Non-linear coherent states associated with conditionally exactly solvable problems

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Abstract. Recently, based on a supersymmetric approach, new classes of conditionally exactly solvable problems have been found, which exhibit a symmetry structure characterized by non-linear algebras. In this paper the associated "non-linear" coherent states are constructed and some of their properties are discussed in detail.

PACS numbers: 03.65.Fd, 02.20.Qs, 42.50.-p

1. Introduction

It is well known that only a few quantum mechanical models admit exact solutions. The class of exactly solvable models can, however, be enlarged by using the technique of generating isospectral Hamiltonians [1]. Recently, another class of problems [2, 3] consisting of so-called conditionally exactly solvable (CES) problems has emerged. The characteristic feature of this class is that their members are exactly solvable problems when the parameters appearing in the potential are fine tuned to assume some specific numerical value or to lie in some range of values.

In some recent papers [4, 5, 6] several classes of CES problems, whose construction is based on supersymmetric (SUSY) quantum mechanics [7] have been found. It was shown [4, 5] that the classes associated with the linear and radial harmonic oscillator admit some non-linear algebra as their symmetry algebra. Here our objective is to construct coherent states corresponding to these CES problems and examine some of their properties. We recall that usually coherent states are constructed using as a basis some Lie algebra [8]. In contrast, here the coherent states are constructed over non-linear algebras and we call them *non-linear coherent states*. In this paper we limit ourselves to the class of CES potentials associated with the radial harmonic oscillator. To be more precise, we shall start with systems having $su(1,1)$ dynamical symmetry

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and then construct coherent states corresponding to the isospectral partners which have a non-linear (i.e. deformed) $su(1,1)$ symmetry. In this context we recall that in ref [9] Nieto et al described a method of constructing coherent and squeezed states for arbitrary quantum mechanical potentials. In the present paper we construct coherent states for hitherto unknown potentials having some particular symmetry properties. To be a bit more explicit, we consider two cases. One in which SUSY is broken and the other in which SUSY is unbroken. In the former case non-linear coherent states can be constructed over the entire Fock space. Whereas in the latter case non-linear coherent states are defined in a subspace of the Fock space.

This paper is organized as follows. In the next section we briefly summarize the essentials of SUSY quantum mechanics. In Section 3 we discuss the CES potentials associated with the radial harmonic oscillator model and its non-linear symmetry algebra. Section 4 is devoted to the construction of the non-linear coherent states. Basic properties of these states are also discussed. Finally, in Section 5 we briefly discuss the case of unbroken SUSY and in Section 6 some discussion and outlook is given.

2. SUSY quantum mechanics

To begin with we note that Witten's model of supersymmetric quantum mechanics consists of a pair of Hamiltonians [7]

$$H_\pm = -\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_\pm(x) \tag{1}$$

acting on some suitable Hilbert space $\mathcal{H}$. For the purpose at hand we take the linear space of square integrable functions on the positive half-line with Dirichlet boundary condition at the origin, $\mathcal{H} = \{\psi \in L^2(\mathbb{R}^+) | \psi(0) = 0\}$. The supersymmetric partner potentials in (1) are given by

$$V_\pm(x) = \frac{1}{2}\left[W^2(x) \pm W'(x)\right] \tag{2}$$

where W is the SUSY potential and $W' = dW/dx$. In terms of the operators

$$A = \frac{1}{\sqrt{2}}\left(\frac{\mathrm{d}}{\mathrm{d}x} + W(x)\right) , \quad A^\dagger = \frac{1}{\sqrt{2}}\left(-\frac{\mathrm{d}}{\mathrm{d}x} + W(x)\right) \tag{3}$$

the Hamiltonians in (1) read $H_+ = AA^\dagger$ and $H_- = A^\dagger A$, respectively. Let us denote the eigenfunctions and eigenvalues of $H_\pm$ by $\psi_n^\pm$ and $E_n^\pm$:

$$H_\pm \psi_n^\pm(x) = E_n^\pm \psi_n^\pm(x) , \quad n = 0, 1, 2, \dots . \tag{4}$$

Then it can be shown [7] that in the case of broken SUSY (we will mainly concentrate on this case) the spectrum of H_- coincides with that of H_+ and both are strictly positive:

$$E_n^+ = E_n^- \equiv E_n > 0 , \quad \psi_n^-(x) = E_n^{-1/2} A^\dagger \psi_n^+(x) , \quad \psi_n^+(x) = E_n^{-1/2} A \psi_n^-(x) . \tag{5}$$

Thus it is clear that if one of the Hamiltonians is exactly solvable then the spectral properties of the other one are also known, that is, it becomes exactly solvable, too. This is the basic idea in the supersymmetric construction methods of CES potentials.

