

SHIFT OF SPECTRAL LINES DUE TO DYNAMIC MULTIPLE SCATTERING AND SCREENING EFFECT : IMPLICATIONS FOR DISCORDANT REDSHIFTS

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Abstract

The frequency shift of spectral lines from astronomical objects is most often explained by the Doppler Effect arising in relative motion and the broadening of a particular line is supposed to depend on the absolute temperature, pressure or the different line of sight velocities. The Wolf effect on the other hand deals with correlation induced spectral changes and explains both the broadening and shift of the spectral lines. In this framework a sufficient condition for redshift has been derived and when applicable the shift is shown to be larger than broadening. Under this condition of larger shift than broadening we find a critical source frequency below which no spectrum is analyzable for a particular medium. This gives rise to new type of screening effect which may play a significant role both at laboratory scale as well as in the astronomical domain. We apply a simple interpretation of the discordant redshifts in galaxy-quasar associations.

Keywords : multiple scattering, spectral line shift, screening effect, critical source frequency.

1 Introduction

In studying the motion of astronomical objects, astrophysicists utilize the study of frequency shift of spectral lines. A redshift of spectral lines is the phenomenon of displacement of the lines towards longer wavelengths. Although for a lot of astronomical objects such as stars, redshifts and blueshifts are easily understood in terms of relative motion, the origin of redshift of quasars has been one of the most controversial topics for the last few decades. Interpretation of redshift as Doppler shift has been broadly accepted, although this could not solve the observed anomalies in the quasar redshifts and some related problems(Wolf,1998).

Recent results in Statistical Optics (Wolf,1986) has made it clear that the shift of the frequency of the spectral lines can be explained without considering the relative motion of the observer with respect to the source. Following this idea, dynamic multiple scattering theory(Datta et al. 1998a,b) has been developed to account for the shift as well as the broadening of the spectral line. It is shown that when light passes through a turbulent(or inhomogeneous) medium, due to multiple scattering effects the shift and the width of the line can be calculated. Here, a sufficient condition for redshift has been derived and when applicable the shift is shown to be larger than broadening. The width of the spectral line can be calculated after multiple scatterings and a relation can be derived between the width and the shift z which applies to any value of z (Roy et al.1999).

Using the above condition it is shown that there exists a lower bound of the source frequency below which no spectra is analyzable in the sense that the broadening will be larger than the shift of the spectral line and the identity of the line becomes confused. This critical source frequency depends very much on the actual nature of the medium through which the light is propagating. We call this the screen effect due to the medium. This new screen effect may play an important role in astronomical domain. This can also be verified in laboratory experiments.

We first discuss briefly the theory of multiple scattering in section 2. In section 3 we shall introduce the critical source frequency and its implications while in section 4 we shall discuss a special type of screening induced by critical source frequency. Finally, a

general discussion will be made on the possible applications of this type of screening.

2 Dynamic Multiple Scattering Theory

First, we briefly state the main results of Wolf's scattering mechanism. Let us consider a polychromatic electromagnetic field of light of central frequency ω_0 and width δ_0 , incident on the scatterer. The incident spectrum is assumed to be of the form

$$S^{(i)}(\omega) = A_0 e^{\left[-\frac{1}{2\delta_0^2}(\omega-\omega_0)^2\right]} \quad (1)$$

The spectrum of the scattered field is given by (Wolf and James,1996)

$$S^{(\infty)}(r\vec{u}', \omega') = A\omega'^4 \int_{-\infty}^{\infty} K(\omega, \omega', \vec{u}, \vec{u}') S^{(i)}(\omega) d\omega \quad (2)$$

which is valid within the first order Born approximation (Born and Wolf ,1997). Here $K(\omega, \omega')$ is the so called scattering kernel and it plays the most important role in this mechanism. Instead of studying $\mathcal{K}(\omega, \omega')$ in detail, we consider a particular case for the correlation function $G(\vec{R}, T; \omega)$ of the generalized dielectric susceptibility $\eta(\vec{r}, t; \omega)$ of the medium which is characterized by an anisotropic Gaussian function

$$\begin{aligned} G(\vec{R}, T; \omega) &= \langle \eta^*(\vec{r} + \vec{R}, t + T; \omega) \eta(\vec{r}, t; \omega) \rangle \\ &= G_0 \exp \left[-\frac{1}{2} \left(\frac{X^2}{\sigma_x^2} + \frac{Y^2}{\sigma_y^2} + \frac{Z^2}{\sigma_z^2} + \frac{c^2 T^2}{\sigma_\tau^2} \right) \right] \end{aligned} \quad (3)$$

