HUMAN CAPITAL AND ECONOMIC GROWTH: THEORY AND POLICY

BIDISHA CHAKRABORTY

Thesis submitted to the Indian Statistical Institute, Calcutta
in Partial Fulfillment of the Requirements for the
Award of the Degree of
Doctor of Philosophy
August, 2008

Dedicated to

My Parents

Contents

Page

Acknowledgements

This is a revised version of the thesis entitled "Human Capital and Economic Growth: Theory and Policy". I have revised the thesis in the light of comments and corrections made by two examiners on the earlier version; I am grateful to them. All the remaining errors are solely mine.

The making of this thesis has made me indebted to many persons in several ways. To list out a few of them who deserves special mention, first comes the name of Professor Manash Ranjan Gupta, my thesis supervisor, who introduced me to the world of research in economics, an untraversed world for me. This work would not have seen the light of day without his constant help and guidance, day in and day out, from the very outset till the final stage.

This thesis has also been benefitted immensely from the invaluable comments made by Professor Dipankar Dasgupta, Professor Amitava Bose, Professor Abhirup Sarkar, Professor Pradip Maity. Let me take this opportunity to convey my regardful thanks to all of them for enriching my thesis offering their innovative ideas. I also thank all members of ERU, ISI, Kolkata for various kinds of help.

I would also like to place on record my sincere thanks to anonymous referees of the *Japanese Economic Review, Hitotsubashi Journal of Economics, Keio Economic Studies, International Journal of Business and Economics* for their comments on the papers on which Chapter 2, Chapter 4, Chapter 5 of this thesis are based.

I would also like to thank the seminar participants at the Indian Statistical Institute (Kolkata and Delhi), Jadavpur University (Kolkata), Burdwan University, South and South East Asia Econometric Society Meeting of 2006 held at Institute of Financial Management and Research where parts of this thesis were read out.

Besides the people already mentioned above, I would like to mention the names of Sonali Roy, Soumyananda Dinda, Debabrata Mukherjee, Parthapratim Pal (whose sudden and sad demise was very shocking), Rituparna Kar, Debasis Mondal, Anup Kumar Bhandari, Sahana Roychowdhury, Shomnath Chattopadhyay, Trishita Roy Barman, Sattwik Santra, Srikanta Kundu, Debasmita Basu, Conan Mukherjee, Lopamudra Chaudhuri, Sanchari Joardar who as friends and colleagues made my stay at Indian Statistical Institute, Kolkata, pleasant and rewarding one. My thanks are also due to the Principal, Pushpita R. Bhattacharya and my other colleagues and students of Bijoygarh Jyotish Roy College, whose indulgence at times helped me to devote some of my precious times for attending the library and Computers Lab at ISI after completing my classes at the College.

Finally, compliments to my parents and love and affection to my sisters, cousins and friends who were constant source of my inspiration towards completion of my work with passion and pleasure.

Economic Research Unit — Bidisha Chakraborty
Indian Statistical Institute
Calcutta.

Chapter 1

Introduction and Literature Survey

Growth theory is one of the most important branches of macroeconomics. Growth theory helps us to understand the intertemporal behaviour of a dynamic economy and to understand the properties of the long-run rate of economic growth. It identifies the factors causing the deviation of the actual rate of growth from the socially efficient rate of growth and analyses the effectiveness of various policies in removing this gap. It analyses the condition of stability of the long-run equilibrium and also attempts to establish links between the long run equilibrium and the persistence of underdevelopment.

With the emergence of the 'new' growth theory, human capital accumulation and its role on economic growth has been placed at the forefront of the research in macroeconomics. The resources embedded in individuals, which make them more productive and equip them to earn higher real income in future, are called human capital. These are individuals' health, acquired skills and learning abilities. While the physical capital goods are owned by the capitalists and the ownership of physical capital is readily transferable by sale, the human capital is inherently embodied in workers and is subject to physiological constraints at the individual level. The social productivity of human capital can be expanded if its accumulation is widely spread among individuals in the society, whereas the aggregate productivity of the stock of physical capital is largely independent of the distribution of its ownership. So human capital needs separate attention while studying its contribution to economic growth.

A lot of works have been done on the theories of human capital accumulation and endogenous economic growth in recent years; and an extensive theoretical literature focusing on the role of human capital accumulation on economic growth has been developed. Our purpose in this chapter is to present a survey of the existing literature.

1.1 Old Growth Theory: Exogenous Growth

The old growth theory starts with the works of Solow (1956) and Swan (1956) who first formalize growth models of an one sector competitive economy. This model is known as the neoclassical one sector growth model. In this model, the steady state growth equilibrium of the economy is defined as a state where its aggregate capital labour ratio is time independent. The steady state growth equilibrium is shown to be stable; and the rate of growth of output in the steady state growth equilibrium is equal to the sum of the rate of labour augmenting technological progress and rate of growth of labour force. Both the rate of technological progress and the rate of growth of labour force are assumed to be exogenous and these make the long run equilibrium rate of growth to be exogenous. Fiscal and monetary policies of the government can not affect this long run rate of growth. However, the short run rate of growth varies inversely with the capital intensity of the economy in the transitional phase of development. While Solow (1956)'s focus was on the adjustment of capital labour ratio in the long run equilibrium, Swan(1956) made a more complete analysis of technical progress. The extension of the one sector Solow (1956) model into a two sector framework with an investment good sector different from the consumption good sector is made by Uzawa (1961, 1963, 1965), Hahn (1965), Takayama (1963, 1965), Drandakis (1963), Inada (1963). Tobin(1965) introduces real balance effect in the savings function of the Solow (1956) model.

In Solow (1956), the rate of savings was exogenously given. Cass (1965) and

Koopmans (1965), following Ramsey (1928), introduces household's intertemporal utility maximization behaviour in an otherwise identical Solow (1956) model and thus endogenized the savings rate. The equilibrium in this Ramsey-Solow model is saddle path stable with only one growth path converging to the unique steady state equilibrium point. However, the long run rate of growth remains exogenous.

1.2 Pioneering Works on Endogenous Growth

The literature on endogenous growth theory (new growth theory) developed in the 1980s to solve the problem of exogenous growth rate in the neo-classical growth model. Endogenous growth theorists try to overcome the shortcoming of the neo-classical growth model by endogenizining the rate of technological progress. The set of earliest part of the literature on endogenous growth consists of the works of Lucas (1988), Romer (1986, 1990), Arrow (1962), Barro (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) etc.

Arrow (1962) endogenizes the rate of technological progress by assuming that the labour productivity grows over time due to the accumulation of experience of the worker which he calls "learning by doing". This learning of the worker takes place through his handling of the machine. So Arrow (1962) considers cumulative gross investment at any point of time as the index of experience. So the technological progress, though endogenously generated, is viewed as an inevitable byproduct of the process of capital accumulation. It is external to the firm but internal to the economy as a whole. So even if marginal productivity of physical capital is diminishing at the firm level, it may not be so at the aggregate level. In Lucas (1988), technological progress is viewed as identical to the accumulation of human capital which is made endogenous by the individual's utility maximizing allocation of resources between the production sector and the education sector. In Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) etc. technological progress is viewed as the product development made by the R & D sector; and the rate of technological progress is endogenously determined by the endogenous resource allocation between the production sector and the R & D sector. All these pioneering models have been extended and reanalysed by various authors in various directions. In this survey, we shall consider only those endogenous growth models which are developed adopting the framework of Lucas (1988) model of human capital accumulation.

1.3 Lucas(1988) model

1.3.1 Special features

The theoretical literature dealing with the role of human capital accumulation on endogenous growth starts with the seminal paper of Lucas (1988) whose model is based on the work of Uzawa (1965). The growth rate of per capita income in the Lucas (1988) model depends on the rate of human capital accumulation which is determined by the labour time allocation of the individuals to acquiring skill.

There are two distinguishing features of the Lucas (1988) model. Firstly, apart from the physical capital, human capital is another accumulable factor in this model; and its accumulation, being a non market activity, takes place through the labour time sacrifice of the individual. Secondly, the production function of the final good is subject to the positive external effect of human capital and so it satisfies private constant returns to scale (CRS) but social increasing returns to scale. However, there is no external effect on human capital accumulation mechanism.

The production function of the Lucas (1988) model is given by

$$Y = K^{\alpha}(uH)^{1-\alpha}(\bar{H})^{\theta} \qquad \text{with} \quad 0 < \alpha < 1 \quad \text{and} \quad \theta > 0. \tag{1}$$

where Y, K and H stand for the level of output, stock of physical capital and the stock of human capital respectively. $u \in [0, 1]$ stands for the labor time allocation to production made by the individual. $\theta > 0$ is a parameter which represents the magnitude of external effect resulting from the average human capital stock of all the individuals denoted by $\bar{H}$. Human capital accumulation function of the individual is given by

$$\dot{H} = m(1-u)H \tag{2}$$

where m is the learning ability of the individual. The budget constraint of the individual is given by

$$\dot{K} = Y - C \tag{3}$$

where C is the level of consumption. The dynamic optimization problem of the representative individual in this model is to maximize the discounted present value of utility over the infinite time horizon, i.e. $\int_0^\infty U(C)e^{-\rho t}dt$, with respect to C and u subject to equations (1), (2) and (3). Here ρ is the constant discount rate and

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$$

is the instantaneous utility function.

1.3.2 Steady state equilibrium and efficiency

Lucas (1988) does not analyse the transitional dynamic properties of his model. He is interested in determining the steady state equilibrium growth rate of a competitive economy and in comparing it to that of the command economy. Economic agents can not internalize the externality in the competitive economy but the social planner can do so in the planned economy. If we assume $U(C) = lnC$ i.e. $\sigma = 1$, and define the steady state growth equilibrium as a state satisfying $\frac{\dot{K}}{K} = \frac{\dot{C}}{C}$ and $\dot{u} = 0$, then the steady state equilibrium growth rate of human capital in the competitive economy in the absence of any fiscal policy is given by

$$g_c = \frac{\dot{H}}{H} = m - \rho;$$

and that in the planned economy is given by

$$g_s = m - \rho\frac{(1-\alpha)}{(1-\alpha+\theta)}.$$

If $\theta = 0$, $g_c = g_s$ and if $\theta > 0$ then $g_s > g_c$. So in the presence (absence) of positive externality of human capital on production, the competitive equilibrium rate of growth falls short of (is equal to) the socially efficient rate of growth. This points out the need of government intervention to correct the inefficiency of dynamic equilibrium in the competitive economy.

1.3.3 Transitional Dynamics

Lucas (1988) does not analyze the transitional dynamic properties of his model. The analysis of transitional dynamic properties of any dynamic model is important for studying the short run intertemporal behaviour of the economy and for studying policy implications. Many authors analyze the transitional dynamic properties of the Lucas(1988) model. The set of literature includes the works of Caballe and Santos (1993), Arnold (1997), Xie (1994), Benhabib and Perli (1994), Alonso-Carrera (2001), Ruiz-Tamarit (2008) etc. Xie (1994) and Ruiz-Tamarit (2008) do the global stability analysis while others do a local stability analysis. In the absence of externalities of human capital on production, the steady state equilibrium is a saddle point and the equilibrium growth path in the transitional phase which converges to the steady state equilibrium point is unique. The result may be true even with a small degree of externality. However, when the external effect is large, the transitional growth path converging to the steady state equilibrium point may be indeterminate. So the countries with different initial conditions and with identical parameters, may move along different growth paths even if they experience same growth rate in the long run. Xie (1994) and Ruiz-Tamarit (2008) do the global stability analysis assuming the inverse of the intertemporal elasticity of substitution parameter to be equal to the elasticity of output with respect to physical capital. Mattana (2003) analyses the global dynamics of the Lucas (1988) model without any external effect and without the restrictive technical condition used by Xie (1994) and Ruiz-Tamarit (2008). Boucekkine and Ruiz Tamarit (2008) analyze the dynamics of the Lucas (1988) model using Gaussian hypergeometric functions without reducing dimension of the dynamic system. The parametric restriction used by Xie (1994) is no more needed to characterize the dynamic system in terms of original variables. Garcia-Belenguer (2007) consider a Lucas (1988) model with aggregate external effects of both human capital and of physical capital on production; and analyze both the local and global dynamics of this model. Chamley (1993) includes the external effect of individuals' labour time allocated to the education sector on the human capital accumulation function in an otherwise identical Lucas (1988) model and shows the possiblity of multiple steady state equilibria as well as the indeterminacy of the transitional growth path in that modified model.

1.3.4 Policy implications

In contrast to the neoclassical growth model, Lucas (1988) model shows that the long-run growth rate of the economy is sensitive to the policy parameters of the government. The policy of subsidization to the education sector raises the growth rate in this model. However, the rate of growth is independent of the proportional tax rate imposed on output or on capital income if the tax revenue is used as lumpsum payment made to individuals. Human capital causes external effects resulting in social increasing returns to scale in production. Due to this, competitive market may fail to provide socially optimal level of human capital. Gomez (2003) and Garcia-Castrillo and Sanso (2000) design optimal fiscal policies in the Lucas (1988) model. In both these two exercises, the dynamic system of equations obtained in the decentralized competitive equilibrium system is compared to that obtained in the centrally planned system to find out the optimal fiscal policies. According to Garcia Castrillo and Sanso (2000), physical capital must be free from taxes but the tax on labour income should finance the educational subsidy. In transition, subsidy can be financed by lumpsum taxation too. Gomez (2003) too finds the tax financed educational subsidy policy to be optimal. However, in his analysis, lumpsum tax is never found to be optimal to finance the subsidy. Gomez (2005b) extends the analysis of Garcia Castrillo and Sanso (2000) and of Gomez (2003) allowing for the average learning time of the individuals to have an external effect on human capital accumulation. He also shows that lumpsum taxation and capital income taxation are not optimal. Alonso Carrera (2000) analyzes the effect of changes in the rate of educational subsidy on the steady state equilibrium as well as on the transitional growth path in the Lucas (1988) model. The effects on the transitional growth paths are conditional on the values of elasticity of output with respect to physical capital and of the intertemporal elasticity of substitution. Lucas (1990) considered a Lucas type model with labour leisure choice and a human capital accumulation function that is concave with respect to time allocated for education. He calibrated the model to show that heavy initial capital taxation should be followed by lower and ultimately zero taxation. Labour income taxation does not affect the learning decision but influences the leisure demand.

1.4 Physical capital mobility across sectors

1.4.1 Rebelo(1991) model and its steady state equilibrium

In Lucas (1988), physical capital is specific to the production sector only. Rebelo (1991) first extends the Lucas (1988) model introducing perfect mobility of physical capital between the production sector and the human capital accumulation sector. Both the sectors have CRS production functions with both human capital and physical capital as inputs. However, there is no external effect of human capital in this model; and hence there is no difference between the socially optimal growth rate and the competitive equilibrium growth rate in the steady state equilibrium. In Rebelo (1991), the steady state equilibrium growth rate is determined by the technological parameters of the production sector in addition to those of the education sector. However, in Lucas (1988), this long run rate of growth is independent of the values of technological parameters of the production sector. An increase in the rate of proportional tax imposed on output or on capital income reduces the growth rate in the Rebelo (1991) model when the tax revenue is used as lumpsum payment. However, in the Lucas (1988) model, imposition of such taxes does not affect the growth rate. So, according to Rebelo (1991), difference in public policies is the main reason of difference in growth rates across countries. Following Rebelo (1991), Mulligan and Sala-i-Martin (1993), Bond, Wang and Yip (1996), Mino (1996, 2001b), Ortigueira (1998), De Hek (2006), Chen and Lee (2007) and many others include physical capital as an input in the production function of the education sector in their two sector endogenous growth models.

1.4.2 Transitional Dynamics and Fiscal Policies

Rebelo (1991) does not analyse the transitional dynamic properties of his model. However, the works of Mulligan and Sala-i-Martin (1993), Mino (1996, 2001b), Bond, Wang and Yip (1996), Song (2000), Ortigueira (1998), Ortigueira and Santos (2002), De Hek (2006), Chen and Lee (2007) etc. deal with transitional dynamic properties of the mobile capital version of Lucas (1988) model. Mulligan and Sala-i-Martin (1993) attempt to analyse the convergence property in an extended Rebelo (1991) model with external effects of the inputs. However, they use numerical simulation techniques and can not find out analytical solution. Mino (1996), who analyses the effects of capital income taxation on the steady state growth equilibrium as well as on the transitional growth path in Rebelo (1991) model, shows that the steady state equilibrium in this model is unique and is always saddle path stable when the education sector is more capital intensive than the production sector. However, in the case of reverse factor intensity ranking, the equilibrium is saddle path stable under some conditions. The effects of capital income taxation on the growth rate also depends on the sectoral capital intensity ranking. If the education sector is relatively more capital intensive than the production sector then the increase in the tax rate lowers the growth rate as well as the physical capital-human capital ratio. Bond, Wang and Yip (1996) analyze the transitional dynamic properties of Rebelo (1991) model and analyse the effects of imposing tax and subsidy. They show that the steady state equilibrium is a saddle point with a unique saddle path if there does not exist any external effect. They find that the imposition of taxes on factors reduce the growth rate while the subsidization policy raises it. Song (2000), using the framework of Bond, Wang and Yip (1996), reconfirms most of the results of Mulligan and Sala-i-Martin (1993) with analytical solutions. Mino (2001b) modifies Rebelo (1991) type framework such that the production technology in both human capital accumulation sector and production sector are subject to social constant returns to scale and private decreasing returns to scale. Increasing returns to scale property of the production technology is not necessary for indeterminacy of transitional growth path in this model. The difference between the private and the social factor intensity rankings play a pivotal role in producing indeterminacy here. Alonso Carrera and Freire Seren (2004) modify the Rebelo (1991) model introducing a third sector producing intermediate goods with capital and labour. The human capital accumulation sector in this model uses the intermediate good as an input but does not use the physical capital. The imperfection generated by taxation policy is responsible for indeterminacy of the transitional growth path in this model. That the intermediate good producing sector should be more physical capital intensive than the final good producing sector is a necessary condition to obtain indeterminacy in this model. Ortigueira (1998) points out that the welfare cost of taxation in the Rebelo (1991) model is higher than that in the Lucas (1988) model. This is so because, in the Rebelo (1991) model, the change in the tax rate affects the long run rate of growth as well as the speed of convergence along the transitional growth path. Gomez(2000) considers income (both physical capital capital income and human capital income) tax and consumption tax in Rebelo (1991) model with leisure in the utility function. Simulation results suggest that higher reliance on consumption taxation would increase in welfare. The optimal tax on capital income is significantly different from zero and optimal tax on human capital depends on the public expenditure on education. He also shows that optimal tax on human capital is higher than that on physical capital. Ferretti and Roubini (1998) also study the effects of factor income taxation and of subsidies to human capital accumulation in Rebelo (1991) model.

1.5 R & D sector and product development

In Romer (1990), Grossman and Helpman (1991), Jones (1995) etc. endogenous economic growth takes place in the long run through product development (technological progress) made by the R & D sector in which the human capital is used as an input. Lucas (1988) and his followers did not introduce R & D sector and product development in their models. However, there exists a few recently developed models in which endogenous growth is driven by the interaction between human capital accumulation and the development of the R& D sector. Human capital, which accumulates over time in these models following the mechanism of

Lucas (1988), not only directly enters as a productive input into the production function of the final good sector but also generates technological progress through the development of R & D activities. The set of works includes Arnold (1998, 2000), Sorensen (1999), Funke and Strulik (2000), Gomez (2005a), Eicher (1996), Redding (1996), Blackburn, Hung and Pozzolo (2000), Bucci (2002) etc.

Sorensen (1999) extends Romer (1990) model introducing a Lucas (1988) type human capital accumulation sector there. He determines the threshold level of human capital below which R& D activities are not profitable; and his model follows Lucas (1988) type structure so long the human capital stock is below that threshold level. Arnold (1998), who focuses on the role of international trade and of international knowledge spillover on the endogenous growth along with the effects of fiscal policies and R & D subsidy policies, combines the human capital accumulation of Lucas (1988) model and the R & D development of Jones (1995b) model and introduces endogenous allocation of human capital among the production sector, R & D sector and education sector. Funke and Strulik (2000) develop an augmented Grossman Helpman model with a R& D sector and a human capital accumulation sector; and show that the economy passes through different stages of development along its adjustment path. In Blackburn, Hung and Pozzolo (2000), human capital acumulates over time following Lucas (1988) mechanism and, at each point of time, is allocated among the final good propduction sector, education sector and a vertically integrated sector producing intermediate goods and doing R & D activities simultaneously. The steady state equilibrium growth rate in this model is determined exclusively by the parameters describing preferences and human capital accumulation technology but is completely independent of R & D activity which, itself, is driven by the human capital accumulation. Gomez (2005a) studies the transitional dynamic properties of Funke and Strulik (2000) model and of Arnold (2000) model. In Bucci (2002), human capital is allocated among final good production sector, intermediate good production sector, R & D sector and education sector. However, the R & D sector and the intermediate good sectors are not vertically integrated. The steady state equilibrium growth rate depends not only on the human capital accumulation technology parameter and preference parameter but also on the monopoly power in the intermediate goods sector. Eicher (1996) and Redding (1996) analyse the interaction between human capital accumulation and technological change using an OLG framework.

1.6 Other Extensions of Lucas (1988) Model

1.6.1 Sector-specific external effect

Lucas (1988) introduces aggregate external effect of human capital on production but does not introduce its sector specific external effect. Ben Gad (2003), Gomez (2004) etc. analyse the role of sector specific external effect of human capital on endogenous growth. Sector specific external effect comes from those individuals who work in that sector only. However, the aggregate external effect in Lucas (1988) model comes from all individuals in the society. Gomez (2004), who introduces sector specific external effect only in the production sector, shows that the competitive equilibrium growth rate is equal to the command economy growth rate even in the presence of this effect. Ben Gad (2003), who introduces the sector specific external effect of human capital in the production sector as well as in the human capital accumulation sector, shows that taxation along with sector specific externality is responsible to generate indeterminacy of the transitional growth path. He also shows that the size of the sector specific externality does not affect the optimal policy in his model; and this is not true in the Lucas (1988) model. Ben Gad (2003) also analyses the role of sector specific external effect of human capital in the Rebelo (1991) model in a section of his paper.

1.6.2 Other inputs in human capital accumulation function

According to Ben Porath (1967) and Heckman (1976), education sector needs some market goods as inputs in addition to the time of the learner. Trostel (1993) develops a model where human capital accumulation needs some goods to be invested for on the job training and the government gives subsidy to the production of these goods. Gomez (2003, 2007) follows Trostel (1993) kind of framework. Alonso Carrera and Freire Seren (2004) assume that human capital accumulation needs some intermediate goods to be purchased from the market. Heckman (1976), Trostel (1993), Gomez (2007), Alonso Carrera and Freire Seren (2004) etc. analyze the effects of taxation rate on the human capital accumulation and on other variables. Pecorino (1995) develops a model where individuals invest a part of their income to accumulate human capital.

1.6.3 Public expenditure

Neither Rebelo (1991) nor Lucas (1988) analyse the role of tax financed public infrastructural investment in their growth models. However, following Barro (1990), there exists a sizeable literature analysing the role of productive public expenditure in the neoclassical Ramsey Solow model; and this set includes the works of Futagami, Morita and Shibata (1993), Dasgupta (1999), Chang (1999), Fernandez, Novales and Ruiz (2004), Woo (2005), Tsoukis and Miller (2003)etc. In recent years, some authors analyse this problem even in the Lucas (1988) type human capital accumulation model. Faig (1995) includes public consumption services as an argument in the utility function in an otherwise identical Lucas (1988) model without external effects. The public expenditure in this model is financed by lumpsum taxes. Faig (1995) shows that the steady state growth equilibrium is saddle point stable. He also analyses the effects of permanent as well as temporary changes in government expenditure and/or taxes and the effects of permanent shocks to production technology on the growth path of the economy.

Chen and Lee (2007) introduce a Barro (1990) type of flow productive public expenditure into the Rebelo (1991) type model. Public expenditure, financed by imposing proportional tax on output, is assumed to be external to the household and is subject to congestion effect. The equilibrium growth path in the transitional phase of development is locally indeterminate when the degree of public spending externalities and congestion effects are large enough even if there is no divergence in factor intensity ranking between the private and social technologies.

Greiner (2006) develops an endogenous growth model where human capital accumulation results from the investment of the public resources financed by the revenue earned from income taxation and from issuing government bonds. The household's labour time allocation between production and education is exogenous; and the household solves only the consumption investment allocation problem. The value of new bonds issued is equal to the excess of interest payment plus educational expenditure over the tax revenue; and the government determines the level of education expenditure and the tax rate. Greiner (2006) shows that neither a too severe nor a too loose budgetary policy is compatible with sustained growth when the government is the debtor.

Hollanders and Weel (2003) analyse the role of public expenditure on the human capital accumulation in a Lucas(1988) type heterogenous agent non scale growth model. Individuals, who differ in terms of skill levels, allocate their labour time between production and human capital accumulation; and firms allocate the individuals' labour time devoted to production between final good production and technology upgradation. The technological progress is biased in favour of the high skilled individuals. The rate of human capital accumulation is determined by the combination of the individuals' labour time allocated to education, the stock of public knowledge capital and the technology level. The government imposes tax on physical capital income and on human capital income to finance the accumulation of public knowledge. It is shown that high skilled individuals are benefitted by the technological progress made by the firms while the relatively less skilled individuals are benefitted by the increase in the public expenditure on education.

1.6.4 Unskilled labour

Robertson (2002) introduces unskilled labour as a separate productive input in a Lucas (1988) model whose skill level remains time independent and whose number grows at an exogenous rate. He analyses the effect of an unanticipated increase in level of unskilled labour on the growth path of the economy.

1.6.5 International capital mobility

Neither Lucas (1988) nor Rebelo (1991) consider international capital mobility. Lahiri (2001), Farmer and Lahiri (2005) etc. incorporate capital mobility in Lucas (1988) framework. Lahiri (2001) considers a small open economy version of Lucas (1988) model; and Farmer and Lahiri (2005) develop a two country Lucas (1988) model. In Lahiri (2001), the excess of the sum of output and of interest income earned from foreign bonds over consumption is invested in foreign bonds paying exogenously given interest rate. He analyzes the transitional dynamic properties of this model and points out the possibilities of indeterminacy of the equilibrium growth path. The range of parameters for which indeterminacy is possible in the closed economy is wider in the open economy in the presence of perfect capital mobility.

In Farmer and Lahiri (2005), two countries, each of whom is modeled as in Lucas (1988), differ only in terms of their initial human capital endowments. They are linked only by capital mobility but not by trade and labour mobility. External effect of human capital on production, which is country specific, generates social increasing returns to scale. The steady state equilibrium appears to be symmetric and unique when the initial human capital endowments of the two countries are very close to each other. However, there may be multiple asymmetric equilibria with one country growing faster than the other when the external effect is strong and when their initial human capital endowments differ substantially.

1.6.6 Non scale growth model

In the Lucas (1988) model, the rate of growth of human capital is independent of its scale. Gong, Greiner and Semmler (2004) find a negative empirical relationship between the growth rate of human capital and its scale. They develop a growth model where human capital accumulation function satisfies this empirical property. This model does not generate endogenous growth in the long run. However, the results of this model is found to be compatible with time series data of US and German economies.

1.6.7 Unemployment

In Lucas (1988) and in Rebelo (1991), the rate of human capital accumulation is independent of the level of unemployment. Mauro and Carmeci (2003) develop a model where unemployment rate of skilled workers retards the human capital accumulation. The equilibrium unemployment rate is found out by assuming wage rate to be a negative function of the unemployment rate. They analyze the long run equilibrium properties and the transitional dynamic properties of their model.

Ortigueira (2006) considers a modified Lucas(1988) model with frictional unemployment and search matching technology. Apart from devoting labour time to learning and to production, individuals devote time to searching job, leisure and home production activities. Multiple equilibria arise in the long run. Countries with high initial levels of human capital converge to an equilibrium with high growth rate and lower unemployment rate. However, countries with low initial level of human capital end up with a low growth rate and a higher unemployment rate in a low level equilibrium trap.

1.6.8 Leisure and/or human capital in the utility function

In Lucas (1988), the representative household derives instantaneous utility only from consumption but neither from leisure nor from human capital. In this model, steady state equilibrium is unique; and the indeterminacy of transitional growth path does not arise here in the absence of external effects of human capital. Solow (2000) shows that the Lucas (1988) model can not explain endogenous growth in the long run when leisure is included in the utility function. Ladron de Guevara et. al (1997, 1999) show that multiple steady state equilibria may arise in a Lucas (1988) type model if the household's utility function includes leisure as an argument. Benhabib and Perli (1994) show that the problem of indeterminacy of the transitional growth path may arise in the Lucas (1988) model even without the external effect of human capital on production when the household's utility function includes leisure as an argument. In Heckman (1976), Ortigueira and Santos (1997), Ortigueira (1998) etc. the marginal utility of leisure varies positively with the individual's level of human capital. Grimauda and Tournemaine (2007) include both human capital and environmental quality in the utility function of the individual. De Hek (2006) introduces leisure in the utility function in a Rebelo (1991) type model and analyses the effects of taxation.

1.6.9 Environment

Neither Lucas (1988) nor many of its extensions consider the problem of interaction between environmental pollution and economic growth. However, many authors have analysed this problem in the Ramsey-Solow framework. Only a few growth models analyse the interaction between environmental pollution and human capital accumulation. Environmental degradation produces negative external effects on the utility level of the household, productivity of the firm and on the learning ability of the individual. These negative external effects can be internalized in a planned economy but not in a market economy. The set of works introducing environmental pollution in the Lucas (1988) type model includes Gradus and Smulders (1993), Rosendahl (1996), Hettich (1998), Schou (2000, 2002) etc.

In Gradus and Smulders (1993), environmental pollution generates negative effects not only on household's utility level but also on the productivity of human capital accumulation technology. They show that the increase in pollution reduces the growth rate. The increase in abatement expenditure crowds out private investment and hence reduces the growth rate.

Rosendahl (1996) extends the Lucas (1988) type model with negative external effect of environmental degradation on the production of final good. Rate of growth in this model depends on the nature of environmental policy; and the external effect emanating from human capital plays a crucial role to make the growth rate of the command economy different from that of the competitive economy.

Hettich (1998) extends the Lucas (1988) type model such that the household's utility level varies negatively with the level of environmental pollution resulting from capital accumulation. The author compares the market economy solution to the planned economy solution; and finds out the optimal pollution tax rate.

In the model of Schou (2000), non renewable resources are also used as inputs of production along with physical capital and human capital; and their use generate environmental pollution which has a negative external effect on production. Like Lucas (1988), he also shows that the market economy growth rate falls short of the command economy growth rate in the presence of positive external effect of human capital. So the government should provide greater emphasis to subsidize education than to preserve natural resources.

Schou (2002) develops an endogenous growth model with environmental pollution and human capital accumulation which focuses on individuals' resource allocation problem not only between production and education but also to child care. Resources allocated to child care determines the birth rate and hence controls the population growth rate. The externality of the negative effect of environmental pollution makes the market economy solution different from the command economy solution. Optimal environmental policy and the optimal population policy are analysed in this model.

1.6.10 Role of money

Pecorino (1995) introduces real money balance in a Rebelo (1991) type model as an additional factor of production in the final goods production sector. Consumers own all assets including money. The rate of expansion of money supply varies positively with wage income and interest income; and the rate of inflation is equal to the difference between the rate of growth of money supply and the real rate of growth. Pecorino (1995) shows that money is not neutral because the rate of inflation caused by monetary expansion affects the real rate of interest and hence the real growth rate.

1.6.11 Heterogenous agents: Difference in human capital accumulation

O'Connell(1998) introduces differences between capitalists and workers in the Lucas (1988) model with a production technology satisfying diminishing returns to scale. Capitalists save and invest a part of their rental income to accumulate the stock of physical capital but workers consume their entire wage and allocate their labour time to human capital accumulation. Their savings is defined as the value of newly accumulated human capital. The author shows that capitalists' and workers' savings rate are equal in the steady state equilibrium and thus claims that the Lucas(1988) model is in clash with Cambridge models of Kaldor (1956) and Pasinetti (1962).

Paquin (1999) modifies Lucas (1988) model introducing householdwise differences in the values of various parameters. He shows that the economy can not reach a steady state equilibrium if the values of the parameters like elasticity of marginal utility of consumption, rate of discount and the marginal tax rate on physical capital income are not equal across the households. The problem does not arise and the growth rate remains unaffected if households differ only in terms of the efficiency parameter of production technology and/or of the initial endowment of human capital. He also shows that the redistributive tax and the transfer program affect the growth rate negatively.

Pecorino (1992), who models rent seeking activities in the Lucas (1988) framework, introduces two types of human capital–productive human capital accumulated through formal education and unproductive human capital accumulated as a byproduct (learning by doing) of rent seeking activities. The production sector producing the final good requires productive human capital as an input. The individual with productive human capital allocates his labour time among final good production, rent seeking effort and acquiring skill through formal education. However, the full time lobbyist spends his entire time in rent seeking activities and builds up unproductive human capital. Rent arising from quota restriction is equal to the tariff revenue earned at a tariff rate at which tariff is equivalent to the quota. He shows that the steady state equilibrium growth rate of the economy varies inversely with the tariff rate on imports. This also varies inversely with the proportion of individuals with unproductive human capital and with the relative share of the rental income owned by them. Removal of quota restriction results in a permanent increase in the growth rate.

Driskill and Horowitz (2002) consider the accumulation of two different types of human capital— advanced human capital and basic human capital. The production sector of the economy requires both types of human capital as inputs. Advanced human capital is made upgrading the basic human capital. So an inflow into the advanced stage implies an outflow from the basic stage; and hence the rate of accumulation of basic human capital is negatively related to the level of investment made for accumulation of the advanced human capital. The authors analyze the transitional dynamic properties of the model and design the optimal investment policies for accumulation of these two types of human capital.

1.6.12 The dual economy

Dualism means coexistence of opposite forces within the same unit. A less developed economy with an institutionally backward agricultural sector and an institutionally advanced manufacturing sector is often called a dual economy. The literature on old dual economy models consists of the works of Lewis (1954), Ranis and Fei (1961), Jorgenson (1961), Bose (1968), Sen (1966), Zarembka (1970), Hornby (1968), Dixit (1969), Bardhan (1971) etc. However, these models are developed on the exogenous growth framework. A few authors like Eicher (1999), Kongsamut, Rebelo and Xie (2001), Matsuyama (1992), Lucas (2004) etc. have developed dynamic models of dual economies using the endogenous growth framework with human capital accumulation. These are two sector models with a manufacturing sector and an agricultural sector; and the latter is an institutionally backward self employment sector.

In Matsuyama (1992), technological progress takes place in the manufacturing sector in the form of learning by doing and the agricultural sector does not face endogenous technological progress. In this model, an exogenous improvement in the agricultural productivity produces a positive (negative) effect on industrial growth in the closed (small open) economy. In Eicher (1999), human capital accumulation takes place through on the 'job training'; and it varies proportionately with the number of workers hired and trained in the manufacturing sector. Individuals are heterogenous in terms of ability which is not observable. Hence there exists adverse selection problem in the manufacturing sector. However, such a problem does not exist in agriculture which is a self employment sector. In order to mitigate the informational asymmetry and to improve the productivity of workers, firms offer efficiency wage in the manufacturing sector. The author shows that the country, with a comparative advantage in agriculture, faces contraction of the manufacturing sector when trade is opened up without international technology spillover. However, in the presence of international technology spillover, the opposite result may hold.

Lucas (2004) develops a dual economy model of rural urban migration and urbanization in which the urban sector follows the Lucas (1988) structure without any physical capital but the rural sector does not experience any human capital accumulation. The human capital has no effect on the productivity of agriculture. Initially everyone is in the rural sector but acquires human capital only after migration to the urban sector. In the steady state equilibrium of his model, the urban sector grows endogenously but the rural sector remains stagnant with a constant fraction of the labour force remaining there.

Kongsamut, Rebelo and Xie (2001) construct a growth model with three sectors– agriculture, manufacturing and service. All these three sectors use physical capital and human capital as inputs. The consumer consumes the products of all these three sectors. However, their income-elasticities of demand are different from each others. They have analyzed the properties of balanced growth equilibrium as well as the transitional dynamic properties of their model.

1.7 The plan of the present thesis

In chapter 2, we extend the Lucas (1988) model in two directions. In section 2.1, we introduce sector specific external effect of human capital on production in an otherwise identical Lucas (1988) model to analyse the transitional dynamic properties of that modified model. In section 2.2, we include human capital as an argument in the utility function of the consumer; and analyse the properties of steady state equilibrium and of the transitional growth path.

In chapter 3, we introduce the negative effect of environmental pollution in the Lucas (1988) model. Like Gradus and Smulders (1993), we assume that the environmental quality positively affects the rate of human capital accumulation. We also assume that the environmental quality itself is positively affected by the size of human capital and is negatively affected by the use of physical capital. We analyse the effects of taxation, abatement expenditure and of educational subsidy on the steady state equilibrium growth rate in this model. We also analyse the transitional dynamic properties of this model.

The efficiency enhancement mechanisms for the rich individual and the poor individual are different. While rich individuals can build up their human capital on their own, poor individuals need the support from exogenous sources in accumulating their human capital. In chapters 4, 5, 6 of the present thesis we develop growth models of a dual economy in which human capital accumulation is viewed as the source of economic growth and in which dualism exists in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor.

In chapter 4, we develop a two sector endogenous growth model of a dual economy focusing on the dualism in the mechanism of human capital accumulation.

While the human capital accumulation mechanism of the rich individual is similar to that in the Lucas (1988) model, the poor individual has a different mechanism of its human capital accumulation. Individuals in the rich(advanced) sector (region) allocate labour time not only to the production sector and to acquire their own knowledge but also provide voluntary labour services to train the individuals in the poor (backward) sector (region). We analyse the properties of the steady state equilibrium as well as of the transitional growth path of a competitive household economy using this two sector dual economy model.

In Chapter 5, we extend the model of chapter 4 to introduce accumulation of physical capital in the advanced sector as well as in the backward sector. We analyse the efficiency property of the steady state equilibrium growth path of a competitive economy using this extended model.

In Chapter 6 we analyse the role of educational subsidy policy of the government who imposes taxes on the rich individuals to finance the educational subsidy given to the poor individuals. We analyse the effects of exogenous changes in the tax financed subsidy rates on the long run equilibrium as well as on the transitional growth path.

1.8 Dualism in human capital accumulation and voluntary labour supply

In chapters 4 and 5 of this thesis, we develop growth models with special consideration to the dualism in human capital accumulation between the rich individual and the poor individual. However, none of the existing dual economy models focuses on the dualism in the mechanism of human capital formation of two different groups of individuals. In this section, we explain the motivation behind the consideration of this dualism with voluntary resource contribution of the riches to the education of the poors. In a less developed economy, the stock of human capital of the poor individual is far lower than that of the rich individual. Also there exists a difference in the mechanism of human capital accumulation between the rich individual and the poor individual. On the one hand, there are rich families who can spend a lot of time and resources for schooling of their children. On the other hand, there are poor families who have neither time nor resources to spend for education of their children. The opportunity cost of schooling of the children of the poors is very high because they can alternatively be employed as child labour. However, they receive support from various exogenous sources. Government sets up free public schools and introduces various schemes of paying book grants and scholarships to the meritorious students coming from the poor families. Government meets the cost of public education program through taxes imposed on rich individuals. In India, the government gives special emphasis on the subsidized education programme for the people belonging to the scheduled castes and scheduled tribes who are economically backward; and their backwardness in education is considered as one of the important causes of their economic backwardness.

Apart from these supports provided by the government, there are some private enterprises who stretch their helping hands towards the poor individuals to acquire education. Rich individuals who are owners of firms or industries open NGO s or give donations to the poor individuals. These NGOs provide free or subsidized educational service to the poor. There are substantial evidences that private individuals and firms provide voluntary services to the education programme of the poors. Corporate giants like Coca-Cola Company, American Express, General Electric Company, Bank of America, Nokia Corporation, Chevron Texaco Corporation are members of CECP (Committee to Encourage Corporate Philanthropy) and are providing various services including providing education to the underprivileged communities. Timberland Co. reports that 95 % of its employees have contributed some 300,000 service hours in 13 countries. 'Make a Connection' program undertaken by Nokia is active in 19 countries including countries of South Africa and Latin America. This program focuses on developing non academic skills like co-operation, communication skills, conflict management etc.[1]

[1] Source: Various newsletters published by CECP

Philanthropic activity in India has a long history. The roots of the development of the voluntary sector in India can be traced in philanthropic and religious obligations imposed on individuals to help the needy (Dadrawala, 2001). A survey done by Indian Centre for Philanthropy covering around 28% of urban India concludes that 96% of upper and middle class households in urban India make charitable contributions (ICP, 2001); and the amount is reported to be around Rs.16 billion (US$34 million) annually. Child Relief and You, Concern India Foundation, UNICEF, Help Age India and many locally operating organisations spend funds to the community development and utilize this expenditure more effectively than the government. Organisations like Swaminarayan Movement, Ramakrishna Mission, Sri Satya Sai Sewa Trust are some examples where volunteers are working on a wide range of socially relevant projects. The Ramkrishna Math and Mission, since its establishment in 1897, has been running thousands of institutions for formal and non-formal education including orphanages, students' homes, a Blind Boys Academy, non-formal and adult education centre, computer training centres, language schools, libraries, librarianship training centres, teachers' training institutes, post-graduate medical research institutes etc. The Math and the Mission, pay special attention to the needs of financially weak individuals. The Mission also runs schools and hostels for tribal boys and girls in Indian states like Arunachal Pradesh, Assam, Madhya Pradesh, Meghalaya, Orissa and in other remote parts of the country. The total number of students in these institutions in 1990-91 was 110,212, of which 82,409 were boys and 27,803 were girls. Sathya Sai Baba has established a number of primary schools, high schools and a university in South India; and these institutions provide education free of tuition charges.

