GLOBAL STABILITY IN SPITE OF "LOCAL INSTABILITY" WITH LEARNING IN GENERAL EQUILIBRIUM MODELS*

Shurojit Chatterji and Subir Chattopadhyay**

WP-AD 97-11

*We wish to thank J. Mora and J.M. Grandmont for useful discussions, and two referees and the Editor for comments which improved the exposition. Part of this research was carried out while the authors were visiting CORE. Financial support from the IVIE, the Spanish government (DGICYT project PB92-0342 and CICYT project 94-1504), and the Commission of the European Communities (SPES CT91-0057), is gratefully acknowledged. Any remaining errors are our own.

**S. Chatterji: Indian Statistical Institute & University of Alicante, S. Chattopadhyay: University of Alicante.

Editor: **Instituto Valenciano de Investigaciones Económicas, S.A.**
Primera Edición Abril 1997.
ISBN: 84-482-1480-3
Depósito Legal: V-1308-1997
Impreso por Key, S.A.
C/. Cardenal Benlloch, 69, 46021-Valencia.
Impreso en España.

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ABSTRACT

It is known through earlier work that deterministic temporary equilibrium dynamics with least squares learning are locally divergent from a steady state whenever the initial parameter estimates of the agents is high. This paper establishes that the learning dynamics may be globally stable *in spite* of displaying "locally unstable" behaviour in realistitc economic models. We identify a simple property of nonlinear temporary equilibrium maps that guarantees that *all* trajectories converge to the steady state under the global dynamics with learning -in particular, the locally divergent trajectories are also driven back to the steady state. These seemingly contradictory results can be reconciled by observing that the dynamics with least squares learning are discontinuous at the steady state. We also identify temporary equilibrium maps for which an open set of locally divergent trajectories escapes to infinity while another open set returns to the steady state.

An application of each result to OLG economies is provided.

KEYWORDS: Least Squares Learning; Global Stability; Local Instability.

1. Introduction

Recent studies (Grandmont-Laroque [7], Grandmont [6]) of the stability of deterministic steady states have shown that the steady state is always locally unstable when agents use least squares regressions on the lagged values of the endogenous variable to predict deviations from the steady state value.[1]

The local instability obtained in [6] and [7] is attributable to the fact that agents extrapolate *all* trends in past data, in particular divergent ones, in deviations from the steady state. This causes a discontinuity in the learning dynamics at the steady state. The dynamics are locally divergent for an open cone of initial perturbations that may be arbitrarily close to the steady state, but generate sufficiently high initial parameter estimates. The dynamics may be locally convergent for another open cone of initial conditions that generate sufficiently low initial parameter estimates.[2]

In this paper we establish that the learning dynamics in general equilibrium models may be globally stable *in spite* of displaying local divergence in the sense described above.[3] So, local instability is not incompatible with agents eventually learning their way to a perfect foresight steady state.

We work with temporary equilibrium maps (TEMs) with the property that when agents form forecasts by simply projecting the last observed value of the state variable, i.e., "naively," the dynamics are "contracting" and converge to the steady state, and this "contracting" property is robust. We emphasize that all the TEMs that we consider exhibit the local divergence phenomenon.

Proposition 1 demonstrates the basic phenomenon by showing the existence of an open set of initial conditions (arbitrarily close to the steady state) that displays local divergence but the trajectories eventually return to the steady state under the global dynamics.

We next impose the restriction that the range of the TEM is bounded below; such a lower bound is typically a consequence of feasibility considerations.

We prove two global stability results. Theorem 1 proves global stability for the simplest specification of least squares learning where agents use only the last two observations of the state variable to form expectations. For this result, we consider the class of "contracting" TEMs whose range is bounded above for positive

[1] See [6] for a justification of this formulation in terms of "decentralized" learning behaviour.

[2] Similar results obtain with Bayesian learning (see [3]). The dynamics with learning are then differentiable. Here too the dynamics diverge locally whenever the prior mean, or, independently, the prior variance, is sufficiently large. Local convergence obtains when agents are sufficiently subjectively certain about low rates of growth.

[3] This seemingly contradictory phenomenon (of global stability in spite of local divergence) is a consequence of the discontinuity of the dynamics with learning at the steady state.

values of the independent variable (in addition to the range being bounded below).[4] Proposition 2 demonstrates the possibility of inflation for an open set of initial conditions if the TEM is allowed to be unbounded above for positive values of the independent variable.

Theorem 2 demonstrates global stability under the simple specification of least squares referred to above and also under recursive ordinary least squares, provided that the TEM is "contracting" and attention is restricted to trajectories along which the state variable remains bounded though, possibly, locally divergent.

The "contracting" property ensures that if the parameter estimate lies in the vicinity of the set of stationary trends [-1,1], i.e, in an interval of the form $[-1-\epsilon, 1+\epsilon]$, $\epsilon > 0$, then convergence occurs. The assumptions of the theorems ensure that the parameter estimate eventually get close to the interval $[-1, 1]$ and this guarantees global stability. Theorems 1 and 2 also prove that the parameter estimate converges.

The methods of the paper can be used to evaluate the stability issue in fully articulated models. Section 5 does so in the simplest specification of overlapping generations economies. The "contracting" property is satisfied for a class of preferences that belong to the gross substitutes variety and also for a class of preferences that lead to backward bending offer curves. It turns out that the critical factor deciding global stability is the endowment of the old. Global stability occurs for boundary endowments while with interior endowments, inflationary paths co-exist with paths that are locally divergent but globally convergent. In the interior endowments case too one can guarantee global stability, but at the cost of a "Projection" that restricts the forecasts to a compact set. The novel feature of our "Projection" is that it does allow for locally divergent behaviour.

Some extensions are discussed in Section 6. Proofs are in the Appendix.

2.1 The Model

In this subsection we specify the reduced form model that is used for the analysis and then present a preliminary lemma.

The primitive of the study is the TEM (Temporary Equilibrium Map) $F(\cdot)$, which describes the dependence of the current value of the state-variable x_t, assumed to be a real number, on its point expectation for the next period, x^e_{t+1},

$$x_t = F(x^e_{t+1}). \tag{1}$$

The steady state value of the state variable is 0 which is the fixed point of the map

[4] The range of the TEM is allowed to be unbounded above for negative values of the independent variable.

$F(\cdot)$.[5] D is the domain of the map $F(\cdot)$ and will be assumed to be unbounded above; the underlying economic model might require D to be bounded below.[6] $F(\cdot)$ will be assumed to be a smooth function around 0.

Assumption F.1: (i) $F : D \to R$, where $D = (-\infty, +\infty)$ or $D = (-d, +\infty)$, $d > 0$, (ii) $F(0) = 0$, (iii) $F(\cdot)$ is continuous on D and is smooth around 0, (iv) $F(D) \subset D$.[7]

The principal property that we require of the map $F(\cdot)$ is phrased in terms of the dynamical system with "naive learning," which obtains when $x^e_{t+1} := x_{t-1}$ for every t. With this specification of expectations formation, the dynamical system with learning is described by the map $x_t = F(x_{t-1})$. Note that $|F(x)| < k|x|$, for every $x \in D$, for some $k \in [0, 1)$, implies that the steady state is globally stable under the dynamics with "naive learning." The property that we require is a slight strengthening of this condition. Formally:

Assumption F.2:[8] There exist $\beta^{*^2} > 1$ and $k^* \in [0, 1)$ such that, (i) $|F(\beta^{*^2} x)| \leq k^*|x|$ for every $x \in F(D)$, where (ii) if $D = (-d, +\infty)$ then $\beta^{*^2} x \in D$ for every $x \in F(D)$.