Here G_0 is a positive constant, $\vec{R} = (X, Y, Z)$, and $\sigma_x, \sigma_y, \sigma_z, \sigma_\tau$ are correlation lengths. The anisotropy is indicated by the unequal correlation lengths in different spatial as well as temporal directions. $\mathcal{K}(\omega, \omega')$ can be obtained from the four dimensional Fourier Transform of the correlation function $G(\vec{R}, T; \omega)$. In this case $\mathcal{K}(\omega, \omega')$ can be shown to be of the form

$$\mathcal{K}(\omega, \omega') = B \exp \left[-\frac{1}{2} \left(\alpha' \omega'^2 - 2\beta \omega \omega' + \alpha \omega^2 \right) \right] \quad (4)$$

where

$$\left. \begin{aligned} \alpha &= \frac{\sigma_x^2}{c^2} u_x^2 + \frac{\sigma_y^2}{c^2} u_y^2 + \frac{\sigma_z^2}{c^2} u_z^2 + \frac{\sigma^2}{c^2} \\ \alpha' &= \frac{\sigma_x^2}{c^2} u_x'^2 + \frac{\sigma_y^2}{c^2} u_y'^2 + \frac{\sigma_z^2}{c^2} u_z'^2 + \frac{\sigma^2}{c^2} \\ \text{and } \beta &= \frac{\sigma_x^2}{c^2} u_x u_x' + \frac{\sigma_y^2}{c^2} u_y u_y' + \frac{\sigma_z^2}{c^2} u_z u_z' + \frac{\sigma^2}{c^2} \end{aligned} \right\} \quad (5)$$

Here $\hat{u} = (u_x, u_y, u_z)$ and $\hat{u}' = (u_x', u_y', u_z')$ are the unit vectors in the directions of the incident and scattered fields respectively.

Substituting (1) and (4) in (2), we finally get

$$S^{(\infty)}(\omega') = A' e^{\left[-\frac{1}{2\delta_0'^2} (\omega' - \bar{\omega}_0)^2 \right]} \quad (6)$$

where

$$\left. \begin{aligned} \bar{\omega}_0 &= \frac{|\beta| \omega_0}{\alpha' + \delta_0^2 (\alpha \alpha' - \beta^2)} \\ \delta_0'^2 &= \frac{\alpha \delta_0^2 + 1}{\alpha' + \delta_0^2 (\alpha \alpha' - \beta^2)} \\ \text{and } A' &= \sqrt{\frac{\pi}{2(\alpha \delta_0^2 + 1)}} A B A_0 \omega_0'^4 \delta_0 \exp \left[\frac{|\beta| \omega_0 \bar{\omega}_0 - \alpha \omega_0^2}{2(\alpha \delta_0^2 + 1)} \right] \end{aligned} \right\} \quad (7)$$

Though A' depends on ω' , it was approximated by James and Wolf (James and Wolf, 1990) to be a constant so that $S^{(\infty)}(\omega')$ can be considered to be Gaussian.

The relative frequency shift is defined as

$$z = \frac{\omega_0 - \bar{\omega}_0}{\bar{\omega}_0} \quad (8)$$

where ω_0 and $\bar{\omega}_0$ denote the unshifted and shifted central frequencies respectively. We say that the spectrum is redshifted or blueshifted according to whether $z > 0$ or $z < 0$, respectively. Here

$$z = \frac{\alpha' + \delta_0^2 (\alpha \alpha' - \beta^2)}{|\beta|} - 1 \quad (9)$$

It is important to note that this z -number does not depend on the incident frequency, ω_0 . This is a very important aspect if the mechanism is to apply in the astronomical domain. Expression (9) implies that the spectrum can be shifted to the blue or to the red, according to the sign of the term $\alpha' + \delta_0^2 (\alpha \alpha' - \beta^2) > |\beta|$. To obtain the

no-blueshift condition, we use Schwarz's Inequality which implies that $\alpha\alpha' - \beta^2 \geq 0$. Thus, we can take

$$\alpha' > |\beta|$$

as the sufficient condition to have only redshift by this mechanism.