In India, Titan, Broadcom, Infosys Foundation, Asea Brown Boveri, Siemens Ltd, Yahoo.com are among the many corporates who are fulfilling part of their social responsibilities by linking up with Akshaya Patra Foundation–a Bangalore based non profit organisation that provides mid day meals to unprivileged children in schools in and around Bangalore. ABB India has identified education as a key area for social and community development activities; and has been helping the teachers of a govt. school of a village close to Peenya, Bangalore, to make their lessons more meaningful and effective[2]. Confederation of Indian Industries (CII) has initiated a program in various parts of India under which a training is imparted to unskilled workers; and a certificate recognising the skill acquired by the worker is also given. These are pure private sector initiatives. There are relatively less number of published works dealing with charitable activities and voluntary labour in less developed countries. The main reason behind this is lack of data.

There exists a substantial literature on the estimates and explanations of voluntary works done by the households. Menchik and Weisbrod (1987) report that, according to a recent survey, over 80 million adults in the US volunteered 8.4 billion hours of labour to non profit organizations in 1980. Other estimates of volunteer workers, relying on non-survey methods, place volunteer labour as high as 8 percent of the total labour force. Andreoni, Gale and Scholtz (1996) find that 40% of households, surveyed by Independent Sector (1990), supply voluntary labour and each volunteer works for an average of 229 hours in a year. Brown and Lankford (1992) observe that most estimates of the value of unremunerated voluntary labour supply exceed estimates of charitable donations of money and property. They use a sample survey data set collected by Florida Bureau of Economics and Business Research in 1983; and it shows that 20% of all respondants volunteer for religious organization and 10% of them volunteer for educational institutions. Using the Health and Retirement Study (2002) report based on the data for the period $1996-2000$, Cao (2006) points out that 40% of households are involved in volunteer works. Freeman(1997) uses data of the May 1989 Current Population Survey (CPS) and 1990 version of the bi- annual Gallup Survey of Charitable Giving and Volunteering to obtain estimates of the extent of volunteering in US. The proportion of individuals volunteering last year is 52.2% and the voluntary labour supply per household is 0.9 hours per week. Volunteering activities account for 7% of US National Income in 1990.

[2]Source: Various issues of Business India.

Kitchen and Dalton (1990) observe that 73% of households in Canada contribute to charitable organizations; and the participation rate of the households to charitable activities varies positively with the increase in the level of income. Bruno and Goette (1999) show that the volunteered labour employment accounts for 6.8% of total employment in USA in 1990. In France, U.K. and Germany the respective shares are 4.2%, 4% and 3.7%. Steinberg (1987) observes that the voluntary labour sector employed 5.7% of compensated labour and produced 3.2% of measured GNP in US in 1975. Day and Devlin (1996, 1998), using the data of the Survey of Volunteer Activity done in Canada in 1987, show that 5.38 million individuals donated 1.018 billion hours of their labour time to welfare organizations in Canada from Nov 1986 to Oct 1987. The value of this volunteered labour time is estimated to be over 2% of GDP. It is also observed that an individual volunteers 178 hours of work per year on the average. Hackl, Halla, and Pruckner (2007) show that the share of the population offering voluntary labour services varies across countries in Europe. The participation rate of the employed workers in voluntary activities are high in Sweden, Slovakia and Great Britain; and are very low in Turkey, Russia, Ukraine, Poland and Hungary. Dolnicar and Randle(2004) observe that the volunteering sector in Australia had an estimated value of 42 billion AUD per annum in 2001 with 4.4 million individuals contributing 704 million voluntary labour hours in that year.

Park and Park (2004) observe that, in Korea, educational institutions rank third in the list of non profit organizations who receive charitable contributions. Freeman (1997) and Cao (2006) also find that, in USA, individuals with higher education levels tend to spend less labour time on family care but spend more labour time for volunteering activities and make more charitable donations. Cao (2006) also provide some explanations for the strong education effect on the voluntary labour supply and donation of resources. Educated individuals can manage their household activities more efficiently. They are more valuable to non profit organizations and are more knowledgeable about the importance of volunteering work and about the utilization of their skill. Nakano (2005), who studies voluntary labour supply in Japan, points out that free rider problem may not arise if the voluntary service providers expect non monetary reward, such as gratitude from the recipients. According to him, free rider problem is more serious in urban areas than in rural areas. Park and Park (2004) observe that 63% of households surveyed in Korea spent money for charitable activities during the year of 1999; and this share was 75% for the highest income group. Andreoni, Gale and Scholtz (1996) also provide some explannations why free riding problem may not exist. Firstly, charitable contributions may not be a purely unselfish phenomenon because it often buys a future service. Secondly, the donor derives utility either from the benefit of the recepient or from the act of giving. Menchik and Weisbrod (1987) point out that individuals view voluntary labour service either as a consumption good or as an investment that improves their working skills. Hackl, Halla and Pruckner (2007) empirically find that volunteering should be considered as an investment good.

These two empirical findings that educated individuals are more involved in voluntary works than the uneducated individuals and providing voluntary labour service is an investment are compatible with the assumptions we make in models developed in chapters 4 and 5. However, various examples of voluntary labour supply presented in this section do not mean that rich individuals are supplying the amount of voluntary labour that is optimal as a class.

Chapter 2

Two Extensions of The Lucas (1988) Model

2.1 Introduction

In chapter 1, we have observed that the Lucas (1988) model has been extended in different directions by various authors. In this chapter, we extend the Lucas (1988) model in two directions. In section 2.2, we introduce sector specific external effect of human capital on production in an otherwise Lucas (1988) model of endogenous growth; and show that the problem of indeterminacy of the transitional growth path does not exist even if the production function satisfies the increasing returns to scale property at the social level. In section 2.3, we include human capital in the utility function in an otherwise identical Lucas (1988) model and show that there may be multiple steady-state equilibria in this extended model. We also analyze the transitional dynamic properties of this extended model.

2.2 Sector Specific Externalities[1]

The Lucas (1988) model assumes the presence of aggregate external effect of human capital on production. It means that all the skilled workers employed in all the sectors generate externalities at equal rates. However, the external effect may

[1] A related version of this section is forthcoming in the journal called'Japanese Economic Review'.

be sector specific in nature; and, in such a case, only the skilled workers employed in the production sector produce external effects on production. Gomez (2004) introduces sector specific externality of human capital in an otherwise identical Lucas(1988) model; and shows that the competitive equilibrium steady state growth path is socially optimal. This result is interesting because the Lucas (1988) model with aggregate external effect shows the competitive equilibrium steady state solution to be sub optimal. Gomez (2004) assumes social constant returns to scale production technology in both the sectors; and hence it is a special case of the models examined by Benhabib et. al. (2000) and Mino (2001b) in which both the sectors use human capital as well as physical capital.

In this section, we want to derive the conditions for uniqueness and indeterminacy of the competitive equilibrium transitional growth path converging to the steady state equilibrium point in the Lucas (1988) model with sector specific externalities. We assume that the production technology satisfies CRS at the private level and IRS at the social level. Benhabib and Perli (1994) have done a similar exercise in the Lucas (1988) model with aggregate externalities. We want to examine how far the properties of transitional growth path analysed by Benhabib and Perli (1994) are different from the corresponding properties obtained from an otherwise identical Lucas (1988) model when the production sector is subject to sector specific externalities.

We obtain an interesting result from the present model. If only sector specific externalities are present in the production function satisfying social IRS, then the steady state equilibrium point in the competitive economy is either a saddle point with a unique saddle path or is an unstable equilibrium point. This rules out the possibility of indeterminacy of the transitional growth path; and hence this is in contrast to the Benhabib-Perli (1994) result that indeterminacy of equilibrium growth path may emerge when external effect of human capital is very strong. Also the conditions for the existence of unique saddle path in this case are different from those derived in Benhabib and Perli (1994).

This section is organized as follows. Section 2.2.1 presents the basic model and derives equations of motion. In section 2.2.2, we derive conditions for uniqueness and indeterminacy of the competitive equilibrium growth path; and compare them to those derived by Benhabib and Perli (1994). Appendix A contains derivations of few results presented in section 2.2.2.

2.2.1 The Model

Size of the labour force is normalized to unity; and the external effect of human capital on production is sector specific in nature. Otherwise the present model is identical to the Lucas (1988) model. The dynamic optimization problem of the representative individual in the competitive economy is given by the following,

$$\text{Maximize } \int_0^\infty U(C)e^{-\rho t}dt$$

subject to the equations

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \sigma > 0[\text{utility function}];$$

$$Y = AK^{\beta}(uh)^{1-\beta}E[\text{Production function}];$$

$$\dot{K} = Y - C[\text{Budget constraint}];$$

and

$$\dot{h} = \delta(1-u)h[\text{Human capital accumulation function}].$$

Here C, Y and K stand for level of consumption, level of output and level of capital stock respectively. h is the level of human capital and u is the fraction of labour allocated to production. δ is the productivity parameter in the human capital accumulation function. U(.) is the utility function and ρ is the rate of discount. E is the external effect of human capital and σ is the elasticity of marginal utility with respect to consumption; β is the capital elasticity of output.

In Lucas (1988), Benhabib and Perli (1994), Xie (1994) etc. external effect is derived from all skilled workers employed in both production sector and human capital accumulation sector. So we have

$$E = \bar{h}^{\epsilon}.$$

Here, $\bar{h}$ is the average human capital of all the workers in the society. However, in Gomez (2004), external effect is derived from those workers who are employed in the production sector. Since u fraction of the identical workers is employed there, we have

$$E = (u\bar{h})^{\gamma}.$$

Here $\epsilon(\gamma) > 0$ represents the elasticity of output with respect to the aggregate (sector specific) external effect of human capital. In this model, we consider both sector specific external effect and aggregative external effect. So we have

$$E = (u\bar{h})^{\gamma}\bar{h}^{\epsilon}.$$

We analyse the transitional dynamic properties of the present model. However, the present model, without aggregative external effect, is a modified version of the model of Gomez (2004). In Gomez (2004), the production function exhibits diminishing returns to scale at the private level and constant returns to scale at the social level. The production function in the present model satisfies the property of constant returns to scale at the private level and of increasing returns to scale at the social level. This property is common to that in the Lucas(1988) model. The representative individual solves this optimization problem with C and u being the two control variables; and we follow the style adopted by Benhabib and Perli (1994). Defining the appropriate current value Hamiltonian, maximizing it with respect to C and u, assuming interior solution and then using the first order optimality conditions[2] we derive the following equations of motion in the case where $E = (uh)^{\gamma}h^{\epsilon}$.

$$\dot{x} = Ax^{\beta}u^{1-\beta+\gamma} + \frac{\delta(1-\beta+\gamma+\epsilon)}{\beta-1}(1-u)x - qx, \tag{1}$$

[2]The optimization exercise is done in the appendix A.

$$\dot{q} = (\frac{\beta}{\sigma} - 1)Ax^{\beta-1}u^{1-\beta+\gamma}q - \frac{\rho}{\sigma}q + q^2, \quad (2)$$

and

$$\dot{u} = \delta\frac{(\beta-\gamma-\epsilon)}{(\beta-\gamma)}u^2 + \frac{\delta(1-\beta+\gamma+\epsilon)}{(\beta-\gamma)}u - \frac{\beta}{(\beta-\gamma}) qu. \quad (3)$$

Here x and q are two ratio variables defined as

$$q = \frac{C}{K}$$

and

$$x = \frac{K}{h^{\frac{1-\beta+\gamma+\epsilon}{1-\beta}}}.$$

Using the 3×3 dynamic system we can examine how far the transitional dynamic properties in the case of pure aggregate externalities are different from those in the case of pure sector specific externalities.

2.2.2 The Results

In the steady state growth equilibrium, $\dot{x} = \dot{q} = \dot{u} = 0$. We denote the steady state equilibrium values of x, q and u by x^*, q^* and u^*. Using equation (1) and $\dot{x} = 0$, we have,

$$x^* = \frac{1}{A}\{q^* + \frac{\delta(1-\beta+\gamma+\epsilon)}{(1-\beta)}(1-u^*)\}^{\frac{1}{\beta-1}}(u^*)^{\frac{1-\beta+\gamma}{1-\beta}}.$$

Then, using equations (1) and (2) and putting $\dot{x} = \dot{q} = 0$, we have

$$q^* = \frac{(\sigma-\beta)}{\beta}[\frac{\rho}{(\sigma-\beta)} + \frac{(1-\beta+\gamma+\epsilon)}{(1-\beta)}\delta(1-u^*)].$$

Using equation (3) and $\dot{u} = 0$, we have,

$$u^* = 1 - \frac{(1-\beta)(\delta-\rho)}{\delta[\sigma(1-\beta+\gamma+\epsilon) - (\gamma+\epsilon)]}.$$

We need $\frac{(\delta-\rho)}{[\sigma(1-\beta+\gamma+\epsilon)-(\gamma+\epsilon)]} > 0$ for $u^* < 1$ and $\frac{(1-\beta)(\delta-\rho)}{\delta[\sigma(1-\beta+\gamma+\epsilon)-(\gamma+\epsilon)]} < 1$ for $u^* > 0$.

The Jacobian matrix, whose elements are evaluated at the steady state equilibrium point corresponding to the dynamic system described by equations (1), (2) and (3), is given by

$$J^* = \begin{bmatrix} J^*_{xx} & \frac{x^*}{(\beta-1)u^*}((1-\beta+\gamma)J^*_{xx} - (1-\beta+\gamma+\epsilon)\delta u^*) & -x^* \\ 0 & \frac{(\gamma-\beta+\epsilon)}{(\gamma-\beta)}\delta u^* & -\frac{\beta}{(\beta-\gamma)}u^* \\ \frac{J^*_{xx}}{x^*}(\frac{\beta}{\sigma}-1)q^* & \frac{J^*_{xx}q^*(1-\beta+\gamma)}{u^*(\beta-1)}(\frac{\beta}{\sigma}-1) & q^* \end{bmatrix}$$

where

$$J^*_{xx} = (\beta - 1)Ax^{*\beta-1}u^{*1-\beta+\gamma} < 0.$$

Here,

$$\text{Trace of } J^* = \delta u^*[\frac{2(\gamma-\beta)+\epsilon}{(\gamma-\beta)}];$$

$$BJ^* = \frac{(\gamma+\epsilon-\beta)}{(\gamma-\beta)}(\delta u^*)^2 + J^*_{xx}q^*\beta[\frac{\sigma(1-\beta+\gamma)-\gamma}{\sigma(1-\beta)(\beta-\gamma)}];$$

and

$$\text{Determinant of} J^* = \frac{\delta u^* q^*}{\sigma(1-\beta)}[\frac{\sigma(1-\beta+\gamma+\epsilon)-(\gamma+\epsilon)}{\beta-\gamma}]\beta J^*_{xx}.$$

Let us first consider the case of pure sector specific external effect on production; i.e. $\epsilon = 0$ and $\gamma > 0$. Here Trace of $J^* > 0$ and this implies that at least one latent root of J matrix is positive. It is a 3×3 system. So if Det.$J^* < 0$, then Trace of $J^* > 0$ implies that there are one negative latent root and two positive latent roots of J^* matrix; and in that case, the steady state equilibrium point is a saddle point and the equilibrium transitional path is locally unique. If Det.$J^* > 0$, then either, all the three latent roots are positive which makes the steady state equilibrium to be unstable, or, two latent roots are negative and one latent root is positive which will lead to indeterminacy of transitional growth paths. Note that

Det. $J^* > (<) 0$ if $\frac{\sigma(1-\beta+\gamma)-\gamma}{\beta-\gamma} < (>) 0$.

When $\frac{\sigma(1-\beta+\gamma)-\gamma}{\beta-\gamma} > 0$, Det $J*$ is negative, and hence the system involves one stable root. So, there exists a unique saddle path converging to the steady state growth equilibrium point.

Note that, if $\sigma = \beta$, then $\frac{\sigma(1-\beta+\gamma)-\gamma}{\beta-\gamma} = (1-\beta) > 0$. In this case, the condition for uniqueness of transitional growth path is always satisfied. Xie (1994) assumes $\beta = \sigma$ from the view point of technical simplicity while analysing the transitional dynamic properties of the Lucas (1988) model; and shows that the equilibrium solution is unique (indeterminate) when $\gamma < (>) \beta$. Benhabib and Perli (1994) do not assume $\beta = \sigma$ in their analysis. However, if $\beta = \sigma$ is assumed in their model, their result is identical to that in Xie (1994). In the present case, with only sector specific external effects of human capital, equilibrium growth path is always unique when $\beta = \sigma$; and the magnitude of external effect parameter has no role to play in this context.

If $\sigma = 1$ then the condition for uniqueness of equilibrium growth path is $\gamma < \beta$ in the case of sector specific external effect; and $\epsilon \in (\beta, 2\beta)$ in the case of aggregative external effect.

Now we turn to analyse the case where the solution is not unique. In the case of pure sector specific external effect, this non-uniqueness problem arises when Det. $J^* > 0$ i.e., $\frac{\sigma(1-\beta+\gamma)-\gamma}{\beta-\gamma} < 0$. Here we apply the Routh (see, Gantmacher (1959)) criterion according to which the number of positive roots of the Jacobian matrix should be equal to the number of variations of sign in the scheme given by

$$\{-1, \text{Trace of} J^*, -BJ^* + \frac{\text{Det of} J^*}{\text{Trace of} J^*}, DetJ^*\}$$

Here Trace of J^* is positive and Det of J^* is positive.

Now

$$-BJ^* + \frac{\text{Det} J^*}{\text{Trace of} J^*} = -(\delta u^*)^2 - J_{xx}{}^* q^* \beta \frac{\sigma(1-\beta+\gamma)-\gamma}{\sigma(1-\beta)(\beta-\gamma)} < 0$$

when $\frac{\sigma(1-\beta+\gamma)-\gamma}{(\beta-\gamma)} < 0$. So the number of variations of sign in that scheme is three. Hence all the three latent roots of J^* matrix are positive when $\frac{\sigma(1-\beta+\gamma)-\gamma}{(\beta-\gamma)} < 0$. So the intertemporal equilibrium point (x^*, q^*, u^*) is unstable. We do not have a case of two negative roots and one positive root which makes the solution indeterminate.

The case of Benhabib and Perli (1994) is identical to the case where $\epsilon > 0$ and $\gamma = 0$. In this case, Trace of $J*$ is negative if $\epsilon > 2\beta$ and Determinant of $J*$ is positive if $\epsilon > \frac{(1-\beta)\sigma}{(1-\sigma)}$. If these two conditions are satisfied then there are two negative roots and one positive root in the system; and this leads to indeterminacy of solution. In this case, if $\sigma = 1$, Determinant of $J*$ is negative and $\epsilon < 2\beta$ is the necessary and sufficient condition for the transitional growth path to be unique. When this condition is violated, then we have Trace of $J^* < 0$, Det. of $J^* < 0$ and $BJ* < 0$. Hence, with $\epsilon > 2\beta$ and with $\sigma = 1$, there is only one variation in sign in the scheme

$$\{-1, \text{Trace of} J^*, -BJ^* + \frac{\text{Det of} J^*}{\text{Trace of} J^*}, DetJ^*\}$$

This means that only one latent root of J^* matrix is positive and its other two latent roots are negative; and this leads to indeterminacy in solution. So indeterminacy of the equilibrium growth path may arise in our model with aggregative external effect but it does not arise in the case of sector specific external effect. Hence we have the following proposition.

Proposition 1 *If the production technology in the Lucas (1988) model is subject to pure sector specific external effect of human capital, then the steady-state equilibrium is either a saddle point with a unique saddle path or is unstable. The possibility of indeterminacy of solution arises only in the presence of aggregative external effect.*

While understanding the intuition behind this result we first consider an arbitrary equilibrium growth path and then another one with higher investment rate. For the second one to be an equilibrium growth path, it's rate of return on the physical capital must be sufficiently high because otherwise it can not justify its higher accumulation rate. Marginal productivity of physical capital varies positively with the stock of human capital. The human capital accumulation can be accelerated by reallocating labour time from the production sector to the human capital accumulation sector. However, this increase in human capital can raise the rate of return on physical capital inspite of its accumulation only if the magnitude of external effect of human capital is strong enough to ensure the high complementarity between physical capital and human capital. In the case of aggregative external effect, the reallocation of workers from the production sector to the human capital accumulation sector does not lower the magnitude of the external effect because then the external effect is derived from all the workers in the economy. So the external effect is very strong there. However, in the case of sector specific external effect, only the workers employed in the production sector contribute to the external effect. So the reallocation of workers from the production sector lowers the magnitude of external effect in this case; and this weak external effect can not ensure the high complementarity between physical capital and human capital.

We give an example where the consideration of sector-specific externality of human capital on production shows the non-existence of indeterminacy of the equilibrium growth path. This is an interesting result because many authors like Benhabib and Farmer (1996), Mino (2001b), Benhabib and Nishimura (1998), Meng (2003), Weder (2001), Nishimura and Venditti (2002) attempt to show that sector-specific externalities can explain indeterminacy of the equilibrium growth path even when the magnitude of external effect is very small. However, those authors consider the sector-specific external effect of physical capital and do not use the Lucas (1988) framework. In our model, sector specific external effect comes from human capital and not from physical capital. So not only the nature of external effect but also the source of external effect are important in explaining indeterminacy of equilibrium.

2.3 Human Capital in the Utility Function[3]

In the Lucas (1988) model, the representative individual allocates her labour time between production and learning; and the steady state equilibrium rate of growth of the economy depends on the allocation of labour time to acquiring education. This optimal allocation of labour time in the steady-state equilibrium is unique in the Lucas (1988) model. However, this model assumes that individuals derive utility only from consumption. The stock of human capital does not enter as an argument in the utility function.

There exists a vast theoretical literature based on the overlapping generation (OLG) framework in which the human capital of the offspring is included as an argument in the parental utility function. This set of literature includes the works of Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994), Selod and Zenou (2003), De la Croix and Doepke (2004), Foster and Rosenzweig (2004) and of

[3]A related version of this section is published in the journal called 'International Journal of Business and Economics'.

many others. In none of these models, multiple equilibria is obtained as a result of the specific nature of the utility function. However, the other set of works dealing with various extensions of the Lucas (1988) model do not introduce human capital in the household's utility function. A few Lucas (1988) type models include leisure as an argument in the utility function. The list includes the works of Mino (1999), Benhabib and Perli (1994), Ben-Gad (2003), De Hek (2006) etc.

In this section, we introduce human capital as an argument in the non separable utility function of the household. The marginal utility of consumption varies positively with the stock of human capital. However, we do not consider her labour-leisure choice. The individual cares for her own human capital because there are many items which can not be consumed by an illiterate person. She can not get the taste of a literary work or of an artistic creation. She can not play computer games. Moreover, she may be also ignorant of why and how the consumption of natural resources and primary sector's products create health hazards and environmental pollution. Schultz (1961) noted that the distinguishing feature of investment in human capital compared to investment in physical capital is that investment in human capital not only enhances capabilities but also satisfy preferences. According to Moretti (2003), the increase in the level of education lowers the crime rate and raises the political awareness of the society. Gradstein and Justman (2000) points out that the human capital helps to build up social capital, enhances social cohesion and reduces ethnic tension. Duczynski (2005) includes capital (human capital and physical capital) in the utility functions of several growth models. He finds that the inclusion of human capital in the separable utility function in the Lucas (1988) model raises the steady state equilibrium growth rate. However, in his model, the inclusion of human capital in the separable utility function does not lead to multiple steady state equilibria. We consider a non separable utility function here. So the inclusion of human capital in the utility function in our model may lead to the existence of multiple steady state equilibria even in the absence of labour-leisure choice.

Kurz (1968) and Liviatan and Samuelson (1969) have shown the possibility of multiple steady state equilibria in the one sector Ramsey Solow model when physical capital stock is introduced into the utility function. In some OLG models, like Galor and Zeira (1993), Banerjee and Newman (1993), Glomm and Ravikumar (1995) etc, possibilities of multiple equilibria exist. However, its explanation does not lie in the inclusion of the human capital of the offspring in the parental utility function. It is explained by other features like credit market imperfection, indivisibilities of investments, endogenization of public policy etc.

This section is organized as follows. In Section 2.3.1, we present the basic model and derive the equations of motion. In section 2.3.2, we analyze the properties of the steady state growth equilibrium. In section 2.3.3, we analyse the transitional dynamic properties of this model.

2.3.1 The Model

We consider an otherwise identical Lucas (1988) model. The dynamic optimization problem of the representative individual in this model is to maximize

$$\int_0^\infty U(C,H)e^{-\rho t}dt$$

subject to the production function given by

$$Y = AK^\alpha (uH)^{1-\alpha} {H_A}^\gamma$$

with $A, \gamma > 0$ and $0 < \alpha < 1$; the dynamic budget constraint given by

$$\dot{K} = Y - C;$$

and the human capital accumulation technology given by

$$\dot{H} = \delta(1-u)H \quad \text{with} \quad \delta > 0.$$

Here A is the level of technology. K is the stock of physical capital. H is the stock of human capital. H_A is the average human capital of all the individuals. C is the level of consumption of the representative household. Y is the Level of output. u

is the fraction of labour time allocated to production. δ is the productivity parameter in the human capital accumulation function. U(.) is the utility function. ρ is the rate of discount. α is the capital elasticity of output. γ is the parameter representing the magnitude of external effect of human capital. So the production function satisfies social IRS in the presence of external effect. If the external effect is absent i.e. if $\gamma = 0$, then the production function satisfies CRS.

We consider the following utility function.

$$U(C, H) = \frac{(C^{\mu} H^{1-\mu})^{1-\sigma}}{(1-\sigma)} \quad \text{with} \quad 0 \leq \mu \leq 1 \quad \text{and} \quad \sigma > 0.$$

If $\mu = 1$, we come back to the original Lucas (1988) model. Here $0 < \mu < 1$ implies that $U_H > 0$. We consider all individuals to be identical so that $H_A = H$. The representative individual solves this optimization problem with respect to control variables C and u. K and H are two state variables. However the individual can not internalize the externality of human capital on production. The current-value Hamiltonian function is given by

$$Z = \frac{(C^{\mu} H^{1-\mu})^{1-\sigma}}{(1-\sigma)} + \lambda_K [AK^{\alpha}(uH)^{1-\alpha} {H_A}^{\gamma} - C] + \lambda_H [\delta(1-u)H].$$

Here λ_K and λ_H are co-state variables representing the shadow prices of physical capital and human capital respectively.

The first order optimality conditions are given by the followings.

$$\frac{\partial Z}{\partial C} = (C^{\mu} H^{1-\mu})^{-\sigma} \mu C^{\mu-1} H^{1-\mu} - \lambda_K = 0; \tag{4}$$

$$\frac{\partial Z}{\partial u} = \lambda_K A K^{\alpha} (1-\alpha) u^{-\alpha} H^{1-\alpha} {H_A}^{\gamma} - \lambda_H \delta H = 0; \tag{5}$$

$$\dot{\lambda_K} = \rho \lambda_K - \lambda_K \alpha A K^{\alpha-1} (uH)^{1-\alpha} {H_A}^{\gamma}; \tag{6}$$

and

$$\dot{\lambda_H} = \rho \lambda_H - (C^{\mu} H^{1-\mu})^{-\sigma} (1-\mu) C^{\mu} H^{-\mu} - \lambda_K (1-\alpha) A K^{\alpha} (u)^{1-\alpha} (H)^{-\alpha} {H_A}^{\gamma} - \lambda_H \delta (1-u). \tag{7}$$

The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_H(t)H(t) = 0.$$

Using equations (4), (5), (6) and (7) and using $H_A = H$, we can derive the following equations of motions.

$$\dot{q} = q[Au^{1-\alpha}z\{\frac{\alpha}{1-\mu(1-\sigma)} - 1\} + q + \frac{(1-\mu)(1-\sigma)}{1-\mu(1-\sigma)\}}\delta(1-u) - \frac{\rho}{\{1-\mu(1-\sigma)\}}]; \tag{8}$$

$$\dot{u} = u[\frac{(1-\mu)\delta u^{\alpha} q}{\mu A(1-\alpha)z\alpha} + \frac{\delta}{\alpha} - \frac{(\alpha-\gamma)}{\alpha}\delta(1-u) - q]; \tag{9}$$

and

$$\dot{z} = z[(\alpha-1)Au^{1-\alpha}z + (1-\alpha)q + (1-\alpha+\gamma)\delta(1-u)]. \tag{10}$$

Here q and z are two ratio variables defined as

$$q = C/K \quad \text{and} \quad z = K^{\alpha-1}H^{1-\alpha+\gamma}$$

Note that we come back to the equations of motion in the Benhabib and Perli (1994) model when $\mu = 1$.

2.3.2 Steady State Growth Equilibrium

In the steady state growth equilibrium, $\dot{q} = \dot{u} = \dot{z} = 0$. We denote the steady state equilibrium values of q, u and z by q^*, u^* and z^*.

Using equations (8), (9) and (10) we obtain the following equations describing the steady state.

$$q^* = \frac{\rho}{\alpha} - \frac{\delta(1-u^*)}{\alpha}[(\alpha-\gamma-\sigma) + (1-\sigma)\frac{\mu\gamma}{(1-\alpha)}] = \frac{\rho}{\alpha} - A_1\delta(1-u^*) \tag{11}$$

where

$$A_1 = \frac{[(\alpha-\gamma-\sigma) + (1-\sigma)\frac{\mu\gamma}{(1-\alpha)}]}{\alpha} .$$

$$z^* = \frac{1}{Au^{*1-\alpha}}[\frac{\rho}{\alpha} + \{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\}\delta(1-u^*)] . \tag{12}$$

Also

$$au^{*2} + bu^* + c = 0 \tag{13}$$

where

$$a = \delta^2[\frac{(1-\mu)A_1}{\mu(1-\alpha)\alpha} - \{(\frac{\alpha-\gamma}{\alpha}) - A_1\}\{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\}],$$

$$b = \frac{(1-\mu)\delta}{\mu(1-\alpha)\alpha}\{\frac{\rho}{\alpha} - A_1\delta\} - (\frac{\delta-\rho}{\alpha})\delta\{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\} + \{(\frac{\alpha-\gamma}{\alpha}) - A_1\}\delta\{\frac{\rho}{\alpha} + \{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\}2$$

and

$$c = \{\frac{\rho}{\alpha} + \{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\}\delta\}\{(\frac{\delta-\rho}{\alpha}) - \{(\frac{\alpha-\gamma}{\alpha}) - A_1\}\delta\}.$$

So, q^* and z^* are uniquely related to u^* by equations (11) and (12). If there exists unique value of u^*, then q^* and z^* are also unique. From the equation (9), we find that, in the steady state,

$$\frac{(1-\mu)\delta q^*}{\mu A u^{*-\alpha}(1-\alpha)\alpha z^*} = \frac{(\alpha-\gamma)}{\alpha}\delta(1-u^*) + q^* - \frac{\delta}{\alpha}.$$

Substituting the steady state equilibrium value of q^* from equation (11) in this equation mentioned above we have

$$\frac{(1-\mu)\delta q^*}{\mu A u^{*-\alpha}(1-\alpha)\alpha z^*} = \frac{\delta(1-u^*)}{\alpha}[\sigma - (1-\sigma)\frac{\mu\gamma}{(1-\alpha)}] + \frac{(\rho-\delta)}{\alpha}. \qquad (14)$$

Since the LHS of equation (14) is positive, the RHS must also be positive. So, we have

$$\delta(1-u^*) > \frac{\delta-\rho}{\sigma - (1-\sigma)\frac{\mu\gamma}{1-\alpha}}.$$

If $\rho < \delta$ and $\sigma \geq 1$, then the RHS of the inequality mentioned above is positive; and hence we have

$$\delta(1-u^*) > \frac{\delta-\rho}{\sigma - (1-\sigma)\frac{\mu\gamma}{1-\alpha}} > \frac{\delta-\rho}{\sigma - (1-\sigma)\frac{\gamma}{1-\alpha}}. \qquad (15)$$

The term in the extreme left of the inequality (15) stands for the rate of growth of human capital in our model; and the term in it's extreme right is the corresponding growth rate in the Lucas (1988) model. If $\rho > \delta$ and $\sigma \geq 1$ then the last two terms of the inequality (15) are negative. However, the term in the extreme left of the inequality (15) is positive if $0 < u^* < 1$; and this may be satisfied because the LHS of equation (14) is positive for $0 < \mu < 1$. So, even in that case, we have

$$\delta(1-u^*) > \frac{\delta-\rho}{\sigma - (1-\sigma)\frac{\gamma}{1-\alpha}}.$$

This is not possible in the Lucas (1988) model; because in that model $\mu = 1$; and hence

$$\delta(1 - u^*) = \frac{\delta - \rho}{\sigma - (1 - \sigma)\frac{\gamma}{(1-\alpha)}}.$$

This implies that $\delta > \rho$ is necessary for u^* satisfying $0 < u^* < 1$ in the Lucas (1988) when $\sigma \geq 1$ and also in the absence of externalities (i.e. $\gamma = 0$).

We assume $\rho > \delta$ and $\sigma < \frac{\gamma}{(1-\alpha+\gamma)}$. In this case,

$$\frac{(\rho - \delta)(1 - \alpha)}{\mu\gamma - \sigma(1 - \alpha + \mu\gamma)} - \frac{(\rho - \delta)(1 - \alpha)}{\gamma - \sigma(1 - \alpha + \gamma)}$$

$$= \frac{(1 - \alpha)(\rho - \delta)\gamma(1 - \mu)(1 - \sigma)}{\{\mu\gamma - \sigma(1 - \alpha + \mu\gamma)\}\{\gamma - \sigma(1 - \alpha + \gamma)\}} > 0$$

So even if $\rho > \delta$ and $\sigma < \frac{\gamma}{1-\alpha+\gamma}$ the inequality of equation (15) holds true. So, in our model, rate of growth of human capital is higher than that in Lucas (1988) model for all interior solutions of u^*. We now have the following proposition.

Proposition 2 *Steady state equilibrium rate of growth of human capital in the present model is higher than that in the Lucas (1988) model for an interior solution of u^*.*

The intuition behind this result is very simple. In our model, human capital enters not only into the production function as a productive input but also into the utility function as an argument with a positive marginal utility. In Lucas (1988) model, marginal utility of human capital is always zero. So the household in the present model allocates greater labour time to human capital accumulation sector than that in the Lucas (1988) model. So human capital grows at a higher rate in the present case than that in Lucas (1988).

Also note that this proposition 1 is valid for values of u^* satisfying $0 < u^* \leq 1$. We have assumed the existence of an interior solution because Lucas (1988) has also done the same. If $u^* = 1$, then $\dot{H} = 0$; and then the rate of growth in the steady state equilibrium is equal to zero in this model as well as in the Lucas (1988) model. If $u^* = 1$, then both these models are identical to the one sector Ramsey Solow model where there is no human capital accumulation.

Two positive solutions of u^* may emerge from equation (13) if (i) $a > 0$, $c > 0$ and $b < 0$; or (ii)if $a < 0$, $c < 0$ and $b > 0$.

The conditions for the larger positive root of the quadratic equation (13) to be less than unity are $(a + b + c) \geq 0$ and $\frac{a}{b} < -1/2$ in the case (i) and are $(a+b+c) < 0$ and $\frac{a}{b} < -1/2$ in the case (ii). Furthermore, in order to obtain real roots, we should have $b^2 \geq 4ac$. In these two situations, both the roots lie between 0 and unity (See Appendix (B.1)). Here

$$a + b + c = \frac{\rho}{\alpha^2}[\frac{\delta(1-\mu)}{\mu(1-\alpha)} + (\delta - \rho)].$$

When $\sigma \geq 1$, a and c are negative and b is positive(See Appendix(B.2)). Both the solutions of u^* are positive in this case. The conditions that both the roots should lie between 0 and unity are $a + b + c \leq 0$ and $\frac{a}{b} < -1/2$; and these are satisfied if

$$\rho \geq \delta[\frac{(1-\mu)}{\mu(1-\alpha)} + 1]$$

and

$$\delta[\{\gamma(1-\mu) + \sigma(1-\alpha+\mu\gamma)\}\{1 + \frac{(1-\mu)}{\mu(1-\alpha)}\} - \frac{(1-\mu)\alpha}{\mu(1-\alpha)}(1-\alpha+\gamma)]$$

$$> \rho[\frac{(1-\mu)}{\mu} + \{2\sigma(1-\alpha+\mu\gamma) + \gamma(1-2\mu)].$$

Proposition 3 *If (i)*$\sigma \geq 1$,
(ii) $\rho \geq \delta[\frac{(1-\mu)}{\mu(1-\alpha)} + 1]$ *and*

$$(iii)\delta[\{\gamma(1-\mu) + \sigma(1-\alpha+\mu\gamma)\}\{1 + \frac{(1-\mu)}{\mu(1-\alpha)}\} - \frac{(1-\mu)\alpha}{\mu(1-\alpha)}(1-\alpha+\gamma)]$$

$$> \rho[\frac{(1-\mu)}{\mu} + \{2\sigma(1-\alpha+\mu\gamma) + \gamma(1-2\mu)]$$

then there exist two solutions of u^* *satisfying* $0 < u^* < 1$.

However, in the Lucas (1988) model, $\mu = 1$; and, in that case, there is a unique solution of u^*. This is clearly understood looking at the equation (14). Note that, when $\sigma \geq 1$ and $\rho \geq \delta[\frac{(1-\mu)}{\mu(1-\alpha)} + 1]$, then the growth rate of human capital in the steady state equilibrium of Lucas (1988) model appears to be negative which is not feasible because $u^* \leq 1$. However, in this model where household derives utility from human capital, the rate of growth of human capital is positive in the steady state equilibrium even in this special case. It is the high positive marginal utility of human capital which induces the individual to allocate a positive fraction of labour time to human capital accumulation sector even when ρ is high and δ is low. Also we are getting multiple positive solutions of u^*. When $\sigma \geq 1$ and $\rho < \delta[\frac{(1-\mu)}{\mu(1-\alpha)} + 1]$, we have $a < 0$, $c < 0$, $b > 0$ and $a + b + c > 0$. In this case, one root of equation (13) lies between 0 and unity and the other root is greater than unity. Since $u^* > 1$ is not feasible, solution is unique.

We now try to provide an intuition behind the existence of multiple equilibria. Inter temporal equilibrium point of a differential equation is unique if the derivative is a monotonic function of the dependent variable. In the Lucas (1988) model, human capital does not enter as an argument in the utility function. So the relative rate of change of the shadow price of human capital, $\frac{\dot{\lambda}_H}{\lambda_H}$, is independent of the marginal rate of indifferent substitution between consumption and human capital in that model; and it is determined only by the marginal productivity of human capital. Once the human capital is efficiently allocated between the production sector and the education sector, the rate of change of shadow price of human capital becomes a constant and hence $\frac{\dot{u}}{u}$ becomes a monotonic function of u. However, in this present model, the marginal rate of indifferent substitution between consumption and human capital appears to be an important determinant of $\frac{\dot{\lambda}_H}{\lambda_H}$; and this affects the time path of the human capital allocation variable, u. This disturbs the monotonic relationship between $\frac{\dot{u}}{u}$ and u; and the possibility of multiple steady state equilibria arises in that case. Even, in an one sector Ramsey-Solow model, multiple steady state equilibria arise when the physical capital stock is introduced into the utility function. Kurz (1968) and Liviatan and Samuelson (1972) have shown this; and this is known as the 'Wealth Effect'. What we analyze in this section is basically the wealth effect of human capital.

2.3.3 Transitional Dynamics

We now turn to analyse the transitional dynamic properties around the steady state equilibrium point(s). We consider the system described by equations (8), (9) and (10). This is a system of 3 differential equations. Initial values of the variable, z, is historically given; and the initial values of the and other two variables, q and u, can be chosen by the individual. So, in order to get the unique saddle path converging to the steady state equilibrium point, we need two latent roots of Jacobian matrix to be positive and the third one to be negative.

Here the Jacobian matrix corresponding to the system of differential equations (8), (9) and (10) is given by the following:

$$J = \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial u} & \frac{\partial \dot{q}}{\partial z} \\ \frac{\partial \dot{u}}{\partial q} & \frac{\partial \dot{u}}{\partial u} & \frac{\partial \dot{u}}{\partial z} \\ \frac{\partial \dot{z}}{\partial q} & \frac{\partial \dot{z}}{\partial u} & \frac{\partial \dot{z}}{\partial z} \end{bmatrix};$$

where the elements of the Jacobian matrix evaluated at the steady-state equilibrium values of the variables are given in Appendix (B.3).

