INNOVATION, IMITATION AND NORTH SOUTH TRADE: ECONOMIC THEORY AND POLICY

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Dedicated to my parents

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Chapter 1

Introduction and Literature Survey

1.1 Intellectual property rights (IPR) protection

Intellectual Property Rights (IPR) gives legal protection to innovators against the imitators by preventing others from using an intellectual creation or by setting the terms on which it can be used. Patent, trademark and copyright are three main areas over which intellectual property rights are assigned. The basic economic argument, used to favour IPR protection, is that the competitive market system would fail to provide private agents sufficient incentives to undertake investment in developing new ideas and informations without such protection because these outcomes have "public good" attributes. Since imitation involves lower cost than innovation, imitating firms gain an advantage over innovating firms unless the IPR can prevent imitation activities. This imitation problem will discourage investment in research and development (R&D). On the otherhand, there is an argument against IPR protection which states that the public good character of the innovation activity calls for greater output when the benefits can be spread across larger number of consumers. Choice of IPR protection policy should then maintain a balance between these two.

IPRs are territorial rights which are conferred by a national government and are valid only within its relevant jurisdiction. National IPR systems are largely designed to take care of the best interest of the country concerned; and this may not be consistent with the best interests of all other countries. Developed countries with many potential innovators have strong IPR protection systems; but many developing countries with a few potential innovators have not strengthened their IPR protection systems. Recently, there has been a general awareness and opinion in favour of strengthening and broadening of IPRs in developed countries. The globalisation of the international economy has also produced an impact on the tightening of IPRs. First, the growing importance of international markets for patented goods has led the innovators of developed countries to demand for similar levels of IPR protection in foreign countries. Secondly, it has been realised that cross-country differences in the designs and enforcements of IPRs would lead to non-tariff barriers to trade and thus would weaken the success of the international trade liberalisation programme.

There have been many international agreements on IPRs since the middle of the nineteenth century. Main instruments of the international law, used for the protection of IPRs until recent years were designed in the Paris convention for the protection of industrial property held in 1883 and in the Berne convention for the protection of literary and artistic works held in 1886. In the last quarter of the twentieth century, U.S.A. expressed concerns over international protection of IPRs; and this led to its inclusion on the agenda of the Uruguay round launched in September 1986. The agreement on 'Trade-Related Aspects of Intellectual Property Rights' (TRIPs) of 1994 provides minimum standards on IPRs to be followed by member countries of the 'World Trade Organisation' (WTO).

1.1.1 Growth effects of IPR protection: Empirical literature

It is generally believed that the strengthening of IPR protection in less developed countries would encourage innovation and technology transfer there and hence foster economic growth. However, the relationship between the strengthening of IPR protection and economic growth is not as clear as widely believed. The empirical literature shows a weak but positive relationship between stronger IPR protection and economic growth. Gould and Gruben (1996) use cross country data on patent protection, trade regime and other country specific characteristics to analyse the determinants of growth rate. Their findings suggest that the IPR protection is a significant determinant of economic growth; and its strength varies positively with the degree of openness of the economy.

Falvey, Foster and Greenaway (2006), who use a panel data for 79 countries, find a positive and generally significant relationship between the extent of IPR protection in a country and its growth rate. Their findings imply that the relationship between IPR protection and economic growth of a country depends upon her level of development, as proxied by initial per capita GDP . They find that a stronger IPR protection significantly improves the growth rate for low and high income countries; but do not find any such relationship for middle income countries. High income countries undertake the majority of innovation activities and stronger IPR protection should encourage further innovation by ensuring higher profit to innovator firms. Strong IPR protection encourages imports and inward foreign direct investment (FDI) in low income countries; and these encourage growth without adversely affecting domestic imitative activities. In middle income countries, the lack of a relationship between the IPR protection and economic growth is likely to reflect two opposing forces. The positive impact of IPR protection on economic growth that works indirectly through trade and FDI is offset by a negative impact coming from the slowing down of knowledge diffusion and reduction in imitation activities. However, these authors do not find any evidence of a negative relationship between IPR protection and economic growth in any of the middle income countries.

Chen and Puttitanun (2006), who use a panel of data for 64 developing countries, find a positive impact of strengthening IPR protection on innovation activities in developing countries and the presence of a U-shaped relationship between the degree of tightening of IPR protection and the level of economic development. Another study made by Patricia (2005) examines the role of high-technology trade, IPR and FDI on the determination of a country's rate of innovation and economic growth. Their empirical analysis is conducted using a unique panel data set of 47 developed and developing countries and the period covers from 1970 to 1990. The findings suggest that (i) high-technology imports are relevant in explaining domestic innovation both in developed and developing countries; (ii) foreign technology has a stronger impact on per capita GDP growth than domestic technology; (iii) stronger IPR protection affects the innovation rate but this impact is more significant for developed countries; (iv) the results regarding the impacts of FDI are inconclusive.

Even though the empirical literature suggests a positive link between IPR protection and economic growth, policy makers and politicians of many developing countries believe that TRIPs agreements are forced upon them by their economically more powerful trading partners. So they are often reluctant to strengthen their IPR protection and so, this issue remains highly contentious in international economic relations between the North and the South. Theoretical models that tries to shed light on this issue generally rely on the North-South framework with an innovating North and an imitating South. This type of framework is important because the design of a system of IPR protection poses a clear trade-off to a welfare-maximizing government. On the one hand, stronger IPR protection provides increased incentives to undertake risky innovative activities; and, on the other hand, this raises the number of monopoly sectors in the economy which limits the aggregate output. Also, in the present day world where the different economies are highly integrated through trade, any policy adoption in one country must affect its trading partners.

1.2 Growth and IPR: Single country models

A few theoretical works analyse how IPR protection affects economic growth in the single country closed economy model. Kwan and Lai (2003) incorporate the exogenous imitation rate into a variety expansion model similar to Romer (1990); and show that it is optimal to protect IPR when the objective of the government is to maximize social welfare. Iwaisako and Futagami (2003) show that a policy of extending patent length enhances the rate of economic growth in the variety expansion model of Romer (1990). However, some other works show that the strengthening of IPR protection does not necessarily enhance economic growth. Horii and Iwaisako (2007) find a positive but very weak empirical relationship between IPR protection and economic growth. In order to explain this fact, Horii and Iwaisako (2007) construct a quality ladder model and show that the strengthening of IPR protection can depress the incentive to innovate. Furukawa (2007) shows that there is an inverse U shaped relationship between the long-run rate of innovation and the rate of imitation. So, either a very strong or a very weak IPR protection policy deters the incentive to innovate; and the long-term rate of economic growth is maximized with an intermediate degree of strengthening IPR protection. Koleda (2005) also shows that the effect of patent novelty requirements on economic growth may be inverse U-shaped; and this implies that a policy of tightening the IPR protection dampens economic growth for a range of stronger novelty requirements.

1.3 IPR protection in static two country partial equilibrium models

Several researchers have analysed the issue of IPR protection using a two country partial equilibrium framework. Chin and Grossman (1990) and Deardorff (1992) analyse the static welfare effects of extending patent protection from an innovator country (North) to another country (South) consuming the innovative products. Both these works treat the investment in R&D as a once-off decision and show that the North always suffers if the

South fails to protect its IPR while the South gains from imitation. Moreover, Deardorff (1992) shows that the level of global welfare varies inversely with the extension of the patent protection to the larger part of the world. Taylor (1993) presents a North-South model of trade and technology transfer where the Southern firm invests resources in imitative activities and the Northern firm invests resources in 'masquing' the production technology. Taylor (1993) shows that a lax of IPR protection made by the South leads to a greater product masquing made by the Northern firm. So, the potential gains from technology transfer with weak IPR protection in the South may be offset by the increase in Northern masking in production. Zigic (1998) uses a North South framework where the representative firms of the two countries are duopolists in the international market. He analyses the distribution of gains of IPR protection between the countries and the optimal level of IPR protection at the world level. The conventional wisdom that the South generally benefits from relaxing its IPR protection and the North suffers, is not supported by the results of Zigic (1998). In Zigic (2000), the North uses tariff as a strategic instrument to reduce the IPR violations made by the South, and, this induces the domestic firm to invest in the socially beneficial R&D activities.

1.4 The North-South model and its importance

North-South models are essentially static or dynamic general equilibrium models of the simplified world economy consisting of two countries who are linked through trade, factor mobility and technology transfer. These two countries are called the North and the South. However, there is a major point of difference between the traditional two country models and the North-South models. In the conventional two country models, the two countries are assumed to be symmetric in nature. However, they are not identical and they differ only in terms of the quantitative magnitudes of some parameters related to technology, tastes or factor endowments. There is no difference in the motivation of the economic agents or in the market structure between the two countries. North-South models focus on the fundamental asymmetries in the structure and performance between a developed economy and a less developed economy. So the South is institutionally and structurally dual to the North and the nature of this dualism varies from models to models. The representative developed country (region) is called the North because the majority of the economically advanced countries lies in the Northern hemisphere of the globe. Similarly, the representative underdeveloped country (region) is called the South because the Southern part of the globe is largely underdeveloped.

A static North-South model can be used to determine the international terms of trade when the North and the South are involved in the trade of commodities; and then it is used to analyse the effects of various trade policies on this terms of trade. If the North and the South are linked by factor mobility, then one can analyse the effects of capital mobility from the developed to the less developed countries or the effect of international migration and brain drain taking place from the underdeveloped to the developed countries. The dynamic North-South model analyses the simultaneous determination of long-run growth rates of different economies with special reference to the role of trade, factor mobility and technology transfer on the growth problem of less developed economies. These dynamic models also attempt to explain the growing imbalance in the levels of development of different countries.

Firms of developed countries, who spend a substantial amount to the R&D activities to innovate new products or to improve the quality of the existing products, often face the problem of imitation activities done in less developed countries. The low labour cost in less developed countries encourages the enterprises in the developed countries to make direct investment or to outsource parts of their production activities in less developed countries. Many recently developed North-South models are used to study the implications of the problems of international imitations and international outsourcing and to analyse the effectiveness of relevant policies like 'Intellectual Property Rights Protection'.

1.5 Old North-South models of trade and growth

The literature on the dynamic general equilibrium models of the world economy, that uses the North-South two country framework, starts with the works of Findlay (1980) who considers a two country free trade world with a Solow (1956) type North producing a manufactured good, and with a Lewis (1954) type South producing a primary product. However, there is no factor mobility or technology transfer between these two countries in the Findlay (1980) model. In spite of the structural asymmetry existing between them, trade acts as an engine of growth to the South; and the world economy grows in a balanced manner in the long-run equilibrium. Benefits of technological progress taking place in the North is not only restricted to the North but also spread over the South through this competitive free trade.

The Findlay (1980) model has been extended and reanalysed by various authors in various directions. Darity (1990) shows that growth rates are not equalised when profit rates in the two countries become uniform in the Findlay (1980) model. There is no adjustment mechanism in that model to allow for simultaneous equalisation of growth rates and profit rates of these two countries. Thus Darity (1990) explains uneven growth of the world economy using a modified Findlay(1980) model. Burgstaller and Saavedra-Rivano (1984) extend the Findlay (1980) model with perfect mobility of capital between the North and the South; and show that this capital mobility reduces employment and relative real income in the South in the long-run equilibrium. Wooton (1982) introduces South-North labour mobility into the North-South model in the form of a guest worker immigration quota set at some fraction of the Northern labour force; and then analyses the comparative steady-state effects of the change in the immigration quota. Kiguel and Wooton(1982) study the incidence of tariff by each region, both in the short-run and in the long-run. Wang (1990) assumes that North South technology transfer takes place through international capital mobility and shows that the shifting from autarky to perfect capital mobility raises the long-run growth rate of the South and lowers the North-South income gap. Burgstaller (1985) develops a neo-Ricardian North-South model where capital stocks are made of working capital (wage fund); and shows that capital accumulation and technical progress may produce negative effects on the terms of trade and employment level in the South in a mobile capital free trade world.

There are many other old North-South models of international trade. Bacha (1978) formalises the idea of unequal exchange of Emmanuel using a neo-Ricardian North-South model with exogenous wages in both the regions and profit rates equated by capital mobility. Chichilnisky (1981) attempts to show that a shift in the composition of the North's demand in favour of the South's exports can worsen the terms of trade of the South. Dixit (1982) introduces asymmetry in the market structure for exportables assuming that the North produces differentiated goods under monopoloistic competition and the South exports an intermediate goods produced under perfect competition. A few structuralist models developed by Taylor (1983), Dutt (1988a, 1988b) etc. assume that the North has a Kelecki-Keynes structure with mark up pricing and imperfect competition in the market structure and with excess capacity in production. They analyse the terms of trade problem of the South and the problem of uneven development between the North and the South. Kaldor (1978), Lewis (1980) and Krugman (1981) also develop North-South models and analyse the problem of uneven development. Krugman (1981) explains uneven development introducing increasing returns to scale production technology in the manufacturing sector which can aggravate the problem of an imbalance in the initial levels of development between the two countries. Lewis (1980) explains this uneven development problem assuming an unidirectional dependency of the South on the North because the earnings made from the Southern exports of primary products is the only source of financing the Southern economic development.

These old North-South models are important in an environment where the rate of growth is exogenous to the system and the trade is inter-industry in nature. However, in reality, a large volume of North-South trade is intra-industry in nature; and the rate of technological progress, the most important determinant of the long run rate of growth, is endogenous being determined by the size of the R&D expenditure which, in turn, is influenced by various government policies.

1.6 New trade and old growth models

1.6.1 Horizontal product innovation

Krugman (1979) introduces the first innovation-imitation North-South model in the literature. In his model, the North has the ability to innovate new differentiated products and the South has the ability to imitate them. The North is a high wage economy and the South is a low wage economy. So, once a product is imitated in the South, the North loses the market for that product. Both the rate of innovation in the North and the rate of imitation in the South are assumed to be exogenous to the system. Krugman's (1979) model features product cycles in trade in the sense that the North exports new products to the South in the initial stage and imports those goods from the South in the later stage when they become old. All these goods are produced under identical technology and with same factor, labour. So the difference in production technologies can not determine the pattern of trade. Trade is intra-industry in nature; and there is a continuous process of North-South technology transfer through imitation activities done in the South. The Krugman (1979) model analyses the role of North-South trade on the world distribution of income. In his model, a technological improvement in the

North (South) raises the Northern (Southern) terms-of-trade; and an expansion of the Northern (Southern) labour endowment lowers the Northern (Southern) relative wage.

Dollar (1986, 1987) extends the Krugman (1979) model to incorporate for two factors of production, capital and labour. Dollar (1986, 1987) not only endogenises the rate of technology transfer from the North to the South but also introduces international capital mobility. In Dollar (1986), trade between the North and the South follows the Heckscher-Ohlin pattern in the long run. The North specialises in the production of capital intensive 'new' goods and exports them to the South; and imports labour intensive 'old' goods from the South who specializes in their production. An increase in the labour endowment in any country lowers its relative wage in the short run but raises it in the long-run. This result is in contrast to the Krugman's (1979) result that relative wage of a region varies inversely with its labour endowment. Dollar (1987) analyses the short-run and the long-run effects of the imposition of Northern import quotas. In the short run, the real wage in the North may rise as a result of the protection. However, in the long run, the imposition of import quotas unambiguously reduces the real wage in the North because the quotas artificially increase the production costs in the North relative to that in the South. This accelerates the transfer of technology and capital flow from the South to the North. So wages in the North decline in terms of all goods in the system.

1.6.2 Vertical product innovation

In Krugman (1979), Dollar (1986, 1987) etc., the technological progress takes the form of horizontal product development. However, the technological progress is viewed as the vertical product development in the model of Flam and Helpman (1987). In this model, the North exports high quality differentiated products and imports low quality differentiated products along with a homogenous product; and the South does the opposite. An increase in the growth rate of population in the South raises the demand for its Northern product; and this raises the wage rate and the level of output in the North. So the terms-of-trade moves against the South. However, the reverse is true in the case of an increase in the population growth rate in the North. An improvement in the level of technology in the South (North) adversely affects the Northern (Southern) wage rate.

Stokey (1991) also presents a North South trade model with vertical product innovation. In Stokey (1991), the North has a comparative advantage in producing new goods; and, in the competitive equilibrium, the South (North) produces a spectrum of lower (higher) quality goods. An increase in the size of the Southern (Northern) labour endowment raises the relative wage in the North (South), raises the welfare in the North (South) and lowers it in the South (North). Technological progress in either of the two countries improves her terms-of-trade and hence raises her social welfare, but lowers the social welfare of the other country.

1.7 New trade and new growth models

'New' (or, endogenous) growth models assume technical change as endogenous to the system. The seminal contribution made by Romer (1990) has paved the way for a new generation of R&D driven endogenous growth theory. Segerstrom et al. (1990), Aghion and Hewitt (1990), Grossman and Helpman (1991a, 1991b, 1991c) etc. also contribute to the development of the R&D driven endogenous growth theory. In these models, technical change results from the allocation of resources to the R&D sector. One set of the existing literature combines the North-South trade models with this R&D driven endogenous growth theory. The R&D sector does the innovation activities in the North and imitation activities in the South; and the North-South trade is characterized by product cycles.

1.7.1 Horizontal product innovation

One subset of this R&D driven North-South endogenous growth models views technical change as horizontal product innovation. The pioneering contribution to this branch of literature comes from Grossman and Helpman (hereafter called GH)(1991b) who develop a North South dynamic general equilibrium model of endogenous product cycle which is essentially a dynamic version of the Krugman (1979) model. Both the rate of product innovation and the rate of imitation in the South are endogenously determined in this model. Also there is a product cycle in the North-South trade. The two countries grow at equal rates in the long-run equilibrium which is assumed to exist; and comparative steady-state exercises with respect to changes in policy parameters are done without analysing the stability property. In this model, technological progress in a country causes the terms of trade to move in her favour; and this result is similar to that in Krugman (1979) model. However, in Krugman's (1979) model, the relative wage of a region varies inversely with the size of its labour endowment; and the opposite happens in the GH (1991b) model. GH (1991b) also use their model to analyse the effectiveness of the policy of strengthening intellectual property rights (IPR) protection in the South; and show that such a policy lowers the balanced growth rate of the world economy and the rate of imitation in the South. Also the North South terms of trade moves against the South as this policy is adopted.

The analysis related to the effect of strengthening IPR protection in the South receives substantial importance in the literature developed following GH (1991b). The GH (1991b) model can not account for the welfare effect of strengthening IPR protection because the welfare calculation needs an explicit account of the transitional behaviour of the economy but GH (1991b) assumes a steady-state equilibrium. Helpman (1993) analyses the welfare effect of strengthening of IPR protection with a complete description of the transitional behaviour of a North South model. However, the imitative activity in the South is costless in his model; and the strengthening of IPR protection implies an exogenous reduction in the rate of imitation. Otherwise, the Helpman (1993) model is similar to the GH (1991b) model. Helpman (1993) shows that the South faces a welfare loss due to the strengthening of IPR protection. However, the North may or may not have a welfare gain in this case. Grinols and Lin (2006) develop an extension of the Helpman (1993) model where the North innovates two types of final consumption goods of which one is consumed in both the regions and the other is consumed only in the South. The result of the Helpman (1993) model is reversed in this model. The South may benefit from the strengthening of IPR protection while the North is hurt by it. Chui et al. (2001) and Currie et al. (1999) extend the GH (1991b) model to account for different stages of development in the South emphasizing the role of North-South knowledge diffusion rate. They show, among many, that a rise in the subsidy to the Southern imitation (or, a lax of IPR protection) raises the steady-state growth of the world economy. Diwan and Rodrik (1991) consider a set up where the Northern consumers and the Southern consumers have different distributions in tastes for goods innovated in the North. They show that the South may be benefitted by the policy of strengthening of IPR protection. All these works assume that the North is the only innovator country and the South can not innovate. However, in the model of Grossman and Lai (2004), both countries can innovate. Using a non cooperative set up, they show that the economy with a lower ability to innovate and with a smaller size of the market for its innovative products would have a lower incentive to strengthen its IPR. They also study the incentives of having an international patent agreements by characterizing an efficient patent regime that provides the optimal aggregate incentives for innovation to inventors throughout the world. They show that the harmonization of patent policies is neither necessary nor sufficient for global efficiency.

1.7.1.1 Multinationalisation

Models surveyed in subsection 1.7.1 of this chapter do not allow for the presence of the Northern multinational firms in the South. However, in reality, multinationalization is an important source of North South technology transfer. Lai (1998) extends the Helpman (1993) model with endogenous North South multinationalization; and assumes that the Southern imitation is possible only after multinationalisation. A stronger IPR protection policy in the South reduces the threat of imitation and thus encourages multinationalisation. This raises the rate of innovation in the North in a new steady-state equilibrium; and causes the terms of trade to move against the North. These results are opposite to those obtained from the Helpman (1993) model. Lai (2001) extends the Lai (1998) model in a way where two Southern countries, who can imitate only, compete with each other to attract the foreign direct investment (hereafter called FDI) from the North who is the only innovator country. The North continuously transfers production to the Southern countries through FDI. This model is used to analyse the effectiveness of the subsidy policy and the import tariff policy adopted in the South. Branstetter et al. (2006) also consider the issue of endogenous multinationalisation of the Northern firms. Unlike Lai (1998), they allow the rate of Southern imitation to be endogenously determined within their model. However, the theoretical results related to the effects of strengthening of IPR protection are similar to those in Lai (1998). They also provide empirical support for their theoretical results.

1.7.2 Vertical product innovation

GH (1991a) develop a quality ladder based dynamic North South model of trade with product cycles; and use it to examine the effects of changes in country size and in subsidy policies of the governments. The North innovates the top quality products and the South imitates those. The Northern innovator is displaced from the market as its product is imitated by a successful Southern firm; and the Southern imitator also faces the risk of losing the market as a new higher quality product is invented in the North. However, the Southern imitator does not face the risk of losing the market in the GH (1991b) horizontal product innovation model; and can continue to produce the product for ever. This difference makes the results of the quality ladder based model different from those of the product variety model. For example, in the 'efficient follower' regime[1] in the quality ladder model, an expansion of the Southern or the Northern labour endowment raises the rate of imitation in the South but produces ambiguous effect on the rate of innovation in the North. However, in the variety based model, this always raises both the Southern rate of imitation and the Northern rate of innovation. Subsidization to the Southern imitative R&D sector, which is considered to be equivalent to a lax of IPR protection, lowers the rate of innovation in the North and raises the rate of imitation in the South. Subsidization to the Northern R&D sector produces the opposite effects. However, in the horizontal innovation model, a subsidy to research in either region lowers the rate of growth (product development) in both the regions. In the 'inefficient follower' regime,[2] results obtained from this quality ladder model are similar to those obtained from the product variety model.

1.7.2.1 Multinationalisation and Licensing

The original GH (1991a) quality ladder model has been extended in various directions by various authors. The GH (1991a) model does not allow for the North-South multinationalization; and the Glass and Saggi (2002) model takes care of that. Glass and Saggi (2002) show that a stronger IPR protection in the South lowers the rate of imitation in the South and hence deters the rate of North-South multinationalization. This also lowers the rate of innovation in the North. In their model, the strengthening of IPR protection acts as a resource wasting activity. Clearly this result is opposite to what Lai

[1] It is the regime where both leaders and the followers in the North are active in the research lab.
[2] It is the regime where only leaders in the North perform active research in the lab.

(1998) obtains in a product variety model. Glass and Wu (2007) also compare the quality ladder model and the product variety model with and without multinationalization and attempt to explain the differences in results originating from the differences in the nature of innovation.

Yang and Maskus (2001) analyse the role of licensing as a mean of technology transfer to the South in the North-South quality ladder model. The North chooses whether to license its technology to the South or not. Licensing not only generates higher profit rate in the South but also raises the risk of imitation. A stronger protection of IPR in the South raises the total return from innovation which is called the size effect. Also, as the imitation risk is reduced, it allows the licensor to deter imitation by giving up a smaller share of the licensing rents to the licensee. So the distribution goes in favour of the licensor (the North) which is called the distribution effect. Combining these two effects, they find that both the rate of licensing and the rate of innovation in the North are increased due to a stronger IPR protection adopted in the South.

Yang and Maskus (2001) analyze only the steady state equilibrium properties but not the transitional dynamic properties of their model[3]. Tanaka et al. (2007) show that the long run equilibrium is unstable in Yang and Maskus (2001) model. Tanaka et. al (2007) modify the Yang and Maskus (2001) model and show that there exists a unique saddle path converging to the steady state equilibrium point in that modified model. They analyse the effect of an increase in the license fee rate; and show that it discourages innovation and technology transfer in that modified model. This result is opposite to what Yang and Maskus (2001) obtains.

Lai and Qiu (2003) investigate the relationship between the trade policy and the IPR

[3]With the exception of Helpman (1993) and Arnold (2002), other studies on international technology transfer also analyse only the steady state equilibrium properties.

protection policy in a North-South model where both regions provide IPR protection as well as trade protection. An increase in the Northern (Southern) tariff rate encourages (discourages) innovation in the North and raises (lowers) the global welfare. So, the North may be benefitted even if it subsidizes the Southern trade liberalization policy. However, Lai and Qiu (2003) do not consider endogenous growth in their model.

1.7.3 Scale effect

In GH (1991a, 1991b) and in many of their extensions, the steady state equilibrium rate of growth of an economy varies positively with the size of its labour endowment. These models can not account for a constant rate of growth when the labour force is growing. The empirical work of Jones (1995a, 1995b) points out that the long run rate of growth of most of the industrialised economies are more or less constant even though the number of skilled workers are growing over time. This criticism leads to the development of some scale free R&D driven endogenous growth models. Segerstrom (1998), Kortum (1997), Dinopoulos and Thompson (1998), Perettoo (1998), Li (2000), Young (1998), Arnold (1998) etc. develop scale free endogenous growth models of closed economies.

Dinopoulos and Segerstrom (2004) develop a quality ladder based North-South model which is free from the problem of scale effect but is otherwise similar to GH (1991a) model. They show that the stronger IPR protection in the South lowers the rate of imitation in the South as well as the rate of innovation in the North but raises the North South wage inequality. These results are similar to those obtained from the GH(1991b) product variety model while the Dinopoulos and Segerstrom (2004) model is of quality ladder type. However, in GH (1991b), the effect on the rate of innovation is permanent but, in Dinopoulos and Segerstrom (2004), this effect is temporary. Dinopoulos and Segerstrom (2004) also allows for endogenous multinationalization in the South following the works of Glass and Saggi (2002); and show that a stronger IPR protection policy, which attracts FDI, has a positive long run effect on the Southern wage rate. Sener (2003) also analyses the effects of strengthening Southern IPR protection on the Northern rate of innovation. In his scale invariant growth model, successful Northern entrepreneurs are engaged in rent protection activities to deter the innovation and the imitation efforts of their rivals. These rent protection activities help in removing the scale effects from the growth structure. A stronger IPR protection policy in the South reduces the rate of imitation in the South as well as the rent protection expenditure in the North. This leads to an increase in the incentive to undertake innovation activities in the North. However, the rate of innovation in the North is reduced in the new equilibrium because a stronger IPR protection in the South expands the production sector in the North causing the wage rate to rise and the R&D activity to be more expensive.

1.7.4 International outsourcing

A few North-South models also analyse the international outsourcing of economic activities. In a static factor endowment based model of North South trade, Feenstra and Hanson (1997) analyse the causes and consequences of international outsourcing. In their model, a single final good is produced in both the regions using a continuum of intermediate goods whose production requires skilled labour and unskilled labour as inputs. Intermediate goods, more intensive in the use of unskilled labor, are produced in the South, and the others are produced in the North. A movement of capital from the North to the South leads to a shift of intermediate goods production from the North to the South. This is a loss of least skilled-intensive activities to the North but is a gain of highest skilled-intensive activities to the South. As a result, the increased outsourcing raises the relative demand for skilled labor as well as the skilled-unskilled wage gap in both the countries.

GH (1991a, 1991b) do not consider the issue of international outsourcing of economic activities. Glass and Saggi (2001) investigate the effects of increased North South outsourcing of production jobs using a quality ladder model similar to GH (1991a). In their model, increased outsourcing raises the Northern innovation (growth) rate and lowers the Northern relative wage. However, the positive growth effect on welfare due to the higher average quality of products may outweigh the negative relative wage effect and thus may bring a welfare gain for the North. Glass (2004) explains the expansion of North South outsourcing in the presence of imperfect protection of IPR in the South. In her model, increased production outsourcing in the South lowers production cost but entails risk of imitation. This model also shows that both the North and the South may have welfare gain due to increased outsourcing resulting from the reduction in the risk of imitation and/or from the expansion of labour endowment. Sayek and Sener (2006) introduce two types of labour - skilled and unskilled - in both the regions in a quality ladder North-South model; and analyse the effect of North-South outsourcing on the Northern rate of innovation and on the skilled unskilled relative wage in both the regions.

All these authors analyse the international outsourcing of production activities and not of R&D activities. The empirical evidence suggests that the extent of North-South outsourcing of R&D activities is also increasing over time[4]. Lai et al. (2003) analyse the outsourcing of R&D activities using a static principal agent model. They consider two types of contracts - fixed and revenue-sharing; and show that the extent of outsourcing is increased leading to an improvement in economic efficiency under revenue sharing contracts. However, in their model, the principal may still find it optimal to choose a contract that allows the leakage of information when it cannot be monitored or verified. Moreover, a stronger protection of IPR neither raises R&D outsourcing nor improves

[4]R&D expenditure by US-owned subsidiaries in China rose from US$ 7 million to US$ 650 million between 1994 and 2002. In Singapore, this jumped from US$ 167 million to US$ 589 million during the same period. R&D investment worth of US$ 1.13 billion has flowed into India during the five year period 1998-2003. More detail evidences of R&D outsourcing are available in the report of OECD science, technology and industry outlook, 2006. Also see R&D Magazine (January 2001) and the work of Lai et al. (2003).

welfare.

1.7.5 International migration

GH (1991a, 1991b) models and their various extensions assume labour to be internationally immobile. Lundborg and Segerstrom (2000, 2002) analyse the effect of international migration using a quality ladder North-South model similar to GH (1991a). In their model, the incentive to migrate is explained by the difference between the levels of utility of the infinitely lived individuals in the two regions. Since the R&D sector is assumed to be more productive in the North than in the South in that model, the growth potential in the world economy is increased when the South-North migration takes place. However, this migration is not beneficial to the Northern consumers. Northern workers are worse affected than Northern capitalists. Southern workers and the migrants are benefitted by this migration.

Bretschger (2001) also analyses the effects of international migration using an expanding product variety North-South model where each of the varieties is produced using skilled labour and unskilled labour. An increase in the skilled labour migration has a positive effect on growth in the host country while the effect of unskilled labour migration depends on the elasticity of technical substitution between the skilled labour and the unskilled labour. The smaller the country size, the higher is the possibility of a negative growth effect of the unskilled labour migration. However, the migration of the skilled labour has a negative growth effect in the source country.

1.7.6 Unemployment

GH (1991a, 1991b) assume full employment of labour in both the regions. There exists a literature explaining unemployment in the GH (1991c) one country closed economy model. Works of De Groot (1998), Van Schauk and De Groot (1998), Staddler (1999), Jurgen (2004) etc. explain this unemployment introducing the efficiency wage hypothesis. Arnold (2002), who first introduces unemployment in the North in a North-South model, considers a labour market that does not adjust instantaneously. He assumes that the imitation in the South causes frictional unemployment in the North. In his model, the relationship between the exogenous rate of imitation and the steady state equilibrium growth rate depends on the degree of labor market flexibility in the North which is measured by the labour absorption rate from the pool of Northern unemployed workers. This relationship is monotonically increasing for high absorption rates, hump-shaped for intermediate absorption rates, and monotonically decreasing for low absorption rates. In this last case, an increase in the exogenous rate of imitation reduces the steady state equilibrium employment level in the North. However, Arnold (2002) does not consider the unemployment problem in the South.

Sener (2001) develops a scale free dynamic general equilibrium model of R&D generated growth with trade of knowledge-based higher quality products between two structurally identical countries. A product replacement mechanism coupled with a time consuming job-matching process generates Schumpeterian unemployment in his model. Trade liberalization in the form of a global reduction of tariff rates raises the unemployment rate of unskilled workers as well as the growth rate of the global economy. Thus Sener (2001) obtains a positive relationship between the long run growth rate and the unemployment rate in the unskilled labour market.

1.8 The plan of the present thesis

The present research work is based on the product variety framework; and this work reanalyses the effects of strengthening of IPR protection in the South in some cases not considered in the existing literature of the works already done in the product variety framework. In chapter 2, we extend the exogenous imitation North-South model of Helpman (1993) in three directions. In section 2.1, we introduce Jacobs (1969) type of localised knowledge spillover in an otherwise identical Helpman (1993) model and analyse the effects of strengthening of IPR protection in the South. In section 2.2, we introduce perfect international labour mobility in an otherwise identical Helpman (1993) model; and, in section 2.3, we allow international outsourcing of production jobs as well as of R&D jobs from the North to the South. In all these sections, we study the effects of stronger IPR protection policy adopted in the South and the effects of changes in the labour endowments of the two countries.

In chapter 3, we analyse the problem of stability and the transitional dynamic properties of the GH (1991b) product variety North-South innovation imitation model. In section 3.1, we study the stability properties of the GH (1991b) model. In section 3.2, we modify the GH (1991b) model allowing for Jacobs (1969) type of localised knowledge spillover in the Northern R&D sector and then reanalyse the stability property of that modified model and also study various comparative dynamic properties of that modified model.

In chapter 4, we study the role of multinationalization on innovation and imitation. We extend the product variety model of Lai (1998) introducing cost of imitation activities in the South; and then compare our results to those obtained in the original model of Lai (1998).

In chapter 5, we study the unemployment problem of the unskilled workers in the South by introducing the efficiency wage hypothesis in the Southern unskilled labour market in an otherwise identical GH (1991b) product variety model. We analyse the effects of stronger IPR protection policy in the South and the effects of changes in labour endowments of the two countries on the level of unemployment in the South.

Chapter 6 presents a general conclusion of the thesis mentioning some of its limitations and the scope for future research.

Chapter 2

Product Cycle Model with Exogenous Imitation

Introduction

In an interesting and widely noted paper published in Econometrica, Helpman (1993) analyses the effect of the tightening of an 'Intellectual Property Rights' (IPR) policy adopted in the South on the growth rate and on the level of social welfare in the North as well as in the South. He uses a dynamic general equilibrium model of a two country world economy where the North innovates and the South imitates. Rate of innovation in the North is endogenous[1] while the imitation rate in the South is exogenous in his model; and the tightening of the IPR protection implies an exogenous reduction in the rate of imitation. This tightening of the IPR protection policy adopted in the South lowers the rate of innovation in the North in the steady-state equilibrium. This policy always lowers the welfare of the South; and also lowers the welfare of the North if the rate of imitation is very small.

In chapter 1 of this thesis we have mentioned that the Helpman (1993) model has been extended by various authors in various directions[2]. In this chapter, we extend the Helpman (1993) model in three directions not considered in the existing literature. In

[1] In section 3 of his paper, innovation rate is endogenous.