Let's now assume that the light in its journey encounters many such scatterers. What we observe at the end is the light scattered many times, with an effect as that stated above in every individual process.

Let there be N scatterers between the source and the observer and z_n denote the relative frequency shift after the n^{th} scattering of the incident light from the $(n-1)^{\text{th}}$ scatterer, with ω_n and ω_{n-1} being the central frequencies of the incident spectra at the n^{th} and $(n-1)^{\text{th}}$ scatterers. Then by definition,

$$z_n = \frac{\omega_{n-1} - \omega_n}{\omega_n}, \quad n = 1, 2, \dots, N$$

or,

$$\frac{\omega_{n-1}}{\omega_n} = 1 + z_n, \quad n = 1, 2, \dots, N$$

Taking the product over n from $n = 1$ to $n = N$, we get,

$$\frac{\omega_0}{\omega_N} = (1 + z_1)(1 + z_2) \dots (1 + z_N)$$

The left hand side of the above equation is nothing but the ratio of the source frequency and the final or observed frequency z_f . Hence,

$$1 + z_f = (1 + z_1)(1 + z_2) \dots (1 + z_N) \quad (10)$$

Since the z -number due to such effect does not depend upon the central frequency of the incident spectrum, each z_i depends on δ_{i-1} only, not ω_{i-1} [here ω_j and δ_j denote the central frequency and the width of the incident spectrum at $(j+1)^{\text{th}}$ scatterer]. To find the exact dependence we first calculate the broadening of the spectrum after N number of scatterings.

Here, we are considering the multiple scattering on the assumption that the scatterers are mutually incoherent. In this case the cross terms in the spectrum of the scattered field are zero. In other words we are not including higher order scatterings.

Moreover, we are also considering small angle scatterings so that there will be small deviation from the actual path of light. As a result the superimposed spectra from different scatterings will result in a signal in the forward direction only. This is similar to the single scattering result of James and Wolf.

2.1 Effect of Multiple Scatterings on the Spectral Line Width

From the second equation in (7), we can easily write,

$$\left. \begin{aligned} \delta_{n+1}^2 &= \frac{\alpha\delta_n^2+1}{\alpha'+(\alpha\alpha'-\beta^2)\delta_n^2} \\ &= \left(\frac{\alpha\delta_n^2+1}{\alpha'}\right) \left[1 + \delta_n^2 \left(\frac{\alpha\alpha'-\beta^2}{\alpha'}\right)\right]^{-1} \end{aligned} \right\} \quad (11)$$

From (13), we can also write

$$\omega_{n+1} = \frac{\omega_n|\beta|}{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2} \quad (12)$$

Then from (11) & (12), we can write

$$\left. \begin{aligned} z_{n+1} &= \frac{\omega_n - \omega_{n+1}}{\omega_{n+1}} \\ &= \frac{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2}{|\beta|} - 1 \\ &= \frac{\alpha'}{|\beta|} \left\{ 1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right) \delta_n^2 \right\} - 1 \end{aligned} \right\} \quad (13)$$

Let's assume that the redshift per scattering process is very small, *i.e.*,

$$0 < \epsilon = z_{n+1} \ll 1$$

for all n .