F.1 and F.2 will be treated as maintained hypotheses; F.2 (ii) requires that if $D = (-d, +\infty)$ then, for some $K_d > 0$, $-d < -K_d \leq F(x^e)$ for every $x^e \in D$.[9]

In fully specified general equilibrium models, feasibility considerations usually imply that the range of the TEM is bounded below. Hence, we impose:

Assumption F.3: There exists $K > 0$ such that $-K \leq F(x^e)$ for every $x^e \in D$.[10]

Under F.1-F.3, the range of the TEM is an interval which is bounded below (no upper bound has been imposed on the range); furthermore, the map has a unique fixed point.

[5] This specification of the TEM corresponds to a situation in which the variable of interest is denoted by, say X, the agents *know* the steady state value, say X^*, and can use the deviation $x := X - X^*$ in their computations since X_t is observed and X^* is known (as in [6], [7], and [8]).

[6] If the variable is a price then it is non-negative so that the state variable x, being a deviation, will be bounded below. If the state variable is the deviation of the logarithm of a price from the logarithm of the steady state value, D will be the entire real line.

[7] Of course, $F(\cdot)$ is assumed to be nontrivial; otherwise, local instability cannot occur.

[8] [9] imposes F.2 to analyse the global dynamics with learning in a one good OLG economy with time invariant differentiable forecasting functions. A local version of F.2 is used in the analysis of linear models in [8] and [10], and [5], [6], and [7] (who linearize a nonlinear model to get a TEM of the form $x_t = ax^e_{t+1}$ which satisfies F.2 when $|a| < 1$).

[9] Note that $F(D) \subset D$, F.1 (iv), is a consistency restriction on beliefs; so the requirement $-d < -K_d$ is a slight strengthening of this condition when D is bounded below.

[10] For $D = (-d, +\infty)$, F.3 is implied by F.2 (ii) as noted above.

The next assumption requires that the range of the TEM be bounded above for *positive* values of x^e; since no restriction is imposed for negative values of x^e, the range may be unbounded above.

Assumption F.4: There exists Q such that $F(x^e) \leq Q$ for every $x^e > 0$.

Given F.1, Q must be non-negative.

The last assumption allows for TEMs that are not bounded above for positive values of x^e and imposes a regularity condition by requiring it to have a positive asymptote (for $x^e > 0$) up to translation.

Assumption F.5: There exist $X^* > 0$ and $\theta \in (0, 1)$ such that
(i) $X^* + F(x^e) > \theta(X^* + x^e)$ for every $x^e \in D$, (ii) $\lim_{x^e \to +\infty} \frac{X^* + F(x^e)}{X^* + x^e} = \theta$.

To give an idea about the underlying economies which generate TEMs satisfying these assumptions, we consider OLG economies (the details are presented in Section 5). The state variable is the deviation of the market clearing price from the price in the monetary steady state; hence, D is bounded below. Figure 1 illustrates. F.1 is always satisfied. F.2 will be satisfied in the gross substitutes case and also when the offer curve bends backwards but income effects are not too strong (Figure 1 (a), (b), and (d)).[11] F.3 is implied by the non-negativity of the consumption of the young (Figure 1 (a)-(d)). F.4 will hold in the case in which the second period endowment is zero and "money is essential"—as in Brock and Scheinkman [2] (Figure 1 (d)). F.5 holds whenever the endowment is interior (Figure 1 (a)-(c)).

We turn to how expectations are formed.

Agents' beliefs about the dynamics will be assumed to be summarized by a model of the form (as in [6], [7], and [8])[12]

$$x_{t+1} = \beta x_t + \epsilon_{t+1} \tag{2}$$

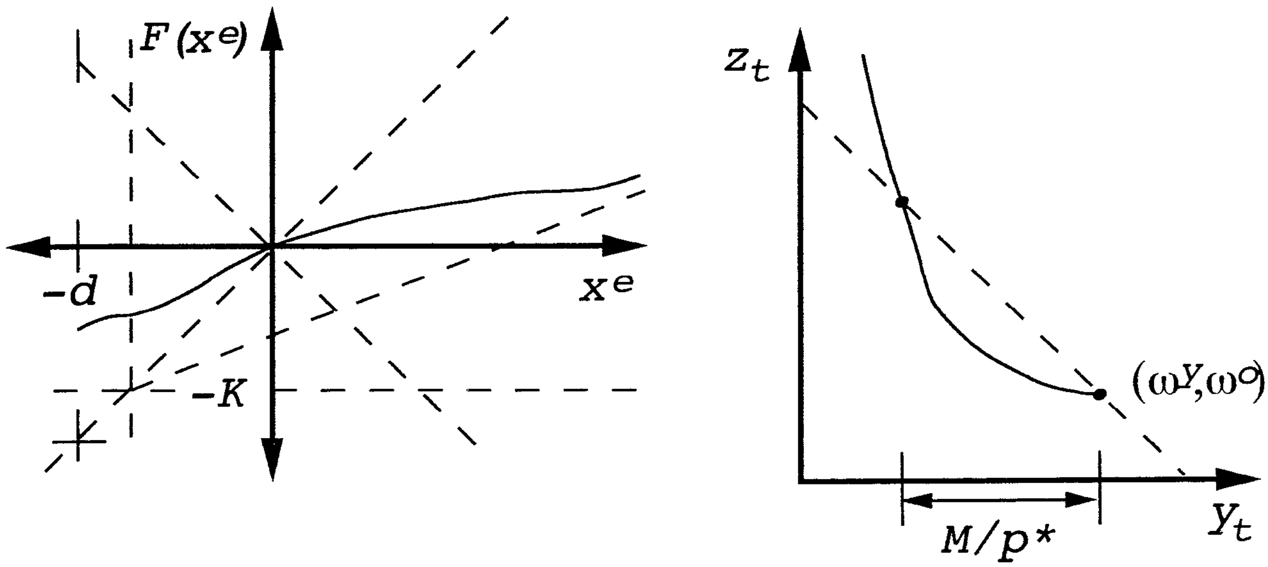
where ϵ is white noise. At time t, agents will be assumed to use information upto $t-1$ (as is standard in the literature) to generate their forecast as follows.

Expectations E.1: Agents' predictions are given by

$$x^e_{t+1} = \hat{x}^e_{t+1} := \beta^2_{t-1} x_{t-1} \qquad \text{if } D = (-\infty, +\infty) \tag{3a}$$

