Self-interaction effects on screening in three-dimensional QED

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Abstract:
We have shown that self interaction effects in massive quantum electrodynamics can lead to the formation of bound states of quark antiquark pairs. A current-current fermion coupling term is introduced, which induces a well in the potential energy profile. Explicit expressions of the effective potential and renormalized parameters are provided.

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In a recent Letter [1] Abdalla and Banerjee have discussed the confinement and screening problems in three dimensional QED. They have studied the inter-"quark" potential between two static test charges in a theory of dynamical fermions of mass m coupled to electromagnetism. Their results indicate that for small separation, the quantum potential tends to the classical logarithmic Coulomb potential. However, for large distance the potential tends to zero. This shows quite conclusively the confining and screening nature of the potential. These results also corroborate the two dimensional results [2] nicely.

The present Letter is aimed at studying the stability of the above scenario, in three dimensions, in the presence of self-interaction among the fermions. In particular, we have chosen the well studied current current Thirring interaction. When the Thirring coupling g is *positive*, the new model shows a marked departure of a qualitative nature from [1] in the short distance regime. In the potential profile, there appears more structure, in the form of a *well*, indicating a strong repulsion below some critical distance. This might lead to stable bound states of the quark antiquark pair. The large distance behaviour shows the expected screening. For negative g, nothing of the above dramatic nature occures, albeit the potential decreases more sharply for short distance.

We formulate the problem along the lines of [1]. The fermion modes in the gauged Thirring model are integrated out to incorporate quantum (fermion loop) effects in the subsequent classical analysis. This bosonization is done in the large m approximation. The auxiliary field B_μ, introduced to linearize the Thirring term, is next integrated, resulting in a generalized Maxwell Chern Simons gauge theory [3] [4]. The theory now contains two independent parameters, m and g. In order to gain further insight, we expand the results in powers of g and keep terms up to $O(1/m,\ g,\ g^2,\ g/m)$. Surprisingly, the terms linear in g does not alter the results very much whereas effect of the higher order corrections is substantial, as mentioned earlier. This is our main result.

The ideas of screening and confinement play a central role in gauge theory. The computational hurdles in four dimensions compel us to study the lower dimensional models. But one has to extract results which are not artefacts of low dimensionality and can be carried on to the real world. Previously, in two dimensional QED, [2] obtained results indicating screening and confinement for massless and massive fermions respectively. QCD was studied by [5], where apart from the dynamical fermion mass, the representations of the dynamical fermions and test charges became important. The problems regarding θ-vacuum, screening, confinement and chiral condensate in two and three dimensional QCD were discussed in [6].

The parent model is

$$L_F = \bar{\psi} i\gamma^\mu(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}\psi + \frac{g}{2} \mid \bar{\psi}\gamma^\mu\psi \mid^2 - \frac{pe^2}{4} \mid A_{\mu\nu} \mid^2 + \frac{qe^2}{2}\epsilon_{\mu\nu\lambda}A^\mu A^{\nu\lambda} + J_\mu A^\mu. \quad (1)$$

Here $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, J_μ is an external conserved current and conventionally one takes $p = 1/e^2$, $q = \mu/(2e^2)$. We have considered them arbitray to keep track of them. The above model is rewritten with the auxiliary field B_μ as

$$L_F = \bar{\psi} i\gamma^\mu(\partial_\mu - ieA_\mu - iB_\mu)\psi - \frac{1}{2g} \mid B_\mu \mid^2 - m\bar{\psi}\psi - \frac{pe^2}{4} \mid A_{\mu\nu} \mid^2 + \frac{qe^2}{2}\epsilon_{\mu\nu\lambda}A^\mu A^{\nu\lambda} + J_\mu A^\mu. \quad (2)$$

The bosonized lagrangian to $O(1/m)$ is

$$L_B = -\frac{a}{4} \mid B_{\mu\nu} \mid^2 + \frac{\alpha}{2}\epsilon_{\mu\nu\lambda}B^\mu B^{\nu\lambda} - \frac{1}{2g} \mid B_\mu \mid^2 - \frac{(a+p)}{4}e^2 \mid A_{\mu\nu} \mid^2$$

$$+\frac{(\alpha+q)}{2}e^2\epsilon_{\mu\nu\lambda}A^\mu A^{\nu\lambda} - \frac{ae}{2}A_{\mu\nu}B^{\mu\nu} + e\alpha\epsilon_{\mu\nu\lambda}B^\mu A^{\nu\lambda} + J_\mu A^\mu, \quad (3)$$

where $\alpha = 1/(8\pi)$ and $a = -1/(6\pi m)$. The above Lagrangian is quadratic in B_μ and after a formal integration of it, we get the gauge invariant effective action,