Then,

$$1 + \epsilon = \frac{\alpha'}{|\beta|} \left\{ 1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right) \delta_n^2 \right\}$$

$$\text{or, } (1 + \epsilon)\frac{|\beta|}{\alpha'} = 1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right) \delta_n^2$$

After N number of scatterings, this sharpness ratio, say Q_N , is given by the following recurrence relation :

$$Q_{N+1} = Q_N \sqrt{\frac{\alpha'}{\alpha' + (\alpha\alpha' - \beta^2)\delta_N^2} - \frac{1}{\alpha\delta_N^2 + 1}}.$$

It is easy to verify that the expression under the square root lies between 0 & 1. Therefore, $Q_{N+1} < Q_N$, and the line is broadened as the scattering proceeds on .

Under the sufficient condition of redshift [i.e., $|\beta| < \alpha'$](Datta et al.,1998c) it was shown that in the observed spectrum

$$\Delta\omega_{n+1} \gg \delta_n \quad (17)$$

if the following condition holds:

$$\frac{\delta_n \omega_0 (\alpha\alpha' - \beta^2)}{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2} \gg 1 \quad (18)$$

where ω_0 is the source frequency.

The relation (18) signifies that the shift is more prominent than the effective broadening so that the spectral lines are observable and can be analyzed. If, on the other hand ,the broadening is higher than the shift of the spectral line, it will be impossible to detect the shift from the blurred spectrum. Hence we can take relation (18) to be one of the conditions necessary for the observed spectrum to be analyzable. For large N (i.e., $N \rightarrow \infty$), the series in the second term of right hand side of (16) converges to a finite sum and we get

$$\delta_{N+1}^2 = \left(\frac{\beta^2}{\alpha'^2}\right)^{N+1} \delta_0^2 + \frac{\alpha'}{\alpha'^2 - \beta^2}.$$

If δ_0 is considered as arising out of Doppler broadening only, we can estimate $\delta_{\text{Dop}} \sim 10^9$ for $T = 10^4$ K. On the other hand, for an anisotropic medium, we can take $\sigma_x = \sigma_y = 3.42 \times 10^{-1}$, $\sigma_z = 8.73 \times 10^{-1}$, $\alpha' = 8.68 \times 10^{-30}$, $\alpha = 8.536 \times 10^{-30}$ and $\beta = 8.607 \times 10^{-30}$ for $\theta = 15^\circ$ (James and Wolf,1990). Then the second term of the above expression will be much larger than the first term, and effectively, Doppler broadening can be neglected in comparison to that due to multiple scattering effect.

Now if we consider the other condition, i.e., $\alpha' < |\beta|$, the series in (16) will be a divergent one and δ_{N+1}^2 will be finitely large for large but finite N . However, if the

condition (17) is to be satisfied, then the shift in frequency will be larger than the width of the spectral lines. In that case the condition $\alpha' < |\beta|$ indicates that blueshift may also be observed but the width of the spectral lines can be large enough depending on how large the number of collisions is. So in general, the blueshifted lines should be of larger widths than the redshifted lines and may not be as easily observable.

3 Critical Source Frequency

Rearranging equation (18) we get,

$$\left(\delta_n - \frac{\omega_0}{2}\right)^2 \ll \frac{\omega_0^2}{4} - \frac{\alpha'}{\alpha\alpha' - \beta^2} \quad (19)$$

Since the left side is non-negative, the right side must be positive. Moreover, since the mean frequency of any source is always positive, *i.e.*, $\omega_0 \geq 0$, we must have

$$\omega_0 \gg \sqrt{\frac{4\alpha'}{\alpha\alpha' - \beta^2}}. \quad (20)$$

We take the right side of this inequality to be the *critical source frequency* ω_c which is defined here as.

$$\omega_c = \sqrt{\frac{4\alpha'}{\alpha\alpha' - \beta^2}}. \quad (21)$$

Thus for a particular medium between the source and the observer, the critical source frequency is the lower limit of the frequency of any source whose spectrum can be clearly analyzed. In other words, the shift of any spectral line from a source with frequency less than the critical source frequency for that particular medium cannot be detected due to its high broadening.