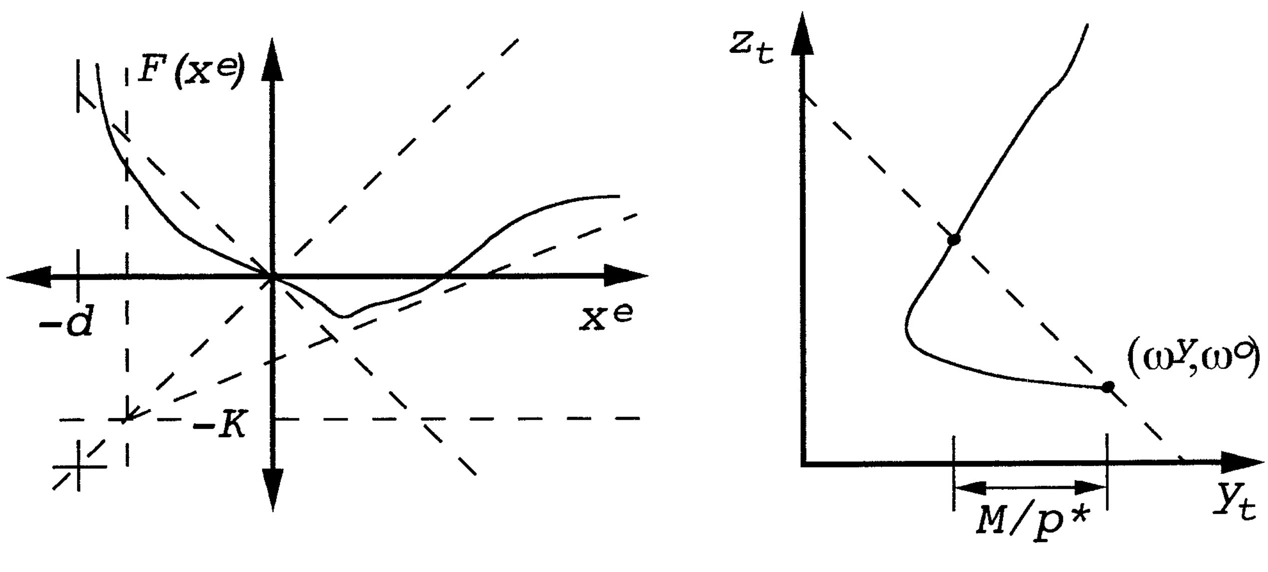
[11] In particular, if cycles of period two exist in the perfect foresight dynamics, then F.2 will *not* hold (Figure 1 (c)).

[12] These models are appropriate for local analysis around the steady state. Since our interest is in global dynamics, it would appear that we should consider more general specifications instead of a "myopic" linear view of the world. Our results indicate that, in spite of the local instability phenomenon associated with these linear models that has been documented in the literature, the global dynamics might well behave nicely so that the agent might well want to continue to believe in a linear world. Nonetheless, further research should consider the case of more general specifications.



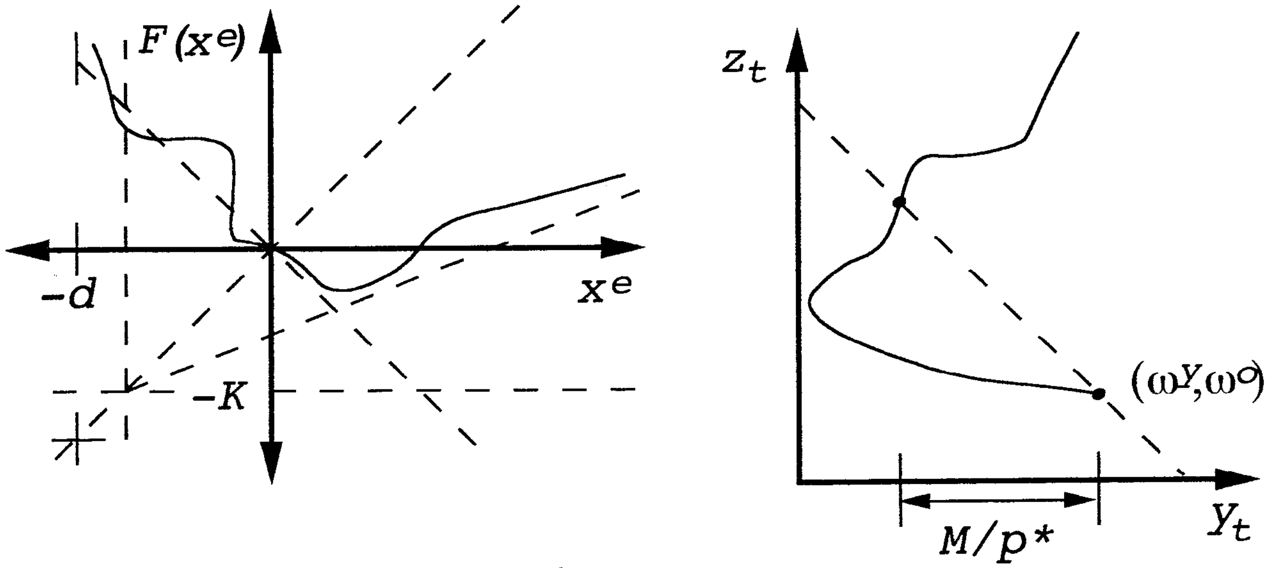
The Gross Substitutes Case

FIGURE 1(a)



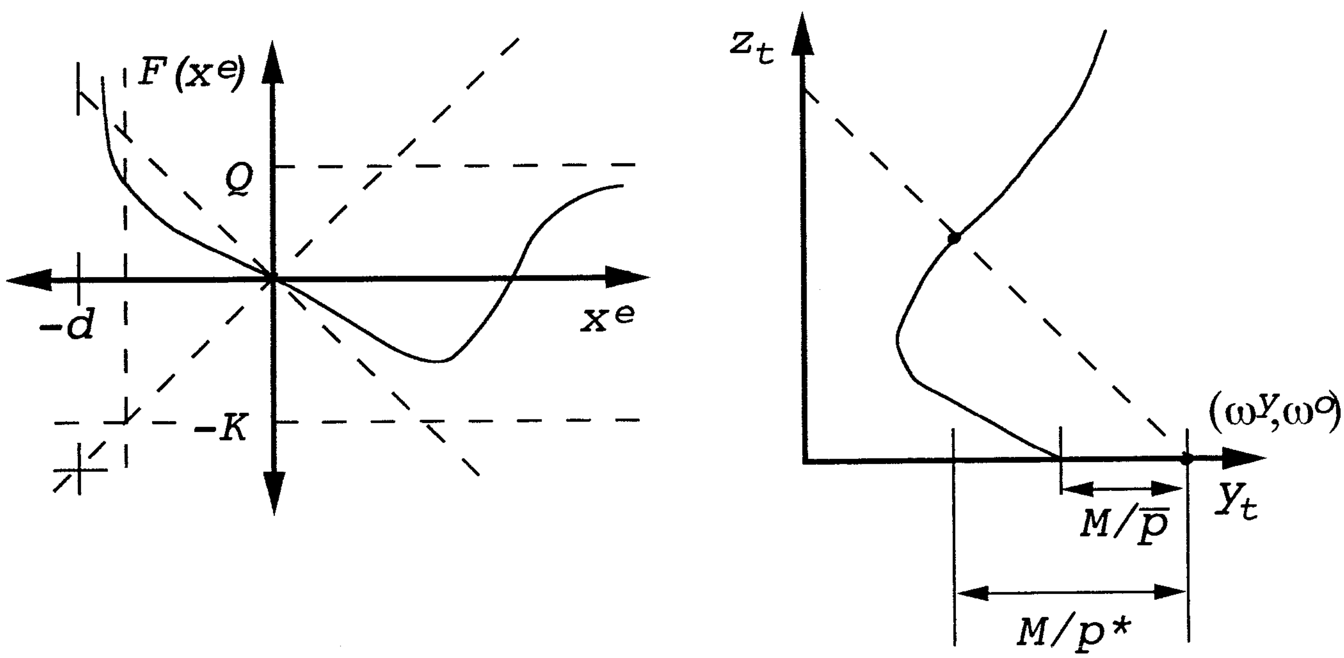
The Backward Bending Case

FIGURE 1 (b)



A 2-Period Cycle

FIGURE 1(c)



Boundary Endowments

FIGURE 1(d)

or $D = (-d, +\infty)$ and $\hat{x}^e_{t+1} > -\delta$ (3a)

$$x^e_{t+1} = \frac{1}{1-(\delta+\hat{x}^e_{t+1})}(-\delta) + \frac{-(\delta+\hat{x}^e_{t+1})}{1-(\delta+\hat{x}^e_{t+1})}(-d) \quad \text{if } D = (-d, +\infty) \text{ and } \hat{x}^e_{t+1} \leq -\delta, \tag{3b}$$

where $\delta > 0$ satisfies $-\delta \in (-d, \beta^{*^2}(-K_d))$.