$$Z(A_\mu) = \int \mathcal{D}\delta(\partial_\mu A^\mu) exp(-i/4)[2e^2 A_\mu \frac{4\alpha^2 g - a + a^2 g\partial^2}{(ag\partial^2-1)^2 + 4\alpha^2 g^2\partial^2}(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu$$

$$+ e^2 A_\mu \frac{\{-4\alpha + 8g(1-\alpha)\partial^2(2\alpha^2 g - a + a^2 g\partial^2)\}}{(ag\partial^2-1)^2 + 4\alpha^2 g^2\partial^2}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda - 4J_\mu A^\mu]. \quad (4)$$

Lorentz gauge is adopted in defining (3). This is a generalized Maxwell Chern Simons type of theory [7, 4, 3]. For $g = q = 0$, this action reduces to the one in [1]. From the A_μ equation of motion, it follows that $\partial_\mu A^\mu = 0$. Let us from now on work with the truncated version of this model keeping only terms of $O(a,\ g,\ g^2,\ ag\ )$. The A_μ-equation in Lorentz gauge is

$$P\epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda + Q\partial^2 A_\mu + J_\mu = 0, \quad (5)$$

$$P = 2e^2[(\alpha+q) - 8(2\alpha^2 g^2 - ag)(1+q)\partial^2];\quad Q = -e^2[(4\alpha^2 g - a - p) + 2p(2\alpha^2 g^2 - ag)\partial^2].$$

The above equation can be rewritten as [1]

$$(\partial^2 + (\frac{P}{Q})^2)(-\epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda) = \frac{P}{Q^2}J_\mu + \frac{1}{Q}\epsilon_{\mu\nu\lambda}\partial^\nu J^\lambda. \quad (6)$$

For $\mu = 2$ in the static case, (6) reduces to

$$(\partial^2 + (\frac{P}{Q})^2)A_0 = \frac{1}{Q}J_0. \quad (7)$$

This equation is inverted to get A_0. We now proceed exactly as in [1]. The potential energy between two external static charges, (emerging from J_0), a distance L apart, is

$$V(L) = -(L_q - L_0) = -q[A_0(x^1 = -L/2,\ x^2 = 0) - A_0(x^1 = L/2,\ x^2 = 0)]. \quad (8)$$

The subscript q denotes the presence of the external charge q. Solving the A_μ-equation for A_0 we get

$$A_0(x) = q_1[\Delta(x^1 + L/2, x^2; N) - \Delta(x^1 - L/2, x^2; N)]$$
$$+ q_2\partial_i\partial_i[\Delta(x^1 + L/2, x^2; N) - \Delta(x^1 - L/2, x^2; N)]. \quad (9)$$

$\Delta(x, m)$ is the two dimensional Euclidean Feynman propagator given by the modified Bessel function

$$\Delta(x, m) = 1/(2\pi)K_0(m\sqrt{(x^1)^2 + (x^2)^2}).$$

Here q_1 and q_2 and N are related to the external charge q and the fermion mass m. Once again for $g = q = 0$ we get back the results of [1]. The interesting term is the latter one in (7) which disappears if *only* $O(a,\ g)$ terms are kept. Because of the derivative operator, it radically alters the functional form of A_0, even though its strength q_2 is much less than q_1. For small separation L, $K_0(L) \approx ln(L)$ and the second term $\approx 1/L^2$, which competes and finally dominates over the first term. This is one of our main results. Finally the potential energy function is expressed in terms of dimensionless variables $M \equiv e^2/m,\ G \equiv ge^2,\ \theta \equiv \mu/e^2,\ X = e^2 L$ as

$$V_{Th} = -\frac{q^2}{\pi}\frac{(1+t)}{1-M/(6\pi)}K_0[\frac{(1+u)X}{4\pi(1-M/(6\pi))}]$$

$$+\frac{q^2}{\pi}\frac{2[G^2/(32\pi^2)+GM/(6\pi)]}{1-M/(3\pi)}(\partial_X)^2 K_0[\frac{(1+u)X}{4\pi(1-M/(6\pi))}]. \tag{10}$$

This expression is to be compared with $V_{classical}$ and V_{AB} in [1]

$$V_{Cl} = \frac{q^2}{\pi}ln(X),$$

$$V_{AB} = -\frac{q^2}{\pi}\frac{1}{1-M/(6\pi)}K_0[\frac{X}{4\pi(1-M/(6\pi))}].$$