[2] See for example, Arnold (2002), Lai (1998) and Grinols and Lin (2006).

section 2.1, we introduce Jacobs (1969) type of localised knowledge spillover in an otherwise identical Helpman (1993) model; and show that a stronger IPR protection policy adopted in the South raises the steady state equilibrium rate of growth (innovation) and may raise the welfare of both the countries in that modified model. In section 2.2, we introduce international migration of labour and show that a stronger IPR protection policy adopted in the South may induce the Southern labour to migrate to the North which in turn may raise the steady state equilibrium rate of growth. In section 2.3, we introduce international outsourcing of production and R&D jobs from the North to the South; and show that the growth effect of IPR protection may depend on the nature of outsourcing of jobs.

This chapter is organised as follows. In section 2.1, we analyse the growth and welfare effects of stronger IPR protection policy in the presence of localised knowledge spillover. In section 2.2, we study the implications of introducing international migration of labour. In section 2.3, we analyse the implications of introducing international outsourcing of jobs.

2.1 Localised knowledge spillover[3]

In Helpman (1993), the knowledge capital stock in the North is assumed to be proportional to the economy's cumulative research experience measured by the number of product designs already developed. This knowledge capital, treated as the public input into the R&D sector, generates positive externalities; and thus lowers the cost of developing new blue prints in the R&D sector. Instead of this so-called Marshall-Arrow-Romer (MAR)[4] type of knowledge spillover, we consider Jacobs (1969) type of localised knowledge spillover in this note. Now the agglomeration of different production units in one region decreases the cost of doing R&D there. Thus here the knowledge spillover

[3] **This section is based on Mondal and Gupta (2006a).**

[4] This terminology is used in Glaeser et al. (1992).

originates from the presence of producers of different goods in one region rather than the experience of the R&D sector of developing product designs in the past. Researchers might benefit from interactions with producers of other goods. They observe the production process directly and find it easier to invent new product designs at cheaper cost.

Empirical supports for Jacobs (1969) types of knowledge spillovers at the level of a city or a region have been documented by Glaeser et al.(1992), Henderson et al.(1995), Feldman et al. (1999) etc. Using a data set on the growth of large industries in 170 U.S. cities between 1956 and 1987, Glaeser et al.(1992) find that local competition and urban variety, but not the regional specialization, encourage employment growth in industries. These evidences, according to them, suggest that important knowledge spillovers might occur between rather within industries; and these findings are consistent with the theories of Jacobs (1969). Henderson et al.(1995) use data set for eight manufacturing industries in U.S. between 1970 and 1987; and show that MAR externalities are important for mature capital goods industries while Jacobs type of externalities are important for new high tech firms. Using the U.S. Small Business Administration's Innovation Data Base (SBIDB), Feldman et al. (1999) show that the diversity of economic activity rather than specialization within a region is more conducive to knowledge spillover and hence product innovation.

In the theoretical literature on North South trade and endogenous growth, these Jacobs type of externalities in the Northern R&D sector have been considered by Dollar (1986, 1987), Martin and Ottaviano (1999), Baldwin et al. (2001) etc. although their focuses are different from those in the Helpman (1993). Following this strand of literature, we assume that the knowledge spillover in the Northern R&D sector is measured by the number of varieties produced in the North.

This is the only minor change in assumption we introduce here. However, we obtain interesting results when we introduce this minor change in an otherwise Helpman (1993) model. We find that the policy of strengthening IPR in the South must raise the rate of innovation in the North in the new steady state growth equilibrium. Also, in this case, both the North and the South may gain in terms of welfare from tightening IPR when the imitation rate is neither very high nor very low. These results are different from those found in Helpman (1993); and are interesting in the context of the debate about the enforcement of IPR in less developed countries. While Helpman's (1993) results go against the adoption of such a policy, our results may advocate this. Also it is the extent of the imitation rate which appears to be crucial factor in determining the desired direction of the policy change.

In subsection 2.1.1, we describe the model. In subsection 2.1.2, we analyse the effect of tightening IPR protection and of the change in factor endowments on the steady state equilibrium rate of growth. In subsection 2.1.3 we analyse the effect of this IPR strengthening policy and of the change in factor endowments on the welfare of the North and of the South.

2.1.1 The basic model

The representative consumer in the North with subscript N, and in the South with subscript S, has the welfare function given by

$$W_i(t) = \int_t^{\infty} e^{-\rho(\tau - t)} log U_i(\tau) d\tau$$

where $U_i(t)$ is the instantaneous utility function given by

$$U_i(t) = (\int_0^{n(t)} x_i(z)^{\alpha} dz)^{\frac{1}{\alpha}} \quad ; \quad 0 < \alpha < 1$$

for i=N, S. Here $n(\tau)$ stands for the number of varieties available at time point τ and $x_i(z)$ represents the amount of zth variety consumed by a representative consumer in the ith region for i=N, S. ρ stands for the constant rate of discount; and α represents the elasticity of substitution between any two varieties.

A representative Northern consumer maximises his welfare subject to the intertemporal budget constraint given by

$$\int_t^\infty e^{-r_N(\tau-t)} E_N(\tau)d\tau = \int_t^\infty e^{-r_N(\tau-t)} I_N(\tau)d\tau + A_N(t) \quad for\, all\, t.$$

Here $E_N(\tau)$, $I_N(\tau)$ and $A_N(\tau)$ stand for instantaneous expenditure, instantaneous income and the current value of assets in the North at time τ. r_N stand for the nominal interest rate in the North.

Note that the representative consumer in the South need not solve any dynamic optimization problem because the South does not have any R&D activity. This consumer maximizes the instantaneous utility function subject to a instantaneous budget constraint which is given by

$$E_S(\tau) = \int_0^{n(t)} p(z)x_S(z)dz.$$

We obtain the following optimality conditions[5]

$$\frac{\dot{E}_N}{E_N} = r_N - \rho; \tag{2.1.1}$$

and

$$x_i(z) = E_i(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}du} \qquad \forall z \in [0, n(t)]. \tag{2.1.2}$$

Here equation (2.1.1) implies the Ramsey rule and equation (2.1.2) represents the demand function for the zth variety of a representative consumer in the ith region for i=N, S. p(z) is the price of the variety z and

$$\varepsilon = \frac{1}{1-\alpha} > 1$$

[5]The derivation of equation (2.1.1) is shown in the Appendix 2.1.

is the price-elasticity of demand for the zth variety. Here,

$$n = n_N + n_S;$$

and n_N (n_S) is the number of varieties produced in the North (South). The North is the innovator country and the South is the imitator country. The producer of the zth variety produced in the North is a profit maximising monopolist while all Southern imitators play Bertrand game. One unit of labour can produce one unit of a product[6]. Labour is internationally immobile but is perfectly mobile among all the sectors within a country. So the price of any Northern product is given by

$$p(z) = p_N = \frac{w_N}{\alpha} \tag{2.1.3}$$

for all $z \in [0, n_N]$; and the price of an imitated Southern product is given by

$$p(z) = p_S = w_S \tag{2.1.4}$$

for all $z \in [0, n_S]$. Here p_N (p_S) and w_N (w_S) represent the equilibrium price[7] of any Northern (Southern) variety and the equilibrium wage[8] of the Northern (Southern) labour respectively. It is also assumed that

$$w_N > w_S. \tag{A}$$

In the North, labour is employed in the production sector as well as in the R&D sector. The labour market equilibrium condition in the North is given by

$$L_N = n_N x_N + L_R \tag{2.1.5}$$

where L_N, $n_N x_N$ and L_R stand for the Northern labour endowment, labour employed in the Northern production sector[9] and labour employed in the Northern R&D sector.

[6]This production technology is the same for all Northern and Southern products.

[7]Price (quantity) of all the varieties produced in a country take the same equilibrium value because utility function is symmetric and technologies are identical.

[8]Wage rate is the marginal cost of production of a variety.

[9]This is equal to total production of all the Northern varieties.

In the South, imitation is costless and labour is employed only in production. Hence

$$L_S = n_S x_S \tag{2.1.6}$$

is the labour market equilibrium condition there.

The R&D sector in the North produces new product designs using labour as the only input; and thus the number of varieties grow over time. This equation of motion is given by

$$\dot{n} = \frac{n_N}{a_N} L_R \tag{2.1.7}$$

where $\frac{a_N}{n_N}$ is the labour requirement to develop a new product-design; and n_N is the knowledge capital. Note that, in Helpman (1993), the knowledge capital was assumed to be equal to the total number of blueprints developed by the R&D sector. We follow Dollar (1986, 1987), Martin and Ottaviano (1999), Baldwin et al. (2001) etc. and assume that the knowledge capital is equal to the number of firms currently producing in the North. This is the only change we introduce in an otherwise identical Helpman (1993) model. We consider Jacobs (1969) type of localised knowledge spillovers. Researchers learn by observing the production process directly and interacting with the local producers.

Note that the formulation in equation (2.1.7) implies that L_R and $\frac{\dot{n}}{n}$ move proportionately in the long-run because the ratio $\frac{n_N}{n}$ is constant in the balanced growth equilibrium. This implication has been criticised by Jones (1995a, 1995b, 1999) because the observed long-term growth rate has been relatively stable despite upward trends in the number of R&D workers. We do not remove the scale effect from the Helpman (1993) model in the present thesis. However it is an interesting area of further research[10].

[10]So we can interpret our model and that of Helpman (1993), as one of medium-term growth. For more on non-scale growth models see Segerstrom (1998) and Arnold (1998). However, we do believe that it would be more interesting (and a much more significant contribution to the literature) to remove the scale effect from the Helpman (1993) model and then to study the effects of strengthening IPR.

Here m stands for the exogenous rate of imitation defined as

$$m = \frac{\dot{n_S}}{n_N} = \hat{m} - \mu \tag{2.1.8}$$

where μ is a parameter representing the degree of tightening the IPR and $\hat{m}$ is the rate of imitation in the absence of IPR.

Also, following Helpman (1993), it can be shown that

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N; \tag{2.1.9}$$

and, in equilibrium,

$$v_N = \frac{w_N a_N}{n_N}. \tag{2.1.10}$$

Here π_N and v_N stand for the Northern firm's instantaneous monopoly profit and its life time discounted present value of profits respectively. Equation (2.1.10) represents the free entry condition in the Northern R&D sector which states that the value of the representative Northern firm is equal to the cost of developing a new blueprint in the Northern R&D sector. The standard no-arbitrage condition in the Northern asset market is given by

$$\frac{\pi_N}{v_N} + \frac{\dot{v_N}}{v_N} = r_N + m. \tag{2.1.11}$$

Also we have

$$E_N = p_N n_N x_N \tag{2.1.12}$$

where E_N stands for expenditure of the representative Northern consumer which is equal to the value of the total product. Like Helpman (1993), we rule out the possibility of international capital mobility.

2.1.2 The steady state growth equilibrium

Following Helpman(1993), we define

$$\xi = \frac{n_N}{n},$$

and

$$g = \frac{\dot{n}}{n}.$$

We also define

$$\theta = \frac{g}{\xi}.$$

Here ξ represents the fraction of goods not imitated so far. The two equations of motion we can derive[11] are given by the following.

$$\dot{\xi} = \xi\theta - (\xi\theta + m)\xi; \tag{2.1.13}$$

and

$$\dot{\theta} = (\frac{L_N}{a_N} - \theta)[\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)]. \tag{2.1.14}$$

The explicit solution of these two differential equations are described in the Appendix (2.3). In this section, we analyse the dynamic properties of the model using a phase diagram shown in the figure 2.1.1.

Note that here

$$(\frac{L_N}{a_N}) > \theta \Rightarrow L_N > a_N \frac{\dot{n}}{n_N} \ ;$$

and this is always true because, $a_N \frac{\dot{n}}{n_N}$ represents the labour employed in the R&D sector which is, in equilibrium, always less than the total labour endowment of the North.

So the equation of the $\dot{\theta} = 0$ stationary locus is given by

$$\theta = (1-\alpha)(\frac{L_N}{a_N}) - \rho\alpha$$

and so it is a horizontal straight line in the figure 2.1.1.

The equation of $\dot{\xi} = 0$ locus is given by the following

$$\theta(1-\xi) = m$$

[11] Derivation is shown in the Appendix (2.2)

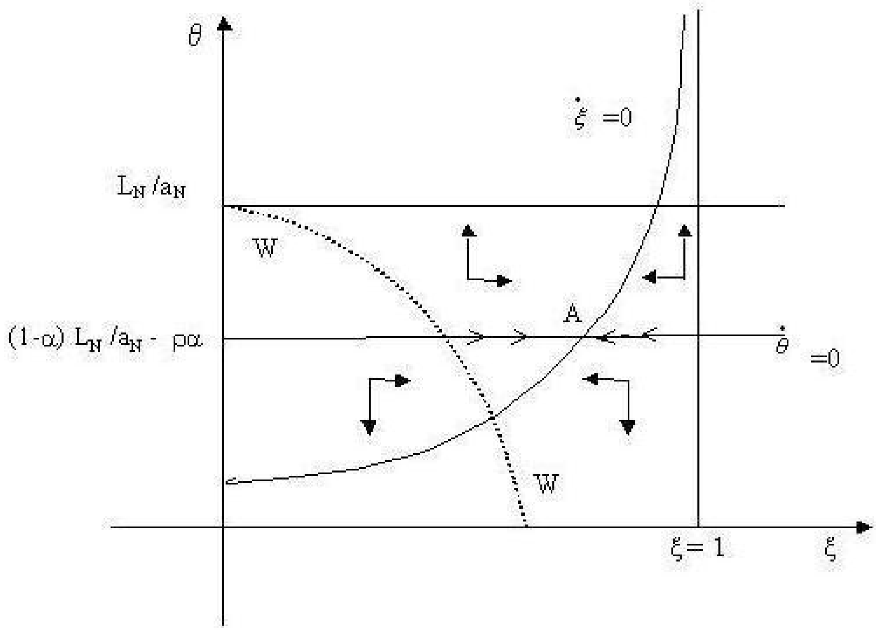

Figure 2.1.1 : The phase diagram

and this curve slopes positively in figure 2.1.1 being asymptotic to the $\xi = 1$ vertical straight line and meeting the vertical axis at $\theta = m$.

The point of intersection of these two curves is the steady state growth equilibrium point. In Appendix (2.3), we show that it is a saddle point and the unique saddle path converging to this equilibrium point coincides with the $\dot{\theta} = 0$ locus. This convergence is guaranteed if and only if $\theta(0) = \theta^*$.

The steady state equilibrium values of ξ and g are given by the followings.

$$\xi^* = 1 - \frac{m}{\theta^*} = 1 - \frac{m}{(1-\alpha)(\frac{L_N}{a_N}) - \alpha\rho};$$

and

$$g^* = \theta^* \xi^* = (1-\alpha)(\frac{L_N}{a_N}) - (\alpha\rho + m).$$

A tightening of IPR protection means a fall in the effective rate of imitation, m. Hence

g^* and ξ^* are increased in the new equilibrium. In figure 2.1.1, $\dot{\xi} = 0$ locus shifts downwards and the $\dot{\theta} = 0$ locus remains unchanged.

An increase in L_N causes the $\dot{\theta} = 0$ locus to shift upward but does not affect the $\dot{\xi} = 0$ locus in figure 2.1.1. So, in the new steady state equilibrium, both g^* and ξ^* are increased. An increase in L_S affects neither the $\dot{\theta} = 0$ locus nor the $\dot{\xi} = 0$ locus in figure 2.1.1. So, a change in L_S does not affect the steady state equilibrium values of g and ξ. We state these results in the following proposition.

Proposition 2.1.1. *(i) A policy of tightening IPR in the South raises the rate of innovation as well as the proportion of unimitated products in the North in the new steady state equilibrium. (ii) An increase in the Northern labour endowment raises the rate of innovation and the proportion of unimitated products in the North in the new equilibrium but a change in the Southern labour endowment does not affect them.*

Results stated in the part (ii) of the above proposition are similar to those obtained by Helpman (1993). However, result (i) is interesting because this is opposite to what Helpman (1993) obtains[12]. In Helpman (1993), a policy of tightening IPR lowers the rate of growth in the new steady state equilibrium. We now turn to provide intuitive explanations of why this effect is opposite in nature to that obtained in the Helpman (1993) model. A stronger IPR protection leads to a reduction in both the effective cost of capital, $(r_N + m)$, as well as the profit rate, $\frac{\pi_N}{v_N}$, in Helpman's (1993) model. Moreover its impact on the effective cost of capital is smaller than the corresponding impact on the profit rate. For this reason the rate of innovation is reduced in his model. However, in the present case, tighter IPR has no effect on the profit rate and only the effective cost of capital is reduced. Thus the positive impact of tightening IPR on the effective cost of capital causes the long run rate of innovation to increase.

[12]Note that Helpman(1993, footnote 19, p. 1261) himself questioned the long run negative relationship between the rate of innovation and the rate of imitation in this product variety framework with a more general functional form of the utility function. However, he was silent about the welfare effect with this more general class of utility function. We are offering here a completely different mechanism that leads to a positive relationship between rate of innovation and rate of imitation.

We now explain, why, in our model, tightening of IPR has no effect on the profit rate. Let us write the expression of the profit rate, $(\frac{\pi_N}{v_N})$, as

$$\frac{\pi_N}{v_N} = \frac{\frac{1-\alpha}{\alpha} w_N x_N}{\frac{a_N}{n_N} w_N} = \frac{1-\alpha}{\alpha a_N} (\frac{n x_N}{\frac{n}{n_N}}) = \frac{1-\alpha}{\alpha a_N} (\frac{\frac{1}{\xi}(n_N x_N)}{\frac{1}{\xi}}).$$

A decrease in m increases ξ in steady state. This decreases per firm profit because the increased competition among firms in the North lowers the market share of every firm. This also decreases the cost of R&D due to the increased knowledge spillover. However, both the numerator and the denominator move proportionately. Hence the profit rate does not depend on m in the steady state equilibrium[13].

We obtain the solution of $\theta(t)$ and $\xi(t)$ as

$$\theta(t) = \theta^* \tag{2.1.15}$$

and

$$\xi(t) = \xi^* + [\xi(0) - \xi^*] e^{a_{22} t} \tag{2.1.16}$$

where

$$a_{22} = m - ((1-\alpha)\frac{L_N}{a_N} - \rho\alpha) = -g^* = -\theta^* \xi^* < 0.$$

The derivation of these solutions are described in Appendix (2.3). Since $a_{22} < 0$, $\xi(t) \to \xi^*$ as $t \to \infty$ whatever be the value of $\xi(0)$.

Now $w_N > w_S$ given by the inequality (A) implies that, in the steady-state equilibrium,

$$m < \theta^* \frac{L_S \alpha^\varepsilon}{L_S \alpha^\varepsilon + L_N - a_N \theta^*}.$$

[13] In Helpman (1993), the expression of profit rate is $\frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha a_N}(\frac{1}{\xi}(n_N x_N))$ and here an increase in ξ (due to a decrease in m) decreases per firm profit only, given the allocation of labour. This results in decreasing the profit rate there.

This condition will always hold true if L_S is large enough relative to L_N. Its derivation is given in Appendix (2.4). Hence $a_{22} < 0$ and $w_N > w_S$ imply that

$$m < min\{(1-\alpha)\frac{L_N}{a_N} - \rho\alpha, \ \ \theta^* \frac{L_S \alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}\}.$$

Since $L_N - a_N\theta^* > 0$, we have $\frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*} < 1$; and hence it is clear that the above inequality will be satisfied if

$$m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}. \tag{B}$$

This is called the feasibility restriction on the rate of imitation.

Using equations (2.1.15) and (2.1.16) we obtain

$$\frac{d\xi(t)}{d\mu}]_{(\theta^*,\xi^*)} = \frac{d\xi^*}{d\mu}(1 - e^{a_{22}t}) > 0 \tag{2.1.17}$$

and

$$\frac{d\theta(t)}{d\mu}]_{(\theta^*,\xi^*)} = 0 \tag{2.1.18}$$

for all $t > 0$. Since $g(t) = \xi(t)\theta(t)$, we have

$$\frac{dg(t)}{d\mu}]_{(\theta^*,\xi^*)} = (1 - e^{a_{22}t}) > 0 \tag{2.1.19}$$

for all $t > 0$. Both the rate of innovation and the fraction of unimitated goods increase at each point of time due to tightening of IPR (except at $t = 0$). Helpman (1993) found that, the rate of innovation is increased in the short run and is decreased in the long run. In our model we do not find any different impact on $g(t)$ in the short run. This is so because, starting from the initial steady state equilibrium, an increase in μ does not affect the $\dot{\theta} = 0$ line. Hence the unique equilibrium trajectory which coincides with the $\dot{\theta} = 0$ locus is not changed in our model when the economy attains a new steady state equilibrium. So $\theta(t)$ remains unchanged and $\xi(t)$ rises for all t until it reaches the new steady state equilibrium point. This ensures that $g(t) = \xi(t)\theta(t)$ will rise for all t till it reaches the new steady state equilibrium point. In Helpman (1993), there is a new saddle path converging to the new steady state equilibrium point obtained for an exogenous and once for all change in μ; and hence $g(t)$ rises initially to reach the new saddle path. However, $g(t)$ falls in the long-run in his model because the saddle path slopes negatively there.

2.1.3 Welfare

2.1.3.1 IPR protection

We now turn to analyse the effects of the policy of tightening IPR in the South on the welfare of a representative worker in the North and in the South. Following Helpman(1993), we define

$$W_N(0) = \int_0^\infty e^{-\rho t} log U_N(t) dt$$

and

$$W_S(0) = \int_0^\infty e^{-\rho t} log U_S(t) dt \, .$$

Here

$$log U_N(t) = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}] + log(1 - \frac{a_N \theta}{L_N}); \qquad (2.1.20)$$

and

$$log U_S(t) = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi(\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]. \qquad (2.1.21)$$

Here $W_i(0)$ is the discounted present value of instantaneous utility flow of the representative worker in the ith region for i=N,S; and $U_i(t)$ is his instantaneous utility function. Derivation of equations (2.1.20) and (2.1.21) are described in the Appendix (2.5). In fact, equations (2.1.20) and (2.1.21) in this note are identical to equations (41) and (16) in Helpman (1993, p. 1265, p. 1254) respectively. Sign of $(\frac{dW_i(0)}{d\mu})$ represents the nature of the welfare effects in the ith country due to tightening of IPR in the South. Following

Helpman (1993), we obtain

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon - 1}(\Delta_N + \Delta_e^S); \tag{2.1.22}$$

and

$$\frac{dW_N(0)}{d\mu} = \frac{1}{\varepsilon - 1}(\Delta_N + \Delta_e^N) + \Delta_S^N. \tag{2.1.23}$$

Here

$$\begin{aligned}
\Delta_N &= \int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{d\mu}dt,\\
\Delta_e^S &= \int_0^\infty e^{-\rho t}[\frac{dlog[\xi(\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]}{d\mu}]dt,\\
\Delta_e^N &= \int_0^\infty e^{-\rho t}[\frac{dlog[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]}{d\mu}]dt,
\end{aligned}$$

and

$$\Delta_S^N = \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{d\mu}]dt.$$

Interpretations of Δ_N, Δ_e^S, Δ_e^N and Δ_S^N are similar to those given in Helpman (1993, p. 1265). Here the number of products, n, grows over time at the rate g. Hence

$$log(n(t)) = log(n(0)) + \int_0^t g(\tau)d\tau.$$

Now, using equation (2.1.19) and the expression of Δ_N, we have,

$$\Delta_N = \frac{-a_{22}}{\rho^2(\rho - a_{22})} > 0 \tag{2.1.24}$$

because $a_{22} < 0$. The derivation is shown in the Appendix (2.6). However, $\Delta_N > 0$ implies that the welfare effect via product availability is positive for both the countries. This is opposite to what Helpman (1993, p. 1265, Proposition 4) obtained because in his model $\Delta_N < 0$. Sign of the welfare effect via product availability is same as the sign of the growth effect.

The derivation of Δ_e^S in detail is given in the Appendix (2.7). It is obtained as

$$\Delta_e^S = [\frac{(\frac{p_S}{p_N})^{\varepsilon-1} - 1 - (\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)}][\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho - a_{22})}] \tag{2.1.25}$$

where $\frac{p_S}{p_N}$ and ξ are measured at their steady state equilibrium values. Here $\Delta_e^S < 0$ because $a_{22} < 0$; and this implies that the welfare effect of tightening IPR due to changes in both the interregional allocation of production and the terms of trade goes against the South when the economies are initially in the steady-state.

Using equation (2.1.22) and using the expressions of Δ_N and Δ_e^S given by equations (2.1.24) and (2.1.25) respectively, we have

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon-1}\frac{-a_{22}}{\rho(\rho-a_{22})}[\frac{1}{\rho} + \frac{1}{\theta^*}\frac{(\frac{p_S}{p_N})^{\varepsilon-1}(1-\frac{\alpha}{1-\xi^*})-1}{\xi^*(\frac{p_N}{p_S})^{1-\varepsilon}+(1-\xi^*)}].$$

Since $a_{22} < 0$ and $\varepsilon > 1$, the right hand side of equation (2.1.22) is positive if the following two sufficient conditions are satisfied.

(i) $1 - \frac{\alpha}{1-\xi^*} \geq 0$; and

(ii) $\theta^*(1-\xi^*) \geq \rho$.

In the steady state growth equilibrium,

$\theta^*(1-\xi^*) = m$;

and hence sufficient conditions (i) and (ii) imply

$m \geq Max\{\rho, \alpha\theta^*\}$.

Hence using the inequality (B) we have[14]

$$\frac{dW_S(0)}{d\mu} > 0 \;\; if \;\; Max\{\rho, \alpha\theta^*\} \leq m < \theta^*\frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

So we have the following proposition.

Proposition 2.1.2. *If the economies are initially in the steady state equilibrium, the South gains in welfare due to tightening of IPR if the imitation rate satisfies the following.*

$$Max\{\rho, \alpha\theta^*\} \leq m < \theta^*\frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon+L_N-a_N\theta^*}.$$

We now note the contrast between Theorem 1 of Helpman (1993, p. 1266) and the proposition 2 of ours here. In the Helpman (1993) model, the South never gains from

[14]See Appendix (2.8) for details of the derivation

tightening of IPR protection. However, we prove here that the South gains from tightening of IPR if the imitation re is neither very low nor very high. This is so because, in this case, the positive welfare effect of product availability dominates the negative welfare effect due to change in the interregional allocation of products and the terms of trade. This special case does not arise in Helpman (1993) model because both these effects are always negative there.

We now analyse the welfare effect in the North. Note that the North also gains due to greater variety of available products because $\Delta_N > 0$. Also the steady state equilibrium value of θ is independent of μ; and hence

$$\Delta_S^N = \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{d\mu}]dt = 0.$$

The expression mentioned above demonstrates that the adjustment of savings rate is not welfare enhancing which is in contrary to that in Helpman (1993, equation (46)). This is so because now the size of the R&D sector ($a_N\theta$) is independent of the change in m. Hence there is no intertemporal reallocation of R&D expenditure. The remaining term in equation (2.1.23) is Δ_e^N which captures the welfare effect in the North due to change in the terms of trade and due to change in the interregional allocation of production. We present the expression of Δ_e^N as follows while the derivation in detail is given in the Appendix (2.9).

$$\Delta_e^N = [\frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}\{1-\frac{\alpha}{\xi}\}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}][\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}].$$

Here $\frac{p_S}{p_N}$ and ξ take their steady-state values. Since $a_{22} < 0$ we find that $\Delta_e^N > 0$ for $\xi^* \leq \alpha$; and here $\xi^* \leq \alpha$ implies $m \geq (1-\alpha)\theta^*$. Here the welfare effect due to change in the terms of trade is positive and the welfare effect due to change in the interregional allocation effect is negative. In Appendix (2.9), we show that the positive terms of trade effect dominates the negative interregional allocation of production effect when $\xi^* \leq \alpha$.

Since $\Delta_S^N = 0$ and $\Delta_N > 0$, we find, from equation (2.1.23), that

$$\frac{dW_N(0)}{d\mu} > 0 \;\; if \;\; \xi^* \leq \alpha.$$

So we have the following proposition.

Proposition 2.1.3. *If the economies are initially in the steady state equilibrium, the North gains in welfare due to tightening of IPR protection if the imitation rate satisfies the following:*

$$(1-\alpha)\theta^* \leq m < \theta^* \tfrac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

Combining propositions 2.1.2 and 2.1.3 we find that both North and South gain in welfare due to tightening of IPR if

$$Max\{\rho, \alpha\theta^*, (1-\alpha)\theta^*\} \leq m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

Helpman (1993) does not find any such special case where both North and South may gain. On the otherhand, he finds that both North and South lose in welfare from tightening of IPR policy when the imitation rate is small.

2.1.3.2 Factor endowment change

We now analyse the effect of changes in the Northern and Southern labour endowments on the welfare of a representative worker in the North and in the South. The expressions of the welfare effects can be written as

$$\frac{dW_S(0)}{dL_i} = \frac{1}{\varepsilon - 1}(\Delta_N^{L_i} + \Delta_e^{SL_i}) \quad \text{for i=N, S ,} \tag{2.1.26}$$

and

$$\frac{dW_N(0)}{dL_i} = \frac{1}{\varepsilon - 1}(\Delta_N^{L_i} + \Delta_e^{NL_i}) + \Delta_S^{NL_i} \quad \text{for i=N, S.} \tag{2.1.27}$$

Here

$$\begin{aligned}
\Delta_N^{L_i} &= \int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{dL_i}dt, \\
\Delta_e^{SL_i} &= \int_0^\infty e^{-\rho t}[\frac{dlog[\xi(\frac{p_N}{p_S})^{1-\varepsilon}+(1-\xi)]}{dL_i}]dt, \\
\Delta_e^{NL_i} &= \int_0^\infty e^{-\rho t}[\frac{dlog[\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]}{dL_i}]dt, \\
\text{and} & \\
\Delta_S^{NL_i} &= \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{dL_i}]dt.
\end{aligned}$$

Here $\Delta_N^{L_i}$ represents the welfare change due to the change in the product variety when the labour endowment of the ith region is changed. $\Delta_e^{SL_i}$ ($\Delta_e^{NL_i}$) represents the welfare change of the South (North) due to the change in the interregional allocation of production and terms-of-trade when the labour endowment of the ith region is changed. $\Delta_S^{NL_i}$ represents the welfare change of the North due to the change in the intertemporal savings rsate when the labour endowment of the ith region is changed.

From equations (2.1.15) and (2.1.16), we know that a change in the Southern labour endowment does not affect the time path of $\xi(t)$ and $\theta(t)$ and hence $g(t)$ remains unchanged. Then the change in L_S only affects the North-South terms-of-trade. An increase in L_S causes the terms-of-trade to move in favour of the North and against the South. In the Appendix (2.10), we have shown that $\Delta_N^{L_i} = 0$, $\Delta_e^{SL_i} < 0$, $\Delta_e^{NL_i} > 0$ and $\Delta_S^{NL_i} = 0$ for $i = S$. So an increase in the Southern labour endowment lowers the welfare level of the South and raises it for the North. This happens only through the channel of terms-of-trade. Welfare effects through the change in product variety and through interregional allocation of production do not exist here.

An increase in the Northern labour endowment raises the steady state equilibrium values of g and ξ. In figure 2.1.1, an increase in L_N causes the $\dot{\theta} = 0$ curve to shift upward.

So, along the transition path, $\theta(t)$ initially jumps to the new saddle path and then remains constant. However, $\xi(t)$ steadily rises along the new saddle path. So, during the transition, both $g(t)$ and $\xi(t)$ go up. This ensures that the levels of welfare of both the North and the South rise because of the availability of greater variety of products and fall because of the interregional allocation of production. This is so because an increase in $\xi(t)$ increases the fraction of products with monopoly pricing and so brings about welfare loss of both countries[15]. In Appendix (2.10), it is shown that $\Delta_S^{NL_N} < 0$ but $\Delta_e^{SL_N}$ and $\Delta_e^{NL_N}$ are ambiguous in sign. So, an increase in L_N affects the Northern welfare negatively through an increase in the intertemporal savings rate. However, the combined effects of the terms-of-trade and of the interregional allocation of production on the welfare level of each of the two countries are ambiguous in sign. We summarize the major results in the following proposition.

Proposition 2.1.4. *If the economies are initially in the steady state equilibrium, then (i) the South loses in welfare and the North gains due to an increase in the Southern labour endowment; and (ii) an increase in the Northern labour endowment has ambiguous effects on the level of welfare in both the countries.*

Helpman (1993) does not analyse the welfare effects of factor endowment changes in his model. However, one can show that the directions of change of the welfare levels through various channels due to factor endowment changes in the Helpman (1993) model are similar to those obtained in this model except for the change in the magnitude of the welfare levels.

2.2 International Migration[16]

North-South models of product development and endogenous growth generally ignore the issue of international labour mobility[17]. These models assume labour endowments to

[15] In Appendix (2.10), we have shown that $\Delta_N^{L_i} > 0$ for $i = N$; and $\frac{d(\xi(t))}{dL_N} > 0$ for all $t > 0$.

[16] **This section is based on Mondal and Gupta (2007b).**

[17] See Grossman and Helpman (1991b), Helpman (1993), Lai (1998), Arnold (2003) etc.

be country specific though the per capita expenditure and hence the instantaneous level of utility of each of the two regions is endogenously determined. There is no international labour mobility even if the per capita real spendings in the two regions are different in equilibrium. There are substantial empirical evidences of international labour mobility taking place in the real world. The static two country competitive equilibrium models as well as the dynamic North-South models of exogenous growth have dealt with this problem. The existing literature consists of the works of Bhagwati and Rodriguez (1976), Kenen (1971), Gruebel and Scott (1966), Rivera-Batiz (1981, 1982), Watanabe (1969), Thompson (1984), Roemer (1983), Saveedra-Rivano and Wooton (1983), Wooton (1982), Mountford (1997), Bhagwati and Hamada (1974), Galor and Stark (1991), Ethier (1985, 1986), Djajic (1989) etc. Macmillan (1982) has made an interesting survey of this literature. Lundborg and Segerstrom (2000, 2002) analyse the growth and welfare effects of international migration using the quality ladder framework developed by Grossman and Helpman (1991a). However, no such analysis has been made in the product variety structure and we plan to fill up this gap in this section.

Following Lundborg and Segerstrom (2000, 2002), we assume that the migration incentive depends on the difference between the per capita real spendings of the two countries. This is so because the agents are infinitely lived in our model and the instantaneous utility of an infinitely lived agent is determined not by the instantaneous wage rate[18] but by the per capita real spending. As a result, in the migration equilibrium, the real per capita expenditure in the North is equal to that in the South in our model; and this equality is not satisfied in Helpman (1993) model because it assumes labour to be internationally immobile. We then analyse the effects of strengthening IPR protection in the South on the rate of innovation in the North in the steady state growth equilibrium. We show that the policy of strengthening IPR protection in the

[18]Existing static and dynamic models where agents are not infinitely lived assume migration incentive to depend on the wage difference.