Discussion of E1: Notice that (3b) of E.1 is required *only if* $D = (-d, +\infty)$ since in this case the dynamics are not defined if $\hat{x}^e_{t+1} \notin D$. The existence of δ follows from F.2 (ii). For $\beta^2 \in [0, \beta^{*^2}]$, and any $x \geq -K_d$, $\hat{x}^e > -\delta$ so that (3b) need not be invoked; consequently, on the set of parameter estimates $[0, \beta^{*^2}]$, (3b) does not interfere with the dynamics, given the maintained hypotheses F.1 and F.2.

$-\delta$ should be interpreted as the agents' believed value of the state variable when the linear law (2) gives way to (3b). Since $-\delta < -K_d$ (since $\delta \in (\beta^{*2}K_d, d)$ and $\beta^{*^2} > 1$), and since $-K_d$ is a lower bound on the realizations of the state variable, the agents' beliefs are never contradicted since the state variable never enters the region where the "linear view of the world" is not valid. (3b) does not artificially restrict the domain as we have allowed forecasts to go to the boundary of D.

Even with (3b), agents *will* extrapolate sufficiently high growth rates in all directions so as to produce local instability. Furthurmore, the mechanics producing global stability or inflation will, in cases where D is bounded below, be the same as in the case where D is the entire real line (and hence (3b) is not invoked). So E.1 does *not* act as a "Projection" (used in earlier literature—see Discussion following E.2 in Section 5) that effectively eliminates certain kinds of divergent behaviour. We need (3b) merely to get a well defined problem.

To fully specify the system with learning, we need to specify how the parameter estimates are updated over time. We consider the following two variants of least squares learning, SL (simple learning) and RL (recursive ordinary least squares):

[**SL**] This is the simplest specification of least squares learning—agents simply extrapolate the most recent adjustment rate in the dynamics of the state variable.

$$\beta_t = \frac{x_t}{x_{t-1}}. \tag{4}$$

[**RL**] In the case where the agents' memory is unbounded, one obtains the following recursive formulation of ordinary least squares learning (e.g., [6])

$$\beta_t = m(\omega_{t-1}x_{t-1})\beta_{t-1} + [1 - m(\omega_{t-1}x_{t-1})]\frac{F(\beta^2_{t-1}x_{t-1})}{x_{t-1}} \tag{5}$$

$$\omega^2_t = m(\omega_{t-1}x_{t-1})\omega^2_{t-1} \tag{6}$$

with $m(z) = \frac{1}{1+z^2}$, and subject to the initial conditions $\omega^2_0 = \frac{1}{\sum_{-L}^{-1} x^2_j}$ and $\beta_0 = \frac{\sum_{-L}^{-1} x_j x_{j+1}}{\sum_{-L}^{-1} x^2_j}$.

For later reference, we note the following standard representation of RL:

$$\beta_t = \frac{\sum_{-L}^{t-1} x_j x_{j+1}}{\sum_{-L}^{t-1} x_j^2}. \tag{5'}$$

Initial conditions for the economy are given by (x_0, β_0) where β_0 is formed using $x_0, \cdots, x_{-L}, x_j \neq 0$ for all j.

The dynamical system with SL estimation is defined by (1), (3), and (4). The dynamical system with RL estimation is defined by (1), (3), (5), and (6).

Remark 1: Note that if, along some trajectory, $x_t = 0$ for some t, then trivially $x_t \to 0$; we do not study such trajectories.

This completes the description of the model.

We end this subsection with a fundamental implication of F.2. Lemma 1 shows the existence of an invariant set (in the vicinity of the set $[-1, 1]$) for the dynamics under SL and RL. This set is labelled I and has the property that, for any initial condition in I, the dynamics of the state variable converge to zero and the parameter estimates also converge. The stability results will follow by ensuring that the parameter estimate enters the set I at some stage.

Lemma 1: *Assume F.1 and F.2. Define the set I by $I := [-\beta^*, \beta^*]$.*
(i) Consider (x_t, β_t) under SL.
$\beta_t \in I$ implies $\beta_{t+j} \in I$ for all $j > 0$, and $x_{t+j} \to 0$, $\beta_{t+j} \to 0$.
(ii) Consider (x_t, β_t, ω_t) under RL.
$\beta_t \in I$ implies $\beta_{t+j} \in I$ for all $j > 0$, and $x_{t+j} \to 0$, $\beta_{t+j} \to \bar{\beta}$, $\omega_{t+j} \to \bar{\omega}$.

2.2 Convergence of an Open Set of Locally Divergent Trajectories

In this subsection we show that under F.1 and F.2, there always exists an open set of initial conditions arbitrarily close to the steady state displaying local instability initially but eventually returning to the steady state. The result is proved for the SL case for TEMs whose range is unbounded above; it holds even if the TEM is a linear function.

Proposition 1: *Assume F.1 and F.2, that $F(D)$ is unbounded above, and consider the SL case. Consider $(-\bar{x}, \bar{x}) \subset F(D)$ for $\bar{x} > 0$ arbitrarily small, and choose any $x_0 \in F(D)$ such that $x_0 \notin (-\bar{x}, \bar{x})$ and $-x_0 \in F(D)$ in case $F(D)$ is bounded below. Then there exists U, an open subset of $\{(x, \beta) |\ x \in (-\bar{x}, \bar{x}),\ |\beta| > \beta^*\}$, in the space of initial conditions (x, β), and V, an open subset of $\{(x_0, \beta) |\ \beta^* > |\beta| > 1\}$, and an integer N, such that in at most N iterates every point in U enters the set V. Moreover, $(x_0, \beta_0) \in V$ implies that $x_t \to 0$ and $\beta_t \to 0$.*

To illustrate the result obtained, we consider a TEM which satisfies F.1, F.2, and F.5 and, in addition, is monotone increasing. It is more convenient to parametrize the system in the space (x_{t-1}, x_t). Figure 2 illustrates. R^2_+ is an invariant set for such a TEM since $x_{t-1} > 0$ implies that $x^e_{t+1} = \beta^2_{t-1} x_{t-1} > 0$, so that $x_t > 0$ due to the monotonicity of the TEM. So, for $x_0 > 0$, $x_t > 0$ and $\beta_t := \frac{x_t}{x_{t-1}} > 0$ for all t.

The subset of the quadrant below the ray $\frac{x_t}{x_{t-1}} = \beta^*$ is the invariant set I; initial conditions in this set are driven to the steady state value. Now fix an $\bar{x} > 0$. The entire open cone (shaded in the figure) can be traced to points in the set labelled $0\bar{x}C$; that is given any point (x_{-1}, x) in the shaded region, there exists a point (x_{-N-1}, x_{-N}) in $0\bar{x}C$ such that the trajectory starting at (x_{-N-1}, x_{-N}) contains the point (x_{-1}, x). In particular, any open set V in the shaded region can be traced back to an open set U in a neighbourhood of the origin indicating local instability. But as remarked earlier, any trajectory that enters I converges to $(0,0)$. Also, as Proposition 2 in Section 3 will show, for sufficiently high values of β_0 the trajectories diverge to infinity; in the diagram this correponds to an open set of points in the set $0\bar{x}C$ for which the induced β values are sufficiently high (labelled W).

