Phase properties of even and odd nonlinear coherent states

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Abstract

Using the Pegg–Barnett formalism we obtain phase probability distributions of the even and odd nonlinear coherent states. These distributions are then used to examine whether or not the even and odd nonlinear coherent states exhibit number/phase squeezing. We also examine whether these states are intelligent states with respect to the number-phase uncertainty relation.

1. Introduction

Coherent states of simple harmonic oscillator [1] as well as coherent states of various Lie algebras [2] have found considerable applications in the study of quantum optics. Superpositions of coherent states in the form of symmetric and antisymmetric combinations were introduced in Ref. [3]. These superpositions, called the even and odd coherent states exhibit different nonclassical effects [4]. Recently another type of coherent states, called nonlinear coherent states (NCS) (corresponding to nonlinear algebras) [5,6] have been introduced to describe the motion of a trapped atom [6]. Subsequently non classical properties of various superpositions of nonlinear coherent states have been studied [7,8].

We recall that till recently the study of optical field phase has been problematic primarily because of the nonexistence of a hermitian phase operator. However, with the advent of the Pegg–Barnett formalism [9–11] (which ensures a hermitian phase operator)the above mentioned difficulty can be avoided. In particular, using the Pegg–Barnett formalism one can obtain the phase probability distribution $P(\theta)$. It may be noted that the phase probability distribution is an essential tool in the study of various phase characteristics. For example, it has been used to study number-phase squeezing [12], number-phase uncertainty states [13] as also states with minimum phase noise [14].

In the present paper we shall obtain the Pegg–Barnett phase probability distribution for the even and odd nonlinear coherent states. We would like to note that there can be any number of nonlinear coherent states corresponding various choices of the nonlinearity function (see below) but we shall confine ourselves to the choice of nonlinearity considered in Ref. [6] to describe the motion of a trapped ion. Subsequently we shall use the phase probability distribution to study $N-\Phi_\theta$ squeezing of these states. In the process we shall also determine whether or not the even and odd NCS are minimum number-phase uncertainty states.

2. Even and odd nonlinear coherent states

Nonlinear coherent states $|\alpha,f\rangle$ are defined as right hand eigenstates of a generalised annihilation operator A:

$$A|\alpha,f\rangle=\alpha|\alpha,f\rangle \tag{1}$$

$$A=af(N),\quad N=a^{\dagger}a \tag{2}$$

where α is an arbitrary complex number and $a(a^{\dagger})$ denotes harmonic oscillator annihilation (creation) operator and $f(x)$ is a reasonably well behaved function.

In the number state representation the nonlinear coherent states are given by

$$|\alpha,f\rangle=C\sum_{n=0}^{\infty}d_n\alpha^n|n\rangle \tag{3}$$

$$d_n=\left[n!\Pi_{i=0}^{\infty}f(i)\right]^{-1/2} \tag{4}$$

where C is a normalisation constant and is given by

$$C=\left[\sum_{n=0}^{\infty}d_n^2|\alpha|^{2n}\right]^{-1/2} \tag{5}$$

Even and odd nonlinear coherent states are defined as

$$|\alpha,f\rangle_{\pm}=N_{\pm}(|\alpha,f\rangle\pm|-\alpha,f\rangle) \tag{6}$$

In the number state representation even NCS are given by

$$|\alpha,f\rangle_{+}=C_{+}\sum_{n=0}^{\infty}d_{2n}\alpha^{2n}|2n\rangle \tag{7}$$

$$C_{+}=\left[\sum_{n=0}^{\infty}d_{2n}^2|\alpha|^{4n}\right]^{-1/2} \tag{8}$$

while the odd NCS are given by

$$|\alpha,f\rangle_{-}=C_{-}\sum_{n=0}^{\infty}d_{2n+1}\alpha^{2n+1}|2n+1\rangle \tag{9}$$

$$C_{-}=\left[\sum_{n=0}^{\infty}d_{2n+1}^2|\alpha|^{4n+2}\right]^{-1/2} \tag{10}$$