South may raise the rate of innovation in the North if the consumers are very patient in their intertemporal choices. Our result may contradict to that of Helpman (1993) who shows that the strengthening of IPR protection in the South never improves the long run rate of innovation in the North. So while the theoretical results of Helpman (1993) go against the policy prescription of strengthening IPR protection in the less developed countries, our exercise points out a case where one can advocate this policy. This extended model also shows that the rate of innovation in the North varies positively with the size of the Southern labour endowment. In Helpman (1993), rate of innovation in the North is independent of the change in the Southern labour endowment.

The basic model and its dynamics are presented in subsections 2.2.1 and 2.2.2. The comparative steady state effects of strengthening IPR protection in the South and of changes in the labour endowments in the two countries are analysed in the next two subsections.

2.2.1 The basic model

The structure of the basic model is similar to the one set out in section 2.1.1; and all the notations have their usual meanings. So we shall be brief in describing the model in this section and in the next section. As earlier, the representative consumer in the North maximises welfare given by

$$W_N = \int_t^{\infty} e^{-\rho(\tau-t)} log U_N(\tau) d\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r_N(\tau-t)} E_N(\tau) d\tau = \int_t^{\infty} e^{-r_N(\tau-t)} I_N(\tau) d\tau + A_N(t) \quad \text{for all } t\ .$$

Here $U_N(\tau)$ is the instantaneous utility function given by

$$U_N(\tau) = \left(\int_0^{n(\tau)} x_N(z)^{\alpha} dz \right)^{\frac{1}{\alpha}} \quad ; \quad 0 \prec \alpha \prec 1\ .$$

Solving this dynamic optimization problem of a price taker consumer we obtain the following optimality conditions.

$$\frac{\dot{E_N}}{E_N} = r_N - \rho \; ; \tag{2.2.1}$$

and

$$x_N(z) = E_N \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon} du} \qquad \forall z \in [0, n] \; . \tag{2.2.2}$$

The monopoly equilibrium price of any Northern product is given by

$$p(z) = p_N = \frac{w_N}{\alpha} \tag{2.2.3}$$

for all $z \in [n_S, n]$; and the price of an imitated Southern product is given by

$$p(z) = p_S = w_S \tag{2.2.4}$$

for all $z \in [0, n_S]$. As earlier, we assume that

$$w_N > w_S \; .$$

The representative consumer in the South maximises welfare given by

$$W_S = \int_t^{\infty} e^{-\rho(\tau - t)} log U_S(\tau) d\tau$$

subject to the instantaneous budget constraint

$$E_S = \int_0^n p(z) x_S(z) dz \; ;$$

where the instantaneous utility function is given by

$$U_S(\tau) = \left(\int_0^n x_S(z)^{\alpha} dz \right)^{\frac{1}{\alpha}} \quad ; \quad 0 \prec \alpha \prec 1 \; .$$

Maximisation of the instantaneous utility subject to the instantaneous budget constraint yields the following demand function given by

$$x_S(z) = E_S \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon} du} \qquad \forall z \in [0, n] \; . \tag{2.2.5}$$

Using the demand functions (2.2.2) and (2.2.5), we obtain the aggregate demand function for the zth variety as

$$x(z) = x_N(z) + x_S(z) = [E_N + E_S]\frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \qquad \forall z \in [0, n] \; .$$

Note that the demand functions are symmetric. Production technologies are identical for all the varieties. So equal quantities of all varieties are produced in each of the two regions; i.e., $x(z)$ is same for all z in a region. Let us use x_N for all $x(z)$ such that $z \in [n_S, n]$ and x_S for all $x(z)$ such that $z \in [0, n_S]$ [19]. Thus, using equations (2.2.3) and (2.2.4), we have

$$x_N = [E_N + E_S]\frac{{p_N}^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \qquad \forall z \in [n_S, n]; \tag{2.2.6}$$

and

$$x_S = [E_N + E_S]\frac{{p_S}^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \qquad \forall z \in [0, n_S]. \tag{2.2.7}$$

Unlike Helpman (1993), labour endowments are no longer country specific here. We incorporate voluntary decision makings of the agents on whether to migrate or not. Labour is perfectly mobile between the two countries. Therefore, there is a force equalizing the welfare of the agents in these two regions (countries). Since the instantaneous utility level of the representative agent in our model depends on the real spending[20], migration equilibrium is attained when the per capita real spendings in the two regions are equalised. Let e_N and e_S stand for the per capita real spendings in the North and in the South respectively. Then the migration equilibrium condition can be written as

$$\frac{e_N}{e_S} = 1. \tag{2.2.8}$$

where

$$e_N = \frac{E_N}{\bar{L}_N + M}, \tag{2.2.9}$$

[19] x_N denotes the level of aggregate demand for a variety faced by a representative Northern firm; and $x_N(z)$ denotes the level of demand of a representative Northern consumer for the zth variety. x_N also stands for the level of total production of that variety.

[20] See Appendix (2.1) for the derivation.

and

$$e_S = \frac{E_S}{\bar{L}_S - M}. \tag{2.2.10}$$

Here E_i is the aggregate instantaneous expenditure of the ith country for $i = N$ and S ; and M is the volume of migration from the South to the North.

The labour market equilibrium condition in the North is given by

$$\bar{L}_N + M = n_N x_N + L_R; \tag{2.2.11}$$

and

$$\bar{L}_S - M = n_S x_S \tag{2.2.12}$$

is the labour market equilibrium condition in the South. Here $\bar{L}_N + M$ and $\bar{L}_S - M$ represent labour availabilities after migration in the North and in the South respectively.

We follow Helpman (1993) in defining the knowledge stock in the North and do not consider Jacobs (1969) type of externalities considered in section 2.1. So the equation of motion describing the time behavior of product development is given by

$$\dot{n} = \frac{n}{a_N} L_R. \tag{2.2.13}$$

The exogenous rate of imitation is once again defined as

$$m = \frac{\dot{n_S}}{n_N} = \hat{m} - \mu. \tag{2.2.14}$$

The maximum monopoly profit of the Northern firm is given by

$$\pi_N = \frac{1 - \alpha}{\alpha} w_N x_N. \tag{2.2.15}$$

The free entry condition in the Northern R&D sector is given by

$$v_N = \frac{a_N}{n} w_N. \tag{2.2.16}$$

The standard no-arbitrage condition in the Northern asset market is given by

$$\frac{\pi_N}{v_N} + \frac{\dot{v_N}}{v_N} = r_N + m. \tag{2.2.17}$$

Also we have

$$E_i = p_i n_i x_i \qquad \text{for } i = N,\ S. \tag{2.2.18}$$

Like Helpman (1993), we assume that there is no movement of financial capital between the two regions. This implies that the North finances its investment in R&D entirely with domestic savings and the trade account is balanced at every time point. The assumption of capital immobility is consistent with perfect international mobility of labour if, as part of the company law, the migrant workers lose the right of ownership of shares of the profit making firms at the time of leaving the job. Their shares are equally distributed among the existing workers[21].

Using equations (2.2.9), (2.2.11), (2.2.13) and (2.2.18) for $i = N$ we obtain the per capita Northern spending as

$$e_N = p_N \left(1 - \frac{a_N \frac{\dot{n}}{n}}{\bar{L}_N + M}\right). \tag{2.2.19}$$

Here, as in Helpman (1993, p-1265), the term $\frac{a_N \frac{\dot{n}}{n}}{\bar{L}_N + M}$ represents the savings rate of a representative Northern consumer. Given other things unchanged, per capita savings rate varies inversely with the volume of South North migration. As the volume of South North migration is increased, the per capita expenditure of a representative Northern consumer is also increased.

Again using equations (2.2.10), (2.2.12) and (2.2.18) for $i = S$ we obtain the per capita Southern spending as

$$e_S = p_S. \tag{2.2.20}$$

[21] If the migrant workers from the North leave the country with their ownership shares then labour mobility leads to capital mobility. However, the workers of the South do not have ownership of profit shares because the Southern firms do not make profit. So migration from the South to the North does not cause capital mobility.

There is no R&D sector in the South; and so a representative Southern consumer does not save and invest. So the per capita Southern expenditure is equal to the per capita income which, in turn, is equal to the price of the representative imitated product produced in the South.

Using equations (2.2.3), (2.2.4), (2.2.19) and (2.2.20), the migration equilibrium condition (2.2.8) can be written as

$$\frac{w_S \alpha}{w_N} = \left(\frac{\bar{L}_N + M - a_N \frac{\dot{n}}{n}}{\bar{L}_N + M} \right). \tag{2.2.21}$$

This equation (2.2.21) shows that an increase in the volume of South North migration raises the relative wage of the South to the North given other things unchanged. This is so because the increase in the volume of South North migration causes an increase in the per capita Northern expenditure. So, in order to maintain the migration equilibrium condition, the per capita expenditure of the South should also go up. This is possible only when the relative wage of the South is increased.

Using equations (2.2.3), (2.2.4), (2.2.6) and (2.2.7) we have

$$\begin{aligned} \frac{x_N}{x_S} &= \left(\frac{p_N}{p_S} \right)^{-\varepsilon} \\ &= \left(\frac{\alpha w_S}{w_N} \right)^{\varepsilon}. \end{aligned} \tag{2.2.22}$$

This equation (2.2.22) represents the relative demand function for the Northern goods. Also, from equations (2.2.11) and (2.2.12), we obtain the relative supply function of the Northern goods as

$$\frac{x_N}{x_S} = \frac{\bar{L}_N + M - a_N \frac{\dot{n}}{n}}{\bar{L}_S - M} \, \frac{n_S}{n_N}. \tag{2.2.23}$$

The product market equilibrium is obtained when the relative demand for the Northern goods is equal to its relative supply. Then, using equations (2.2.14), (2.2.22) and (2.2.23), we have

$$\frac{w_S \alpha}{w_N} = \left(\frac{\bar{L}_N + M - a_N \frac{\dot{n}}{n}}{\bar{L}_S - M} \frac{m}{\left(\frac{\dot{n_S}}{n_S}\right)} \right)^{1-\alpha}. \tag{2.2.24}$$

This equation (2.2.24) shows that the relative wage of the South, $\frac{w_S}{w_N}$, varies positively with the volume of migration, M, and with the imitation rate, m. As M is increased, the relative supply of labour in the North is increased and this lowers the relative wage there. Also a decrease in m raises the share of the products not yet imitated by the South. So more firms stay in the North; and this raises the demand for Northern labour. This, in turn, raises the relative wage of the North.

So, from equations (2.2.21) and (2.2.24), we find that an increase in the volume of South North migration, M, raises the Southern relative wage, $\frac{w_S}{w_N}$, through two different channels. However, the effect of M on $\frac{w_S}{w_N}$ obtained through the channel of product market equilibrium condition (equation (2.2.24)) is stronger than that effect obtained through the migration equilibrium condition (equation (2.2.21)) if [22]

$$\bar{L}_N + M \geq \frac{a_N \frac{\dot{n}}{n}}{1-\alpha}. \tag{2.2.25}$$

Now using equations (2.2.21) and (2.2.24) we obtain

$$\left(\frac{\bar{L}_N + M - a_N \frac{\dot{n}}{n}}{\bar{L}_N + M} \right) = \left(\frac{\bar{L}_N + M - a_N \frac{\dot{n}}{n}}{\bar{L}_S - M} \frac{m}{\left(\frac{\dot{n_S}}{n_S}\right)} \right)^{1-\alpha}. \tag{2.2.26}$$

Equilibrium volume of migration is determined by equation (2.2.26) in terms of the values of various parameters. This equation (2.2.26) also shows that, given other things unchanged, a decrease in the imitation rate, m, causes an increase in M when condition (2.2.25) is satisfied. This is so because a decrease in m lowers the R.H.S. of equation (2.2.26) and leaves the L.H.S. unaffected. Also an increase in M raises the R.H.S. of

[22] Condition (2.2.25) can also be written as $\left(\frac{n_N x_N}{a_N \frac{\dot{n}}{n}}\right) \geq \frac{\alpha}{1-\alpha}$. This can be interpreted as follows: the relative size of the Northern production sector compared to the R&D sector is higher than a critical minimum level.

equation (2.2.26) at a higher rate than the L.H.S. when condition (2.2.25) is satisfied. So, a decrease in the rate of imitation raises the volume of South North migration.

2.2.2 The dynamics of the model

Following Helpman (1993) we define

$$\xi = \frac{n_N}{n} \; ; \tag{2.2.27}$$

and

$$g = \frac{\dot{n}}{n} \; . \tag{2.2.28}$$

Here ξ denotes the share of the products not yet imitated by the South and g represents the rate of product innovation made in the North. Using equations (2.2.13) and (2.2.28) we have

$$L_R = a_N g \; ;$$

and using equations (2.2.14), (2.2.26), (2.2.27) and (2.2.28), we have

$$\left(\frac{\bar{L}_N + M}{\bar{L}_N + M - a_N g} \right)^{\frac{1}{1-\alpha}} = \left(\frac{\bar{L}_S - M}{\bar{L}_N + M - a_N g} \, \frac{\xi}{1-\xi} \right) . \tag{2.2.29}$$

Using this equation (2.2.29) we can solve M implicitly in terms of g and ξ as

$$M = M(g, \; \xi) \tag{2.2.30}$$

with $M_g < 0$ and $M_\xi > 0$ [23].

Using equation (2.2.27) we have the following equation of motion:

$$\dot{\xi} = g - (g + m)\xi. \tag{2.2.31}$$

This equation (2.2.31) is similar to the equation (2) in Helpman (1993, page-1251). Using the no arbitrage condition (2.2.17) and equations (2.2.1), (2.2.11), (2.2.13), (2.2.14),

[23] $M_g < 0$ and $M_\xi > 0$ are proved under the sufficient condition given by equation (2.2.25). Derivations are done in Appendix (2.11).

(2.2.15), (2.2.16), (2.2.18), (2.2.27), (2.2.28) and (2.2.30) we derive the following equation of motion of the Northern rate of innovation[24] :

$$\dot{g}(M_g - a_N) + M_\xi[g - (m+g)\xi] = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right) - (\rho + m + g)\right]. \tag{2.2.32}$$

If $M \equiv 0$ and hence $M_g \equiv M_\xi \equiv 0$, then equation (2.2.32) in this model is similar to the equation (2.2.24) in Helpman (1993, page-1259). The dynamic properties of the system can be studied solving these two equations of motion (2.2.31) and (2.2.32). Here, we shall analyse the effects of strengthening IPR protection in the South on the steady state equilibrium values of g and ξ. A local stability analysis of the steady state equilibrium is made in Appendix (2.13).

At the steady state equilibrium point, $\dot{g} = \dot{\xi} = 0$. Thus, from equations (2.2.31) and (2.2.32), we obtain the steady state equilibrium conditions given by

$$\frac{1-\alpha}{\alpha\, a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right) - (\rho + m + g) = 0; \tag{2.2.33}$$

and

$$g - (g+m)\xi = 0. \tag{2.2.34}$$

Equation (2.2.33) is obtained because $(\bar{L}_N + M - a_N g) > 0$. Using equations (2.2.33) and (2.2.34) we obtain

$$\frac{1-\alpha}{\alpha\, a_N}(\bar{L}_N + M - a_N g)\frac{m+g}{g} = \rho + m + g. \tag{2.2.35}$$

The L.H.S. of equation (2.2.35) represents the profit rate[25] of a typical Northern firm and the R.H.S. represents its cost of capital. This equation (2.2.35) is otherwise identical to the equation (29) in Helpman (1993, p-1261) with only one point of difference. Profit rate is a positive function of the volume of migration, M, in this model where as, in Helpman (1993), $M \equiv 0$. An increase in the volume of migration from the South to

[24] The derivation of equation (2.2.32) is shown in Appendix (2.12).

[25] Profit rate is defined as the ratio of the per period profit and the value of the firm, $\frac{\pi_N}{v_N}$.

the North raises the effective labour endowment in the North. This raises the labour availability for the Northern production sector. Given full employment of labour, this raises the per firm level of production in the North; and so the profit rate is increased.

Using equations (2.2.29) and (2.2.34) we have

$$\left(\frac{\bar{L}_N + M}{\bar{L}_N + M - a_N g}\right) = \left(\frac{\bar{L}_S - M}{\bar{L}_N + M - a_N g}\,\frac{g}{m}\right)^{1-\alpha}. \tag{2.2.36}$$

Here

$$\frac{dM}{dg} = \frac{\frac{1-\alpha}{g} - \frac{\alpha\, a_N}{\bar{L}_N+M-a_N g}}{\frac{1}{\bar{L}_N+M} - \frac{\alpha}{\bar{L}_N+M-a_N g} + \frac{1-\alpha}{\bar{L}_S-M}} > 0$$

if the condition (2.2.25) is satisfied. Here equation (2.2.36) is obtained combining the Northern product market equilibrium condition and the North South migration equilibrium condition. An increase in M lowers the relative wage of the North through both these channels; and its effect through the product market equilibrium channel is stronger than the effect through the migration equilibrium channel. Also an increase in g raises the North South relative wage through both these channels and its effect through the product market equilibrium channel is again stronger than its effect through the other channel. So there should be a positive relationship between M and g along the equation (2.2.36).

2.2.2.1 Existence of equilibrium

Using equations (2.2.35) and (2.2.36) we have

$$\frac{\left(\frac{\alpha}{1-\alpha}a_N\frac{\rho+m+g}{m+g} + a_N\right)}{\left(\frac{\alpha}{1-\alpha}a_N\frac{\rho+m+g}{m+g}\right)^{\alpha}\left[\bar{L}_N + \bar{L}_S - a_N g - \frac{\alpha}{1-\alpha}a_N g\ \frac{\rho+m+g}{m+g}\right]^{1-\alpha}} = \frac{1}{m^{1-\alpha}}. \tag{2.2.37}$$

The L.H.S. of equation (2.2.37) can be written as

$$\left(\frac{\frac{\alpha}{1-\alpha}a_N\frac{\rho+m+g}{m+g}}{\bar{L}_N + \bar{L}_S - a_N g - \frac{\alpha}{1-\alpha}a_N g\ \frac{\rho+m+g}{m+g}}\right)^{1-\alpha} + \frac{a_N}{\left(\frac{\alpha}{1-\alpha}a_N\frac{\rho+m+g}{m+g}\right)^{\alpha}\left[\bar{L}_N + \bar{L}_S - a_N g - \frac{\alpha}{1-\alpha}a_N g\ \frac{\rho+m+g}{m+g}\right]^{1-\alpha}}.$$

The second term of the above expression is always an increasing function of g. The first term of the above expression is a positive function of g if

$$\frac{a_N(\rho + g + m)(g + m)}{\bar{L}_N + \bar{L}_S - a_N g} > \rho. \tag{2.2.38}$$

Note that if ρ is sufficiently small then the condition (2.2.38) is always satisfied. Then, for a small positive value of ρ, the L.H.S. of equation (2.2.37) is an increasing function of g. The R.H.S. of equation (2.2.37) does not depend on g. Then the equality of the L.H.S. and the R.H.S. of equation (2.2.37) at some unique positive value of g is guaranteed if

$$\frac{\frac{a_N}{1-\alpha} m + \frac{\alpha}{1-\alpha} a_N \rho}{\{\frac{\alpha}{1-\alpha} a_N(\rho + m)\}^{\alpha}} < (\bar{L}_N + \bar{L}_S)^{1-\alpha}. \tag{2.2.39}$$

Once g is solved, we can solve for M from equation (2.2.35) and for ξ from equation (2.2.34). So we have the following proposition.

Proposition 2.2.1. *The existence of a unique steady state equilibrium solution is guaranteed when ρ is very small and when condition (2.2.39) is satisfied.*

A graphical presentation of the equilibrium is shown in figure 2.2.1 Here the L.H.S. of equation (2.2.37) is represented by the positively sloped NN curve; and the horizontal XX curve represents the R.H.S. of equation (2.2.37). The steady state equilibrium is determined at the point E. For a very high value of ρ, the NN curve may become negatively sloped; and hence it may not intersect the XX curve[26].

2.2.3 IPR tightening

Equations (2.2.35) and (2.2.36) are two equations to determine two unknowns, M and g. In Helpman (1993), $M \equiv 0$; and equation (2.2.35) determines the value of g but equation (2.2.36) does not exist. We now turn to analyse the effect of a change in m on the equilibrium values of g and M in this model. Helpman (1993) defines the strengthening of IPR protection as an exogenous reduction in the value of m. The

[26]In Helpman (1993), $g = (1 - \alpha)\frac{L_N}{a_N} - \rho\alpha$; and hence g may be negative for a very high value of ρ.

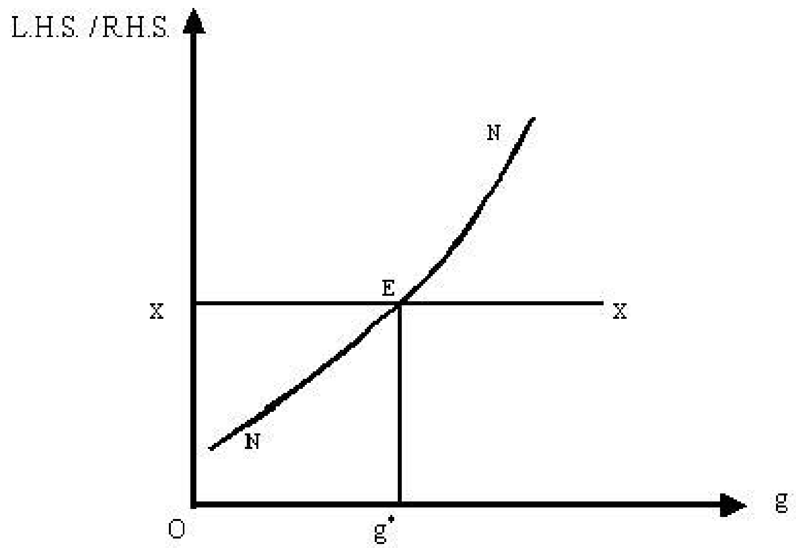

Figure 2.2.1 : Existence of the steady-state equilibrium.

comparative steady state effects on g and M with respect to change in m are derived in Appendix (2.14). We show that

$$\frac{\partial g}{\partial m} = \frac{-\left(\Delta - \frac{1}{m}\frac{1}{\bar{L}_N+M-a_N g}\right)}{\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}} \ ;$$

$$\frac{\partial M}{\partial m} = \frac{\frac{(\bar{L}_N+M)}{g(\bar{L}_N+M-a_N g)}\frac{1}{m} - \frac{\rho}{(\rho+g+m)(g+m)}\left(\frac{1}{m} + \frac{\bar{L}_N+M-\frac{a_N g}{1-\alpha}}{(\bar{L}_N+M-a_N g)g}\right)}{\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}} \ ;$$

and

$$\Delta = \frac{\rho}{(\rho+g+m)(g+m)}\left\{\frac{\bar{L}_N+M-\frac{a_N g}{1-\alpha}}{(\bar{L}_N+M-a_N g)(\bar{L}_N+M)} + \frac{1}{\bar{L}_S-M}\right\} > 0 \ .$$

Here, $\lim_{\rho\to 0}\Delta = 0$; and hence

$$\lim_{\rho\to 0}\left(\frac{\partial g}{\partial m}\right) = -\frac{\bar{L}_S-M}{m(\bar{L}_N+M)} < 0 \ ;$$

and

$$\lim_{\rho \to 0} \left(\frac{\partial M}{\partial m} \right) = -\frac{\bar{L}_S - M}{m} < 0 \ .$$

In figure 2.2.1, the horizontal XX curve shifts upward when m is reduced. However, the positively sloped NN curve does not shift in this case when $\rho = 0$. So we can establish the following proposition.

Proposition 2.2.2. *A policy of strengthening IPR protection in the South may lead to an increase in the rate of innovation as well as in the volume of South North migration if the representative consumer discounts the future at a very low rate.*

So we find a possibility of a positive effect of the policy of IPR tightening in the South on the rate of innovation in the presence of perfect international mobility of labour in an otherwise identical Helpman (1993) model. Helpman (1993) finds a negative effect on the innovation rate in a world of labour immobility; and he claims that the South can never benefit from this policy[27]. We are not analysing the welfare impact of this policy in this extended model because it involves a lot of technical complications. However, our result points out that the strengthening of IPR protection in the South may lead to welfare gains in both the regions (countries) via its positive effect on product availability[28].

We now try to understand the intuition of this innovative result. This can be clearly understood if we assume ρ to be zero[29]. $M \equiv 0$ in Helpman (1993); and so, with $\rho = 0$, g becomes independent of m in his model[30]. This is so because the profit rate of the Northern firm, $\frac{\pi_N}{v_N}$, and the cost of capital, $\rho + m + g$, move proportionately in this case. However, in the present model, M is endogenously determined and there is a positive

[27]See section 5 of Helpman (1993), page-1274.

[28]The welfare gain of the South due to product availability may outweigh all other adverse welfare effects of stricter IPR protection and the South may be a net welfare gainer. See Mondal & Gupta (2006a) for such a result.

[29]We are aware of the fact that $\rho = 0$ assumption disturbs the boundedness property of the objective functional.

[30]See equation (2.2.35) in the present model.

migration equilibrium partial effect[31] on M. So the reduction in m raises $\bar{L}_N + M$; and hence the profit rate is reduced at a lower rate than the cost of capital. Hence there should be an increase in g to satisfy the equation (2.2.35). However, the migration equilibrium condition requires a positive relationship between g and M to be satisfied. So, in the new steady state equilibrium, both the volume of South North migration, M, and the rate of innovation, g, are increased simultaneously due to a strengthening of IPR protection. This positive effect exists even for a small positive value of ρ provided that the equilibrium exists. When ρ takes a high value, the equilibrium may not exist.

2.2.4 Changes in labour endowments

We have derived the comparative steady state effects of changes in the labour endowments in the two countries in Appendix (2.14). The results are summarized here.

$$\frac{\partial g}{\partial \bar{L}_N} = \frac{\partial g}{\partial \bar{L}_S} = \frac{-\left(\frac{1}{\bar{L}_N+M-a_N g}\right)\left(\frac{1-\alpha}{\bar{L}_S-M}\right)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)};$$

$$\frac{\partial M}{\partial \bar{L}_N} = \frac{\left(\frac{1}{m+g} - \frac{1}{\rho+m+g}\right)(-\Delta_1)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)};$$

and

$$\frac{\partial M}{\partial \bar{L}_S} = \frac{\frac{1-\alpha}{\bar{L}_S-M}(\Delta_2)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)};$$

where Δ is already defined in section 3.2. Also,

$$\Delta_1 = \frac{(1-\alpha)(\bar{L}_N+M) - a_N g}{(\bar{L}_N+M)(\bar{L}_N+M-a_N g)} > 0;$$

and

$$\Delta_2 = \left(\frac{1}{m+g} - \frac{1}{\rho+m+g} - \frac{a_N}{\bar{L}_N+M-a_N g} - \frac{1}{g}\right) < 0.$$

It can be easily shown that

$$\lim_{\rho\to 0}(\Delta) = 0;$$

[31] It is the effect of a change in m on M taking place through equation (2.2.36) and keeping g unchanged.

and

$$\lim_{\rho \to 0}(\Delta_2) = -\frac{\bar{L}_N + M}{\left(\bar{L}_N + M - a_N g\right) g} < 0.$$

Then we obtain

$$\lim_{\rho \to 0}\left(\frac{\partial g}{\partial \bar{L}_N}\right) = \lim_{\rho \to 0}\left(\frac{\partial g}{\partial \bar{L}_S}\right) = \frac{g}{\bar{L}_N + M} > 0;$$

$$\lim_{\rho \to 0}\left(\frac{\partial M}{\partial \bar{L}_N}\right) = 0;$$

and

$$\lim_{\rho \to 0}\left(\frac{\partial M}{\partial \bar{L}_S}\right) = 1 > 0.$$

However, for any value of ρ, we have

$$\left(\frac{\partial M}{\partial \bar{L}_S}\right) - \left(\frac{\partial M}{\partial \bar{L}_N}\right) = 1.$$

The increase in either $\bar{L}_N$ or $\bar{L}_S$ produces a symmetric rightward shift of the positively sloped NN curve in the figure 2.2.1 while the horizontal XX curve remains unchanged. The effects of changes in the labour endowments of North and South on the rate of innovation are identical but their effects on the volume of South North migration are different. We summarize the above results in the following proposition.

Proposition 2.2.3. *An increase in the labour endowment of each of the two regions has the identical positive effect on the rate of innovation in the North if the representative consumer discounts the future at a very low rate. However, the change in the Southern labour endowment affects the volume of South North migration positively and at a higher rate than the change in the Northern labour endowment.*

The result is interesting because this goes against what we find in Helpman (1993). In the steady state growth equilibrium in Helpman (1993), the rate of innovation is independent of the change in the Southern labour endowment and varies positively with the size of the Northern labour endowment only. We now provide the intuition behind this new result summarized in proposition 3. Here labour is perfectly mobile between the two regions. So it is the aggregate (Northern plus Southern) labour endowment that matters in determining the rate of innovation in the steady state growth equilibrium. So the effect of a change in the labour endowment on the rate of innovation is always symmetric whatever be its origin[32]. Hence the increase in the Southern labour endowment must have a positive effect on the rate of innovation through international migration. However, the effect of a change in the labour endowment on the volume of migration is not symmetric. Volume of South North migration varies positively with the rate of innovation and negatively with the size of the Northern labour endowment[33]. The increase in the Southern labour endowment has only a positive effect on the level of South North migration through its positive effect on the innovation rate. However, the increase in the Northern labour endowment has an additional negative effect on the volume of migration apart from the symmetric positive innovation effect. So the net positive effect of the increase in the Northern labour endowment is smaller than that of the increase in the Southern labour endowment.

2.3 International outsourcing

In this section, we introduce outsourcing of jobs from the North to the South in an otherwise identical Helpman (1993) model. We assume that the North outsources an exogenous fraction of the R&D sector jobs as well as of the production jobs to the South. Labour endowment is country specific and the definition of the knowledge stock in the North follows Helpman (1993). Though outsourcing of production (or, manufacturing) jobs from the North to the South are common phenomenon today, outsourcing of R&D jobs are less well known. However, according to a report of the R&D magazine 2001 issue, more than 25% of the total R&D jobs of the largest US corporations will be performed with the outside partners. Also increasing number of Northern companies are engaged in outsourcing of information technology jobs to countries such as India and

[32]See equation (2.2.37).
[33]See equation (2.2.35).

China, as well as manufacturing jobs to China and South-East Asia.

We abstract from the issues regarding the causes of outsourcing and the determination of the rate of outsourcing. Our focus is on the analysis of the growth effects of exogenous outsourcing. We show that in the presence of outsourcing, the growth effect of increased outsourcing and of stronger IPR protection in the South crucially depends on the nature of outsourcing. In the case of innovation outsourcing, the steady state equilibrium rate of innovation may be reduced due to increased volume of outsourcing of R&D jobs from the North to the South. However, just the opposite result is true in the case of production outsourcing. Also the strengthening of IPR protection in the South may raise the steady state equilibrium rate of innovation in the presence of innovation outsourcing and may lower it in the case of production outsourcing. We do not want to mean that both types of outsourcing are equally important in reality. We want to highlight on the point that the effects of outsourcing and of IPR strengthening are related to the nature of outsourcing. So there should be a proper coordination between the Northern policy of outsourcing and the Southern policy of strengthening IPR protection.

This section is organised as follows. In subsection 2.3.1, we present the basic model. The comparative steady state effects on the growth (innovation) rate with respect to change in the parameters are analysed in subsection 2.3.2; and the results related to transitional dynamic properties of the model are summarized in subsection 2.3.3.

2.3.1 The basic model

The behavior of the household is identical to that described in two earlier sections. The optimality conditions are

$$\frac{\dot{E}}{E} = r - \rho; \tag{2.3.1}$$

and

$$x(z) = E(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}du} \qquad \forall z \in [0, n(t)]. \tag{2.3.2}$$

We assume that the Northern producers produce only γ fraction of each unit of the product at home and outsource the remaining $(1-\gamma)$ fraction of the production to the South. Here γ is exogenously given. So the marginal cost of production of a representative variety produced by a Northern firm is given by

$$MC_N = \gamma w_N + (1-\gamma)w_S.$$

For $\gamma = 1$, we have $MC_N = w_N$ which is obtained in Helpman (1993). The price of any Northern product is given by

$$p(z) = p_N = \frac{\gamma w_N + (1-\gamma)w_S}{\alpha} \tag{2.3.3}$$

for all $z \in [0, n_N]$. The price of an imitated Southern product is given by

$$p(z) = p_S = w_S \tag{2.3.4}$$

for all $z \in [0, n_S]$. If $\gamma = 1$, equation (2.3.3) is same as that obtained in the Helpman (1993) model. We also assume that

$$w_N > w_S.$$

The labour market equilibrium condition in the North is given by

$$L_N = \gamma n_N x_N + L_R^N. \tag{2.3.5}$$

The Southern labour, L_S, is employed in outsourced R&D and production jobs and in producing the imitated varieties. Hence

$$L_S = (1-\gamma)n_N x_N + n_S x_S + L_R^S \tag{2.3.6}$$

is the labour market equilibrium condition in the South. Here $(1-\gamma)n_N x_N$, and L_R^S stand for the Southern labour employed in doing the outsourced Northern production jobs and in doing the outsourced Northern R&D jobs respectively. If $\gamma = 1$ and $L_R^S = 0$, then equations (2.3.5) and (2.3.6) are same as the corresponding ones in the Helpman (1993) model.

R&D sector in the North produces new product designs using labour; and thus the number of varieties grow over time. We assume that β fraction of every product design is made in the North and the remaining $(1-\beta)$ fraction is outsourced to the South. β and γ are exogenous to our model; and we interpret them either as the South's absorptive capacity of the R&D jobs and the production jobs or as the outcomes of the policy decisions of the Northern government. Since β fraction of each design is made in the North and the remaining fraction is outsourced to the South, we can obtain the following equations.

$$\beta \dot{n} = \frac{n}{a_N} L_R^N, \tag{2.3.7}$$

and

$$(1-\beta)\dot{n} = \frac{n}{a_S} L_R^S. \tag{2.3.8}$$

Here $\frac{\beta a_N}{n}$ is the labour requirement in the North and $\frac{(1-\beta)a_S}{n}$ is the labour requirement in the South to develop a new product design; and n is the stock of knowledge capital. If $\beta = 1$, equation (2.3.7) is same as the equation (2.1.7) in section 2.1 and equation (2.3.8) does not exist. We assume that

$$\frac{w_S}{w_N} < \frac{a_N}{a_S} \leq 1.$$

Here the first inequality implies that the unit cost of producing a new product design in the North is greater than that in the South; and the second inequality implies that the productivity of the R&D workers in the North is not lower than that in the South.

The no-arbitrage condition in the Northern asset market is given by

$$\frac{\pi_N}{v_N} + \frac{\dot{v}_N}{v_N} = r + m; \tag{2.3.9}$$

and the maximum profit of the Northern monopolist producing any variety is given by

$$\pi_N = \frac{1-\alpha}{\alpha}(\gamma w_N + (1-\gamma)w_S)x_N. \tag{2.3.10}$$

Here also

$$m = \frac{\dot{n_S}}{n_N}.$$

The free entry condition in the R&D sector in the North with outsourcing of R&D jobs is now modified as follows

$$\frac{\beta a_N}{n}w_N + \frac{(1-\beta)a_S}{n}w_S = v_N. \tag{2.3.11}$$

Here the left hand side of equation (2.3.11) is the cost of developing a new variety whose $(1-\beta)$ fraction is developed in the South. If $\beta = 1$, equation (2.3.11) is same as equation (2.1.10) in section 2.1.