To be a bit more explicit, in [4, 5, 6] it has been suggested to construct SUSY potentials W in such a way that V_+ becomes (under certain conditions imposed on the parameters involved) one of the well-known exactly solvable (shape-invariant) potentials and thus giving rise, in general, to a class of CES potentials V_-.

3. A model with broken SUSY

Now as a specific model we consider the following SUSY potential

$$W(x) = x + \frac{\gamma+1}{x} + \frac{u'(x)}{u(x)}\,, \tag{6}$$

where $u(x) = {}_1F_1(\frac{1-\varepsilon}{2}, \gamma+\frac{3}{2}, -x^2)$ is a confluent hypergeometric function and the two potential parameters have to obey the conditions $\gamma \geq 0$ and $\varepsilon > -2\gamma - 2$. This SUSY potential can be shown [5, 6] to give rise to

$$V_+(x) = \frac{x^2}{2} + \frac{\gamma(\gamma+1)}{2x^2} + \varepsilon + \gamma + \frac{1}{2}\,. \tag{7}$$

Clearly, V_+ represents the generalised radial harmonic oscillator (this can be regarded as the potential corresponding to the two body Calogero-Sutherland model) and the associated spectral properties of H_+ are well known

$$E_n = 2n + 2\gamma + 2 + \varepsilon\,, \quad \psi_n^+(x) = \left[\frac{2n!}{\Gamma(n+\gamma+\frac{3}{2})}\right]^{1/2} x^{\gamma+1}\,\mathrm{e}^{-x^2/2}\,L_n^{\gamma+\frac{1}{2}}(x^2)\,. \tag{8}$$

Here L_n^ν denotes a generalised Laguerre polynomial and we also note that SUSY is broken, that is, $\exp\left\{\pm\int \mathrm{d}x\,W(x)\right\} \notin \mathcal{H}$. As a consequence, the SUSY partner Hamiltonian H_- has the same eigenvalues E_n and its eigenfunctions can be obtained from (8) via (5):

$$\begin{aligned}\psi_n^-(x) &= \frac{1}{\sqrt{4n+4\gamma+4+2\varepsilon}}\left(-\frac{\mathrm{d}}{\mathrm{d}x} + x + \frac{\gamma+1}{x} + \frac{u'(x)}{u(x)}\right)\psi_n^+(x)\\ &= \left[\frac{2\,n!}{(n+\gamma+1+\frac{\varepsilon}{2})\Gamma(n+\gamma+\frac{3}{2})}\right]^{1/2} x^{\gamma+2}\,\mathrm{e}^{-x^2/2}\left(L_n^{\gamma+3/2}(x^2) + \frac{u'(x)}{2\,x\,u(x)}\right)\,.\end{aligned} \tag{9}$$

The corresponding CES potential explicitly reads

$$V_-(x) = \frac{x^2}{2} + \frac{(\gamma+1)(\gamma+2)}{2x^2} + \gamma - \varepsilon + \frac{3}{2} + \frac{u'(x)}{u(x)}\left(2x + 2\frac{\gamma+1}{x} + \frac{u'(x)}{u(x)}\right)\,. \tag{10}$$

In ref. [5] we have shown that the symmetry algebra underlying the eigenvalue problem associated with H_- is a non-linear one. To be more explicit, with the help of the ladder operators for H_+ given by $c = (\mathrm{d}/\mathrm{d}x + x)^2/2 - (\gamma+1)(\gamma+2)/2x^2$, which together with its adjoint $c^\dagger$ and H_+ close a (linear) Lie algebra, one can introduce similar ladder operators for H_- defined by $D = A^\dagger c A$ and its adjoint $D^\dagger = A^\dagger c^\dagger A$. These operators act on eigenstates of H_- as follows:

$$D^\dagger\psi_n^-(x) = f_{n+1}\psi_{n+1}^-(x)\,, \quad D\psi_n^-(x) = f_n\psi_{n-1}^-(x)\,, \quad D\psi_0^-(x) = 0\,, \tag{11}$$

where f_n is given by

$$f_n = -2\sqrt{n(n+\gamma+\tfrac{1}{2})(2n+2\gamma+2+\varepsilon)(2n+2\gamma+\varepsilon)}\,. \tag{12}$$