3. Global Stability Under SL

In this section we consider a class of TEMs for which global stability holds when agents use SL to form expectations. Assumptions F.3 and F.4 ensure that at some stage the β estimates "jump" into I thereby guaranteeing convergence. It is important to note that the range of the TEM is allowed to be unbounded above.

Theorem 1: *Assume F.1-F.4, and consider the SL case. For any initial condition (x_0, β_0) such that $x_0 \neq 0$, $x_t \to 0$, $\beta_t \to 0$.*

Proof: It suffices to show that every trajectory must enter the set I of Lemma 1. If not, then $|\frac{x_{j+1}}{x_j}| > \beta^* > 1$ for all j. But since F.3 imposes a lower bound, $x_j \geq -K$ for all j, the divergence of $|x_j|$ implies that eventually x_j exceeds any positive number, in particular, $\max\{Q, K\}$. Choose J such that $x_J > \max\{Q, K\}$ and note that $x^e_{J+2} := \beta^2_J x_J > 0$. Hence, from F.4 we have $|x_{J+1}| = |F(x^e_{J+2})| \leq \max\{Q, K\} < x_J$. It follows that, $|\frac{x_{J+1}}{x_J}| \leq 1 < \beta^*$. ∎

Remark 2: F.3 and F.4 do not interfere with the argument given in Proposition 1 about the existence of locally divergent trajectories. So Theorem 1 shows that, so long as the range of the TEM is bounded for positive values of x^e *all* of these locally divergent trajectories must go back to the steady state (under SL).

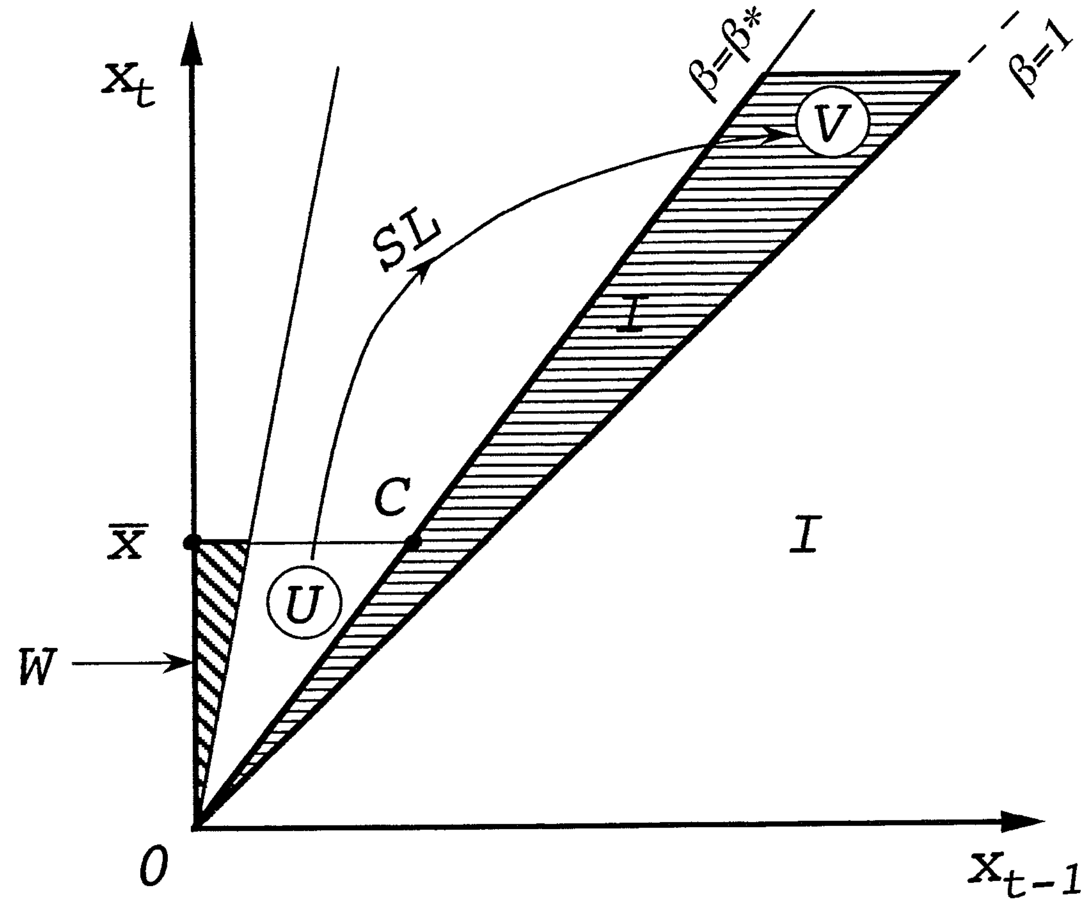


FIGURE 2

F.4 is crucial in ensuring global stability. If we replace F.4 by F.5, which makes the TEM unbounded for positive values of x^e, we obtain Proposition 2 which shows that an open set of locally divergent trajectories diverges to infinity. However, by Proposition 1, not all locally divergent trajectories diverge to infinity.

Proposition 2:[13] *Under F.1, F.2, and F.5 (i) there exists a $B > 0$ such that if for some T, $x_T > B$ and $\beta_T > 1/\theta$, then x_t diverges under SL; (ii) for any $x_0 > 0$, there exists β_0 such that (i) holds.*

4. A General Global Stability Result

In this section we show that, under F.1 and F.2, *any* trajectory along which the state variable remains bounded must converge to the steady state under both the learning schemes introduced in Subsection 2.1. So, one gets global stability and this in spite of the possiblity of local instability which occurs even if the state variable remains bounded.

Theorem 2: *Assume F.1 and F.2 and restrict attention to trajectories such that the sequence x_t is bounded.*
(i) Consider a sequence (x_t, β_t) under SL. Then $x_t \to 0$ and $\beta_t \to 0$.
(ii) Consider a sequence (x_t, β_t, ω_t) under RL. Then $x_t \to 0, \beta_t \to \bar{\beta}, \omega_t \to \bar{\omega}$.

For the proof, it suffices to show that parameter estimate β eventually enters the set I of Lemma 1. In the SL case, this follows directly from the assumption that x is bounded. In the RL case, we use the following auxilliary result.

Lemma 2:[14] *Consider the sequence $q_t := \frac{\sum_{-L}^{t-1} x_j x_{j+1}}{\sum_{-L}^{t} x_j^2}$, $L \geq 0$, where $x_j \in R$ and $x_j \neq 0$ for some j. Then $|q_t| \leq \cos(\frac{\pi}{L+t+2}) \leq 1$, $L + t \geq 1$.*

(5') indicates that q_t differs from the RL estimator β_t, which uses data upto time t, because of the presence of the additional term x_t^2 in the sum in the denominator of q_t. The boundedness of the state variable allows one to deduce the limiting behaviour of β_t from that of q_t thereby showing that β eventually enters the set I.

For TEMs whose range is bounded, Theorem 2 implies global stability. For TEMs whose range is not bounded, Theorem 2 rules out the possibility of obtaining, in the limit, complicated dynamics restricted to a bounded set. In particular, under the conditions of Theorem 2, all limit cycles and chaotic attractors with compact support can be safely ignored as possible outcomes under the learning dynamics.[15]

[13] It is easy to show that the argument in Proposition 2 can be replicated in the RL case to show divergence to infinity.