We assume that the world is in a steady-state growth equilibrium; and hence

$$\frac{\dot{n}}{n} = \frac{\dot{n_N}}{n_N} = \frac{\dot{n_S}}{n_S} = g$$

where g is the balanced growth rate. The market value of each of the Northern firms is normalised to unity[34]. We do it following Lai (1998, p. 137, footnote no. 5). Hence we have

$$\frac{\dot{E}}{E} = \frac{\dot{n}}{n}.$$

Walras law is to be satisfied and labour-endowments L_N and L_S are exogenously given. So, in the steady state growth equilibrium, we have

$$g = \frac{\dot{n}}{n} = \frac{\dot{n_N}}{n_N} = \frac{\dot{n_S}}{n_S} = \frac{\dot{w_S}}{w_S} = \frac{\dot{w_N}}{w_N} = \frac{\dot{E}}{E}. \tag{2.3.12}$$

This completes the equational structure of the model. Now we solve for the long run equilibrium rate of innovation, g.

[34]In the original paper of Helpman, $\frac{\dot{v_N}}{v_N} = -g$, because there expenditure is normalised to unity.

In the steady state growth equilibrium, we have $\dot{v_N} = 0$; and hence using equations (2.3.9), (2.3.10) and (2.3.11), we have

$$\frac{\frac{1-\alpha}{\alpha}[\gamma + (1-\gamma)\frac{w_S}{w_N}](n_N x_N)}{\beta a_N(\frac{n_N}{n}) + (1-\beta)a_S(\frac{w_S}{w_N})(\frac{n_N}{n})} = (r+m). \tag{2.3.13}$$

Here the left hand side of equation (2.3.13) shows the profit rate of the Northern firm and the right hand side represents its cost of capital. Note that, other things remaining unchanged, the profit rate is an increasing function of outsourcing rate of the production jobs, γ, and is a decreasing function of that of the R&D jobs, β. This is so because we have already assumed that $\frac{w_S}{w_N} < \frac{a_N}{a_S} \leq 1$. Given $0 < \gamma < 1$ and $0 < \beta < 1$, the relationship between the profit rate and the North-South relative wage, $\frac{w_N}{w_S}$, is indeterminate. If $\gamma = 1$ and $0 < \beta < 1$, i.e., if there is only R&D outsourcing, then the profit rate varies inversely with $\frac{w_S}{w_N}$. On the other hand, if $\beta = 1$ and $0 < \gamma < 1$, i.e., if there is only outsourcing of production jobs, then the profit rate varies directly with $\frac{w_S}{w_N}$. Helpman(1993) model is a special case of it when $\beta = \gamma = 1$, i.e., when there is no outsourcing. In this case, the profit-rate is independent of $\frac{w_N}{w_S}$. Any policy that raises the profit rate at a higher rate than the cost of capital will result in a higher value of g in the new steady state equilibrium.

Using equations (2.3.5), (2.3.6) and (2.3.12) we have

$$\gamma n_N x_N = L_N - \beta a_N g,$$

and

$$n_S x_S = L_S - (1-\beta)a_S g - (1-\gamma)n_N x_N.$$

Using equations (2.3.2), (2.3.3), (2.3.4) and the above mentioned expressions of $n_N x_N$ and $n_S x_S$, we can solve for the South-North relative wage as

$$\frac{w_S}{w_N} = \frac{\gamma.\Omega^{1-\alpha}}{\alpha - (1-\gamma).\Omega^{1-\alpha}} \tag{2.3.14}$$

where[35]

$$\Omega = \left[\frac{(L_N - \beta a_N g)}{L_S - (1-\beta)a_S g - \frac{1-\gamma}{\gamma}(L_N - \beta a_N g)} \right] \frac{m}{\gamma g}. \tag{2.3.15}$$

Using equations (2.3.13) and (2.3.14) we have

$$\frac{\gamma + (1-\gamma)[\frac{\gamma.\Omega^{1-\alpha}}{\alpha-(1-\gamma).\Omega^{1-\alpha}}]}{\beta a_N + (1-\beta)a_S[\frac{\gamma.\Omega^{1-\alpha}}{\alpha-(1-\gamma).\Omega^{1-\alpha}}]} \frac{1-\alpha}{\alpha\gamma}(L_N - \beta a_N g)(1 + \frac{m}{g}) = (r+m). \tag{2.3.16}$$

This equation (2.3.16) solves for the equilibrium value of g. If $\beta = \gamma = 1$, we have

$$\frac{1-\alpha}{\alpha a_N}(L_N - a_N g)(1 + \frac{m}{g}) = r + m$$

which is identical to equation (29) in the Helpman (1993, p. 1261) model. With $0 < \beta, \gamma < 1$, it is very difficult to analyse the existence, uniqueness and stability of the equilibrium and to find out the comparative steady state effects. So we simplify our analysis considering the two types of outsourcing separately.

2.3.2 Comparative steady-state effects

2.3.2.1 Only innovation outsourcing

Here we assume that there is only innovation outsourcing and no production outsourcing. So $\gamma = 1$ and $0 < \beta < 1$. Then putting $\gamma = 1$ in equation (2.3.16) and then using equation (2.3.15), we have

$$(1-\alpha)g^{1-\alpha} = \frac{g(r+m)}{g+m}\frac{(a_N \alpha\beta)g^{1-\alpha}}{L_N - \beta a_N g} + \frac{g(r+m)}{g+m}\frac{(1-\beta)a_S}{(L_N - \beta a_N g)^{\alpha}}(\frac{m}{L_S - (1-\beta)a_S g})^{1-\alpha} \tag{2.3.17}$$

From equations (2.3.1) and (2.3.12) we have $r = \rho + g$. Figure 2.3.1 shows the L.H.S. and the R.H.S. of equation (2.3.17) as a function of g. The L.H.S. of equation (2.3.17)

[35] In deriving the equation (2.3.15) we have used the fact that in the steady-state

$$\frac{n_N}{n_S} = (\frac{n_N}{\dot{n_S}})(\frac{\dot{n_S}}{n_S}) = \frac{g}{m}.$$

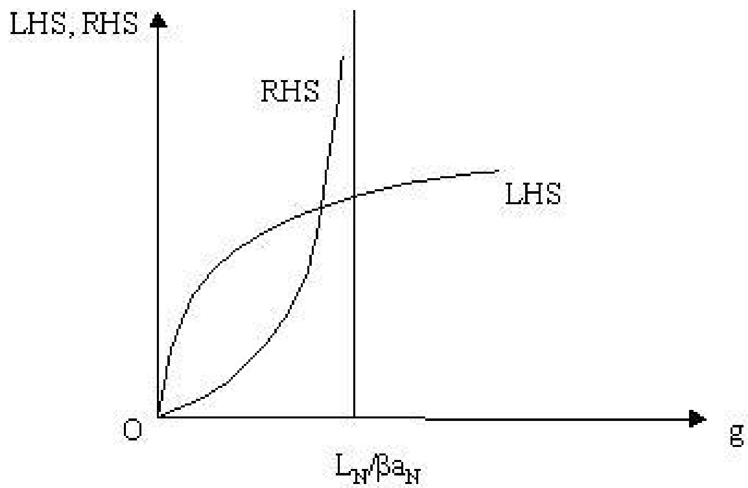

Figure 2.3.1 : Determination of g in case of R&D outsourcing

is an increasing and concave function of g whose corresponding curve starts from the origin because $0 < \alpha < 1$. At $g = \frac{L_N}{\beta a_N}$, the L.H.S. is positive and finite. The R.H.S. of equation (2.3.17) is an increasing function of g and its corresponding curve starts from the origin and is asymptotic[36] to the vertical straight line $g = \frac{L_N}{\beta a_N}$. It satisfies the property that

$$\lim_{g \to 0^+} \frac{d(L.H.S.)}{dg} > \lim_{g \to 0^+} \frac{d(R.H.S.)}{dg}.$$

So the R.H.S. curve must intersect the L.H.S. curve from below in figure 2.3.1; and the two curves must intersect once. Thus the existence of a unique steady-state equilibrium is ensured. We can now perform the comparative steady-state exercise[37] with respect to changes in the two parameters, m and β. A decrease in m implies the stronger protection of IPR in the South and a decrease in β represents increased rate of outsourcing of

[36]We assume that $\frac{L_S}{(1-\beta)a_S} > \frac{L_N}{\beta a_N}$. This implies that the relative size of the South is bigger than that of the North. The curvature of the R.H.S. function is analysed in Appendix (2.15).

[37]Mathematical derivation is shown in Appendix (2.16). Stability analysis of the steady-state equilibrium is shown in Appendix (2.18).

Northern R&D jobs to the South. The L.H.S. of equation (2.3.17) is independent of changes in m and β. A decrease in m leads to a downward shift of the R.H.S. curve in the figure 2.3.1 when $\rho \to 0$ and thus g is increased in the new equilibrium. However, a decrease in m leads to an upward shift of the R.H.S. curve in figure 2.3.1 when $m \to 0$, and then the equilibrium value of g is reduced. This leads to the following proposition.

Proposition 2.3.1. *In the case of only innovation outsourcing, strengthening of IPR in the South (i) raises the long run equilibrium rate of innovation in the North when consumers are very patients in their intertemporal choice and (ii) lowers that rate of innovation when the rate of imitation in the South is negligible.*

We now turn to provide the intuition behind this result. As m is decreased, the cost of capital of the Northern firm is reduced. However, the profit rate is increased when $\rho \to 0$ i.e., when the consumer is very patient in her intertemporal choice. There are two separate effects of the reduction in m on the profit rate in this model. The first one is the direct negative effect which Helpman (1993) considers. This effect operates through an increase in $\frac{n_N}{n}$ which lowers the per firm share of production in the North. The second one is an indirect positive effect working through the change in the North-South relative wage. This effect does not exist in the Helpman(1993) model due to the absence of outsourcing. The former effect tends to lower the profit rate while the later has a favourable impact on the profit rate. Cost of capital is reduced due to a decrease in the imitation rate. The disincentive to do R&D in the North resulting from this former negative effect on the profit rate is outweighed by the reduction in the cost of capital when ρ is very low. In fact, g becomes independent of m when $\rho = 0$ in the Helpman (1993) model. However, in our model the second positive effect on the profit rate working through the channel of North-South relative wage raises the profit rate relative to the cost of capital when $\rho \to 0$; and thus the rate of innovation is increased in the new steady state equilibrium. The second positive effect on the profit rate becomes very weak when the rate of imitation is negligible; and hence it is dominated by the first negative effect. So the rate of innovation in the North is reduced in that case.

A reduction in m causes the R.H.S. curve in figure 2.3.1 to shift upward when $\beta \to 1$; and so the value of g is reduced in the new equilibrium. This confirms the Helpman(1993) result that strengthening IPR protection in the South lowers the Northern rate of innovation in the absence of outsourcing.

A decrease in β leads to a downward shift of the R.H.S. curve in figure 2.3.1 when $m \to 0$. So the value of g is increased in the new steady state equilibrium. This leads to the following proposition.

Proposition 2.3.2. *An increase in the rate of outsourcing of the Northern innovation activity raises the long run equilibrium rate of innovation in the North if the rate of imitation in the South is negligible.*

We provide the intuition of this result. A decrease in β affects the profit rate of the Northern firm through two channels. (i) It raises the size of the Northern production sector which, in turn, raises the profit rate. (ii) It also raises the relative demand for the Southern labour which raises the South-North relative wage and thus lowers the profit rate. The relative wage effect does not work when the rate of imitation is negligible. So a decrease in β raises the profit rate by increasing the size of the Northern production sector in this case. Thus the equilibrium value of g is increased.

We now turn to analyse the effect of strengthening IPR protection in the South on the North South relative wage in the steay state equilibrium. Using equation (2.3.14) and $\gamma = 1$, we have

$$\frac{w_S}{w_N} = \frac{\Omega^{1-\alpha}}{\alpha}$$

where

$$\Omega = \frac{(L_N - \beta a_N g)m}{[L_S - (1-\beta)a_S g]g}.$$

Here $\frac{w_S}{w_N}$ is an increasing function of Ω because $0 < \alpha < 1$; and Ω is an increasing function of m because $L_N > \beta a_N g$. Ω is a decreasing function of g if

$$\frac{L_S}{(1-\beta)a_S} > \frac{L_N}{\beta a_N} .$$

The proposition 2.3.1 states that a decrease in m raises g for a sufficiently small value of ρ. This implies that Ω is decreased due to a decrease in m for a sufficiently small value of ρ. Hence a stronger IPR protection in the South raises the relative wage of the North when the consumers are very patient in their intertemporal choices. The proposition 2.3.1 also states that a decrease in m lowers g when m is sufficiently close to zero. This implies that Ω is decreased due to a reduction in m when m is sufficiently small. So a policy of stronger IPR protection adopted in the South raises the relative wage of the North when the rate of imitation is negligible.

The proposition 2.3.2 states that a decrease in β raises g when m is sufficiently small. So an increase in the rate of R&D outsourcing affects the North South relative wage in two opposite ways. First, given g, a decrease in β raises the Southern relative wage by raising the relative demand for the Southern labour. Secondly, an increase in g raises the proportion of products not yet imitated by the South; and this lowers the Southern relative wage. However, both these two effects are negligible when m is sufficiently small. So the North South relative wage remains independent of β in this case. Otherwise, the net effect is indeterminate. The effects on North-South relative wage are summarized in the following proposition.

Proposition 2.3.3. *(i) A policy of stronger IPR protection adopted in the South raises the North-South relative wage when the consumers are very patient in their intertemporal choices or when the rate of imitation is negligible. (ii) The increase in the rate of outsourcing of R&D jobs does not affect the North-South relative wage when the rate of imitation is negligible.*

2.3.2.2 Only production outsourcing

Here we assume that there is only production outsourcing and no innovation outsourcing. So $\beta = 1$ and $0 < \gamma < 1$. Then putting $\beta = 1$ in equation (2.3.16) we have

$$1 + \frac{(1-\gamma)\Omega^{1-\alpha}}{\alpha - (1-\gamma).\Omega^{1-\alpha}} = \frac{(r+m)g}{g+m}\frac{\alpha}{1-\alpha}\frac{a_N}{L_N - a_N g} \tag{2.3.18}$$

where

$$\Omega = \frac{L_N - a_N g}{L_S - \frac{1-\gamma}{\gamma}(L_N - a_N g)}.\frac{m}{\gamma g}.$$

This is the expression of Ω obtained from equation (2.3.15) using $\beta = 1$. Using equations (2.3.1) and (2.3.12) we have $r = \rho + g$. Here Ω is a decreasing function of g and the L.H.S. of equation (2.3.18) is an increasing function of Ω. Hence the L.H.S. of equation (2.3.18) is a decreasing function of g. The R.H.S. of equation (2.3.18) is an increasing function of g whose corresponding curve in the figure 2.3.2 is starting from the origin and is asymptotic to the vertical line $g = \frac{L_N}{a_N}$[38]. So the two curves representing the L.H.S. and the R.H.S. of equation (2.3.18) and drawn in figure 2.3.2 must intersect once and thus the existence of an unique steady state equilibrium value of g is ensured[39]. A decrease in m causes the L.H.S. curve in figure 2.3.2 to shift downward because $0 < \gamma < 1$ and because the L.H.S of equation (2.3.18) is a positive function of Ω which, in turn, varies directly with m. However, in Helpman(1993), $\gamma = 1$; and hence the L.H.S. is independent of the change in m. It also causes the R.H.S. curve to shift upward because $r = \rho + g$. So $(r + m)$ falls at a lower rate than $(g + m)$ when m falls with given g. This shift is similar to that in Helpman (1993). So in the new steady state equilibrium, the value of g must be decreased.

Also a decrease in γ causes the L.H.S. curve in figure 2.3.2 to shift upward while the R.H.S. curve remains unaltered. This is so because Ω varies inversely with γ and so the

[38] It is shown in the Appendix (2.17).

[39] Stability analysis of the steady state equilibrium is shown in Appendix (2.19).

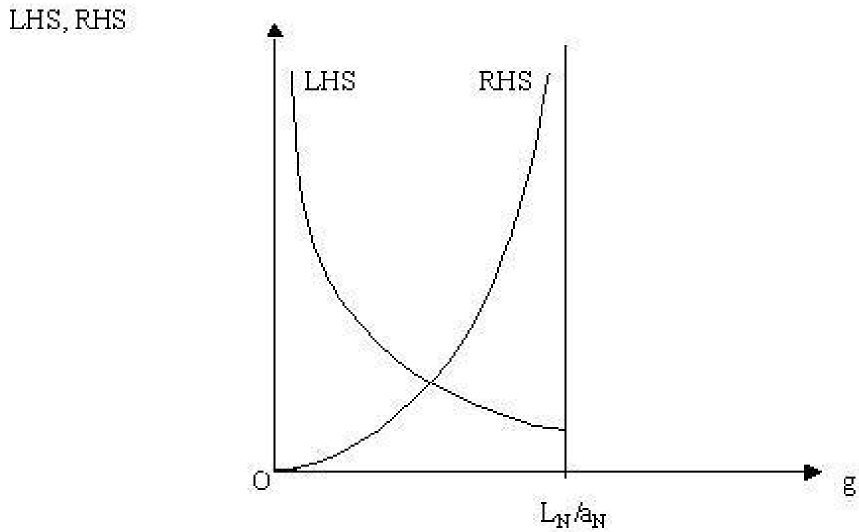

Figure 2.3.2 : Determination of g in case of production outsourcing

L.H.S. of equation (2.3.18) varies negatively with γ. However, the R.H.S. of equation (2.3.18) is independent of γ. So g must increase in the new equilibrium. These results are summarised in the following proposition.

Proposition 2.3.4. *In the case of only production outsourcing, (i) strengthening of IPR protection in the South always lowers the steady state equilibrium rate of innovation in the North at a higher rate than that in the Helpman(1993) model; and (ii) an increase in the degree of outsourcing always raises that rate of innovation.*

In the present model, with North-South outsourcing in production, the profit rate is an increasing function of the South North relative wage. Given g, a decrease in m causes the South North relative wage to fall and thus the profit rate is decreased. This increase in the North South relative wage acts as an additional channel through which the profit rate is reduced. So the profit rate is reduced at a higher rate in this model than that in the Helpman (1993) model because, in Helpman(1993), profit rate is independent of the relative wage. So the rate of innovation in this model is also reduced at a higher rate compared to that in the Helpman (1993) model.

Using equation (2.3.14) we have

$$\frac{1-\gamma}{\gamma}\frac{w_S}{w_N} = \frac{1}{\frac{\alpha}{(1-\gamma)\Omega^{1-\alpha}} - 1} .$$

We have already noted that Ω varies inversely with γ. So a decrease in γ causes an increase in $\frac{1-\gamma}{\gamma}\frac{w_S}{w_N}$. The increase in $\frac{1-\gamma}{\gamma}\frac{w_S}{w_N}$ is interpreted as an increase in the average cost of production of an outsourced product in the South relative to that of its non outsourced component in the North. This increase in the relative cost of production raises the profit rate, given g. However, given g, the cost of capital is not affected by a decrease in γ. Hence the rate of innovation, g, is increased in the new steady state equilibrium.

We now turn to study the effect of strengthening IPR protection on the North South relative wage in the steady state equilibrium. We express equation (2.3.14) as

$$\frac{w_N}{w_S} = \frac{\alpha}{\gamma\,\Omega^{1-\alpha}} - \frac{1}{\gamma} + 1$$

where

$$\Omega = \frac{(L_N - a_N g)\, m}{\left[L_S - \frac{1-\gamma}{\gamma}(L_N - a_N g)\right]\gamma g}.$$

Ω is an increasing function of m and a decreasing function of g. Also $\frac{w_N}{w_S}$ is a decreasing function of Ω. However, from the proposition 2.3.2, we know that a reduction in m lowers g. So the reduction in m affects the North South relative wage in two opposite ways. Hence the net effect of adopting a policy of stronger IPR protection in the South on the North South relative wage is ambiguous in our model. However, it is easy to verify that the indirect effect of the reduction in m through lowering g is negligible when the rate of imitation is very low. Thus the introduction of a policy of stronger IPR protection in the South raises the North South relative wage in this case.

The effect of the increase in the rate of production outsourcing on the North South relative wage can also be divided into two parts. First, given g, a reduction in γ raises the relative demand for the Southern labour and this raises the South North relative wage. Secondly, a reduction in γ raises g; and this, in turn, lowers the South North relative wage because then more firms stay in the North raising the relative demand for the Northern labour. These two effects operate in opposite directions and the net effect becomes ambiguous. The effects on the North-South relative wage are summarized in the following proposition.

Proposition 2.3.5. *In the case of only production outsourcing, (i) a policy of stronger IPR protection adopted in the South raises the North-South relative wage when the rate of imitation is negligible and (ii) the increase in the rate of outsourcing produces ambiguous effect on the North-South relative wage.*

2.3.3 Transitional dynamic properties

We summarize the major results in the form of the following proposition. The mathematical derivations are given in the Appendix (2.18) and Appendix (2.19).

Proposition 2.3.6. *The steady-state equilibrium is saddle point stable with a unique saddle path converging to the steady state equilibrium point if the imitation rate is very low.*

Our result is valid in both the cases - R&D outsourcing and production outsourcing. However, we can not prove this result when imitation rate is high. Helpman (1993) has proved the saddle point stability of the steady-state equilibrium in his model and it was independent of the value of the rate of imitation. We can not find the explicit solutions of the equation of motions and hence can not find out the welfare effects what Helpman (1993) could find out in his simplified model. However, the results related to welfare effects in the Helpman (1993) model are conditional on the assumption that the rate of imitation is very small.

Chapter 3

Product Cycle Models with Endogenous Imitation

Introduction

The model of Grossman and Helpman (1991b) has been one of the most complete and influential analyses of Vernon's (1966) idea of international product cycle. A developed group of countries, called the North, first invent new products and supply them in the international market. The developing countries, called the South, follow up through imitations and gradually specialize in the production of these goods over time. Capturing and explaining product cycles is crucial for understanding the gains from trade across developed and developing countries. Grossman and Helpman (1991b) took this empirically plausible idea into a rigorous theoretical analysis, utilizing state-of-the-art tools and concepts of modern endogenous-growth theory and bringing incentives into the picture.

Due to the fact that the North-South trade model of Grossman and Helpman (1991b) captures how economic incentives respond to primitives, it provides a platform for undertaking a wide range of extensions, such as introducing tariff policies, migration, trade unions etc. However, the model is highly complicated and so the comparison of two alternative trade policies is typically possible only in the steady state. But evaluating the welfare effects of policies requires to account for the transition to a new steady state; for this, it is important to know whether the new steady state is stable. However, Grossman and Helpman assumed the existence of a steady state equilibrium of the global economy; and remained silent about the stability properties of that equilibrium in their paper (1991b), as well as in their book (1991c, chapter 11).

The task of the present chapter is two fold. First, in section 3.1, we provide a local stability analysis of the steady state equilibrium of the GH (1991b) model[1] which assumes the so-called Marshall-Arrow-Romer (MAR) kind of spillover in the Northern R&D sector. Secondly, in section 3.2, we modify the GH (1991b) model replacing MAR spillover by Jacob's (1969) type of localised konwledge spillover in the Northern R&D sector and analyse various comparative steady state and dynamic properties of this modified model. We also analyse the local stability properties of the steady state equilibrium in this modified model.

3.1 Stability analysis of the Grossman-Helpman (1991b) model[2]

In this section, we provide a local stability analysis of the steady state equilibrium in the GH (1991b) product variety model. It is shown that there exists a unique saddle path converging to the steady state equilibrium in this model in both the wide gap equilibrium case and in the narrow gap equilibrium case. It is true that our analysis has not progressed up to global stability because of the difficulty of the problem[3]. Even the finding that this model is locally saddle point stable may be interesting to scholars of trade theory. Ultimately, researchers, while considering alternative trade policies, resort to numerical analysis through calibration exercises. However, the prior information that the model is well-behaved is important for conducting calibration exercises. This

[1] Hereafter we often refer 'GH' as 'Grossman and Helpman'.

[2] **This section is based on Mondal (2007).**

[3] For example, Arnold (2000a) has achieved more, but it must be understood that the Romer (1990) model of technical change is far simpler than the GH (1991b) model of endogenous product cycles.

would convince researchers that, applying, for example, the time-elimination method may deliver a refined equilibrium addressing the structure of the model properly, instead of leaving open the question of whether numerical analysis solves for one of many possible trajectories near the steady-state.

Initially, authors of endogenous growth models were more interested in determining the rate of growth in the steady state equilibrium but did not investigate the stability properties of the equilibrium. However, some researchers in recent times have shown interest in this direction. Benhabib and Perili (1994) and Xie (1994) have analysed the stability property of the long-run equilibrium in the Lucas (1988) model. Arnold (2000a, 2000b) has analysed the stability of equilibrium in the Romer (1990) model. Arnold (2006) has done the same for the Jones (1995b) model. The works of Devereux and Lapham (1994) and of Segerstrom (1994) have provided a similar stability analysis for the Rivera-Batiz and Romer (1991) model and for the GH (1991a) quality ladder model respectively. However, the stability properties of GH (1991b) product-variety model of Endogenous Product Cycles have not been fully explored yet. Though there exists a few product-variety type North-South models in the literature enriched with a local stability analysis, these models do not deal with endogenous imitation rate in the South. For example, Helpman (1993) who shows that the long-run equilibrium is a saddle point deals with exogenous imitation rate in the South. GH (1991c, section 8.1) and Walde (1996) analyze the stability issue in two-country dynamic models where both countries innovate but none of them imitates.

3.1.1 Grossman and Helpman (1991b) model

There are two countries in the world - the North and the South denoted by the subscripts N and S respectively. They are linked by free trade in differentiated products which are invented in the North and imitated by the South.

3.1.1.1 The demand for goods

The representative household maximises the intertemporal utility function given by

$$W_i = \int_t^{\infty} e^{-\rho(\tau-t)} log\left(U_i(\tau)\right) d\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r_i(\tau-t)} E_i(\tau) d\tau = \int_t^{\infty} e^{-r_i(\tau-t)} I_i(\tau) d\tau + A_i(t) \quad \text{for all } t.$$

Here $E_i(\tau)$, $I_i(\tau)$, $U_i(\tau)$ and $A_i(\tau)$, stand for the instantaneous expenditure, instantaneous income, instantaneous utility and current value of assets at time τ of the representative consumer in the ith region for i= N, S. ρ and r_i stand for the rate of time preference and the nominal interest rate in the ith region respectively. There is no international mobility of financial capital.

The instantaneous utility function is assumed to have the following form.

$$U_i(t) = \left(\int_0^{n(t)} x_i(z)^{\alpha} dz \right)^{\frac{1}{\alpha}} \quad \text{with } 0 < \alpha < 1.$$

Here $n(t)$ and $x_i(z)$ stand for the number of varieties (products) at time t and the quantity of the zth variety consumed by the representative consumer in the ith region.

Solving the optimisation problem we obtain the following demand function for the zth variety

$$x_i(z) = E_i(t) \frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \tag{3.1.1}$$

for $i = N,\ S$. Here $n(t) = n_N(t) + n_S(t)$ and $\varepsilon = \frac{1}{1-\alpha} > 1$ is the constant price elasticity of demand. The aggregate demand function for product z is given by

$$x(z) = x_N(z) + x_S(z) = (E_N(t) + E_S(t)) \frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du}.$$

The demand function faced by the representative Northern producer is given by

$$x_N = (E_N(t) + E_S(t)) \frac{p_N^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \quad for \ \ z \in [0, n_N] \ ; \tag{3.1.2}$$

and the demand function faced by the representative Southern producer is given by

$$x_S = (E_N(t) + E_S(t)) \frac{p_S^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \quad for \;\; z \in [0, n_S] \, . \tag{3.1.3}$$

We also obtain the following optimal time path of expenditure given by

$$\frac{\dot{E_i}(t)}{E_i(t)} = r_i - \rho \quad \text{for } i = N, \ S. \tag{3.1.4}$$

3.1.1.2 The North

There are two sectors in the North - a competitive R&D sector and a production sector. In the production sector there are $n_N(t)$ firms each producing one differentiated product and every firm is a monopolist on its own product. There is perfect intersectoral mobility of labour leading to the same wage in equilibrium. The R&D sector produces the blueprints of new goods.

The production function in the R&D sector takes the following form[4]:

$$\dot{n} = \left(\frac{n}{a_N} \right) L_N^R \, . \tag{3.1.5}$$

Here L_N^R and $\frac{a_N}{n}$ stand for the level of employment and the per unit labour requirement in the R&D sector respectively. $a_N > 0$ is a technological parameter.

One unit of labour can produce one unit of any variety in the North. So, using equation (3.1.5), the Northern labour market clearing equation can be expressed as

$$L_N = a_N \left(\frac{\dot{n}}{n} \right) + n_N x_N \tag{3.1.6}$$

where L_N stands for the labour endowment in the North[5].

The monopoly price and the monopoly profits of the representative Northern firm are

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[4]From here onward the time argument of functions is suppressed when it is convenient to do so.

[5]All varieties in the North are produced in equal quantities because the utility function is symmetric and technologies are identical.

given by the following:

$$p_N = \frac{w_N}{\alpha}, \tag{3.1.7}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N. \tag{3.1.8}$$

The free entry condition in the Northern R&D sector with positive rate of innovation implies

$$v_N = \frac{a_N}{n} w_N, \tag{3.1.9}$$

where $\frac{a_N}{n} w_N$ is the cost of developing a new design in the R&D sector and v_N stands for the value of the Northern firm. The no-arbitrage condition in the Northern asset market is given by

$$\frac{\dot{v_N}}{v_N} + \frac{\pi_N}{v_N} = r_N + m. \tag{3.1.10}$$

The rate of imitation, m, is defined as

$$m = \frac{\dot{n_S}}{n_N}. \tag{3.1.11}$$

We also have the trade balance condition in the North as

$$E_N = p_N n_N x_N \ . \tag{3.1.12}$$

Let $\frac{\dot{n}}{n} = g$ and $\frac{n_N}{n} = \xi$. From equation (3.1.12) we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(n_N \dot{x}_N)}{n_N x_N} \ .$$

Using equations (3.1.6), (3.1.7), (3.1.9) and the definition of g we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{v_N}}{v_N} + g - \frac{a_N \ \dot{g}}{L_N - a_N g} \ . \tag{3.1.13}$$

Using equations (3.1.6), (3.1.8), (3.1.9) and the definitions of g and ξ we obtain

$$\frac{\pi_N}{v_N} \quad = \quad \frac{1-\alpha}{\alpha a_N} \, \frac{L_N - a_N g}{\xi} \ . \tag{3.1.14}$$

Using equations (3.1.4), (3.1.10) and (3.1.14) we can express equation (3.1.13) as follows.

$$\dot{g} = \left(\frac{L_N}{a_N} - g\right) \left[\rho + m + g - \frac{1-\alpha}{\alpha} \left(\frac{L_N}{a_N} - g\right) \frac{1}{\xi}\right] \ . \tag{3.1.15}$$

3.1.1.3 The South

The South has a competitive imitative R&D sector and a production sector producing the imitated products. The production function of the imitative R&D sector takes the following form.

$$\dot{n_S} = \frac{n_S}{a_S} L_S^R \; . \tag{3.1.16}$$

Here L_S^R, $\dot{n_S}$ and (a_S/n_S) stand for the amount of labour used in the imitative R&D sector, the number of new imitated products and the effective labour output coefficient in the R&D sector respectively. The labour market clearing equation of the South is given by the following.

$$L_S = a_S \left(\frac{\dot{n_S}}{n_S} \right) + n_S x_S. \tag{3.1.17}$$

Here L_S stands for the labour endowment in the South. The monopoly price, p_S, and the monopoly profit, π_S, of a typical Southern imitator (producer) are given by

$$p_S = \frac{w_S}{\alpha} \tag{3.1.18}$$

and

$$\pi_S = \frac{1-\alpha}{\alpha} w_S x_S \; . \tag{3.1.19}$$

The Northern wage rate, w_N, is higher than the Southern wage rate, w_S. The Southern imitator charges the monopoly price if this does not exceed the marginal cost of production (wage-rate) in the North. This implies that

$$p_S = \frac{w_S}{\alpha} \leq w_N \; .$$

This is the *wide gap* equilibrium case. However when $\frac{w_S}{\alpha} > w_N$, the Southern firm charges the limit price as

$$p_S = w_N. \tag{3.1.20}$$

This is known as the *narrow gap* equilibrium case. Here

$$\pi_S = (w_N - w_S) x_S \; . \tag{3.1.21}$$

3.1.1.4 Wide gap case

The free entry condition in the Southern imitative R&D sector with positive rate of imitation is given by

$$v_S = \frac{a_S}{n_S} w_S \ . \tag{3.1.22}$$

Here $\frac{a_S}{n_S} w_S$ is the cost of developing an imitative blue print and v_S stands for the value of the Southern firm. The no-arbitrage condition in the Southern asset market is given by

$$\frac{\dot{v_S}}{v_S} + \frac{\pi_S}{v_S} = r_S \ . \tag{3.1.23}$$

The trade balance condition in the South is given by

$$E_S = p_S n_S x_S \ . \tag{3.1.24}$$

Using equation (3.1.11) and the definition of ξ, we write $\frac{\dot{n_S}}{n_S} = m\frac{\xi}{1-\xi}$. Then, from equation (3.1.17), we have

$$n_S x_S = L_S - m a_s \frac{\xi}{1-\xi} \ . \tag{3.1.25}$$

From equations (3.1.18) and (3.1.22), we have

$$p_S = \frac{v_S}{\alpha} \frac{n_S}{a_S} \ . \tag{3.1.26}$$

Using equation (3.1.4) and differentiating both sides of each of the equations (3.1.24), (3.1.25) and (3.1.26) with respect to time, we have

$$r_S - \rho = \frac{\dot{v_S}}{v_S} + \frac{\dot{n_S}}{n_S} - a_S \frac{\dot{\left(m\frac{\xi}{1-\xi}\right)}}{L_S - m a_s \frac{\xi}{1-\xi}} \ .$$

Using equations (3.1.11), (3.1.19), (3.1.22), (3.1.23) and (3.1.25), the above mentioned equation can be written as

$$-\rho \ = \ -\frac{1-\alpha}{a_S \alpha}\left(L_S - m a_s \frac{\xi}{1-\xi}\right) + m\frac{\xi}{1-\xi} - a_S \frac{\dot{m}\left(\frac{\xi}{1-\xi}\right) + \dot{\xi}\frac{m}{(1-\xi)^2}}{L_S - m a_s \frac{\xi}{1-\xi}} \ . \tag{3.1.27}$$

Again, using the definition of ξ, we obtain[6]

$$\dot{\xi} = g - (g+m)\xi \ . \tag{3.1.28}$$

Using equation (3.1.28), equation (3.1.27) can be written as

$$\begin{aligned}\dot{m} = \frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\left[\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right] \\ - \frac{m}{\xi(1-\xi)}\left\{g - (g+m)\xi\right\} \ .\end{aligned} \tag{3.1.29}$$

Equations (3.1.15), (3.1.28) and (3.1.29) represent the dynamic equations in the *wide gap equilibrium* case.