From these relations it also follows that

$$\begin{aligned}\psi_n^-(x) &= (f_1 f_2 \cdots f_n)^{-1}\left(D^\dagger\right)^n \psi_0^-(x)\\ &= (-\tfrac{1}{4})^n\left[n!\,(\gamma+\tfrac{3}{2})_n(\gamma+1+\tfrac{\varepsilon}{2})_n(\gamma+2+\tfrac{\varepsilon}{2})_n\right]^{-1/2}(D^\dagger)^n\psi_0^-(x)\,.\end{aligned} \tag{13}$$

The non-linear algebra closed by the operators D, $D^\dagger$ and H_- explicitly reads

$$[H_-, D] = -2D\,, \quad \left[H_-, D^\dagger\right] = 2D^\dagger\,, \quad \left[D, D^\dagger\right] = \Phi(H_-)\,, \tag{14}$$

where the non-linear structure function¶ Φ is given by

$$\Phi(H_-) = 8H_-^3 - 12(\gamma+\varepsilon+\tfrac{1}{2})H_-^2 + 4(2\varepsilon\gamma+\varepsilon^2+\varepsilon+1)H_-\,. \tag{15}$$

Actually, these types of algebras (having as structure function a polynomial of degree $p-1$ in one of the generators) are called W_p algebras. More explicitly, the above algebra (14) is a polynomial deformed $su(1,1)$ algebra and has first been discussed in some detail by Roček [10]. For a discussion within a more general approach see also Karassiov [11] and Katriel and Quesne [12].

The quadratic Casimir operator for the non-linear (cubic) algebra (14) reads

$$C = DD^\dagger - \Psi(H_-) \tag{16}$$

where

$$\Phi(H_-) = \Psi(H_-) - \Psi(H_- - 2)\,. \tag{17}$$

We note that in the above Fock space representation (11)-(13) we have the relations

$$\begin{aligned}&\Psi(H_-) = f^2_{H_-/2-\gamma-\varepsilon/2} = (H_- - 2\gamma - \varepsilon)(H_- + 1 + \varepsilon)(H_- + 2)H_-\,,\\ &DD^\dagger = \Psi(H_-)\,, \quad D^\dagger D = \Psi(H_- - 2)\,,\end{aligned} \tag{18}$$

and, therefore, the Casimir operator (16) vanishes as expected [11, 12]. This, however, will in general not be the case for non-Fock space representations of the algebra (14) [10, 11, 12].

4. The non-linear coherent states

We shall now construct coherent states corresponding to the algebra in (14). At this point we note that coherent states can be constructed following any of the three methods [13]: (i) By applying the unitary displacement operator to the ground state. (ii) Defining coherent states as eigenstate of the lowering operator. (iii) Defining coherent states as minimum uncertainty states. These three methods are generally not equivalent and only in the case of the standard harmonic oscillator, where the commutator of the raising and lowering operator is the unit operator, these three methods are equivalent. Since the symmetry algebra in the present case is a non-linear one, the Baker-Campbell-Hausdorff

¶ We note that in the case of Lie algebras the structure function would have been a linear one.

disentangling theorem cannot be used and so we shall follow the second approach to construct non-linear coherent states. Note that coherent states obtained in this way are essentially Barut-Girardello coherent states [14]. We also remark that the procedure following below is very similar to the construction of coherent states associated with quantum groups [15].

Thus we define coherent states as

$$|\mu\rangle = \sum_{n=0}^{\infty} c_n\, \mu^n\, |n\rangle\ , \tag{19}$$

where the c_n's are real constants to be determined, μ is an arbitrary complex number, and the ket $|n\rangle$ is a short-hand notation for the eigenstate ψ_n^- of H_-. Now, by our definition $|\mu\rangle$ should be an eigenstate of the lowering operator D and so we have

$$D|\mu\rangle = \mu\,|\mu\rangle = \sum_{n=0}^{\infty} c_{n+1}\, \mu^{n+1}\, f_{n+1}\, |n\rangle\ . \tag{20}$$

