algebra by assuming a specific distribution function for $z$. We will indicate the results for more general distributions, wherever necessary.

*(A.4:) z is uniformly distributed between 0 and 1.*

Observe that A.4 satisfies both A.1 and A.3.

Assuming interior solutions for the maximization of the bank's expected return, and using the first order conditions derived from equations (1), (4) and (5), we have

$$\delta_1^* = D_{11}^* = 1 - \theta_1, \qquad \delta_2^* = D_1^* = 1 - \theta_2, \qquad D_{12}^* = \theta_1 - \theta_2, \text{ with } 0 < \theta_2 < \theta_1 < 1.$$

The optimal investment levels by the bank(s) are:

$$\eta_1^* = \frac{(1-\theta_1)^2}{2r_1}, \qquad \eta_2^* = \frac{(1-\theta_2)^2}{2r_2},$$

$$I_{11}^* = \frac{(1-\theta_1)^2}{2r_1} \qquad I_{12}^* = \frac{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}{2r_2}.$$

Therefore,

$$I_1^* = I_{11}^* + I_{12}^* = \frac{(1-\theta_1)^2}{2r_1} + \frac{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}{2r_2}.$$

(Recall from Proposition 1 that the bank with the lower auditing cost is never the senior lender; hence, there are no values for $I_{21}^*$ and $I_{22}^*$.) If $I_{12}^* = 0$, we are in the case where bank 1 is the only lender. On the other hand, observe that, $I_{12}^* > 0 \Rightarrow \theta_1 > [\theta_2(2 - \theta_2)]^{(1/2)}$.

Let $\pi^*$ measure the expected profit to the promoter when the entire project is funded by equity. Given propositions 1–3, we need to consider the relative value of $\Pi_1^*$ with those of $\pi^*$, $\pi_1^*$ and $\pi_2^*$ to check when multiple lending is optimal. Using A.4,

$$\Pi_1^* = 0.5 - qX - \sum_{i=1}^{2} \theta_i(1-\theta_i) + \frac{q - r_1}{r_1}\left\{\frac{(1-\theta_1)^2}{2}\right\}$$

$$+ \frac{q - r_2}{r_2}\left\{\frac{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}{2}\right\},$$

$$\pi_1^* = 0.5 - qX - \theta_1(1-\theta_1) + \frac{q - r_1}{r_1}\left\{\frac{(1-\theta_1)^2}{2}\right\},$$

$$\pi_2^* = 0.5 - qX - \theta_2(1-\theta_2) + \frac{q - r_2}{r_2}\left\{\frac{(1-\theta_2)^2}{2}\right\}$$

$$\pi^* = 0.5 = qX.$$

**Proposition 4.** *Let A.2 and A.4 hold. The promoter borrows from lender i only if*

$$r_i \le \bar{r}_i \equiv \frac{q(1-\theta)}{1+\theta_i} \qquad \forall\, i = 1, 2.$$

Proposition 4 gives us a necessary condition. If $r_i \le \bar{r}_i$, then it is profitable for the promoter to at least borrow from $L_i$ rather than finance the project entirely through equity. However, even if $r_i \le \bar{r}_i$, it is possible that the promoter does not borrow from $L_i$. This could be the case if borrowing from $L_j$ alone is more profitable.

**Proposition 5.** *Let $r_i \le \bar{r}_i$. Under assumptions A.2 and A.4, multiple lending is optimal if and only if*

$$q\left\{1 - \frac{2\theta_2(1-\theta_2)}{\theta_1^2 - \theta_2^2}\right\} \ge r_2 \ge r_1\left\{\frac{1+\theta_1}{1-\theta_1}\right\}.$$