We can now classify the spectra of the different sources, from which light comes to us after passing through a scattering medium characterized by the parameters α , α' , β . If we allow only small angle scattering in order to get prominent spectra, according to the Wolf mechanism, they will either be blueshifted or redshifted. The redshift of spectral lines may or may not be detected according to whether the condition (20) does or does not hold. In this way those sources whose spectra are redshifted, are classified in two cases, *viz.*, $\omega_0 > \omega_c$ and $\omega_0 \leq \omega_c$. In the first case, the shifts of the spectral

lines can be easily detected due to condition (17). But in the later case, the spectra will suffer from the resultant blurring.

3.1 Critical Source Frequency and Anisotropy

The anisotropy discussed in this paper is statistical in nature. We have considered both temporal as well as spatial correlations to induce such spectral changes. The spatial anisotropy has been characterized by unequal correlation lengths along three mutually perpendicular directions. We have used the term *strong anisotropy* in a particular direction (among the above three) to specify that the correlation length along that direction is very large compared to that of the other two directions. We now find the critical source frequency in both the isotropic and anisotropic cases.

Spatially Isotropic Case

If the spatial correlation lengths are equal in every direction , i.e. $\sigma_x = \sigma_y = \sigma_z = \sigma$ then the parameters α, α', β reduce to

$$\alpha = \alpha' = \frac{\sigma^2}{c^2} + \frac{\sigma_\tau^2}{c^2}; \quad \beta = \frac{\sigma^2}{c^2} \cos \theta + \frac{\sigma_\tau^2}{c^2} \quad (22)$$

Therefore from (21) we get

$$\omega_c = \frac{2c}{\sigma} \sqrt{\frac{\sigma^2 + \sigma_\tau^2}{(1 - \cos \theta)\sigma^2(1 + \cos \theta) + 2\sigma_\tau^2}} \quad (23)$$

Therefore,

$$\begin{aligned} \omega_c &= \frac{2c}{\sigma} \operatorname{cosec} \theta; \quad \text{if } \sigma \gg \sigma_\tau \\ \omega_c &= \frac{2c}{\sigma} \sqrt{\frac{1}{2(1 - \cos \theta)}} \quad \text{if } \sigma \ll \sigma_\tau \end{aligned}$$

In both cases σ_c varies inversely with σ . This is equivalent to taking $\sigma_\tau = 0$ i.e. for white noise.

Spatially Anisotropic Case

Let there be a strong anisotropy along, say, the x-axis. Then $\sigma_x \gg \sigma_y, \sigma_z$. Consequently,

$$\alpha = \frac{\sigma^2}{c^2} + \frac{\sigma_\tau^2}{c^2}; \quad \alpha' = \frac{\sigma^2}{c^2} u_x'^2 + \frac{\sigma_\tau^2}{c^2}; \quad \beta = \frac{\sigma^2}{c^2} u_x u_x' + \frac{\sigma_\tau^2}{c^2} \quad (24)$$

We can define

$$k = \frac{\sigma_x}{\sigma_\tau}$$

as a ratio of the correlation lengths along the spatial and temporal directions. We now consider the variation of ω_c with k in the following two case :

1. σ_τ is fixed ; k varies with σ_x
2. σ_x is fixed ; k varies inversely with σ_τ .

The nature of the variation of ω_c against k in the above two cases has been studied by Datta et al.(Datta et al.1998a,b). We can easily find the critical source frequency for $k = 1$ as

$$\omega_c = \frac{2c\sqrt{u_x'^2 + 1}}{\sigma|u_x - u_x'|} \quad (25)$$

From (24) and (25) we can say that the stronger the anisotropy, the lower the critical source frequency and hence the greater the scope of more and more spectra to be analyzable. This induces a special type of screening that we are now going to discuss.

4 Screening Effect Induced by the Critical Source Frequency

Collisional mechanism of light beams give some sort of screening which may or may not show frequency shift of the spectral lines. Here the scatterer is the medium with its dielectric susceptibility randomly fluctuating both spatially and temporally. The critical source frequency induces a special type of screening which not only changes the colour of the incident lines but also broadens their widths. Specifically speaking, when an incident line with its peak frequency ω_0 encounters the scattering medium whose critical source frequency is ω_c , then

1. frequency shift occurs
2. the spectral width increases and this increased width will be less than or greater than the frequency shift according as ω_0 is greater or less than ω_c .