Comparing this result with definition (19) we obtain the recurrence relation

$$c_{n+1} = \frac{c_n}{f_{n+1}}\ , \quad n = 0, 1, 2, \ldots\ , \tag{21}$$

and consequently the constants c_n for $n \geq 1$ are given by

$$c_n = c_0 \prod_{i=1}^{n} (f_i)^{-1}\ . \tag{22}$$

The remaining constant c_0 is determined via the normalisation of the coherent states:

$$\begin{aligned}\langle\mu|\mu\rangle &= c_0^2 \left[1 + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} f_i^{-2} \right) |\mu|^{2n} \right] \\ &= c_0^2 \sum_{n=0}^{\infty} \frac{(|\mu|^2/16)^n}{n!\,(\gamma+\frac{3}{2})_n (\gamma+1+\frac{\varepsilon}{2})_n (\gamma+2+\frac{\varepsilon}{2})_n} = 1\ .\end{aligned} \tag{23}$$

Hence, the normalisation constant $c_0 = c_0(\mu)$ can be expressed in terms of a generalised hypergeometric function

$$c_0^{-2}(\mu) = {}_0F_3 \left(\gamma + \frac{3}{2}, \gamma + 1 + \frac{\varepsilon}{2}, \gamma + 2 + \frac{\varepsilon}{2}; \frac{|\mu|^2}{16} \right)\ . \tag{24}$$

Similarly we can show that these non-linear coherent states are not orthogonal for $\mu \neq \nu$,

$$\langle\mu|\nu\rangle = c_0(\mu) c_0(\nu)\, {}_0F_3 \left(\gamma + \frac{3}{2}, \gamma + 1 + \frac{\varepsilon}{2}, \gamma + 2 + \frac{\varepsilon}{2}; \frac{\mu^* \nu}{16} \right) \neq 1\ , \tag{25}$$

and, therefore, form an over-complete basis in the Hilbert space.

Another important property, namely, the resolutions of unity can also be obtained for these non-linear coherent states. Let us assume that we have a positive measure ρ on the complex plane such that

$$\int_{\mathbb{C}} \mathrm{d}\rho(\mu^*, \mu)\, |\mu\rangle\langle\mu| = 1\ . \tag{26}$$

Making the polar decomposition $\mu = \sqrt{x}\,\mathrm{e}^{\mathrm{i}\varphi}$ and the ansatz $\mathrm{d}\rho(\mu^*,\mu) = \mathrm{d}\varphi\mathrm{d}x\,\sigma(x)/2\pi c_0^2(\sqrt{x})$, with σ being a yet unknown density on the positive half-line, the above resolution of unity (26) reduces to the relations

$$\int\limits_0^\infty dx\, x^n \sigma(x) = 16^n\, n!\,(\gamma+\tfrac{3}{2})_n\,(\gamma+1+\tfrac{\varepsilon}{2})_n\,(\gamma+2+\tfrac{\varepsilon}{2})_n\,, \quad n = 0,1,2,\ldots\,. \tag{27}$$

In other words, σ is a probability density on the positive half-line defined via the moments given above. For technical details on this so-called Stieltjes moment problem see [16]. Here we note that the integral (27) may be viewed as Mellin transformation [17] of the density σ. In other words, σ is given via the inverse Mellin transformation of its moments. For the above moments this inverse transformation leads (see ref [17] p 353) to a Meijer G-function [18] and we explicitly have

$$\sigma(x) = \frac{G_{04}^{40}\left(\frac{x}{16}\,\middle|0,\gamma+\frac{1}{2},\gamma+\frac{\varepsilon}{2},\gamma+1+\frac{\varepsilon}{2}\right)}{16\,\Gamma(\gamma+\frac{3}{2})\,\Gamma(\gamma+1+\frac{\varepsilon}{2})\,\Gamma(\gamma+2+\frac{\varepsilon}{2})}\,. \tag{28}$$

In Figure 1 we plot the radial density $f(x) = \sigma(x)/c_0^2(\sqrt{x})$ for fixed $\gamma = 1$ and various values of $\varepsilon > -2\gamma - 2$. Figure 2 presents the same quantity now, however, with fixed $\varepsilon = 1$ and various values of $\gamma \geq 0$.