Observe that, given risk neutrality the returns to each agent can be added up to get the total surplus. Thus, each dollar going to the bank implies a dollar being lost by the promoter. The promoter has to give enough to the bank to cover its capital cost, plus the auditing cost in the event of default. Consider the senior lender, bank 1. The audit cost per unit of return is $[\theta_1 F(D^*_{11})]/(R^*_1)$. The effective cost to the bank, per unit of loan, is, therefore,

$$r_1\left[1 + \frac{\theta_1 F(D^*_{11})}{R^*_1}\right]$$

With banks making zero profits, the payment to the bank must exactly cover this cost. If this payment is less than the equity cost, $q$, then the bank is an attractive investor to the promoter. Similarly, the effective cost per unit of return to bank 2 is

$$r_2\left[1 + \frac{\theta_2 F(D^*_1)}{R^*_{12}}\right]$$

Putting in the explicit solutions for the uniform case, worked out above, we get the respective effective costs for banks 1 and 2 as

$$r_1\frac{1+\theta_1}{1-\theta_1}, \qquad r_2\frac{\theta_1^2 - \theta_2^2}{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}$$

We can rewrite the condition in Proposition 5 in two parts:

$$q \ge \max\left\{r_1\frac{1+\theta_1}{1-\theta_1}, \qquad r_2\frac{\theta_1^2 - \theta_2^2}{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}\right\}$$

$$r_1\frac{1+\theta_1}{1-\theta_1} \le r_2\frac{\theta_1^2 - \theta_2^2}{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}$$

The first part guarantees that lending by both banks is feasible; the second generates the required hierarchy.

Given below are a set of numerical values for the parameters to show that the condition in proposition 5 is satisfied.

**Example 1:** Let $\theta_1 = 0.5$, $\theta_2 = 0.125$, $r_2 = \{\gamma q\}/15$ and $r_1 = \{\phi r_2\}/3$, where, $\gamma, \phi \in (0, 1]$. Further, let $q < 0.5/X$. The above parametric configurations satisfy A.4. Also, $\theta_1$, $\theta_2$, $r_1$ and $r_2$ satisfy the conditions in proposition 5. Note

$$\Pi_1^* - \pi_1^* = \frac{(1-\gamma)}{128\gamma} \geq 0 \qquad \forall\ \gamma \in (0, 1],$$

$$\Pi_1^* - \pi_2^* = \frac{45(1-\phi)}{2\gamma\phi} \geq 0 \qquad \forall\ \phi \in (0, 1].$$

The above parametric values imply that $\Pi_1^* \geq Max\ \{\pi_1^*, \pi_2^*\}$. Therefore, the promoter borrows from both the lenders making $L_1$ senior. ■

For a more general distribution function, we cannot get explicit values for $\bar{r}_i$. However, for completeness, we state the following result:

**Proposition 6.** *Let A.1–A.3 hold.*

*(a) The promoter borrows from lender i only if*

$$r_i \leq \bar{r}_i \equiv \frac{qR_i^*}{R_i^* + \theta_i F(\delta_i^*)} \qquad \forall\, i = 1, 2.$$

*(b) If the condition in (a) is satisfied, then multiple lending is optimal if and only if*

$$q\,\frac{R_{12}^*}{R_{12}^* + \theta_2 F(D_1^*)} \geq r_2 \geq r_1\,\frac{R_1^* + \theta_1 F(\delta_1^*)}{R_1^*}.$$

## 4. Communicating banks

So far, we have implicitly assumed that the information obtained from auditing is available to the auditing lender alone. Now suppose that the information obtained by the auditor is also observable by the other lender. Thus, as before, the senior lender audits whenever its claim is not met. The junior lender observes this, and *learns* that the realization from the project is not sufficient to meet the senior claimant's debt obligation. Hence, the junior lender does not expect to get paid and, most importantly, does not audit in these states. This allows it to save on auditing costs.

Observe that, in a hierarchy, once default has occurred, there is no incentive for the senior lender to hide its audit information from the junior creditor. With equal priority in claims, there is an incentive for the auditing lender to hide the information regarding the actual realization of the state. This is because, with equal priority, each lender gets a fraction of the realized value in default states. If one lender does not know the true realization, the other can get a higher value.