3.1.1.5 Narrow gap case

We define k such that $k = \frac{w_N}{w_S}$. By assumption, $k > 1$. Then, from equations (3.1.20) and (3.1.22), we have

$$p_S = w_N = \frac{v_S n_S k}{a_S} \ . \tag{3.1.30}$$

Using equations (3.1.21), (3.1.22) and (3.1.25) and the definition of k, we obtain

$$\frac{\pi_S}{v_S} = \frac{k-1}{a_S}\left(L_S - a_S m\frac{\xi}{1-\xi}\right) \ . \tag{3.1.31}$$

In this *narrow gap* equilibrium case, the relative demand for the Northern product to the Southern product can be obtained from equations (3.1.2), (3.1.3), (3.1.7) and (3.1.20). It is given by

$$\frac{x_N}{x_S} = \alpha^{\varepsilon} \ .$$

Then using equations (3.1.6) and (3.1.25) and the definitions of g and ξ, we express the above mentioned equation as

$$\left(\frac{L_N - a_N g}{L_S - m a_s \frac{\xi}{1-\xi}}\right)\frac{1-\xi}{\xi} = \alpha^{\varepsilon} \ . \tag{3.1.32}$$

[6] We have $\xi = \frac{n_N}{n}$. This implies $\frac{\dot{\xi}}{\xi} = \frac{\dot{n_N}}{n_N} - \frac{\dot{n}}{n} = \frac{\dot{n} - \dot{n_S}}{n_N} - \frac{\dot{n}}{n} = \frac{\dot{n}}{n_N} - \frac{\dot{n_S}}{n_N} - \frac{\dot{n}}{n} = \frac{g}{\xi} - m - g$. Hence we obtain equation (3.1.28).

Differentiating both sides of equation (3.1.32) with respect to time, we have

$$a_S \frac{\dot{\left(m \frac{\xi}{1-\xi}\right)}}{L_S - m a_s \frac{\xi}{1-\xi}} = a_N \frac{\dot{g}}{L_N - a_N g} + \frac{\dot{\xi}}{\xi(1-\xi)} \; . \tag{3.1.33}$$

From equation (3.1.24), we have

$$\frac{\dot{E}_S}{E_S} = \frac{\dot{p_S}}{p_S} + \frac{\dot{(n_S x_S)}}{n_S x_S} \tag{3.1.34}$$

which can be further simplified (see Appendix 3.3), using equation (3.1.33), as

$$\dot{k} = \left[k^2 \frac{a_N}{a_S} \alpha^{-\varepsilon} \left(\frac{L_N}{a_N} - g\right) \frac{1-\xi}{\xi}\right] - \frac{L_S}{a_S} k - \rho k + a_N \frac{k \dot{g}}{L_N - a_N g} + \frac{k \dot{\xi}}{\xi(1-\xi)}. \tag{3.1.35}$$

Equations (3.1.15), (3.1.28) and (3.1.35) represent the dynamic equations in the *narrow gap equilibrium* case.

3.1.2 Stability in the wide gap case

Equations (3.1.15), (3.1.28) and (3.1.29) are the equations of motions in this case. At the steady state equilibrium point, $\dot{g} = \dot{\xi} = \dot{m} = 0$. So the steady state system of equations can be written as follows.

$$\rho + m + g - \frac{1-\alpha}{\alpha} \left(\frac{L_N}{a_N} - g\right) \frac{1}{\xi} = 0 \; , \tag{3.1.15.1}$$

$$\xi = \frac{g}{g+m} \; , \tag{3.1.28.1}$$

and

$$\rho + m \frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha} \left(\frac{L_S}{a_S} - m \frac{\xi}{1-\xi}\right) = 0 \; . \tag{3.1.29.1}$$

In deriving equations (3.1.15.1) and (3.1.29.1) we have used the fact that

$$L_N > L_N^R = a_N g$$

and

$$L_S > L_S^R = a_S m \frac{\xi}{1-\xi} \; .$$

From equation (3.1.28.1), we have

$$\frac{\xi}{1-\xi} m = g.$$

Using this equation and equation (3.1.29.1), we solve for the steady state equilibrium value of g as

$$g^* = \frac{L_S}{a_S}(1-\alpha) - \rho\alpha \ . \tag{3.1.36}$$

Using equations (3.1.36), (3.1.28.1) and (3.1.15.1) we can solve for ξ^* and m^* uniquely[7]. Note that, from equation (3.1.15.1), after replacing ξ in terms of g and m from equation (3.1.28.1), we can solve for g at $m = 0$ as

$$g = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha. \tag{3.1.37}$$

Also, from equation (3.1.15.1), after replacing ξ from equation (3.1.28.1), we find that g and m are positively related. Thus, the existence of a steady state equilibrium with positive imitation rate is ensured if

$$\frac{L_S}{a_S} > \frac{L_N}{a_N} .$$

Linearising the system of equations (3.1.15), (3.1.28) and (3.1.29) around the steady state equilibrium values of the variables we obtain

$$\begin{bmatrix} \dot{m} \\ \dot{\xi} \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{m}}{\partial g}]_{(m^*,\xi^*,g^*)} \\ \frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{\xi}}{\partial g}]_{(m^*,\xi^*,g^*)} \\ \frac{\partial \dot{g}}{\partial m}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{g}}{\partial \xi}]_{(m^*,\xi^*,g^*)} & \frac{\partial \dot{g}}{\partial g}]_{(m^*,\xi^*,g^*)} \end{bmatrix} \cdot \begin{bmatrix} m(t) - m^* \\ \xi(t) - \xi^* \\ g(t) - g^* \end{bmatrix} \tag{3.1.38}$$

From equations (3.1.15), (3.1.28) and (3.1.29) we can obtain the following elements of

[7]Superscript * denotes the steady-state value of a variable. This notation is followed everywhere in the rest of this chapter.

the Jacobian matrix evaluated at the steady state equilibrium point (see Appendix 3.1).

$$
\begin{aligned}
\frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,g^*)} &= \left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) - \rho + m \; ; \\
\frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,g^*)} &= \frac{(g+m)^2}{g}\left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) - \rho + m\right] \; ; \\
\frac{\partial \dot{m}}{\partial g}]_{(m^*,\xi^*,g^*)} &= -\frac{m}{g}(g+m) \; ; \\
\frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,g^*)} &= -\frac{g}{g+m} \; ; \\
\frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,g^*)} &= -(g+m) \; ; \\
\frac{\partial \dot{\xi}}{\partial g}]_{(m^*,\xi^*,g^*)} &= \frac{m}{g+m} \; ; \\
\frac{\partial \dot{g}}{\partial m}]_{(m^*,\xi^*,g^*)} &= \left(\frac{L_N}{a_N} - g\right) \; ; \\
\frac{\partial \dot{g}}{\partial \xi}]_{(m^*,\xi^*,g^*)} &= (\frac{L_N}{a_N} - g)\frac{(\rho+m+g)(g+m)}{g} \; ; \\
\frac{\partial \dot{g}}{\partial g}]_{(m^*,\xi^*,g^*)} &= \left(\frac{L_N}{a_N} - g\right) + \rho + m + g \; .
\end{aligned}
$$

Let us denote the Jacobian matrix of the right hand side of equation (3.1.38) as A. Then the trace of A is given by

$$
Tr(A) = \left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m + \left(\frac{L_N}{a_N} - g\right) > 0 \; .
$$

It is positive because $\frac{L_S}{a_S} > \frac{L_N}{a_N}$ by assumption and $\frac{L_N}{a_N} > g$. The determinant (see Appendix 3.1) of A is given by

$$
Det(A) = \; - \, \rho \, \frac{m}{g}\left(\frac{L_N}{a_N} - g\right)\left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) - \rho - g\right] \; .
$$

The upper limit of g is $\frac{L_N}{a_N}$. Thus $Det(A) < 0$ if

$$
\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) \;\; > \;\; \rho + g,
$$

or if,

$$
g < \frac{(2-\alpha)\frac{L_S}{a_S} - \rho\alpha}{2} \; .
$$

Comparing equation (3.1.36) and the above mentioned inequality we find that the steady state equilibrium value of g ($= g^*$), always satisfies this restriction. Hence we have $Tr(A) > 0$ and $Det(A) < 0$ evaluated at the steady state equilibrium point. This implies that exactly one of the three latent roots of A is negative and the other two roots are positive[8]. Hence we have the following proposition.

Proposition 3.1.1. *The steady state equilibrium in the wide gap case of Grossman and Helpman (1991b) model is saddle point stable with a unique trajectory converging to the steady state equilibrium point.*

3.1.3 Stability in the narrow gap case

The dynamic equations in this *narrow gap* case are given by the following.

$$\dot{g} = \left(\frac{L_N}{a_N} - g\right)\left[\rho + g - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi} + \frac{1-\xi}{\xi}\left\{\frac{L_S}{a_S} - \frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}\right\}\right], \tag{3.1.39}$$

$$\dot{\xi} = g - \xi\left[g + \frac{1-\xi}{\xi}\left\{\frac{L_S}{a_S} - \frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}\right\}\right], \tag{3.1.40}$$

and

$$\dot{k} = \left[k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi}\right] - \frac{L_S}{a_S}k - \rho k + a_N\frac{k\dot{g}}{L_N - a_N g} + \frac{k\dot{\xi}}{\xi(1-\xi)}. \tag{3.1.35}$$

Here equations (3.1.39) and (3.1.40) are obtained from equations (3.1.15) and (3.1.28) respectively, replacing m in terms of g and ξ from equation (3.1.32). The steady state system of equations is given by the following.

$$\rho + g - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi} + \frac{1-\xi}{\xi}\left\{\frac{L_S}{a_S} - \frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}\right\} = 0, \tag{3.1.39.1}$$

$$\frac{L_S}{a_S} - \frac{\alpha^{-\varepsilon}a_N}{a_S}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi} = g, \tag{3.1.40.1}$$

and

$$k\frac{\alpha^{-\varepsilon}a_N}{a_S}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi} = \rho + \frac{L_S}{a_S}. \tag{3.1.35.1}$$

[8]The other two roots may also be imaginary. However, the negative determinant implies that at least one latent root is negative.

If $\frac{L_S}{a_S} > \frac{L_N}{a_N}$, then a steady state equilibrium exists with ongoing rate of imitation in this *narrow gap* case (see Appendix 3.4). Linearising the dynamic equations (3.1.35), (3.1.39) and (3.1.40) around the steady state equilibrium point we obtain

$$\begin{bmatrix} \dot{k} \\ \\ \dot{\xi} \\ \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{k}}{\partial k}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{k}}{\partial \xi}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{k}}{\partial g}]_{(k^*,\xi^*,g^*)} \\ \\ \frac{\partial \dot{\xi}}{\partial k}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{\xi}}{\partial \xi}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{\xi}}{\partial g}]_{(k^*,\xi^*,g^*)} \\ \\ \frac{\partial \dot{g}}{\partial k}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{g}}{\partial \xi}]_{(k^*,\xi^*,g^*)} & \frac{\partial \dot{g}}{\partial g}]_{(k^*,\xi^*,g^*)} \end{bmatrix} \cdot \begin{bmatrix} k(t) - k^* \\ \\ \xi(t) - \xi^* \\ \\ g(t) - g^* \end{bmatrix} . \tag{3.1.41}$$

From equations (3.1.35), (3.1.39) and (3.1.40) we calculate the following derivatives of the Jacobian matrix evaluated at the steady state equilibrium point (see Appendix 3.2).

$$\begin{aligned}
\frac{\partial \dot{k}}{\partial k}]_{(k^*,\xi^*,g^*)} &= \frac{L_S}{a_S} + \rho \; ; \\
\frac{\partial \dot{\xi}}{\partial \xi}]_{(k^*,\xi^*,g^*)} &= -\frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right) \; ; \\
\frac{\partial \dot{g}}{\partial g}]_{(k^*,\xi^*,g^*)} &= \frac{L_N}{a_N} + \rho + \frac{L_S}{a_S}\frac{1-\xi}{\xi} \; ; \\
\frac{\partial \dot{\xi}}{\partial g}]_{(k^*,\xi^*,g^*)} &= (1-\xi)\left[\frac{\frac{L_N}{a_N} - \frac{L_S}{a_S}}{\frac{L_N}{a_N} - g}\right] \; ; \\
\frac{\partial \dot{g}}{\partial \xi}]_{(k^*,\xi^*,g^*)} &= \frac{1}{\xi^2}\left(\frac{L_N}{a_N} - g\right)\left(\rho\xi + \frac{L_S}{a_S} - g\right) \; ; \\
\frac{\partial \dot{\xi}}{\partial k}]_{(k^*,\xi^*,g^*)} &= \frac{\partial \dot{g}}{\partial k}]_{(k^*,\xi^*,g^*)} = 0 \; .
\end{aligned}$$

Here $\frac{\partial \dot{\xi}}{\partial k} = \frac{\partial \dot{g}}{\partial k} = 0$. So we need not calculate $\frac{\partial \dot{k}}{\partial \xi}$ and $\frac{\partial \dot{k}}{\partial g}$ to evaluate the determinant of the Jacobian. Let us denote the Jacobian matrix of the right hand side of equation (3.1.41) as B. So we have (see Appendix 3.2):

$$Tr(B) = 2\rho + \frac{g}{\xi} + \frac{L_N}{a_N} > 0;$$

and (see Appendix 3.2):

$$Det(B) = \left(\frac{L_S}{a_S} + \rho\right)\left[-\frac{\rho}{\xi}\left(\frac{L_N}{a_N} - g\right) - \rho\left(\frac{L_S}{a_S} - \frac{L_N}{a_N}\right) - \frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\frac{1}{\xi}\frac{L_N}{a_N}\right] .$$

Here $\frac{L_N}{a_N} > g$ and by assumption, $\frac{L_S}{a_S} > \frac{L_N}{a_N}$. Hence $Det(B)$ is negative. Hence we have the following proposition:

Proposition 3.1.2. *The steady-state equilibrium in the narrow gap case of Grossman and Helpman (1991b) model is saddle point stable with a unique saddle path.*

In this section, we have analysed the local stability properties of the GH (1991b) model. We have shown that the steady state equilibrium of this model is saddle point stable in both the wide gap and the narrow gap case. In the case of saddle point stability of the long-run equilibrium point, comparative static exercises with respect to the policy parameters make sense only if the short run effects throw the system from the initial equilibrium point to the saddle path of the new equilibrium point. The Helpman (1993) model satisfies this property. Unfortunately we can not verify this property in the GH (1991b) model because it is technically more complicated than the Helpman (1993) model and we can not derive the explicit solution of the saddle path. However, we can do so if we modify the GH (1991b) model assuming Jacobs (1969) type of localised knowledge spillover in the Northern R&D sector. So we can analyse the transitional dynamic properties of this modified model. This motivates us to modify the GH (1991b) model in the next section.

3.2 Modified Grossman-Helpman (1991b) model

In Grossman and Helpman (1991b), the stock of knowledge capital in the North is assumed to be proportional to the economy's cumulative research experience measured by the number of product designs already developed. This knowledge capital, treated as the public input in the R&D sector, generates positive externalities; and thus lowers the cost of developing new blue prints in the R&D sector. Instead of this so called Marshall-Arrow-Romer (MAR) type of knowledge spillover, we consider Jacobs (1969) type of localised knowledge spillover in the Northern R&D sector. Already Helpman (1993) model has been modified in this direction in section 2.1 in chapter 2 of the present thesis.

With this change in assumption in the GH (1991b) model, the steady state equilibrium of the world economy satisfies saddle point stability in the narrow gap case and becomes unstable in the wide gap case. We analyse the comparative steady state properties in the narrow gap equilibrium and find that many of the comparative steady state results of this modified model differ from the corresponding properties obtained in the original GH (1991b) model. First, the strengthening of IPR protection in the South raises the rate of innovation in the North in this modified model but lowers it in the original GH (1991b) model[9]. An increase in the Southern (Northern) labour endowment lowers (raises) the rate of innovation in the North and produces positive (ambiguous) effect on the rate of imitation in the South in the present model. However, in the GH (1991b) model, an increase in the labour endowment of each of the two regions has positive effects on the rate of innovation and on the rate of imitation. Thirdly, the economic integration between the North and the South is growth reducing in the present model but is growth enhancing in the GH (1991b) model.

We also analyse the comparative transitional dynamic effects with respect to changes in the policy parameters in this modified model. GH (1991b, 1991c, Ch-11) could not analyse any such transitional dynamic effects. However, Helpman (1993) did this in his exogenous imitation model. In such a case one can distinguish between the short run effect and the long run effect of the change in the policy parameters. In the present model, short run effects and the long run effects on the North-South relative wage are not qualitatively similar. The relative wage of a region varies directly with the size of its labour endowment in the long run but varies inversely with that in the short run. This short run result is consistent with that in Krugman (1979) while the long run result is similar to that in GH (1991b). Also, as the IPR protection is strengthened in the South,

[9]We have defined stringent IPR protection in the South as increasing the labour requirement in imitation. This definition has been taken from Glass and Saggi (2002). According to this definition, a stronger IPR in the South leads to a decrease in the rate of innovation in the GH (1991b) model.

the North-South relative wage overshoots on impact in the short run and rises steadily in the long run.

We then, following Helpman (1993), analyse some welfare effects of changes in policy parameters. No other works in the existing literature have analysed the welfare effects in the endogenous imitation model. We find that a policy of strengthening IPR protection in the South may lead to welfare gain in both the countries and the marginal welfare gain in the North is higher than that in the South. In Helpman (1993), the South always faces a welfare loss. The increase in the size of the Southern labour endowment may raise the welfare of each of the two regions in this model. However, this always lowers the welfare of the South in the Helpman (1993) model.

3.2.1 The model

The behaviour of the household in this modified model is similar to the one described in subsection 3.1.1 of this chapter. So all the equations from (3.1.1) to (3.1.4) remains unchanged here. However, as far as the description of the North is concerned, instead of equation (3.1.5), we assume that the production function in the R&D sector in the North takes the following form.

$$\dot{n} = (\frac{n_N}{a_N})L_N^R. \tag{3.2.1}$$

Here L_N^R and $\frac{a_N}{n_N}$ stand for the level of employment and the per unit labour requirement in the R&D sector there. Here $a_N > 0$ is a technological parameter. We can justify this modification in the case of localised knowledge spillovers. Here the externalities come from the presence of different producers in a locality and not from the number of blue prints developed by the R&D sector. R&D sector derives benefit from the interaction with the producers of different goods. These benefits may come from the direct observation of the production process by which the researchers learn how to invent a new good at a cheaper cost.

The labour market clearing equation in the North is given by

$$L_N = a_N(\frac{\dot{n}}{n_N}) + n_N x_N. \tag{3.2.2}$$

Here L_N stands for the labour endowment in the North[10].

The monopoly price and the monopoly profit of the Northern firm producing each of the n_N varieties are given by

$$p_N = \frac{w_N}{\alpha} \tag{3.2.3}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N. \tag{3.2.4}$$

The free-entry condition in the R&D sector in the North is given by

$$v_N = \frac{a_N}{n_N} w_N \tag{3.2.5}$$

where $\frac{a_N}{n_N} w_N$ is the cost of developing a new product design in the R&D sector and v_N is the value of the Northern firm. The Northern no-arbitrage condition is given by

$$\frac{\dot{v_N}}{v_N} + \frac{\pi_N}{v_N} = r_N + m. \tag{3.2.6}$$

We define the rate of imitation, m, as

$$m = \frac{\dot{n_S}}{n_N};$$

and the fraction of products staying in the North, ξ, as

$$\xi = \frac{n_N}{n}. \tag{3.2.7}$$

The trade balance equation in the North is given by

$$E_N = p_N n_N x_N. \tag{3.2.8}$$

[10] All the commodities in the North are produced in equal quantities because the utility function is symmetric and the technologies are identical.

Using equation (3.2.1), we have

$$L_N^R = a_N \frac{\dot{n}}{n} \frac{n}{n_N} = a_N \frac{g}{\xi} \tag{3.2.9}$$

where

$$g = \frac{\dot{n}}{n}.$$

We define

$$\frac{g}{\xi} = \theta. \tag{3.2.10}$$

Then the labour market clearing condition in the North given by equation (3.2.2) can be modified as

$$L_N = a_N\theta + n_N x_N. \tag{3.2.11}$$

Using equations (3.2.4), (3.2.5) and (3.2.11), we have

$$\frac{\pi_N}{v_N} = \frac{1-\alpha}{a_N\alpha} n_N x_N = \frac{1-\alpha}{a_N\alpha}(L_N - a_N\theta).$$

Since the equation (3.2.8) is satisfied at each point of time, we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{\dot{(n_N x_N)}}{n_N x_N}.$$

Using equations (3.2.1) to (3.2.11), we have[11]

$$\dot{\theta} = [\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)](\frac{L_N}{a_N} - \theta). \tag{3.2.12}$$

Also using equations (3.2.7) and (3.2.10), we have[12]

$$\dot{\xi} = (\theta - m - \theta\xi)\xi. \tag{3.2.13}$$

Equation (3.2.12) is an equation of motion describing the dynamics of the North. Equation (3.2.13) is another equation of motion. However, this would be modified when the determination of m would be analysed in the next section.

[11] See Appendix 3.6 for the derivations.

[12] Note that $\frac{\dot{\xi}}{\xi} = \frac{\dot{n_N}}{n_N} - \frac{\dot{n}}{n} = \frac{\dot{n} - \dot{n_S}}{n_N} - g = \theta - m - \theta\xi$.

3.2.1.1 The South

The description of the South is similar to that described in the earlier section. The production function in the imitative R&D sector is given by

$$\dot{n_S} = \frac{n_S}{a_S} L_S^R; \tag{3.2.14}$$

and the Southern labour market clearing condition is given by

$$L_S = a_S(\frac{\dot{n_S}}{n_S}) + n_S x_S.$$

Using equation (3.2.7) and the definition of m we have

$$\frac{\dot{n_S}}{n_S} = m\frac{n_N}{n_S} = m\frac{\xi}{1-\xi}.$$

Hence the labour market clearing equation can be written as

$$L_S = a_S m\frac{\xi}{1-\xi} + n_S x_S. \tag{3.2.15}$$

The monopoly price and the monopoly profit of a typical Southern producer are given by

$$p_S = \frac{w_S}{\alpha} \tag{3.2.16}$$

and

$$\pi_S = \frac{1-\alpha}{\alpha} w_S x_S. \tag{3.2.17}$$

We always assume that $w_N > w_S$. In the *wide gap* case, the equilibrium price in the South, p_S, satisfies the condition

$$p_S = \frac{w_S}{\alpha} < w_N.$$

However, in the *narrow gap* case, we have $\frac{w_S}{\alpha} > w_N$; and the Southern firm charges the limit price given by

$$p_S = w_N. \tag{3.2.18}$$

In this case its profit is given by

$$\pi_S = (w_N - w_S)x_S. \tag{3.2.19}$$

In the *narrow gap* case we have

$$\frac{w_S}{\alpha} > w_N > w_S. \tag{3.2.20}$$

3.2.1.2 Wide gap case

In the wide gap case, the profit rate for a representative Southern firm is given by

$$\frac{\pi_S}{v_S} = \frac{\frac{1-\alpha}{\alpha}x_S w_S}{\frac{a_S}{n_S}w_S} = \frac{1-\alpha}{\alpha}n_S x_S.$$

Here v_S is the value of a representative Southern firm. The no-arbitrage condition in the Southern asset market is given by

$$\frac{\dot{v_S}}{v_S} + \frac{\pi_S}{v_S} = r_S; \tag{3.2.21}$$

and the trade balance condition in the South is given by

$$E_S = p_S n_S x_S.$$

Then

$$\frac{\dot{E_S}}{E_S} = \frac{\dot{p_S}}{p_S} + \frac{\dot{n_S x_S}}{n_S x_S}. \tag{3.2.22}$$

Using equations (3.1.4), (3.2.15) and the fact that $p_S = \frac{n_S v_S}{\alpha a_S}$, equations (3.2.21) and (3.2.22) imply that

$$\dot{m}\frac{\xi}{1-\xi} = (\rho + m\frac{\xi}{1-\xi})(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}) - \frac{1-\alpha}{\alpha}(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})^2 - \frac{m}{(1-\xi)^2}\{\xi\theta - (\xi\theta + m)\xi\}. \tag{3.2.23}$$

So, in the wide gap case, equations of motion are given by differential equations (3.2.12), (3.2.13) and (3.2.23).

3.2.1.3 Narrow gap case

We define k such that

$$k = \frac{w_N}{w_S} > 1.$$

Since in the narrow gap case, $p_S = w_N$, using equations (3.1.2) and (3.1.3) we have,

$$\frac{x_N}{x_S} = \alpha^{\varepsilon}. \tag{3.2.24}$$

Using equations (3.2.11), (3.2.15), (3.2.24) and the fact that $\frac{n_S}{n_N} = \frac{1-\xi}{\xi}$, we have

$$\frac{L_N - a_N\theta}{L_S - ma_s\frac{\xi}{1-\xi}}\frac{1-\xi}{\xi} = \alpha^{\varepsilon};$$

and this can be reexpressed as

$$m\frac{\xi}{1-\xi} = \frac{L_S}{a_S} - \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}. \tag{3.2.25}$$

The free entry condition in the South implies that

$$v_S = \frac{a_S}{n_S}\frac{w_N}{k}.$$

Using equation (3.2.18), the equation mentioned above can be written as

$$w_N = p_S = \frac{v_S n_S k}{a_S}. \tag{3.2.26}$$

Using equation (3.2.21), we have,

$$\frac{\pi_S}{v_S} = \frac{(w_N - w_S)x_S}{\frac{a_S}{n_S}w_S} = \frac{k-1}{a_S}(L_S - a_S m\frac{\xi}{1-\xi}). \tag{3.2.27}$$

Then using equations (3.1.4), (3.2.15), (3.2.22) and (3.2.26), we have

$$r_S - \rho = \frac{\dot{v_S}}{v_S} + \frac{\dot{n_S}}{n_S} + \frac{\dot{k}}{k} - a_S\frac{(\dot{m\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}};$$

and this can be further simplified as[13]

$$\dot{k} = k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - k[\frac{L_S}{a_S} + \rho] + [\frac{k\dot{\theta}}{\frac{L_N}{a_N} - \theta} + \frac{k\dot{\xi}}{\xi(1-\xi)}]. \tag{3.2.28}$$

[13]See Appendix 3.5 for the detail derivation.

Also replacing the value of m in terms of ξ and θ from equation (3.2.25) in equation (3.2.13), we have

$$\dot{\xi} = (1-\xi)[\xi\theta - \frac{L_S}{a_S} + \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}]. \tag{3.2.29}$$

Equations of motion in the narrow gap case are given by differential equations (3.2.12), (3.2.28) and (3.2.29).

3.2.2 The steady state equilibrium in the wide gap case

3.2.2.1 Existence of an unique equilibrium

The steady state equilibrium system of equations in the wide gap case are obtained by putting $\dot{\theta} = \dot{\xi} = \dot{m} = 0$ in equations (3.2.12), (3.2.13) and (3.2.23). Then, we have[14]

$$\rho + \theta^* - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta^*) = 0, \tag{3.2.30}$$

$$\rho + m^*\frac{\xi^*}{1-\xi^*} - \frac{1-\alpha}{\alpha}(\frac{L_S}{a_S} - m^*\frac{\xi^*}{1-\xi^*}) = 0, \tag{3.2.31}$$

and

$$\theta^* = \frac{m^*}{1-\xi^*}. \tag{3.2.32}$$

From equation (3.2.30) we can solve for θ^* uniquely. Also using equation (3.2.32), we can rewrite equation (3.2.31) as a function of θ^* and ξ^*. This solves for an unique value of ξ^* given the unique value of θ^*. Once θ^* and ξ^* are solved, equation (3.2.32) solves for an unique value of m^*. So, we have an unique steady state equilibrium in the wide gap case.

[14]Superscript * of a variable denotes its steady state value. This notation is followed everywhere in the rest of this chapter.

3.2.2.2 Stability of equilibrium

Linearising equations (3.2.12), (3.2.13) and (3.2.23) around their steady state equilibrium values, we obtain the following.

$$\begin{bmatrix} \dot{m} \\ \\ \dot{\xi} \\ \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{m}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \\ \\ \frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\xi}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \\ \\ \frac{\partial \dot{\theta}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\theta}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\theta}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \end{bmatrix} \cdot \begin{bmatrix} m(t) - m^* \\ \\ \xi(t) - \xi^* \\ \\ \theta(t) - \theta^* \end{bmatrix} \tag{3.2.33}$$

We also have,

$$\frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,g^*)} = (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi};$$

$$\frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,g^*)} = (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi};$$

$$\frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} = -\theta\xi \quad ; \qquad \frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,\theta^*)} = -\xi;$$

$$\frac{\partial \dot{\theta}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} = (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \quad ; \qquad \text{and} \qquad \frac{\partial \dot{\theta}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} = \frac{\partial \dot{\theta}}{\partial m}]_{(m^*,\xi^*,\theta^*)} = 0.$$

Here we note that all the above mentioned derivatives are evaluated at the steady state equilibrium values of g, ξ and m. Let us denote the Jacobian matrix of the right hand side of the set of equations (3.2.33) as C. Then we have[15],

$$C = \begin{bmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi} & ? \\ \\ -\xi & -\theta\xi & ? \\ \\ 0 & 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{bmatrix}_{(m^*,\xi^*,\theta^*)}$$

Trace of the matrix C is given by

$$Tr(C) = \left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\frac{1}{\alpha} + \left(\frac{m}{1-\xi} - \theta\xi\right) + \left(\frac{L_N}{a_N} - \theta\right)\frac{1}{\alpha};$$

[15] Notation '?' in a matrix mean that we do not need to calculate those derivatives to get the trace and the determinant of the matrix. This interpretation of '?' is followed everywhere in the rest of this chapter.

and the determinant of the matrix C is given by

$$Det(C) = \frac{1}{\alpha}\left(\frac{L_N}{a_N} - \theta\right)\frac{m}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right).$$

From equation (3.2.32) we find that $\theta\xi < m\frac{\xi}{1-\xi}$. Also, equations (3.2.11) and (3.2.15) imply that $\left(\frac{L_N}{a_N} - \theta\right) > 0$ and $\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right) > 0$. Then both the trace and the determinant of the matrix C are positive. So, in order to determine the sign of the roots of the matrix C, we apply the Routh-Hurwitz Theorem. The characteristic equation associated with the matrix C is given by

$$-q^3 + Tr(C)q^2 - M(C)q + Det(C) = 0. \qquad (3.2.34)$$

where

$$M(C) = \begin{vmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi} \\ -\xi & -\theta\xi \end{vmatrix}_{(m^*,\xi^*,\theta^*)}$$

$$+\begin{vmatrix} -\xi\theta & ? \\ 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{vmatrix}_{(m^*,\xi^*,\theta^*)} + \begin{vmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & ? \\ 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{vmatrix}_{(m^*,\xi^*,\theta^*)}$$

Routh-Hurwitz Theorem states that the number of positive roots of the characteristic equation (3.2.34) is equal to the number of variations of signs in the scheme given by

$$\{\,-1,\ \ Tr(C),\ \ -M(C) + \frac{Det(C)}{Tr(C)},\ \ Det(C)\,\}. \qquad (3.2.35)$$

Now,

$$-M(C) + \frac{Det(C)}{Tr(C)} = -\frac{m}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right) - \left(\frac{L_N}{a_N} - \theta\right)\frac{1}{\alpha}\left[\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\frac{1}{\alpha} + m\right]$$

$$+\frac{\frac{1}{\alpha}(\frac{L_N}{a_N} - \theta)\frac{m}{\alpha}(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})}{(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + m + (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha}}$$

$$= -\frac{m}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right) - \left(\frac{L_N}{a_N} - \theta\right)\frac{m}{\alpha} - \frac{1}{\alpha^2}\left(\frac{L_N}{a_N} - \theta\right)\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)$$

$$\left[1 - \frac{m}{(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + m + (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha}}\right].$$

Here, $\frac{L_S}{a_S} > \frac{m\xi}{1-\xi}$ and $\frac{L_N}{a_N} > \theta$. Hence,

$$\left[-M(C) + \frac{Det(C)}{Tr(C)}\right] < 0.$$

So the number of variations in sign in (3.2.35) is equal to three, which means that all the three roots of the equation (3.2.34) are positive. This proves that the steady state equilibrium in the wide gap case is unstable. We can summarize the major result in the following proposition.

Proposition 3.2.1. *The steady state equilibrium in the wide gap case of the modified Grossman-Helpman (1991b) model with localized knowledge spillover is unstable.*

3.2.3 The steady state equilibrium in the narrow gap case

3.2.3.1 Existence of an unique equilibrium

The steady state equilibrium system of equations in the narrow gap case are obtained by putting $\dot{\theta} = \dot{k} = \dot{\xi} = 0$ in equations (3.2.12), (3.2.28) and (3.2.29). Then, we have

$$\theta^* = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha, \tag{3.2.36}$$

$$k^* a_N \alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} = L_S + \rho a_S, \tag{3.2.37}$$

and

$$a_N \alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} = L_S - \theta^*\xi^* a_S. \tag{3.2.38}$$

Using equation (3.2.25), the steady state equilibrium value of m is obtained as

$$m^* = \frac{1-\xi^*}{\xi^*}\{L_S - \frac{1-\xi^*}{\xi^*}(\frac{L_N}{a_N} - \theta^*)a_N\alpha^{-\varepsilon}\}\frac{1}{a_S} = \theta^*(1-\xi^*). \tag{3.2.39}$$

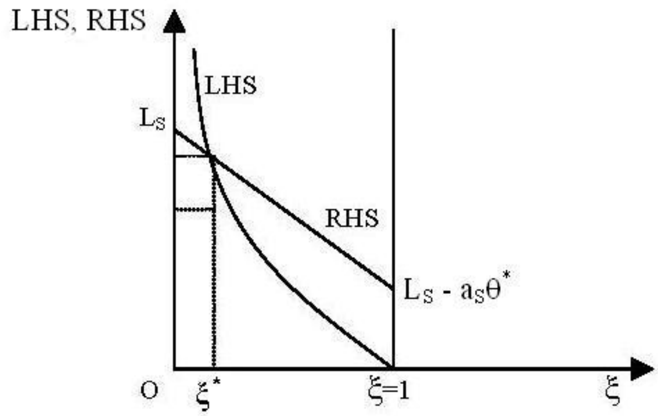

Figure - 3.2.1 : Existence of the equilibrium

where the last equality follows from equation (3.2.38). Also, using equations (3.2.37) and (3.2.38), we have

$$(k^* - 1)L_S = \rho a_S + k^*\theta^*\xi^* a_S. \tag{3.2.40}$$

We first show that a unique steady state equilibrium exists. From equation (3.2.36), θ^* is determined uniquely. If ξ^* is unique then equations (3.2.37) and (3.2.39) show that k^* and m^* are also unique. The existence of a unique ξ^* is ensured by equation (3.2.38); and we show it using figure 3.2.1. The Left Hand Side (LHS) of equation (3.2.38) is shown by the LHS curve which slopes negatively being asymptotic to the vertical axis and meeting the horizontal axis at $\xi = 1$. The Right Hand Side (RHS) of this equation is shown by the negatively sloped RHS curve. It meets the vertical axis because L_S is finite and also meets the horizontal axis at $\xi = \frac{L_S}{\theta^* a_S} > 1$[16].