[14] We thank J. Mora for bringing this result to our attention. The proof is available upon request.

[15] In [1] complex dynamics are obtained due to the interaction between the stabilizing effect of

Theorem 2 is tight in the RL case since Proposition 2 continues to apply, showing that the boundedness condition cannot be dispensed with.

5. OLG Economies

We apply the results of the paper to pure exchange OLG economies with two period lived agents, one good in each period, and money. For the sake of ease of exposition, we assume that the economy is stationary with only one type of household in each generation with a utility function which is additively separable across time.

Let $(y_t, z_t) \in R^2_+$ denote the consumption vector of the agent born in period t, let the function $u(y) + v(z)$, defined on R^2_+, represent preferences, and let endowments be denoted by $(\omega^y, \omega^o) \in R^2_+/\{(0,0)\}$. We assume:

Assumption OLG.1: $u : R_+ \to R$ and $v : R_+ \to R$ satisfy
(i) $u(\cdot)$ and $v(\cdot)$ are C^2
(ii) $u'(\cdot) > 0$, $v'(\cdot) > 0$, $u''(\cdot) < 0$, and $v''(\cdot) < 0$, on R_{++},
(iii) $\lim_{y \to 0^+} u'(y) = +\infty$,
(iv) $\frac{u'(\omega^y)}{v'(\omega^o)} < 1$ for $(\omega^y, \omega^o) \in R^2_{++}$,
(v) $\liminf_{z \to 0^+} zv'(z) > 0$ in case $\omega^o = 0$ (boundary endowments).

OLG.1 (i)-(iv) are standard assumptions. OLG.1 (v) (see Brock and Scheinkman [2]) guarantees that there are no perfect foresight equilibria which converge to the autarchic steady state; it can be interpreted as requiring that "money is essential in the economy." OLG.1 (iv) and (v) correspond to the "Samuelson case."

Let p_t denote the money price of the commodity in period t and p^e_{t+1} the point expectation of the money price of the commodity in the next period. Given $(p_t, p^e_{t+1}) \in R^2_{++}$, agents' maximize utility subject to the constraints $(y_t, z_t) \in R^2_+$, $y_t \leq \omega^y$, and $p_t(y_t - \omega^y) + p^e_{t+1}(z_t - \omega^o) \leq 0$; this generates the money demand function defined as $m^d(p_t, p^e_{t+1}) := p_t \cdot (\omega^y - y(p_t, p^e_{t+1}))$.

Define $\theta := \frac{u'(\omega^y)}{v'(\omega^o)}$ for interior endowments (so $\theta \in (0,1)$ by OLG.1 (ii) and (iv)), and $\theta := 0$ for boundary endowments (when $\omega^o = 0$). On R^2_{++}, the function $m^d(\cdot)$ is differentiable with the possible exception of (p_t, p^e_{t+1}) such that $p_t = \theta \cdot p^e_{t+1}$.

There is a fixed supply of money denoted by $M > 0$. Given p^e_{t+1}, a temporary equilibrium obtains if $m^d(p_t, p^e_{t+1}) = M$ for some p_t. The relevant properties of the equilibrium price are summarized in the following lemma (the proof is routine and hence omitted).

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global nonlinear forces, and the destabilizing effect of local instability. As a referee observed, our result of global stability in spite of local instability is based on a similar interaction and shows how learning could be a potential source of complicated fluctuations.

Lemma 3: *Under OLG.1, given* $p^e_{t+1} > 0$,

(i) there exists a unique $p_t > \underline{p} > 0$ *solving* $m^d(p_t, p^e_{t+1}) = M$.

(ii) (Golden Rule) there exists a unique $p^* > 0$ *solving* $m^d(p^*, p^*) = M$.

(iii) $p_t \geq \theta \cdot p^e_{t+1}$.

(iv) $\lim_{p^e_{t+1} \to \infty} \frac{p_t}{p^e_{t+1}} = \theta < 1$ *if* $(\omega^y, \omega^o) \in R^2_{++}$.

(v) if $\omega^o = 0$ *then there exists a* $\bar{p}$ *such that* $p^e_{t+1} > p^* \quad \Rightarrow \quad p_t \leq \bar{p} < +\infty$.

(vi) $\frac{dp_t}{dp^e_{t+1}} = \frac{y_2(\cdot)p_t}{(\omega^y - y(\cdot)) - y_1(\cdot)p_t}$.

It can be shown that $\frac{dp_t}{dp^e_{t+1}}|_{p^e_{t+1}=p^*} < 1$ (this follows from OLG.1 (iv)).

Our purpose is to assess the stability of the Golden Rule under learning. Define $x_t := p_t - p^*$. x_t will be our state variable. The TEM is induced from the temporary equilibrium price, p_t, given p^e_{t+1}, by subtracting p^* from both the variables and has the following properties:

The domain of the TEM is $D = (-p^*, +\infty)$. *The TEM satisfies F.1. Lemma 3 (i) gives a lower bound for market clearing prices; this comes from feasibility. Hence, the TEM satisfies F.3 with* $K = K_d := (p^* - \underline{p})$. *In addition, it satisfies F.4 when endowments are on the boundary (by Lemma 3 (v)), and F.5 when endowments are interior (by Lemma 3 (iv)).*

The next assumption rules out preferences with strong income effects; it includes the gross substitutes case ($\frac{dp_t}{dp^e_{t+1}} > 0$ for all $p^e_{t+1} > 0$) as a special case.

Assumption OLG.2: (i) $-1 < \frac{dp_t}{dp^e_{t+1}}|_{p^e_{t+1}=p^*}$.
(ii) for $p^e_{t+1} \geq \underline{p}$, $m^d(2p^* - p^e_{t+1}, p^e_{t+1}) = M$ only for $p^e_{t+1} = p^*$.

Lemma 4 provides sufficient conditions under which F.2 holds (the proof is tedious and is omitted, but see Figure 1).

Lemma 4: *Under OLG.1 and OLG.2, F.2 holds.*

We now specify expectation formation.

Agents are assumed to know the value p^*. They generate their forecasts through E.1 where $D = (-p^*, +\infty)$ and $-\delta \in (-p^*, {\beta^*}^2(\underline{p} - p^*))$; Discussion E.1 applies.

We turn to the results. With boundary endowments, one gets global stability under SL.

Corollary 1: *Take any economy with boundary endowments satisfying OLG.1-2. F.1-4 are satisfied. Under E.1, the dynamics are well defined. So, Theorem 1 applies and all trajectories, including the locally divergent ones, return to the steady state under SL.*

Under OLG.1 and OLG.2, there exists an open set of locally divergent trajectories which return to the steady state under SL. If endowments are interior, then this set coexists with another open set of trajectories which diverge to infinity.

Corollary 2: *Take any economy satisfying OLG.1-2. F.1-3 are satisfied. Assume in addition that endowments are interior so that F.5 is satisfied. Furthermore, under E.1, the dynamics are well defined. Proposition 1 applies and there are locally divergent trajectories which return to the steady state under SL.*[16]
Proposition 2 applies and there are locally divergent trajectories which escape to infinity under SL and RL.

We note that under perfect foresight, the maximal rate of inflation is $\frac{1}{\theta}$. It can be shown that if x diverges then necessarily $\beta_t > \frac{1}{\theta}$ for all $t \geq T$, some T.