In this section, variables with superscript '$C$' indicate communication among lenders. The expected revenue of the senior claimant is the same as in the previous sections, even with communication. However, the junior claimant gains from communication. The junior lender will now have to audit in only those states where $D^C_{11} \leq z < D^C_1$. Therefore, the expected returns to $L_1$ and $L_2$ will be:

$$R^C_{11} = z^e - \int_{D^C_{11}}^{\bar{z}} (z - D^C_{11})\, dF(z) - \theta_1 F(D^C_{11}).$$

$$R^C_{12} = \int_{D^C_{11}}^{\bar{z}} (z - D^C_{11})\, dF(z) - \int_{D^C_1}^{\bar{z}} (z - D^C_1)\, dF(z) - \theta_2[F(D^C_1) - F(D^C_{11})].$$

With communication, $R^C_{11}$ will remain the same, i.e., $R^C_{11} = R_{11}$. Proposition 1 suggests that in the event of strict hierarchy, the lender with the higher auditing cost will be the senior claimant. Communication among the lenders will not affect this result. This can be seen as follows. Consider any lender. The lender increases its debt claims from zero that level where the marginal revenue from an additional unit of debt equals the marginal cost of auditing the additional default state. Note that, the marginal revenue for the junior claimant will always be lower than the senior claimant. In addition, if the junior claimant has higher auditing costs, then its marginal cost will also be higher than the senior claimant. Now, suppose, $L_2$ is the senior lender. Denote the optimal debt claim by $L_2$ as $D^C_{22}$. Note that for $L_2$, any additional debt claim over $D^C_{22}$ would imply that the marginal revenue is lower than the marginal costs of doing so. For $L_1$ who is junior, any positive debt claims would mean that it would get paid only if returns exceed $D^C_{22}$. Therefore, the marginal revenues to $L_1$ will be lower than the marginal cost for any positive debt claims it has. This implies that if $L_1$ is junior, it will not have any positive debt claims and hence will not invest at all. Note that this reasoning is independent of the communicating abilities of the banks. Therefore, if the promoter borrows from both the lenders, the optimal arrangement is to make $L_1$ senior. The expected return to the junior lender $L_2$, will be $R^C_{12} = R_{12} + \theta_2 F(D_{11})$. This implies that $I^C_{12} > I_{12}$ and the expected auditing cost decreases with communication. The following results are readily obtained.

**Proposition 7.** *Let A.1–A.3 hold. With multiple lenders, the expected profit to the promoter is more when the lending institutions communicate with each other.*

**Proposition 8.** *Under A.2, A.4 and communicating banks, multiple lending is optimal if and only if*

$$q\left\{1 - \frac{2\theta_2(\theta_1 - \theta_2)}{\theta_1^2 - \theta_2^2}\right\} \geq r_2 \geq r_1\left\{\frac{1 + \theta_1}{1 - \theta_1}\right\}.$$

It is interesting to note that, the upper bound on $r_2$ is greater in proposition 8 than in proposition 5. This implies that the probability of obtaining multiple lending increases with communication. Since communication reduces the (total) expected auditing costs, the aggregate net surplus is increased. The promoter, therefore, finds it optimal to borrow from multiple lenders for a wider range of parametric values. Alternatively, the overall efficiency improves with communication since the duplication of monitoring effort is avoided. Two facts emerge particularly interesting in this context—the 'disclosure rules' of the firm and *group lending*.

An implication of our result pertains to group lending—and within it, the fact that syndicated arrangements are increasingly replacing the existing consortium arrangements. Ravishanker (1998) and Megginson et al. (1985) study the emergence of the syndicated loan system in India and in the international context, respectively. Under group lending, the banks can lend either as a *consortium* or, as a *syndicate*. In a consortium loan, after independently assessing a firm's project and its credentials, a group of banks lend collectively to it. Under the syndicated loan system, a lead bank determines the exposure level of bank finance for a particular borrower. Here, it is usually the lead bank that appraises and monitors the project, unlike the consortium lending case, where all banks individually appraise and monitor the borrower. In the lead bank system, coordination among the banks in a group ensures that duplication of verification costs does not take place. More importantly, the more efficient auditor does the auditing. The importance of lead bank monitoring and its effect on project financing has been more recently dealt with in Hansen and Terregrosa (1992) and Jain and Kini (1999).