Let a spectral line with wavelength greater than the critical wavelength of the medium propagate and hence be scattered. The scattered line, according to the screening effect, will be of width greater than the shift of that line. Therefore, the observer will find the broadening of the line more prominent than its shift and hence it will be difficult

to analyze the line. On the other hand, if the wavelength of the line is less than the critical wavelength, its shift will dominate the broadening. In the case of strong spatial anisotropy (without loss of generality, we may assume it along the x-axis) it can be shown that the maximum value of this critical wavelength for a particular medium (i.e. for a particular set of the spatial correlation lengths) is given by the following relation:

$$\lambda_c = \pi \sqrt{\sigma_x^2 + \sigma_\tau^2} \quad (26)$$

Thus this type of screening involves only two parameters - the spatial and temporal correlation lengths. If we take a particular medium and may know its spatial and temporal correlation lengths , we can easily find that particular wavelength from (26) which enables us to get the nature of the screening. To be more specific, we can find out the wavelength zone beyond which the anlyzability of the spectral lines will be difficult.

5 Possible Implications for Quasar Redshifts

The screening effect as discussed above might play a significant role in observations of quasars as well as in the laboratory. If we consider the spectra originating from distant quasars, it might be possible to explain the large redshift and broadening by this type of screening. We may try to visualize this at least in a simple fashion for the redshifts in the case of NGC 4319 Galaxy and Markarian 205 Quasar pair as follows.

5.1 *Screening Effect and Galaxy-Quasar Pair*

The above mentioned screening effect gives in this case the simple explanation for the redshift controversy in a galaxy-quasar pair (Sulentic, 1989) [NGC 4319 is the Galaxy and Markarian 205 the quasar]. They appear to be connected in very deep photographs but the quasar's redshift is found to be much higher (z -number is almost 12 times) than that of the galaxy. This empirical evidence is normally taken as

one of the strongest arguments in favour of non-cosmological redshifts. Weedman (Weedman,1986) made a critical survey of this situation using statistical arguments for comparisons of positions in the sky for quasars and galaxies. It has been claimed (Weedman, 1986) that if one uses this kind of statistical test for several samples, none shows evidence for association of high redshift quasars with low redshift galaxies. This impacts on hypotheses of both non-cosmological redshifts and gravitational lensing. Here, we want to emphasize that this kind of association of high redshift quasars with low redshift galaxies is a possibility in our theory of shift of the spectral lines by multiple scatterings. We can try to visualise the situation in the following way :

We know very little little about the atmospheres surrounding those distant galaxies and quasars. We nevertheless, assume that they are very different. Therefore the medium through which we observe the quasar is much different from that for the galaxy. Two sets of parameters (α , α' , β) characterising the media thus give rise to two different critical source frequencies, say ω_G (for galaxy) and ω_Q (for quasar). Now if the lines emitted from the galaxy have frequencies less than ω_G , then the Wolf-shift cannot be observed. On the other hand, if those lines emitted from the quasar have greater frequencies than ω_Q , prominent Wolf-shifts will be observed. The Doppler shift (or the shift as interpreted in the expanding Universe picture) might be present in both cases. Thus the net shift observed in the case of the quasar will be higher than in the case of the galaxy. Let the z -number of the galaxy be (say $z_1 =$) a and the z -number of the quasar due to the Wolf effect be (say $z_2 =$) b . Again the resultant z number can be written as $1 + z = (1 + Z_1)(1 + z_2)$. Then

$$\begin{aligned} 12a &= a + b + ab \\ \Rightarrow b &= \frac{11a}{1+a} \end{aligned}$$

This indicates that the distance of the quasar is just the same as that of the galaxy, but the shift in the first case is much higher than that of the later. We can envisage this phenomena as some sort of *screening* operation for the Wolf Effect. In other words, new type of screening arises due to the nature of the medium . This will be studied in details in future work. Lastly, we should mention that the critical source frequency relating to the screening effect which plays a crucial role in explaining both redhsift

and spectral width can be tested in *laboratory experiments*.

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