3. Phase properties

We now turn to phase distributions of nonlinear even and odd NCS. In the Pegg–Barnett approach [9–11] we start with a finite dimensional $(s+1)$ dimensional Hilbert space spanned by the number states $|0\rangle,|1\rangle,\cdots,|s\rangle$. In this space a complete orthonormal set of phase states $|\theta_m\rangle, m=0,1,\cdots,s$ is defined by

$$|\theta_m\rangle=\frac{1}{\sqrt{s+1}}\sum_{n=0}^{s}\exp(in\theta_m)|n\rangle \tag{11}$$

where θ_m are given by

$$\theta_m=\theta_0+\frac{2\pi m}{s+1},\quad m=0,1,\cdots,s \tag{12}$$

The value of θ_0 is arbitrary and defines a particular basis in the phase space. A hermitian phase operator Φ_θ is defined as

$$\Phi_\theta=\sum_{m=0}^{s}\theta_m|\theta_m\rangle\langle\theta_m| \tag{13}$$

For superposition states of the form $|\psi\rangle=\sum b_n e^{in\phi}|n\rangle$ the phase probability distribution is given by

$$|\langle\theta_m|\psi\rangle|^2=\frac{1}{s+1}+\frac{2}{s+1}\times\sum_{n>k}b_nb_k\cos[(n-k)(\phi-\theta_m)] \tag{14}$$

Now choosing θ_0 as

$$\theta_0=\phi-\frac{\pi s}{s+1} \tag{15}$$

we obtain from (13)

$$|\langle\theta_m|\psi\rangle|^2=\frac{1}{s+1}+\frac{2}{s+1}\times\sum_{n>k}b_nb_k\cos(n-k)\frac{2\pi\mu}{s+1} \tag{16}$$

where $\mu=m-\frac{s}{2}$.

The continuous phase probability distribution $P(\theta)$ can now be obtained as

$$P(\theta)=\lim_{s\to\infty}\frac{s+1}{2\pi}|\langle\theta_m|\psi\rangle|^2=\frac{1}{2\pi}\left(1+2\sum_{n>k}b_nb_k\cos[(n-k)\theta]\right) \tag{17}$$

We can now calculate various quantum mechanical averages. For instance the phase variance is given by (after symmetrizing the phase window)

$$\langle(\Delta\Phi_\theta^2)\rangle=\int_{-\pi}^{\pi}\theta^2P(\theta)=\frac{\pi^2}{3}+\sum_{n>k}b_nb_k\frac{(-1)^{n-k}}{(n-k)^2} \tag{18}$$

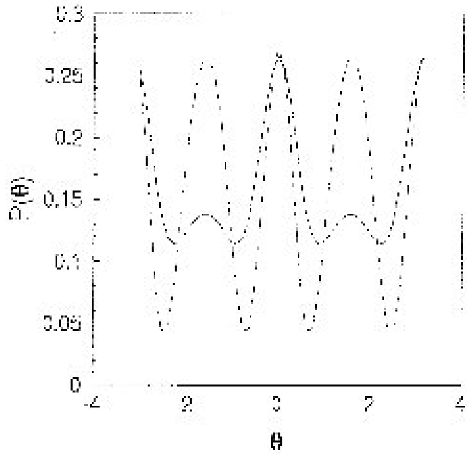

Fig. 1. Phase distribution of even NCS for $\alpha = .4$ and $\eta = .3$ (solid curve) and $\eta = .6$ (broken curve).

Another relation which we shall use concerns number-phase uncertainty. In the present case it is given by

$$\left(\langle(\Delta N)^2\rangle\langle(\Delta\Phi_\theta)^2\rangle\right)^{1/2} \geq \tfrac{1}{2}|\langle[N,\Phi_\theta]\rangle| \tag{19}$$

where

$$[N,\Phi_\theta] = i[1 - 2\pi P(\theta_0)] \tag{20}$$

From (18) it follows that there is squeezing in the number variable if

$$F(\alpha) = \langle(\Delta N)^2\rangle - \tfrac{1}{2}|\langle[N,\Phi_\theta]\rangle| < 0 \tag{21}$$

while the condition for squeezing in the phase variable is given by

$$G(\alpha) = \langle(\Delta\Phi_\theta)^2\rangle - \tfrac{1}{2}|\langle[N,\Phi_\theta]\rangle| < 0 \tag{22}$$

We now proceed to the computation of Pegg–Barnett phase probability distributions using (6)–(10). The particular class of nonlinearity we choose here was considered in Ref. [6] to describe the motion of trapped ion. In the present case the nonlinearity function $f(n)$ is given by [6]

$$f(n) = L_n^1(\eta^2)\left[(n+1)L_n^0(\eta^2)\right]^{-1} \tag{23}$$

where η is known as the Lamb–Dicke parameter and $L_n^\alpha(x)$ denotes the generalised Lagurre polynomials [15].