In our paper, syndicated lending allows information flows from the lead bank to other lenders in the syndicate. This reduces the total expected auditing cost as all banks do not need to audit. Any auditing cost is a deadweight loss to the system. Since banks make zero profits, anything that increases the net surplus (for instance, through lower total auditing costs), improves the return to the promoter. Hence the promoter prefers syndicated lending to one where banks lend as independent entities.

## 5. Conclusion

We consider lenders auditing the promoter in the default state to prevent *strategic default*. The lenders choose the debt claims so as to reduce the occurrence of default states. This reduces the expected auditing costs. This formulation is similar to Moore (1993). The optimal hierarchy depends solely upon the differential auditing costs of the lenders. The optimal number of lenders chosen by the promoter depends upon the following tradeoff—between high debt claims and high bank investment, on the one hand and low debt claims and low bank investment, on the other.

The two crucial features distinguishing our model from that of Winton (1995) are (a) differential auditing costs across lenders and (b) the calculation of optimal loan supply by the lenders. The differences in auditing costs form the basis for a hierarchical claim structure. The optimal loan supply by the lenders, along with a difference in auditing costs, determines the optimal number of lenders.

Our results are summarized as follows. For multiple lending to occur, it is necessary that the lenders have different auditing and capital costs. Our model predicts that the lender with the higher auditing cost is the senior claimant. Taken to its logical extreme, this result also explains why firms are the residual claimants. Our results do not support the case for an equal priority in claims.

We also establish that group lending is an efficient outcome, and that a system of *syndicated* lending is more efficient than a system of *consortium* lending.

## 6. Appendix

The following lemmas will be used during the course of proving the results.

**Lemma 1.** *The optimal debt claim to $L_1$ when it is the senior claimant is equal to the debt claim if $L_1$ were lending alone. Further, the aggregate debt claim with $L_1$ as the senior claimant is equal to the debt claim when $L_2$ alone lends to the project. I.e., $D^*_{11} = \delta^*_1$ and $D^*_1 = \delta^*_2$.*

**Proof:** From equations (1) and (4), we obtain that,

$$R_{11} = z^e - \int_{D_{11}}^{\bar{z}} (z - D_{11})\, dF(z) - \theta_1 F(D_{11})$$

$$R_1 = z^e - \int_{\delta_1}^{\bar{z}} (z - \delta_1)\, dF(z) - \theta_1 F(\delta_1).$$

Therefore, the conditions for the interior solutions for $D^*_{11}$ and $\delta^*_1$ are:

$$\theta_1 h(D^*_{11}) = 1 = \theta_1 h(\delta^*_1) \Rightarrow \delta^*_1 = D^*_{11}.$$

Further, from equation (5), we obtain

$$R_{12} = \int_{D_{12}}^{D_1} (z - D_{11})\, dF(z) + \int_{D_1}^{\bar{z}} D_{12}\, dF(z) - \theta_2 F(D_1).$$

The condition for the interior solution of $D^*_1$ is $\theta_2 h(D^*_1) = 1$. Also, $\theta_2 h(\delta^*_2) = 1$. Thus, $\delta^*_2 = D^*_1$. ■

**Lemma 2.** *The optimal debt claim with $L_2$ as the sole claimant is more than the optimal debt claim when $L_1$ is the sole claimant, i.e., $\delta^*_2 > \delta^*_1$.*

**Proof:** From Lemma 1 we have $h(\delta^*_1) = 1/\theta_1$ and $h(\delta^*_2) = 1/\theta_2$. As $\theta_1 > \theta_2$ (from A.2) and $h'(.) > 0$ (from A.3), we have $\delta^*_2 > \delta^*_1$. ■

**Lemma 3**

$$F(D^*_0) \geq \frac{1}{D^*_0} \int_0^{D^*_0} F(z)\, dz$$

**Proof:**

$$F(D_0^*) \equiv \frac{1}{D_0^*} F(D_0^*) D_0^* = \frac{F(D_0^*)}{D_0^*} \int_0^{D_0^*} dz \geq \frac{1}{D_0^*} \int_0^{D_0^*} F(z)\, dz$$

as $F(.)$ is a non-decreasing function. ■

**Lemma 4.** *The aggregate debt claim with equal priority in claims is more than the optimal debt claim with any lender lending alone. I.e., $D_0^* \geq \delta_i^*$, $i = 1, 2$.*