We now proceed to examine some further properties of these non-linear coherent states. To do this we define the following hermitian operators:

$$X_1 = \frac{D + D^\dagger}{2}\,, \quad X_2 = \frac{D - D^\dagger}{2\mathrm{i}}\,. \tag{29}$$

In terms of these operators the non-linear algebra (14) reads

$$[H_-, X_1] = -2\mathrm{i}X_2\,, \quad [H_-, X_2] = 2\mathrm{i}X_1\,, \quad [X_1, X_2] = \frac{\mathrm{i}}{2}\,\Phi(H_-)\,. \tag{30}$$

The uncertainty relation for the two operators X_1 and X_2 in some state $|\psi\rangle \in \mathcal{H}$ reads

$$(\Delta X_1)^2_\psi\,(\Delta X_2)^2_\psi \geq \frac{1}{4}\,|\,\langle\psi|[X_1, X_2]|\psi\rangle\,|^2\,, \tag{31}$$

where $(\Delta X_i)^2_\psi = \langle\psi|X_i^2|\psi\rangle - \langle\psi|X_i|\psi\rangle^2$. We note that the non-linear coherent states $|\mu\rangle$ in (19) having property (20) always satisfy the equality sign in (31). Note that in the notation used in [19] these states are called intelligent states. However, it can be shown that when the functional $F(\mu) = \left(\langle\mu|DD^\dagger|\mu\rangle - |\mu|^2\right)$ attains its minimum for some value of μ, say μ_0, then the non-linear coherent state $|\mu_0\rangle$ is a minimum uncertainty state corresponding to the non-linear algebra (30).

5. The case of unbroken SUSY

Let us now briefly describe the situation when SUSY is unbroken. In this case we choose

$$W(x) = x - \frac{\gamma+1}{x} + \frac{u'(x)}{u(x)}\,, \quad \gamma \geq 0\,, \tag{32}$$

where now $u(x) = {}_1F_1(\frac{1-\varepsilon}{2}, -\gamma - \frac{1}{2}, -x^2)$. For a more general case and the conditions on the parameters ε and γ see ref [5]. It turns out that V_+ again represents the radial