The characteristic equation of the J matrix is given by

$$|J - \lambda I_3| = 0$$

where λ is an eigenvalue of the Jacobian evaluated at the steady state. The three charateristic roots can be solved from the equation

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where

$$a_0 = -1,$$

$$b_0 = \text{Trace of } J,$$

$$a_1 = \frac{\partial \dot{z}}{\partial x}\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{x}}{\partial a}\frac{\partial \dot{a}}{\partial x} + \frac{\partial \dot{a}}{\partial z}\frac{\partial \dot{z}}{\partial a} - \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{x}}{\partial x}$$

and

$$b_1 = \text{Determinant of} J.$$

Trace of J is given by

$$b_0 = 2[\rho - \delta(1-u^*)(1-\sigma)(1+\frac{\mu\gamma}{(1-\alpha)})] - \frac{\delta\gamma}{\alpha}u^*. \qquad (16)$$

Using equation (14), we find that, when $0 < \mu < 1$,

$$\rho + \delta(1-u^*)[\sigma - (1-\sigma)\frac{\mu\gamma}{(1-\alpha)}] - \delta > 0;$$

Or,

$$\rho - \delta(1-u^*)(1-\sigma)(1+\frac{\mu\gamma}{(1-\alpha)}) - \delta u^* > 0.$$

Now equation (16) shows that $b_0 > 0$ when $2\alpha > \gamma$. This means that the Trace of J matrix is positive when external effect is very weak.

Here,

$$b_1 = J_{uu}(\alpha-1)z^*Au^{*1-\alpha}q^*\alpha + J_{uz}z^*q^*[A(1-\alpha)^2u^{*-\alpha}z^*\frac{\alpha}{\{1-\mu(1-\sigma)\}}$$

$$+\delta\{(1-\alpha+\gamma) - \frac{(1-\mu)(1-\sigma)(1-\alpha)}{\{1-\mu(1-\sigma)\}}\}]$$

$$-J_{uq}Au^{*1-\alpha}z^*q^*\delta[\frac{(1-\alpha)(1-\mu)(1-\sigma)}{1-\mu(1-\sigma)} - (1-\alpha+\gamma)(1-\frac{\alpha}{1-\mu(1-\sigma)})].$$

It can be shown that, under the sufficient conditions $\sigma \geq 1$ and $\alpha > \gamma$, b_1 is negative (See Appendix(B.4).). Note that the conditions ensuring $b_0 > 0$ and $b_1 < 0$ are independent of the values of u^*, q^* and z^*. So they apply to each of the two steady state equilibria. So we have the following proposition.

Proposition 4 *There exists unique equilibrium growth path converging to each of the two steady state equilibrium points, if $\alpha > \gamma$ and if $\sigma \geq 1$.*

So, for each of the two steady state equilibria, the transitional growth path is unique when the external effect of human capital on production is very weak. This result is similar to that obtained by Benhabib and Perli (1994) and Xie (1994) etc though they analyzed Lucas (1988) model where steady state equilibrium point is unique. Unfortunetely, it is very difficult to derive a meaningful condition of the indeterminacy of the transitional growth path when $0 < \mu < 1$. However, we can not rule out the possibility of indeterminacy when γ takes a high value. Here $z(0)$ is historically given while $q(0)$ and $u(0)$ are chosen. Depending on the choice of

$q(0)$ and $u(0)$, the initial state trajectory will meet one of the two saddle paths of the two equilibrium points and will converge to the corresponding equilibrium point. Unfortunately we can not use a phase diagram to explain the transitional dynamics because it is a 3×3 system.

This model introduces human capital as an argument in the utility function of the household in an otherwise identical Lucas (1988) model; and shows that the multiple steady state growth equilibria may exist when the discount rate is very high and /or when the productivity parameter in the human capital accumulation function takes a very low value. So our work stengthens the importance of wealth effect of a stock variable in generating multiple equilibria as shown by Kurz (1968). In a less developed country, mortality rate is higher than that in a developed economy. So the discount rate is also higher. The human capital accumulation technology is more efficient in an economically advanced economy than that in a backward economy due to differences in educational infrastructural facilities. Hence the productivity coefficient of human capital accumulation technology takes a very low value in a backward economy. Hence the possibility of multiple steady state growth equilibria appears to be stronger in a less developed economy.

Appendix A

The current value Hamiltonian corresponding to the dynamic optimization problem of the representative individual in the competitive economy is given by

$$H = \frac{C^{1-\sigma}}{1-\sigma} + \lambda_K[AK^\beta(uh)^{1-\beta}E - C] + \lambda_h[\delta(1-u)h]$$

where

$$E = (uh)^\gamma h^\epsilon.$$

The first order conditions for an interior optimum solution are given by the followings.

$$C^{-\sigma} = \lambda_1; \tag{A.1}$$

$$A(1-\beta)\lambda_1 K^\beta h^{1-\beta+\gamma+\epsilon}u^{\beta+\gamma} = \lambda_2\delta h; \tag{A.2}$$

$$\dot{\lambda_1} = \rho\lambda_1 - \lambda_1 A\beta K^{\beta-1}h^{1-\beta+\gamma+\epsilon}u^{1-\beta+\gamma}; \tag{A.3}$$

and

$$\dot{\lambda_2} = \rho\lambda_2 - \lambda_1 A(1-\beta)K^\beta h^{(\beta+\gamma+\epsilon)}u^{1-(\beta+\gamma)} - \lambda_2\delta(1-u). \tag{A.4}$$

Substituting for λ_1 from equation (A.1) in equation (A.2), we have,

$$\frac{A(1-\beta)}{\delta}C^{-\sigma}K^\beta h^{-\beta+\gamma+\epsilon}u^{-\beta+\gamma} = \lambda_2.$$

Taking logarithms on both sides and then differentiating with respect to time, we have

$$\frac{\dot{\lambda_2}}{\lambda_2} = \beta\frac{\dot{K}}{K} + (\gamma-\beta+\epsilon)\frac{\dot{h}}{h} - \sigma\frac{\dot{C}}{C} - (\beta-\gamma)\frac{\dot{u}}{u}. \tag{A.5}$$

Next we derive another expression for the rate of growth of λ_2 from equation (A.4) and set it equal to the expression in equation (A.5). Thus we have

$$\rho - \delta = \delta(\gamma-\beta+\epsilon)(1-u) + \rho - \beta q + (\gamma-\beta)\frac{\dot{u}}{u}$$

where

$$q = \frac{C}{K}.$$

The equation described above is same as equation (3) in the basic model. From the budget constraint and the production function, we have

$$\dot{K} = AK^\beta(uh)^{1-\beta+\gamma}h^\epsilon - C. \tag{A.6}$$

From equation (A.1), we have

$$-\sigma\frac{\dot{C}}{C} = \frac{\dot{\lambda_1}}{\lambda_1}.$$

Using (A.3) we have

$$\dot{C} = \frac{A\beta}{\sigma}K^{\beta-1}h^{1-\beta+\gamma+\epsilon}u^{1-\beta+\gamma}C - \frac{\rho}{\sigma}C. \tag{A.7}$$

Using equations (A.6) and (A.7), we have

$$\frac{\dot{q}}{q} = (\frac{\beta}{\sigma} - 1)Ax^{\beta-1}u^{1-\beta+\gamma} - \frac{\rho}{\sigma} + q.$$

This equation is same as equation (2) in the basic model. Using equation (A.6) and the human capital accumulation function $\dot{h} = \delta(1-u)h$, we obtain

$$\frac{\dot{x}}{x} = \frac{\dot{K}}{K} - \frac{(1-\beta+\gamma+\epsilon)}{(1-\beta)}\frac{\dot{h}}{h} = Ax^{\beta-1}u^{1-\beta+\gamma} + \frac{\delta(1-\beta+\gamma+\epsilon)}{(\beta-1)}(1-u) - q$$

where $x = \frac{K}{h^{\frac{1-\beta+\gamma+\epsilon}{1-\beta}}}$

This equation is same as equation (1) in this paper. $\dot{u} = 0$ in the steady state equilibrium. Then, using equation (3), we have,

$$q^* = \frac{\delta u^*(\beta-\gamma)}{\beta} + \frac{\delta(1-\gamma+\beta+\epsilon)}{\beta}. \tag{A.8}$$

Using $\dot{x} = 0$ in the steady state, we have

$$Ax^{*\beta-1}u^{*1-\beta+\gamma} = q^* - \frac{(1-\beta+\gamma+\epsilon)}{(\beta-1)}(1-u^*). \tag{A.9}$$

The Jacobian matrix, with elements evaluated at the steady state equilibrium point, is given by

$$J = \begin{bmatrix} J^*{}_{xx} & \frac{x^*}{(\beta-1)u^*}((1-\beta+\gamma)J^*xx - (1-\beta+\gamma+\epsilon)\delta u^*) & -x^* \\ 0 & \frac{(\gamma-\beta+\epsilon)}{(\gamma-\beta)}\delta u^* & \frac{-\beta}{(\beta-\gamma)}u^* \\ \frac{J^*{}_{xx}}{x^*}(\frac{\beta}{\sigma}-1)q^* & J^*{}_{xx}(\frac{\beta}{\sigma}-1)\frac{(1-\beta+\gamma)q^*}{(\beta-1)u^*} & q^* \end{bmatrix}.$$

$$\text{Trace of} J^* = J^*{}_{xx} + \frac{(\gamma-\beta+\epsilon)}{(\gamma-\beta)}\delta u^* + q^*$$

Using equations (A.8) and (A.9) we have

$$\text{Trace of} J^* = \delta u^*[\frac{2(\gamma-\beta)+\epsilon}{(\gamma-\beta)}] > 0.$$

The determinant of J^* is given by

$$Det.J* = \frac{\delta u^* q^*}{\sigma(1-\beta)}[\frac{\sigma(1-\beta+\gamma+\epsilon)-(\gamma+\epsilon)}{\beta-\gamma}]\beta J^*_{xx}.$$

Also we define BJ^* as the sum of minors of the diagonal elements of the matrix J^*. Hence,

$$BJ^* = \begin{bmatrix} J^*{}_{xx} & \frac{x^*}{(\beta-1)u^*}((1-\beta+\gamma)J^*xx-(1-\beta+\gamma+\epsilon)\delta u^*) \\ 0 & \frac{(\gamma-\beta+\epsilon)}{(\gamma-\beta)}\delta u^* \end{bmatrix} + \begin{bmatrix} J^*{}_{xx} & -x^* \\ \frac{J^*{}_{xx}}{x^*}(\frac{\beta}{\sigma}-1)q^* & q^* \end{bmatrix} +$$

$$\begin{bmatrix} \frac{(\gamma-\beta+\epsilon)}{(\gamma-\beta)}\delta u^* & \frac{-\beta}{(\beta-\gamma)}u^* \\ J^*{}_{xx}(\frac{\beta}{\sigma}-1)\frac{(1-\beta+\gamma)q^*}{(\beta-1)u^*} & q^* \end{bmatrix},$$

Or,

$$BJ^* = \frac{(\gamma+\epsilon-\beta)}{(\gamma-\beta)}(\delta u^*)^2 + J^*_{xx}q^*\beta[\frac{\sigma(1-\beta+\gamma)-\gamma}{\sigma(1-\beta)(\beta-\gamma)}].$$

Appendix B

Appendix B.1

When $a < 0$, $c < 0$ and $b > 0$, then the higher of the positive root is

$$\frac{-b-\sqrt{b^2-4ac}}{2a}.$$

This is less than unity if

$$\frac{-\sqrt{b^2-4ac}}{2a} < 1 + \frac{b}{2a}.$$

Multiplying both sides by $-2a$ we have

$$\sqrt{b^2-4ac} < -2a - b. \qquad (C.1)$$

[Since $-2a$ is positive the direction of inequality remains same.] Here, LHS is positive. $LHS < RHS$ implies that RHS is also positive.

Hence,

$$-2a - b > 0,$$

Or,

$$\frac{a}{b} < -1/2.$$

Squaring both sides of the inequality (C.1) we have

$$4a(a+b+c) > 0.$$

So when $a < 0$, $c < 0$ and $b > 0$, then higher of the positive root is less than unity if $(a+b+c) < 0$ and $\frac{a}{b} < -1/2$. When $a > 0$, $c > 0$ and $b < 0$, then higher of the positive root is less than unity if $(a+b+c) > 0$ and $\frac{a}{b} < -1/2$. Furthermore, in order to obtain real roots, the discriminant must be non negative, i.e., $b^2 - 4ac \geq 0$.

Appendix B.2

From the expression of A_1, given in section 2.3.2 of the paper, we have

$$(\frac{\alpha-\gamma}{\alpha}) - A_1 = \frac{1}{\alpha(1-\alpha)}[\sigma(1-\alpha) - (1-\sigma)\mu\gamma].$$

This is positive for $\sigma \geq 1$. Also note that

$$\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1 = \frac{1}{\alpha(1-\alpha)}[\gamma+\sigma(1-\alpha)-(1-\sigma)\mu\gamma] = \frac{1}{\alpha(1-\alpha)}[\gamma(1-\mu)+\sigma(1-\alpha+\mu\gamma)] > 0$$

Now using the expression of b mentioned in section 2.3.2 and the expression mentioned above we have

$$b = \frac{(1-\mu)\delta}{\mu(1-\alpha)\alpha}\{\frac{\rho}{\alpha} - A_1\delta\} + \{(\frac{\alpha-\gamma}{\alpha}) - A_1\}\delta\frac{\rho}{\alpha} + \{\frac{(1-\alpha+\gamma)}{(1-\alpha)} - A_1\}\delta[2\delta\{\frac{\alpha-\gamma}{\alpha}) - A_1\} - (\frac{\delta-\rho}{\alpha})].$$

When $\sigma \geq 1$, the first two terms of this expression are positive. The sign of the third term depends on the sign of

$$[2\delta\{\frac{\alpha-\gamma}{\alpha}) - A_1\} - (\frac{\delta-\rho}{\alpha})].$$

This can be simplified as

$$\frac{\rho}{\alpha} + \frac{\delta}{\alpha}[(2\sigma-1) - 2(1-\sigma)\frac{\mu\gamma}{(1-\alpha)}].$$

If $\sigma \geq 1$, this term is positive. Hence $\sigma \geq 1$ is the sufficient condition for b to be positive.

Appendix B.3

The elements of the Jacobian matrix are the following:

$$J_{qq} = \frac{\partial \dot{q}}{\partial q} = q^*,$$

$$J_{qu} = \frac{\partial \dot{q}}{\partial u} = q^*[A(1-\alpha)u^{*-\alpha}z^*\{\frac{\alpha}{1-\mu(1-\sigma)} - 1\} - \frac{\delta(1-\mu)(1-\sigma)}{\{1-\mu(1-\sigma)\}}],$$

$$J_{qz} = \frac{\partial \dot{q}}{\partial z} = q^*Au^{*1-\alpha}\{\frac{\alpha}{1-\mu(1-\sigma)} - 1\},$$

$$J_{uq} = \frac{\partial \dot{u}}{\partial q} = u^*[\frac{(1-\mu)\delta u^{*\alpha}}{\mu Az^*(1-\alpha)\alpha} - 1],$$

$$J_{uu} = \frac{\partial \dot{u}}{\partial u} = u^*[\frac{(1-\mu)\delta u^{*\alpha-1}}{\mu Az^*(1-\alpha)}q^* + \delta\frac{(\alpha-\gamma)}{\alpha}],$$

$$J_{uz} = \frac{\partial \dot{u}}{\partial z} = u^*[-\frac{(1-\mu)\delta u^{*\alpha}}{\mu A(1-\alpha)\alpha z^{*2}}q^*],$$

$$J_{zu} = \frac{\partial \dot{z}}{\partial u} = z^*[-(1-\alpha)^2 A u^{*-\alpha} z^* - \delta(1-\alpha+\gamma)],$$

$$J_{zz} = \frac{\partial \dot{z}}{\partial z} = A(\alpha-1)u^{*1-\alpha}z^*,$$

and

$$J_{zq} = \frac{\partial \dot{z}}{\partial q} = (1-\alpha)z.$$

Using equations (11) and (14) we have

$$J_{uq} = \frac{\delta}{\alpha q^*}[(\alpha-\gamma)(1-u^*)-1] < 0.$$

Appendix B.4

The determinant of the Jacobian matrix is given by the following.

$$b_1 = J_{uu}(\alpha-1)z^* A u^{*1-\alpha} q^* \alpha + J_{uz} z^* q^* [A(1-\alpha)^2 u^{*-\alpha} z^* \frac{\alpha}{\{1-\mu(1-\sigma)\}}$$
$$+\delta\{(1-\alpha+\gamma) - \frac{(1-\mu)(1-\sigma)(1-\alpha)}{\{1-\mu(1-\sigma)\}}\}]$$
$$-J_{uq} A u^{*1-\alpha} z^* q^* \delta[\frac{(1-\alpha)(1-\mu)(1-\sigma)}{1-\mu(1-\sigma)} - (1-\alpha+\gamma)(1-\frac{\alpha}{1-\mu(1-\sigma)})].$$

The first term is negative because $J_{uu} > 0$ at steady state if $\alpha > \gamma$. The second term of the determinant is negative because $\{1-\mu(1-\sigma)\} > 0$,

$$(1-\alpha+\gamma) - \frac{(1-\mu)(1-\sigma)(1-\alpha)}{1-\mu(1-\sigma)} = \frac{(1-\alpha)\sigma}{1-\mu(1-\sigma)} + \gamma > 0$$

and

$$J_{uz} = \frac{\partial \dot{u}}{\partial z} = -\frac{(1-\mu)\delta u^{*\alpha} u^* q^*}{\mu A(1-\alpha)\alpha z^{*2}} < 0.$$

Also it can be shown that

$$\frac{(1-\alpha)(1-\mu)(1-\sigma)}{1-\mu(1-\sigma)} - (1-\alpha+\gamma)(1-\frac{\alpha}{1-\mu(1-\sigma)}) = \frac{1}{1-\mu(1-\sigma)}[(1-\alpha)(\alpha-\sigma-\gamma)+\gamma\mu(1-\sigma)].$$

which is negative if $\sigma \geq 1$. So the third term of the determinant is also negative. Hence the entire determinant is negative if $\alpha > \gamma$ and if $\sigma \geq 1$.

Chapter 3

Human Capital Accumulation, Environmental Quality, Taxation and Endogenous Growth

3.1 Introduction

In this chapter, we plan to analyse the role of environmental pollution on the human capital accumulation and economic growth. We consider a Lucas (1988) type model of endogenous growth in which the environmental quality positively affects the rate of human capital accumulation and the level of environmental quality varies positively with the size of human capital and negatively with the use of physical capital. We analyse the effects of taxation on the steady state equilibrium growth rate in this modified Lucas (1988) model. We also analyse the transitional dynamic properties of this model.

There exists a substantial theoretical literature focusing on the interaction between the economic growth and the environmental degradation[1]. In these models, the degradation of environmental quality either lowers the utility of the consumer or lowers the productivity of the factors. Most of these models are built in an

[1]See, for example, Mohtadi (1996), Dinda (2005), Gradus and Smulders (1993), Hettich (1998), Rosendahl (1996), Perez and Ruiz (2007), Endress, Roumasset and Zhou (2005), Grimaud(1999), Ricci (2007), Grimaud and Tournemaine (2007) etc.

one sector Ramsey-Solow framework. Environmental degradation is viewed as the social byproduct of the use of modernized machineries in the production sector because the operation of these modenized machines requires the use of pollution enhancing raw materials like oil, coal etc. Some authors like Mohtadi (1996), Bretschger and Smulders (2007), Perez and Ruiz (2007), Hettich (1998) etc. assume a direct relation between the level of environmental pollution and the stock of physical capital when entire physical capital stock is fully utilized[2]. Other authors like Grimaud and Tournemaine (2007), Hettich (1998), Grimaud (1999) etc. assume the level of environmental pollution to be a function of the level of output of the aggregate production sector.

There exists another set of theoretical literature focusing on the role of human capital accumulation on economic growth[3]. The literature starts with the Lucas (1988) model; and this model has been extended and reanalysed by various authors in different directions. The rate of labour augmenting technical progress, i.e., the rate of human capital accumulation is endogenous to the analysis; and the productivity parameter of the human capital accumulation technology is an important determinant of the rate of growth. Some of the works focusing on the interaction between economic growth and environmental pollution are based on the Lucas (1988) framework. In Hettich (1998), environmental pollution negatively affects the welfare of the household; and, in Rosendahl (1996), environmental quality produces a positive effect on the productivity of capital. Ricci (2007) makes a survey of the literature. However, in none of these existing works, except of Gradus and Smulders (1993), environmental quality affects the learning ability of the individuals.

When human capital accumulation is the engine of economic growth, the learn-

[2]If capital accumulation means replacement of old machines by more eco-friendly machines, then environmental pollution should vary negatively with capital accumulation.

[3]See for example, Lucas (1988), Rebelo (1991), Bengad (2003), Caballe and Santos (1993), Ortigueira (1998), Faig (1995), Mino(1996), Greiner and Semmler (2002), Alonso-Carrera and Freire- Seren (2004), Chamley (1993) etc.

ing ability of the individual becomes an important determinant of the rate of human capital accumulation. Environmental pollution produces negative effects on the health of the individual; and this lowers the ability to learn. Noise pollution disturbs the academic environment. Margulis (1992) finds significant empirical correlation between lead in air and blood lead levels. Next, he shows that children with higher blood lead levels have a lower cognitive development and requires supplemental education. Kauppi (2006) shows that methyl mercury, whose exposure to human comes from fish consumption, may lower the learning ability of the children. Air pollution also causes problems related to eye sight and functioning of the brain. Gradus and Smulders (1993) consider this negative effect of environmental pollution in an otherwise identical Lucas (1988) model. However, they do not analyse the effects of various fiscal policies and the transitional dynamic properties of that model.

Human capital accumulation also has a positive effect on the upgradation of the environmental quality. Education makes the people aware of the environmental problems and of the importance of protecting environment; and the educated people can protect the environment in a scientific way. This positive effect of human capital accumulation on the environmental quality is ignored not only by Gradus and Smulders(1993) but also in the other theoretical models like of Mohtadi (1996), Dinda (2005), Hettich (1998), Rosendahl (1996), Ricci (2007)[4]etc. However, there are empirical supports in favour of this positive relationship. Torras and Boyce (1998) regress environmental pollution on income, on literacy rate, Gini coefficient of income inequality etc; and find that the literacy rate has a significant negative effect on pollution particularly in low income countries. Petrosillo and Zurlini et.al. (2007) find that the attitudes of the tourists, who visit Marine protected area, are highly dependent on their education level. Clarke and Maantay (2006) find that the participation rate of the people in the recycling program counducted in New York city and its neighbourhood is highly dependent on the education level of the

[4]Some authors e.g. Grimaud (1999), Goulder and Mathai (2000), Hart (2004) study the issue of environment in R& D driven growth model where innovations help to improve the environment.

participators.

In this chapter, we consider a modified version of the Lucas (1988) model with two special features. (i)Environmental quality positively affects the marginal return to education; and (ii) Environmental quality varies positively with the stock of human capital and negatively with the stock of physical capital whose full utilization is ensured by the perfect flexibility of factor prices. We analyse the effect of taxation on the steady state equilibrium rate of growth of the economy. The interesting results obtained in this chapter are as follows. Firstly, the steady state equilibrium rate of growth in this model varies positively with the proportional tax rate imposed on output or on capital income when tax revenue is spent as lumpsum payment. However, this rate of growth is independent of the tax rate imposed on labour income. In the Lucas (1988) model, this rate of growth is independent of the tax rate imposed either on output or on capital income. In the Rebelo (1991) model, the rate of growth varies inversely with the tax rate imposed on output or on capital income. Secondly, there exists a unique saddle path converging to the unique steady state equilibrium point in this modified model. Thirdly, the positive effect of output taxation on the steady state equilibrium rate of growth is strengthened when tax revenue is spent as abatement expenditure. Fourthly, the optimum output tax rate, which is obtained maximizing the steady-state equilibrium rate of growth, varies proportionately with the competitive output share of human capital when tax revenue is spent as educational subsidy.

Section 3.2 presents the basic model and contains the analysis of the effect of output taxation on the steady state equilibrium rate of growth when tax revenue is distributed among the individuals as lumpsum payment. Section 3.3 presents the analysis related to the transitional dynamic properties of the model. Section 3.4 contains the analysis related to the effects of factor income taxation. In section 3.5 we analyse the basic model when tax revenue has alternative uses. In section 3.5.1, tax revenue is spent on abatement activities. Section 3.5.2 contains the analysis with tax revenue financing the educational subsidy, Section 3.6 analyzes the relationship between growth and social welfare. Concluding remarks are made in section 3.7.

3.2 The model

The model presented in this chapter is an extension of the Lucas (1988) model. The government imposes a proportional tax on output; and the tax revenue is distributed among the individuals as lumpsum payment. The dynamic optimization problem of the representative individual is to maximize

$$\int_0^{\infty} U(C)e^{-\rho t}dt$$

subject to the production function given by

$$Y = A(aH)^{\gamma}K^{1-\gamma} \tag{1}$$

with $A > 0$ and $0 < \gamma < 1$; the dynamic budget constraint given by

$$\dot{K} = (1-\tau)Y - C + P \tag{2}$$

with $0 \leq \tau \leq 1$; the human capital accumulation function given by

$$\dot{H} = mE^{\delta}(1-a)H \tag{3}$$

with $\delta > 0$; and the environmental stock accumulation function given by

$$E = E_0 K^{-\beta} H^{\beta} \tag{4}$$

with E_0, $\beta > 0$. Here A is the technology parameter. K is the stock of physical capital. H is the stock of human capital and τ is the proportional output tax rate. E is the environmental quality. P is the lumpsum income transfer resulting from the distribution of tax revenue and C is the level of consumption of the representative household. Y is the Level of output and a is the fraction of labour time allocated to production. m is the productivity parameter in the human capital accumulation function. u(.) is the utility function. ρ is the rate of discount and γ is the elasticity of output with respect to human capital. Equations (3) and

(4) make the present model different from the Lucas (1988) model. Equation (4) with $\beta > 0$ implies that environmental quality varies positively with the stock of human capital and negatively with the stock of physical capital. Equation (3) with $\delta > 0$ implies that the positive external effect of environmental quality is present in the human capital accumulation function. If $\delta = 0$, or $\beta = 0$, then we come back to the original Lucas (1988) model. The representative individual solves this optimization problem with respect to the control variables C and a. K and H are two state variables. However the individual can not internalize the externality.

We assume $U(C) = lnC$ for the sake of simplicity. We also impose a restriction on the parameters given by

$$\gamma > \frac{\beta\delta}{1-\beta\delta}.$$

For the saddle path stability of the system we need this condition $\gamma > \frac{\beta\delta}{1-\beta\delta}$ that in turn implies $\beta\delta < \frac{\gamma}{1+\gamma}$. This condition implies that the magnitude of external effect ($\beta\delta$) should not be very high. Benhabib and Perli (1994) also found similar result in the Lucas (1988) model. Since $\frac{\gamma}{1+\gamma}$ is a positive function of γ the above condition also implies elasticity of output with respect to human capital (γ) must be high to justify taxation.

The current value Hamiltonian is given by

$$Z = lnC + \lambda_K[A(1-\tau)(aH)^{\gamma}K^{1-\gamma} - C + P] + \lambda_H[m(E)^{\delta}(1-a)H].$$

Here λ_K and λ_H are the two co state variables.

The first order optimality conditions are given by the following.

$$\frac{\partial Z}{\partial C} = \frac{1}{C} - \lambda_K = 0, \tag{5}$$

and

$$\frac{\partial Z}{\partial a} = \lambda_K A(1-\tau)\gamma K^{1-\gamma}a^{\gamma-1}H^{\gamma} - \lambda_H m(E)^{\delta}H = 0. \tag{6}$$

Time behaviour of the co state variables along the optimum growth path should satisfy the following.

$$\dot{\lambda_K} = \rho\lambda_K - \lambda_K A(1-\tau)(1-\gamma)(aH)^{\gamma}K^{-\gamma}, \tag{7}$$

and

$$\dot{\lambda_H} = \rho\lambda_H - \lambda_K A(1-\tau)\gamma(a)^\gamma H^{\gamma-1}K^{1-\gamma} - \lambda_H m(E)^\delta(1-a). \tag{8}$$

Transversality conditions are given by the followings.

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_H(t)H(t) = 0.$$

The budget of the government is balanced; and hence

$$P = \tau Y.$$

Hence, at the aggregate level, equation (2) is modified as follows.

$$\dot{K} = Y - C. \tag{9}$$

3.2.1 Steady State Equilibrium

Along the steady state equilibrium growth path, $\frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{C}}{C}$ and $\frac{\dot{a}}{a} = 0$. Using equations (5) and (7), we have

$$\frac{\dot{C}}{C} = A(1-\tau)(1-\gamma)a^\gamma(\frac{H}{K})^\gamma - \rho. \tag{10}$$

From the equation (6), we have

$$\frac{\lambda_K}{\lambda_H} = \frac{m(E)^\delta H}{A(1-\tau)\gamma K^{1-\gamma}a^{\gamma-1}H^\gamma}. \tag{11}$$

Differentiating both sides of equation (11) with respect to time and then using the steady state equilibrium condition, we have

$$(\frac{H}{K})^{\gamma-\beta\delta} = \frac{m(E_0)^\delta}{A(1-\tau)a^\gamma(1-\gamma)}. \tag{12}$$

Along the steady state equilibrium growth path, $\frac{\dot{C}}{C} = \frac{\dot{H}}{H}$. So using equations (3), (4), (10) and (12) we have

$$a = [\frac{\rho^{\gamma-\beta\delta}(A(1-\tau)(1-\gamma))^{\beta\delta}}{m^\gamma(E_0)^{\delta\gamma}}]^{\frac{1}{(\gamma-\beta\delta-\gamma\beta\delta)}}. \tag{13}$$

We assume $\gamma > \beta\delta + \gamma\beta\delta$. Thus we find a negative relationship between optimum a and τ in the steady state growth equilibrium. Note that optimum $a < 1$; and this is guaranteed if

$$\frac{\rho^{\gamma-\beta\delta}(A(1-\tau)(1-\gamma))^{\beta\delta}}{m^{\gamma}(E_0)^{\delta\gamma}} < 1. \tag{14}$$

Hence, using equations (10), (12) and (13), the balanced growth rate of the economy, denoted by g, is obtained as

$$g = [A(1-\tau)(1-\gamma)a^{\gamma}]^{-\frac{\beta\delta}{\gamma-\beta\delta}}[m(E_0)^{\delta}]^{\frac{\gamma}{\gamma-\beta\delta}} - \rho. \tag{15}$$

From equation (13) we have,

$$a = [\frac{\rho^{\gamma-\beta\delta}(A(1-\tau)(1-\gamma))^{\beta\delta}}{m^{\gamma}(E_0)^{\delta\gamma}}]^{\frac{1}{(\gamma-\beta\delta-\gamma\beta\delta)}}.$$

$$loga = \frac{1}{\gamma-\beta\delta(1+\gamma)}[(\gamma-\beta\delta)log\rho + \beta\delta log\{A(1-\tau)(1-\gamma)\} - \gamma log(m{E_o}^{\delta})]$$

Since a lies between 0 and 1, $loga$ is negative. Since $\gamma - \beta\delta(1+\gamma)$ is positive $[(\gamma-\beta\delta)log\rho + \beta\delta log\{A(1-\tau)(1-\gamma)\} - \gamma log(m{E_o}^{\delta})]$ must be negative.

$$\frac{dloga}{d\gamma} = \frac{-(1-\beta\delta)}{\{\gamma-\beta\delta(1+\gamma)\}^2}[(\gamma-\beta\delta)log\rho + \beta\delta log\{A(1-\tau)(1-\gamma)\} - \gamma log(m{E_o}^{\delta})]$$

$$+\frac{1}{\{\gamma-\beta\delta(1+\gamma)\}}[log\rho - \frac{A(1-\tau)\beta\delta}{A(1-\tau)(1-\gamma)} - log(m{E_0}^{\delta})]$$

The first term is positive and we can not say anything about the sign of the second term. Hence we can not say anything about the direction of change in a with respect to change in γ. Equation (13) shows that a varies positively with the discount rate, ρ, production technology parameter, A. This equation also shows that a varies negatively with the tax rate, τ, productivity parameter of the human capital accumulation function, m, and the initial environmental quality level, E_0.

Substituting $A(1-\tau)(1-\gamma)$ from equation (13) in equation (15) we have

$$g = \rho(\frac{1}{a} - 1).$$

So the steady state equilibrium growth rate, g, varies negatively with a. As a varies negatively with the tax rate, τ, the growth rate varies positively with the tax rate, τ. So we have the following proposition.

Proposition 1 *When human capital accumulation function in the Lucas (1988) model receives the negative external effect from environmental pollution, the steady state equilibrium rate of growth of the economy varies positively with the tax rate on output.*

If either $\delta = 0$, or, $\beta = 0$ then we go back to the Lucas (1988) model without external effect. In this case

$$a = \frac{\rho}{m}; \quad \text{and} \quad g = m - \rho.$$

Hence the tax rate on output can not affect the growth rate in this case. In the model of Rebelo (1991), the increase in output tax rate reduces the rate of growth of the economy. However, in this present model, the rate of growth varies positively with the output tax rate.

As the tax rate on output is increased, the post tax marginal productivity of physical capital is reduced. This reduces the net rate of return on physical capital; and hence the physical capital is accumulated at a lower rate. As a result, the rate of upgradation of environmental quality is increased. This produces positive external effect on human capital accumulation. So, in the new steady state equilibrium, the rate of growth of human capital as well as the rate of growth of income are increased.

In the model of Rebelo (1991), an increase in the proportional output tax rate causes a decline in the rate of growth. So the optimum tax rate is zero in the Rebelo (1991) model. In Rebelo (1991), physical capital accumulation positively affects the human capital accumulation. So the increase in the output tax rate reduces the rate of growth of human capital and the rate of growth of the economy in the steady state equilibrium. In Mohtadi (1996), an increase in the output tax rate reduces the rate of growth of physical capital. Although there exists negative external effect of physical capital accumulation on the environmental quality in his model, the rate of growth of output is positively related to rate of growth of physical capital accumulation. Hence, in his model, an increase in output tax rate reduces the rate of growth of output. So our result contradicts the results obtained by Rebelo (1991) and Mohtadi (1996). This model points out a case where growth rate is positively related to the tax rate. This is so because physical capital accumulation has no positive effect on the human capital accumulation in this model. If we assume Rebelo (1991) type of human capital accumulation function where physical capital contributes positively to the human capital accumulation and if we also consider the negative effect of physical capital accumulation on environmental quality, then we may not have a monotonic relationship between the growth rate and the tax rate. On the contrary, we may have an interior optimal tax rate which would maximize the balanced growth rate of the economy.

3.3 Transitional Dynamics

We now turn to analyse the transitional dynamic properties of the model around the steady state equilibrium point. We derive the equations of motion which describe the dynamics of the system.

We define two new variables x and y such that $x = \frac{C}{K}$ and $y = \frac{H}{K}$.

Using equations (1), (9) and (10) we have

$$\frac{\dot{x}}{x} = Aa^{\gamma}y^{\gamma}[(1-\tau)(1-\gamma)-1] - \rho + x. \tag{16}$$

Using equations (1), (3), (4) and (9) we have

$$\frac{\dot{y}}{y} = mE_0{}^{\delta}y^{\beta\delta}(1-a) - Aa^{\gamma}y^{\gamma} + x. \tag{17}$$

Differentiating both sides of the equation (11) with respect to time, t, and then using equations (3), (7), (8) and (9), we have

$$\frac{\dot{a}}{a} = \frac{\{1-(1-\gamma+\beta\delta)(1-a)\}}{(1-\gamma)}m(E_0)^{\delta}y^{\beta\delta} + Aa^{\gamma}y^{\gamma}\{\frac{\beta\delta}{1-\gamma}+\tau\} - \frac{(1-\gamma+\beta\delta)}{(1-\gamma)}x. \tag{18}$$

The dynamics of the system is now described by the differential equations (16), (17) and (18). Their solutions describe the time path of the variables x, y and a. When either $\beta = 0$, or $\delta = 0$, and $\tau = 0$, these equations of motion become identical to those obtained in Benhabib and Perli (1994).

Along the steady state equilibrium growth path, $\dot{x} = \dot{y} = \dot{a} = 0$. Their steady state equilibrium values are denoted by x^*, y^* and a^*. From equation (16), we have

$$x^* = \rho - A{a^*}^{\gamma}{y^*}^{\gamma}\{(1-\tau)(1-\gamma) - 1\}.$$

From equations (17) and (18) we have

$$y^* = [\frac{A(1-\tau)(1-\gamma)\rho^{\gamma}}{(m{E_0}^{\delta})^{1+\gamma}}]^{\frac{1}{\beta\delta-\gamma+\gamma\beta\delta}};$$

and

$$a^* = [\frac{\rho^{\gamma-\beta\delta}\{A(1-\tau)(1-\gamma)\}^{\beta\delta}}{(m{E_0}^{\delta})^{\gamma}}]^{\frac{1}{\gamma-\beta\delta-\gamma\beta\delta}}.$$

So the steady state equilibrium point is unique. If either $\beta = 0$, or, $\delta = 0$, then

$$y^* = [\frac{A(1-\tau)(1-\gamma)\rho^{\gamma}}{m^{(1+\gamma)}}]^{\frac{1}{-\gamma}};$$

and

$$a^* = \frac{\rho}{m}.$$

These expressions are similar to those obtained in Benhabib and Perli (1994). We now turn to show that there exists a unique saddle path converging to the unique steady state equilibrium point. Note that it is a system of 3 differential equations. Initial value of the variable, y, is historically given; and the values of other two variables x and a can be chosen by the controller. So if the roots are real, then, in order to get the unique saddle path converging to the steady state equilibrium point, we need exactly one latent root of the Jacobian matrix corresponding to the system of differential equations to be negative and the other two roots to be positive.

We can show that[5]

$$\text{Trace of} J = x^* + A(a^*y^*)^{\gamma}\gamma[\tau + \frac{\beta\delta}{(1-\gamma)} - 1] + m(E_0y^{\beta})^{\delta}[(1-a^*)\beta\delta + \frac{(1-\gamma+\beta\delta)}{(1-\gamma)}a^*];$$

and

$$\text{Det of} J = m(E_0y^{\beta})^{\delta}x^*\frac{\rho}{(1-\gamma)}[\beta\delta(1+\gamma) - \gamma].$$

[5]Derivation in detail is shown in the Appendix (A).

Here J is the 3×3 Jacobian matrix corresponding to the system of three differential equations.

Since we have assumed $[\beta\delta(1+\gamma)-\gamma] < 0$, the Determinant of J is always negative. Note that, if there does not exist any external effect of aggregate human capital on production, then the determinant of the Jacobian matrix of corresponding differential equations in the Lucas (1988) model is always negative. The negative sign of the determinant of J implies that either all the three latent roots are negative or only one root is negative with other two roots being positive. So we have to look at the sign of the trace of J. The trace of J is positive if $\beta\delta < \frac{\gamma}{1-\gamma}$. The Trace is positive and Determinant is negative if $(1-\tau)(1-\gamma) < \beta\delta < \frac{\gamma}{(1-\gamma)}$. So all the roots can not be negative when $(1-\tau)(1-\gamma) < \beta\delta < \frac{\gamma}{(1-\gamma)}$. Hence, only one latent root is negative and the other two roots are positive.

Hence, there is a unique saddle path converging to the unique steady state equilibrium point in this case. So we have the following proposition.

Proposition 2 *There exists a unique saddle path converging to the unique steady state equilibrium point if* $(1-\tau)(1-\gamma) < \beta\delta < \frac{\gamma}{(1-\gamma)}$.

Using equations (4), (17) and the definition of y, we have

$$\frac{\dot{E}}{E} = \beta[m{E_0}^{\delta}y^{\beta\delta}(1-a) - Aa^{\gamma}y^{\gamma} + x].$$

So it is clear from this differential equation that, once we obtain time behaviour of y, x and a along the unique saddle path, we can easily solve for the intertemporal transitional behaviour of the environmental quality, E. Since it is a 3×3 dynamic system, we can not use the phase diagram to examine the transitional dynamics of environmental quality. However, it is clear that $\frac{\dot{E}}{E} > (<)0$ for $\frac{\dot{y}}{y} > (<)0$. So it is the time behaviour of the capital intensity of production, $\frac{H}{K}$, which determines the time behaviour of environmental quality, E.

3.4 Factor Income Taxation

Now we consider taxation on factor income at different rates. Suppose that a tax at the rate of τ_K and a tax at the rate of τ_l are imposed on capital income and labour income respectively. If $\tau_K = \tau_l$, then it is equivalent to taxing output at that rate. The budget constraint of this individual in this case is given by

$$\dot{K} = (1 - \tau_K)rK + (1 - \tau_l)waH - C + P. \tag{19}$$

Here r and w are rental rate on capital and wage rate respectively. The dynamic optimization problem of the representative individual in this model is to maximize

$$\int_0^\infty U(C)e^{-\rho t}dt,$$

through the choice of C and a, with

$$U(C) = lnC,$$

and subject to the equations (1), (3), (4) and (19).

The competitive equilibrium conditions of the profit maximizing firm are given by

$$r = A(1 - \gamma)(aH)^\gamma K^{-\gamma};$$

and

$$w = A\gamma(aH)^{\gamma-1}K^{1-\gamma}.$$

From the first order optimality conditions[6], we have

$$\frac{\dot{C}}{C} = (1 - \tau_K)r - \rho; \tag{20}$$

and, in the steady state equilibrium, we have

$$(1 - \tau_K)r = m(E)^\delta. \tag{21}$$

Substituting $r = A(1 - \gamma)(aH)^\gamma K^{-\gamma}$ in equation (21), we have

$$(\frac{H}{K})^{\gamma-\beta\delta} = \frac{m{E_0}^\delta}{(1 - \tau_K)A(1 - \gamma)a^\gamma}. \tag{22}$$

[6]The optimality conditions are given in the Appendix B.