So the two curves must intersect at only one point satisfying $0 < \xi^* < 1$. ξ^* can not be zero in the steady state equilibrium. So there can not exist any steady state

[16] A sufficient condition for this to happen is $\frac{L_S}{a_S} > \frac{L_N}{a_N}$. Note that this condition is also sufficient to ensure the local saddle point stability of the steady state equilibrium in the narrow gap case. Also note that this condition ensures the relative wage bound in the norrow gap case which is $\frac{w_N}{w_S} < \frac{1}{\alpha}$. This is proved in Appendix 3.9.

equilibrium where the North specializes in R&D and the South manufactures all the varieties.

Using equations (3.2.36) and (3.2.38) we solve for the unique[17] equilibrium value of ξ^* as

$$\xi^* = \frac{(L_S + A) - \sqrt{(L_S - A)^2 + 4A(L_S - B)}}{2B}.$$

Here

$$A = a_N \alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*);$$

and

$$B = \theta^* \; a_S \; .$$

3.2.3.2 Stability of equilibrium

To check the local stability of the steady state equilibrium in the narrow gap case, we evaluate the partial derivatives at the steady state equilibrium point and obtain,

$$\frac{\partial \dot{\theta}}{\partial \theta} = (\frac{L_N}{a_N} - \theta)[1 + \frac{1-\alpha}{\alpha}] = (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \; ;$$

$$\frac{\partial \dot{\theta}}{\partial k} = \frac{\partial \dot{\theta}}{\partial \xi} = \frac{\partial \dot{\xi}}{\partial k} = 0 \; ;$$

$$\frac{\partial \dot{k}}{\partial k} = 2k\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - [\frac{L_S}{a_S} + \rho] = \frac{L_S}{a_S} + \rho; \qquad \text{[using equation (3.2.37)]}$$

and

$$\frac{\partial \dot{\xi}}{\partial \xi} = -(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi}) \qquad \text{[using equations (3.2.37) and (3.2.40)].}$$

The linearised dynamic system is given by

$$\begin{bmatrix} \dot{\theta} \\ \\ \dot{\xi} \\ \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha}(\frac{L_N}{a_N} - \theta) & 0 & 0 \\ \\ ? & -(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi}) & 0 \\ \\ ? & ? & \rho + \frac{L_S}{a_S} \end{bmatrix}_{(\theta^*, \xi^*, k^*)} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \\ \xi(t) - \xi^* \\ \\ k(t) - k^* \end{bmatrix}$$

[17]Using equation (3.2.36), it can be proved that $L_S - B > 0$ under the condition that $\frac{L_S}{a_S} > \frac{L_N}{a_N}$. Hence $\xi^* = \frac{(L_S+A)+\sqrt{(L_S-A)^2+4A(L_S-B)}}{2B} > 1$ is ruled out on the ground that $\xi^* \leq 1$.

We define a_{11}, a_{22}, a_{33} and a_{32} such that[18]

$$a_{11} = \frac{\partial \dot{\theta}}{\partial \theta} = \frac{1}{\alpha}(\frac{L_N}{a_N} - \theta);$$

$$a_{22} = \frac{\partial \dot{\xi}}{\partial \xi} = -(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi});$$

$$a_{33} = \frac{\partial \dot{k}}{\partial k} = \rho + \frac{L_S}{a_S};$$

and

$$a_{32} = \frac{\partial \dot{k}}{\partial \xi} = k^2 \frac{a_N}{a_S} \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta)(-\frac{1}{\xi^2}) - (\rho + \frac{L_S}{a_S}).$$

Since $\left(\frac{L_N}{a_N} - \theta\right) > 0$ and $\frac{L_S}{a_S} > \frac{L_N}{a_N}$ by assumption, we have $a_{11} > 0$, $a_{22} < 0$, $a_{33} > 0$ and $a_{32} < 0$. The roots of the characteristic equation of the Jacobian matrix are a_{11}, a_{22} and a_{33}. Since exactly one root is negative and the other two roots are positive, the steady state equilibrium in the narrow gap case is locally saddle point stable with a unique saddle path converging to the steady state equilibrium point. We summarize this result in the following proposition.

Proposition 3.2.2. *The steady state equilibrium in the narrow gap case of the modified Grossman-Helpman (1991b) model with localized knowledge spillover is locally saddle point stable with a unique trajectory converging to the steady state equilibrium point.*

To determine the solution of the linearized version of the equations of motion along the unique saddle path, we choose the eigenvectors corresponding to the two positive roots as zero. This procedure leads us to the solution

$$\theta(t) = \theta^*, \tag{3.2.41}$$

$$\xi(t) = \xi^* - [\xi^* - \xi(0)]e^{a_{22}t}, \tag{3.2.42}$$

and

$$k(t) = k^* + [\xi^* - \xi(0)] \left(\frac{a_{32}}{a_{33} - a_{22}} \right) e^{a_{22}t}. \tag{3.2.43}$$

[18] All derivatives are evaluated at the steady state equilibrium point.

We shall use these equations in later sections to determine the transitional dynamic properties in the narrow gap case. Since we prove that the steady state equilibrium in the wide gap case is locally unstable but is locally saddle point stable in the narrow gap case, we are interested in analysing the comparative steady state effects in the narrow gap equilibrium case only.

3.2.4 Comparative steady state effects

3.2.4.1 Strengthening IPR protection

We consider

$$a_S = a_m + \lambda$$

where a_m is the technology parameter and λ is a policy parameter representing the degree of strengthening IPR protection in the South. The stronger the IPR protection, the greater is the value of λ ; and hence the greater is the effective labour requirement[19] in the imitative R &D sector. Thus strengthening IPR protection in the South raises the value of a_S.

Equation (3.2.36) shows that θ^* is independent of a_S. An increase in a_S causes the BB curve in the figure 3.2.1 to shift downward. However, the AA curve remains unchanged. Thus ξ^* is increased in the new equilibrium. So the LHS of equation (3.2.37) is decreased for given k^*. The RHS of this equation is increased due to the increase in a_S. Thus k^* should rise in the new equilibrium. Equation (3.2.39) implies that m^* should fall. Also, g^* $(= \theta^* \xi^*)$ should rise in the new steady state equilibrium. Hence we can establish the following proposition.

Proposition 3.2.3. *A policy of strengthening IPR protection in the South raises the rate of innovation and the proportion of unimitated products in the North and lowers the*

[19] The increase in the labour requirement means the increase in the cost of imitation. So strengthening IPR protection means the increase in the cost of imitation. We follow Glass and Saggi (2002) for this kind of definition of IPR tightening in the South.

rate of imitation in the South and the South-North relative wage in the new steady-state growth equilibrium.

The result is important because it is not identical to that obtained in the GH (1991b) model. The effect on the rate of imitation in the South and on the North-South wage gap are qualitatively similar in both the cases. However, the effect on the Northern rate of innovation is negative in the GH (1991b) model but is positive in the present model. The strengthening of IPR protection makes imitation costly in the South. So the rate of imitation is decreased and the fraction of the imitated products produced in the South is reduced. The North produces a higher fraction of products and so its demand for labour in the production sector is increased. However, the opposite happens in the South. This leads to an increase in the North South relative wage because labour is internationally immobile. Also, the cost of capital of a Northern firm is reduced due to the decline in the rate of imitation. However its profit rate remains unchanged because both the instantaneous profit of the firm and the cost of developing a blue print fall at equal rates. So the incentive to innovate in the North is increased and this leads to an increase in the rate of innovation there. In the GH (1991b) model, a decrease in the imitation rate lowers the profit rate; and the magnitude of this reduction is higher than that in the cost of capital. This generates a negative incentive to innovate in the North.

3.2.4.2 Changes in labour endowments

The increase in the Southern labour endowment, L_S, has no impact on θ^*. In Figure 3.2.1, BB curve shifts upward and AA curve remains unchanged when L_S is increased. This lowers ξ^* in the new equilibrium. Now equation (3.2.40) shows that k^* falls in the new equilibrium. Also, $g^* = \theta^* \xi^*$ is decreased. Equation (3.2.39) shows that m^* rises.

Equation (3.2.36) shows that an increase in L_N raises θ^* as well as $(\frac{L_N}{a_N} - \theta^*)$ because $0 < \alpha < 1$. This causes the AA curve to shift upward while the BB curve remains the same. This raises ξ^* in the new equilibrium. k^* is increased while the effect on m^* is ambiguous. $g^* = \theta^*\xi^*$ must go up. We can now summarize these effects in the following proposition.

Proposition 3.2.4. *The increase in the size of the labour endowment in the South (North) lowers (raises) the rate of innovation, the proportion of unimitated products, and the North South relative wage but produces positive (indeterminate) effect on the rate of imitation in the South in the new steady-state growth equilibrium.*

An increase in the Southern labour endowment raises its availability to both the production sector and the imitative R&D sector. This leads to an increase in the rate of imitation. So the fraction of products manufactured in the South is also increased. This lowers the demand for labour in the Northern production sector. As the rate of imitation is increased, the incentive to innovate in the North is reduced; and this leads to a decrease in the rate of innovation. This raises the supply of labour to the Northern production sector. However, the demand for Southern labour is increased. This explains the increase in the South North relative wage.

Similarly, an increase in the Northern labour endowment makes more labour available to the R&D sector and the Northern production sector. This raises the share of products produced in the North and the size of its R&D sector (measured by $a_N\theta^*$ in the steady state). Increased R&D sector's size leads to an increase in the rate of innovation. Since the share of products produced in the North is increased, an increase in the demand for labour takes place in the Northern labour market. The South faces a decline in the demand for labour from the production sector. So the relative North South wage is increased. However, the effect on the rate of imitation is ambiguous[20].

The result regarding the relationship between a region's (country's) size of its labour endowment and its relative wage is consistent with that of GH (1991b). However, the effect of the increase in the Southern labour endowment on the rate of innovation in

[20]Since in the steady state $m^* = \theta^*(1-\xi^*)$ and both θ^* and ξ^* increases, effect on m^* is not clear.

this model is opposite to that obtained in the GH (1991b) model because the rate of Northern innovation varies positively with the size of Southern labour endowment in that model. The effect of the increase in the Northern labour endowment on the rate of imitation is ambiguous in our model but the rate of imitation is increased in the GH (1991b) model in this case.

3.2.5 Transitional dynamic effects

We now analyse how the variables θ, ξ and k behave in transition from one steady state to another with respect to the once for all change in a parameter. Equations (3.2.41), (3.2.42) and (3.2.43) gives the general solution of the endogenous variable along the saddle path. We also know from subsection 3.2.3 that $a_{11} > 0, \quad a_{22} < 0, \quad a_{33} > 0$ and $a_{32} < 0$. We are now in a position to analyse various transitional dynamic effects of changes in policy parameters

3.2.5.1 Strengthening IPR protection

We assume that the system is initially in the steady state equilibrium; and then analyse the effects of parametric change on its transitional behaviour. Note that if $\xi(0) = \xi^*$, then the entire system is in steady state equilibrium initially. The first order response of tightening of IPR can be evaluated at $\xi(0) = \xi^*$ as

$$\frac{d\theta(t)}{da_S} = 0\,, \tag{3.2.44}$$

$$\frac{d\xi(t)}{da_S} = (1 - e^{a_{22}t})\frac{d\xi^*}{da_S}\,, \tag{3.2.45}$$

and,

$$\frac{dk(t)}{da_S} = \frac{dk^*}{da_S} + \frac{d\xi^*}{da_S} e^{a_{22}t}\left(\frac{a_{32}}{a_{33} - a_{22}}\right). \tag{3.2.46}$$

Here $\frac{d\xi^*}{da_S} > 0$; and hence

$$\frac{d\xi(0)}{da_S} = 0,$$

and

$$\frac{d\xi(t)}{da_S} > 0$$

for any $t > 0$. Using equation (3.2.37) and the values of a_{22}, a_{33} and a_{32}, we obtain

$$(\frac{a_{32}}{a_{33} - a_{22}}) = \frac{\frac{k^*}{\xi^*(1-\xi^*)}[-k^* \frac{a_N}{a_S} \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta^*) \frac{1-\xi^*}{\xi^*} + \frac{\partial \dot{\xi}}{\partial \xi}]}{[\rho + \frac{L_S}{a_S}] - [\frac{\partial \dot{\xi}}{\partial \xi}]} = -\frac{k^*}{\xi^*(1-\xi^*)}. \tag{3.2.47}$$

Differentiating equation (3.2.37) with respect to a_S, we have

$$\frac{1-\xi^*}{\xi^*}[\frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)} \frac{d\xi^*}{da_S}] a_N \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta^*) = \rho. \tag{3.2.48}$$

Then, using equations (3.2.47), equation (3.2.46) can be written as

$$\frac{dk(t)}{da_S} = \frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)} \frac{d\xi^*}{da_S} e^{a_{22}t}.$$

Then evaluating the above mentioned derivative at $t = 0$ and using equation (3.2.48), we obtain

$$\frac{dk(0)}{da_S} = [\frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)} \frac{d\xi^*}{da_S}] = \frac{\rho}{a_N \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta^*) \frac{1-\xi^*}{\xi^*}} > 0.$$

As $t \to \infty$, $\frac{dk(t)}{da_S} \to \frac{dk^*}{da_S}$ because $a_{22} < 0$. Using figutre 3.2.2 we now can describe how ξ and k behave during transition from one steady state to another due to an increase in a_S. As a_S is increased, k jumps from point A to the new saddle path and then rises over time to reach the new steady state equilibrium at point B. ξ increases steadily from A to B. Since $\theta = \theta^*$ for all t, g= $\theta^* \xi$ rises proportionately with ξ and $m = \theta(1 - \xi)$ falls with ξ. $1 < k < \frac{1}{\alpha}$ is to be satisfied at the new equilibrium point B. Otherwise we shall violate the narrow gap condition. However, we can establish the following proposition from this comparative dynamic analysis.

Proposition 3.2.5. *As IPR protection is strengthened once for all in the initial steady state equilibrium, (i) North-South relative wage initially overshoots on impact and then rises steadily along the saddle path to reach the new steady state equilibrium but (ii) the proportion of unimitated products and the rate of innovation rise steadily over time along the saddle path.*

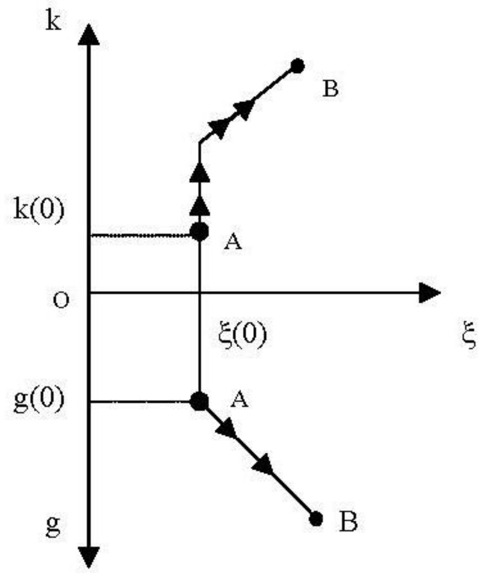

Figure - 3.2.2 : Transitional movement due to stronger IPR protection

3.2.5.2 Changes in labour endowments

Differentiating equations (3.2.41), (3.2.42) and (3.2.43) with respect to L_S and using the initial condition $\xi(0) = \xi^*$, we have

$$\frac{d\theta(t)}{dL_S} = 0\,, \tag{3.2.49}$$

$$\frac{d\xi(t)}{dL_S} = (1 - e^{a_{22}t})\frac{d\xi^*}{dL_S}\,, \tag{3.2.50}$$

and

$$\frac{dk(t)}{dL_S} = \frac{dk^*}{dL_S} - \frac{d\xi^*}{dL_S}e^{a_{22}t}(\frac{k^*}{\xi^*(1-\xi^*)}). \tag{3.2.51}$$

From the comparative steady state exercises worked out in subsection 3.2.4.2, we have

$$\frac{d\xi^*}{dL_S} < 0 \;\; and \;\; \frac{dk^*}{dL_S} < 0.$$

Differentiating equation (3.2.37) with respect to L_S we have

$$\frac{1-\xi^*}{\xi^*}a_N\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)[\frac{dk^*}{dL_S} - \frac{k^*}{\xi^*(1-\xi^*)}\frac{d\xi^*}{dL_S}] = 1.$$

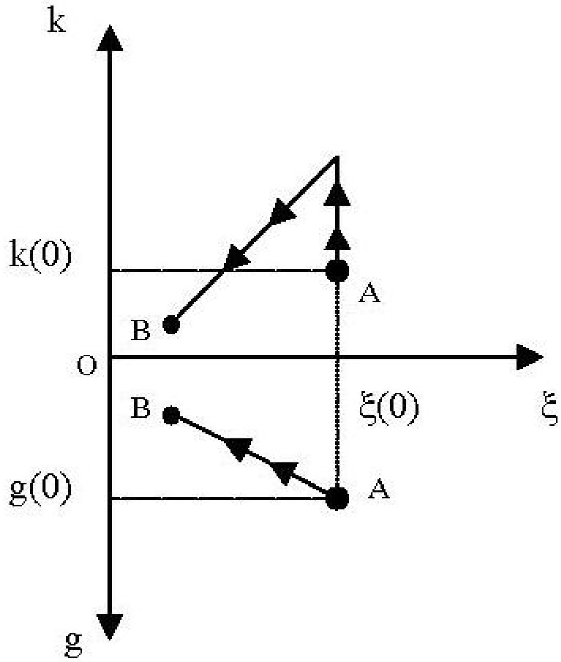

Figure - 3.2.3 : Transitional movement due to change in L_S

This equation implies that the Right Hand Side of equation (3.2.51) is positive at $t = 0$ which, in turn, implies that $\frac{dk(0)}{dL_S} > 0$. Equation (3.2.50) clearly shows that $\frac{d\xi(0)}{dL_S} = 0$. However $a_{22} < 0$, and this means that $\frac{dk(t)}{dL_S} < 0$ for all $t > T_1 > 0$; and $\frac{d\xi(t)}{dL_S} < 0$ for all $t > 0$. Using figure 3.2.3, we can now describe how $g(t)$, $k(t)$ and $\xi(t)$ behave in the transitional phase following an once for all increase in L_S. In transition from A to B, $k(t)$ first jumps to reach the new saddle path given $\xi(0) = \xi^*$; and then falls to B along the saddle path. However, $\xi(t)$ falls steadily over time; and so $g(t)$ also falls over time proportionately to B along the saddle path. This leads to the following proposition.

Proposition 3.2.6. *As the size of the Southern labour endowment is increased once for all in the initial steady state equilibrium, (i) the North-South relative wage rises initially and then falls to reach the new steady state equilibrium along the saddle path; but (ii) the proportion of unimitated products and the rate of innovation falls steadily over time along the saddle path.*

Differentiating equations (3.2.41), (3.2.42) and (3.2.43) with respect to L_N and using the initial condition $\xi(0) = \xi^*$, we have

$$\frac{d\theta(t)}{dL_N} = \frac{1-\alpha}{a_N} > 0\,, \tag{3.2.52}$$

$$\frac{d\xi(t)}{dL_N} = (1 - e^{a_{22}t})\frac{d\xi^*}{dL_N}\,, \tag{3.2.53}$$

and

$$\frac{dk(t)}{dL_N} = \frac{dk^*}{dL_N} - \frac{d\xi^*}{dL_N}e^{a_{22}t}(\frac{k^*}{\xi^*(1-\xi^*)}). \tag{3.2.54}$$

From the comparative steady state exercises we have

$$\frac{d\xi^*}{dL_N} > 0, \quad \frac{dk^*}{dL_N} > 0 \;\; and \;\; \frac{dg^*}{dL_N} > 0.$$

Differentiating equation (3.2.37) with respect to L_N we have

$$\frac{1-\xi^*}{\xi^*}a_N\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)[\frac{dk^*}{dL_N} - \frac{k^*}{\xi^*(1-\xi^*)}\frac{d\xi^*}{dL_N}] + k^*\frac{1-\xi^*}{\xi^*}\alpha^{1-\varepsilon} = 0.$$

Using the above mentioned equation and equations (3.2.53) and (3.2.54), we have

$$\frac{d\xi(0)}{dL_N} = 0 \quad and \quad \frac{dk(0)}{dL_N} < 0.$$

However, $a_{22} < 0$; and this means that $\frac{dk(t)}{dL_N} > 0$ for all $t > T_2 > 0$ and $\frac{d\xi(t)}{dL_N} > 0$ for all $t > 0$. Evaluating $g(t) = \theta(t)\xi(t)$ at $t = 0$ and then differentiating this with respect to L_N and using the initial condition, $\xi(0) = \xi^*$, we have

$$\frac{dg(0)}{dL_N} = \frac{d\theta^*}{dL_N}\xi^* = \xi^*\frac{1-\alpha}{a_N} > 0 \quad and \quad \frac{dg(t)}{dL_N} > 0\,,\; for\, all\; t.$$

Using figure 3.2.4 we can now describe how $g(t)$, $k(t)$ and $\xi(t)$ behave in the transitional phase following an once for all increase in L_N. $g(t)$ rises on impact and then goes on steadily over time to attain the new steady state equilibrium. $k(t)$ falls on impact and then rises steadily to reach the new equilibrium. $\xi(t)$ increases steadily. This leads to the following proposition.

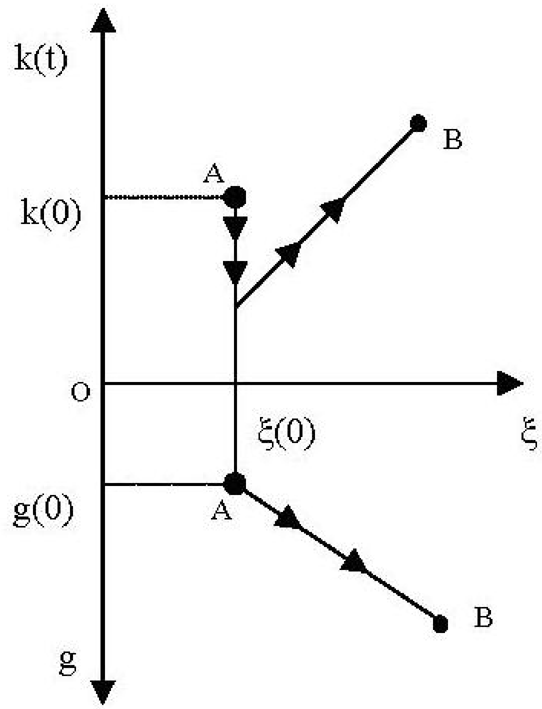

Figure - 3.2.4 : Transitional movement due to change in L_N

Proposition 3.2.7. *As the size of the Northern labour endowment is increased once for all in the initial steady state equilibrium, then, along the saddle path, (i) the North-South relative wage falls initially and then rises to reach the new steady state equilibrium, (ii) the proportion of unimitated products increases steadily over time; and (iii) the rate of innovation in the North overshoots on impact and then increases steadily over time.*

From propositions 3.2.6 and 3.2.7, it now follows that, in the short run, the relative wage of a region varies inversely with the size of the labour endowment of that region and directly with that of the other region. However, in the long-run, it varies directly with its size and inversely with that of the other. GH (1991c) have found that the relative wage of a region varies directly with the size of that region and inversely with the size of the other region. In their own words, *"This result may be surprising to readers versed in the neo-classical growth model, and it stands in sharp contrast to the findings reported by Krugman (1979)"* (GH (1991c), page-304, last paragraph). However, GH (1991c) have dealt with the comparative steady state properties only; and did not analyse the transitional dynamic properties. We do the transitional dynamic analysis in the narrow gap equilibrium in this modified GH (1991c) model and show that it is possible to reconcile both Krugman (1979) result and GH (1991c) result. Thus the short run impact of a change in labour endowment on the relative wage is consistent with the result of Krugman (1979); and the long run impact is consistent with that of GH (1991c)[21].

Our result regarding the impact of a change in the region's size of the labour endowment on its relative wage is consistent with that of Dollar (1986). He uses a dynamic general equilibrium model of North-South trade and shows that the short-run effect of an increase in the Southern labour endowment is to raise the relative wage of the North by raising the demand for Northern products. He mentions this as the "classical result". However, in the long-run, relative wage of the North is decreased in his model by accelerating the transfer of technology and capital flow from the North to the South. We do not have capital in our model as another factor of production. However, it is the faster imitation rate in the South that gradually raises the share of imitated products produced in the South; and this, in turn, raises the Southern relative wage in the long run.

3.2.6 Welfare effects

Our analysis in this section is similar to that in Helpman (1993). The instantaneous utility function of the representative individual of the ith region is given by

$$U_i(t) = E_i[n_N p_N^{1-\varepsilon} + n_S p_S^{1-\varepsilon}]^{\frac{1}{\varepsilon-1}} = E_i[p_N^{-1} n^{\frac{1}{\varepsilon-1}}\{\xi + (1-\xi)\alpha^{1-\varepsilon}\}^{\frac{1}{\varepsilon-1}}]$$

[21]It is worthwhile to report one result of Lai (1995) in this context. Lai (1995), using a product-variety endogenous growth model like GH (1991c), finds that an increase in the supply of unskilled labor in a country lowers its steady state equilibrium relative wage while an increase in supply of skilled labor in a country raises its steady-state equilibrium relative wage when the elasticity of substitution between the goods is sufficiently high. However, Lai (1995) deals with the comparative steady state effects only.

for $i = N, S$. Here E_i represents the per capita income in the ith region with

$$E_N = \frac{p_N n_N x_N}{L_N} = p_N(1 - \frac{a_N \theta}{L_N})$$

and

$$E_S = \frac{p_S n_S x_S}{L_S} = p_S(1 - \frac{a_S m^{\frac{\xi}{1-\xi}}}{L_S}) = p_S\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\}.$$

Using the above mentioned equations we have

$$log(U_N(t)) = log(1 - \frac{a_N \theta}{L_N}) + \frac{1}{\varepsilon - 1} log(n(t)) + \frac{1}{\varepsilon - 1} log\{\xi + (1-\xi)\alpha^{1-\varepsilon}\} \quad (3.2.55)$$

and

$$log(U_S(t)) = log\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\} + \frac{1}{\varepsilon - 1} log(n(t)) + \frac{1}{\varepsilon - 1} log\{\xi + (1-\xi)\alpha^{1-\varepsilon}\} + log(\alpha). \quad (3.2.56)$$

Using equations (3.2.55) and (3.2.56), we have

$$log(U_S(t)) - log(U_N(t)) = log(\frac{1-\xi}{\xi}\frac{L_N}{L_S}) - (\varepsilon - 1) log(\alpha). \quad (3.2.57)$$

Equation (3.2.57) implies that the relative instantaneous utility of a representative individual in any region depends on the relative size of its labour endowment, $\frac{L_N}{L_S}$, interregional allocation of production, ξ and the monopoly power of the representative Northern producer. Various parametric changes affect the relative instantaneous utility through the endogenous variable, ξ.

The discounted present value of welfare of the representative individual in the ith region is given by

$$W_i = \int_0^\infty e^{-\rho t} log(U_i(t)) dt$$

for i= N, S. Using equation (3.2.57) we have

$$W_N - W_S = \int_0^\infty e^{-\rho t}[(\varepsilon - 1) log(\alpha) - log(\frac{1-\xi}{\xi}\frac{L_N}{L_S})] dt. \quad (3.2.58)$$

3.2.6.1 Strengthening IPR protection

Differentiating both sides of equation (3.2.58) with respect to a_S and evaluating the derivatives at the steady state equilibrium values, we have

$$\frac{dW_N}{da_S} - \frac{dW_S}{da_S} = \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})} > 0. \tag{3.2.59}$$

Here $\frac{dW_i}{da_S}$ is the marginal welfare change in the ith region (i=N, S) due to strengthening of IPR protection in the South. So the North has a higher marginal welfare gain than the South in this case.

In our present model with endogenous imitation and localised knowledge spillover, the absolute welfare effect in each of the two regions is ambiguous. Using equation (3.2.56), it can be shown that[22]

$$\frac{dW_S}{da_S} = -\frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})} + \frac{1}{\varepsilon-1}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}[\frac{\theta^*}{\rho} + \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}].$$

Here the first term in the RHS is negative and it represents the marginal welfare loss through the endogenous reallocation of intertemporal R&D expenditure. The second term is positive and it represents the marginal welfare gain through the availability of greater variety of products. The third term is negative and it represents the marginal welfare loss through the inter-regional allocation of production. Similarly, from equation (3.2.55), it can be shown that

$$\frac{dW_N}{da_S} = \frac{1}{\varepsilon-1}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}[\frac{\theta^*}{\rho} + \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}].$$

Here the first term in the RHS is positive and it represents the marginal welfare gain through the availability of greater variety of products. The second term is negative and it represents the marginal welfare loss through the inter regional allocation of production. Both $\frac{dW_S}{da_S}$ and $\frac{dW_N}{da_S}$ may have positive signs. So both the countries may have welfare gain in this case. Using equation (3.2.59), we have the following proposition.

[22] Derivations are done in Appendix 3.7

Proposition 3.2.8. *Both the regions (countries) may gain in welfare due to strengthening of IPR protection in the South; and the North always has a higher marginal welfare gain than the South in this case.*

This result is interesting in the light of the results obtained from Helpman (1993). In an exogenous imitation model with MAR kind of knowledge spillover, Helpman (1993) has shown that the South always faces welfare loss due to stronger IPR protection there. However, the North may or may not have welfare gain in his model. This difference arises because the strengthening of IPR protection raises the steady state equilibrium rate of growth in both the countries in the present model but lowers this rate of growth in the Helpman (1993) model. Hence, in this modified model, both the countries gain in welfare due to increased variety in consumption. If this positive effect outweighs the negative effect of inter-regional allocation of production and intertemporal reallocation of R&D expenditure, then there is net welfare gain of each of the two countries. There is no terms of trade effect in this narrow gap equilibrium case. The North derives higher marginal welfare gain than the South because there is no intertemporal reallocation of R&D expenditure there; and hence the welfare of the North is not affected through this channel which causes welfare loss to the South.

3.2.6.2 Changes in labour endowments

The welfare effect in the ith region with respect to change in the size of the labour endowment of the jth region can be derived as

$$\frac{dW_i}{dL_j} = \int_0^\infty e^{-\rho t}[\frac{dlog(U_i(t))}{dL_j}]dt = \frac{1}{\varepsilon - 1}(\Delta_N^{L_j} + \Delta_e^{L_j}) + \Delta_s^{iL_j} \tag{3.2.60}$$

for i, j=N, S. Here $\Delta_N^{L_j}$ and $\Delta_e^{L_j}$ represent the welfare effects taking place through a change in the variety in consumption and that through a change in the interregional allocation of production respectively. $\Delta_s^{iL_j}$ represents the welfare effect occurring through a change in the ith region's worker's savings rate. In Appendix 3.8, we show that

$$\Delta_N^{L_j} = \theta^* \frac{-a_{22}}{\rho^2(\rho - a_{22})} \frac{d\xi^*}{dL_j} \quad > \quad 0 \text{ (and } < 0) \qquad \text{for j=N (and j=S) },$$

$$\Delta_e^{L_j} = \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}} \frac{1}{\rho} \frac{d\xi^*}{dL_j} \quad < \quad 0 \text{ (and } > 0) \qquad \text{for j=N (and j=S)},$$

$$\Delta_s^{NL_j} \quad = \quad 0 \text{ (and } < 0 \text{)} \qquad \text{for j= S (and j=N)}$$

and

$$\Delta_s^{SL_j} \qquad \text{is ambiguous in sign for j= S, N.}$$

From the above mentioned expressions, we find that both the direct marginal effect and the cross marginal effect on welfare with respect to change in labour endowment in either region may take any sign. So both the North and the South may either gain or lose in welfare.

Helpman (1993) did not analyse the welfare effect of changes in the size of the factor endowments in his model. However, in the Helpman (1993) model, it can be shown that an increase in the size of the Northern labour endowment raises the rate of innovation, North-South relative wage, the fraction of unimitated products and the savings rate in the new steady state equilibrium. Out of these four effects, the first two effects cause welfare gain and the last two effects cause welfare loss in the case of North. The South gains in welfare only due to the first effect and faces welfare loss due to the second and the third effect. The fourth effect does not apply to the South because imitation is costless and exogenous there. Hence the net welfare effects of changes in the size of labour endowments are ambiguous for both the regions in the Helpman (1993) model.

However, an increase in the Southern labour endowment only raises the North-South relative wage and does not affect any other variable in the Helpman (1993) model. This raises the level of welfare in the North and lowers that in the South. Thus the level of welfare in the South (North) varies inversely (directly) with the size of the Southern labour endowment in the Helpman (1993) model. Our analysis is important because we have shown that this size-welfare relationship obtained in the Helpman (1993) model is not necessarily valid once we endogenise the rate of imitation and introduce localised knowledge spillover.

Chapter 4

Multinationalisation and Endogenous Imitation

Introduction[1]

Technological knowledge can be diffused internationally in various alternative ways - namely through: (i) international labour mobility; (ii) communication patterns; (iii) foreign direct investment; and (iv) imitation. However, in the GH (1991a, 1991b) model and in the Helpman (1993) model, imitation is the only channel of transfer; and Southern firms invest resources only to imitate the products of the Northern firms.

Lai (1998) extends the Helpman (1993) model to allow for multinational firms as the source of technology transfer from the North to the South. Lai (1998) shows that the tightening of IPR protection in the South raises the long-run equilibrium rate of growth (innovation) and the rate of multinationalisation when the technology transfer takes place through multinational firms. This result is opposite to that of Helpman (1993). However, both Helpman (1993) and Lai (1998) assume that the imitation activity in the South is costless. Imitation rate is exogenous and strengthening of IPR protection in their models implies an exogenous fall in the imitation rate.

In GH (1991a, 1991b) models, imitation from the North to the South is direct and there is no multinationalisation from the North to the South. Glass and Saggi (2002) extend

[1]**This chapter is based on Mondal and Gupta (2007c).**

the GH (1991a) quality ladder model incorporating the endogenous multinationalisation from the North to the South; and have come to a result completely opposite to that of Lai (1998). Glass and Saggi (2002) define strengthening of IPR protection as equivalent to an exogenous increase in the cost of imitation. Glass and Wu (2007) also find results similar to those obtained in Lai (1998) and Glass and Saggi (2002). In fact, Glass and Wu (2007) point out the importance of reanalysing the results of Lai (1998) in a product variety model with endogenous rate of imitation.