The correction terms in (10) are

$$u = \frac{4}{1-M/(2\pi)}[\frac{G}{64\pi^2}+G^2(-\frac{1}{32\pi^3}-\frac{\theta}{128\pi^3}+\frac{\theta}{8\pi^2}$$

$$-\frac{3\theta^2}{512\pi^3})-MG(\frac{1}{96\pi^3}-\frac{1}{6\pi^2}+\frac{\theta}{24\pi^2}-\frac{2\theta}{3\pi}-\frac{\theta^2}{4\pi})]$$

$$t = \frac{4}{1-M/(2\pi)}[\frac{G}{64\pi^2}+G^2(-\frac{1}{1024\pi^4}-\frac{3}{512\pi^3}-\frac{\theta}{64\pi^3}+\frac{\theta}{4\pi^2}+\frac{3\theta^2}{32\pi^2})$$

$$-GM(\frac{1}{64\pi^3}+\frac{1}{3\pi^2}+\frac{\theta}{12\pi^2}-\frac{4\theta}{3\pi}-\frac{\theta^2}{2\pi})].$$

This constitues our main result.

Notice that θ does not play any significant role in the present context and so we put $\theta = 0$. In Figure (1), for $M = 0.1,\ G = \pm 0.1,\ V_{Cl},\ \ V_{AB}$ and $V_{Th}^{\pm}$ are plotted. For this case, the potential well in $VThP(G = +0.1)$ is at $X \approx 2\sqrt{G^2/(32\pi^2)+GM/(6\pi)} \approx 0.046$ and for $X < 0.003$, $VThP$ becomes positive. $VThN(G = -0.1)$ shows a sharper descent at short distance. Figure (2) shows only $O(G)$ corrections to the V_{AB} result. The fact that $V_{Th} \approx V_{AB}$ up to $O(G)$ is quite insensitive for a wide range of values of G and M. Here, an unrealistic value $G = \pm 50$ is chosen just to separate the V_{Th} and V_{AB} lines as well as to stress the stability of this formulation up to $O(M,\ G)$. The shift of V_{Th} with respect to V_{AB} in the upward or downward direction direction is dictated by the sign of G.

Up to $O(M,\ G)$ the effective mass of the gauge particle A_μ and the renormalized charge q^2 are

$$M_{Th} = \frac{P}{Q} \approx \frac{e^2/(4\pi)}{1-M/(6\pi)}(1-\frac{G}{2\pi}) = M_{AB}(1-\frac{G}{2\pi}),$$

$$(q^2)_{Th} \approx \frac{q^2}{1-M/(6\pi)}(1+\frac{G}{16\pi^2}) = (q^2)_{AB}(1+\frac{G}{16\pi^2}).$$

Higher order momentum dependent terms have been left out.

Let us summarise our results. The potential energy between the external charges at large distance is screened as the gauge particles aquire mass from the Chern Simons term. Without the Thirring interaction, at short distance, one gets a decreasing negative potential, logarithmic in nature. The positive Thirring coupling term introduces a sort of centrifugal barrier in the effective potential, which leads to the potential well formation. Expressing the centrifugal term as $(angularmomentum)^2/(2ML^2)$ shows clearly that *only* the Thirring term contributes to the correction in the angular momentum of the "bound state". The physical reason is the following. From (2), notice that B_μ/g is identified as the fermion current [4]. The *vector nature* of the interaction leads to the derivative-term upon integration, which subsequently changes the angular momentum. Estimates of effective mass of the state can be obtained from harmonic oscillator excitations around the well minimum.

Acknowledgment: I am grateful to Professor S. Dutta Gupta, Director, S. N. Bose National Centre for Basic Sciences, Calcutta, for allowing me to use the Institution facilities.

References

[1] E. Abdalla and R. Banerjee, Phys. Rev. Lett. 80 (1998) 238.

[2] H. J. Rothe, K. D. Rothe and J. A. Swieca, Phys. Rev. D19 (1979) 3020.

[3] K.-I. Kondo, Prog. Theor. Phys. 94 (1995) 889; N. Banerjee, R. Banerjee and S. Ghosh, Nucl. Phys. B481 (1996) 421.

[4] S. Ghosh, Phys. Rev. D59 (1999) 045014.

[5] D. J. Gross, I. R. Klebanov, A. V. Matytsin and A. V. Smilga, Nucl. Phys. B461 (1996) 109; hep-th/9511104.

[6] G. Grignani, L. Paniak, G. W. Semenoff, hep-th/9705102; L. D. Paniak, G. W. Semenoff and A. R. Zhitnitsky, Nucl. Phys. B487 (1997) 191; A. R. Zhitnitsky, Phys. Lett. B291 (1992) 465.

[7] S. Deser, R. Jackiw and S. Templeton, Ann. Phys. 140 (1982) 372.