Equating the rate of growth of consumption to the rate of growth of human capital in the steady state equilibrium, we have

$$m(E)^{\delta}a = \rho. \tag{23}$$

Substituting E and $\frac{H}{K}$ from equations (4) and (22) in the equation (23), we have

$$a = [\frac{\rho^{\gamma-\beta\delta}(A(1-\tau_K)(1-\gamma))^{\beta\delta}}{m^{\gamma}(E_0)^{\delta\gamma}}]^{\frac{1}{(\gamma-\beta\delta-\gamma\beta\delta)}}. \tag{24}$$

Note that this equation (24) is same as equation (13) with τ replaced by τ_K. a varies inversely with τ_K. Also note that a is independent of τ_l. Hence $g = \rho(\frac{1}{a} - 1)$ also varies positively with τ_K and is independent of the change in τ_l.

Proposition 3 *The steady state equilibrium growth rate of the economy varies positively with the tax rate imposed on capital income and is invariant to the tax rate imposed on labour income.*

If the tax rate imposed on physical capital income is increased, then the post tax marginal productivity of capital is reduced. This lowers the rate of growth of consumption. So the rate of growth of physical capital stock is reduced. As a result, environmental quality is improved; and this, in turn, exerts a positive external effect on human capital accumulation. Hence the rate of growth of human capital is increased. In the steady state growth equilibrium, this causes the rate of growth of output to rise. However, the change in the tax rate on labour income does not affect the marginal productivity of capital; and so it keeps the rate of growth unchanged.

3.5 Alternative Uses of Tax Revenue

3.5.1 Abatement Expenditure

The abatement expenditure is an important factor determining the quality of the environment. Authors like Hettich (1998), Dinda (2005), Ricci (2007)etc. analyse the role of abatement expenditure on environmental pollution and on economic growth. In this section, we assume that the abatement activity is undertaken by the government. Tax revenue is not distributed as lumpsum payment. It is used to finance the abatement expenditure denoted by S. Hence $S = \tau Y$ and $P = 0$. Our modified environmental quality function is given by

$$E = E_0 K^{-\beta} H^{\beta-\theta} S^{\theta} \tag{25}$$

with $\beta > \theta > 0$. Here $\theta > 0$ implies that the level of environmental quality varies positively with the size of abatement expenditure; and $\beta > \theta$ implies that the former varies positively with the stock of human capital.

The budget constraint of the household is given by

$$\dot{K} = (1-\tau)Y - C. \tag{26}$$

It is same as equation (2) with $P = 0$. The dynamic optimization problem of the representative individual in this model is to maximize

$$\int_0^\infty U(C)e^{-\rho t}dt$$

through the choice of C and a, with

$$U(C) = lnC,$$

and subject to the equations (1), (3), (25) and (26). S is treated as given in the optimization process because it is external to the individual in a competitive economy.

The optimality conditions remain same as obtained in section 2 and hence these are represented again by equations (5), (6), (7) and (8).

Here also we obtain

$$\frac{\dot{x}}{x} = -A(1-\tau)\gamma a^{\gamma} y^{\gamma} - \rho + x. \tag{27}$$

Using equations (1), (3), (25) and (26), we have

$$\frac{\dot{y}}{y} = m{E_0}^{\delta}(\tau A a^{\gamma})^{\theta\delta} y^{\delta(\beta+(\gamma-1)\theta)}(1-a) - A(1-\tau)a^{\gamma}y^{\gamma} + x. \tag{28}$$

Differentiating both sides of the equation (11) with respect to time, t, and then using equations (2), (3), (7) and (8), we have

$$\frac{\dot{a}}{a}=\frac{\{1-\{(1-\gamma)(1-\theta\delta)+\delta\beta\}(1-a)\}}{\{(1-\gamma(1-\theta\delta)\}}m(E_0)^{\delta}(\tau A a^{\gamma})^{\theta\delta}y^{\delta(\beta+(\gamma-1)\theta\delta))}+$$

$$A(1-\tau)a^{\gamma}y^{\gamma}\frac{\delta[\beta-\theta(1-\gamma)]}{\{1-\gamma(1-\theta\delta)\}}-\frac{\{(1-\gamma)(1-\theta\delta)+\delta\beta\}}{\{1-\gamma(1-\theta\delta)\}}x. \quad (29)$$

In the steady state equilibrium, from equation (27), we have

$$x^{*}=\rho+Aa^{*\gamma}y^{*\gamma}(1-\tau)\gamma.$$

From equations (28) and (29), we have

$$y^{*}=[\frac{\rho}{(1-\gamma)(1-\tau)Aa^{*(\gamma+1)}}]^{\frac{1}{\gamma}};$$

and

$$a^{*}=[\frac{\rho^{\gamma-\beta\delta+\theta\delta(1-\gamma)}\{A(1-\tau)(1-\gamma)\}^{(\beta\delta+\theta\delta(\gamma-1))}}{(m{E_0}^{\delta}(A\tau)^{\theta\delta)})^{\gamma}}]^{\frac{1}{\gamma+\{\theta-(1+\gamma)\beta\}\delta}}.$$

We have already assumed $\beta>\theta$ which implies $\beta>(1-\gamma)\theta$. Here Trace of the Jacobian matrix corresponding to the dynamic system given by the differential equations (27), (28) and (29) is

$$\text{Trace of J}=\frac{\rho\delta}{a(1-\gamma)\{1-\gamma(1-\theta\delta)\}}[\{\beta-\theta(1-\gamma)\}\{1-\gamma(1-\gamma)\}+\gamma^{2}\theta(1-\gamma)]+\frac{\rho(1+\gamma\beta\delta)}{\{1-\gamma(1-\theta\delta)\}}.$$

Here Trace is positive if $\beta>\theta(1-\gamma)$.

Similarly it can be shown that Determinant of J

$$=m{E_o}^{\delta}xy(\tau Aa^{\gamma})^{\theta\delta}y^{(\gamma-1)\theta+\beta\}\delta-1}\rho[(-\theta+\gamma\theta+\beta)\delta(\gamma-1)+N\{\gamma-\delta(-\theta+\gamma\theta+\beta)+a\delta(-\theta+\gamma\theta+\beta)\}]$$

where

$$N=2(\gamma-1)+(-\theta+\gamma\theta+\beta)\delta-\gamma+\frac{1}{a}[\gamma\delta\theta\{(-\theta+\gamma\theta+\beta)\delta-\gamma\}-2\gamma\theta\delta a+2(\theta-\beta)\delta a].$$

If $(-\theta+\gamma\theta+\beta)>0$ and $\gamma>\delta(-\theta+\gamma\theta+\beta)$ then N is negative. Hence the condition of saddle path stability i.e. the condition of Jacobian determinant being negative, in this case is satisfied when $\beta>\theta(1-\gamma)$ and $\gamma>\frac{\delta(\beta-\theta)}{(1-\theta\delta)}$.

When the tax revenue is spent to finance the abatement expenditure (AE) then

$$\frac{d(lna)}{d\tau}|AE=-\frac{\delta[\tau(\beta-\theta)+\theta\gamma]}{\tau(1-\tau)[\gamma(1-\beta\delta)-\delta(\beta-\theta)]}.$$

Hence the condition, that is required to ensure that the tax rate affects a^* negatively, is $\gamma > \frac{\delta(\beta-\theta)}{(1-\beta\delta)}$. Since we have assumed $\beta > \theta$, $\gamma > \frac{\delta(\beta-\theta)}{(1-\beta\delta)}$implies that $\gamma > \frac{\delta(\beta-\theta)}{(1-\theta\delta)}$. So the condition $\gamma > \frac{\delta(\beta-\theta)}{(1-\beta\delta)}$ is needed to satisfy saddle path stability as well as to ensure that the tax rate affects a^* negatively. Hence a^* is negatively related to the tax rate and the growth rate, g, is positively related to the tax rate provided that the condition $\gamma > \frac{\delta(\beta-\theta)}{(1-\beta\delta)}$ is fulfilled.

When the tax revenue is spent to provide lumpsum transfer (LT) then

$$\frac{d(lna)}{d\tau}|LT = -\frac{\beta\delta}{(\gamma-\beta\delta-\gamma\beta\delta)(1-\tau)}.$$

$$|\frac{d(lna)}{d\tau}|AE - |\frac{d(lna)}{d\tau}|LT = \frac{\theta\delta\gamma[(\gamma-\tau)(1-\beta\delta)-\beta\delta]}{(1-\tau)\tau(\gamma-\gamma\beta\delta-\beta\delta+\theta\delta)(\gamma-\beta\delta-\gamma\beta\delta)}.$$

The mathematical sign of the LHS depends on the sign of $[(\gamma-\tau)(1-\beta\delta)-\beta\delta]$. Hence the effect of τ on the labour allocation to the education sector can not be unambiguously compared in these two systems even when all the parameters take same values. So we have the following proposition.

Proposition 4 *When the environmental quality varies positively with the abatement expenditure financed by a proportional output tax, an increase in the tax rate raises the steady state equilibrium growth rate of the economy if* $\gamma > \frac{\delta(\beta-\theta)}{(1-\beta\delta)}$.

3.5.2 Educational Subsidy

In this section, we assume that the tax revenue is spent to finance the educational subsidy only. The modified human capital accumulation function is given by

$$\dot{H} = mE^{\delta}(1-a)HG^{\phi} \tag{30}$$

with $\phi > 0$. Here

$$G = \frac{\tau Y}{H};$$

and G denotes the effectiveness of the educational subsidy that varies positively with the level of subsidy and inversely with the stock of human capital. If $\phi = 0$, we come back to the equation (3) of the basic model in section 2. Since G is proportional to Y and since Y varies positively with capital stock, K, the rate of human capital accumulation also receives a positive external effect from physical capital accumulation. The budget constraint of the household is given by equation (26). The representative individual maximizes

$$\int_0^\infty lnCe^{-\rho t}dt$$

with respect to C and a subject to the equations (1), (4), (26) and (30). G is treated as given in the maximization process. Note that, $\theta\delta$ of th model with abatement expenditure is replaced by ϕ in this model and all other things remain unchanged.

3.6 Growth and Social Welfare

If the social welfare is a positive function of the balanced growth rate, then there is no conflit between the growth rate maximization and the social welfare maximization. Fortunately, this is true in all the models described in the earlier sections.

Here W stands for the level of social welfare. So

$$W = \int_0^\infty lnCe^{-\rho t}dt = g\int_0^\infty te^{-\rho t}dt + lnC(0)\int_0^\infty e^{-\rho t}dt = \frac{lnC(0)}{\rho} + \frac{g}{\rho^2}$$

because $C(t) = C(0)e^{gt}$ along the balanced growth path. In section 3.2 and in section 3.4, where the tax revenue is returned to the individuals as lumpsum income transfer, we have $C(0) = Y(0) - gK(0)$. Hence, we have

$$\frac{dW}{dg} = \frac{1}{\rho^2} - \frac{K(0)}{\rho\{Y(0) - gK(0)\}} = \frac{1}{\rho^2}[\frac{\frac{C(0)}{K(0)} - \rho}{\frac{C(0)}{K(0)}}]$$

Here, $\frac{dW}{dg} > 0$, because

$$\frac{C(0)}{K(0)} - \rho = (mE_0{}^{\delta})^{\frac{-\gamma}{\beta\delta-\gamma+\gamma\beta\delta}}\rho^{\frac{\gamma\beta\delta}{\beta\delta-\gamma+\gamma\beta\delta}}(A(1-\tau)(1-\gamma))^{\frac{\beta\delta}{\beta\delta-\gamma+\gamma\beta\delta}}[\frac{1-(1-\tau)(1-\gamma)}{(1-\tau)(1-\gamma)}] > 0.$$

In section 3.5 and in section 3.6, we have $C(0) = (1-\tau)Y(0) - gK(0)$. In this case also, we have

$$\frac{dW}{dg} = \frac{1}{\rho^2}[\frac{\frac{C(0)}{K(0)} - \rho}{\frac{C(0)}{K(0)}}]$$

In these two cases, along the steady state growth path, $\frac{C(0)}{K(0)}$ is given by

$$\frac{C(0)}{K(0)} = \rho + \frac{\gamma\rho}{(1-\gamma)a^*}.$$

Hence, in section 3.5, $\frac{C(0)}{K(0)}$ is given by

$$\frac{C(0)}{K(0)} - \rho = \frac{\gamma\rho}{(1-\gamma)}\left[\frac{\rho^{\gamma-\beta\delta+\theta\delta(1-\gamma)}\{A(1-\tau)(1-\gamma)\}^{(\beta\delta+\theta\delta(\gamma-1))}}{(mE_0{}^{\delta}A\tau^{\theta\delta})^{\gamma}}\right]^{\frac{-1}{\gamma+\{\theta-(1+\gamma)\beta)\}\delta}} > 0$$

and in section 3.6, $\frac{C(0)}{K(0)}$ is given by

$$\frac{C(0)}{K(0)} - \rho = \frac{\gamma\rho}{(1-\gamma)}\left[\frac{\rho^{\gamma-\beta\delta+\phi(1-\gamma)}\{A(1-\tau)(1-\gamma)\}^{(\beta\delta+\phi(\gamma-1))}}{(mE_0{}^{\delta}(A\tau)^{\phi})^{\gamma}}\right]^{\frac{-1}{\gamma+\phi-(1+\gamma)\beta\delta}} > 0.$$

Hence $\frac{dW}{dg} > 0$. So the social welfare along the balanced growth path varies positively with the balanced growth rate, g. However, we can not derive the socially optimal tax rate along the transitional growth path[7].

3.7 Conclusion

We have developed an endogenous growth model where the environmental quality varies negatively with the stock of physical capital and varies positively with the size of human capital. The rate of human capital accumulation is positively affected by the external effect emanating from environment. The interesting results obtained in this model are as follows. Firstly, the steady state equilibrium rate of growth, in this model, varies positively with the proportional tax rate imposed on output or on capital income when tax revenue is spent as lumpsum payment. This result holds even if the environmental quality is positively related to the abatement expenditure and the entire tax revenue is spent as abatement expenditure. However, this rate of growth is independent of the tax rate imposed on labour income. In Lucas (1988), this rate of growth is independent of the tax rate imposed either on output or on capital income. In Rebelo (1991), Mohtadi (1995) etc. the rate of growth varies inversely with the tax rate. Garcia Castrillo Sanso (2000), Gomez

[7] The detailed derivations are shown in Appendix C.

(2003) etc. find the optimal physical capital tax rate to be zero and the optimal labour tax rate to be positive in the Lucas (1988) model when tax revenue is spent as educational subsidy. However, none of these models considers the negative effect of environmental degradation on the human capital accumulation. However, in this model, we have considered the negative effect of environmental degradation on the human capital accumulation and have shown that the steady state equilibrium growth rate would receive a positive effect from taxation either on output or on capital income. Existing literature does not point out such a possibility.

Appendix A

Here the Jacobian matrix corresponding to the system of differential equations (16), (17), (18) is given by:

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{x}}{\partial a} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial a} \\ \frac{\partial \dot{a}}{\partial x} & \frac{\partial \dot{a}}{\partial y} & \frac{\partial \dot{a}}{\partial a} \end{bmatrix};$$

and the elements of the Jacobian matrix evaluated at the steady state equilibrium values of the variables are given as follows.

$$\frac{\partial \dot{x}}{\partial x} = x^*;$$

$$\frac{\partial \dot{x}}{\partial y} = Aa^{*\gamma} x^* \gamma y^{*\gamma - 1}\{(1-\tau)(1-\gamma) - 1\};$$

$$\frac{\partial \dot{x}}{\partial a} = Aa^{*\gamma - 1} x^* \gamma y^{*\gamma}\{(1-\tau)(1-\gamma) - 1\};$$

$$\frac{\partial \dot{y}}{\partial x} = y^*;$$

$$\frac{\partial \dot{y}}{\partial y} = m(E_0)^\delta (1-a)\beta\delta y^{\beta\delta} - Aa^\gamma \gamma y^\gamma;$$

$$\frac{\partial \dot{y}}{\partial a} = -A\gamma a^{\gamma-1} y^{\gamma+1} - m(E_0)^\delta y^{*(\beta\delta+1)};$$

$$\frac{\partial \dot{a}}{\partial x} = -\frac{(1-\gamma+\beta\delta)a^*}{(1-\gamma)};$$

$$\frac{\partial \dot{a}}{\partial y} = \frac{\{1-(1-\gamma+\beta\delta)(1-a)\}}{(1-\gamma)} m(E_0)^\delta y^{\beta\delta-1} a\beta\delta + \{\tau + \frac{\beta\delta}{(1-\gamma)}\} Aa^{\gamma+1}\gamma y^{\gamma-1};$$

and

$$\frac{\partial \dot{a}}{\partial a} = \frac{(1-\gamma+\beta\delta)}{(1-\gamma)} m(E_0)^\delta y^{\beta\delta} a + \{\tau + \frac{\beta\delta}{(1-\gamma)}\} A\gamma a^\gamma y^\gamma.$$

The characteristic equation of the J matrix is given by

$$|J - \lambda I_3| = 0;$$

where λ is an eigenvalue of the Jacobian matrix with elements being evaluated at the steady state equilibrium values. The three charateristic roots can be solved from the equation

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where

$$a_0 = -1,$$

$$b_0 = \text{Trace of } J,$$

$$a_1 = \text{sum of the minors of diagonal terms of } J,$$

and

$$b_1 = \text{Determinant of} J.$$

Clearly a_0 is negative. We can derive that

$$b_0 = \text{Trace of } J = J_{xx}+J_{aa}+J_{yy} = x^* + \frac{(1-\gamma+\beta\delta)}{(1-\gamma)} m(E_0)^\delta y^{\beta\delta} a + \{\tau + \frac{\beta\delta}{(1-\gamma)}\} A\gamma a^\gamma y^\gamma$$

$$+m(E_0)^\delta (1-a)\beta\delta y^{\beta\delta} - Aa^\gamma \gamma y^\gamma$$

$$= x^* + A(a^* y^*)^\gamma \gamma[\tau + \frac{\beta\delta}{(1-\gamma)} - 1] + m(E_0 y^\beta)^\delta [(1-a^*)\beta\delta + \frac{(1-\gamma+\beta\delta)}{(1-\gamma)} a^*].$$

Also it can be shown that

$$b_1 = \text{Determinant of } J = J_{xx}[J_{yy}J_{aa} - J_{ya}J_{ay}] - J_{xy}[J_{yx}J_{aa} - J_{ya}J_{ax}] + J_{xa}[J_{yx}J_{ay} - J_{yy}J_{ax}]$$

$$= m^2 {E_0}^{2\delta} a(1-a) y^{2\beta\delta} x\beta\delta \frac{(\beta\delta + 1 - \gamma)}{(1-\gamma)} - AmE_0{}^\delta a^{\gamma+1} y^{\gamma+\beta\delta} x(1-\tau)\gamma(1-\gamma)$$

$$+mE_0{}^\delta y^{\beta\delta+1} x[mE_0{}^\delta \beta\delta y^{\beta\delta-1} a \frac{\{1-(\beta\delta+1-\gamma)(1-a)\}}{(1-\gamma)} + A\gamma a^{\gamma+1} y^{\gamma-1}(1-\tau)(\beta\delta-\gamma)].$$

From the steady state equilibrium values – x^*, y^* and a^*, obtained from equations (16), (17), (18), we have the following equations.

$$Aa^{*\gamma+1} y^{*\gamma} = \frac{\rho}{(1-\tau)(1-\gamma)};$$

and

$$mE_0{}^{\delta}y^{*\beta\delta} = \frac{Aa^{*\gamma}y^{*\gamma}(1-\tau)(1-\gamma)-\rho}{(1-a^*)}.$$

Using the two equations mentioned above we find that

$$b_1 = \text{Determinant of } J = \frac{mE_0{}^{\delta}y^{*\beta\delta}x^*\rho}{(1-\gamma)}[\beta\delta(1+\gamma)-\gamma].$$

Appendix B

The current value Hamiltonian function is given by

$$Z = lnC + \mu_K[(1-\tau_K)rK + (1-\tau_l)waH - C + P] + \mu_H[m(E)^{\delta}(1-a)H]$$

where μ_K and μ_H are the co state variables.

The first order optimality conditions are given by the following.

$$\frac{\partial Z}{\partial C} = \frac{1}{C} - \mu_K = 0, \qquad (B.1)$$

and

$$\frac{\partial Z}{\partial a} = \mu_K(1-\tau_l)wH - \lambda_H m(E)^{\delta}H = 0. \qquad (B.2)$$

Time behaviour of the co state variables along the optimum growth path should satisfy the following.

$$\dot{\mu_K} = \rho\mu_K - \mu_K(1-\tau_K)r, \qquad (B.3)$$

and

$$\dot{\mu_H} = \rho\mu_H - \mu_K(1-\tau_l)wa - \mu_H m(E)^{\delta}(1-a). \qquad (B.4)$$

Transversality conditions are given by the followings.

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_H(t)H(t) = 0.$$

From equations (B.1) and (B.3) we have

$$\frac{\dot{C}}{C} = \frac{(1-\tau_K)r-\rho}{\sigma}.$$

From equation (B.2) we have

$$\frac{\mu_K}{\mu_H} = \frac{mE^\delta}{(1-\tau_l)w}.$$

Since w is constant in the steady state equilibrium, we have

$$\frac{\dot{\mu_K}}{\mu_K} = \frac{\dot{\mu_H}}{\mu_H}.$$

From the equation mentioned above, we have

$$(1-\tau_K)r = m(E)^\delta.$$

Appendix C

In section 3.2 and in section 3.4, we have

$$\dot{K} = Y - C.$$

Hence, along the steady state growth path,

$$\frac{\dot{K}}{K} = g = \frac{Y(0)}{K(0)} - \frac{C(0)}{K(0)}.$$

Now, from equation (1) we find that, along the steady state growth path,

$$\frac{Y(0)}{K(0)} = Aa^{*\gamma}(\frac{H}{K})^{*^{\gamma}}.$$

Using equation (10) we have,

$$g = A(1-\tau)(1-\gamma)a^{*\gamma}(\frac{H}{K})^{*^{\gamma}} - \rho.$$

Hence

$$\frac{C(0)}{K(0)} = \frac{Y(0)}{K(0)} - g = \rho + Aa^{*\gamma}(\frac{H}{K})^{*^{\gamma}}[1-(1-\tau)(1-\gamma)].$$

Substituting the values of a^* and $(\frac{H}{K})^*$ from section 3.3 we obtain the expression of $(\frac{C(0)}{K(0)} - \rho)$ in section 3.7.

In section 3.5 and in section 3.6, we have

$$\dot{K} = (1-\tau)Y - C.$$

Hence,

$$\frac{C(0)}{K(0)} = (1-\tau)\frac{Y(0)}{K(0)} - g.$$

In these sections also, we find that

$$g = A(1-\tau)(1-\gamma)a^{*\gamma}(\frac{H}{K})^{*^{\gamma}} - \rho.$$

The expression of $\frac{Y(0)}{K(0)}$ remains same as above. Hence,

$$\frac{C(0)}{K(0)} - \rho = \frac{\gamma\rho}{(1-\gamma)a^*}.$$

Chapter 4

Dualism in the Human Capital Accumulation and Transitional Dynamic Properties of a Growing Economy*

4.1 Introduction

In chapter 1, we have discussed how the Lucas (1988) model has been extended in various directions. A few dual economy models have been developed to analyse endogenous growth in less developed countries. However, none of the existing models focuses on the dualism in the mechanism of human capital formation of two different groups of individuals. In this chapter, we analyze transitional dynamic properties of a growth model of a dual economy in which dualism exists in the mechanism of human capital accumulation of two types of individuals. We derive conditions under which the saddle path converging to the steady-state growth equilibrium point is unique and the conditions under which the transitional growth path may be indeterminate. Importance of this dualism has been explained in section 1.6.12 in chapter 1 of the present thesis.

In this chapter, we develop a growth model of an economy in which human capital accumulation is viewed as the source of economic growth and in which dif-

*A related version of this chapter is published in the journal called 'Keio Economic Studies' ference exists in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor. The poor individual lags behind the rich individual in terms of the initial endowment of human capital and in terms of the efficiency of the human capital accumulation technology. The rich individual not only allocates his labour time to his production and to own skill accumulation but also allocates a part of his labour time to the training of the poor people[1]. We assume the presence of external effect of the human capital on production as well as on the human capital accumulation of the poor individual[2].

The analysis of the transitional dynamic properties of the growth models have received substantial attention of the researchers in recent years[3]. A number of studies have analyzed the transitional dynamic properties in Lucas (1988) model. Xie (1994) considers the Lucas (1988) model to examine global stability properties and shows that the Lucas (1988) model generates indeterminacy in the transitional growth path in the presence of strong external effects on production. Benhabib and Perli (1994) do the local stability analysis and come to similar results. In this chapter, we do the local stability analysis of the steady state equilibrium in our

[1]This voluntary allocation of labour time to the training of the poor individual can not be supported in a world where the contribution comes mainly in the form of tax payment. However we have mentioned evidences of voluntary contributions too in section 1.8 in chapter 1.In the examples presented in section 1.8, we are providing various examples of voluntary labour supply by rich individuals in different countries at different point of time. We do not have any time series data or panel data on this. So we can not test whether this amount of voluntary labour supply is optimal as a class or not.

[2]There exists a large theoretical literature in both urban economics and in macroeconomics that has considered external effects emanating from human capital in explaining growth of cities, religions and countries e.g. Glaeser and Mare (1994), Glaeser (1997), Peri (2002), Ciccone and Peri (2002). In some other literature, it is found that education generates very little externalities. e.g Rudd (2000), Acemoglu and Angrist (2000). Moretti (2003) rightly points out that the empirical literature on the subject is still very young and more work is needed before we can draw convincing conclusions about the size of human capital externalities.

[3]This includes the work of Caballe and Santos (1993), Arnold (1997), Xie (1994), Benhabib and Perli (1994), Alonso-Carrera (2001), Ruiz-Tamarit (2002), Mulligan and Sala-i-Martin (1993), Mino (1996, 2001b), Bond, Wang and Yip (1996), Song (2000) and Ortigueira (1998) etc.

more general model and derive some interesting transitional dynamic results. We show that a social IRS production technology with external effects of human capital can not explain indeterminacy in this model. The result seems to be interesting because it is contrary to that obtained from Xie (1994) and from Benhabib and Perli (1994).

The rest of the chapter is organized as follows. Section 4.2 presents the basic model. Section 4.3 presents the transitional dynamic analysis of the basic model. Concluding remarks are made in Section 4.4.

4.2 The basic dual economy model

We consider an economy with two types of individuals –rich individuals and poor individuals. Poor individuals lag behind rich individuals in terms of initial endowment of human capital and in terms of the efficiency of the human capital accumulation technology. All workers (individuals) are employed in a single aggregative sector that produces a single good. By human capital we mean the set of specialized skills or efficiency level of workers that accumulate over time. The mechanisms of human capital accumulation are different for two types of individuals. There is external effect of human capital of the rich individuals on the production and on the human capital accumulation of the poor individuals. Population size of either type of individual is normalised to unity. All individuals belonging to each group are assumed to be identical. There is full employment of both types of labour and the labour market is competitive.

The single production sector is owned by rich individuals and they employ poor individuals as wage labourers. Rich individuals and poor individuals have different types of human capital which are imperfectly substitute. The rich individual allocates 'a' fraction of the total non-leisure time to production. Let H_R and H_P be the skill level of the representative rich and poor individual (worker) respectively.

The production function takes the following form.

$$Y = A(aH_R)^{\alpha} {H_P}^{1-\alpha} \bar{H_R}^{\epsilon_R} \bar{H_P}^{\epsilon_P} \quad (1)$$

where $0 < \alpha < 1$. Here $\bar{H_R}$ and $\bar{H_P}$ represent the average levels of human capital of all the individuals belonging to the rich (R) group and to the poor (P) group respectively. $\epsilon_R > 0$ and $\epsilon_P > 0$ are parameters representing the magnitude of the external effect of their human capital on production respectively. Production function satisfies CRS in terms of private inputs but shows social IRS if external effect is taken into consideration. Y stands for the level of output.

The representative rich individual (worker) owns the advanced type of human capital and his income is given by αY. $(1-\alpha)Y$ is the wage income of the poor workers because the labour market is competitive. Both the rich individual and the poor individual consume whatever they earn and hence they do not save (or invest). So there is no accumulation of physical capital in this model; and so capital does not enter as an input in the production function[4]. So we have

$$C_R = \alpha Y; \quad (2)$$

and

$$C_P = (1-\alpha)Y. \quad (3)$$

Here C_P and C_R are the levels of consumption of the representative poor worker and of the representative rich worker respectively. The representative rich individual (worker) maximizes his discounted present value of utility over the infinite time horizon with respect to the labour time allocation variables. His instantaneous utility function is given by

$$U(C_R) = \frac{C_R^{1-\sigma}}{1-\sigma}. \quad (4)$$

[4]Though it is assumed for simplicity, it is a serious limitation of the exercise. However, the model becomes highly complicated when physical capital accumulation is introduced. There are number of authors who did not consider physical capital in growth models e.g. Pecorino (1992), Rosendahl (1996), Lucas (2004), Driskill and Horowitz (2002) etc.

Here $\sigma > 0$ is the constant elasticity of marginal utility of consumption.

4.2.1 Difference in the mechanism of human capital accumulation

Mechanism of the human capital accumulation of the representative rich individual is assumed to be similar to that in the Lucas (1988) model. The rate at which his human capital is formed is proportional to the labour time or effort devoted to acquire skill.
Hence

$$\dot{H}_R = mbH_R \tag{5}$$

where b is the fraction of the non-leisure time devoted to acquiring his own skill. Here $0 \leq b \leq 1$; and m is a positive constant representing the productivity parameter of the human capital formation function of the rich individual.

However, mechanisms of human capital formation for the two classes of individuals are different. The skill formation of a poor individual takes place through the training program conducted by the rich individual who wants to make the poor individual more efficient and productive. Every rich individual spends $(1 - a - b)$ fraction of its labour time in this training. Individuals of the rich region have incentive to train the individuals of the poor region because they work as labourers in the rich sector[5]. For the sake of simplicity, it is assumed that poor individuals

[5] This story is valid when the process of human capital accumulation refers to internal training provided by the employing firm. In the case of formal schooling, every rich individual may deviate unilaterally from contributing to educational services. However, this is not true in a situation where some kind of Folk Theorem holds. For example, all rich individuals may co-operate among themselves and may come to an agreement that each of them would employ equal number of educated poor workers. In that case, equal distribution of benefit provided by formal schooling is ensured for rich individuals. All rich individuals are identical in terms of their preference, capital endowment, production technology and skill. Similarly all poor individuals are identical in terms of skill. So equal allocation is the optimum allocation in this case. However, there is a problem in justifying that they will not be able to internalize externalities present in the production function

have surplus labour time and they improve their skill in leisure time (in the evening or in the slack season). So they do not devote any fraction of non-leisure time to learning[6]. The additional skill acquired by the representative poor worker (individual) is assumed to be a linear homogeneous function in terms of the effort level of the rich individual and of the skill level already attained by the poor individual.

However, we assume that there exists a positive external effect of the average skill level of all rich individuals on the human capital accumulation of the representative poor individual. Hence we have

$$\dot{H_P} = \{(1-a-b)H_R\}^{\delta} {H_P}^{1-\delta-\gamma} \bar{H_R}^{\gamma}. \tag{6}$$

Here $0 < \delta < 1$; and $\gamma > 0$ is the parameter representing the magnitude of the external effect on the skill formation of the poor individual. The accumulation function of H_P satisfies private DRS and social CRS. However, the accumulation function of H_R given by equation (5) satisfies CRS at the private level as well as at the social level. In models of Tamura (1991), Eaton and Eckstein (1997), Lucas (2004) etc. the human capital accumulation technology is subject to external effects. In models of Eaton and Eckstein (1997) and Tamura (1991), average human capital stock is affecting human capital accumulation technology where as, in the model of Lucas (2004), human capital level of the leader affects the human capital accumulation of all other individuals (followers). Leader is the individual with the highest skill level. In our model, the rich individual has already attained high level of human capital and the poor individual is lagging behind. Rich individuals and poor individuals are assumed to be identical within their respective groups. So it is justified to assume that the human capital of the rich individual should have external effect on the poor individual's human capital accumulation technology; and it should not be the other way round.

---

even when they co operate.

[6]This is a simplifying assumption. However, if the time devoted for production and human capital accumulation by the poor individuals are assumed to be exogenously given, that will yield the same result.

4.3 Growth in the competitive economy

4.3.1 The optimization problem

The objective of the representative rich individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional is given by

$$J_H = \int_0^\infty U(C_R)e^{-\rho t}dt.$$

This is to be maximized with respect to a and b subject to the equations of motion given by

$$\dot{H_R} = mbH_R;$$

$$\dot{H_P} = \{(1-a-b)H_R\}^\delta {H_P}^{1-\delta-\gamma}\bar{H_R}^\gamma;$$

and given the initial values of H_R and H_P. Here $U(C_R)$ is given by equation (4) and Y is given by equation (1). Here ρ is the constant positive discount rate. The control variables are a and b where $0 \leq a \leq 1$, $0 \leq b \leq 1$, $0 \leq a+b \leq 1$. The state variables are H_R and H_P. The current value Hamiltonian is given by

$$H^c = \frac{C_R^{1-\sigma}}{1-\sigma} + \lambda_R mbH_R + \lambda_P\{(1-a-b)H_R\}^\delta {H_P}^{1-\delta-\gamma}\bar{H_R}^\gamma$$

where λ_R and λ_P are co-state variables of H_R and H_P respectively representing shadow prices of the human capital of rich individuals and of the human capital of poor individuals. C_R is given by the equation (2). The representative individual can not internalise the externalities. However, $\bar{H_R} = H_R$ because all rich individuals are identical.

4.3.2 Optimality conditions

(A) First order conditions necessary for this optimization problem with respect to control variables a and b are given by the following.

$$(\alpha Y)^{-\sigma}\alpha^2\frac{Y}{a} - \lambda_P\delta\frac{\dot{H_P}}{(1-a-b)} = 0; \tag{7}$$

and

$$\lambda_R m H_R - \lambda_P \delta \frac{\dot{H}_P}{(1-a-b)} = 0. \tag{8}$$

(B) Time derivatives of co-state variables satisfying the optimum growth path are given by the following.

$$\dot{\lambda_R} = \rho\lambda_R - (\alpha Y)^{-\sigma}\alpha^2 \frac{Y}{H_R} - \lambda_R m b - \lambda_P \delta \frac{\dot{H}_P}{H_R}; \tag{9}$$

and

$$\dot{\lambda_P} = \rho\lambda_P - (\alpha Y)^{-\sigma}\alpha(1-\alpha)\frac{Y}{H_P} - \lambda_P(1-\delta-\gamma)\frac{\dot{H}_P}{H_P}. \tag{10}$$

(C) Transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_P(t)H_P(t) = 0.$$

Using equations (7) and (8) we have

$$(\alpha Y)^{-\sigma}\alpha^2\frac{Y}{a} = \lambda_R m H_R; \tag{11}$$

and, using equations (7), (8) and (9), we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - m. \tag{12}$$

Now, from equations (10) and (7), we have

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - \frac{\delta(1-\alpha)ar}{\alpha(1-a-b)} - (1-\delta-\gamma)r \tag{13}$$

where r is the rate of growth of H_P.

4.3.3 The Transitional Dynamics

We now turn to analyse transitional dynamic properties around the steady state equilibrium point. We derive equations of motion which describe the dynamics of the system. We define $z = \frac{H_R}{H_P}$ and $x = (1-a-b)$. Using equations (5) and (6), we have

$$\frac{\dot{z}}{z} = m(1-a-x) - x^{\delta}z^{\delta+\gamma}. \tag{14}$$

Differentiating the log of both sides of the equation (11) with respect to time and then, using equations (1), (5) and (12), we have

$$\frac{\dot{a}}{a} = \frac{1}{1-\alpha(1-\sigma)}[m-\rho-\{1-(\alpha+\epsilon_R)(1-\sigma)\}(1-a-x)m+(1-\alpha+\epsilon_P)(1-\sigma)x^{\delta}z^{\delta+\gamma}]. \tag{15}$$

Similarly differentiating the log of both sides of equation (8) with respect to time and then using equations (5), (6), (12) and (13) we have

$$\frac{\dot{x}}{x} = \frac{1}{(1-\delta)}[m - \frac{a(1-\alpha)\delta}{\alpha}x^{\delta-1}z^{\delta+\gamma} - (1-\delta-\gamma)m(1-a-x)]. \tag{16}$$

The dynamics of the system is now described by differential equations (14), (15) and (16). They solve for the time path of the variables z, a and x.

Steady state equilibrium

Equating the growth rates of z, x and a equal to zero we obtain the steady state equilibrium values of respective variables denoted by z^*, x^* and a^*. From equation (14), we have

$$z^* = (m(1-a^*-x^*)x^{*-\delta})^{\frac{1}{(\delta+\gamma)}}. \tag{17}$$

Substituting z^* from equation (17) into equation (15) and using $\frac{\dot{a}}{a} = 0$, we have

$$m(1-a^*-x^*) = \frac{m-\rho}{[1-(1-\sigma)(1+\epsilon_P+\epsilon_R)]}. \tag{18}$$

Here the LHS of equation (18) represents $\frac{\dot{H}_R}{H_R}$. So equation (18) shows that $\frac{\dot{H}_R}{H_R}$ is independent of γ but is dependent on ϵ_P and ϵ_R in the steady state equilibrium when $\sigma \neq 1$. $\frac{\dot{H}_R}{H_R}$ varies positively (negatively) with ϵ_P and ϵ_R if $\sigma < (>)1$.

So the rate of human capital accumulation of the rich individual is independent of the degree of externality in the human capital accumulation of the poor individual. The value of mb should be less than the highest possible value of the growth rate of human capital, m; and the restriction required for this is given by

$$\sigma > 1 - \frac{\rho}{m[1+\epsilon_P+\epsilon_R]}.$$

If the condition mentioned above is satisfied, then the condition for positive mb is also satisfied because we assume $m > \rho$. We also find that the growth rate of

H_R is positively affected by the increase in the intensity of external effects in the production sector if $\sigma < 1$ and is negatively affected by that if $\sigma > 1$.

The steady state equilibrium rate of growth of income is denoted by χ; and it can be shown that

$$\chi = \frac{\dot{Y}}{Y} = \frac{(1+\epsilon_R+\epsilon_P)(m-\rho)}{[1-(1-\sigma)(1+\epsilon_P+\epsilon_R)]}.$$

Hence χ also varies positively with ϵ_R and ϵ_P.

Note that, if there is no externality, i.e. if $\epsilon_R = 0$, $\epsilon_P = 0$ and $\gamma = 0$, then we have

$$\chi = mb = \frac{m-\rho}{\sigma}.$$

In this case, income and human capital of both type of individuals grow at the common rate, mb. This is the growth rate obtained in the Lucas (1988) model in the absence of external effect on production.

Substituting z^* from equation (17) in equation (16) and using $\frac{\dot{x}}{x} = 0$, we have

$$a^* = \frac{\alpha}{\delta(1-\alpha)}[\frac{m\{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)\}}{(m-\rho)} - (1-\delta-\gamma)]x^*. \quad (19)$$

Using equations (18) and (19) we can solve for x^*; and the solution is given by

$$x^* = \frac{\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\}\delta(1-\alpha)(m-\rho)}{[\alpha\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\} + (m-\rho)(\delta+\alpha\gamma)]m\{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)\}}. \quad (20)$$

Now equations (17), (18), (19) and (20) show that values of z^*, x^* and a^* are uniquely determined given the pre determined values of parameters. a^* and z^* are given by the following expressions.

$$a^* = \frac{\alpha\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R) + (\delta+\gamma)(m-\rho)\}\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\}}{[\alpha\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\} + (m-\rho)(\delta+\alpha\gamma)]m\{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)\}};$$

and

$$z^* = [\frac{m-\rho}{[1-(1-\sigma)(1+\epsilon_P+\epsilon_R)]}x^{*-\delta}]^{\frac{1}{(\delta+\gamma)}} \quad (21)$$

where x^* is given by the equation (20). Using equation (18) we have

$$a^* + x^* = \frac{\frac{\rho}{m} - (1-\sigma)(1+\epsilon_P+\epsilon_R)}{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)}.$$

Also using equations (18) and (19), we have

$$\frac{a^*}{x^*} = \frac{\alpha}{\delta(1-\alpha)}[\frac{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)}{(m-\rho)} + (\delta+\gamma)].$$

If we assume

$$m > \rho \text{ and } \sigma > 1 - \frac{\rho}{m(1+\epsilon_P+\epsilon_R)}$$

then, using the expressions of $(a^* + x^*)$ and $(\frac{a^*}{x^*})$, it can be easily shown that

$$0 < a^* + x^* < 1 \text{ and } (a^*/x^*) > 0.$$

Hence we can show that $0 < x^* < 1$ and $0 < a^* < 1$. Equation (21) now clearly shows that $z^* > 0$ in this case. So we have the following proposition.