In this chapter, we consider an otherwise identical North-South product variety model of Lai (1998) except for two features.(i) We endogenise the imitation rate in the South by making the imitation activity costly. Mansfield et. al (1981) estimate that the cost of imitation is about 60% of the cost of innovation. The endogenity of the imitation rate plays a crucial role to analyse the growth effect of strengthening IPR protection. (ii) We introduce two types of labour in the South - skilled and unskilled. The skilled labour in the South performs two tasks - R&D work and production in multinational firms. Unskilled workers are used only for the production of imitated goods. Skilled workers are those persons who have formal scientific training. Imitation requires scientific knowledge and so skilled workers are required to do it. Skilled workers are paid higher wages by multinational firms. So Southern imitators, being small farms, employ unskilled workers at low wage and they learn the technique by working and experience. Unskilled workers do not have the formal scientific education.

Our results differ from those obtained in the model of Lai (1998). In our model, the policy of strengthening IPR protection in the South lowers the rate of innovation in the North and the rate of multinational flow to the South[2]. Also, it raises the imitation rate in the South and the domestic skilled-unskilled wage ratio in the South. So our results are consistent with the results of Helpman (1993) and of GH (1991b) which are

[2]Glass and Saggi (2002) have got similar results using a quality-ladder based North-South growth model. Both Lai (1998) and Glass and Saggi (2002) have assumed that there is only one type of labour in the South. Our results may be valid even with one type of Southern labour. However, the sufficient conditions are hard to interpret.

questioned by Lai (1998).

This chapter is organised as follows. Section 4.1 presents the basic product cycle model with multinationalisation as the channel of production transfer from the North to the South. Section 4.2 presents the working of the model in the steady state equilibrium; and the comparative steady state results are summarized in section 4.3.

4.1 The model

We assume a free trade world consisting of two countries (regions) - the North and the South. Northern firms invent products by incurring an upfront innovation cost; and each of the successful firms produces and sells the product in the world market and earns a monopoly profit. Northern firms have a choice between producing the product in the North and shifting production plants to the South being multinational in nature. The life of the patent on every product in each of the two regions is infinitely long. However, the patent law enforcement in the South is not perfect but it is perfect in the North. Though the marginal cost of production in the South is lower than that in the North, there is a risk of imitation faced by the Northern multinational firm and it comes from local Southern firms. Both the rate of innovation in the North and the rate of imitation in the South are endogenously determined in the model. However, Southern firms can not innovate.

In the North, there is only one type of labour; and it is used in both the activities: R&D and production. In the South, there are two types of labour - skilled and unskilled. Skilled workers are required in the imitative R&D sector and in producing products of multinational firms. However, by imitative R&D, Southern firms become able to use unskilled labour in production[3]. One unit of each of the varieties can be produced using one unit of labour in both the countries. Southern firms invest resources to imitate

[3]Otherwise, there would be no difference in costs between multinationals and Southern firms; and so the Southern firms would lack the incentive to imitate.

the products of multinational firms (MNC). There is Bertrand competition between the MNC and the successful local imitator in each of the differentiated product markets. Since the marginal cost of production of the MNC is higher than that of the imitator firm, MNC is driven out of the market in equilibrium. The optimal pricing decision of the successful imitating firm depends on the skilled-unskilled wage gap in the South. If this gap is high enough then the imitating firm can charge the profit maximising monopoly price. This is the case of wide gap equilibrium. However, if the skilled-unskilled wage gap is narrow, then the Southern imitating firm charges an equilibrium price equal to the rival's marginal cost of production. This is the so-called narrow-gap equilibrium case[4]. Strengthening the IPR protection in the South means increasing the cost of imitation by incurring more labour to imitate a single product. This is due to the stricter uniqueness criteria imposed by the local Southern Government[5].

4.1.1 The demand for goods

The demand side of the model is identical to those presented in subsection 2.1.1 in chapter 2 and in subsection 3.1.1.1 in chapter 3. This is also identical to that in Lai (1998) and to that in Helpman (1993). Hence, we obtain the following optimality conditions:

$$\frac{\dot{E}}{E} = r - \rho; \tag{4.1.1}$$

and

$$x(z) = E(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}du} \qquad \forall z \in [0, n(t)]. \tag{4.1.2}$$

4.1.2 The production of goods

Here

$$n = n_N + n_S$$

[4]In GH (1991b), the wide-gap or the narrow-gap case arises depending on whether the North-South wage gap is wide or narrow. However, in our model, this arises when the domestic wage gap between the skilled worker and the unskilled worker in the South is wide or narrow.

[5]This kind of definition of stronger IPR protection is borrowed from Glass and Saggi (2002).

and

$$n_S = n_M + n_I$$

where n_N, n_M and n_I stand for the number of products produced by the Northern firms, by the multinationals in the South and by the Southern imitators respectively. The producer of each of the varieties is a profit maximising monopolist; and hence the equilibrium price of a typical jth producer is given by

$$p_j = \frac{c_j}{\alpha} \quad \text{for j= } N,\ M,\ I. \tag{4.1.3}$$

Here c_j is the marginal cost of production of the jth type of firm. Here $c_N = w_N$, $c_M = w_H$ and $c_I = w_L$ where w_N, w_H and w_L stand for wage rates of Northern workers, Southern skilled workers and Southern unskilled workers respectively.
We assume the following.

$$w_N > w_H > w_L\ . \tag{4.1.4}$$

Due to Bertrand price competition between a successful Southern imitator and the multinational firm, the imitating firm can charge the monopoly price given by equation (4.1.3) for $j = I$ if

$$\frac{w_L}{\alpha} < w_H. \tag{4.1.5}$$

This is the case of wide gap equilibrium. However, when $\frac{w_L}{\alpha} > w_H$, the Southern imitating firm charges a limit price equal to its rival's marginal cost of production. So

$$p_I = w_H \tag{4.1.6}$$

in this case; and it is known as the narrow gap equilibrium. It should be noted that the multinational firm can not compete with the local imitating firm in equilibrium in both the cases and is driven out of the market.
Profit of the jth type of firm is given by

$$\pi_j = (p_j - c_j)x_j \quad \text{for j= } N,\ M,\ I.$$

Using equations (4.1.3), (4.1.6) and the expression of profit of the jth type of firm we obtain the monopoly equilibrium profit of different types of producers as follows.

$$\pi_N = \frac{(1-\alpha)}{\alpha} w_N x_N, \tag{4.1.7}$$

$$\pi_M = \frac{(1-\alpha)}{\alpha} w_H x_M, \tag{4.1.8}$$

$$\pi_{IN} = (w_H - w_L) x_I, \tag{4.1.9a}$$

and

$$\pi_{IG} = \frac{(1-\alpha)}{\alpha} w_L x_I. \tag{4.1.9b}$$

Here π_{IN} and π_{IG} are the profit of the Southern imitator in the narrow gap equilibrium and that in the wide gap equilibrium respectively.

4.1.3 R&D technology

New products in the North grow over time according to

$$\dot{n} = n \frac{L_N^R}{a_N}. \tag{4.1.10}$$

Similarly the new imitative products in the South grow over time according to

$$\dot{n_I} = n_I \frac{H_S^R}{a_I}. \tag{4.1.11}$$

Here L_N^R and H_S^R are the amount of Northern labour hired for R&D in the North and the amount of Southern skilled labour hired for imitation in the South respectively. $\frac{a_N}{n}$ and $\frac{a_I}{n_I}$ are the effective per unit labour requirements in the Northern R&D sector and in the Southern imitative R&D sector respectively. The cost of innovating a product in the North is given by

$$C_N = \frac{a_N}{n} w_N; \tag{4.1.12}$$

and the cost of imitating a multinational product in the South is given by

$$C_I = \frac{a_I}{n_I} w_H. \tag{4.1.13}$$

An increase in a_I is interpreted as the strengthening of the IPR protection in the South[6].

4.1.4 Steady state equilibrium conditions

The world economy is in a steady state growth equilibrium. Hence[7]

$$\frac{\dot{E}}{E} = \frac{\dot{n}}{n} = \frac{\dot{n_S}}{n_S} = \frac{\dot{n_N}}{n_N} = \frac{\dot{n_I}}{n_I} = \frac{\dot{n_M}}{n_M} = g. \tag{4.1.14}$$

Here g is the rate of growth.

4.1.5 Free entry condition

Let Π_N be the discounted present value of the stream of profits of a typical Northern firm over the infinite time horizon. Then, in the steady state equilibrium, $\Pi_N = \frac{\pi_N}{r}$[8]. Hence, the free entry condition in the North is given by

$$\frac{a_N}{n} w_N = \Pi_N = \frac{\pi_N}{r}. \tag{4.1.15}$$

Similarly the condition for free entry into the imitation activity in the South is given by

$$\frac{a_I}{n_I} w_H = \frac{\pi_I}{r} \tag{4.1.16}$$

where the left hand side (L.H.S.) of equation (4.1.16) is the cost of imitation and its right hand side (R.H.S.) is the discounted present value of profits of the imitator in the steady state equilibrium.

[6]Glass and Saggi (2002) define $(1+k)a_I$ as the resource requirement in the imitation sector; and a higher value of k represents a stronger IPR protection in their model. This is so because a stronger IPR protection can reduce imitation efficiency through various channels e.g., the copying firm may need to distinguish the imitation in the view of legal authorities, they may have to fight in the court to prove that their product is sufficiently unique. Also as IPR protection is strengthened, aspects of the design that would have been copied may have to be innovated anew. Thus, k can be thought of as measuring how much of the design must be unique to satisfy the standard.

[7]We normalise the value of a Northern firm to be unity. This implies that $\frac{\dot{E}}{E} = g$ in the balanced growth equilibrium.

[8]$\Pi_N = \int_0^\infty e^{-rt} \pi_N(t) dt = \frac{\pi_N}{r}$. The last step follows because $\pi_N(t)$ is constant in the steady state equilibrium.

4.1.6 Multinationalisation equilibrium

Following Lai (1998) we define the rate of imitation in the South as

$$\iota = \frac{\dot{n_I}}{n_M}$$

and the rate of multinationalisation[9] in the South as

$$\omega = \frac{\dot{n_S}}{n_N}.$$

Each of all the multinational firms faces the threat of imitation from its Southern local competitors; and it discounts its future profits at the rate $(r+\iota)$. The discounted present value of profits of a multinational firm in the steady state equilibrium[10] is given by

$$\Pi_M = \frac{\pi_M}{r+\iota}.$$

In equilibrium, a typical Northern firm is indifferent between multinationalisation and continuing production in the North. So the discounted present value of profits of a Northern firm should be equal to that of an MNC. Hence, in equilibrium,

$$\frac{\pi_N}{r} = \frac{\pi_M}{r+\iota}. \tag{4.1.17}$$

Equation (4.1.17) is same as equation (14) in Lai (1998) (see p. 142 of Lai (1998)). It shows that the instantaneous profit of a Northern firm must be smaller than that of a Southern MNC in the multinationalisation equilibrium because MNC faces the risk of imitation what the Northern firm does not face. This is ensured only if the Northern wage, w_N, is always higher than the wage of the Southern skilled worker, w_H.

4.1.7 Labour market equilibrium

The demand for skilled labour from the multinationals and imitative R&D sector in the South are $n_N x_M$ and $a_S \frac{\dot{n_I}}{n_I}$ respectively. So the skilled labour market equilibrium

[9] A more natural measure of multinationalisation rate is $\iota = \frac{\dot{n_M}}{n_N}$. However, we follow Lai (1998). Even if this more natural measure is used, the results are marginally different.

[10] $\Pi_M = \int_0^\infty e^{-rt}\{e^{-\iota t}\pi_M(t)\}dt$. Here the expression $\{e^{-\iota t}\pi_M(t)\}$ is the expected profit of the multinational firm at period t. Then $\Pi_M = \int_0^\infty e^{-(r+\iota)t}\pi_M dt = \frac{\pi_M}{r+\iota}$.

condition in the South with endowment, H_S, is given by

$$H_S = n_M x_M + a_I \frac{\dot{n_I}}{n_I} \, . \tag{4.1.18}$$

The demand for unskilled labour in the South is $n_I x_I$. So the unskilled labour market equilibrium condition in the South with endowment, L_S, is given by

$$L_S = n_I x_I \, . \tag{4.1.19}$$

The demand for labour from the Northern R&D sector and from the Northern production sector are given by $a_N \frac{\dot{n}}{n}$ and $n_N x_N$ respectively. Thus the Northern labour market equilibrium condition is given by

$$L_N = a_N \frac{\dot{n}}{n} + n_N x_N \tag{4.1.20}$$

where L_N represents the labour endowment in the North.

4.2 Existence of steady state growth equilibrium

First, we describe the equilibrium condition in the North. Using equations (4.1.7) and (4.1.15), we have

$$\frac{\alpha}{1-\alpha} a_N r = n_N x_N \left(\frac{n}{n_N} \right) .$$

Using equations (4.1.1), (4.1.14), (4.1.20) and the fact that[11] $\frac{n}{n_N} = 1 + \frac{\omega}{g}$, we reexpress the equation mentioned above as

$$\frac{\alpha}{1-\alpha} a_N (\rho + g) = (L_N - a_N g) \left(1 + \frac{\omega}{g} \right) \, . \tag{4.1.21}$$

Equation (4.1.21) describes the relationship between the rate of innovation in the North, g, and the rate of multinationalisation, ω, when Northern product markets and the labour market are in equilibrium. From equation (4.1.21), it is clear that ω is a positive function of g. This is so because, as the rate of multinationalisation is increased, firms

[11] $\frac{n}{n_N} = 1 + \frac{n_S}{n_N} = 1 + \frac{\dot{n_S}}{n_N} \frac{n_S}{\dot{n_S}} = 1 + \frac{\omega}{g}$, using the definitions of ω and g.

shift their production base to the South. Full employment of Northern labour then causes its reallocation from the Northern production sector to the R&D sector. This raises the rate of innovation there. This equation (4.1.21) is identical to equation (22) of Lai (1998) (see p. 143 of Lai (1998)). Rate of imitation in the South, ι, does not enter into the Northern equilibrium condition because imitation from the North to the South is not direct.

4.2.1 Narrow gap equilibrium

We now turn to describe the equilibrium condition in the South. We first describe the case of narrow gap equilibrium. In this case, using equations (4.1.9a) and (4.1.16) we have

$$\frac{a_I}{n_I} w_H = \frac{(w_H - w_L)x_I}{r}.$$

Using equations (4.1.1), (4.1.14) and (4.1.19), the above mentioned equation can be writen as

$$a_I(\rho + g) = (1 - \frac{w_L}{w_H})L_S. \tag{4.1.22}$$

Equation (4.1.22) shows that the skilled-unskilled wage ratio in the South varies positively with g. This is so because an increase in g raises the cost of capital of a Southern imitating firm in the steady state equilibrium. So, in equilibrium, the profit rate should also go up; and this is possible when the ratio of the marginal cost of production to the price of the imitated product is decreased. Since the price of the imitated product is equal to the skilled wage, w_H, in this narrow gap equilibrium, this increase in g then implies a rise in the skilled-unskilled wage ratio.

Using equations (4.1.2), (4.1.3) and (4.1.6), we have

$$\frac{x_I}{x_M} = \alpha^{-\varepsilon}. \tag{4.1.23}$$

Equation (4.1.23) shows that the demand for the imitating firm's product is proportional to the demand for the multinational firm's product. This is so because, in the narrow gap equilibrium, the relative price between these two types of products is always constant. Using equations (4.1.18) and (4.1.19), equation (4.1.23) can be written as[12]

$$\frac{L_S}{H_S - a_I g}\frac{g}{\iota} = \alpha^{-\varepsilon}. \tag{4.1.24}$$

This equation (4.1.24) shows a positive relationship between the rate of growth and the rate of imitation. An increase in the growth rate of the imitated products raises the size of the imitative R&D sector in the South and hence lowers the number of skilled workers available for multinational firms. This lowers the level of production of the multinationals. The rate of imitation must increase to keep the relative production unchanged, i.e., to satisfy equation (4.1.23). This equation (4.1.24) does not exist in the Lai (1998) model because imitation is costless there.

We now describe the multinationalisation equilibrium condition. Using equations (4.1.1), (4.1.2), (4.1.3), (4.1.7), (4.1.8), (4.1.14), (4.1.18), (4.1.20) and (4.1.21) and the expressions of ι and ω, the multinational equilibrium equation (4.1.17) can be written as[13]

$$\left[\frac{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g}{H_S - a_I g}\ \frac{g}{\iota+g}\right]^{\alpha} = \frac{\rho+g}{\rho+g+\iota}. \tag{4.1.25}$$

This equation (4.1.25) is similar but not identical to the equation (23) in Lai (1998) (see p. 143 of Lai (1998)). The left hand side of equation (4.1.25) is the relative profit rate of the Northern firm to that of the multinational firm and the right hand side is the relative cost of capital of the Northern firm to that of the multinational firm. There is a negative relationship between g and ι as implied by equation (4.1.25)[14]. An increase in ι lowers the relative cost of capital of the Northern firm and also lowers its relative

[12]We have $\frac{n_M}{n_I} = \frac{n_M}{\dot{n}_I}\frac{\dot{n}_I}{n_I} = \frac{g}{\iota}$, using the definitions of g and ι.

[13]See Appendix 4.1 for the detail derivation of equation (4.1.25).

[14]This holds under the necessary and sufficient condition $(g+\iota) > \rho\frac{\alpha}{1-\alpha}$. We know from inequality (A8) of Appendix 4.1 that $g > (1-\alpha)\frac{L_N}{a_N} - \rho\alpha$. Since $\iota \geq 0$, this necessary and sufficient condition is always satisfied if we assume that $\frac{L_N}{a_N} \geq \frac{\alpha\rho(2-\alpha)}{(1-\alpha)^2}$. This condition is crucial for the results. Otherwise part of the AA curve slopes positively and our results may not be valid if the point of intersection is on the positively sloped segment. We like to focus on the negative slope of the AA curve because Lai (1998) derived a negative relationship between g and ι under a set of restrictions. See Appendix 4.1 for the mathematical derivation.

profit rate. However, its effect on the relative cost of capital is higher and hence the rate of multinationalisation is decreased due to an increase in ι. Since more firms stay in the North, the demand for labour from the production sector is increased there. This reduces the availability of labour to the Northern R&D sector; and hence the Northern rate of innovation is reduced.

Equations (4.1.24) and (4.1.25) are two equations to determine the two unknowns, g and ι. The negatively sloped curve AA in the figure 4.1.1 represents equation (4.1.25) and the positively sloped curve OB represents equation (4.1.24). The existence of an unique steady state equilibrium solution for g and ι is shown in the figure 4.1.1. In Lai (1998), ι is exogenous and the OB curve does not exist. So the equilibrium value of g is obtained in terms of ι according to the AA curve. Once the equilibrium value of g is obtained, $\frac{w_L}{w_H}$ is solved from equation (4.1.22) and ω is solved from equation (4.1.21). Also the relative wage of the Northern worker to that of the Southern skilled worker, $\frac{w_N}{w_H}$, can be solved from equation (4.1.25). The LHS of equation (4.1.25) is the relative profit of the Northern firm to that of the multinational firm; and this varies negatively with $\frac{w_N}{w_H}$. So, using the RHS of equation (4.1.25), we can solve[15] for the $\frac{w_N}{w_H}$.

4.2.2 Wide gap equilibrium

In the case of wide gap equilibrium, the Southern imitator charges the monopoly price given by $p_I = \frac{w_L}{\alpha}$. So the relative price of the imitator's product to that of the multinational's product is not constant here. The per period profit of the imitating firm is represented by equation (4.1.9b) in this case. So equations (4.1.22), (4.1.23) and (4.1.24) are not valid here. However, equations (4.1.21) and (4.1.25) remain unchanged in the

[15] The left hand side of equation (4.1.25) can be written as $\left(\frac{w_N}{w_H}\right)^{\frac{-\alpha}{1-\alpha}}$. Then the solution of the relative wage is given by

$$\frac{w_N}{w_H} = \left(\frac{\rho + g}{\rho + g + \iota}\right)^{-\frac{1-\alpha}{\alpha}}.$$

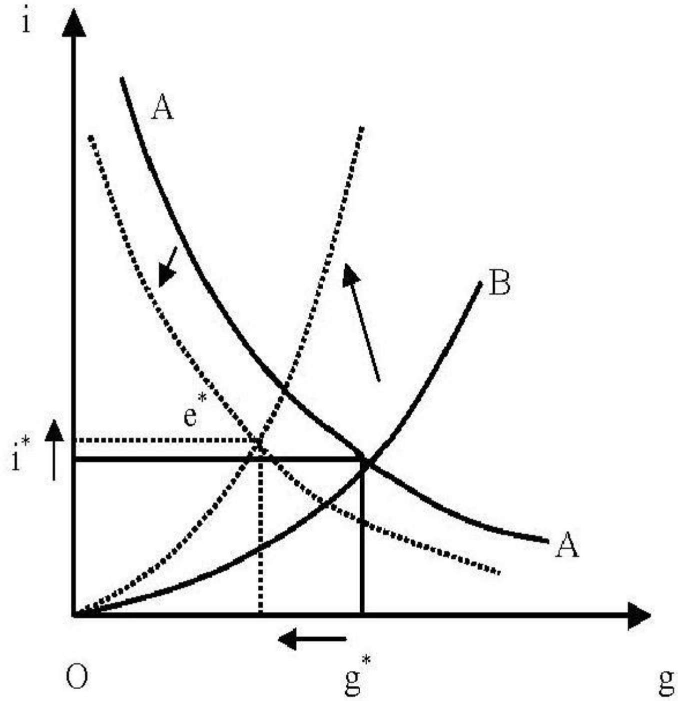

Figure 4.1.1: Steady state equilibrium

wide gap equilibrium. Using equations (4.1.9b) and (4.1.16), the free entry condition of the Southern imitating firm in this wide gap equilibrium can be written as

$$\frac{\frac{1-\alpha}{\alpha} w_L x_I}{\frac{a_I}{n_I} w_H} = r. \tag{4.1.26}$$

The left hand side of equation (4.1.26) represents the profit rate of a typical Southern imitating firm and the right hand side stands for its cost of capital. The profit rate is an increasing function of $\frac{w_L}{w_H}$ and the cost of capital is an increasing function of g. Using equations (4.1.19) and (4.1.26), we have

$$\frac{w_L}{w_H} = \frac{\frac{\alpha}{1-\alpha} a_I(\rho + g)}{L_S}; \tag{4.1.27}$$

and this equation (4.1.27) solves for $\frac{w_L}{w_H}$ in terms of g.

Now using equations (4.1.1), (4.1.2), (4.1.3), (4.1.14), (4.1.18), (4.1.19) and the expressions for ι and g, we can reexpress the equation (4.1.27) as

$$\left\{ (H_S - a_I g)\frac{\iota}{g} \right\}^{1-\alpha} L_S^{\alpha-1} = \frac{\frac{\alpha}{1-\alpha} a_I(\rho + g)}{L_S}. \tag{4.1.28}$$

The left hand side of equation (4.1.28) stands for the domestic relative wage of the unskilled workers in the South. So $\frac{w_L}{w_H}$ varies inversely with g and positively with ι. Equation (4.1.28) shows that g and ι are positively related. An increase in the rate of imitation in the South raises the relative demand for the unskilled labour there. This raises the relative wage of the unskilled labour. So the profit rate of the imitating firm is increased because it is a positive function of $\frac{w_L}{w_H}$. Then the cost of capital must also increase to restore equilibrium, i.e., to satisfy equation (4.1.28); and this is possible only when the rate of innovation, g, is increased.

In the wide gap equilibrium case, equations (4.1.25) and (4.1.28) are two equations to determine two unknowns g and ι. Equation (4.1.28) can be represented by a positively sloped curve similar to OB curve in figure 4.1.1. We call it OB curve again. AA curve represents equation (4.1.25). So the same figure 4.1.1 can also be used here to show the existence of an unique steady-state equilibrium solution for g and ι in this wide gap equilibrium case. Once g is solved, $\frac{w_L}{w_H}$ is solved from equation (4.1.27) and ω is solved from equation (4.1.21). Also $\frac{w_N}{w_H}$ can be solved using equation (4.1.25) (see footnote 15).

4.3 Comparative steady state analysis

4.3.1 Stronger IPR protection

We first consider the case of a narrow gap equilibrium. Strengthening of IPR protection in the South is defined as an increase in a_I. Equation (4.1.24) shows that ι will rise, given g when a_I is increased. So OB curve shifts upward. Also, given g, an increase in a_I causes an increase in the left hand side of equation (4.1.25); and hence ι should fall to satisfy this equation. So AA curve shifts towards the origin. In the new equilibrium i.e., at point e^* in the figure 4.1.1, g will fall and ι will rise because AA curve shifts by a larger magnitude than the OB curve[16]. Equation (4.1.22) shows that $\frac{w_L}{w_H}$ is decreased

[16]See Appendix 4.2 for mathematical derivations of the comparative static results.

and equation (4.1.25) shows that $\frac{w_N}{w_H}$ is increased in this case. Equation (4.1.21) shows that ω is decreased. We summarise these results in the following proposition.

Proposition 4.3.1. *Strengthening of IPR protection in the South, measured by an increase in the cost of imitation, lowers the rate of innovation in the North as well as the rate of multinationalisation. This also raises the Southern rate of imitation, relative wage of the North*[17] *and the skilled-unskilled wage ratio in the South.*

We now try to provide the intuition behind these results; and, in this context, we compare the mechanism of this model to those of the other related models. Given other things unchanged, an increase in the unit skilled labour requirement in the imitative R&D sector in the South raises the size of this sector and leaves fewer skilled workers available for the multinational firms. This has the following two effects: (1) the relative wage of the skilled worker is increased in the South and (2) the level of production of the representative multinational firm is decreased. The first effect raises the marginal cost of production of the multinational firm and makes it less efficient compared to its Northern counterpart. Other things remaining unchanged, the rate of imitation in the South should fall following the first effect. The second effect tends to raise the rate of imitation in the South, given g, because the relative sale of the multinational firm to that of the local imitating firm is constant in the narrow gap equilibrium. However, this later effect dominates the former; and hence the rate of imitation is increased due to an increase in the cost of imitation.

As the rate of imitation is increased, the multinational firm operating in the South faces a higher threat of its product being imitated. So the expected monopoly duration of each of the multinationals is reduced. So its discounted present value of profit relative to that of the Northern firm is reduced. This lowers the rate of multinationalisation in the South. When more firms stay in the North, demand for Northern labour from the production sector is increased; and so the labour allocation to the R&D sector is

[17] *This is defined as $\frac{w_N}{w_H}$. See footnote (15) for its exact expression.*

reduced. So the rate of product development (innovation) in the North is reduced.

There is a common property of all the related models. An increase in the pace of North-South production transfer raises the rate of Northern innovation because Northern labour force is reallocated from the production sector to the R&D sector. In Helpman (1993), imitation is direct. So a reduction in the imitation rate causes a reduction in the pace of North-South production transfer. In Lai (1998), imitation takes place through multinationalisation and the reduction in the imitation rate raises the relative profitability of the multinational firm by lowering the South-North relative wage. So the multinationalisation rate is increased; and this raises the pace of North-South production transfer. In the present model, cost of imitation is increased. So the demand for skilled labour is increased in the imitative R&D sector which, in turn, raises its wage rate. So multinationalisation becomes less attractive and the North South production transfer takes place at a lower rate.

In the case of a wide gap equilibrium, the shift of AA curve remains same as in the case of a narrow gap equilibrium. However, equation (4.1.28) shows that ι rises following an increase in a_I, given g; and this implies that BB curve shifts upward. Once again, in the new equilibrium i.e., at point e^* in the figure 4.1.1, g will fall and ι will rise because AA curve shifts by a larger magnitude than BB curve[18]. Then equation (4.1.21) shows that ω is decreased and equation (4.1.25) shows that $\frac{w_N}{w_H}$ is increased. However, the effect on $\frac{w_L}{w_H}$ is ambiguous in this case (see equation (4.1.27)). So the direction of the effect of the stronger IPR protection in the South on endogenous variables like g, ω and ι does not depend on the nature of the equilibrium.

We can compare our results to those obtained from the GH (1991b) model which considers endogenous imitation but does not consider multinationalisation. In GH (1991b), an increase in the cost of imitation, defined as an increase in the per unit labour requirement in the imitative R&D activity in the South, lowers both the rate of innovation and the

[18]See Appendix 4.2 for mathematical derivations of comparative static results.

rate of imitation in the wide gap equilibrium as well as in the narrow gap equilibrium. This is so because strengthening IPR protection acts as a resource wasting effect in the South[19] in that model. In the present model, the rate of innovation and the rate of imitation may move in the opposite directions due to strengthening of IPR protection in the South. While the rate of innovation is reduced, the rate of imitation may go up.

4.3.2 Change in factor endowments

4.3.2.1 Narrow gap equilibrium

In the narrow gap equilibrium, an increase in the Northern labour endowment, L_N, causes an upward shift of AA curve in figure 4.1.1. However, OB curve remains unaffected because equation (4.1.24) does not include L_N. So, in the new steady state equilibrium, both g and ι rise. Equation (4.1.21) shows that ω rises in this case. An increase in the Southern unskilled labour endowment, L_S, causes OB curve to shift upward, but does not affect AA curve. So, in the new steady state equilibrium, g falls and ι rises. ω also falls in this case. An increase in the Southern skilled labour endowment, H_S, causes AA curve to shift upward and OB curve to shift downward. However, OB curve shifts by a larger magnitude than AA curve such that g and ω rise and ι fall in the new steady state equilibrium[20]. We summarize the major results in the following proposition.

Proposition 4.3.2. *An increase in the Northern labour endowment raises the Northern rate of innovation, the Southern rate of imitation and the rate of multinationalisation. An increase in the Southern unskilled (skilled) labour endowment lowers (raises) the Northern rate of innovation and the rate of multinationalisation but raises (lowers) the Southern rate of imitation.*

We now try to provide the intuition behind these results. An increase in the Northern

[19]In GH (1991b), both the rate of innovation and the rate of imitation would be reduced if the size of the South is reduced.

[20]This has been shown mathematically in Appendix 4.3 of chapter 4.

labour endowment raises the Northern terms of trade (North-South relative wage). This is a standard result in the GH (1991b) framework and our description of the North is identical to that in GH (1991b). As the relative wage of the North is increased, larger fraction of Northern firms shift their production base to the South. So the rate of multinationalisation is increased. In the North, the size of the production sector shrinks and the size of the R&D sector is enlarged; and this raises the rate of innovation. In the South, the demand for skilled labour is increased and so the relative wage of the skilled worker to that of the unskilled worker goes up. This raises the profit rate of the local Southern imitator. So, the rate of imitation in the South goes up. In the narrow gap equilibrium, relative sale of the imitated firm compared to the multinational firm is constant. So an increase in the Southern unskilled labour endowment raises the rate of imitation. This reduces the expected monopoly duration of the multinational firm in the South. So, the rate of multinationalisation is decreased. This raises the demand for Northern labour in the Northern production sector and thus lowers the size of the Northern R&D sector. Thus the rate of innovation in the North is lowered. An increase in the skilled labour endowment in the South raises the allocation of skilled labour to multinational firms and thus the per-firm sale of the MNC goes up. This lowers the rate of imitation in the South. As the rate of imitation is reduced, the rate of multinationalisation is raised. So, the rate of innovation in the North is also increased.

4.3.2.2 Wide gap equilibrium

In the wide gap equilibrium, an increase in the Northern labour endowment, L_N, causes an upward shift of AA curve but keeps OB curve unaffected. So, in the new steady state equilibrium, both g and ι rise; and ω rises following a rise in g. An increase in L_S makes OB curve shift downward but does not affect AA curve. So, in the new steady state equilibrium, g and ω rise and ι falls. An increase in H_S causes OB curve to shift downward and AA curve to shift upward. g and ω rise in the new equilibrium but the effect on ι is ambiguous[21]. We summarize the major results in the following proposition.

Proposition 4.3.3. *An increase in the Northern labour endowment raises the Northern rate of innovation, the Southern rate of imitation and the rate of multinationalisation. An increase in the Southern unskilled and/or skilled labour endowment raises the Northern rate of innovation and the rate of multinationalisation. The Southern rate of imitation varies inversely with the Southern unskilled labour endowment but may vary either way with Southern skilled labour endowment.*

Comparing propositions 4.3.2 and 4.3.3, we find that the effects of a change in the Northern labour endowment on various endogenous variables in our model does not depend on the skilled-unskilled wage gap in the South. Also the effects of a change in the skilled labour endowment on the rate of innovation and on the rate of multinationalisation does not depend on the nature of the equilibrium (wide gap vs. narrow gap). However, results differ in the case of a change in the unskilled labour endowment. In the narrow gap equilibrium case, an increase in the unskilled labour endowment lowers the rate of innovation and the rate of multinationalisation but raises the rate of imitation. However, just an opposite result is obtained in the wide gap equilibrium.

4.3.3 Tax on imitation sector[22]

Grossman and Helpman (1991b) do not define the strengthening of IPR protection as the increase in the labour requirement in the imitation sector. They define it as an imposition of a tax on this sector. If such a tax is imposed at the rate ϕ as percentage of the selling price of the imitated blueprint, then equation (4.1.16) is modified as follows.

$$(1+\phi)\,\frac{a_I}{n_I} w_H = \frac{\pi_I}{r}.$$

[21]This has been shown mathematically in Appendix 4.3 of chapter 4.

[22]**We assume that the tax-revenue is spent in such a manner that the demand side of the model remains undisturbed. If a public good is produced using the tax revenue and if the utility function of the consumer described in subsection 2.1.1 in chapter 2 is separable in terms of the private goods (varieties) and the public good, then the demand functions for the varieties are independent of tax-rate.**

The left hand side of the equation mentioned above represents the post-tax buying price of the blueprint because the market for the blueprint is perfectly competitive. Then equation (4.1.22) is modified as follows.

$$(1+\phi)\{a_I(\rho+g)\} = \left(1 - \frac{w_L}{w_H}\right) L_S \ . \tag{4.1.29}$$

However, ϕ, does not enter into the equations (4.1.21), (4.1.24) and (4.1.25). So a policy of changing ϕ does not affect the equilibrium values of g, ι and ω which are determined simultaneously by the equations (4.1.21), (4.1.24) and (4.1.25) in the narrow gap equilibrium. So the policy of imposing a tax on the Southern imitative R&D sector raises $\frac{w_H}{w_L}$; and this is shown by equation (4.1.29). This only hurts the Southern unskilled workers relatively. We summarise this result in the following proposition.

Proposition 4.3.4. *A policy of imposing a tax on the Southern imitative R&D sector in the narrow gap equilibrium case has no effect on the rate of product development in the North, on the rate of imitation in the South and on the rate of multinationalisation. Only the skilled-unskilled wage inequality in the South is increased by this policy.*

Our results are similar to those in the narrow gap equilibrium in the GH (1991b) model where a tax policy in the South also does not have any impact on the rate of innovation and on the rate of imitation.