One way to rule out the inflationary paths of Corollary 2 is to introduce a cash-in-advance constraint on consumption when old which can be invoked to guarantee that $y_t \leq \omega^y - \epsilon$, $\epsilon > 0$, and this bounds the TEM. An alternative is to replace E.1 by a "Projection," E.2 below. E.2 restricts forecasts to lie in a compact set P which satisfies the consistency requirement that if agents constrain their forecasts to lie in P then the actual realizations of the state variable indeed lie in P. Convergence now follows as a corollary to Theorem 2.

Expectations E.2: ("Projection") Agents' predictions are given by

$$x^e_{t+1} = \hat{x}^e_{t+1} \quad \text{if } \hat{x}^e_{t+1} \in P$$
$$x^e_{t+1} = x^e_{\min} \quad \text{if } \hat{x}^e_{t+1} < x^e_{\min}$$
$$x^e_{t+1} = x^e_{\max} \quad \text{if } x^e_{\max} < \hat{x}^e_{t+1},$$

where $\hat{x}^e_{t+1} := \beta^2_{t-1} x_{t-1}$, $P := [x^e_{\min}, x^e_{\max}]$, $x^e_{\min} := -p^* + \bar{\delta}$ for $\bar{\delta} \in (0, \underline{p})$, and $x^e_{\max}$ is chosen to satisfy $+\infty > x^e_{\max} \geq \max_{x^e \in [x^e_{\min}, 0]} F(x^e)$.

Note that $x^e_{\min} < -p^* + \underline{p} = -K_d$. Also, under the boundary assumption on preferences, OLG.1 (iii), the maximization determining $x^e_{\max}$ is well defined. Under OLG.1-2, there is a $\beta^{*2} > 1$ such that $\beta^{*2} F(x^e_{\max}) < x^e_{\max}$ so that E.2 does not interfere with the dynamics on the set I.

Corollary 3: *Consider any economy satisfying OLG.1-2. F.1-2 are satisfied. Let $x_0 \in P$. Under E.2 the dynamics are well defined and the state variable is bounded under SL and RL (since the TEM is continuous). Hence, Theorem 2 applies and global stability obtains.*

[16] There is an exception: if $u(x) = v(x) = \ln x$ and $\omega^o = 0$ then $F(D) = \{0\}$. Of course, in this case there are *no* locally divergent trajectories.

Discussion of E.2: The term "Projection Facility," as used in the extant literature refers to restrictions imposed on parameter estimates to guarantee *local* convergence to the rational expectations value of the parameter. In stochastic models, even under a local version of the "contracting" condition F.2, bad realizations of the shocks can lead the parameter estimate away from the region in which convergence obtains; hence, a "Projection Facility" is invoked to guarantee local stability.[17] Evans and Honkapoja [5] presents an analysis of the role of these "Projection Facilities" in the case of regression on *exogenous* variables with noise.[18]

However, in the case of regression on lagged values of the *endogenous* variable, as in the deterministic models of [6], [7] and this paper, the local instability phenomenon is *always* present and one cannot dispense with a "Projection Facility" and get local stability. Though E.2 implicitly restricts the parameter estimates, it does not bind locally (around the steady state); hence, it permits local instability to occur. It is a "Projection" but of a much weaker form than the earlier versions of the "Projection Facility" that in effect rule out the local instability phenomenon. Woodford [10] uses similar bounds and gets global convergence but his model too is one of regression on *exogenous* variables so that the local instability we have in our framework is absent.

6. Extensions

We mention some extensions.

Theorem 2 can be proved for the case of Bayesian learning studied in [3].

We can consider the case in which there is a predetermined variable so that the TEM takes the form $x_t = F(x^e_{t+1}, x_{t-1})$ (as in [3], [6] and [7]). Let forecasts be formed as postulated in E.1. If we impose the "contracting" condition uniformly for all values of the predetermined variable, our analysis can be replicated.

The general multidimentional case is cumbersome to handle but it is immediate that with a linear TEM, our analysis can be replicated.

The case in which the TEM is subject to i.i.d. shocks is of considerable interest. We do not enter into the complications that noise causes; an extension of Theorem 2 in the case of recursive learning is studied in [4].

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[17] See [8] for an early application of "Projection Facilities;" see [6] and [7] for a criticism.

[18] The model considered in [5] assesses the stability of stationary sunspot equilibria and periodic allocations in one good OLG economies using stochastic approximation techniques. They show that, under a local version of our "contracting" condition F.2, local convergence obtains without invoking the "Projection Facility" if the support of the shock is sufficiently small.

APPENDIX

Proof of Lemma 1: By the Discussion following E.1, on the set I, (3b) can be ignored.

Since $F(\beta^2 x) = F(\beta^{*2} \cdot (\beta^2/\beta^{*2}) \cdot x)$, F.2 implies that

$$|\frac{F(\beta^2 x)}{x}| = \frac{\beta^2}{\beta^{*2}} \cdot |\frac{F(\beta^{*2} \cdot (\beta^2/\beta^{*2}) \cdot x)}{(\beta^2/\beta^{*2})x}| \leq k^* < 1 \text{ for all } \beta^2 \leq \beta^{*2},\ x \neq 0. \quad (7)$$

In the SL case, since $\beta_{t+1} = \frac{x_{t+1}}{x_t}$, by (7) $|\beta_{t+1}| < 1$. But $\beta^{*2} > 1$, so that $\beta_{t+1} \in I := [-\beta^*, \beta^*]$. $\beta_{t+j} \in I$ follows by induction.

In the RL case, β_{t+1} is a convex combination of $\beta_t \in I$ and $\frac{x_{t+1}}{x_t}$ where the latter, by (7), lies I. Hence, β_{t+1} lies in I. Again, $\beta_{t+j} \in I$ follows by induction.

Since $x_{t+j} \neq 0$ for all j, (7) implies that it obeys $|x_{t+j+1}| \leq k^*|x_{t+j}|$ and hence $x_{t+j} \to 0$ since $k^* \in [0,1)$. Thus $x_{t+j} \to 0$ has been proved in both the cases.

We turn to convergence of the parameter in the SL case. The updating rule for β can be written as $\beta_{t+j+1} = \frac{\beta^2_{t+j} F(\beta^2_{t+j} x_{t+j})}{\beta^2_{t+j} x_{t+j}}$. Since $x_{t+j} \neq 0$ for all j and $x_t \to 0$, by L'Hospital's rule, $\lim_{t\to\infty} \frac{F(\beta^2_{t+j} x_{t+j})}{\beta^2_{t+j} x_{t+j}} = F'(0)$. F.2 implies that $-1 < F'(0) < 1$. Moreover (7) implies that $|\beta_{t+j}| \leq k^* < 1$ for all j. Hence, for some J large enough, β obeys $|\beta_{t+j+1}| \leq C|\beta_{t+j}|$, $0 \leq C < 1$, $j > J$, and hence converges to 0.

Consider the RL case. Since the function $m(z) < 1$ for all $z > 0$, ω^2_{t+j} is non-negative and non-increasing and hence ω_{t+j} converges to a finite limit $\bar{\omega}$.