Proposition 1 *If $m > \rho > m(1-\sigma)(1+\epsilon_P+\epsilon_R)$ then the steady state growth equilibrium of this model is unique satisfying $0 < a^*, x^* < 1$ and $z^* > 0$.*

Uniqueness of the saddle path

We now turn to prove the uniqueness of the saddle path converging to the steady state equilibrium point. Note that it is a system of 3 differential equations. Initial values of the variable, z, is historically given; and those of other two variables, x and a, can be chosen by the controller. So if the roots are real then, in order to get the unique saddle path converging to the steady state equilibrium point, we need exactly one latent root of the Jacobian matrix to be negative and the other two roots to be positive.

Here the Jacobian matrix corresponding to the system of differential equations (14), (15) and (16) is given by the following

$$J = \begin{bmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial a} \\ \frac{\partial \dot{x}}{\partial z} & \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial a} \\ \frac{\partial \dot{a}}{\partial z} & \frac{\partial \dot{a}}{\partial x} & \frac{\partial \dot{a}}{\partial a} \end{bmatrix},$$

Here, the elements of the Jacobian matrix evaluated at the steady-state equilibrium values of the variables are given in Appendix (A).

The characteristic equation of the J matrix is given by

$$|J - \lambda I_3| = 0$$

where λ is an eigenvalue of the Jacobian matrix with elements being evaluated at the steady state equilibrium values. Its three characteristic roots can be solved from the equation

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where

$$a_0 = -1,$$

$$b_0 = \text{Trace of } J,$$

$$a_1 = \frac{\partial \dot{z}}{\partial x}\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{x}}{\partial a}\frac{\partial \dot{a}}{\partial x} + \frac{\partial \dot{a}}{\partial z}\frac{\partial \dot{z}}{\partial a} - \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{x}}{\partial x},$$

and

$$b_1 = \text{Determinant of} J.$$

Clearly a_0 is negative. We can derive that

$$b_1 = \frac{ma^*\delta(1-\alpha)x^{*2\delta-1}z^{*2(\delta+\gamma)}(\delta+\gamma)}{\alpha(1-\delta)\{1-\alpha(1-\sigma)\}}(a^*+x^*)[(1-\sigma)(1+\epsilon_P+\epsilon_R)-1].$$

This is negative if

$$[(1-\sigma)(1+\epsilon_P+\epsilon_R)-1] < 0$$

which is always true because $m(1-a^*-x^*) > 0$ and $m > \rho$ by assumption. So the negative sign of the determinant of J implies that either all the three roots are negative or only one root of J is negative with other two roots being positive. So we have to look at the sign of b_0 which is trace of J. If $b_0 > 0$, then all the roots can not be negative. Hence, only one latent root is negative and the other two are positive.

Now it can be shown that

$$b_0 = m(a+x) + mx\frac{(1-\delta-\gamma)}{(1-\delta)} + ma\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}.$$

If $m(1-a^*-x^*) > 0$ and if $m > \rho$ then equation (18) shows that

$$1-(1+\epsilon_P+\epsilon_R)(1-\sigma) > 0; \quad \text{and hence} \quad 1-(\alpha+\epsilon_R)(1-\sigma) > 0$$

and $1-\alpha(1-\sigma) > 0$. Also $(1-\delta-\gamma) > 0$, by assumption.

So b_0 is always positive. Hence, in this case, there is a unique saddle path converging to the steady state equilibrium point; and this result is independent of the values of ϵ_P and ϵ_R. So we have the following proposition.

Proposition 2 *There exists a unique saddle path converging to the unique steady state equilibrium point whatever be the magnitude of the external effect of human capital on production.*

So far we have considered the case of three real roots. However, $b_1 < 0$ may imply a possibility of one negative latent root and two imaginary latent roots. Since $b_0 > 0$, the sum of the two imaginary latent roots is positive[7]. In that case too, we should have only one saddle path converging to the equilibrium point. Other trajectories may move cyclically around the equilibrium point. However, they will not converge.

The above mentioned result is important. We consider a production function satisfying private CRS and social IRS. However, the presence of this aggregate external effect of human capital on production can not explain indeterminacy of the transitional growth path in this model whatever be the magnitude of this external effect. Xie (1994), Benhabib and Perli (1994) etc. have shown that the social IRS property of the production technology may explain indeterminacy of equilibria in Lucas (1988) model. We now turn to provide the intuitive explanations of the result summarized in the above mentioned proposition. Since entire income is consumed and there is no accumulation of physical capital, economic growth is explained only by the accumulation of two human capital inputs. In a standard growth model, social IRS property of the production technology helps to raise the investment on physical capital at a very high rate because the agent makes the consumption-savings allocation rationally. With no scope of physical capital to accumulate over time, the social IRS property of the production technology loses its sharpness.

4.4 Conclusion

Existing endogenous growth models dealing with the role of human capital accumulation on economic growth have not considered dualism in the nature of human capital formation among different types of individuals. On the other hand old dual

[7]A numerical example is given in the Appendix (B).

economy models considering institutional dualism in less developed countries do not focus on human capital accumulation and endogenous growth. This chapter attempts to bridge the gap. In this chapter, we analyze the model of an economy with two different types of individuals in which growth originates from human capital accumulation and the dualism exists in the nature of human capital accumulation of two types of individuals. Like Lucas (1988) and Benhabib and Perli (1994), we analyze steady state equilibrium properties and transitional dynamic properties of the model; and put special emphasis on the role of externalities. We consider the role of externality of human capital on the production function.

We derive some interesting transitional dynamic properties of this model. External effects on production and the social increasing returns to scale property of the production technology can not explain the indeterminacy of the transitional growth path if there is constant returns in the human capital accumulation function. This is an interesting result because Xie (1994), Benhabib and Perli (1994) etc. have shown that a strong external effects on production can explain indeterminacy in the Lucas (1988) model. Our assumption that the rich will be rich forever and the poor will be poor forever is somewhat unsatisfactory. A valuable extension would be to introduce intersectoral labour mobility as found in Lucas (2004).

Appendix A

The elements of the Jacobian matrix evaluated at the the steady-state equilibrium values of the variables are given as follows.

$$\frac{\partial \dot{z}}{\partial z} = -(\delta+\gamma)x^{*\delta}z^{*\delta+\gamma};$$

$$\frac{\partial \dot{z}}{\partial x} = -mz^{*} - \delta x^{*\delta-1}z^{*\delta+\gamma+1};$$

$$\frac{\partial \dot{z}}{\partial a} = -mz^{*};$$

$$\frac{\partial \dot{x}}{\partial z} = -a^{*}\frac{(1-\alpha)\delta}{\alpha(1-\delta)}(\delta+\gamma)x^{*\delta}z^{*\delta+\gamma-1};$$

$$\frac{\partial \dot{x}}{\partial x} = \frac{x^{*}}{(1-\delta)}[a^{*}\frac{-(1-\alpha)(\delta-1)\delta}{\alpha}x^{*\delta-2}z^{*\delta+\gamma} + (1-\delta-\gamma)m];$$

$$\frac{\partial \dot{x}}{\partial a} = \frac{(1-\delta-\gamma)}{(1-\delta)}mx^{*} - \frac{(1-\alpha)\delta}{\alpha(1-\delta)}x^{*\delta}z^{*(\delta+\gamma)};$$

$$\frac{\partial \dot{a}}{\partial z} = \frac{(1-\alpha+\epsilon_P)(1-\sigma)(\delta+\gamma)x^{*\delta}a^{*}z^{\delta+\gamma-1}}{\{1-\alpha(1-\sigma)\}};$$

$$\frac{\partial \dot{a}}{\partial x} = ma^{*}\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}} + \frac{(1-\alpha+\epsilon_P)(1-\sigma)\delta a^{*}}{\{1-\alpha(1-\sigma)\}}x^{*\delta-1}z^{*\delta+\gamma};$$

and

$$\frac{\partial \dot{a}}{\partial a} = a^{*}m\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}.$$

Appendix B

We consider the possibility of two positive characteristic roots being complex conjugates. This is well known that an equation of an odd degree must have at least one real root, opposite in sign to that of the last term, the leading term being positive. The charateristic equation is given by

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where $a_0 = -1$. If we divide the both sides of the equation by a_0, b_1 being negative, the last term becomes positive. So there exists one negative real root. There exists a possibility that other two roots are complex conjugates with positive real parts. $Trace(J) > 0 > Det(J)$ ensures that the two roots are not purely imaginary and they have positive real part. Consider the following numerical specifications. For instance, let $m = 2$, $\rho = 0.3$, $\sigma = 2$, $\alpha = 0.7$, $\delta = 0.4$ $\gamma = 0.2$, $\epsilon_P = 0.01$, $\epsilon_R = 0.2$. Under this specification,$(1 - a^* - x^*) = 0.384615385$, $\frac{a^*}{x^*} = 12.83$, $x^* = 0.045$, $a^* = 0.57$, $z^* = 5.14$ and three eigen values are -0.7669, 1.92 and 1.42.

As another example, again let, $m = 2$, $\rho = 0.3$, $\sigma = 3$, $\alpha = 0.7$, $\delta = 0.4$ $\gamma = 0.2$, $\epsilon_P = 0.01$, $\epsilon_R = 0.2$. Under this specification,$(1 - a^* - x^*) = 0.25$, $\frac{a^*}{x^*} = 21.14$, $x^* = 0.03$, $a^* = 0.72$, $z^* = 2.98$ and three eigen values are -0.5411, $1.8817+0.0521i$ and $1.8817-0.0521i$. In both the examples, there exists a unique saddle path which will converge to the steady state equilibrium point because the stable (negative) root is real. Diverging trajectories, on the other hand, are monotonic in the first example where all the roots are real; and is cyclical in the second example where two roots are complex conjugates with positive real parts.

Chapter 5

Physical capital accumulation and the social efficiency of the steady-state equilibrium*

5.1 Introduction

In chapter 4, we have developed a growth model of an economy focusing on the dualism in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor. In this chapter we generalize that growth model considering physical capital accumulation; and here physical capital is used as an input to produce the final good. We also disaggregate the economy into two sectors producing the same commodity with different production techniques and organizations. Otherwise the present model is similar to that of chapter 4. We analyse properties of the steady state growth equilibrium of a competitive household economy and compare them to those of a command (planned) economy. In the competitive economy, the externalities can not be internalized and the labour time allocation between the two sectors is made through the income maximizing behaviour of the migrant. However, in the planned economy, this allocation is directly controlled by the dynamic optimization exercise of the planner who can

*A related version of this chapter is published in the journal called 'Hitotsubashi Journal of Economics'.

also internalize the externalities. We show that steady-state growth equilibrium in the competitive economy is not socially efficient. However, we can not analyse the transitional dynamic properties of this complicated model.

We derive some important results from this model. First, externality parameters of both the sectors play an important role in determining the long run rate of growth of different macro economic variables. Secondly, the steady state equilibrium rate of growth of the human capital in the competitive economy is always less than that in the command economy if there is no external effect of rich sector's human capital on the human capital accumulation in the poor sector. However, in the presence of that externality, we may get the opposite result. In Lucas (1988), rate of growth in the competitive economy is always less than that in the planned economy because Lucas (1988) does not consider human capital accumulation in the poor sector. Thirdly, if there is no externality in either of the two sectors, steady state equilibrium rates of growth are same in both the systems and are equal to that obtained in the Lucas (1988) model. Lastly, the external effect of the poor sector's human capital accumulation is important only if there is external effect of the rich sector's human capital. If this externality comes from the human capital accumulation in the poor sector only and not from the human capital accumulation in the rich sector, then the steady-state equilibrium rate of growth in the planned economy is always higher than that in the competitive economy.

This chapter is organized as follows. Section 5.2 discusses the assumptions of the model with specified focus on the nature of the dualism. In section 5.3, we present the steady state growth rates of the macroeconomic variables in the household (competitive) economy; and in section 5.4 we do the same for the planned (command) economy. In section 5.5, we consider an extension of the basic model introducing accumulation of the physical capital in the poor region too. Concluding remarks are made in section 5.6.

5.2 The dual economy model

We consider a closed economy with two sectors (regions) - a rich sector and a poor sector. In both the regions (sectors), the same and single commodity is produced. By human capital we mean the set of specialized skills or efficiency level of the workers what they can acquire by devoting time to an activity called schooling. This skill level (human capital stock) of the representative worker (individual) of any region accumulates over time. There are external effects of human capital on the production technology in both the regions and on the human capital accumulation function in the poor region. Total number of workers in each of the two regions is normalised to unity. All individuals in a region are assumed to be identical. There is full employment of labour and capital; and the factor markets are competitive.

5.2.1 Dualism in the production technology and organization

The rich region undertakes the capitalist mode of production. Workers of the poor region are employed as wage labourers in the rich region. Physical capital is an essential input in producing the commodity there; and individuals invest a part of their income to augment the stock of physical capital. Labour of individuals (workers) of the rich region and of the poor region are treated as two imperfectly substitute factors of production in the rich sector. An individual of the rich region allocates 'a' fraction of the total non-leisure time in the production sector in that region. Labour originating from the poor region is perfectly mobile between the two regions. The representative worker of the poor region allocates 'u' fraction of his non-leisure time to working in the poor region and v fraction of that time to learning and the remaining fraction to working in the rich region. Let H_R and H_P be the skill level of the representative individual (worker) of the rich region and that of the poor region respectively.

The production function in the rich region takes the following form.

$$Y_R = A_R(aH_R)^{\alpha}\{(1-u-v)H_P\}^{\beta}K^{1-\alpha-\beta}\bar{H_R}^{\epsilon_R}\bar{H_P}^{\epsilon_P} \tag{1}$$

where $0 < \alpha < 1$, $0 < \beta < 1$, $0 < \alpha + \beta < 1$, $0 < a < 1$, and $0 \leq u < 1$. Here

$\epsilon_R > 0$ and $\epsilon_P > 0$ are parameters representing the magnitudes of external effects of H_R and H_P on the production technology in the rich region. A_R is the technology parameter of the advanced region. $\bar{H_R}$ and $\bar{H_P}$ are the average levels of human capital of these two types of individuals from which the external effects come [1]. K is the stock of physical capital specific to the rich sector. The production function satisfies CRS in terms of the private inputs while it is subject to social IRS when the external effects are internalized.

On the other hand, there is family farming in the poor region and the labour expressed in terms of human capital is the only input there [2]. Total output produced in the poor region is equally divided among the workers employed. The production function of the poor region is given by the following.

$$Y_P = A_P(uH_P)\bar{H_R}^{\eta_R}\bar{H_P}^{\eta_P}. \tag{2}$$

This also satisfies CRS at the private level and IRS at the social level. η_R and $\eta_P > 0$ are parameters representing the magnitudes of external effects of H_R and H_P on the production technology in the poor region. A_P is the technology parameter of the poor region.

βY_R is the wage income of workers of the poor region who are employed in the rich region. So $(1-\beta)Y_R$ is the income of individuals of the rich region. A part of $(1-\beta)Y_R$ is consumed and the other part is saved (invested). So the budget constraint of the representative individual of the rich region is given by

$$\dot{K} = (1-\beta)Y_R - C_R. \tag{3}$$

Here C_R is the level of consumption of the representative individual of the rich region. It is assumed that there is no depreciation of physical capital. The individual of the poor region consumes whatever he earns; and this assumption is borrowed from Lewis (1954). They earn the competitive labour income in the rich sector,

[1]We consider aggregate external effects and not sector specific external effects.

[2]It is a simplifying assumption. In the next section, we introduce physical capital in this production function.

βY_R, and the entire income obtained from the production in the poor region, Y_P. Hence, we have

$$Y_P + \beta Y_R = C_P \tag{4}$$

where C_P is the level of consumption of the representative worker in the poor region. However, the representative individual (worker) in the rich sector allocates income between savings and consumption maximizing his discounted present value of utility over the infinite time horizon. The representative individual individual of the ith region has the instantaneous utility function given by

$$U(C_i) = \frac{C_i^{1-\sigma}}{1-\sigma}, \tag{5}$$

Here $\sigma > 0$ is the elasticity of marginal utility of consumption; and $i = R, P$.

5.2.2 Dualism in the mechanism of human capital accumulation

Mechanism of the human capital accumulation in the rich sector is assumed to be similar to that in Lucas (1988). The relative rate of human capital formation varies proportionately with the time or effort devoted to acquire skill. Hence

$$\dot{H}_R = mbH_R \tag{6}$$

where b is the fraction of the non-leisure time devoted to acquiring own skill. Here $0 \leq b \leq 1$; and m is a positive constant representing the productivity parameter of the human capital accumulation technology.

However, mechanisms of human capital formation in the two regions are different. The skill formation of a poor individual takes place through a training programme conducted by individuals in the rich region. Poor individuals need outside assistance provided by rich individuals because they lag behind rich individuals in terms of initial human capital endowment and the knowledge accumulation technology. The knowledge trickles down from the more knowledgeable person to the inferiors[3]. Every individual of the rich region spends $(1-a-b)$ fraction of its labour time in this training. The individual of the rich region has an incentive to train workers of the poor region because they work as labourers in the rich sector[4]. Every worker in the poor region devotes v fraction of time for acquiring skill. We assume that there exists a positive external effect of the average skill level of rich individuals and of poor individuals on the human capital accumulation in the poor region. Hence we have

$$\dot{H_P} = m_P\{(1-a-b)H_R\}^{\delta}(vH_P)^{1-\delta-\gamma}\bar{H_R}^{\mu\gamma}\bar{H_P}^{(1-\mu)\gamma}. \tag{7}$$

Here $0 < \delta < 1$, $0 < \mu < 1$ and $\gamma > 0$. Here γ is the parameter representing the magnitude of the external effect on the skill formation of the poor individual and $m_P > 0$ is the efficiency parameter of the education technology of the poor individual. The human capital accumulation function of the poor individual follows DRS at the private level and CRS at the social level.

In models of Tamura (1991), Eaton and Eckstein (1997), Lucas (2004) etc. the human capital accumulation technology is subject to external effects. In models of Eaton and Eckstein (1997) and of Tamura (1991), the average human capital stock produces external effect on human capital accumulation technology. However, in the model of Lucas (2004), the human capital level of the leader generates the external effect on the human capital accumulation of all other individuals. Leader is the individual with the highest skill level. In our model, rich individuals have

[3]In reality, poors need assistance of the riches also due to credit market imperfection. This is not applicable here because the process of human capital accumulation does not require non labour input.

[4]This story is valid when the process of human capital accumulation refers to internal training provided by the employing firm. In the case of formal schooling, each rich individual may deviate unilaterally from contributing to educational services. However, this is not true in a situation where some kind of Folk Theorem holds. For example, all the rich individuals may co-operate among themselves and may come to an agreement that each of them would employ equal number of educated poor workers. In that case, equal distribution of benefit provided by formal schooling is ensured for the rich individuals. All the rich individuals are identical in terms of their preference, capital endowment, production technology and skill. Similarly all the poor individuals are identical in terms of skill. So equal allocation is the optimum allocation in this case.

high level of human capital and poor individuals are lagging behind. Rich individuals and poor individuals are assumed to be identical within their representative groups. So the human capital stock of the representative rich individual should have external effect on the poor individual's human capital accumulation technology; and it should not be the other way round.

We assume that the rich individual provides labour time to educate the poor and does not provide output or capital. Marginal productivity of labour of the rich individual is always positive in this model and so the sacrifice of labour time indirectly implies a sacrifice of income. In reality, contributions are often made in terms of non labour resources. Our objective is to reanalyse results of the Lucas (1988) model; and so we follow the framework of Lucas (1988) which also solves a labour time allocation problem between production and education. For the sake of simplicity we do not consider the capital allocation problem that one may find in Rebelo (1991). It should also be noted that, in many adult education programmes organized in India, teachers and organizers donate labour time; and these are more important than monetary contributions.

5.3 Growth in the household economy

5.3.1 The optimization problem of the rich individual

The objective of the representative individual of the rich region is to maximize the discounted present value of utility over the infinite time horizon given by:

$$J_H = \int_0^\infty U(C_R)e^{-\rho t}dt.$$

This is to be maximized with respect to C_R,a and b subject to the equations of motion given by

$$\begin{aligned}\dot{K} &= (1-\beta)Y_R - C_R;\\ \dot{H}_R &= mbH_R;\end{aligned}$$

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^{\delta}(vH_P)^{1-\delta-\gamma}\bar{H_R}^{\mu\gamma}\bar{H_P}^{(1-\mu)\gamma};$$

and given the initial values of K, H_R and H_P. Here $U(C_R)$ is given by equation (5); and Y_R is given by equation (1). Here ρ is the positive rate of discount. Control variables are C_R, a and b, where $0 \leq C_R < \infty$, $0 \leq a \leq 1$, $0 \leq b \leq 1$ and $0 \leq a + b \leq 1$. State variables are K, H_R and H_P. The household can not internalise the external effects. If $a + b = 1$, this optimization problem is identical to that in Lucas (1988).

5.3.2 The optimization problem of the poor indivdual

The representative poor individual maximizes the objective functional given by

$$J_{H_P} = \int_0^\infty U(C_P)e^{-\rho t}dt$$

with respect to control variables u and v subject to the equation of motion given by

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^\delta (vH_P)^{1-\delta-\gamma} \bar{H_R}^{\mu\gamma} \bar{H_P}^{(1-\mu)\gamma};$$

and given the initial values of H_R and H_P. Here H_P is the state variable; and u and v are control variables satisfying $0 \leq u \leq 1$, $0 \leq v \leq 1$, and $0 \leq u + v \leq 1$. Here C_P is given by equation (4); and Y_R and Y_P are given by equations (1) and (2). The individual can not internalise the external effect.

5.3.3 The steady state growth equilibrium

Optimality conditions of these two optimization problems are presented in Appendix (A). We analyze the steady state growth properties of the system using those conditions. Along the steady state growth path (SGP), C_R, K, Y_R, H_R, H_P, Y_P grow at constant rates; and a, b and u are time independent. At this stage, we assume the existence of the steady state growth equilibrium. The condition [5] for the growth rate of Y_R to be equal to the growth rate of Y_P is given by

$$(\eta_R + \eta_P) - \frac{(\epsilon_P + \epsilon_R)}{(\alpha + \beta)} = 0.$$

[5] In our future research, we shall try to drop this assumption and analyze the implied dynamics of the model. However, this extension is beyond the scope of the present revision.

We assume that this condition holds through out the analysis. It can be shown that the movement along the steady state growth path is optimal because it satisfies the transversality conditions[6]. Rates of growth of major macroeconomic variables can be derived[7] as follows.

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = mb, \tag{8}$$

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{C}_R}{C_R} = \frac{\dot{K}}{K} = \frac{(\alpha + \beta + \epsilon_P + \epsilon_R)}{(\alpha + \beta)} mb, \tag{9}$$

and

$$\frac{\dot{Y}_P}{Y_P} = (1 + \eta_R + \eta_P) mb. \tag{10}$$

Here

$$mb = \frac{m - \rho}{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}}; \tag{11}$$

and

$$a = \frac{\alpha(1-b)[\frac{m}{mb} - (1-\delta-\gamma)]}{\delta\beta + \alpha[\frac{m}{mb} - (1-\delta-\gamma)]}. \tag{12}$$

From equation (12), we find that $a = (1-b)$ when $\beta = 0$. This implies that the individual of the rich region would not allocate any labour time to educate an individual of the poor region if the workers from the poor region are not required as input in the rich sector's production technology.

Here

$$v = \frac{(m-\rho)(1-\delta-\gamma)}{m\{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}}. \tag{13}$$

As the magnitude of the external effect on the human capital accumulation of the poor individual is increased, v falls. If $\sigma(<) > 1$, v varies (positively) negatively with ϵ_P and ϵ_R.

Note that, if there is no externality, i.e., if $\epsilon_R = \epsilon_P = \eta_R = \eta_P = \gamma = 0$, then equation (11) shows that

$$mb = \frac{m-\rho}{\sigma}.$$

[6]It is shown in the Appendix (A).

[7]The derivation in detail is given in Appendix (A).

Equation (9) shows that, in this case,

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{C}_R}{C_R} = \frac{\dot{K}}{K} = mb.$$

In this case, consumption, income and human capital of both the regions and physical capital of the rich region grow at the common rate mb. This is same as the growth rate obtained in the Lucas(1988) model in the absence of external effect. We need to assume $m > \rho$ because b can not take a negative value.

If $\sigma = 1$ i.e. if $U(C_R) = log_e C_R$, then we have

$$\frac{\dot{H}_R}{H_R} = mb = m - \rho$$

even in the presence of external effects. This is same as the rate of human capital accumulation in the Lucas (1988) model with $\sigma = 1$. However, all other macro-economic variables like K, C_P, C_R, Y_P, Y_R do not necessarily grow at this rate when $\sigma = 1$ and when externalities exist.

In this case, the rate of human capital accumulation in the rich sector is independent of the degrees of various types of externalities. However, the common balanced growth rate of other macro-economic variables as shown by the equation (9) varies positively with the degree of externality in the production and/or with that in the human capital accumulation function of the poor individual. Similarly equation (10) shows that the rate of growth of output in the poor sector varies positively with the degree of externality in the production technology of the poor sector. In the Lucas (1988) model, there is no poor sector; and hence the role of external effects in the poor sector can not be analyzed there.

5.4 Growth in the Command Economy

In a command economy the social planner maximizes the discounted present value of the instantaneous social welfare function over the infinite time horizon. The instantaneous social welfare is assumed to be a positive function of the level of consumption of the representative individual in the rich region as well as of that in the poor region. This function is defined as

$$W = \frac{({C_R}^{\theta}{C_P}^{1-\theta})^{1-\sigma}}{1-\sigma}, \qquad \text{with} \qquad 0 \leq \theta \leq 1 \tag{14}$$

where θ and $(1-\theta)$ are the weights given to consumption of the representative individual in the rich region and in the poor region respectively. If $\theta = 1(0)$, it is same as the utility function of the representative individual in the rich (poor) region which we have considered in section 3.

5.4.1 The optimisation problem

The objective of the social planner is to maximize

$$J_P = \int_0^{\infty} W e^{-\rho t} dt$$

with respect to C_R, C_P, u, v, a and b subject to constraints given by

$$\begin{aligned}
\dot{K} &= Y_R + Y_P - C_R - C_P, \\
\dot{H}_R &= m b^* H_R, \\
&\textit{and} \\
\dot{H}_P &= m_P \{(1 - a^* - b^*) H_R\}^{\delta} (v^* H_P)^{1-\delta-\gamma} {H_R}^{\mu\gamma} {H_P}^{(1-\mu)\gamma}.
\end{aligned}$$

Here Y_R and Y_P are given by equations (1) and (2);and W is given by equation (14). Here control variables are C_R, C_P, a^*, b^*, u^* and v^*. The social planner can internalise all types of externalities what the individual in the competitive economy can not do.

5.4.2 The steady state growth equilibrium

We define the steady state growth equilibrium following the same style adopted in section 5.3.3. Along the SGP, rates of growth of major macroeconomic variables are derived[8] as follows.

$$\frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = \frac{\dot{Y}_R}{Y_R} = \frac{\dot{Y}_P}{Y_P} = \frac{\dot{K}}{K} = \frac{(\alpha + \beta + \epsilon_R + \epsilon_P)}{(\alpha + \beta)} m b^*; \tag{15}$$

[8]The derivtion in detail is given in the appendix (B).

and

$$\frac{\dot{H}_R}{H_R} = \frac{\dot{H}_P}{H_P} = mb^*. \tag{16}$$

Here

$$mb^* = \frac{(m-\rho) + m[a^*\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)}\} + \frac{\mu\gamma}{\delta}(1-a^*)]}{[1 + \frac{\mu\gamma}{\delta} - \frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}]}; \tag{17}$$

and b^* and a^* are the optimum values of b and a in the command economy.

5.4.3 Command Economy Vs Household Economy

The presence of externality creates divergence between the socially optimum growth rate in the command economy and the competitive equilibrium growth rate in the household economy. If there is no externality, then equations (11) and (17) show that

$$mb^* = mb = \frac{(m-\rho)}{\sigma}.$$

So the growth rate in the competitive economy is socially efficient in the absence of external effects. This result is similar to that obtained in Lucas (1988).

Comparing equations (11) and (17) we find that $(mb^* - mb)$ may take any sign in the presence of external effects. However, if $\gamma = 0$, then

$$mb^* = \frac{(m-\rho) + ma^*\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)}\}}{[1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}]}.$$

From equation (11), we find that mb is independent of γ. So, $mb^* > mb$ if $\gamma = 0$ and if either ϵ_R or η_R is positive.

Setting $\sigma = 1$ and using equations (11) and (17), we have

$$mb = m - \rho;$$

and

$$mb^* = \frac{(m-\rho) + m[a^*\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)}\} + \frac{\mu\gamma}{\delta}(1-a^*)]}{(\frac{\mu\gamma}{\delta} + 1)}.$$

Hence, with $\sigma = 1$, we have

$$mb^* - mb = \frac{[ma^*\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)} - \frac{\mu\gamma}{\delta}\} + \frac{\rho\mu\gamma}{\delta}]}{(\frac{\mu\gamma}{\delta} + 1)}.$$

The above term may be positive or may be negative. If there does not exist any kind of external effect i.e. if $\epsilon_R = \eta_R = \gamma = 0$ then $mb^* = mb$. If $\gamma = 0$ but η_R or η_P is positive and if we have an interior solution such that $\frac{u^*}{(1-u^*-v^*)} > 0$, then $mb^* > mb$. It is negative if the following condition is satisfied[9].

$$a^* < \frac{\rho\mu\gamma}{m\delta\{\frac{\mu\gamma}{\delta} - \frac{\epsilon_R}{\alpha} - \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)}\}} = \bar{a}.$$

If $\gamma = 0$, a^* can never be lower than $\bar{a}$.

This leads to the following proposition.

Proposition 1 *Suppose that $\sigma = 1$. (i) If $\gamma = 0$ then $mb* > mb$ provided either ϵ_R or η_R or both are positive; (ii) $(mb^* - mb)$ may take any sign with $\mu > 0$ and $\gamma > 0$; and $mb > mb^*$ if $a^* < \bar{a}$*

So the socially efficient rate of growth of the human capital is always higher than its competitive equilibrium growth rate if there is no externality in the human capital accumulation in the poor sector. This is the generalization of the result of the Lucas (1988) model. Lucas (1988) has shown that the competitive equilibrium growth rate of human capital falls short of the socially efficient rate. However, his result was proved in the one sector (region) model with externality in the production function. The present chapter shows that the Lucas (1988) result is valid even in a dual economy with production externality in the rich sector as well as in the poor sector provided that there is no externality on the human capital accumulation.

However, if there is externality on the human capital accumulation in the poor sector, then the result may be reversed. In the presence of positive externality on the human capital accumulation in the poor sector, the labour time allocation of the rich individual to the training of the poor region workers is higher in

[9]When $\epsilon_R = \eta_R = 0$,

$$mb^* - mb = \frac{\mu\gamma}{\delta}\frac{[-(m-\rho)+m]}{(\frac{\mu\gamma}{\delta}+1)} - \frac{m\mu\gamma a^*}{\delta(\frac{\mu\gamma}{\delta}+1)} = \frac{\mu\gamma[\rho - ma^*]}{(\mu\gamma+\delta)}$$

Hence $\rho < ma^*$ is the sufficient condition for $mb^* - mb$ to be negative.

a command economy than that in the household economy because the command economy can internalise the externality. So the time allocated to acquiring his own skill of the individual in the rich region may be lower in the command economy than that in the competitive economy. So the socially optimum growth rate of the human capital may be lower than its competitive equilibrium growth rate in this case.

If $\mu = 0$, then from equation (17), we have

$$mb^* = \frac{(m-\rho) + m[a^*\{\frac{\epsilon}{\alpha} + \frac{\beta\eta_R u^*}{\alpha(1-u^*-v^*)}\}]}{[1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}]};$$

and comparing with mb given by the equation (11) we find that $mb^* > mb$ in this case. Here $\mu = 0$ implies that there is no external effect of H_R on the accumulation of H_P. So we have the following proposition.

Proposition 2 *If $\mu = 0$ then $mb^* > mb$.*

If the human capital of the rich sector does not create any externality on the human capital accumulation in the poor sector and if the entire external effect comes from the human capital of the poor sector, then the rate of growth of the human capital in the household economy is less than that in the command economy. Here $\bar{H_R}^{\mu\gamma}\bar{H_P}^{(1-\mu)\gamma}$ represents the total external effect on the human capital accumulation of the poor individual. $\bar{H_R}^{\mu\gamma}$ is the external effect of teaching and $\bar{H_P}^{(1-\mu)\gamma}$ represents the external effect of learning. It is the external effect of teaching which matters in this case. $\mu = 0$ implies the absence of the externalities of teaching.

Let us consider the following numerical specification:

$m = 3; m_P = 1; \rho = 0.33; \epsilon_R = 0; \epsilon_P = 0.3; \alpha = 0.55; \beta = 0.2; \eta_R = 0; \mu = 0.5; \gamma = 0.1; \delta = 0.8; A_P = 1; A_R = 2.5; \sigma = 1.5; \eta_P = 0.4$. This specification yields the steady state solution of the variables a, u, v, y, z, b given by $b^* = 0.529; v^* = 0.114; z^* = 14.01; y^* = 42.446; a^* = 0.33; u^* = 0.11$ in the command economy and $b = 0.524; a = 0.410; v = 0.052; z = 18.564; y = 76.3; u = 0.81$ in the competitive economy. In this case, the growth rate of human capital in the command economy is given by $mb^* = 1.587$ and that in the competitive economy is given by $mb = 1.572$. So here the growth rate of the competitive economy is marginally lower than that of the command economy[10].

5.5 Capital formation in the rural sector

5.5.1 The household economy

We now consider capital formation in the poor sector which takes place through investment of the poor individuals. The representative individual in the poor sector maximizes $\int_0^\infty e^{-\rho t} U(C_P) dt$ with respect to u, v and C_P subject to equations of motion given by (7) and

$$\dot{K_P} = Y_P + \beta Y_R - C_P. \quad (18)$$

Here $U(C_P)$ is given by equation (4) and K_P represents the level of capital stock of the poor sector. Here C_P, u and v are control variables; and K_P and H_P are state variables. Capital stock now enters as an input into the production function of the poor sector which is given by

$$Y_P = A_P (u H_P)^{\phi} {K_P}^{1-\phi} \bar{H_R}^{\eta_R} \bar{H_P}^{\eta_P}. \quad (19)$$

The optimization problem of the representative individual in the rich region remains same as in section 3.1. Following the same style adopted in the earlier section we derive[11] the steady state equilibrium rates of growth of the different macro economic variables. They are given by

$$\frac{\dot{Y_P}}{Y_P} = \frac{\dot{C_P}}{C_P} = \frac{\dot{K_P}}{K_P} = \frac{(\phi + \eta_P + \eta_R)}{\phi} mb; \quad (20)$$

and

$$\frac{\dot{Y_R}}{Y_R} = \frac{\dot{C_R}}{C_R} = \frac{\dot{K_R}}{K_R} = \frac{(\alpha + \beta + \epsilon_P + \epsilon_R)}{(\alpha + \beta)} mb. \quad (21)$$

Here

$$mb = \frac{m - \rho}{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}}. \quad (22)$$

[10] I have made a number of numerical simulations. However, I have not found a case where the growth rate of the command economy is lower than that in the competitive economy. Lack of numerical support weakens the importance of the theoretical result but does not disprove it.

[11] Derivation in detail is given in the Appendix (C).

Note that the optimum value of mb as given by equation (22) is independent of $(1-\phi)$ which represents the physical capital elasticity of output in the poor sector. This expression of mb is same as that given by equation (11).

5.5.2 The command economy

The social planner solves the same problem analysed in section 5.4. However, the planner now controls the capital allocation between the two sectors in addition to controlling the labour allocation and the consumption-investment allocation. The optimization problem to be solved is given by the following. The planner's objective is to maximize

$$J_P = \int_0^\infty W e^{-\rho t} dt$$

subject to the constraints given by

$$\begin{aligned} \dot{K} &= Y_P + Y_R - C_P - C_R, \\ \dot{H}_R &= mb^* H_R, \\ and & \\ \dot{H}_P &= m_P\{(1-a^*-b^*)H_R\}^\delta v^* {H_P}^{1-\delta-\gamma} \bar{H_R}^{\mu\gamma} \bar{H_P}^{(1-\mu)\gamma} \end{aligned}$$

with respect to control variables, a^*, b^*, C_R, C_P, u^*, v^* and x. Here

$$Y_R = A_R(a^* H_R)^\alpha \{(1-u^*-v^*)H_P\}^\beta \{xK\}^{1-\alpha-\beta} {H_R}^{\epsilon_R} {H_P}^{\epsilon_P}; \tag{23}$$

and

$$Y_P = A_P(u^* H_P)^\phi ((1-x)K)^{1-\phi} {H_R}^{\eta_R} {H_P}^{\eta_P}. \tag{24}$$

Here x is the additional control variable satisfying the property $0 \le x \le 1$. x represents the fraction of physical capital allocated to the rich sector (region).

In the steady-state growth equilibrium, we can derive[12] rates of growth of different macro economic variables as follows.

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = mb^*; \tag{25}$$

[12]Derivation in detail is given in the Appendix (D).

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{Y}_P}{Y_P} = \frac{\dot{K}}{K} = \frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = [\alpha + \beta + \epsilon_R + \epsilon_P]\frac{mb^*}{(\alpha + \beta)}; \tag{26}$$

and

$$mb^* = \frac{(m - \rho) + m[a^*\{\frac{\epsilon_R}{\alpha} + \frac{\eta_R u^* \beta}{\alpha\phi(1-u^*-v^*)}\} + \frac{\mu\gamma}{\delta}(1 - a^*)]}{\frac{\mu\gamma}{\delta} + \sigma - \frac{(\epsilon_R+\epsilon_P)}{(\alpha+\beta)}}. \tag{27}$$

If we compare equations (17) and (27)we find that they are identical when $\phi = 1$. Also equation (27) clearly shows that mb^* varies negatively with ϕ. However, in section 5.1, we have found that mb is independent of ϕ. This leads to the following proposition.

Proposition 3 *If physical capital is used as an input in the poor sector's production function then the socially efficient rate of growth of human capital varies positively with the physical capital elasticity of output in the poor sector while its competitive equilibrium rate of growth in the household economy is independent of that elasticity.*

Its explanation lies in the assumption of the model. In the household economy, entire surplus originating from a sector is invested to that sector itself; and there is no intersectoral capital mobility. So the capital accumulation in the poor sector does not affect the labour-time allocation problem of the rich individual. So the rate of growth of the human capital being determined by the rich individual's labour time allocation to the human capital accumulation sector is independent of the capital elasticity of output in the poor sector. However, in the planned economy, the planner allocates the total capital stock between the two sectors. Investment of the surplus of any sector is not sector specific. The planner controls total investment which is the sum of surplus originating from both the sectors.

Comparing equations (11) and (27), we find that $mb^* > mb$ when $\gamma = \mu = 0$; and mb may be greater than mb^* when $\gamma > 0$ and $\mu > 0$. So the central points of the results summarized in propositions 1 and 2 remain unchanged even in this extended model.

5.6 Conclusion

Existing endogenous growth models do not focus on the dualism in the human capital accumulation and old two sector dual economy models did not consider human capital accumulation and endogenous growth. This chapter tries to bridge the gap. In this chapter, we have developed the model of a two sector dual economy in which growth stems from human capital accumulation and the dualism exists in the nature of human capital accumulation between the two sectors. Like Lucas (1988), we have analyzed the steady state growth equilibrium properties of the model and have put special emphasis on the role of externalities. We have considered not only the role of human capital's externality on the rich sector's production function but also its role on the production function as well as on the human capital accumulation function in the poor sector.

We have derived some interesting results from this model. First, externality parameters in the poor sector appear to be important determinants of the long run rate of growth of the different macro economic variables in this model. Secondly, the rate of growth of human capital in the competitive economy is always less than that in the command economy if there is no externality in the human capital accumulation in the poor sector. However, in the presence of that externality, we may obtain an opposite result. Competitive equilibrium growth rate may exceed the growth rate obtained in the command economy. Lucas (1988) and its extended models did not find this possibility; and so this is an important result. Lastly, if there is no externality in either sector, rates of growth are same in both the systems. In a command economy, the planner has the power of allocating poor workers between the two sectors along with the power of maximizing an welfare function which takes care of consumption of the people of both the sectors. However, this power does not help the planner to achieve a higher rate of growth than that obtained in the competitive equilibrium in the absence of externalities. Also the external effect on the poor sector's human capital accumulation is important only if this external effect comes from the rich sector's human capital. If externalities come from the human capital of the poor sector only and not from the human capital of the rich sector, then the rate of growth in the command economy exceeds that obtained in the competitive equilibrium.

These results have important implications in the context of educational subsidy policies. Lucas (1988) has advocated for an educational subsidy policy because the competitive equilibrium rate of growth of human capital in the Lucas (1988) model falls short of its socially efficient rate of growth. However, this is not necessarily true in the present model when rich individuals provide training to poor individuals and the human capital accumulation of the poor individual is subject to external effects. So results of this model may question the necessity of subsidizing the higher education sector which generally benefits the rich and not the poors.