**Proof:** The condition for an interior solution for $D_{0i}^*$ is

$$[1 - \theta_j f(D_0^*) - F(D_0^*)] + \alpha_j \left[ F(D_0^*) - \frac{1}{D_0^*} \int_0^{D_0^*} F(z)\, dz \right] = 0.$$

Lemma 3 implies $[1 - \theta_j f(D_0^*) - F(D_0^*)] \leq 0$. Combining this with equation (3) and A.3, the result follows. ■

**Corollary 1.** *Denote $R_i \equiv R_i(\theta_i, \delta_i)$, where*

$$R_i = z^e - \int_{\delta_i}^{\bar{z}} (z - \delta_i)\, dF(z) - \theta_i F(\delta_i).$$

*Therefore, $R_1^* \equiv R_i(\theta_i, \delta_i^*) = R_{11}^*$ and $R_2^* \equiv R^*(\theta_2, \delta_2^*) = R_{12}^* + R_1^* + \theta_1 F(D_{11}^*)$.*

**Proof:** The result is obtained by applying Lemma 1 in equations (1), (4) and (5). ■

- **Proof of Theorem 1.**

The proof will be organized as follows. Denote $\{\Delta\}$ as the SDC and $\{\Phi\}$ as an alternative contract offered by the promoter to the lender(s). The result will be established by showing that for any contract $\{\Phi\}$, $\Pi(\{\Delta\}) \geq \Pi(\{\Phi\})$, where $\Pi$ is the expected profit to the promoter. Note that the optimal contract—the one which maximizes the net surplus—is also the one which maximizes the promoter's profit.

We prove the result by considering the two possible cases—(i) the promoter borrows from $L_j$ alone and (ii) the promoter borrows from both $L_i$ and $L_j$.

**Case I:** The promoter signs with $L_j$ alone.

Describe the alternate contract $\{\Phi'\}$ as a combination of debt and equity. Given $\delta_j$, let $\beta' \in [0, 1]$ be the the equity share of $L_j$. Therefore, $\forall z \geq \delta_j$, $\beta'(z - \delta_j)$ is the return from equity to $L_j$, while $(1 - \beta')(z - \delta_j)$ is the promoter's return.

From (1) we have

$$R_j(\{\Phi'\}) = \int_0^{\delta_j} z\, dF(z) + \int_{\delta_j}^{\bar{z}} \delta_j\, dF(z) + \int_{\delta_j}^{\bar{z}} \beta'(z - \delta_j)\, dF(z) - \int_0^{\bar{z}} \theta_j\, dF(z)$$

$$= \int_0^{\delta_j} z\, dF(z) + \int_{\delta_j}^{\bar{z}} \{\beta' z + (1 - \beta')\delta_j\}\, dF(z) - \theta_j.$$

Note that if $\beta' > 0$, then $L_j$ has to incur $\theta_j$ in all the states. The lender does not know the true realization of $z$ without auditing. However, with only debt, once the lender's claim is paid, he does not audit. With equity, it has to audit even when $z \geq \delta_j$. Therefore, the optimal debt claim set by $L_j$, denoted by $\delta_j^*$ is given by:

$$\frac{\partial R_j(\{\Phi'\})}{\partial \delta_j^*} = (1 - \beta')[1 - F(\delta_j^*)] = 0,$$

implying in equilibrium, either $\beta' = 1$ or $\delta_j^* = \bar{z}$. Note that with either $\beta' = 1$ or $\delta_j^* = \bar{z}$,

$$R_j(\{\Phi'\}) = \int_0^{\delta_j} z dF(z) + \int_{\delta_j}^{\bar{z}} \{\beta' z + (1 - \beta')\delta_j\}\, dF(z) - \theta_j = z^e - \theta_j.$$