We now turn to the figures. In Figs. 1 and 2, we have plotted Pegg–Barnett phase probability distributions for the even and odd nonlinear coherent states. From Fig. 1 we find that for small value of η the phase distribution for even NCS has a central peak at $\theta = 0$ while there are two not so well developed peaks at $\theta = \pm\frac{\pi}{2}$. For a larger value of η the peaks at $\theta = \pm\frac{\pi}{2}$ become prominent. On the other hand from Fig. 2 we find that for η small the phase distribution for odd NCS has only a central peak at $\theta = 0$ and as η is increased apart from the central peak two more peaks develop at $\theta = \pm\frac{\pi}{2}$. Thus for both the even as well as the odd NCS quantum interference effects become more prominent for relatively larger values of η.

In Fig. 3 we plot phase distributions of the even and odd nonlinear coherent states. From the figure it is clear that the distributions for the even and odd NCS are rather different, the former having well developed peaks and the latter having not so well developed peaks at $\theta = \pm\frac{\pi}{2}$. Thus unlike the case of ordinary even and odd coherent states [16] the Pegg–Barnett distribution clearly reflects the different character of quantum interference in the case of even and odd NCS.

In Fig. 4 we have plotted the function $F(\alpha)$ for the even and odd NCS against α keeping η fixed (we note that the reverse i.e., keeping α fixed and varying η is also possible. But there is no change in

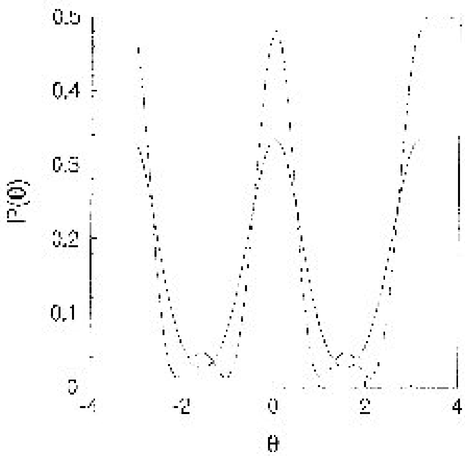

Fig. 2. Phase distribution of odd NCS for $\alpha = .4$ and $\eta = .3$ (solid curve) and $\eta = .6$ (broken curve).

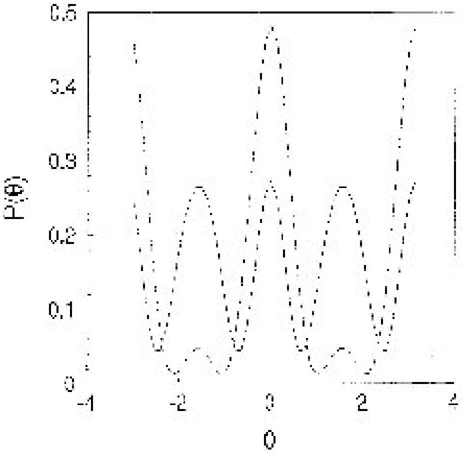

Fig. 3. Phase distribution of even NCS (solid curve) and odd NCS (broken curve) for $\eta = .6$ and $\alpha = .4$.

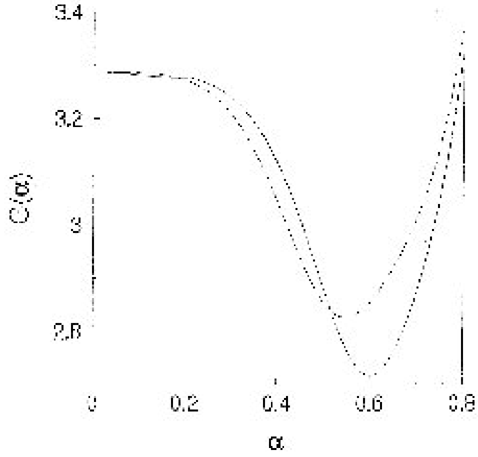

Fig. 5. Plot of $G(\alpha)$ of even NCS (solid curve) and odd NCS (broken curve) for $\eta = .8$.

qualitative behaviour of the function F). Clearly if $F(\alpha)$ is less than zero then there will be number squeezing. From Fig. 4 it is clear that both the even and the odd NCS exhibit number squeezing. However, number squeezing is more profound in the case of odd NCS. We have checked that for larger values of η number squeezing is more prominent.

Fig. 5 shows plot of the function $G(\alpha)$ for the even and the odd NCS. From Fig. 5 we find that for both even and odd NCS $G(\alpha)$ decreases upto a certain value of α and then again increases as α increases. This behaviour remains the same when α and/or η is changed. Therefore we conclude that neither the even nor the odd NCS exhibit phase squeezing.