Appendix A

Optimality conditions of the dynamic optimization problem solved by the rich individual

The current value Hamiltonian is given by

$$Z^R = \frac{C_R^{1-\sigma}}{1-\sigma} + {\lambda_K}^R[(1-\beta)A_R(aH_R)^\alpha\{(1-u-v)H_P\}^\beta K^{1-\alpha-\beta}\bar{H_R}^{\epsilon_R}\bar{H_P}^{\epsilon_P} - C_R]$$

$$+{\lambda_H}^R mbH_R + {\lambda_P}^R[m_P\{(1-a-b)H_R\}^\delta(vH_P)^{1-\delta-\gamma}\bar{H_R}^{\mu\gamma}\bar{H_P}^{(1-\mu)\gamma}]$$

where ${\lambda_K}^R$, ${\lambda_H}^R$ and ${\lambda_P}^R$ are co state variables.

(A) First order optimality conditions necessary for this optimization problem with respect to control variables C_R, a, b are given by the following.

$$C_R^{-\sigma} - {\lambda_K}^R = 0; \qquad (A.1)$$

$${\lambda_K}^R\alpha(1-\beta)\frac{Y_R}{a} - {\lambda_P}^R\delta\frac{\dot{H_P}}{(1-a-b)} = 0; \qquad (A.2)$$

and

$${\lambda_H}^R mH_R - {\lambda_P}^R\delta\frac{\dot{H_P}}{(1-a-b)} = 0. \qquad (A.3)$$

(B)Time derivatives of co-state variables satisfying the optimum growth path are given by the following.

$$\dot{{\lambda_K}^R} = \rho{\lambda_K}^R - {\lambda_K}^R(1-\alpha-\beta)(1-\beta)\frac{Y_R}{K}; \qquad (A.4)$$

$$\dot{{\lambda_H}^R} = \rho{\lambda_H}^R - {\lambda_K}^R\alpha(1-\beta)\frac{Y_R}{H_R} - {\lambda_P}^R\delta\frac{\dot{H_P}}{H_R} - {\lambda_H}^R mb; \qquad (A.5)$$

and

$$\dot{{\lambda_P}^R} = \rho{\lambda_P}^R - {\lambda_K}^R\beta(1-\beta)\frac{Y_R}{H_P} - {\lambda_P}^R(1-\delta-\gamma)\frac{\dot{H_P}}{H_P}. \qquad (A.6)$$

(C) Transversality conditions are given by the followings.

$$\lim_{t\to\infty} e^{-\rho t}{\lambda_H}^R H_R(t) = 0; \lim_{t\to\infty} e^{-\rho t}{\lambda_P}^R H_P(t) = 0; \lim_{t\to\infty} e^{-\rho t}{\lambda_K}^R K(t) = 0.$$

Optimality conditions of the dynamic optimization problem solved by the poor individual

The current value Hamiltonian is given by

$$Z^P = \frac{C_P^{1-\sigma}}{1-\sigma} + {\lambda_H}^P[m_P\{(1-a-b)H_R\}^\delta (vH_P)^{1-\delta-\gamma}\bar{H_R}^{\mu\gamma}\bar{H_P}^{(1-\mu)\gamma}]$$

where ${\lambda_H}^P$ is the costate variable.

(A) First order conditions necessary for this optimization problem with respect to control variables u and v are given by the following.

$$(\beta Y_R + Y_P)^{-\sigma}[-\beta^2 \frac{Y_R}{(1-u-v)} + \frac{Y_P}{u}] = 0; \qquad (A.7)$$

and

$$(\beta Y_R + Y_P)^{-\sigma}[-\beta^2 \frac{Y_R}{(1-u-v)}] + {\lambda_H}^P(1-\delta-\gamma)\frac{\dot{H}_P}{v} = 0. \qquad (A.8)$$

(B) Time derivative of co-state variable satisfying the optimum growth path is given by the following.

$$\dot{\lambda_H}^P = \rho{\lambda_H}^P - (\beta Y_R + Y_P)^{-\sigma}[\beta^2 \frac{Y_R}{H_P} + \frac{Y_P}{H_P}] - {\lambda_H}^P(1-\delta-\gamma)\frac{\dot{H}_P}{H_P}. \qquad (A.9)$$

(C) The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}{\lambda_H}^P H_P(t) = 0.$$

The steady state growth equilibrium

We define a new set of variables $z = \frac{H_R}{H_P}$ and $y = {H_R}^{\alpha+\beta+\epsilon_P+\epsilon_R}K^{-(\alpha+\beta)}$.

From equation (7), we find that the growth rate of the human capital of the poor region is given by

$$\frac{\dot{H}_P}{H_P} = m_P(1-a-b)^\delta H_R^{\delta+\mu\gamma} v^{1-\delta-\gamma} H_P^{-\delta-\gamma+(1-\mu)\gamma}. \qquad (A.10)$$

Since on SGP a, b, v and r are constant, the growth rate of H_P is given by

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a-b)^\delta v^{1-\delta-\gamma} z^{\delta+\mu\gamma} = mb. \qquad (A.11)$$

Using equations (A.1) and (A.4) we have

$$\frac{\dot{\lambda_K}^R}{\lambda_K{}^R} = -\sigma\frac{\dot{C_R}}{C_R} = \rho - (1-\alpha-\beta)(1-\beta)\frac{Y_R}{K}. \tag{A.12}$$

Since $\frac{\dot{C_R}}{C_R}$ is constant along SGP, $\frac{Y_R}{K}$ is also constant.

Using equations (3) and (A.12)we obtain the growth rate of physical capital stock as follows.

$$\frac{\dot{K}}{K} = (1-\beta)\frac{Y_R}{K} - \frac{C_R}{K},$$

Or,

$$\frac{\dot{K}}{K} = \frac{(\rho+\sigma\chi)}{(1-\alpha-\beta)} - \frac{C_R}{K},$$

where $\chi = \frac{\dot{C_R}}{C_R}$

Since $\frac{\dot{K}}{K}$ and the first term in the RHS of the equation mentioned above are constant, $\frac{C_R}{K}$ is also constant.

Hence,

$$\frac{\dot{Y_R}}{Y_R} = \frac{\dot{C_R}}{C_R} = \frac{\dot{K}}{K} = \chi.$$

Taking log and then differentiating both sides of equation (A.12) and using (A.11) we obtain the common rate at which the consumption of the rich region, physical capital and output of the rich region would grow; and that is given by equation (9).

From equation (A.7), we find that, in migration equilibrium,

$$\frac{\beta^2 Y_R}{(1-u-v)} = \frac{Y_P}{u}. \tag{A.13}$$

From equation (A.13), we find that, if Y_P grows at a higher rate than Y_R, then $(1-u-v)$ will tend to zero and, if Y_R grows at higher rate than Y_P, then u tends to zero. We obtain an interior solution of u and $(1-u-v)$ if and only if growth rates of Y_R and Y_P are equal. The condition for the growth rate of Y_R to be equal to the growth rate of Y_P is given by

$$(\eta_R + \eta_P) - \frac{(\epsilon_P + \epsilon_R)}{(\alpha+\beta)} = 0. \tag{A.14}$$

So, if equation (A.14) holds, then u is constant; and satisfies $0 < u < 1$ which means the incomplete specialization of the intersectoral allocation of labour of the poor region. This implies that, in the steady state growth equilibrium, workers of the poor region work in both the sectors. From equations (A.2) and (A.3), we have,

$$\frac{\lambda_K{}^R}{\lambda_H{}^R} = \frac{mH_R a}{\alpha(1-\beta)Y_R}.$$

Differentiating both sides of this equation and using equations (8), (9), (A.12) and (A.17), we have the solution of mb given by the equation (11). From equations (A.6) and (A.2) we have

$$\frac{\dot{\lambda_P}^R}{\lambda_P{}^R} = \rho - [\frac{\delta\beta a}{\alpha(1-a-b)} + (1-\delta-\gamma)]\frac{\dot{H}_P}{H_P}. \quad (A.15)$$

Differentiating both sides of this equation (A.3) with respect to time and using equation (8) we have

$$\frac{\dot{\lambda_H}^R}{\lambda_H{}^R} = \frac{\dot{\lambda_P}^R}{\lambda_P{}^R}. \quad (A.16)$$

From this equation the equation (12) follows.

From equations (A.5), (A.2) and (A.3) we have

$$\frac{\dot{\lambda_H}^R}{\lambda_H{}^R} = \rho - m. \quad (A.17)$$

From equations (A.15), (A.16), (A.17) and (8) we can solve for a which is given by equation (12).

From equation (A.9), we have

$$\frac{\dot{\lambda_H}^P}{\lambda_H{}^P} = \rho - (1-\delta-\gamma)[\frac{(1-u-v)}{v\beta^2 Y_R}(\beta^2 Y_R + Y_P) + 1]\frac{\dot{H}_P}{H_P}.$$

Now, using equation (A.13), we have

$$\frac{\dot{\lambda_H}^P}{\lambda_H{}^P} = \rho - (1-\delta-\gamma)[\frac{(1-v)}{v} + 1]\frac{\dot{H}_P}{H_P}. \quad (A.18)$$

Differentiating both sides of equation (A.8) and using equations (9) and (8) we have

$$\frac{(1-\delta-\gamma)}{v} = \frac{\rho}{mb} - (1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)} + 1. \quad (A.19)$$

Substituting the expression of mb from equation (11) in equation (A.19) we have the solution of v which is given by equation (13). From equations (A.13) and (A.14) we have

$$(1-u-v)^{\beta-1} = \frac{A_P}{\beta^2 A_R a^\alpha} z^{\beta+\epsilon_P-1-\eta_P} y^{\frac{(1-\alpha-\beta)}{(\alpha+\beta)}}. \quad (A.20)$$

From equations (A.12) and (9) we have,

$$y = \frac{\sigma z^{(\beta+\epsilon_P)}[\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)} mb + \frac{\rho}{\sigma}]}{(1-\alpha-\beta)(1-\beta)A_R a^{\alpha}(1-u-v)^{\beta}}. \quad (A.21)$$

where z can be derived from the condition that H_R and H_P grow at equal rate. The expression of z in terms of a, b and v can be obtained from equation (A.11) and is given by

$$z = [\frac{mb}{m_P(1-a-b)^{\delta} v^{1-\delta-\gamma}}]^{\frac{1}{(\delta+\mu\gamma)}}.$$

Transversality Conditions

Now we shall show that the balanced growth path described above satisfies the transversality conditions.

From equation (A.16) we have

$$\frac{\dot{\lambda_H}^R}{\lambda_H{}^R} = \frac{\dot{\lambda_P}^R}{\lambda_P{}^R};$$

and, in the steady state,

$$\frac{\dot{H_R}}{H_R} = \frac{\dot{H_P}}{H_P}.$$

From equations (A.17) and (6) we have

$$-\rho + \frac{\dot{\lambda_P}^R}{\lambda_P{}^R} + \frac{\dot{H_P}}{H_P} = -\rho + \frac{\dot{\lambda_H}^R}{\lambda_H{}^R} + \frac{\dot{H_R}}{H_R} = -m(1-b) < 0.(\text{constant})$$

Hence we can prove that

$$\lim_{t\to\infty} e^{-\rho t}\lambda_H{}^R H_R(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_P{}^R H_P(t) = 0$$

Here

$$\frac{\dot{\lambda_K}^R}{\lambda_K{}^R} + \frac{\dot{K}}{K} = (1-\sigma)\frac{\dot{K}}{K} = (1-\sigma)\chi$$

Hence, $\lim_{t\to\infty} e^{-\rho t}\lambda_K{}^R K(t) = 0$ when $(1-\sigma)\chi < \rho$.

Appendix B

Optimality conditions of the dynamic optimization problem solved by the social planner

The current value Hamiltonian is given by

$$Z^W = \frac{({C_R}^{\theta}{C_P}^{1-\theta})^{1-\sigma}}{1-\sigma} + \lambda_K[(1-\beta)A_R(a^*H_R)^{\alpha}\{(1-u^*-v^*)H_P\}^{\beta}K^{1-\alpha-\beta}{\bar{H_R}}^{\epsilon_R}{\bar{H_P}}^{\epsilon_P} - C_R]$$

$$+\lambda_R m b^* H_R + \lambda_P[m_P\{(1-a^*-b^*)H_R\}^{\delta}(v^*H_P)^{1-\delta-\gamma}{\bar{H_R}}^{\mu\gamma}{\bar{H_P}}^{(1-\mu)\gamma}]$$

where λ_K, λ_R, λ_P are the costate variables.

(A) First order necessary conditions of optimization with respect to C_R, C_P, a, b, u and v are given by the following.

$$({C_R}^{\theta}{C_P}^{1-\theta})^{-\sigma}\theta {C_R}^{\theta-1}{C_P}^{1-\theta} - \lambda_K = 0; \qquad (B.1)$$

$$({C_R}^{\theta}{C_P}^{1-\theta})^{-\sigma}(1-\theta){C_R}^{\theta}{C_P}^{-\theta} - \lambda_K = 0; \qquad (B.2)$$

$$\lambda_K\alpha\frac{Y_R}{a^*} - \lambda_P\delta\frac{\dot{H_P}}{(1-a^*-b^*)} = 0; \qquad (B.3)$$

$$\lambda_R m H_R - \lambda_P\delta\frac{\dot{H_P}}{(1-a^*-b^*)} = 0; \qquad (B.4)$$

$$\lambda_K\frac{Y_P}{u^*} - \lambda_K\beta\frac{Y_R}{(1-u^*-v^*)} = 0; \qquad (B.5)$$

and

$$-\lambda_K\frac{\beta Y_R}{(1-u^*-v^*)} + \lambda_P(1-\delta-\gamma)\frac{\dot{H_P}}{v^*} = 0. \qquad (B.6)$$

(B) Time derivative of co-state variables which satisfy their time behaviour along the optimum growth path are given by the followings.

$$\dot{\lambda_K} = \rho\lambda_K - \lambda_K(1-\alpha-\beta)\frac{Y_R}{K}; \qquad (B.7)$$

$$\dot{\lambda_R} = \rho\lambda_R - \lambda_K(\alpha + \epsilon_R)\frac{Y_R}{H_R} - \lambda_R mb^* - \lambda_P(\delta + \mu\gamma)\frac{\dot{H}_P}{H_R} - \lambda_K\eta_R\frac{Y_P}{H_R}; \qquad (B.8)$$

and

$$\dot{\lambda_P} = \rho\lambda_P - \lambda_K(\beta+\epsilon_P)\frac{Y_R}{H_P} - \lambda_P\{(1-\delta-\gamma)+(1-\mu)\gamma\}\frac{\dot{H}_P}{H_P} - \lambda_K(1+\eta_P)\frac{Y_P}{H_P}. \quad (B.9)$$

(C) Transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_P(t)H_P(t) = 0.$$

The steady state growth equilibrium

From equation (B.5), we have

$$\frac{(1-u^*-v^*)}{u^*} = \beta\frac{Y_R}{Y_P}. \qquad (B.10)$$

For $\frac{(1-u^*-v^*)}{u^*}$ to be a constant, Y_R and Y_P must grow at equal rates.

Since on SGP the growth rate of H_R, H_P, a, b and v are constant, the following equation holds true in this case too.

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a^*-b^*)^\delta {v^*}^{1-\delta-\gamma}{z^*}^{\delta+\mu\gamma} = mb^*. \qquad (B.11)$$

From equations (B.1) and (B.2), we have

$$\frac{C_R}{C_P} = \frac{\theta}{1-\theta}. \qquad (B.12)$$

We consider the case where $0 < \theta < 1$. Differentiating equation (B.1) with respect to time we have

$$\frac{\dot{\lambda_K}}{\lambda_K} = \{(1-\sigma)\theta - 1\}\frac{\dot{C}_R}{C_R} + (1-\sigma)(1-\theta)\frac{\dot{C}_P}{C_P}. \qquad (B.13)$$

Equation (B.12) shows that $\frac{C_R}{C_P}$ is constant. Hence using equations (B.1), (B.2) and (B.7) we have,

$$\frac{\dot{\lambda_K}}{\lambda_K} = -\sigma\frac{\dot{C}_R}{C_R} = -\sigma\frac{\dot{C}_P}{C_P} = \rho - (1-\alpha-\beta)\frac{Y_R}{K} \qquad (B.14)$$

Since growth rates of C_R and C_P are constant along the steady-state growth path, $\frac{Y_R}{K}$ is also constant. Using the budget constraint of the planner, we now obtain

$$\frac{\dot{K}}{K} = \frac{Y_R}{K}[1 + \frac{Y_P}{Y_R}] - \frac{C_R}{K}[1 + \frac{C_P}{C_R}].$$

Along the SGP, $\frac{\dot{K}}{K}$, $\frac{Y_R}{K}$, $\frac{C_P}{C_R}$ are constants. We have assumed that $\frac{Y_P}{Y_R}$ is constant. So $\frac{C_R}{K}$ must be constant. Hence along the SGP,

$$\frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = \frac{\dot{K}}{K} = \frac{\dot{Y}_R}{Y_R} = \frac{\dot{Y}_P}{Y_P}.$$

Using this equation, equation (1) and equation (B.11), we obtain the growth rate as given by equation (15).

From equation (B.8), we have

$$\frac{\dot{\lambda}_R}{\lambda_R} = \rho - \frac{ma^*(\alpha + \epsilon_R)}{\alpha} - mb^* - \frac{m(\delta + \mu\gamma)}{\delta}(1 - a^* - b^*) - \frac{ma^*\eta_R Y_P}{\alpha Y_R}. \quad (B.15)$$

From equations (B.3) and (B.4), we have

$$\frac{\lambda_K}{\lambda_R} = \frac{mH_R a^*}{\alpha Y_R}.$$

Differentiating both sides of this equation and using equations (B.14), (B.15), (B.11) and (15) we obtain the expression for mb^* given by equation (17).

From equations (B.3) and (B.6), we have

$$\frac{\delta a^*}{\alpha(1 - a^* - b^*)} = \frac{(1 - \delta - \gamma)(1 - u^* - v^*)}{\beta v^*}. \quad (B.16)$$

From equations (B.9) and (B.3), we have

$$\frac{\dot{\lambda}_P}{\lambda_P} = \rho - [\frac{\delta a^*(\beta + \epsilon_P)}{\alpha(1 - a^* - b^*)} + \frac{\delta a^*(1 + \eta_P)Y_P}{(1 - a^* - b^*)\alpha Y_R} + \{(1-\delta-\gamma)+(1-\mu)\gamma\}]\frac{\dot{H}_P}{H_P}. \quad (B.17)$$

Differentiating both sides of equation (B.4) with respect to time, we have

$$\frac{\dot{\lambda}_R}{\lambda_R} = \frac{\dot{\lambda}_P}{\lambda_P}. \quad (B.18)$$

This equation (B.18) is same as equation (A.16)

We now analyze how optimum values of a, b and u are determined in the command economy. Using equations (B.17), (B.18), (B.15) and (B.16) we have

$$[\frac{(\beta+\epsilon_P)(1-\delta-\gamma)(1-u^*-v^*)}{\beta v^*}+\frac{(1-\delta-\gamma)(1+\eta_P)u^*}{v^*}+(1-\delta-\gamma)+(1-\mu)\gamma+\frac{\mu\gamma}{\delta}]mb^*$$

$$= ma^*[\frac{\epsilon_R}{\alpha}-\frac{\mu\gamma}{\delta}+\frac{\eta_R\beta u^*}{\alpha(1-u^*-v^*)}]+m\frac{(\delta+\mu\gamma)}{\delta}.$$

From equation (17), b can be expressed in terms of a^*, u^*, v^*. Substituting that value of b^* in the equation just above this paragraph we get a^* in terms of u^* and v^*. Once a^* and b^* are determined in terms of u^* and v^*, z^* can be determined in terms of u^* and v^* by using the fact that H_R and H_P grow at equal rate. The expression of z^* is given by the following.

$$z^* = [\frac{mb^*}{m_P(1-a^*-b^*)^\delta v^{*1-\delta-\gamma}}]^{\frac{1}{(\delta+\mu\gamma)}}.$$

Now using equations (B.14) and (B.7) we have

$$(1-\alpha-\beta)A_R a^{*\alpha}(1-u^*-v^*)^\beta y^* z^{*-(\beta+\epsilon_P)} = \rho+\sigma\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)mb^*}{(\alpha+\beta)}.$$

From this equation, y^* can be determined in terms of u^* and v^*. Then from equations (B.16) and (B.10), u^* and v^* can be determined. Substituting Y_P and Y_R, equation (B.10) can be written as

$$(1-u^*-v^*)^{\beta-1} = \frac{A_P}{\beta A_R a^{*\alpha}} z^{*\beta+\epsilon_P-1-\eta_P} y^{*(\frac{1-\alpha-\beta}{\alpha+\beta})}.$$

This equation holds if Y_R and Y_P grow at equal rates and the condition for that is given by (A.14).

Transversality Conditions

Now we shall show that the balanced growth path described above satisfies the transversality condition.

Using equations (6) and (B.15) we find that

$$\frac{\dot{\lambda}_R}{\lambda_R}+\frac{\dot{H}_R}{H_R} < \rho$$

$$\Longrightarrow (\rho-\frac{ma^*(\alpha+\epsilon_R)}{\alpha}-mb^*-\frac{m(\delta+\mu\gamma)}{\delta}(1-a^*-b^*)-\frac{ma^*\eta_R Y_P}{\alpha Y_R})+mb^* < \rho.$$

This is always true because a^* and $(1 - a^* - b^*)$ are positive. Hence

$$\lim_{t \to \infty} e^{-\rho t} \lambda_R(t) H_R(t) = 0.$$

The condition

$$\lim_{t \to \infty} e^{-\rho t} \lambda_P(t) H_P(t) = 0$$

is also satisfied because in the steady state growth equilibrium,

$$\frac{\dot{H}_R}{H_R} = \frac{\dot{H}_P}{H_P};$$

and equation (B.18) shows that

$$\frac{\dot{\lambda}_R}{\lambda_R} = \frac{\dot{\lambda}_P}{\lambda_P}.$$

The condition

$$\lim_{t \to \infty} e^{-\rho t} \lambda_K(t) K(t) = 0$$

is satisfied if $\{(1-\sigma)\chi - \rho\}$ is negative. Using equations (15) and (17) we find that

$$(1 - \sigma)\chi - \rho < 0.$$

Appendix C

First order optimality conditions derived from the dynamic optimization problem solved by the poor individual in section 5.1 are given by the following.

$$C_P^{-\sigma} - \lambda_{K_P} = 0; \qquad (C.1)$$

$$\lambda_{H_P}(1 - \delta - \gamma)\frac{\dot{H}_P}{v} - \lambda_{K_P}\beta^2 \frac{Y_R}{(1 - u - v)} = 0; \qquad (C.2)$$

$$\phi\frac{Y_P}{u} - \beta^2 \frac{Y_R}{(1 - u - v)} = 0; \qquad (C.3)$$

$$\dot{\lambda}_{K_P} = \rho\lambda_{K_P} - \lambda_{K_P}\frac{(1 - \phi)Y_P}{K_P}; \qquad (C.4)$$

and

$$\dot{\lambda}_{H_P} = \rho\lambda_{H_P} - \lambda_{H_P}(1 - \delta - \gamma)\frac{\dot{H}_P}{H_P} - \lambda_{K_P}\phi\frac{Y_P}{H_P} - \lambda_{K_P}\beta^2\frac{Y_R}{H_P}. \qquad (C.5)$$

Along the SGP, $\frac{\dot{H}_P}{H_P}$, $\frac{\dot{H}_R}{H_R}$, a, b and v are constant. So the following equation holds true in this case also

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a-b)^\delta v^{1-\delta-\gamma} z^{\delta+\mu\gamma} = mb. \qquad (C.6)$$

As the optimization problem of the representative rich individual remains unchanged, optimality conditions given by equations (A.1)-(A.6) are also valid here. Hence the expression for mb and the growth rates given by equations (22) and (21) remain same. From equation (C.3), migration equilibrium condition of the workers of the poor region is now given by

$$\frac{u}{1-u-v} = \frac{\phi Y_P}{\beta^2 Y_R}. \qquad (C.7)$$

So, in order to obtain an interior solution for u and $(1-u-v)$, Y_P and Y_R must grow at equal rates. From equations (C.1) and (C.4), we have

$$\frac{\dot{\lambda}_K}{\lambda_K} = -\sigma\frac{\dot{C}_P}{C_P} = \rho - (1-\phi)\frac{Y_P}{K_P}.$$

Since along the SGP $\frac{\dot{C}_P}{C_P}$ is constant, $\frac{Y_P}{K_P}$ is also constant. Now

$$\frac{\dot{K}_P}{K_P} = \frac{Y_P}{K_P}[1+\beta\frac{Y_R}{Y_P}] - \frac{C_P}{K_P}.$$

Since along SGP $\frac{\dot{K}_P}{K_P}$ and $\frac{Y_P}{K_P}$ are constant and we have assumed that $\frac{Y_R}{Y_P}$ is constant, $\frac{C_P}{K_P}$ must be constant there. Hence we have

$$\frac{\dot{C}_P}{C_P} = \frac{\dot{Y}_P}{Y_P} = \frac{\dot{K}_P}{K_P}.$$

Using this equation, equation (19) and equation (C.6) we obtain the common growth rate of Y_P, K_P and C_P given by equation (20). Here two sectors grow at the same rate if the following condition is satisfied.

$$(\alpha+\beta+\epsilon_P+\epsilon_R)\phi = (\alpha+\beta)(\phi+\eta_P+\eta_R). \qquad (C.8)$$

If there do not exist externalities, i.e., if $\epsilon_R = \epsilon_P = \eta_R = \eta_P = 0$, then the above condition is always satisfied. If $\phi = 1$, then production functions (2) and (19) are same; and then the condition given by equation (C.8) is same as condition (A.14) used in the basic model.

Appendix D

First order optimality conditions derived from the dynamic optimization problem solved by the social planner in section 5.2 are given by the following.

$$({C_R}^{\theta}{C_P}^{1-\theta})^{-\sigma}\theta {C_R}^{\theta-1}{C_P}^{1-\theta} - \lambda_K = 0; \qquad (D.1)$$

$$({C_R}^{\theta}{C_P}^{1-\theta})^{-\sigma}(1-\theta){C_R}^{\theta}{C_P}^{-\theta} - \lambda_K = 0; \qquad (D.2)$$

$$\lambda_K \alpha \frac{Y_R}{a^*} - \lambda_P \delta \frac{\dot{H}_P}{(1-a^*-b^*)} = 0; \qquad (D.3)$$

$$\lambda_R m H_R - \lambda_P \delta \frac{\dot{H}_P}{(1-a^*-b^*)} = 0; \qquad (D.4)$$

$$\lambda_K \phi \frac{Y_P}{u^*} - \lambda_K \beta \frac{Y_R}{(1-u^*-v^*)} = 0; \qquad (D.5)$$

$$-\lambda_K \frac{\beta Y_R}{(1-u^*-v^*)} + \lambda_P (1-\delta-\gamma)\frac{\dot{H}_P}{v^*} = 0; \qquad (D.6)$$

$$\lambda_K(1-\alpha-\beta)\frac{Y_R}{x} - \lambda_K(1-\phi)\frac{Y_P}{(1-x)} = 0; \qquad (D.7)$$

$$\dot{\lambda_K} = \rho\lambda_K - \lambda_K(1-\alpha-\beta)\frac{Y_R}{K} - \lambda_K(1-\phi)\frac{Y_P}{K}; \qquad (D.8)$$

$$\dot{\lambda_R} = \rho\lambda_R - \lambda_K(\alpha+\epsilon_R)\frac{Y_R}{H_R} - \lambda_R m b^* - \lambda_P(\delta+\mu\gamma)\frac{\dot{H}_P}{H_R} - \lambda_K \eta_R \frac{Y_P}{H_R}; \qquad (D.9)$$

and

$$\dot{\lambda_P} = \rho\lambda_P - \lambda_K(\beta+\epsilon_P)\frac{Y_R}{H_P} - \lambda_P\{(1-\delta-\gamma)+(1-\mu)\gamma\}\frac{\dot{H}_P}{H_P} - \lambda_K(\phi+\eta_P)\frac{Y_P}{H_P}. \qquad (D.10)$$

From equation (D.5), we obtain

$$\frac{(1-u^*-v^*)}{u^*} = \frac{\beta Y_R}{\phi Y_P}. \qquad (D.11)$$

From equations (D.1) and (D.2), we obtain

$$\frac{C_R}{C_P} = \frac{\theta}{1-\theta}. \tag{D.12}$$

From equation (D.7), we obtain

$$\frac{x}{(1-x)} = \frac{(1-\alpha-\beta)}{(1-\phi)}\frac{Y_R}{Y_P}. \tag{D.13}$$

To have an interior solution of u^*, $(1-u^*-v^*)$ and x, equation (D.11) implies that Y_R and Y_P must grow at equal rate. From equation (D.12) we find that $\frac{C_R}{C_P}$ is constant. Since on the SGP, the growth rates of H_R, and H_P, a^*, b^* and v^* are time independent, the following equation holds true in this case too

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a^*-b^*)^{\delta}{v^*}^{1-\delta-\gamma}{z^*}^{\delta+\mu\gamma} = mb^*. \tag{D.14}$$

From equation (D.8), we have

$$\frac{\dot{\lambda}_K}{\lambda_K} = \rho - [(1-\alpha-\beta)\frac{Y_R}{K} + (1-\phi)\frac{Y_P}{K}].$$

Using equations (D.13), (D.1) and (D.2) we now have

$$\frac{\dot{\lambda}_K}{\lambda_K} = \rho - \frac{(1-\alpha-\beta)}{x}\frac{Y_R}{K} = -\sigma\frac{\dot{C}_R}{C_R} = -\sigma\frac{\dot{C}_P}{C_P}. \tag{D.15}$$

Since on the SGP, x and growth rates of C_R and C_P are constant, $\frac{Y_R}{K}$ must be constant. Using equations (D.15), (D.12), (D.14) and (23) we now obtain the common growth rate of Y_R, Y_P, C_R, C_P, K given by equation (26). From equation (D.9), we have

$$\frac{\dot{\lambda}_R}{\lambda_R} = \rho - \frac{ma^*(\alpha+\epsilon_R)}{\alpha} - mb^* - \frac{m(\delta+\mu\gamma)}{\delta}(1-a^*-b^*) - \frac{ma^*\eta_R Y_P}{\alpha Y_R}. \tag{D.16}$$

From equations (D.3) and (D.4), we have

$$\lambda_K\alpha\frac{Y_R}{a} = \lambda_R m H_R.$$

Differentiating both sides of this equation and using equations (D.15), (D.16), (25) and (26), we obtain equation (27).

Chapter 6

Dualism in the Human Capital Accumulation and the Role of Educational Subsidy

6.1 Introduction

In chapters 4 and 5, we have developed growth models dealing with dualism in the human capital accumulation. Rich individuals voluntarily allocate their resources (labour) to the training of poor individuals in those models, but the government does not provide any tax financed educational subsidy for the education of these poor individuals. This chapter attempts to develop a theoretical model of endogenous growth involving redistributive taxation and educational subsidy to build up human capital of poor individuals. The government of India has given special emphasis on the subsidized general education programme and special training programmes for the people beloging to scheduled castes and scheduled tribes. Majority of them are economically backward. The fruitfulness of this policy is subject to a lot of debates among the intellectuals. Here we analyze the model of a dual economy with two different classes of individuals in which dualism exists in the nature of their human capital accumulation; and the government imposes a proportional tax on the resources of rich individuals to finance the educational subsidy given to the poors.

Lucas (1990) has already drawn our attention to "increased subsidies to schooling, that would.....have potentially large effects on human capital accumulation and long term growth rates.....[It] might well be an interesting subject for future research." Many authors have analysed the issue of education subsidy in recent years. The set of literature includes the works of Zhang (2003), Blankenau and Simpson (2004), Bovenberg and Jacobs (2003, 2005), Boskin (1975), Blankenau (2005), Brett and Weymark (2003) and of many others. Most of them deal with the effects of subsidies and public expenditures on education and growth using an overlapping generation framework but do not consider a Lucas (1988) type model.

In this chapter, we develop a growth model of a dual economy in which human capital accumulation is viewed as the source of economic growth and dualism exists in the mechanism of human capital accumulation of two types of individuals — the rich and the poor. We assume that the rich individual has a high initial level of human capital endowment and an efficient human capital accumulation technology[1]. The poor individual lags behind the rich individual both in terms of the initial human capital endowment and in terms of the productivity of human capital accumulation technology. We call them rich and poor because human capital is an important determinant of income[2]. Rich individuals acquire knowledge spending their own resources. However, poor individuals are benefitted by the teaching of rich individuals; and redistributive taxes are imposed by the government on rich individuals to finance the educational subsidy given to poor individuals. The government taxes a fraction of resources of rich individuals and spends the tax rev-

[1]It means that the rich individual has a higher ability of learning and a larger stock of secondary inputs.

[2]The empirical works on the skilled-unskilled wage inequality in different countries, i.e., the works of Robbins(1994a, 1994b), Lachler (2001), Beyer, Rojas and Vergara (1999), Marjit and Acharyya (2003), Wood (1997) etc. have a debate over this hypothesis. Beyer, Rojas and Vergara(1999) have shown that the extent of wage inequality and the proportion of the labour force with college degrees in the post liberalization period in Chile were negatively related. According to the World Development Report (1995), increased educational opportunities exerted downward pressures on wage inequality in Columbia and Costa Rica. Many other works have shown the opposite empirical picture in many other countries.

enue to meet the cost of training of poor individuals. We consider the rate of tax cum educational subsidy to be exogenously given; and our objective is to analyse the effects of the introduction of this tax cum subsidy policy on the steady state equilibrium growth path and on the transitional growth path of the economy. We assume the presence of external effect of human capital on the production technology of both types of individuals.

We derive some interesting results from this model. An exogenous and once for all increase in the rate of tax financed educational subsidy raises the labour time allocation to production for both types of individuals and lowers the balanced growth rate of the stock of their human capitals in the new long run equilibrium. However, the short term (transitional dynamic) and long term impacts of the adoption of tax financed educational subsidy policy on the labour allocation to the human capital accumulation sector are identical for the rich individuals but not for the poor individuals. Following an exogenous and once for all increase in the tax financed educational subsidy rate, the labour time allocated to production of the poor individual first falls over time from the initial equilibrium level and then starts rising along the transitional path till the new steady state equilibrium point is reached. So the educational subsidy policy can induce the poor individual to acquire more human capital in the short-run. However, its benefit does not exist in the long run. Also there is a conflict between the growth rate maximization and the social welfare maximization. Social welfare maximizing subsidy rate may be positive though the steady state equilibrium growth rate is maximized in the absence of this subsidy.

Section 6.2 presents the basic model. Section 6.3 solves the dynamic optimization problems of two types of individuals and derives the properties of the steady state growth equilibrium. Section 6.4 analyzes the effects of the exogenous and once for all change in the rate of tax financed educational subsidy. Section 6.5 analyzes the properties of the optimal educational subsidy policy. In section 6.6 the external effect of poor individual's human capital is included in the rich individual's human capital accumulation function. Concluding remarks are made in section 6.7.

6.2 The basic dual economy model

We consider a one commodity model of a closed economy with two types of individuals –the rich and the poor. Human capital accumulation is a non market activity like that in Lucas (1988). However, the mechanisms of human capital accumulation are different for two types of individuals. There is external effect of human capital on production as well as on the human capital accumulation. Population size of either type of individual is normalised to unity. All individuals belonging to each group are assumed to be identical. There is full employment of both types of labour; and the labour markets are competitive.

The government taxes $(1-x)$ fraction of the labour time endowment of the rich individual to finance the cost of the training programme of the poor individual[3]. Labour endowment is the only resource of the individual. Out of the remaining x fraction of labour time, a rich individual allocates 'a' fraction to production and $(1-a)$ fraction to his own human capital accumulation. The poor individual allocates u fraction of labour time to production. Let H_R and H_P be levels of human capital of the representative rich and the poor individual (worker) respectively. We assume that $H_R(0) > H_P(0)$. This means that the poor individual lags behind the rich individual in terms of initial human capital endowment.

Both type of individuals produce the product using their labour as the only input; and this labour input is expressed in efficiency (human capital) unit. The

[3]Park and Phillippopoulos (2004), Benhabib et.al (1996) consider taxation on resources. Generally taxes are imposed on income. However, taxes are also imposed on land and many other assets in less developed countries. In this model, labour is the only resource and marginal productivity of labour is positive. So imposing tax on labour time endowment is equivalent to imposing tax on income. Due to technical complications, we can not consider the capital accumulation dynamics in this model. So we can not consider taxes on capital income of rich individuals.

production functions of the rich worker (individual) and of the poor worker (individual) are given by

$$Y_R = AaxH_R\bar{H_R}^{\epsilon_R} \tag{1}$$

and

$$Y_P = AuH_P\bar{H_P}^{\epsilon_P}. \tag{2}$$

Here $0 \leq x \leq 1$. ϵ_R and $\epsilon_P > 0$ are parameters representing the magnitudes of external effect of human capital in these two production technologies. Production function satisfies CRS in terms of private inputs but shows social IRS if external effect is taken into consideration. Y_R and Y_P stand for the levels of output of the rich individual and of the poor individual.

Both the rich individual and the poor individual consume whatever they produce; and hence they do not save (invest). So there is no accumulation of physical capital in this model; and hence physical capital does not enter as an argument in the production function[4]. We have

$$C_R = AaxH_RH_R^{\epsilon_R}; \tag{3}$$

and

$$C_P = AuH_PH_P^{\epsilon_P}. \tag{4}$$

Here C_P and C_R are the levels of consumption of the representative poor individual and of the representative rich individual respectively. The representative individual of either type maximizes his discounted present value of utility over the infinite time horizon with respect to labour time allocation variables. The instantaneous utility functions of the rich individual and of the poor individual are given by

$$U(C_R) = lnC_R \tag{5}$$

[4]Though it is assumed for simplicity, it is a serious limitation of the exercise. However, the model becomes highly complicated when physical capital accumulation is introduced. There are number of authors who did not consider physical capital in growth models e.g. Pecorino (1992), Rosendahl (1996), Lucas (2004), Driskill and Horowitz (2002) etc.

and

$$U(C_P) = lnC_P \tag{6}$$

respectively. Mechanism of the human capital accumulation of the rich individual is assumed to be similar to that in Lucas (1988). Hence

$$\dot{H}_R = m_R(1-a)xH_R. \tag{7}$$

Here $0 < a < 1$; and m_R is a positive constant representing the productivity parameter of the human capital formation function of the rich individual. Broadly speaking, the human capital accumulation technology depends on the ability of the learner and on other secondary inputs; and here m_R depends on all of those factors.

However, mechanisms of human capital formation of two classes of individuals are different. The skill formation of a poor individual takes place through the training program conducted by the government. The government taxes $(1-x)$ fraction of the available labour time of the rich individual and spends this $(1-x)$ fraction in this training. So this $(1-x)$ fraction can also be interpreted as the rate of educational subsidy given to poor individuals. The poor individual devotes $(1-u)$ fraction of non-leisure time to learning. The human capital accumulation function of the representative poor individual is assumed to take the following form.

$$\dot{H}_P = m_P(1-u)H_P[(\frac{\bar{H}_R}{\bar{H}_P} - 1)(1-x) + 1]. \tag{8}$$

Here $m_P > 0$ is a parameter representing the productivity of the human capital accumulation technology of the representative poor individual. We assume $m_R > m_P$ because the poor individual owns a smaller stock of secondary inputs required in learning. Note that

$$\frac{\partial \dot{H}_P}{\partial(1-x)} = m_P(1-u)H_P(\frac{\bar{H}_R}{\bar{H}_P} - 1) \geq 0$$

for $\bar{H}_R \geq \bar{H}_P$. So this training program is productive when poor individuals lag behind rich individuals in terms of their human capital endowments. Here the knowledge accumulation technology is such that the knowledge trickles down from more knowledgeable persons to inferiors. $(\frac{\bar{H}_R}{\bar{H}_P} - 1)$ can be interpreted as the degree of effectiveness of the teaching program. So the higher the extent of the knowledge gap between the rich individual and the poor individual, the more effective will be the teaching programme made for poors. In models of Tamura (1991), Eaton and Eckstein (1997), Lucas (2004) etc. the human capital accumulation technology is subject to external effects. In models of Eaton and Eckstein (1997) and Tamura (1991), average human capital stock of the society brings external effect on the human capital accumulation of every individual. However, in the model of Lucas (2004), human capital stock of the leader causes external effect on the human capital accumulation of all other individuals. Leader is that individual whose human capital is at the highest level. In our model, the representative rich individual has already attained a higher level of human capital and the representative poor individual is lagging behind. Rich individuals and poor individuals are assumed to be identical within their respective groups. So the rich individual may be treated as the leader; and hence the average human capital of rich individuals relative to that of poor individuals should have a positive external effect on poor individual's human capital accumulation technology.

6.3 The dynamics of the model

6.3.1 The optimization problem of the rich individual

The objective of the representative rich individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional of the rich individual is given by

$$J_R = \int_0^\infty U(C_R)e^{-\rho t}dt.$$

This is to be maximized with respect to the control variable a satisfying $0 \leq a \leq 1$ subject to the equation of motion given by

$$\dot{H}_R = m_R(1-a)xH_R;$$

and given the initial value of the state variable, H_R. Here $U(C_R)$ is given by equation (5) and C_R is given by equation (3). Here ρ is the constant positive discount rate. The current value Hamiltonian is given by

$$H^R = lnC_R + \lambda_R m_R(1-a)xH_R$$

where λ_R is the co state variable and it is interpreted as the shadow price of the human capital of the rich individual. C_R is given by the equation (3).