With $\beta' = 1$, we have

$$\Pi(\{\Phi'\}) = -qX + \frac{q}{r_j}\{z^e - \theta_j\}.$$

With the SDC, $\{\Delta\}$, we have

$$R_j(\{\Delta\}) = R_j^*$$

$$\Pi(\{\Delta\}) = \int_{\delta_j^*}^{\bar{z}} (z - \delta_j^*)\, dF(z) + \frac{q}{r_j}\{R_j^*\}.$$

Therefore,

$$\Pi(\{\Delta\}) = \Pi^{max}(\{\Phi^*\})$$

$$= \int_{\delta_j^*}^{\bar{z}} (z - \delta_j^*)\, dF(z) + \frac{q}{r_j}\{R_j^* - z^e + \theta_j\}$$

$$> \frac{q}{r_j}\{R_j^* - z^e + \theta_j\} > 0.$$

The last inequality follows from the fact that, $R_j^* \equiv R_j(\delta_j^*) > R_j(\delta_j = \bar{z}) = z^e - \theta_j$, and $\delta_j^* = arg\ \max_{\delta_j} = R_j(\delta_j)$.

**Case II:** The promoter signs with both the lenders.

In this case, the alternative contracts, could be either $\{\Phi_A\}$, such that $L_j$ is offered SDC while $L_i$ is offered equity for $i \neq j$ or $\{\Phi_B\}$, such that both $L_i$ and $L_j$ are offered equity.

The lender that is offered the equity contract, will have to audit in all the states. This is similar to Case I. As established earlier, the promoter will earn greater profit by offering that lender $\{\Delta\}$.

Thus, the SDC, $\{\Delta\}$, is the optimal contract. ■

- **Proof of Proposition 1.**

It will be sufficient for the proof to show that $L_1$ cannot be the junior claimant. This will leave us with the only other possible option, that $L_1$ is the senior claimant. Suppose, $L_1$ is the junior claimant, since, $D_{22}^*$ maximizes $R_{22}$ and from $h'(.) > 0$, we have

$$1 - \theta_2 f(D_{22}^*) - F(D_{22}^*) = 0$$

$$\{1 - F(D_{22}^*)\}[1 - \theta_2 h(D_{22}^*)] = 0$$

$$\{1 - F(D_{22}^*)\}[1 - \theta_1 h(D_{22}^*)] < 0 \qquad \theta_1 > \theta_2$$

$$1 - \theta_1 f(z) - F(z) < 0, \qquad \forall z \geq D_{22}^*.$$

$$\int_{D_{22}^*}^{D_2^*} [1 - \theta_1 f(z) - F(z)]\, dz < 0 \qquad \text{since } D_2^* > D_{22}^*.$$

From (5), we have $\forall D_{21}^* > 0$,

$$\begin{aligned}
R_{21}^* &= \int_{D_{22}^*}^{D_2^*} (z - D_{22}^*)\, dF(z) + \int_{D_2^*}^{\bar{z}} D_{21}^*\, dF(z) - \theta_1 F(D_2^*), \\
&< \int_{D_{22}^*}^{D_2^*} (z - D_{22}^*)\, dF(z) + \int_{D_2^*}^{\bar{z}} D_{21}^*\, dF(z) - \theta_1\{F(D_2^*) - F(D_{22}^*)\}, \\
&= \int_{D_{22}^*}^{D_2^*} [1 - \theta_1 f(z) - F(z)]\, dz \\
&< 0.
\end{aligned}$$

Thus $L_1$ gets negative returns when it is junior. The zero profit condition ensures that $L_1$ will not lend any positive amount to the project. This contradicts the fact that both the lenders are lending positive amounts. ■

- **Proof of Proposition 2.**

From equations (6) and (8), we obtain

$$\Pi_1^* - \Pi_0^* = (q - r_1)(I_{11}^* - I_{01}^*) + (q - r_2)(I_{12}^* - I_{02}^*)$$

$$+ \theta_1[F(D_0^*) - F(D_{11}^*)] + \theta_2[F(D_0^*) - F(D_1^*)]$$

$$= (q - r_1)(I_{11}^* - I_{01}^*) + (q - r_2)(I_{12}^* - I_{02}^*)$$

$$+ \theta_1[F(D_0^*) - F(\delta_1^*)] + \theta_2[F(D_0^*) - F(\delta_2^*)]$$

$$\geq (q - r_1)(I_{11}^* - I_{01}^*) + (q - r_2)(I_{12}^* - I_{02}^*).$$

The inequality follows from Lemma 4. Therefore,

$$\Pi_1^* - \Pi_0^* \geq (q - r_1)(I_{11}^* - I_{01}^*) + (q - r_2)(I_{12}^* - I_{02}^*)$$