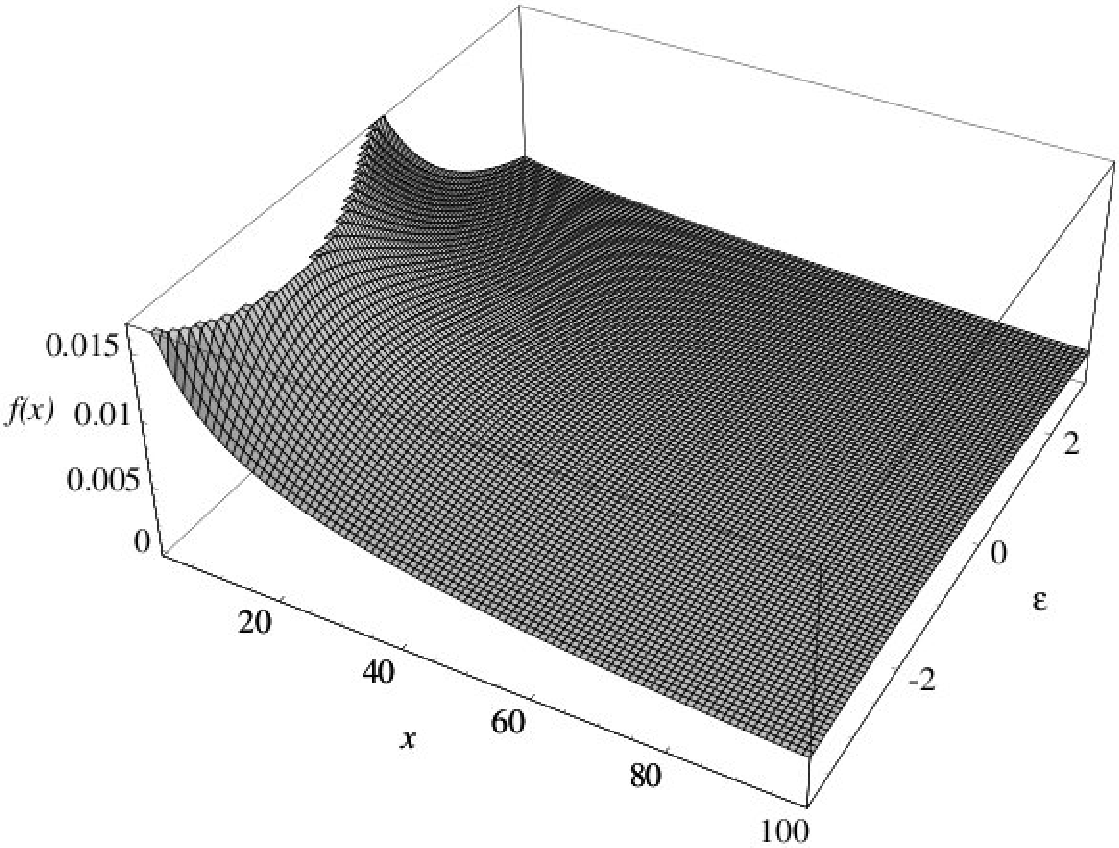

Figure 1. A plot of the radial density $f(x) = \sigma(x)/c_0^2(\sqrt{x})$ entering the resolution of unity (26) with σ and c_0 given by (28) and (24), respectively. Here we have fixed the parameter $\gamma = 1$.

oscillator while V_- is a CES potential. Note that essential details of this problem can be obtained from the broken SUSY case by replacing γ by $-\gamma-2$. However, the eigenvalues for H_- are now given by

$$E_0 = 0\ , \quad E_{n+1} = 2n + 1 + \varepsilon\ , \tag{33}$$

which coincides with the spectrum of H_+ with the exception of the vanishing ground-state energy, which is missing in H_+ due to unbroken SUSY. For the explicit form of the corresponding eigenstates we refer to [5].

Again we may define ladder operators $D = A^\dagger c A$ and $D^\dagger = A^\dagger c^\dagger A$ where the SUSY operators A and $A^\dagger$ are now defined with the new SUSY potential (32). They act on the eigenstates of H_- in the following way:

$$D^\dagger|n+1\rangle = g_{n+1}|n+2\rangle\ , \quad D|n+1\rangle = g_n|n\rangle\ , \quad D|0\rangle = 0 = D^\dagger|0\rangle\ , \tag{34}$$

where

$$g_n = -2\sqrt{n(n+\gamma+\tfrac{3}{2})(2n-1+\varepsilon)(2n+1+\varepsilon)}\ . \tag{35}$$

From the last relation in (34) it is clear that the ground state is isolated in the sense that the non-linear algebra is (non-trivially) realised over the excited states only. Note that