The first order condition necessary for this optimization problem with respect to the control variable, a, is given by the following.

$$\frac{1}{a} = \lambda_R m_R H_R x. \tag{9}$$

Time derivative of the co-state variable satisfying the optimum growth path is given by the following:

$$\dot{\lambda_R} = \rho\lambda_R - \frac{1}{H_R} - \lambda_R m_R(1-a)x. \tag{10}$$

The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t) = 0.$$

Using equations (9) and (10) we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - m_R x. \tag{11}$$

Differentiating the equation (9) with respect to time, t, and then, using equation (10), we have

$$\dot{a} = m_R a^2 x - \rho a. \tag{12}$$

6.3.2 The optimization problem of the poor individual

The representative poor individual also maximizes the discounted present value of utility over the infinite time horizon. The objective functional is given by

$$J_P = \int_0^\infty U(C_P)e^{-\rho t}dt$$

which is to be maximized with respect to control variable u satisfying $0 \leq u \leq 1$ subject to the equation of motion given by

$$\dot{H_P} = m_P(1-u)H_P[(\frac{\bar{H_R}}{\bar{H_P}} - 1)(1-x) + 1]$$

and given the initial value of the state variable, H_P. Here $U(C_P)$ is given by equation (6) and C_P is given by equation (4). Here ρ is the constant positive discount rate. The current value Hamiltonian is given by

$$H^P = lnC_P + \lambda_P m_P(1-u)H_P[(\frac{\bar{H_R}}{\bar{H_P}} - 1)(1-x) + 1]$$

where λ_P is the costate variable representing the shadow price of human capital of the poor individual.

Here $\bar{H_R} = H_R$ and $\bar{H_P} = H_P$ because individuals are identical within their groups. However, the individual can not internalize the externality. The first order condition necessary for this optimization problem with respect to the control variable, u, is given by the following.

$$\frac{1}{u} = \lambda_P m_P H_P[(\frac{H_R}{H_P} - 1)(1-x) + 1]. \tag{13}$$

Time derivative of the costate variable satisfying the optimum growth path is given by the following.

$$\dot{\lambda_P} = \rho\lambda_P - \frac{1}{H_P} - \lambda_P m_P(1-u)[(\frac{H_R}{H_P} - 1)(1-x) + 1]. \tag{14}$$

The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_P(t)H_P(t) = 0.$$

Using equations (13) and (14) we have

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - m_P[(\frac{H_R}{H_P} - 1)(1-x) + 1]. \tag{15}$$

Differentiating equation (13) with respect to time, and then using equation (14), we have

$$\dot{u} = m_P u^2[(\frac{H_R}{H_P} - 1)(1-x) + 1] - \rho u - \frac{(H_R/H_P)(1-x)(\frac{\dot{H_R}}{H_R} - \frac{\dot{H_P}}{H_P})}{[(\frac{H_R}{H_P} - 1)(1-x) + 1]}u. \tag{16}$$

We define $z = \frac{H_R}{H_P}$. Using equations (7) and (8) we have

$$\dot{z} = m_R(1-a)xz - m_P(1-u)[(z-1)(1-x)+1]z. \tag{17}$$

Using equations (16) and (17) we have

$$\dot{u} = m_P u^2[(z-1)(1-x)+1] - \rho u - \frac{m_R(1-x)(1-a)xzu}{[(z-1)(1-x)+1]} + m_P(1-x)(1-u)zu. \tag{18}$$

6.3.3 The steady state growth equilibrium

Along a steady state equilibrium growth path, labour time allocations remain unchanged and the human capital stock of two types of individuals grow at equal rates. So we have

$$\dot{a} = \dot{u} = \dot{z} = 0.$$

From equations (12), (17) and (18), we determine steady state equilibrium values of a, u and z in terms of x. They are given by the followings.

$$a^* = \frac{\rho}{m_R x}; \tag{19}$$

$$u^* = \frac{\rho}{m_P[(z^*-1)(1-x)+1]} = \frac{\rho}{m_R x}; \tag{20}$$

and

$$z^* = 1 + \frac{1}{(1-x)}[\frac{m_R(1-a^*)x}{m_P(1-u^*)} - 1] = \frac{x(m_R - m_P)}{(1-x)m_P}. \tag{21}$$

Note that $u^* = a^*$; and z^* is a positive constant for given x satisfying $0 < x < 1$ because $m_R > m_P$. $z^* > 1$ implies $H_R > H_P$ in the steady state growth equilibrium and this is possible if $x > \frac{m_P}{m_R}$. We assume

$$x > \frac{m_P}{m_R}$$

to be always satisfied. This implies that $m_R x - m_P > 0$. The balanced growth rate of human capital is denoted by g. Using equation (7) and the expression of a^*, we have

$$g = \frac{\dot{H}_R}{H_R} = \frac{\dot{H}_P}{H_P} = m_R x - \rho. \tag{22}$$

6.4 Comparative dynamic effects of educational subsidy policy

6.4.1 Long-run effects

We first consider the impacts of a change in the rate of tax financed educational subsidy on steady state equilirium values of variables. The results of comparative steady state exercises with respect to x are summarized as follows:

$$\frac{da^*}{dx} = -\frac{\rho}{m_R x^2} < 0; \tag{23}$$

$$\frac{du^*}{dx} = -\frac{\rho}{m_R x^2} < 0; \tag{24}$$

$$\frac{dz^*}{dx} = \frac{(m_R - m_P)}{m_P (1-x)^2} > 0; \tag{25}$$

and

$$\frac{dg}{dx} = m_R.$$

A decrease in x causes a reallocation of labour time to the production sector from the human capital accumulation sector; and this reduces the balanced growth rate of human capital of the two groups of individuals. Here $m_R > m_P$; and equations (23) and (24) show that marginal effects of change in the educational subsidy rate on labour allocations are same for two groups of individuals. So a fall in x causes $\frac{\dot{H_R}}{H_R}$ to grow at a lower rate than $\frac{\dot{H_P}}{H_P}$ along the transitional phase. So z^* takes a lower value in the new steady state equilibrium. Here $z = \frac{H_R}{H_P}$ is an index of inter group income inequality in the economy because income in this model depends only on the stock of human capital and individuals are identical within their respective groups. We now state the main result in the form of the following proposition.

Proposition 1 *The increase in the rate of tax financed educational subsidy raises the labour time allocation to production of both types of individuals in the new steady-state growth equilibrium; and this lowers the balanced growth rate of the stock of human capitals of two groups of individuals as well as the degree of inter group income inequality.*

There exists a substantial theoretical literature related to the effects of taxation on the endogenous growth rate in human capital accumulation models; and in most of these models, the increase in the tax rate produces a negative effect on the growth rate. However, the tax revenue in these models are spent either as lumpsum payment or to build public infrastructure. Alonso Carrera (2000) analyse the effect of educational subsidy policies in Lucas (1988) type model; and finds that the steady state equilibrium rate of growth varies positively with the rate of subsidy when education subsidy is financed by lumpsum tax. Bond, Wang and Yip (1996) obtains the same result when education subsidy is financed by factor income taxation and lumpsum taxation. Garcia-Castrillo and Sanso (2000), Gomez (2003) etc. analyse optimal policy in the Lucas (1988) model. They also show that the educational subsidy raises the rate of growth in the steady state equilibrium and prescribe joint application of taxation on labour and subsidization to education as the optimal policy. Our result that the tax financed educational subsidy lowers the steady state equilibrium rate of growth is of significance in the context of the existing literature. Also these models in the existing literature do not consider the problem of dualism in human capital accumulation; and hence do not analyse the effectiveness of group specific educational subsidy policy financed by redistributive taxation.

6.4.2 Transitional dynamic effects

In this subsection, we analyze the effects of an exogenous and once for all change in the rate of tax financed educational subsidy on the transitional growth path of the economy. We consider the dynamic equations (12), (17) and (18).

Linearizing these equations around the steady state growth equilibrium point, (a^*, u^*, z^*) we obtain the following differential equations.

$$\begin{bmatrix} \dot{a} \\ \dot{u} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{a}}{\partial a} & \frac{\partial \dot{a}}{\partial u} & \frac{\partial \dot{a}}{\partial z} \\ \frac{\partial \dot{u}}{\partial a} & \frac{\partial \dot{u}}{\partial u} & \frac{\partial \dot{u}}{\partial z} \\ \frac{\partial \dot{z}}{\partial a} & \frac{\partial \dot{z}}{\partial u} & \frac{\partial \dot{z}}{\partial z} \end{bmatrix} \begin{bmatrix} (a - a^*) \\ (u - u^*) \\ (z - z^*) \end{bmatrix}.$$

Here $\frac{\partial \dot{a}}{\partial u} = \frac{\partial \dot{a}}{\partial z} = 0$; and other coefficients of the Jacobian matrix evaluated at the steady state equilibrium are displayed in Appendix (A). The characteristic equation of the Jacobian matrix is given by

$$(\frac{\partial \dot{a}}{\partial a} - \lambda)[(\frac{\partial \dot{u}}{\partial u} - \lambda)(\frac{\partial \dot{z}}{\partial z} - \lambda) - \frac{\partial \dot{u}}{\partial z}\frac{\partial \dot{z}}{\partial u}] = 0.$$

So two of the characteristic roots[5] are equal to ρ and the remaining one is $(m_P - m_R)x$. Here we have two control variables, a and u, and one predetermined variable, z. Two charateristic roots are positive and one is negative. So the steady state equilibrium is saddle path stable with only one trajectory converging to the unique steady state equilibrium point. Hence we have the following proposition.

Proposition 2 *There exists a unique saddle path converging to the unique steady state equilibrium point.*

The solution to the differential equation (12) is given by

$$a(t) = a^* + (a(0) - a^*)e^{m_R a^* xt}. \quad (26)$$

Here $m_R a^* x > 0$. Hence $a(t)$ converges either to minus infinity or to infinity unless $a(0) = a^*$. However, $a(t) < 0$ implies that the production level and consequently the consumption level are negative, which are not feasible. So $a(t) \geq 0$ along the equilibrium growth path. Also $a(t)$ can not increase unboundedly since its upper limit is unity. So $a(t)$ must satisfy the following equation along the saddle path.

$$a(t) = a^* \quad \text{for all} \quad t \geq 0. \quad (27)$$

So the effect of the change in x on $a(t)$ along the transitional path is same as that in the steady state growth equilibrium. This leads to the following proposition.

Proposition 3 *Short term and long term impacts of a tax financed educational subsidy policy on the resource allocation to the human capital accumulation sector are identical for the rich individual.*

This property makes the analysis easier when we turn to find out transitional dynamic effects on u and z. Along this saddle path, the time path of u and z must satisfy the following[6] equations.

[5] It is shown in the Appendix(B)

[6] These are derived in the Appendix(C)

$$u(t) = u^* - \frac{\rho m_P(1-x)}{{m_R}^2 x^2}(z(0) - z^*)e^{-(m_R - m_P)xt}; \tag{28}$$

and

$$z(t) = z^* + (z(0) - z^*)e^{-(m_R - m_P)xt}. \tag{29}$$

We assume that the economy initially is at the steady state equilibrium. i.e, $z(0) = z^*$. So the effect of the exogenous change in x on the time behaviour of u along the transitional path around the steady state equilibrium point is given by

$$\frac{\partial u}{\partial x} = \frac{\rho}{m_R x^2}[\frac{(m_R - m_P)}{m_R(1-x)}e^{-(m_R - m_P)xt} - 1].$$

For $t = 0$, we have

$$\frac{\partial u}{\partial x} = \frac{\rho(m_R x - m_P)}{{m_R}^2 x^2(1-x)} > 0.$$

Note that $\frac{\partial u}{\partial x} \gtreqless 0$ for all $t \lesseqgtr t^*$ where

$$t^* = \frac{1}{(m_R - m_P)x} ln[\frac{(m_R - m_P)}{m_R(1-x)}].$$

For t^* to be positive, $(m_R - m_P)$ should be greater than $m_R(1-x)$ which is always true if $x > \frac{m_P}{m_R}$. So we find that the new saddle path of u resulting from a fall in x lies below the old saddle path initially and then crosses it.

The effect of the exogenous change in x on the time behaviour of z along the transitional path is given by

$$\frac{\partial z}{\partial x} = \frac{(m_R - m_P)}{m_P(1-x)^2}[1 - e^{-(m_R - m_P)xt}].$$

For $t = 0$, we have $\frac{\partial z}{\partial x} = 0$; and $\frac{\partial z}{\partial x} > 0$ for all $t > 0$. So the time path of z shifts downward along the transitional path while moving towards the new long run equilibrium value when x is decreased. Mathematical signs of $\frac{\partial u}{\partial x}$ and $\frac{\partial z}{\partial x}$ evaluated at $t = 0$ give us the direction of immediate effects of the change in x. We can now establish the following proposition.

Proposition 4 *If the economy is initially in a steady state growth equilibrium, then, following an once for all increase in the rate of tax financed educational subsidy, (i) the poor individual's labour time allocation to production denoted by u is first reduced and then is increased along the new saddle path converging to* *the new steady state equilibrium point; and (ii) the degree of inter group income inequality measured by z is reduced along the new saddle path converging to the new steady state equilibrium point.*

We analyze time behaviours of u and z using equations (17) and (18) and $a = a^*$. Figure 6.1 depicts the phase diagram where demarkation curves representing $\dot{u} = 0$ and $\dot{z} = 0$ are obtained from differential equations (17) and (18)[7]. The steady state equilibrium is a saddle point and the unique saddle path is shown by the downward sloping stable arm VV. If the economy initially stays on VV path then it would monotonically converge to the unique equilibrium point E. Using equations (28) and (29), we find that the equation of the saddle path in the $u - z$ plane is given by

$$u(t) - u^* = -\frac{\rho m_P(1-x)}{{m_R}^2 x^2}(z(t) - z^*);$$

and its slope is negative.

The effects of the exogenous and once for all change in x on $\dot{u} = 0$ and $\dot{z} = 0$ curves are displayed in Figure 6.2. The initial $\dot{u} = 0$ and $\dot{z} = 0$ curves are denoted by U_1U_1 and Z_1Z_1. When x is reduced, $\dot{u} = 0$ curve shifts to U_2U_2 and $\dot{z} = 0$ curve shifts to Z_2Z_2. The equilibrium point shifts from E_1 to E_2; and the saddle path V_1V_1 shifts to V_2V_2 in this case. However, there may be an upward shift of $\dot{u} = 0$ locus though E_2 lies left to E_1 even in this case.

Figure 6.3 shows the effect of the decrease in x on the saddle path in $a - z$ plane and in $u - z$ plane. The upper panel (a) of Figure 6.3 depicts the relation

between a and z and the lower panel (b) shows the relation between u and z along the saddle path. Since a is time independent in the transitional phase, the saddle path in $a - z$ plane is a horizontal straight line parallel to z axis. An once for all increase in the educational subsidy rate (a decrease in x) produces an increase in a^* so that the saddle path S_1S_1 shifts up to S_2S_2 and the equilibrium point E_1 shifts to E_2. Hence, as the tax financed educational subsidy rate is increased, rich individuals allocate more labour time to production and less labour time to education in the transitional phase.

[7] The properties of $\dot{u} = 0$ and $\dot{z} = 0$ curves are displayed in Appendix (D).

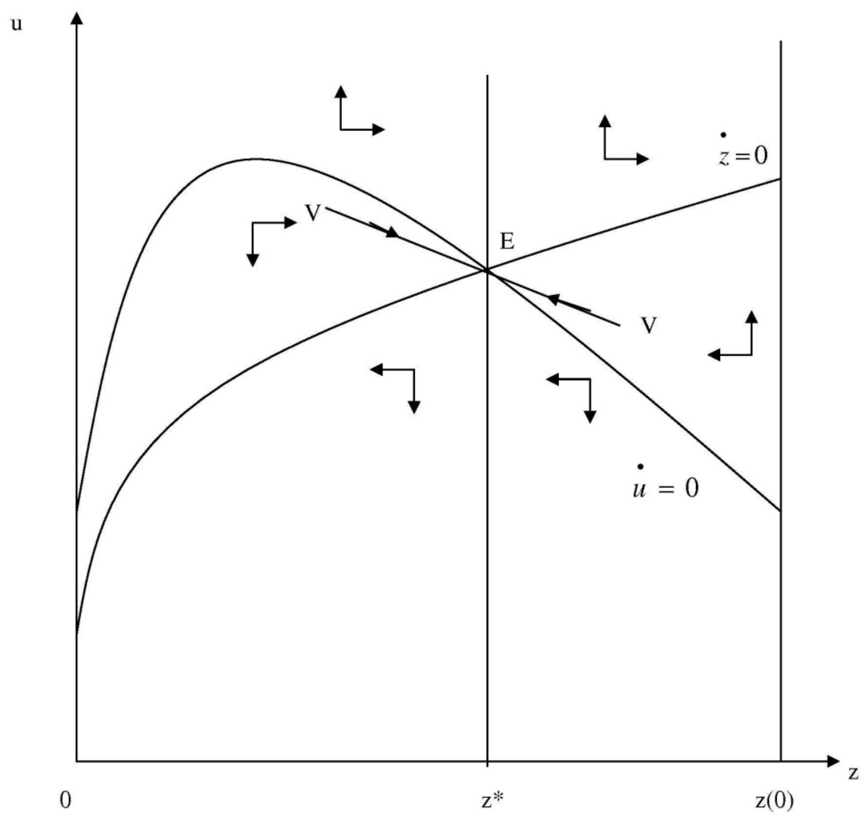


Figure 6.1 : The phase diagram

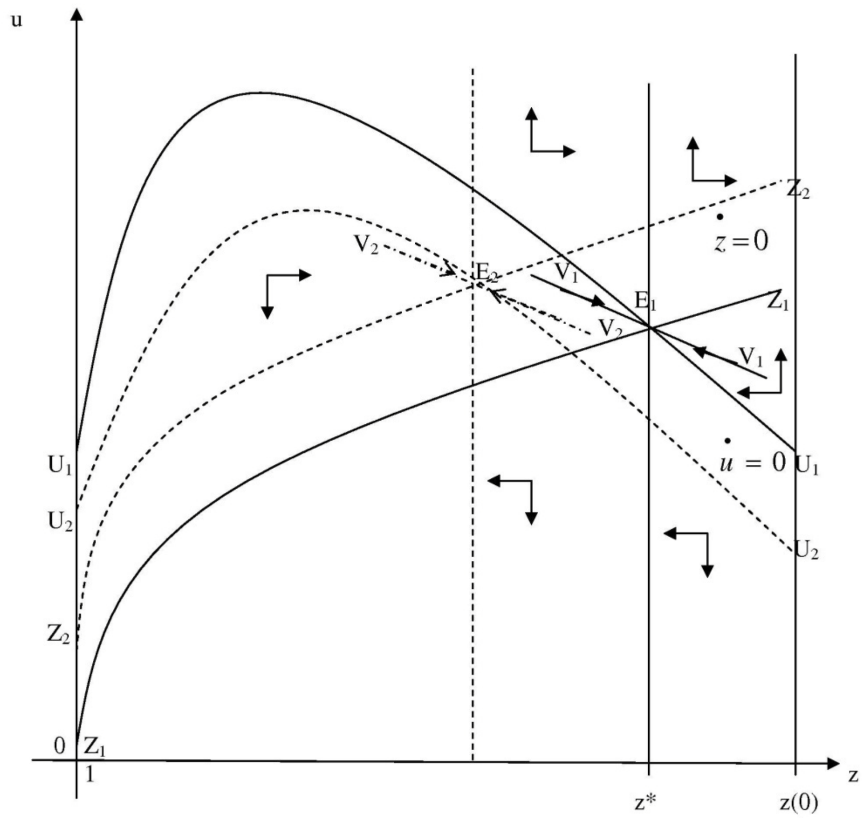
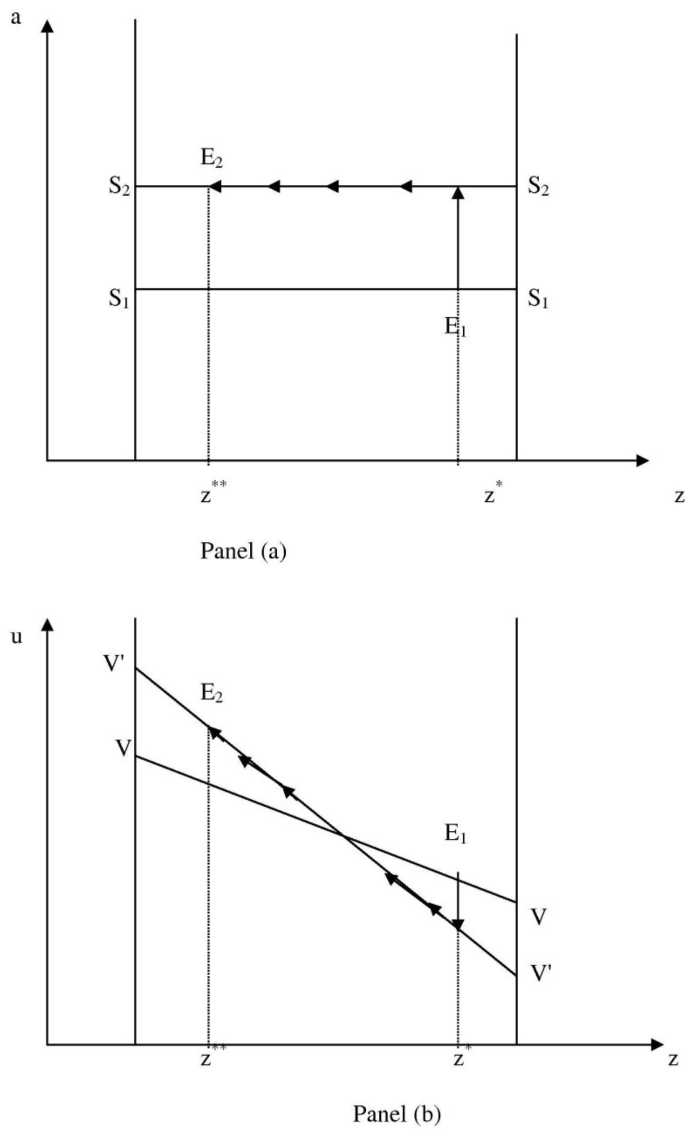


Figure 6. 2 : Effect of change in x on phase diagram



Effect of decrease in x on the saddle path

Figure 6. 3

As x is decreased, V_1V_1 shifts to V_2V_2 and the equilibrium point shifts from E_1 to E_2 in the $u - z$ plane. The adjustment is not instantaneous but it is a gradual change. Although a decrease in x may reduce u initially, it would increase u after some time point is reached. So, as the tax financed educational subsidy rate is increased, poor individuals may initially devote lower time to production and more time to education. However, in the long run, they also devote more time to production and less time to education like rich individuals.

6.5 Optimal subsidy policy

We assume that the government maximizes the discounted present value of instantaneous social welfare over the infinite time horizon with respect to the tax rate.

The instantaneous social welfare function is defined as follows.

$$W = blnC_R + (1-b)lnC_P \quad \text{with} \quad 0 < b < 1. \tag{30}$$

Here b and $(1-b)$ are weights given to the consumption of the rich individual and to the consumption of the poor individual respectively. Substituting C_R and C_P from equations (3) and (4)into equation (30) we have

$$W = b[lnA+ln(ax)+(1+\epsilon_R)lnH_R+(1-b)[lnA+lnu+(1+\epsilon_P)lnH_R+(1+\epsilon_P)ln(\frac{1}{z})].$$

Now, in this above mentioned equation, we substitute H_R by $H_R(0)e^{m_R(1-a)xt}$, H_P by $\frac{H_R}{z}$, a from the equation (19), u from the equation (20) and z from equation (21). Thus we have

$$W = lnA + ln(\frac{\rho}{m_R}) - (1-b)lnx + \{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\}lnH_R(0)+$$

$$\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\}(m_Rx - \rho)t + (1-b)(1+\epsilon_P)ln\{\frac{m_P(1-x)}{(m_R - m_P)x}\}.$$

This equation shows the steady state equilibrium level of instantaneous social welfare in terms of parameters of the model. The effect of a change in x on welfare is determined not only by its effect on g given by equation (22) but also by its effects on u and z given by equations (24) and (25).

The discounted present value of instantaneous social welfare is given by

$$SW = \int_0^\infty We^{-\rho t}dt = \frac{1}{\rho}[lnA + ln(\frac{\rho}{m_R}) - (1-b)lnx + (1-b)(1+\epsilon_P)ln\{\frac{m_P(1-x)}{(m_R - m_P)x}\}$$

$$+\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\}lnH_R(0)] + \frac{(m_R x - \rho)}{\rho^2}\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\}. \quad (31)$$

Here, SW and x do not have any monotonic relationship[8] for $0 < b < 1$.

If we consider $b = 1$, then, from equation (31), we have

$$SW = \frac{1}{\rho}[lnA + ln(\frac{\rho}{m_R}) + (1+\epsilon_R)lnH_R(0)] + \frac{(m_R x - \rho)(1+\epsilon_R)}{\rho^2}.$$

Then SW is a monotonically increasing function of x. So, with $b = 1$, social welfare is maximized at $x = 1$, i.e. in the absence of a tax financed educational subsidy policy. However, when $0 < b < 1$, $\frac{\partial SW}{\partial x}$ may be equal to zero for some value of x satisfying $0 < x < 1$; and $\frac{\partial SW}{\partial x} = 0$ implies that

$$\frac{(1-b)}{\rho x} + \frac{(1-b)(1+\epsilon_P)}{\rho x(1-x)} = \frac{m_R}{\rho^2}\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\}.$$

The effects of exogenous changes in ϵ_R, ϵ_P and m on the optimum value of x are derived in the Appendix (E) where it is shown that the optimum x varies positively with ϵ_R and m; and may vary either way with ϵ_P. This leads to the following proposition.

Proposition 5 *If $0 < b < 1$, then, in the steady state growth equilibrium, optimum subsidy rate may be positive; and this positive optimum subsidy rate varies negatively with m_R and ϵ_R and may vary either way with ϵ_P. However, it is not optimal to provide this subsidy when $b = 1$.*

However, from equation (22), we find that g is maximized when $x = 1$. So the steady state equilibrium rate of growth is maximized in the absence of the subsidy but, the social welfare may not be maximum in this case. This is so because we consider $b < 1$; and this implies that the consumption of the poor individual, who is benefitted by this tax cum educational subsidy programme, has a positive marginal contribution on the social welfare. The social welfare maximization policy

[8]The first order and second order differentiation of SW with respect to x are given in Appendix (E)

takes care of interests of poor individuals but the rate of growth maximizing policy does not do that. When $b = 1$, the social welfare function does not take care of interests of poor individuals and so there is no conflict between the social welfare maximization and the growth rate maximization in this case.

6.6 External effect of the poor individual's human capital on the rich individual's human capital accumulation

In this section, we include external effect of the representative poor individual's human capital on the rich individual's human capital accumulation function. The human capital accumulation function of the rich individual takes the following form in the modified version:

$$\dot{H}_R = m_R(1-a)x{H_R}^{1-\delta}\overline{H_P}^{\delta}. \tag{32}$$

This function satisfies social CRS and private DRS. The human capital accumulation function of the representative poor individual is assumed to take the following form.

$$\dot{H}_P = m_P(1-u)H_P[(\frac{\bar{H}_R}{\bar{H}_P}-1)(1-x)+1]. \tag{33}$$

The representative rich individual maximizes

$$J_R = \int_0^\infty U(C_R)e^{-\rho t}dt$$

with respect to the control variable a satisfying $0 \leq a \leq 1$ subject to the equation of motion given by

$$\dot{H}_R = m_R(1-a)x{H_R}^{1-\delta}{H_P}^{\delta}.$$

The current value Hamiltonian is given by

$$H^R = lnC_R + \lambda_R m_R(1-a)xH_R$$

where λ_R is the co state variable and it is interpreted as the shadow price of the human capital of the rich individual. As before C_R is given by the equation

$$C_R = AaxH_R{H_R}^{\epsilon_R};$$

From the optimization problem of the rich individual we have

$$a = \frac{(\frac{H_R}{H_P})^{\delta}}{\lambda_R H_R m_R x};$$

and

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - \frac{1}{H_R} - \lambda_R m_R (1-a) x (1-\delta) {H_R}^{-\delta} {H_P}^{\delta} = \rho - (\frac{H_R}{H_P})^{-\delta} m_R x [\delta a + (1-\delta)]. \quad (34)$$

From the two equations mentioned above we have

$$\frac{\dot{a}}{a} = z^{-\delta} m_R x a - \rho - \delta m_P (1-u)[(z-1)(1-x)+1]. \quad (35)$$

We define $z = \frac{H_R}{H_P}$. Using equations (32) and (33) we have

$$\dot{z} = m_R (1-a) x z^{1-\delta} - m_P (1-u)[(z-1)(1-x)+1] z. \quad (36)$$

The optimization problem of the poor individual remains same. From this optimization problem we obtain

$$\dot{u} = m_P u^2 [(\frac{H_R}{H_P} - 1)(1-x) + 1] - \rho u - \frac{(H_R/H_P)(1-x)(\frac{\dot{H_R}}{H_R} - \frac{\dot{H_P}}{H_P})}{[(\frac{H_R}{H_P} - 1)(1-x) + 1]} u. \quad (37)$$

Using equations (36) and (37) we have

$$\dot{u} = m_P u^2 [(z-1)(1-x)+1] - \rho u - \frac{m_R (1-x)(1-a) x z^{1-\delta} u}{[(z-1)(1-x)+1]} + m_P (1-x)(1-u) z u. \quad (38)$$

Along the steady state equilibrium growth path, we have

$$\dot{a} = \dot{u} = \dot{z} = 0.$$

So the steady state equilibrium values of a and u are given by

$$a^* = \frac{\rho + \delta m_P (1-u)[(z^* - 1)(1-x) + 1]}{z^{*-\delta} m_R x};$$

and

$$1 - u^* = \frac{m_R x z^{*-\delta} - \rho}{m_P (1+\delta)\{(z^* - 1)(1-x) + 1\}}.$$

Setting $\frac{\dot{u}}{u} = 0$ and substituting the values of a^* and u^* in equation (38) we have,

$$[x + z^*(1-x)][m_P - \frac{\rho\delta - m_R x z^{*-\delta}}{(1+\delta)\{(z^* - 1)(1-x) + 1\}}] = 0.$$

z^* can be obtained from the above equation. Since $x + z^*(1-x) \neq 0$ we have

$$m_P\{(z-1)(1-x)+1\}(1+\delta) = m_R x z^{-\delta} - \rho\delta. \tag{39}$$

Since the LHS of equation (39) is a positive function of z and the RHS is a negative function of z there exists a unique solution of z.

The growth rate is

$$g = \frac{m_R x z^{-\delta} - \rho}{(1+\delta)}.$$

$$\frac{dg}{dx} = \frac{m_R z^{*-\delta-1}[m_P(1+\delta)\{\delta x(z^*-1)+(1-x)z^*\}]}{(1+\delta)[m_P(1+\delta)(1-x)+m_R x\delta z^{*-\delta-1}]} > 0$$

because $0 \leq x < 1$ and $z^* \geq 1$.

Hence the increase in the tax rate produces a negative effect on the growth rate even in this extended model. This is one example where the long-run growth rate varies inversely with the tax financed educational subsidy rate even if the poor's human capital has some externality on the rich's human capital accumulation. In this case, externality does not generate social increasing returns to scale (IRS) in the human capital accumulation. We have social CRS and private DRS in equation (2).

6.7 Conclusion

In this chapter, we analyse the effects of an exogenous and once for all change in the rate of tax financed educational subsidy and derive some interesting results. The increase in the rate of subsidy raises the labour time allocation to production for both types of individuals and lowers the balanced growth rate of their stock of human capitals in the new long run equilibrium. However, the short term and long term impacts of tax financed educational subsidy policy on the resource allocation to the human capital accumulation sector are same for one group but not for the other group. These two effects are identical for rich individuals. However, for poor individuals, it is not the same. Following an increase in the subsidy rate, the labour time allocated to production of the representative poor individual first falls from the initial equilibrium level and then starts rising along the transitional path till the new steady state equilibrium point is reached. This assymetry in the transitional effects for these two groups is explained by differences in their human capital accumulation functions. As the labour time allocation to the human capital accumulation sector is increased, the rate of human capital accumulation of the poor individual may also rise initially following the introduction of the educational subsidy policy. However, in the long run, it must fall. So the education subsidy policy would not provide any benefit to the backward section of the population in the long run.

Also the policy prescription involves a conflict between the social welfare maximization objective. It is optimal not to adopt this educational subsidy policy when the government wants to maximize the balanced rate of growth. However the social welfare maximizing subsidy rate may be positive.

Appendix A

Elements of the Jacobian matrix, corresponding to equations of motion (12), (17)) and (18) given by

$$J = \begin{bmatrix} \frac{\partial \dot{a}}{\partial a} & \frac{\partial \dot{a}}{\partial u} & \frac{\partial \dot{a}}{\partial z} \\ \frac{\partial \dot{u}}{\partial a} & \frac{\partial \dot{u}}{\partial u} & \frac{\partial \dot{u}}{\partial z} \\ \frac{\partial \dot{z}}{\partial a} & \frac{\partial \dot{z}}{\partial u} & \frac{\partial \dot{z}}{\partial z} \end{bmatrix}$$

and evaluated at the steady state equilibrium point, are given by the followings.

$$\frac{\partial \dot{a}}{\partial a} = \rho,$$
$$\frac{\partial \dot{a}}{\partial u} = 0,$$
$$\frac{\partial \dot{a}}{\partial z} = 0,$$

$$\frac{\partial \dot{u}}{\partial a} = \rho(1 - \frac{m_P}{m_R}),$$
$$\frac{\partial \dot{u}}{\partial u} = \frac{\rho(m_R - m_P)}{m_R(1-x)},$$
$$\frac{\partial \dot{u}}{\partial z} = \frac{m_P(1-x)\rho}{{m_R}^2 x}[(m_R - m_P) + \frac{m_P \rho}{m_R x}],$$
$$\frac{\partial \dot{z}}{\partial a} = -m_R x^2 \frac{(m_R - m_P)}{m_P(1-x)},$$
$$\frac{\partial \dot{z}}{\partial u} = m_R x^2 \frac{(m_R - m_P)}{m_P(1-x)}$$

and

$$\frac{\partial \dot{z}}{\partial z} = -(m_R x - \rho)(1 - \frac{m_P}{m_R}).$$

Appendix B

The characteristic equation of the Jacobian matrix is given by

$$(\frac{\partial \dot{a}}{\partial a} - \lambda)[\lambda^2 - \lambda(\frac{\partial \dot{u}}{\partial u} + \frac{\partial \dot{z}}{\partial z}) + \frac{\partial \dot{u}}{\partial u}\frac{\partial \dot{z}}{\partial z} - \frac{\partial \dot{u}}{\partial z}\frac{\partial \dot{z}}{\partial u}] = 0.$$

Here, using steady state equilibrium values of u and z, we have

$$(\frac{\partial \dot{u}}{\partial u} + \frac{\partial \dot{z}}{\partial z}) = \rho - x(m_R - m_P);$$

and

$$\frac{\partial \dot{u}}{\partial u}\frac{\partial \dot{z}}{\partial z} - \frac{\partial \dot{u}}{\partial z}\frac{\partial \dot{z}}{\partial u} = -\rho x(m_R - m_P).$$

So the characteristic equation reduces to the following.

$$(\rho - \lambda)[\lambda^2 + \lambda\{x(m_R - m_P) - \rho\} - \rho x(m_R - m_P)] = 0.$$

Solving this cubic equation we find that the three roots are ρ, ρ and $-x(m_R - m_P)$.

Appendix C

The solutions to differential equations (12), (17) and (18) are given by

$$\begin{bmatrix} a(t) - a^* \\ u(t) - u^* \\ z(t) - z^* \end{bmatrix} = C_1 v_1 e^{\lambda_1 t} + C_2 v_2 e^{\lambda_2 t} + C_3 v_3 e^{\lambda_3 t}$$

where v_1, v_2 and v_3 are eigen vectors corresponding to characteristic roots λ_1, λ_2 and λ_3 of the Jacobian matrix. Here C_1, C_2 and C_3 are coefficients to be determined by initial conditions. In order to ensure the convergence of the transitional path to the steady state equilibrium point, we need to choose $C_1 = C_2 = 0$ when $\lambda_3 < 0$. Then we have

$$\begin{bmatrix} a(t) - a^* \\ u(t) - u^* \\ z(t) - z^* \end{bmatrix} = C_3 v_3 e^{\lambda_3 t}.$$

We consider $v_3 = (v_{31}, v_{32}, v_{33})'$. Elements of v_3 can be found out by solving the following system of equations.

$$\begin{bmatrix} \frac{\partial \dot{a}}{\partial a} - \lambda_3 & 0 & 0 \\ \frac{\partial \dot{u}}{\partial a} & (\frac{\partial \dot{u}}{\partial u} - \lambda_3) & \frac{\partial \dot{u}}{\partial z} \\ \frac{\partial \dot{z}}{\partial a} & \frac{\partial \dot{z}}{\partial u} & \frac{\partial \dot{z}}{\partial z} - \lambda_3 \end{bmatrix} \begin{bmatrix} v_{31} \\ v_{32} \\ v_{33} \end{bmatrix} = 0.$$

Using elements of the Jacobian matrix shown in Appendix (A) and considering that $\lambda_3 = -x(m_R - m_P)$, the above system of equations can be written as follows.

$$\{\rho + x(m_R - m_P)\}v_{31} = 0; \tag{C.1}$$

$$\rho(1 - \frac{m_P}{m_R})v_{31} + \{\frac{\rho(m_R - m_P)}{m_R(1-x)} - \lambda_3\}v_{32} + \frac{m_P(1-x)\rho}{{m_R}^2 x}[(m_R - m_P) + \frac{m_P\rho}{m_R x}]v_{33} = 0; \tag{C.2}$$

and

$$-m_R x^2 \frac{(m_R - m_P)}{m_P(1-x)}v_{31} + m_R x^2 \frac{(m_R - m_P)}{m_P(1-x)}v_{32} + \{-(m_R x - \rho)(1 - \frac{m_P}{m_R}) - \lambda_3\}v_{33} = 0. \tag{C.3}$$

From equation (C.1), we find that $v_{31} = 0$. Since u is a jump variable and z is a state variable, we have to choose $v_{33} = 1$ to solve these equations. From equations (C.2) and (C.3), we have

$$v_{32} = -\frac{\frac{\partial \dot{u}}{\partial z}}{(\frac{\partial \dot{u}}{\partial u} - \lambda_3)},$$

or equivalently

$$v_{32} = \frac{\lambda_3 - \frac{\partial \dot{z}}{\partial z}}{\frac{\partial \dot{z}}{\partial u}}.$$

[This is so because $(\frac{\partial \dot{u}}{\partial u} - \lambda_3)(\frac{\partial \dot{z}}{\partial z} - \lambda_3) - \frac{\partial \dot{u}}{\partial z}\frac{\partial \dot{z}}{\partial u} = 0$]

So we have

$$v_{32} = \frac{m_P\rho(1-x)}{{m_R}^2 x^2}. \tag{C.4}$$

Hence the general solution of $a(t)$, $u(t)$ and $z(t)$ are given as follows.

$$a(t) = a^*, \tag{C.5}$$

$$u(t) - u^* = C_3 v_{32} e^{-x(m_R - m_P)t}, \tag{C.6}$$

and

$$z(t) - z^* = C_3 e^{-x(m_R - m_P)t}. \tag{C.7}$$

When $t = 0$, from equation (C.7), we have $C_3 = z(0) - z^*$. Using equations (C.4), (C.5), (C.6) and (C.7), we obtain equations (24), (25) and (26).

Appendix D

$\dot{z} = 0$ demarkation curve is obtained by substituting $a = a^*$ in equation (17). The equation of $\dot{z} = 0$ curve is given by

$$u = 1 - \frac{(m_R x - \rho)}{m_P\{(z-1)(1-x)+1\}}.$$

Here, along the $\dot{z} = 0$ curve,

$$\frac{\partial u}{\partial z} = \frac{(m_R x - \rho)(1-x)}{m_P\{(z-1)(1-x)+1\}^2} > 0,$$

and

$$\frac{\partial^2 u}{\partial z^2} = -2\frac{(m_R x - \rho)(1-x)^2}{m_P\{(z-1)(1-x)+1\}^3} < 0.$$

So $\dot{z} = 0$ curve is positively sloped and is concave to the z axis in the Figure 6.1. At $z = 1$, the intercept of $\dot{z} = 0$ curve is given by

$$u = 1 + \frac{\rho}{m_P} - \frac{m_R x}{m_P}.$$

$\dot{u} = 0$ demarkation curve is obtained by substituting $a = a^*$ in equation (18). The equation of $\dot{u} = 0$ curve is given by

$$u = \frac{1}{m_P x}[\rho + \frac{(m_R x - \rho)(1-x)z}{\{(z-1)(1-x)+1\}} - m_P(1-x)z].$$

Here

$$\frac{\partial u}{\partial z} = (1-x)[\frac{(m_R x - \rho)x}{\{(z-1)(1-x)+1\}^2} - m_P].$$

Hence

$$\frac{\partial u}{\partial z} = 0$$

at

$$(z-1) = \frac{\sqrt{(m_R x - \rho)x} - \sqrt{m_P}}{(1-x)\sqrt{m_P}}. \qquad (E.1)$$

Also, along the $\dot{u} = 0$ curve,

$$\frac{\partial^2 u}{\partial z^2} = -\frac{2(1-x)^2 x(m_R x - \rho)}{\{(z-1)(1-x)+1\}^3} < 0.$$

If the RHS of equation (E.1) is positive, then $\dot{u} = 0$ curve is inverted U shaped in the diagram given by Figure 6.1; and if the RHS of equation (E.1) is negative, then $\dot{u} = 0$ curve is downward sloping. At $z = 1$, the intercept of $\dot{u} = 0$ curve is given by

$$u = 1 + \frac{\rho}{m_P} - \frac{m_R x}{m_P} + \frac{(m_R x - m_P)}{m_P x}.$$

So the intercept of $\dot{u} = 0$ curve is higher than that of $\dot{z} = 0$ curve. When x is reduced, the intercept of $\dot{z} = 0$ curve goes up; but the effect on the intercept term of $\dot{u} = 0$ curve remains ambiguous.