$$\geq \frac{q - r_2}{r_2}\{R_{11}^* - R_{01}^* + R_{12}^* - R_{02}^*\},$$

$$= \frac{q - r_2}{r_2}\left\{R_2^* - \left[\int_0^{D_0^*} z dF(z) + \int_{D_0^*}^{\bar{z}} D_0^* \, dF(z) - \theta_2 F(D_0^*)\right]\right\}$$

$$+ \frac{q - r_2}{r_2}\theta_1[F(D_0^*) - F(D_1^*)],$$

$$\geq \frac{q - r_2}{r_2}\{R_2^* - R(\theta_2, D_0^*)\}$$

$$\geq 0.$$

The second inequality follows from $r_1 > r_2$. The third inequality is obtained using $D_0^* \geq D_1^*$. The last inequality follows from the definition of $R_2^*$. ■

- **Proof of Proposition 3.**

From the Proof of proposition 1, it is clear that, if the lenders have the same auditing costs, i.e., if $\theta_1 = \theta_2$, then maintaining strict hierarchy is not possible. The only case that remains to be examined is the equal priority case. The lenders $L_i$ and $L_j$, differ only in their capital costs. Therefore, it is sufficient for the proof to show that with $\theta_1 = \theta_2 = \theta$, $\pi_1(\theta) \geq \Pi_0(\theta)$. With $\theta_1 = \theta_2 = \theta$, we have,

$$\pi_1^*(\theta) = \Pi_0^*(\theta) = \theta[2F(D_0^*) - F(D^*)]$$

$$+ \frac{q - r_1}{r_1}(R^* - R_{01}^*) - \frac{q - r_2}{r_2}R_{02}^*$$

$$\geq \frac{q - r_2}{r_2}\left\{R^* - \left[\int_0^{D_0^*} z dF(z) + \int_{D_0^*}^{\bar{z}} D_0^* \, dF(z) - 2\theta F(D_0^*)\right]\right\}$$

$$\geq \frac{q-r_2}{r_2}\left\{R^* - \left[\left[\int_0^{D_0^*} z\,dF(z) + \int_{D_0^*}^{\bar{z}} D_0^*\,dF(z) - \theta F(D_0^*)\right]\right]\right\}$$

$$= \frac{q-r_2}{r_2}\{R^* - R(\theta, D_0^*)\}$$

$$\geq 0,$$

where, $D^*$ is obtained from the fact that, $\theta h(D^*) = 1$ and $R^* = R(\theta, D^*)$. Note that, $\theta_1 = \theta_2 \Rightarrow R^*_{01} = R^*_{02}$. The first inequality is obtained from A.2, while the last inequality follows from the definition of $R^*$. ■

- **Proof of Proposition 4.**

From the set of equations in (15) we get,

$$\pi_i^* = 0.5 - qX - \theta_i(1-\theta_i) + \frac{q-r_i}{r_i}\left\{\frac{(1-\theta_i)^2}{2}\right\},$$

$$\pi^* = 0.5 - qX.$$

Therefore,

$$\pi_i^* \geq \pi^* \Rightarrow r_i \leq \bar{r}_i \equiv \frac{q(1-\theta_i)}{1+\theta_i}.$$

■

- **Proof of Proposition 5.**

From the set of equations given in (15), we obtain

$$\Pi_1^* - \pi_1^* = (q-r_2)\,\frac{\theta_1^2 + \theta_2^2 - 2\theta_2}{r_2} - \theta_2(1-\theta_2)$$

$$= \frac{q}{2r_2}\{\theta_1^2 + \theta_2^2 - 2\theta_2\} - \frac{\theta_1^2 - \theta_2^2}{2}$$

$$\Pi_1^* - \pi_2^* = (q-r_1)\,\frac{(1-\theta_1)^2}{2r_1} + (q-r_2)(I_{12}^* - \eta_2^*) - \theta_1(1-\theta_1)$$

$$= q(1-\theta_1)\left\{\frac{1-\theta_1}{2r_1} - \frac{1+\theta_1}{2r_2}\right\}.$$