Finally in Fig. 6 we plot the function $H(\alpha)$. Note that when $H(\alpha) = 0$ the corresponding state is an intelligent state. From the figure we find that for both the even and the odd NCS the function $H(\alpha)$ starts from zero (and thus are intelligent states) and as α increases $H(\alpha)$ also increases. However compared to the odd NCS the even NCS remain intelligent over a larger range of α.

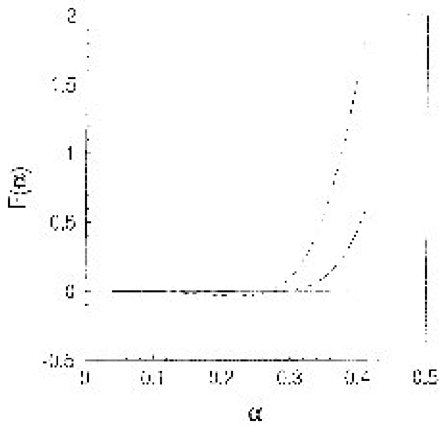

Fig. 4. Plot of $F(\alpha)$ of even NCS (broken curve) and odd NCS (dotted curve) for $\eta = .9$.

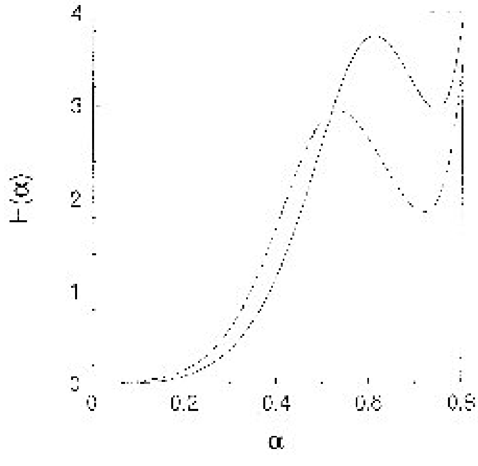

Fig. 6. Plot of $H(\alpha)$ of even NCS (solid curve) and odd NCS (broken curve) for $\eta = .8$.

4. Conclusions

In this article we have examined phase properties of even and odd nonlinear coherent states. In particular we have the Pegg–Barnett phase distributions for these states and compared them. It has also been shown that while the two superpositions exhibit number squeezing, phase squeezing has been absent in both the cases. However, both the even and odd NCS have been shown to be intelligent states with respect to the number-phase uncertainty relation (19).

References

[1] R.J. Glauber, Phys. Rev 131 (1963) 2766.
[2] A.P. Perelomov, Generalised Coherent States and Their Applications, Springer, Berlin 1986.
[3] V.V. Dodonov, I.A. Malkin, V.I. Man'ko, Physica 72 (1974) 597.
[4] M. Hillery, Phys. Rev. A 36 (1987) 3796; Y. Xia, G. Guo, Phys. Lett. A 136 (1989) 281; C.C. Gerry, J. Mod. Opt. 40 (1993) 1053.
[5] V.I. Man'ko, G. Marmo, E.C.G. Sudarshan, F. Zaccaria, Phys. Scr. 55 (1997) 528.
[6] R.L. de Matos, W. Vogel, Phys. Rev. A 54 (1996) 4560.
[7] S. Mancini, Phys. Lett. A 233 (1997) 291.
[8] B. Roy, Phys. Lett. A 249 (1998) 25.
[9] D.T. Pegg, S.M. Barnett, Europhys. Lett. 6 (1988) 483.
[10] S.M. Barnett, D.T. Pegg, J. Mod. Opt. 36 (1989) 7.
[11] D.T. Pegg, S.M. Barnett, A 39 (1989) 1665.
[12] R. Tanas, Ts. Gantsog, Quant. Opt. 4 (1992) 245.
[13] R. Tanas, A. Miranowicz, Ts. Gantsog, in: A. Wolf (Ed.), Progress in Optics, vol. 35, 1996, p. 355.
[14] B.M. Garraway, P.L. Knight, Phys. Rev. A 46 (1992) R5346; Phys. Scr. T 48 (1993) 66.
[15] M. Abramowitz, I.A. Stegun (Eds.), Handbook of Mathematical Functions, Dover, NY, 1972.
[16] V. Buzek, P.L. Knight, in: E. Wolf (Ed.), Progress in Optics, vol. 34, 1995.