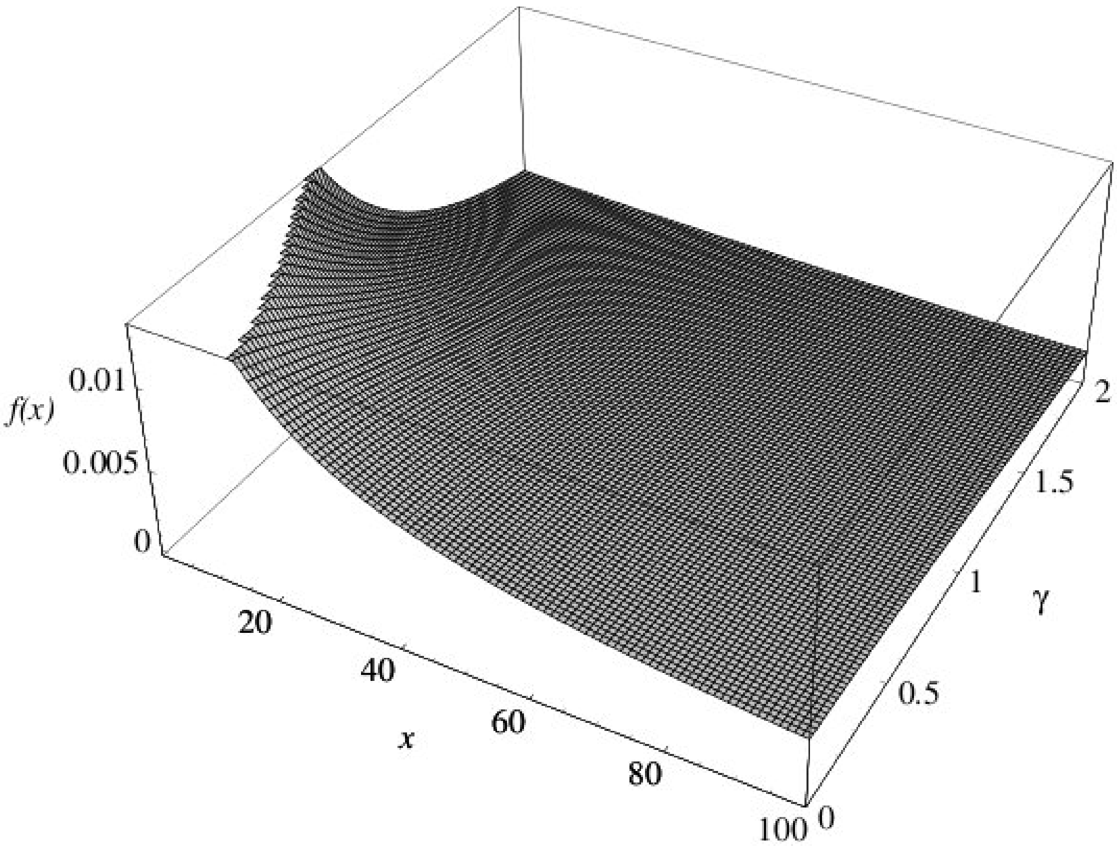

Figure 2. Same as Figure 1 but now with fixed $\varepsilon = 1$ and various values of $\gamma \geq 0$.

the non-linear algebra closed by D, $D^\dagger$ and H_- is identical in form with (14). However, in the structure function (15) we have to replace γ by $-\gamma - 2$ [5].

Now proceeding as in the case of broken SUSY, we can find a superposition state which is an eigenstate of the annihilation operator D. However, this non-linear coherent state is now given by a superposition of the excited energy eigenstates:

$$|\eta\rangle = \sum_{n=0}^{\infty} d_n \, \eta^n \, |n+1\rangle \,, \tag{36}$$

where η is a complex number and the d_n's are given by

$$\begin{aligned} & d_n = d_0 \prod_{i=1}^{n} g_i^{-1} \,, \quad n = 1, 2, 3, \ldots \,, \\ & d_0^{-2}(\eta) = {}_0F_3\left(\gamma + \tfrac{5}{2}, \tfrac{\varepsilon}{2} + \tfrac{1}{2}, \tfrac{\varepsilon}{2} + \tfrac{3}{2}; \tfrac{|\eta|^2}{16}\right) \,. \end{aligned} \tag{37}$$

We note that the states $|\eta\rangle$ in (36) are very similar to coherent states although they are not coherent states. In particular, the states $|\eta\rangle$ are not complete because of the absence of the ground state in the superposition (36). We can, however, call these states excited coherent states or photon-added coherent states [20] because $|\langle 0|\eta\rangle|^2 = 0$ for all $\eta \in \mathbb{C}$.

Note that $\lim_{\eta\to 0}|\eta\rangle = |1\rangle$. The corresponding resolution of unity reads in this case

$$\int_{\mathbb{C}} \mathrm{d}\rho(\eta^*,\eta)\,|\eta\rangle\langle\eta| = 1 - |0\rangle\langle 0|\,, \tag{38}$$

where $\eta = \sqrt{x}\,\mathrm{e}^{\mathrm{i}\varphi}$, $d\rho(\eta^*,\eta) = \mathrm{d}\varphi\mathrm{d}x\,\sigma(x)/2\pi d_0^2(\sqrt{x})$ and the probability density σ is again given via its moments:

$$\int_0^\infty \mathrm{d}x\, x^n \sigma(x) = 16^n\, n!\,(\gamma+\tfrac{5}{2})_n\,(\tfrac{1}{2}+\tfrac{\varepsilon}{2})_n\,(\tfrac{3}{2}+\tfrac{\varepsilon}{2})_n\,, \quad n = 0,1,2,\ldots\,. \tag{39}$$

As in the case of broken SUSY σ can be expressed in terms of a Meijer G-function and explicitly reads

$$\sigma(x) = \frac{G^{40}_{04}\left(\frac{x}{16}\,\middle|\,0,\gamma+\frac{3}{2},\frac{\varepsilon}{2}-\frac{1}{2},\frac{\varepsilon}{2}+\frac{1}{2}\right)}{16\,\Gamma(\gamma+\frac{5}{2})\,\Gamma(\frac{\varepsilon}{2}+\frac{1}{2})\,\Gamma(\frac{\varepsilon}{2}+\frac{3}{2})}\,. \tag{40}$$

6. Final remarks

Starting from the cubic algebra formed by the ladder operators of CES Hamiltonians related to the radial harmonic oscillator we have constructed the associated non-linear coherent states. These states are different to those obtained recently [21] via the Darboux transformation from standard (linear) coherent states [8]. The present non-linear coherent states have been shown to be minimum uncertainty states with respect to the X_1-X_2 uncertainty relation.