To show convergence of β_{t+j} in the RL case, consider the subset of the equilibrium manifold defined as $M_s = \{(0, \beta, \omega) | \beta \in I\}$. As in [6] (Section 4), by a Center Manifold argument, M_s is locally stable. Thus there exists V, an open neighbourhood of M_s, such that all trajectories originating in V converge to an element of M_s. Since we have already established that $x_{t+j} \to 0$, the dynamics are bound to enter V and thus the sequence β_{t+j} converges to some $\bar{\beta}$. ∎

Proof of Proposition 1: We prove the proposition by showing the existence of $x_{-N} \in (-\bar{x}, \bar{x})$ and β_{-N}, where $|\beta_{-N}| > \beta^*$, such that the trajectory which starts at (x_{-N}, β_{-N}) reaches x_0 in N iterates. The proof is completed by noting that the map $F(\cdot)$ is continuous, so that the dynamical system obtained by iterating SL a finite number of times is also continuous, with the implication that one can extend the construction to an open set containing x_0.

So given x_0, consider x^e such that $F(x^e) = x_0$ (since $F(\cdot)$ need not be monotone, there may be more than one candidate value; in such an event any of them can be chosen for the purpose of the construction). Choose β_0 as follows: a) if $x^e \cdot x_0 > 0$ then $\beta_0 > 1$ if $x^e \cdot x_0 < 0$ then $\beta_0 < -1$. Now define $x_{-1} := \frac{x_0}{\beta_0}$. $|x_{-1}| = |\frac{x_0}{\beta_0}| < |x_0|$, since $|\beta_0| > 1$, so that $x_{-1} \in F(D)$ since $x_0 \in F(D)$ and $-x_0 \in F(D)$, by hypothesis, and the range is an interval under F.1. If x^e is no less than $-\delta$, then $\beta_{-1}^2 := \frac{x^e}{x_{-1}}$. If $x^e < -\delta$, then β_{-1}^2 is the value of β^2 at which the right hand side of (3b) equals x^e (such a value of β^2 exists since the right hand side varies continuously in β^2 via $\hat{x}^e$).

By construction, $F(\beta_{-1}^2 \cdot x_{-1}) = x_0$ and $|x_0| > |x_{-1}|$, so that $\beta_{-1}^2 > \beta^{*2} > 1$ (if not, F.2 would imply that $|F(\beta_{-1}^2 \cdot x_{-1})| = |x_0| < |x_{-1}|$). If $x_{-1} \in (-\bar{x}, \bar{x})$ then we are done. If not, proceed by induction noting that given x_{-j}, $|\beta_{-j}|$ is also given, so that looking at the inverse image of x_{-j} we can determine x_{-j-1} (which will be in $F(D)$ by repeating the argument given above for x_{-1} and using F.2), hence the sign of β_{-j} (hence a specific value), and $|\beta_{-j-1}|$. For $j \geq 1$, β_{-j} so defined satisfies $\beta_{-j}^2 > \beta^{*2} > 1$ (by the argument given in the first step of the induction argument) so that $|x_{-j-1}| < \frac{|x_{-j}|}{\beta^*}$ for $j \geq 1$, since $x_{-j-1} := \frac{x_{-j}}{\beta_{-j}}$. Thus, since $\beta^* > 1$, for some finite N, x_{-N} is driven arbitrarily close to 0, in particular to a point $x_{-N} \in (-\bar{x}, \bar{x})$.

Finally, the initial β_0 can be chosen so that $|\beta_0| \in (1, \beta^*]$ so that using Lemma 1 one gets convergence to the steady state from the initial condition (x_0, β_0).

As indicated earlier, the proof is completed by noting that the dynamical system obtained by iterating SL is continuous and this lets us construct the required open sets around (x_0, β_0) (the set V) and around $(x_{-N}, |\beta_{-N}|)$ (the set U). ■

Proof of Proposition 2: (i) Using F.2 (for $x^e > 0$) and F.5 (i), one gets $X^* + x^e > X^* + F(x^e) > \theta(X^* + x^e)$. Note that for $\beta_t > 1$ and $x_t > 0$ and large, F.5 guarantees that $x_{t+1} > 0$; we work with such a pair of β_t and x_t. So we have $1 > \frac{X^* + x_{t+1}}{X^* + \beta_t^2 x_t} > \theta$. The second inequality can be rewritten as $\frac{F(\beta_t^2 x_t)}{x_t} > \frac{\theta - 1}{x_t} X^* + (\theta \beta_t)\beta_t$ where, for $\beta_t^2 x_t$ large, F.5 (ii) implies that the left hand side approaches the right. Restrict attention to $\beta_t > 1$. Evidently, $\beta_{t+1} > \beta_t$ if and only if $\frac{x_{t+1}}{x_t} > \beta_t > 1$. A sufficient condition for this to happen is that $\frac{\theta - 1}{x_t} X^* + (\theta \beta_t)\beta_t > \beta_t$, or dropping the subscript t, that $\beta(\theta\beta - 1) > (1 - \theta)\frac{X^*}{x}$. Given that $\beta\theta > 1$, where $\theta < 1$, the inequality will be satisfied for $x > B$, some large B, and consequently $\beta_{t+1} > \beta_t$. But then $\beta_{t+1}^2 x_{t+1} > \beta_t^2 x_t$ and since for x_t large the TEM is monotone increasing (by F.5 (ii)) $x_{t+2} > x_{t+1} > B$ and now by an induction argument $\beta_{t+j} > \beta_t$, $x_{t+j} > B$. But then $x_{t+j}/x_{t+j-1} > \beta_t > 1$ for all $j > 0$, and x_t diverges.

(ii) Given any $x_0 > 0$, choose $\beta_0(x_0)$ so that it satisfies (a) $\frac{F([\beta_0(x_0)]^2 x_0)}{x_0} > \frac{1}{\theta}$, (b) $F([\beta_0(x_0)]^2 x_0) > B$, where B is as specified in (i), (c) $\beta_0(x_0)\theta > 1$. Then $\beta_1 > \frac{1}{\theta}$ and $x_1 > B$ so that (x_1, β_1) satisfies the conditions of (i). ■

Proof of Theorem 2: It suffices to show that in each of the two specifications for parameter updating, the dynamics *must* enter the invariant set I identified in

Lemma 1.

Lemma A.1: *Consider SL. Let (x_0, β_0), such that $x_0 \neq 0$, be any initial condition. Then there exists $J > 0$ such that $\beta_J \in I := [-\beta^*, \beta^*]$.*

Proof: If not $|\beta_j| > \beta^* > 1$ for all j. Since $\beta_j = \frac{x_j}{x_{j-1}}$, one has $\left|\frac{x_j}{x_0}\right| > (\beta^*)^j$. Therefore, x_j must eventually become unbounded. ■

Lemma A.2 and Lemma A.3 complete the proof for the RL case.

Lemma A.2: *Let q_t be as defined in Lemma 2 and consider any bounded sequence x_t. Then for the induced sequences q_t and β_t, $\frac{q_t}{\beta_t} \to 1$.*

Proof: Using (5'), $\frac{q_t}{\beta_t} = \frac{\sum_{-L}^{t-1} x_j^2}{\sum_{-L}^{t-1} x_j^2 + x_t^2}$. Consider the sum $\sum_{-L}^{t} x_j^2$. This sum can either converge to $M > 0$ or diverge to infinity. Consider the case in which it converges so that $x_t^2 \to 0$ necessarily holds. The result follows. Now suppose that the sum diverges. Since, by hypothesis, x_t is bounded, so is x_t^2, and the result follows. ■

Lemma A.3: *Consider RL. Given $\beta^* > 1$, there exists T such that $\beta_t \in [-\beta^*, \beta^*]$ for all $t > T$.*

Proof: The result follows since the upper bound on $|q_t|$ converges to one from below (Lemma 2) and the limiting behaviour of β_t mimics that of q_t (Lemma A.2). ■

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