Appendix E

The discounted present value of instantaneous social welfare over the infinite horizon is defined as

$$SW = \int_0^\infty W e^{-\rho t} dt;$$

and it is given by equation (31). Now

$$\frac{\partial SW}{\partial x} = -\frac{(1-b)}{\rho x} - \frac{(1-b)(1+\epsilon_P)}{\rho x(1-x)} + \frac{m_R}{\rho^2}\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\};$$

and

$$\frac{\partial^2 SW}{\partial x^2} = \frac{(1-b)}{\rho x^2(1-x)^2}[(2+\epsilon_P)(1-2x) + x^2].$$

Here, $\frac{\partial^2 SW}{\partial x^2} < 0$ when

$$\frac{x^2}{(2x-1)} < (2+\epsilon_P);$$

and $(2x-1)$ must be positive. We assume these to be satisfied for the optimal value of x.

Now $\frac{\partial SW}{\partial x} = 0$ implies that

$$\frac{(1-b)}{\rho x} + \frac{(1-b)(1+\epsilon_P)}{\rho x(1-x)} = \frac{m_R}{\rho^2}\{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)\};$$

and its total differential is given by

$$(-\frac{\partial^2 SW}{\partial x^2})dx = \{(1+\epsilon_R)b+(1-b)(1+\epsilon_P)\}\frac{dm_R}{\rho^2} + \frac{m_R b}{\rho^2}d\epsilon_R + (1-b)\{\frac{m_R}{\rho^2} - \frac{(1-b)}{\rho x(1-x)}\}d\epsilon_P.$$

Hence

$$\frac{dx}{dm_R} = \frac{(1+\epsilon_R)b + (1-b)(1+\epsilon_P)}{-\rho^2 \frac{\partial^2 SW}{\partial x^2}} > 0;$$

$$\frac{dx}{d\epsilon_R} = \frac{m_R b}{\rho^2 \frac{\partial^2 SW}{\partial x^2}} > 0;$$

and

$$\frac{dx}{d\epsilon_P} = \frac{(1-b)(\frac{m_R}{\rho^2} - \frac{1}{\rho x(1-x)})}{-\frac{\partial^2 SW}{\partial x^2}}.$$

Note that $\frac{dx}{d\epsilon_P} > (<) 0$ for $\frac{m_R}{\rho^2} > (<) \frac{1}{x(1-x)}$.

Chapter 7

Conclusion

In this thesis, we have analysed several theoretical aspects related to the human capital accumulation and endogenous economic growth. In this chapter we summarize the main results of the work, and also mention the limitations of our present study with a discussion on some ideas for further research on this area.

7.1 Summary of the work

In this section, we shall present a summary of the works done in the previous chapters. In chapter 1, we have presented a survey of the existing theoretical literature on the role of human capital accumulation on endogenous economic growth. Since this is a vast literature, we have surveyed only those theoretical works which have been developed adopting the framework of Lucas (1988).

In chapter 2, we have extended the Lucas (1988) model in two directions. In one section, we introduce sector specific external effect of human capital on production in an otherwise Lucas (1988) model of endogenous growth. We show that, whatever be the magnitude of sector specific external effect, the problem of indeterminacy of the transitional growth path does not exist even if the production function satisfies the increasing returns to scale property at the social level. In the other section, we include human capital as an argument in the non separable utility function of the household in an otherwise identical Lucas (1988) model; and show that there may be multiple steady-state equilibria when the discount rate is very high and /or when the productivity parameter in the human capital accumulation function takes a very low value. So such a possibility of multiple steady state growth equilibria appears to be stronger in a less developed economy.

Chapter 3 sheds light on the relationship between human capital accumulation and environmental quality. We modify the Lucas (1988) model assuming that the accumulation of physical capital leads to environmental pollution which, in turn, lowers the learning ability of the individual. The interesting result obtained in this chapter is that the steady state equilibrium rate of growth in this model varies positively with the proportional tax rate imposed on output or on capital income when the tax revenue is spent either as lumpsum payment or as abatement expenditure. However, this rate of growth is independent of the tax rate imposed on labour income. The optimum tax rate is positive when the tax revenue is spent as educational subsidy; and this optimum tax rate varies proportionately with the competitive output share of human capital.

The efficiency enhancement mechanisms for rich individuals and poor individuals are different in less developed countries. While rich individuals can build up their human capital on their own, poor individuals need the support from exogenous sources. In chapters 4, 5 and 6 of the present thesis, we have developed growth models of a dual economy in which human capital accumulation is viewed as the source of economic growth and in which dualism exists in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor. While the human capital accumulation mechanism of the rich individual is similar to that in the Lucas (1988) model, the poor individual has a different mechanism of its human capital accumulation. Rich individuals allocate labour time not only to the production sector and to acquire their own knowledge but also to train the poor individuals. This feature is considered in chapters 4 and 5.

In chapter 4, we analyse the properties of the steady state growth equilibrium as well as of the transitional growth path of a competitive household economy model which focuses on the dualism in the human capital accumulation. We show that a social IRS production technology with aggregate external effects of human capital on production can not explain indeterminacy of the transitional growth path in this model. However, we do not consider accumulation of physical capital in this chapter.

In chapter 5, we generalize the growth model developed in chapter 4 introducing physical capital accumulation; and disaggregating the economy into two sectors producing the same commodity with different production techniques and organizations. We analyse properties of the steady state growth equilibrium of a competitive household economy in this model and compare them to those of a command (planned) economy. Externality parameters of both the sectors appear to be important determinants of the long run rates of growth of different macro economic variables. The steady state equilibrium rate of growth of the human capital in the competitive economy is always less than that in the command economy if there is no external effect of the rich sector's human capital on the human capital accumulation in the poor sector. However, in the presence of that externality, we may obtain an opposite result.

In the models developed in chapters 4 and 5, rich individuals voluntarily allocate their resources (labour) to the training of poor individuals, but the government does not play any role in providing education to them. In chapter 6, we have developed a theoretical model of endogenous growth involving redistributive taxation and educational subsidy to build up human capital of poor individuals. Here the government imposes a proportional tax on the resources (labour endowment) of rich individuals to finance the educational subsidy given to poor individuals. In this chapter, we analyse the effects of exogenous and once for all changes in the tax financed educational subsidy rates on the long run equilibrium as well as on the transitional growth path of the economy. We derive some interesting results from this model. An exogenous and once for all increase in the rate of tax financed educational subsidy raises the labour time allocation to production for both types of individuals and lowers the balanced growth rate of the stock of their human capitals in the new long run equilibrium. However, the short term (transitional dynamic) and long term (steady state) impacts of the adoption of tax financed educational subsidy policy on the labour allocation to the human capital accumulation sector are identical for the rich individuals but not for the poor individuals. Following an exogenous and once for all increase in the tax financed educational subsidy rate, the labour time allocated to production of the poor individual first falls over time from the initial equilibrium level and then starts rising along the transitional path till the new steady state equilibrium point is reached. So the educational subsidy policy can induce the poor individual to acquire more human capital in the short-run. However, its benefit does not exist in the long run. Also there is a conflict between the growth rate maximization and the social welfare maximization. Social welfare maximizing subsidy rate may be positive though the steady state equilibrium growth rate is maximized in the absence of this subsidy.

7.2 Limitations and the scope for further research

The present work is subject to various limitations; and we discuss them in this section. First, we have adopted Lucas (1988) framework. In the Lucas (1988) model, physical capital is used as an input only in the final goods production sector and not in the human capital accumulation sector. Rebelo (1991) has introduced perfect physical capital mobility between the production sector and the human capital accumulation sector; and his analysis has different policy implications. In none of the models developed in different chapters of the thesis, physical capital is required for human capital accumulation; and we should overcome this limitation in our future works.

Secondly, the models developed in this thesis are closed economy models. Extending these models to a two country North-South world with international spillover effect of human capital accumulation would be an interesting addition to the existing literature. Human capital accumulation of poor individuals in the South (less developed country) should receive some externalities from the human capital accumulation in the North (developed country), in the presence of the spillover effect. Farmer and Lahiri (2005) have analysed the role of spillover effect but not the dualism of human capital accumulation.

Thirdly, we have assumed that the government imposes tax on the resources of the rich individual to finance the educational subsidy given to the poor individual. Though some authors like Park and Phillippopoulos (2004), Benhabib et.al (1996)etc. consider taxation on resources, generally taxes are imposed on the levels of income or consumption. Gomez (2003a) and Garcia-Castrillo and Sanso (2000) consider taxation on income to design optimal tax policies in the Lucas (1988) model. Gomez(2000) considers income (both physical capital income and human capital income) tax and consumption tax in Rebelo (1991) model. Considering income tax or consumption tax as the instrument of financing the subsidy in our models may bring different results.

Fourthly, we have neglected the importance of the role of population size and population growth on the quality of education received by the poor individuals. If the size of the population is vast and the majority of them are poor, then the population size may produce a negative external effect on the human capital accumulation; and thus a high rate of population growth may be an obstacle to a high rate of economic growth.

Fifthly, we should consider the role of labour unions on economic growth. Unionization of the workers on the one hand raises the rate of human capital accumulation making the workers disciplined, motivated and interactive with others. However, this lowers the rate of physical capital accumulation raising the wage rate and thus lowering the surplus for investment.

Sixthly, we should study the role of child labour market on the human capital accumulation of the poor individuals. If the children of poor families get jobs in the child labour market, the rate of human capital accumulation of the poor individuals can not be increased simply providing educational subsidies.

Seventhly, we have not analysed the role of public capital accumulation on economic growth in this thesis. Futagami et.al (1993), Dasgupta (1998) etc. have analysed the role of public capital on economic growth in the Ramsey-Solow model; and Faig (1995) and Chen and Lee (2007) have analysed this in the Rebelo (1991) model. In a two sector dual economy model, intersectoral allocation of public investment would be an interesting topic for analysis.

Lastly, a less developed economy also suffers from the unemployment problem in the skilled labour market and in the unskilled labour market; and the rate of unemployment is an important determinant of the demand for education. We should consider this aspect in the future research.

Bibliography

Acemoglu D and J. Angrist, (2000), "How large are human capital externalities? Evidence from compulsory schooling laws", *NBER Macroeconomics Annual*, 15, 9-58.

Aghion, P. and P. Howitt (1992), "A model of growth through creative destruction", *Econometrica*, 60(2), 323-351.

Alonso Carrera, J (2001), "More on the dynamics in the endogenous growth model with human capital", *Investigaciones Econmicas*, 25(3), 561-583.

Alonso-Carrera J (2000), "The subsidy to human capital accumulation in an endogenous growth model: A comparative dynamics analysis", *Journal of Macroeconomics*, 22 (3), 409–431.

Alonso-Carrera, J. and M. Freire-Seren, (2004), "Multiple equilibria, fiscal policy, and human capital accumulation", *Journal of Economic Dynamics and Control*, 28(4), 841-856.

Andreoni, J., W. Gale and J. Scholz, (1996), "Charitable contributions of time and money", Madison: University of Wisconsin-Madison, Department of Economics.

Arnold, L. G. (1997), "Stability of the steady-State equilibrium in the Uzawa-Lucas model: A simple proof", *Zeitschrift fur Wirtschafts- und Sozialwissenschaften*, 117(2), 197-207.

Arnold, L.G. (1998). "Growth, welfare, and trade in an integrated model of human-capital accumulation and research", *Journal of Macroeconomics*, 20 (1), 81-105.

Arnold, L. G., (2000), "Stability of the market equilibrium in Romer's model of endogenous technological change: A complete characterization", *Journal of Macroe-*

conomics, 22(1), 69-84.

Arrow, K. J. (1962), "The economic implications of learning by doing", *Review of Economic Studies*, 29(3), 155-73.

Banerjee A.V. and A.F. Newman, (1993), "Occupational choice and the process of development", *Journal of Political Economy*, 101 , 274-298.

Bardhan, P. (1971), "Optimum trade policy in a dual economy", in Bhagwati, Jones, Mundall and Vanek (eds.), *Trade, Balance of Payments and Growth, Essays in Honor of C.P. Kindleberger*, North Holland Publishing Co.

Barro, R. J, (1990), "Government spending in a simple model of endogenous growth", *Journal of Political Economy*, 98 (5), S103-S125.

Ben-Gad, M.,(2003), "Fiscal policy and indeterminacy in models of endogenous growth", *Journal of Economic Theory*, 108(2), 322-344.

Benhabib, J. and R.E. Farmer (1996), "Indeterminacy and sector-specific externalities", *Journal of Monetary Economics*, 37 (3), 421-443.

Benhabib J, Q. Meng and K. Nishimura (2000) "Indeterminacy Under Constant Returns to Scale in Multisector Economies", *Econometrica*, 68 (6), 1541-1548.

Benhabib, J. and K. Nishimura (1998), "Indeterminacy and sunspots with constant returns", *Journal of Economic Theory*, 81, 58-96.

Benhabib, J. and R. Perli, (1994), "Uniqueness and indeterminacy: On the dynamics of endogenous growth", *Journal of Economic Theory*, 63(1), 113-142.

Ben-Porath, Y. (1967), "The production of human capital and the life cycle of earnings", *Journal of Political Economy*, 75(4), 352-365.

Beyer,H., P. Rojas and R. Vergara, (1999), "Trade liberalization and wage inequality", *Journal of Development Economics*, 59 (1), 103-123.

Blackburn, K., V. Hung, and A. F. Pozzolo, (2000), "Research, Development and Human Capital Accumulation", *Journal of Macroeconomics*, 22(2), 189-206.

Blankenau, W, (2005), "Public schooling, college subsidies and growth", *Journal of Economic Dynamics and Control*, 29 (3), 487-507.

Blankenau, W. F. and N. B. Simpson, (2004), "Public education expenditures and growth", *Journal of Development Economics*, 73 (2), 583-605.

Bond E., W. Wang and C.K. Yip, (1996), "A general two sector model of endogenous growth with human capital and physical capital: balanced growth and transitional dynamics", *Journal of Economic Theory*, 68(1), 149-173.

Bose, S. (1968), "Optimal growth and investment allocation", *Review of Economic Studies*, 35, 465-80.

Boskin, M, (1975), "Notes on the tax treatment of human capital", NBER Working Paper No. 116.

Boucekkine, R. and Ruiz Tamarit, (2008), "Special functions for the study of economic dynamics: The case of the Lucas-Uzawa model", *Journal of Mathematical Economics*, 44, 33-54.

Bovenberg, A. L and B. Jacobs, (2003), "On the optimal distribution of education and income", mimeo, University of Amsterdam/ Tilburg.

Bovenberg, A. L and B. Jacobs, (2005), "Redistribution and education subsidies are siamese twins", *Journal of Public Economics*, 89(11-12), 2005-2035.

Bretschger, L. and S. Smulders (2007), "Sustainable resource use and economic dynamics", *Environmental and Resource Economics*, 36 (1),1-13.

Brett, C and J. A. Weymark, (2003), "Financing education using optimal redistributive taxation", *Journal of Public Economics*, 87 (11), 2549-2569.

Brown, E., and H. Lankford (1992), "Gifts of money and gifts of time: Estimating the effects of tax prices and available time", *Journal of Public Economics*, 47 (3), 321-341.

Bruno S. F. and L. Goette, (1999), "Does Pay Motivate Volunteers?," IEW - Working Papers #7, Institute for Empirical Research in Economics, University of Zurich.

Bucci, A. (2002)."When Romer meets Lucas: On human capital, imperfect competition and growth" Department of Economics, University of Milan, Italy, *Departmental Working Papers with number 2002-06.*

Caballe, J. and M. Santos, (1993), "On endogenous growth with physical and human capital", *Journal of Political Economy*, 101(6), 1042-1067.

Cao, H. (2006), "Time and financial transfers within and beyond the family", *Journal of Family and Economic Issues*, 27(2), 155-184.

Cass, D. (1965), "Optimum growth in an aggregative model of capital accumulation", *Review of Economic Studies*, 32(3), 233-240.

Chakraborty, B. and M.R. Gupta, (2006), " A note on the inclusion of human capital in the Lucas model", *International Journal of Business and Economics*, 5(3), 211-224.

Chamley, C, (1993),“Externalities and dynamics in models of learning or doing”, *International Economic Review*, 34(3), 583-610.

Chang, W., (1999), “Government spending, endogenous labor, and capital accumulation”, *Journal of Economic Dynamics and Control*, 23(8), 1225-1242.

Chen, B. and S. Lee, (2007),“Congestible public goods and local indeterminacy: A two-sector endogenous growth model”, *Journal of Economic Dynamics and Control*, 31 (7), 2486-2518.

Ciccone, A. and G. Peri, (2002), “Identifying human capital externalities: Theory with an application to US cities”; IZA Discussion Paper, 488, Institute for the Study of Labor (IZA).

Clarke, M. and J. Maantay (2006),“Optimizing recycling in all of New York City's neighborhoods: Using GIS to develop the REAP index for improved recycling education, awareness, and participation,Resources”, *Conservation and Recycling*, 46 (2), 128-148.

Dadrawala, N. M., (2001), *A Handbook on Administration of Trusts*, Centre for Advancement of Philanthropy. Mumbai.

Dasgupta, D., (1999), “Growth versus welfare in a model of nonrival infrastructure”, *Journal of Development Economics*, 58(2), 359-385.

Day, K. M. and R. A. Devlin, (1996), “Volunteerism and crowding out: Canadian econometric evidence”, *Canadian Journal of Economics*, 29(1), 37-53.

Day, K. M. and R. A. Devlin, (1998), “The payoff to work without pay: Volunteer work as an investment in human capital", *Canadian Journal of Economics*, 31(5), 1179-1191.

De Hek, P. A, (2006), "On taxation in a two-sector endogenous growth model with endogenous labor Supply", *Journal of Economic Dynamics and Control*, 30 (4), 655-685.

De la Croix, D. and M. Doepke, (2004), "Public versus private education when differential fertility matters", *Journal of Development Economics*, 73, 607-629.

Dinda, S. (2005), "A theoretical basis for environmental Kuznets curve", *Ecological Economics*, 53, 403-413.

Dixit A.K, (1969), "Marketable surplus and dual development", *Journal of Economic Theory*, 1, 203-219.

Dolnicar, S. and M. Randle (2004), "What moves which volunteers to donate their time? An investigation of psychographic heterogeneity among volunteers in Australia", School of Management and Marketing, University of Wollongong, Faculty of Commerce- Papers.

Drandakis E.M.(1963), "Factor substitution in the two-sector growth model", *Review of Economic Studies*, 30 (2), 217-28.

Driskill, R. A. and A. W. Horowitz, (2002), "Investment in hierarchical human capital", *Review of Development Economics*, 6(1), 48-58.

Duczynski (2005), "Models of growth with capital contributing to utility", *Bulletin of the Czech Econometric Society*, 12, 22, 61-86.

Eaton J. and Z. Eckstein, (1997), "Cities and growth: Theory and evidence from

France and Japan", *Regional Science and Urban Economics*, 27, 443-474.

Eckstein, Z. and I. Zilcha, (1994), "The effects of compulsory schooling on growth, income distribution and welfare", *Journal of Public Economics*, 54 (3), 339-359.

Eicher, T.S. (1996). "Interaction Between Endogenous Human Capital and Technological Change," *Review of Economic Studies*, 63 (1), 127-144.

Eicher, T.S. (1999), "Training, adverse selection and appropriate technology: Development and growth in a small open economy", *Journal of Economic Dynamics and Control*, 23 (5-6), 727-746.

Endress, L.H., J. A. Roumasset , T. Zhou (2005), "Sustainable growth with environmental spillovers", *Journal of Economic Behavior & Organization*, 58 (4), 527-547.

Faig, M. (1995), "A simple economy with human capital: Transitional dynamics, technolgy shocks, and fiscal policies", *Journal of Macroeconomics*, 17(3), 421-446.

Farmer R. and A. Lahiri, (2005), "A two-country model of endogenous growth", *Review of Economic Dynamics*, 8(1), 68-88.

Ferretti, M. and N. Roubini, (1998), "On the taxation of human and physical capital in models of endogenous growth", *Journal of Public Economics*, 70, 237-254.

Fernandez, E., A. Novales and J. Ruiz (2004), "Indeterminacy under non-separability of public consumption and leisure in the utility function", *Economic Modelling*, 21(3), 409-428.

Foster, A.D. and M.R. Rosenzweig, (2004), "Technological change and the distribution of schooling: evidence from green-revolution India", *Journal of Development Economics*, 74, 87-111.

Freeman, R. (1997), "Working for Nothing: The Supply of Volunteer Labor", *Journal of Labor Economics*, 15(1), S140-66.

Funke, M. and H. Strulik (2000), " On Endogenous Growth with Physical Capital, Human Capital and Product Variety", *European Economic Review*, 44(3), 491-515.

Futagami, K., Y. Morita, and A. Shibata, (1993), "Dynamic analysis of an endogenous growth model with public capital", *Scandinavian Journal of Economics*, 95(4), 607-25.

Galor, O. and J. Zeira, (1993), "Income distribution and macroeconomics", *Review of Economic Studies*, 60, 35-52.

Gantmacher, F.R, (1959), *Applications of the theory of matrices*, London: Interscience Publication.

Garcia-Belenguer, F., (2007), "Stability, global dynamics and Markov equilibrium in models of economic growth", *Journal of Economic Theory*, 136, 392-416.

Garcia-Castrillo, P and M. Sanso, (2000), "Human capital and optimal policy in a Lucas-type model", *Review of Economic Dynamics*, 3(4), 757-770.

Glaeser, E.L. and D.C.Mare, (1994), "Cities and skills", NBER Working Paper No. 4728.

Glaeser, E.L., (1997), "Learning in cities", NBER Working Paper No.6271.

Glomm, G. and B. Ravikumar, (1995), "Endogenous public policy and multiple equilibria", *European Journal of Political Economy*, 11(4), 653-662.

Gollin, D., S. Parente and R. Rogerson. (2002), "The role of agriculture in development", *American Economic Review Papers and Proceedings*, 92(2), 160-64.

Gomez, M.A (2000), "Welfare-maximizing tax structure in a model with human capital", *Economics Letters*, 68(1), 95-99.

Gomez, M. A., (2003), "Optimal fiscal policy in the Uzawa-Lucas model with externalities", *Economic Theory*, 22(4), 917-925.

Gomez M. A. (2004), "Optimality of the competitive equilibrium in the Uzawa-Lucas model with sector-specific externalities", *Economic Theory*, 23(4), 941-948.

Gomez M. A. (2005a), "Transitional dynamics in an endogenous growth model with physical capital, human capital and R & D", *Studies in nonlinear Dynamics and Econometrics*, 9(1), 1-20.

Gomez, M. A., (2005b), "Externalities and fiscal policy in a Lucas-type model", *Economics Letters*, 88(2), 251-259.

Gomez, M. A., (2007), "Optimal tax structure in a two-sector model of endogenous growth", *Journal of Macroeconomics*, 29(2), 305-325.

Gong, G., A. Greiner and W. Semmler, (2004), "The Uzawa-Lucas model without scale effects: Theory and empirical evidence", *Structural Change and Economic Dynamics*, 15(4), 401-420.

Goulder, L.H. and K. Mathai, (2000), "Optimal CO2 abatement in the presence of induced 2 technological change", *Journal of Environmental Economics and Management*, 39 (1), 1-38.

Gradstein, M. and M. Justman,(2000), “Human Capital, Social Capital, and Public Schooling”, *European Economic Review*, 44(4-6), 879-890.

Gradus R. and S. Smulders,(1993), “The tradeoff between environmental care and long term growth - pollution in three prototype growth models”, *Journal of Economics*, 58(1), 25-51.

Greiner A (2006), “Human capital formation, public debt and economic growth”, *Journal of Macroeconomics*, 30 (1), 415-427.

Greiner, A. and W. Semmler, (2002), “Externalities of investment, education and economic growth,” *Economic Modelling*, 19(5), 709-724.

Grimaud, A. (1999), “Pollution permits and sustainable growth in a Schumpeterian Model”, *Journal of Environmental Economics and Management*, 38 (3), 249-266.

Grimaud, A. and F. Tournemaine, (2007), “Why can an environmental policy tax promote growth through the channel of education?”, *Ecological Economics*, 62(1), 27 –36.

Grossman, G. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, Cambridge: MIT Press.

Gupta, M.R. and B. Chakraborty (2007), “Uniqueness and indeterminacy of the equilibrium growth path in the Uzawa-Lucas model with sector specific externalities” *The Japanese Economic Review, 58(3)*, 400-406.

Gupta, M.R. and B. Chakraborty (2006), “Human capital accumulation, economic growth and transitional dynamics in a dual economy”, *Keio Economic Studies*, 43(1), 43-59.

Gupta, M.R. and B. Chakraborty (2006), “Human capital accumulation and endogenous growth in a dual economy”, *Hitotsubashi Journal of Economics*, 47(2), 169-195.

Hackl, F., M. Halla, and G. J. Pruckner (2007), “Volunteering and income - the fallacy of the good samaritan?”, *KYKLOS*, 60 (1), 77-104.

Hahn, F.H. (1965), “On two-sector growth models”, *Review of Economic Studies*, 32(4), 339-46.

Hart, R., (2004), “Growth, environment and innovation: a model with production vintages and environmentally oriented research”, *Journal of Environmental Economics and Management*, 48 (3),1078 - 1098.

Heckman, J, (1976), “A life-cycle model of earnings, learning, and consumption”, *Journal of Political Economy*, 84(4), S11-S44.

Hettich, F. (1998), “Growth effects of a revenue-neutral environmental tax reform” *Journal of Economics*, 67(3), 287-316.

Hollanders, H. and B. Weel, (2003), “Skill-biased technical change: On endogenous growth, wage inequality and government intervention”, in: H.Hagemann and S.Seiter (eds.), Growth Theory and Growth Policy, Routledge: London, 156-171.

Hornby, J. M.(1968), “Investment and trade policy in the dual economy”, *The Economic Journal*, 78 (309), 96-107.

Horii, R. and M. Ikefuji, (2007), “Wealth heterogenity and escape from the poverty-environment trap”, *Journal of Public Economic Theory*, 9(6), 1041-1068.

Horii, R., A. Kitagawa and K.Futagami, (2007),“Availability of higher education and long term economic growth”, *Japanese Economic Review*, forthcoming.

Inada, K. (1963), “On a two-sector model of economic growth: comments and a generalization”, *Review of Economic Studies*, 30(2),119-127.

Independent Sector, (2001), “Giving and volunteering in the United States”, Retrieved, July 22, 2007. *From http://www.independentsector.org/PDFs/GV01keyfind.pdf.*

Indian Centre for Philanthropy (ICP), (2001), *Giving and Fundraising in India,* Sampradaan Indian Centre for Philanthropy. New Delhi.

Itaya, J. (1991), “Tax incidence in a two sector growing economy with perfect foresight: Long run analysis”, *Journal of Public Economics*, 44(1), 95-118.

Itaya, J. (2007), “Can environmental taxation stimulate growth? The role of indeterminacy in endogenous growth models with environmental externalities”, *Journal of Economic Dynamics and Control*, forthcoming.

Jones, C. I., (1995), “R & D-based models of economic growth”, *Journal of Political Economy*, 103(4), 759-84.

Jorgenson D. W, (1961), “The development of a dual economy”, *The Economic Journal*, 71, 309-334.

Kaldor, N.(1956), “Alternative theories of distribution”, *The Review of Economic Studies*, 23(2), 83-100.

Kauppi, L (2006), “Mercury-background for abating mercury pollution”, Presentation for the International Mercury Conference, Brussels, 26-27 Oct 2006.

Kitchen, H. and R. Dalton, (1990), "Determinants of charitable donations by families in Canada: A regional analysis", *Applied Economics*, 22 (3), 285-299.

Kongsamut, P., S. Rebelo, and D. Xie, (2001), "Beyond balanced growth", *Review of Economic Studies*, 68(4), 869-82.

Koopmans, T. C. (1965), "On the concept of optimal economic growth", *The Econometric Approach to Development Planning*, ch 4, 225-87. North-Holland Publishing Co., Amsterdam.

Kurz, M. (1968), "Optimal economic growth and wealth effects", *International Economic Review*, 9 (3), 348-357.

Lachler, U., (2001), "Education and earnings inequality in Mexico", World Bank Working Paper.

Ladron-de-Guevara, A., S. Ortigueira and M. Santos. (1997), "Equilibrium dynamics in two-sector models of endogenous growth", *Journal of Economic Dynamics and Control*, 21(1), 115-143.

Ladron-de-Guevara, A., S. Ortigueira and M. Santos. (1999), "A two-sector model of endogenous growth with leisure", *Review of Economic Studies*, 66(3), 609-31.

Lahiri, A. (2001), "Growth and equilibrium indeterminacy: the role of capital mobility", *Economic Theory*, 17(1), 197-208.

Lewis, W.A, (1954), "Economic development with unlimited supplies of labour", *Manchester School of Economic and Social Studies*, 22 (2), 139-191.

Liviatan, N. and P. A. Samuelson, (1969), "Notes on turnpikes: Stable and unstable"; *Journal of Economic Theory*, 1 (4), 454-475.

Lucas, R. E (1988), "On the mechanics of economic development", *Journal of Monetary Economics*, 22 (1), 3-42.

Lucas, R. E, (1990), "Supply-side economics: An analytical review", *Oxford Economic Papers*, 42(2), 293-316.

Lucas, R. E, (2004), "Life earnings and rural-urban migration", *Journal of Political Economy*, 112(S1), S29-S59.

Margulis, S. (1992), "Back- of- the- envelope estimates of environmental damage costs in Mexico", World Bank Policy Research Working Paper # 824.

Marjit, S and R. Acharyya, (2003), *International Trade, Wage inequality and the Developing Economy*, Physica Verlag, New York.

Matsuyama, K. (1992), "Agricultural productivity, comparative advantage and economic growth", *Journal of Economic Theory*, 58 (2), 317-34.

Mattana, P, (2003), "A Global Characterization of the Transitional Dynamics of the Two-Sector Capital Accumulation Growth Model Without Externality", Universita di Cagliari - Department of Economics, Working Paper No. 23.

Mauro L. and G. Carmeci, (2003), "Long run growth and investment in education: Does unemployment matter?", *Journal of Macroeconomics*, 25 (1), 123-137.

Menchik, P. L. and B. A. Weisbrod, (1987), "Volunteer labor supply", *Journal of Public Economics*, 32, 159-183.

Meng, Q (2003), "Multiple transitional growth paths in endogenously growing open economies", *Journal of Economic Theory*, 108 (2), 365-376.

Mino, K (1989), "Income taxation and endogenous growth", *Economics Letters*, 31 (4), 325-329.

Mino, K, (1996), "Analysis of a Two-Sector Model of Endogenous Growth with Capital Income Taxation", *International Economic Review*, 37(1), 227-51.

Mino, K., (1999), "Non-separable utility function and indeterminacy of equilibrium in a model with human capital", *Economics Letters*, 62 (3), 311-317.

Mino, K, (2001a). "Optimal taxation in dynamic economies with increasing returns", *Japan and the World Economy*, 13(3), 235-253.

Mino, K (2001b), "Indeterminacy and endogenous growth with social constant returns", *Journal of Economic Theory*, 97(1), 203-222.

Mohtadi,H.(1996), "Environment, growth and optimal policy design", *Journal of Public Economics*, 63 (1), 119-140.

Moretti, E, (2003), "Human capital externalities in cities", NBER Working Paper No.9641.

Mulligan C.B. and X. Sala-i-Martin, (1993), "Transitional dynamics in two sector models of endogenous growth", *Quarterly Journal of Economics*, 108(3), 739-773.

Nakano, S (2005), "Who volunteers in japan and how?-Basic findings from micro data", *Keio Economic Observatory Discussion Paper* No.96.

Nishimura, K. and A. Venditti (2002), "Intersectoral externalities and indeterminacy", *Journal of Economic Theory*, 105 (1), 140-157.

O' Connell, J. (1998), "Savings in the Uzawa Lucas model of economic growth", *Journal of Macroeconomics*, 20(2), 413-422.

Ortigueira S, (1998), "Fiscal policy in an endogenous growth model with human capital accumulation - Transitional dynamics with multiple equilibria", *Journal of Monetary Economics*, 42(2), 323-355.

Ortigueira, S. and M. S. Santos (1997), "On the speed of convergence in endogenous growth models", *American Economic Review*, 87(3), 383-99.

Ortigueira, S. and M. S. Santos (2002), "Equilibrium dynamics in a two-sector model with taxes", *Journal of Economic Theory*, 105(1), 99-119.

Ortigueira, S., (2006), "Skills, search and the persistence of high unemployment", *Journal of Monetary Economics*, 53(8), 2165-2178.

Paquin, L. (1999), "Heterogeneous agents, distribution and growth", *Universit Laval - Departement d'economique*, Cahiers de recherche, No. 9913.

Park, H and A. Philippopoulos, (2003), "On the dynamics of growth and fiscal policy with redistributive transfers", *Journal of Public Economics*, 87 (3-4), 515-538.

Park, H and A. Philippopoulos, (2004), "Indeterminacy and fiscal policies in a growing economy", *Journal of Economic Dynamics and Control*, 28 (4), 645-660.

Park, T and Park, S. (2004)," An economic study on charitable giving of individuals in Korea: Some new findings from 2002 survey data", *Department of Economics, Yonsei University*, Seoul, Korea.

Pasinetti, L. (1962),"The rate of profit and income distribution in relation to the rate of economic growth", *Review of Economic Studies*, 29(4), 267-279.

Pecorino, P. (1992), "Rent seeking and growth: The case of growth through human capital accumulation", *Canadian Journal of Economics*,25(4), 944-56.

Pecorino, P., (1995)," Inflation, human capital accumulation and long-run growth", *Journal of Macroeconomics*, 17(3), 533-542.

Pecorino, P., (1995), "Tax rates and tax revenues in a model of growth through human capital Accumulation", *Journal of Monetary Economics*, 36(3), 527-539.

Perez, R. and J. Ruiz (2007), "Global and local indeterminacy and optimal environmental public policies in an economy with public abatement activities", *Economic Modelling*, 24 (3), 431-452.

Peri, G, 2002, "Young workers, learning and agglomeration", *Journal of Urban Economics*, 52 (3), 582-607.

Petrosillo, I., G. Zurlini, M. Corliano, N. Zaccarelli and M. Dadamo (2007), "Tourist perception of recreational environment and management in a marine protected area", *Landscape and Urban Planning*, 79(1), 29-37.

Ramsey F.P. (1928), "A mathematical theory of saving", *Economic Journal*, 38(152), 543-559.

Ranis, G and C.H. Fei, (1961), "A theory of economic development", *American Economic Review*, 51, 533-556.

Rebelo, S, (1991), "Long-run policy analysis and long run growth", *Journal of Political Economy*, 99(3), 500-521.

Redding, S.(1996), "Low-skill, low-quality trap: strategic Complementarities between human capital and R&D", *Economic Journal*, 106 (435), 458-70.

Ricci, F, (2007), "Channels of transmission of environmental policy to economic growth: A survey of the theory", *Ecological Economics*, 60 (4), 688 - 699.

Robbins, D., (1994a), "Malaysian wage structure and its causes", Harvard Institute for International Development.

Robbins, D., (1994 b), "Philippine wage structure and its causes", Harvard Institute for International Development.

Robertson, P.E., (2002), "Demographic shocks and human capital accumulation in the Uzawa-Lucas Model", *Economics Letter*, 74(2), 151-156.

Romer, P. M, (1986), "Increasing Returns and Long run Growth", *Journal of Political Economy*, 94 (5), 1002-37.

Romer, P. M, (1990), "Endogenous technological change", *Journal of Political Economy*, 98(5), S71-S102.

Rosendahl, K.E (1996), "Does improved environmental policy enhance economic growth?", *Environmental and Resource Economics*, 9(3), 341-364.

Rudd, J. B., (2000), "Empirical evidence on human capital spillovers", FEDS Discussion Paper No. 2000-46, Board of Governors of the Federal Reserve - Macroeconomic Analysis Section.

Ruiz-Tamarit, J(2008), "The closed-form solution for a family of four-dimension nonlinear MHDS," Journal of Economic Dynamics and Control, 32(3), 1000-1014.

Saint-Paul, G. and T. Verdier, (1993), "Education, democracy and growth", *Journal of Development Economics*, 42 (2), 399-407.

Schou, P (2000), "Polluting non-renewable resources and growth", *Environmental and Resource Economics*. 16(2), 211-227.

Schou, P (2002),"Pollution externalities in a model of endogenous fertility and growth", *International Tax and Public Finance*, 9(6), 709-725.

Schultz, T.W, (1961), "Investment in human capital", *The American Economic Review*, 1(2), 1-17.

Selod, H and Y. Zenou, (2003), "Private versus public schools in post-Apartheid South African cities: Theory and policy implications", *Journal of Development Economics*, 71 (2), 351-394.

Sen, A. K. (1966), "Peasants and dualism with or without surplus labor", *The Journal of Political Economy*, 74(5), 425-450.

Solow, R.M, (1956), "A contribution to the theory of economic growth", *Quarterly Journal of Economics*, 70(1), 65-94.

Solow, R.M, (2000), *Growth theory: An exposition*, MIT Press.

Song,Y (2000), "The return to capital and convergence in a two sector model of endogenous growth", *International Economic Journal*, 14(4), 143-163.

Sorensen, A. (1999), "R&D, learning, and phases of economic growth", *Journal of Economic Growth*, 4(4), 429-45.

Steinberg, R. S., (1987), "Voluntary donations and public expenditures in a federal system", *American Economic Review*, 77(1), 24-36.

Swan, T.W., (1956), "Economic growth and capital accumulation," Economic Record, 32, 334-61.

Takayama A.(1963), "On a two-sector model of economic growth: A comparative statics analysis", *Review of Economic Studies*, 30(2), 95-104.

Takayama, A.(1965), "On a two-sector model of economic growth with technological progress", *Review of Economic Studies*, 32(3), 251-62.

Tamura, R, (1991), "Income convergence in an endogenous growth model", *Journal of Political Economy*, 99 (3), 522-540.

Tobin, J. (1965), "Money and economic growth", *Econometrica*, 33(4), 671–684.

Torras, M and J, Boyce,(1998), "Income, inequality, and pollution: a reassessment of the environmental Kuznets Curve", *Ecological economics*, 25, 147-160.

Todaro,M, (1969), "A model of labour migration and urban unemployment in less developed countries", *American Economic Review*, 59 (1), 138-148.

Trostel, P. A. (1993), "The effect of taxation on human capital", *Journal of Political Economy*, 101(2), 327-50.

Tsoukis, C. and N. J. Miller (2003), "Public services and endogenous growth", *Journal of Policy Modeling*, 25(3), 297-307.

Uhlig, H and N. Yanagawa, (1996)." Increasing the capital income tax may lead to faster growth", *European Economic Review*, 40(8), 1521-1540.

Uzawa, H. (1961), "On a two-sector model of economic growth", *Review of Economic Studies*, 29(1), 117-124.

Uzawa, H. (1963), "On a two-sector model of economic growth: II", *Review of Economic Studies*, 30(2), 105-118.

Uzawa, H (1965) "Optimum technical change in an aggregative model of economic growth", *International Economic Review*, 6(1), 18-31.

Weder, M. (2001), "Indeterminacy in a small open economy Ramsey growth model", *Journal of Economic Theory* , 98 (2), 339-356.

Woo, J (2005), "Social polarization, fiscal instability and growth", *European Economic Review*, 49 (6), 1451-1477.

Wood, A, (1997), "Openness and wage inequality in developing countries: The Latin American challenge to east asian conventional wisdom", *World Bank Economic Review*, 11(1), 33-57.

Xie, D (1991),"Increasing returns and increasing rates of growth", *Journal of Political Economy*, 99(2), 429-435.

Xie, D (1994) "Divergence in economic performance: Transitional dynamics with multiple Equilibria", *Journal of Economic Theory*, 63(1), 97-112.

Zarembka, P., (1970), "Marketable surplus and growth in the dual economy", *Journal of Economic Theory*, 2(2), 107-121.

Zhang, J., (2003), "Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests", *Journal of Public Economics*, 87(7-8), 1825-1835.