Therefore,

$$\Pi_1^* \geq \pi_1^* \Leftrightarrow q\left\{1 - \frac{2\theta_2(1-\theta_2)}{\theta_1^2 - \theta_2^2}\right\} \geq r_2, \text{ and } \Pi_1^* \geq \pi_2^* \Leftrightarrow r_2 \geq r_1\left\{\frac{1+\theta_1}{1-\theta_1}\right\}.$$

Further,

$$q\left\{1 - \frac{2\theta_2(1-\theta_2)}{\theta_1^2 - \theta_2^2}\right\} \geq r_2 > 0 \Rightarrow R_{12} > 0.$$

■

- **Proof of Proposition 6.**

The proof is similar to that of proposition 5. The conditions in (a) and (b) of the proposition is readily obtained by using the following inequalities:

$$\pi_j^* \geq \pi^*; \qquad \forall j = 1, 2 \text{ and } \Pi_1^* \geq \max\{\pi_1^*, \pi_2^*\}.$$

The final expressions in (a) and (b) are obtained by using the fact that the net surplus in the system is the same as the expected profits to the promoter. Therefore, the values of $\Pi_1^*$, $\pi_1^*$ and $\pi_2^*$ are obtained from equations (9), (12) and (13) respectively. Finally, note that, with general distribution forms, $\pi^* = z^e - qX$. ■

- **Proof of proposition 8 is identical to that of proposition 5.**

## Notes

1. See also Hart and Moore (1994, 1995), Berglöf and Thadden (1994), Diamond (1991b, 1992).
2. The Narasimhan Committee was a high-powered committee set up by the Indian government to recommend reforms in the Indian financial sector. One of its recommendations was to move towards syndicated lending by banks.
3. We restrict our model to only two lenders, for the sake of simplicity. The model can easily be extended to include more lenders.
4. Park (2000) has a model where our first and second stages are collapsed into one, and the promoter decides on the amount of the loan from the bank(s). As we will show, the debt claim of each bank is a function of the actual value of the monittoring cost. For the firm to decide on the bank debt claim, it will have to know this exact value. For deciding the seniority of the creditors' claims, the promoter need to know only the relative monitoring costs of the banks, not their exact values. With full information on the bank's costs, the two approaches will give identical results.
5. *Ex post* it may not always be in the interest of the lenders to audit the promoter when she defaults. However, without any such commitment on the part of the lenders, the promoter may not have any incentive to report the true realization. Hence, she may want to default even when the project realization does not warrant such default. In other words, such a commitment rules out *strategic defaults* referred to in Bolton and Scharfstein (1995) and Hart and Moore (1994). We are implicitly assuming that the promoter knows that she will be 'audited' with probability one, if she defaults. Consequently, she will be caught and then be required to pay heavy fines. Alternatively, this audit probability may be less than one (Mookherjee and Png, 1989) and yet deter the promoter from lying provided the fines are sufficiently high. This does not affect the nature of our results.
6. Variables with asterisks denote optimal values.

7. This is a form of credit rationing (Moore, 1993; Stiglitz and Weiss, 1983).
8. Since, by assumption, $q > r_j$, the first best surplus is $z^3 - min(r_1, r_2)X$. However, because of the auditing costs, we are in the second best.
9. Being junior, he has to audit in more states than the senior claimant, as $D_j = D_{jj} + D_{ji}$.
10. The sharing rule, $\alpha_i$, is obtained once the lenders decide upon $D_{0i}$, $i = 1, 2$.
11. The proofs of all the results are in the appendix.
12. A possible scenario involving the non-optimality of debt contracts is the case where auditing is stochastic, e.g., Mookherjee and Png (1989). In this setup, they proceed to show that 'equity like' contracts are optimal. However, their model addresses different economic problems.

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