In the present approach we have constructed non-linear coherent states as eigenstates of the annihilation operator (method (ii)), which turn out to be equivalent to those defined as minimum uncertainty states (method (iii)). It would also be of interest to find similar states which equalise other uncertainties like H_--X_1 or H_--X_2, and find their relations to the present one. Another interesting possibility is to construct in a similar way coherent states related to other CES potentials. For example, those related to the CES potentials which are SUSY partners of the linear harmonic oscillator. Here the algebra formed by the ladder operators closes a quadratic algebra and SUSY is unbroken [4, 5]. In fact, in doing so [22] one finds other non-linear coherent states which generalise those previously constructed by Fernández et al [23].

Acknowledgments

One of the authors (GJ) would like to thank P.P. Kulish, H. Leschke, A. Odzijewicz, J. Trost and S. Warzel for valuable comments and suggestions.

References

[1] Abraham P B and Moses H E 1980 *Phys. Rev.* A **22** 1333
[2] de Souza Dutra A 1993 *Phys. Rev.* A **47** R 2435
[3] Dutt R, Khare A and Varshni Y P 1995 *J. Phys. A: Math. Gen.* **28** L 107

[4] Junker G and Roy P 1997 *Phys. Lett.* A **232** 155
[5] Junker G and Roy P 1997 *quant-ph/9709021*
[6] Junker G and Roy P 1998 *quant-ph/9803024* to appear in *Ann. Phys., NY*
[7] Junker G 1996 *Supersymmetric Methods in Quantum and Statistical Physics* (Berlin: Springer-Verlag)
[8] Klauder J R and Skagerstam B-S 1985 *Coherent States, Applications in Physics and Mathematics* (Singapore: World Scientific)
Perelemov A 1986 *Generalized Coherent States and Their Applications* (Berlin: Springer-Verlag)
[9] Nieto M M and Simmons Jr L M 1979 *Phys. Rev.* A **19** 438
[10] Roček M 1991 *Phys. Lett.* B **255** 554
[11] Karassiov V P 1994 *J. Phys. A: Math. Gen.* **27** 153
[12] Katriel J and Quesne C 1996 *J. Math. Phys.* **37** 1650
[13] Zhang W-M, Feng D H and Gilmore R 1990 *Rev. Mod. Phys.* **62** 867
[14] Barut A O and Girardello L 1971 *Comm. Math. Phys.* **21** 41
[15] Biedenharn L C 1989 *J. Phys. A: Math. Gen.* **22** L 873
Odzijewicz A 1998 *Comm. Math. Phys.* **192** 183
[16] Akhiezer N I 1965 *The Classical Moment Problem and some related questions in analysis* (Edinburgh: Oliver & Boyd)
[17] Erdélyi A, Magnus W, Oberhettinger F and Tricomi F G 1954 *Tables of Integral Transforms*, vol I, (New York: McGraw-Hill)
[18] Erdélyi A, Magnus W, Oberhettinger F and Tricomi F G 1953 *Higher Transcendental Function*, vol I, (New York: McGraw-Hill)
[19] Aragone C, Guerri G, Salamó S and Tani J L 1974 *J. Phys. A: Math. Gen.* **7** L 149
[20] Agarwal G S and Tara K 1991 *Phys. Rev.* A **43** 492
[21] Bagarov V G and Samsonov B F 1996 *J. Phys. A: Math. Gen.* **29** 1011
Samsonov B F 1998 *J. Math. Phys.* **39** 967
[22] Cannata F, Junker G and Trost J 1998 *quant-ph/9806080*
[23] Fernández C D J, Hussin V and Nieto L M 1994 *J. Phys. A: Math. Gen.* **27** 3547
Fernández C D J, Nieto L M and Rosas-Ortiz O *J. Phys. A: Math. Gen.* **28** 2693