However, in the wide gap equilibrium, this modifies the Southern free entry condition, i.e., equation (4.1.26), as follows.

$$\frac{\frac{1-\alpha}{\alpha} w_L x_I}{(1+\phi)\frac{a_I}{n_I} w_H} = r. \tag{4.1.30}$$

Then equation (4.1.28) is also modified as follows.

$$\frac{1}{1+\phi} \left\{(H_S - a_I g)\frac{\iota}{g}\right\}^{1-\alpha} L_S^{\alpha-1} = \frac{\frac{\alpha}{1-\alpha} a_I(\rho+g)}{L_S}. \tag{4.1.31}$$

An increase in ϕ reduces the left hand side of equation (4.1.31), given other things unchanged. So given g, ι must increase to satisfy equation (4.1.31) following an increase in ϕ; and hence OB curve should shift rightwards in the figure 4.1.1. However, AA curve is not affected by a change in ϕ. So, in the new equilibrium, g should fall and ι should rise. Hence equation (4.1.21) shows that ω is decreased and equation (4.1.25) shows that $\frac{w_N}{w_H}$ is increased (see footnote 15). Since the left hand side of equation (4.1.28) gives the expression for the $\frac{w_L}{w_H}$, a decrease in g and/or an increase in ι raise the unskilled-skilled wage ratio in the South.

Proposition 4.3.5. *A policy of imposing a tax on the Southern imitative R&D sector in the wide gap equilibrium case lowers the rate of innovation in the North and the rate of multinationalisation; and raises the Southern rate of imitation, Southern unskilled-skilled wage ratio and the North-South relative wage.*

Comparing propositions 4.3.4 and 4.3.5 we find that the effects of the imposition of a tax on the Southern imitative R&D sector crucially depends on the domestic skilled-unskilled wage gap. In the case of narrow gap equilibrium, this policy would only hurt the Southern unskilled workers relatively and would not produce any effect on g, ω and ι. However, in the case of wide gap equilibrium, this policy would not only benefit the unskilled workers relatively but also would produce negative effects on the rate of innovation and on the rate of multinationalisation and positive effects on the rate of imitation and on the North-South relative wage.

Chapter 5

Unemployment in the South

Introduction[1]

Models developed in earlier chapters of this thesis assume full employment of labour in both the countries. In this chapter, we analyse the unemployment problem of unskilled workers in the less developed countries introducing unemployment in the South in a North South product cycle model as developed by GH (1991b). Endogenous growth is driven by the introduction of the new differentiated products in the North which are later imitated in the South; and the unemployment in the unskilled labour market in the South is explained by the efficiency wage hypothesis.

There are two branches of the existing literature to which this chapter is related. The first one focuses on the relationship between the long run rate of economic growth and the level of unemployment in a closed economy framework[2]. A subset of these works considers unemployment resulting from efficiency wage[3]. However, these papers do not consider the North South product transfer through imitation in the South; and hence do not analyse the role of IPR protection in the South. The other branch of the literature deals with the effects of strengthening IPR protection in the less developed countries

[1]**This chapter is based on Mondal and Gupta (2007a).**

[2]See, for example, Bean and Pissarides (1993), Aghion and Howitt (1994), Palokangas (1996), Van Schauk and De Groot (1998), Staddler (1999) etc.

[3]See, for example, De Groot (1998), Van Schauk and De Groot (1998), Staddler (1999) etc.

on the rate of economic growth and on the welfare in the different trading countries of the world. However, these papers use North South models based on GH (1991a, 1991b) framework and assume full employment of labour in all the countries[4]. The model of Arnold (2002) is the only exception in the North South literature because it deals with the unemployment problem in the North. However, Arnold (2002) does not consider unemployment in the South caused by the efficiency wage hypothesis. In reality, less developed countries suffer from severe unemployment and underemployment problems of unskilled (uneducated) labour in agricultural sectors and in urban informal sectors.

In this chapter, we extend the GH (1991b) model introducing unemployment in the South caused by the efficiency wage hypothesis. We consider a South with two types of labour - skilled and unskilled; and introduce efficiency wage hypothesis in the unskilled labour market. Mirrlees (1976), Stiglitz (1976), Dasgupta and Ray (1986) and many others explain unemployment of unskilled labour in less developed countries using the efficiency wage hypothesis in static models. However, the North in this model has only skilled labour. In the Western developed countries, the illiteracy rate is negligible and the percentage of unskilled (uneducated) workers is very low. So we do not consider unskilled labour in the North. However, illiteracy is a serious problem in poor countries of South Asia and Africa. Agricultural sectors and Urban informal sectors in less developed countries are mainly dependent on unskilled (uneducated) workers. Size of the formal sector in a less developed country is far lower than the size of the unorganised (agriculture and urban informal) sector. Efficiency wage hypothesis is generally valid for those workers who are underpaid because the improvement in their income raises their levels of consumption and working abilities. Rodgers (1975), Bliss and Stern (1978) etc. provide empirical evidences in favour of this hypothesis. Unskilled workers of rural and urban informal sectors earn substantially less than skilled workers of urban formal

[4]See, for example, GH (1991b), Helpman (1993), Lai (1998), Yang and Maskus (2001), Glass and Saggi (2002) etc.

sectors.

This framework allows us to analyse the effects of strengthening IPR protection in the South not only on the rate of economic growth (innovation) but also on the level of unemployment in the South in the steady state equilibrium of the world economy. We show that the movements of growth rate and unemployment level due to strengthening of IPR protection may not be unidirectional; and the nature of their movements depends on the North South wage gap. In both the wide gap and the narrow gap cases, stronger IPR protection in the South lowers the balanced rate of growth in both the regions and raises the North South relative wage in the new steady state equilibrium. However, the level of unemployment in the South is increased in the narrow gap case and is decreased in the wide gap case.

We also analyse the effects of changes in factor endowments in both the regions. The expansion of the Southern skilled (Northern) labour endowment raises the rate of innovation (growth) and raises (lowers) the rate of imitation. A similar result is also obtained from the GH (1991b) model. The level of unemployment in the South is increased (decreased) due to an expansion of the Northern (Southern skilled) labour endowment in the narrow gap case. However, in the wide gap case, the expansion of the Northern labour endowment does not affect the Southern unemployment at all. So the effect of the change in the Northern labour endowment on the level of Southern unemployment crucially depends on the North South wage gap. The expansion of the unskilled labour endowment in the South does not affect the innovation rate and the imitation rate and only raises the level of unemployment there. Like GH (1991b), the North South relative wage varies directly (inversely) with the size of the Northern (Southern skilled) labour endowment in this model. However, the change in the unskilled labour endowment in the South does not affect the North South relative wage at all.

This chapter is organised as follows. The model of the international product cycle with unemployment in the South is presented in section 5.1. Section 5.2 presents the reduced form steady state equilibrium conditions and analyses the effects of strengthening IPR protection and of changes in labour endowments on the unemployment level and on the growth rate. In subsection 5.2.1, we analyse these effects in the wide gap case; and, in subsection 5.2.2, we do the same in the narrow gap case. Concluding remarks are made in section 5.3.

5.1 The model

There are two countries in the world - the North and the South. They are linked by free trade in differentiated products which are invented in the North and imitated in the South. A representative Northern firm incurs an upfront innovation cost to invent a new product and then earns a stream of monopoly profits from that product until it gets imitated by a potential Southern firm. Patents are perfectly protected in the North but are imperfectly protected in the South which leads to imitation there. Due to lower labour cost, a successful imitator from the South earns an infinite stream of positive profit which it balances against the positive imitation cost. The structure of this international product cycle model is adapted from GH (1991b). However, unlike GH (1991b), we introduce two types of labour - skilled and unskilled - in the labour market of the South; and assume that the efficiency of the unskilled worker varies positively with the relative wage of the unskilled labourer to that of the skilled worker[5]. Thus the level of endowment of the Southern unskilled labour expressed in efficiency unit is endogenous to this model. The introduction of the efficiency wage function leads to an

[5]Many models of efficiency wage hypothesis assume worker's efficiency to be a function of relative wage. See, for example, Summers (1988), Agell and Lundborg (1992, 1995), Akerlof and Yellen (1990) etc.

unemployment equilibrium in the unskilled labour market of the South[6]. The level of endowment of Southern skilled labour is exogenously given and is fully employed. The skilled labour is used in imitation as well as in production. However, the unskilled labour is used only in the production sector. The North has only skilled labour and it is used in production as well as in R&D. Its level of endowment is given and it is fully employed.

5.1.1 The demand for goods

The demand side of the model is standard as used throughout this thesis. The representative household maximises the intertemporal utility function given by

$$W = \int_t^\infty e^{-\theta(\tau-t)} log[U(\tau)]d\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^\infty e^{-r(\tau-t)} E(\tau)d\tau = \int_t^\infty e^{-r(\tau-t)} I(\tau)d\tau + A(t) \qquad for\ all\ t.$$

Here E(τ), I(τ), U(τ) and A(τ) stand for the level of instantaneous expenditure, level of instantaneous income, level of instantaneous utility and current stock of assets at time τ. θ and r stand for the rate of time preference and the nominal interest rate respectively. The instantaneous utility function is assumed to have the following form.

$$U(\tau) = \left[\int_0^n x(z)^\alpha dz\right]^{\frac{1}{\alpha}} \qquad with\ \ 0 < \alpha < 1.$$

Here n and $x(z)$ stand for the number of varieties and the level of consumption of the zth variety. It is assumed that the proportions of unemployed members are same for all the households in the South; and thus we ignore the income distributional aspect of unemployment.

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[6]The idea that efficiency wage hypothesis leads to an unemployment equilibrium is well known in the literature on the theory of unemployment.

Solving the optimisation problem we obtain the following demand function for the zth variety.

$$x(z) = \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon} du} E. \tag{5.1.1}$$

This is true for all z $\in [0, n]$. Here $\varepsilon = \frac{1}{1-\alpha} > 1$ is the constant price elasticity of demand. We also obtain the following optimal time path of expenditure given by

$$\frac{\dot{E}}{E} = r - \theta. \tag{5.1.2}$$

Its derivation is given in the Appendix 2.1 of chapter 2. From Appendix 2.1, it can be easily shown that (see equations (A.6) and (A.9))

$$\frac{\dot{U}}{U} = \frac{\dot{E}}{E} - \frac{\dot{P}}{P}$$

where

$$P^{(1-\varepsilon)} = \int_0^n p(u)^{(1-\varepsilon)} du.$$

Here the subscripts N and S stand for the North and the South respectively. Also we have

$$n = n_N + n_S \tag{5.1.3}$$

where n_i is the number of products produced in the ith region for $i = N, S$. The description of the demand side is similar to that in the GH (1991b) model.

5.1.2 Production in the North

There are two sectors in the North - a competitive R&D sector and a production sector. In the production sector, n_N firms produce n_N differentiated products; and each of those firms is a monopolist on its own product. Labour is the only input used in both the sectors; and there is perfect intersectoral mobility of labour leading to the same equilibrium wage in all the sectors. In the R&D sector, the blue prints of the new products are developed.

The production function in the R&D sector takes the following form.

$$\dot{n} = \left(\frac{n}{a_N}\right) L_N^R \tag{5.1.4}$$

where L_N^R and $\frac{a_N}{n}$ stand for the level of employment and the per unit labour requirement in the R&D sector. Here $a_N > 0$ is a technological parameter. The number of products, n, rises over time if $L_N^R > 0$; and hence the labour productivity, $\frac{n}{a_N}$, rises over time. Northern labour market is competitive; and hence the Northern wage rate, w_N, is equal to the value of the marginal productivity of labour in the R&D sector in the North. Value of the Northern firm is normalised to unity[7]. So w_N is proportional to $\frac{n}{a_N}$. Hence

$$\frac{\dot{w_N}}{w_N} = \frac{\dot{n}}{n}. \tag{5.1.5}$$

It is assumed that one unit of labour is required to produce one unit of product of any variety produced in the North. Then, using equation (5.1.4), we can express the labour market clearing equation as

$$L_N = a_N\left(\frac{\dot{n}}{n}\right) + n_N x_N. \tag{5.1.6}$$

Here L_N and x_N stand for the level of Northern labour endowment and the level of output of any Northern variety[8].

The monopoly price and the monopoly profit of the Northern firm producing each of the n_N varieties are given by the following.

$$p_N = \frac{w_N}{\alpha}; \tag{5.1.7}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N. \tag{5.1.8}$$

[7]GH(1991b) does not make this assumption. We borrow it from Lai (1998). However, major results of this chapter are independent of the normalizing assumption.

[8]All the commodities in the North are produced in equal quantities because the utility function is symmetric and the technologies are identical.

Here the Northern wage rate, w_N, is the marginal cost of production of each of these varieties. It turns out in this model that π_N is constant in the steady state equilibrium. The free-entry condition in the R&D sector in the North is given by

$$\frac{a_N w_N}{n} = \frac{\pi_N}{r+m} \tag{5.1.9}$$

where the left hand side of the equation (5.1.9) is the cost of developing a new variety and its right hand side is the value of the Northern firm defined as the discounted present value of expected stream of its monopoly profits over the infinite time horizon. Here (r+m) is the effective rate of discount; and

$$m = \frac{\dot{n_S}}{n_N}$$

is the rate of imitation in the South. It represents the risk premium to be paid by the Northern firm. Using equations (5.1.6), (5.1.8) and (5.1.9), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N(\frac{\dot{n}}{n})}{a_N}\right)\left(1+\frac{m}{(\dot{n_S}/n_S)}\right) = r+m. \tag{5.1.10}$$

So far the description of the North is concerned, there is no major difference between the present model and the original GH(1991b) model.

5.1.3 Production in the South

The South does not innovate but imitates the Northern products. It has a competitive imitative R&D sector and a production sector producing imitated products. The skilled labour whose endowment is assumed to be exogenously given is used in both the sectors and is perfectly mobile. The unskilled labour whose endowment is measured in efficiency unit is used only in the production sector.

$$h = h\left(\frac{w_{SL}}{w_R}\right) \quad with\ h'(.) > 0,\ and\ with\ \ h''(.) > (<)\ 0\ \ for\ \left(\frac{w_{SL}}{w_R}\right) < (>)\ \gamma > 0,$$

is the wage-efficiency function of the representative unskilled worker. $h(\frac{w_{SL}}{w_R}).L_S$ is the aggregate endowment of the unskilled labour expressed in efficiency unit and L_S stands for the number of unskilled workers. This wage-efficiency function also satisfies the properties like $h(0) = 0$ and $h(1) = 1$. Here w_R and w_{SL} are the reference wage and the wage rate of the unskilled worker respectively. We assume the reference wage to be proportional to the skilled worker's wage, w_{SH}. Hence

$$w_R = \phi w_{SH} \qquad for\ \ 0 < \phi < 1.$$

We now try to explain this specification of the wage efficiency function[9]. The worker works harder as his current wage relative to the reference wage is higher. This reference wage may be either external or internal to the firm. Conventional models like Shapiro and Stiglitz (1984), Salop (1979), Akerlof (1982) etc. interpret the reference wage as external to the firm. It is the average of wages paid in all other firms weighted by probabilities of being employed there plus the unemployment benefit weighted by the probability of remaining unemployed. This is the expected wage of the worker when he is sacked from the present firm. However, some other models like Akerlof and Yellen (1990), Agell and Lundborg (1992, 1995), Danthine and Kurmann (2004, 2006) etc. interpret the reference wage as internal to the firm. Price of the complementary factor of production, e.g., wage rate of the co-worker with different skill[10], rental rate on capital[11], may be interpreted as the internal reference wage. Danthine and Kurmann (2004) considers the worker's past wage in the same firm as the reference wage. In the present model, the skilled labour is the only complementary factor to the unskilled labour in the production sector of the South. So, following the tradition of the existing literature, the reference wage in the efficiency function of the unskilled worker is assumed to be proportional to the wage-rate of the skilled worker[12]. In GH (1991b) model, $h(.) \equiv 1$. So there is no difference between the endowment of labour and the number of workers.

[9] This is different from the explanations of Mirrlees (1976), Stiglitz (1976), Dasgupta and Ray (1986) etc.

[10] See Akerlof and Yellen (1990)

[11] See Agell and Lundborg (1992, 1995), Danthine and Kurmann (2006) etc.

[12] The results of this model are conditional on this assumption. Additional remarks are made in the conclusion section in chapter 6 (see pages 175-177).

The production function in the imitative R&D sector is given by

$$\dot{n_S} = \frac{n_S}{a_S} H_R \tag{5.1.11}$$

where H_R, $\dot{n_S}$ and (a_S/n_S) stand for the amount of skilled labour used in the imitative R &D sector, the number of new imitated products and the effective labour output coefficient in the imitative R&D sector. Here

$$a_S = a_m + \lambda$$

where a_m is the technology parameter and λ is a policy parameter representing the degree of strengthening the IPR protection in the South. The stronger the IPR protection, the greater is the value of λ and hence the greater is the effective per unit labour requirement[13] in the imitative R &D sector.

All the imitated products in the South are produced under identical technology; and the production function of the representative imitated product is given by

$$x_S = (\delta L_D{}^{-\rho} + (1-\delta) H_p{}^{-\rho})^{-\frac{1}{\rho}}.$$

Here $0 < \delta < 1$ and $\rho > 0$ are two technological parameters. Here x_S, L_D, H_p and $\frac{1}{(1+\rho)}$ stand for level of output, level of unskilled labour employment expressed in efficiency unit, amount of skilled labour input and the elasticity of substitution between L_D and H_p. In GH (1991b), $H_p = 0$ by assumption. If $\delta = 1$, then we can come back to the production function in the GH(1991b) model.

A typical Southern firm, assumed to be a monopolist on its own imitated product,

[13]The increase in the labour requirement means the increase in the cost of imitation because skilled labour is the only input. We follow Glass and Saggi (2002) for this kind of definition of IPR protection in the South.

maximises profit given by

$$\pi_S = p_S x_S - \left(w_{SH} H_p + w_{SL} \frac{L_D}{h\left(\frac{w_{SL}}{w_R}\right)} \right)$$

with respect to w_{SL}, H_p and L_D subject to the demand function for x_S given by equation (5.1.1). Here w_{SL}, w_{SH}, and p_S represent the prices of the unskilled labour, of the skilled labour and of the representative imitated product. From the solution to this optimisation exercise, we obtain following equations[14].

$$H_p = x_S(\Omega)^{\frac{1}{\rho}}; \tag{5.1.12}$$

$$L_D = x_S(\Omega)^{\frac{1}{\rho}}.K^{\frac{-1}{1+\rho}}; \tag{5.1.13}$$

$$p_S = \frac{w_{SH}}{(1-\delta)\alpha}(\Omega)^{\frac{1+\rho}{\rho}}; \tag{5.1.14}$$

and

$$\frac{h'(.)}{h(.)}\frac{w_{SL}}{w_R} = 1. \tag{5.1.15}$$

Here

$$K = \frac{1-\delta}{\delta}\frac{w_{SL}}{h(.)w_R} \quad \textit{and} \quad \Omega = (\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta).$$

Here equations (5.1.12) and (5.1.13) represent the demand functions for skilled labour and unskilled labour. Equation (5.1.14) shows the equilibrium (monopoly) price for the representative Southern imitated product; and equation (5.1.15) shows the equilibrium condition of efficiency wage stating that the elasticity of efficiency with respect to relative wage is equal to unity. Note that this equilibrium condition (15) does not exactly correspond to the equilibrium condition of Solow (1979) which is concerned with absolute real wage but not with relative wage. Here equation (5.1.15) solves for $\left(\frac{w_{SL}}{w_{SH}}\right)$ uniquely because $w_R = \phi w_{SH}$[15]. So w_{SL} and w_{SH} always move proportionately and the equilibrium value of $\left(\frac{w_{SL}}{w_{SH}}\right)$ is independent of any parametric change that does not affect

[14]See Appendix 5.1 for the detail derivation.

[15]We assume that the solution of $\left(\frac{w_{SL}}{w_{SH}}\right)$ satisfies the condition that $w_{SH} > w_{SL}$. This ensures that the skilled workers do not join the unskilled labour force in equilibrium.

the efficiency function. So the expressions K and Ω, being functions of $\left(\frac{w_{SL}}{w_{SH}}\right)$ only, are also invariant to these parametric changes. Hence equations (5.1.12) and (5.1.13) show that levels of demand for inputs vary proportionately with the level of output; and equation (5.1.14) shows that the monopoly price varies proportionately with w_{SH}. Hence we have

$$\frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{p_S}}{p_S}. \tag{5.1.16}$$

In the imitative R&D sector, wage rate of skilled labour is equal to its value of marginal productivity given by $v_S.(\frac{n_S}{a_S})$ where v_S is the value of a Southern firm. Using equation (5.1.1) and the North South labour market clearing conditions, it can be shown that $\frac{w_N}{w_{SH}}$ and $\frac{w_N}{w_{SL}}$ are constant in the steady state equilibrium. So using equation (5.1.5) we have

$$\frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_N}}{w_N} = \frac{\dot{n}}{n}. \tag{5.1.17}$$

Equation (5.1.17) implies that, the value of the Southern firm (v_S) is time-independent in the steady state equilibrium. The equilibrium condition in the skilled labour market and that in the unskilled labour market are given by

$$H_S = a_S \frac{\dot{n_S}}{n_S} + n_S H_p \tag{5.1.18}$$

and

$$L_S = \frac{n_S L_D}{h(.)} + U_S \tag{5.1.19}$$

respectively. Here H_S and U_S represent the number of available skilled workers and the number of unemployed unskilled workers respectively. Since the efficiency wage is rigid downwards, there should be a positive level of unemployment of unskilled labour in equilibrium. At a particular point of time, n_S is given. So equations (5.1.18) and (5.1.19) solve for $(\frac{\dot{n_S}}{n_S})$ and U_S because values of (w_{SL}/w_{SH}), L_D and H_p are obtained from equations (5.1.12), (5.1.13) and (5.1.15).

In the GH(1991b) model, $H_S = 0$. So H_p and w_{SH} do not exist. Since $h(.) \equiv 1$, equation (5.1.15) does not exist too. w_{SL} is flexible and hence $U_S = 0$ in equilibrium. Only one type of labour is employed in production sector and in imitative R&D sector. Equations (5.1.12) and (5.1.18) do not exist. Equations (5.1.13) and (5.1.14) are modified as follows.

$$L_D = x_S;$$

and

$$p_S = \frac{w_{SL}}{\alpha}.$$

Equations (5.1.11) and (5.1.19) are modified as follows.

$$\dot{n_S} = \frac{n_S}{a_S}.L_R;$$

and

$$L_S = n_S L_D + L_R.$$

It can be shown that π_S remains constant in the steady state equilibrium of this model. The free entry condition in the imitative R&D sector in the South is given by

$$\frac{\pi_S}{r} = \frac{a_S w_{SH}}{n_S}. \tag{5.1.20}$$

Here the R.H.S. of equation (5.1.20) represents the cost of imitating a new variety; and the L.H.S. of equation (5.1.20) represents the value of the Southern firm defined as the discounted present value of its profits from its production over the infinite time horizon when r is the rate of discount. The value of the Southern firm is also time independent in the steady state equilibrium. In GH(1991b), equation (5.1.20) remains otherwise identical with the only modification that w_{SH} is replaced by w_{SL}. This is so because, in GH(1991b), same labour is used in the production sector as well as in the imitative R&D sector.

Note that the North as well as the South faces the same rate of interest, r, because it is assumed that there exists a competitive world capital market.

5.1.4 Wide gap equilibrium vs. Narrow gap equilibrium

We assume that the marginal cost of production of a variety in the South is always lower than that in the North and there is Bertrand price competition between a successful Southern imitator and its Northern counterpart. The Southern firm can charge the monopoly price on its imitated product as given in equation (5.1.14) if the following condition holds true in equilibrium.

$$p_S = \frac{w_{SH}}{(1-\delta)\alpha}(\Omega)^{\frac{1+\rho}{\rho}} < w_N.$$

This is possible if the North South wage gap is very high. This is the case of wide gap equilibrium. However, when the above inequality is satisfied in the reverse order we have the case of narrow gap equilibrium. Here the equilibrium price charged by a typical Southern firm is given by

$$p_S = w_N < \frac{w_{SH}}{(1-\delta)\alpha}(\Omega)^{\frac{1+\rho}{\rho}}. \tag{5.1.21}$$

The possibility of this case arises when the North South wage gap is very narrow. Henceforth, in this chapter, North South relative wage will be defined by $\left(\frac{w_N}{w_{SH}}\right)$ which represents the Northern wage rate relative to the wage rate of the Southern skilled labour[16]. The instantaneous profit of the representative Southern firm in the wide gap case is given by

$$\pi_S = \frac{1-\alpha}{\alpha}\frac{w_{SH}}{(1-\delta)}(\Omega)^{\frac{1+\rho}{\rho}} x_S; \tag{5.1.22}$$

and that in the narrow gap case is given by

$$\pi_S = \left[w_N - \frac{w_{SH}}{(1-\delta)}(\Omega)^{\frac{1+\rho}{\rho}}\right] x_S. \tag{5.1.23}$$

From equation (5.1.1) we have

$$\frac{x_N}{x_S} = \left(\frac{p_N}{p_S}\right)^{-\varepsilon}.$$

[16] Ratio of the skilled wage to the unskilled wage in the South is fixed by the efficiency function (see equation (5.1.15)).

This is the relative demand function for the Northern product. In the narrow gap case, using equations (5.1.7) and (5.1.21), we have,

$$\frac{x_N}{x_S} = \alpha^{\varepsilon}.$$

This implies that the relative demand for Northern product is constant in the narrow gap case. However, it varies with the North South relative wage in the wide-gap case. Using equations (5.1.7) and (5.1.14) we have

$$\frac{w_N}{w_{SH}} = \frac{\Omega^{\frac{1+\rho}{\rho}}}{1-\delta}\left(\frac{x_N}{x_S}\right)^{\varepsilon}. \tag{5.1.24}$$

While analysing the effects of strengthening the IPR protection on the growth rate and on the unemployment level, we shall consider these two cases separately.

5.1.5 Steady state equilibrium growth

It is assumed that the economy is in the steady state growth equilibrium where n_N and n_S grow at equal rate, g. Hence

$$\frac{\dot{n}}{n} = \frac{\dot{n_N}}{n_N} = \frac{\dot{n_S}}{n_S} = g. \tag{5.1.25}$$

Value of g is determined endogenously in this model and so it is an endogenous growth model. We have normalised the value of a Northern firm to unity . This implies that the expenditure in the North would grow at the rate of new product development there[17]. Now using equations (5.1.5), (5.1.7), (5.1.16), (5.1.17), and (5.1.25) we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{E_S}}{E_S} = \frac{\dot{n}}{n} = \frac{\dot{n_S}}{n_S} = \frac{\dot{n_N}}{n_N} = \frac{\dot{w_N}}{w_N} = \frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{p_S}}{p_S} = \frac{\dot{p_N}}{p_N} = g. \tag{5.1.26}$$

Equations (5.1.6) and (5.1.25) together imply that $n_N x_N$ is constant in the steady state growth equilibrium. Similarly equations (5.1.12), (5.1.18) and (5.1.25) imply that $n_S x_S$ is

[17]Normalising the value of a Northern firm (which is $\frac{a_N}{n} w_N$) to unity, we get $\frac{\dot{w_N}}{w_N} = \frac{\dot{n}}{n}$. We have $E_N = p_N n_N x_N$. Now replacing $n_N x_N$ from equation (5.1.6) we can easily show that in the steady state $\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} = \frac{\dot{n}}{n} = g$. Also $E_S = p_S n_S x_S$ and from equation (5.1.18) we get $(n_S x_S)$ is constant in the steady state. Now using Equation (5.1.16) and (5.1.17) we get $\frac{\dot{E_S}}{E_S} = \frac{\dot{p_S}}{p_S} = \frac{\dot{n}}{n} = g$.

also constant in the steady state growth equilibrium. So, in the steady state equilibrium, we have

$$\frac{\dot{x_N}}{x_N} = \frac{\dot{x_S}}{x_S} = -g. \tag{5.1.27}$$

From the instantaneous utility function we obtain

$$U = (n_N {x_N}^\alpha + n_S {x_S}^\alpha)^{\frac{1}{\alpha}}.$$

Then using equations (5.1.25) and (5.1.27) and also considering the fact that $n_N x_N$ and $n_S x_S$ are constant in the steady state growth equilibrium, we have

$$\frac{\dot{U}}{U} = \left(\frac{1-\alpha}{\alpha}\right) g.$$

Like GH(1991b) model, the above equation implies that the long run rate of product development and the long run growth rate of utility are proportional (see page 1221 in GH(1991b)). Any parametric change should have similar effects on these two.

5.2 The existence of steady-state equilibrium and the comparative statics

We now derive the reduced form equations of the model which can be used to determine the rate of innovation (growth), rate of imitation and the level of unemployment in the steady state equilibrium. Using equations (5.1.2) and (5.1.26) and $E = (E_N + E_S)$, we have

$$r = \theta + g. \tag{5.2.1}$$

Again using equations (5.1.12), (5.1.13), (5.1.18), (5.1.19) and (5.1.26) we have

$$H_S = a_S g + n_S x_S (\Omega)^{\frac{1}{\rho}}; \tag{5.2.2}$$

and

$$L_S - U_S = \frac{n_S x_S (\Omega)^{\frac{1}{\rho}} . K^{\frac{-1}{1+\rho}}}{h(.)}. \tag{5.2.3}$$

Also using equations (5.1.10) and (5.1.26), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N g}{a_N}\right)\left(1+\frac{m}{g}\right) = r + m. \tag{5.2.4}$$

In the wide gap case, we use equations (5.1.20) and (5.1.22); and obtain

$$r a_S \frac{\alpha}{1-\alpha} = \frac{n_S x_S}{(1-\delta)}(\Omega)^{\frac{1+\rho}{\rho}}. \tag{5.2.5}$$

In the narrow gap case, we use equations (5.1.20) and (5.1.23) and obtain

$$\left(w_N - \frac{w_{SH}}{(1-\delta)}(\Omega)^{\frac{1+\rho}{\rho}}\right) x_S n_S = a_S w_{SH} r. \tag{5.2.6}$$

Again using equations (5.1.6) and (5.2.2) in the narrow gap case, we have

$$\alpha^{\varepsilon} = \frac{x_N}{x_S} = \frac{L_N - a_N g}{H_S - a_S g}\frac{n_S}{n_N}(\Omega)^{\frac{1}{\rho}}. \tag{5.2.7}$$

In the wide gap case, we have a set of four equations (5.2.2), (5.2.3), (5.2.4), (5.2.5) with four unknowns - $n_S x_S$, g, m and U_S. In the narrow gap case, our equations are (5.2.2), (5.2.3), (5.2.4) and (5.2.7). Equation (5.2.6) is used to solve for the North South relative wage, $\left(\frac{w_N}{w_{SH}}\right)$, in the narrow gap case. In the wide gap case, this relative wage is obtained from the equation given by

$$\frac{w_N}{w_{SH}} = \frac{\Omega^{\frac{\alpha+\rho}{\rho}}}{1-\delta}\left(\frac{H_S - a_S g}{L_N - a_N g}\ \frac{g}{m}\right)^{1-\alpha} \tag{5.2.8}$$

which we obtain using equations (5.1.6), (5.1.24) and (5.2.2).

5.2.1 The wide gap equilibrium case

Using equations (5.2.1), (5.2.2) and (5.2.5), we obtain

$$g = \frac{H_S.(\Omega) - \frac{\alpha(1-\delta)}{1-\alpha}a_S\theta}{a_S.\left(\frac{\alpha(1-\delta)}{1-\alpha} + \Omega\right)}. \tag{5.2.9}$$

This is the equation of SS curve which represents the relationship between the rate of innovation, g, and the rate of imitation, m, satisfying equilibrium in the South. This curve is drawn horizontal in the figure 5.1. Then, combining equations (5.2.2) and (5.2.4), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N g}{a_N}\right)\left(1+\frac{m}{g}\right) = \theta + g + m \tag{5.2.10}$$

which can also be written as

$$m = \frac{\theta + \frac{g}{\alpha} - \frac{1-\alpha}{\alpha}\frac{L_N}{a_N}}{\frac{1-\alpha}{\alpha}\frac{L_N}{a_N g} - \frac{1}{\alpha}}. \tag{5.2.11}$$

This is the equation of NN curve which shows the relationship between g and m satisfying equilibrium in the North. Note that $m > 0$ for

$$g_b = (1-\alpha)\frac{L_N}{a_N} - \alpha\theta < g < (1-\alpha)\frac{L_N}{a_N} = g_u$$

NN curve slopes positively in the figure 5.1 starting from the point

$$g_b = (1-\alpha)\frac{L_N}{a_N} - \alpha\theta$$

on the g-axis. This NN curve is identical to that in GH(1991b) because descriptions of the North are same in two models. The equilibrium values of g and m are determined at the point of intersection of NN curve and SS curve[18]. Since $\frac{n_S}{n_N} = \frac{m}{g}$, we now can determine the equilibrium value of $\frac{w_N}{w_{SH}}$ using equation (5.2.8). Then, using equations (5.2.2) and (5.2.3), we have

$$\frac{H_S - a_S g}{L_S - U_S} = K^{\frac{1}{1+\rho}} h(.)\ . \tag{5.2.12}$$

This is the equation of UU curve which shows the relationship between unemployment level and the growth rate in the South. Here both K and $h(.)$ are functions of the relative wage, $\frac{w_{SL}}{w_{SH}}$, and the equilibrium value of $\frac{w_{SL}}{w_{SH}}$ is uniquely determined by the equation (5.1.15). So $\frac{w_{SL}}{w_{SH}}$ and hence $h(.)$ and K are independent of the changes in a_S, U_S and g because a_S, U_S and g do not enter the efficiency function. We find a positive

[18]The existence of this intersection depends on a mild parametric restriction. See Appendix 5.2 for the details.

relationship between the growth rate and the unemployment level from this equation (5.2.12). Here $U_S > 0$ is guaranteed by the following assumption

$$\left(1 + \frac{\Omega}{\frac{\alpha(1-\delta)}{1-\alpha}}\right).K^{\frac{1}{1+\rho}}.h(.) > \frac{H_S - a_S\theta}{L_S}. \tag{5.2.13}$$

This inequality (5.2.13) implies that the demand for unskilled labour falls short of this labour endowment when the efficiency wage is binding.

5.2.1.1 IPR protection

We now turn to analyse the effects of the strengthening of IPR protection on the equilibrium growth rate, g, on the imitation rate, m, and on the level of unemployment, U_S. Here, the strengthening of IPR protection implies an increase[19] in the value of a_S. From equation (5.2.9), we find that g as well as $a_S g$ varies inversely with a_S. From equation (5.2.11), we find that m and g are positively related. However, a_S does not enter the equation (5.2.11). So an increase in a_S causes the horizontal SS curve to shift downward and does not cause any shift of the positively sloped NN curve. So both g and m take lower values in the new equilibrium shown in the figure 5.1. Note that a_S does enter the equation (5.2.12). So UU curve in the figure 5.1 does shift in this case with changes in the slope as well as in the intercept. From equation (5.2.9), we find that $a_S g$ varies inversely with a_S. So equation (5.2.12) shows that U_S falls in the new equilibrium when a_S is increased. In order to find out the effect on the North South relative wage, we can express equation (5.2.8) as follows.

$$\frac{w_N}{w_{SH}} = \frac{\Omega^{\frac{\alpha+\rho}{\rho}}}{1-\delta}\left(\frac{H_S - a_S g}{(L_N - a_N g)\frac{m}{g}}\right)^{1-\alpha}. \tag{5.2.14}$$

Also, using equation (5.2.10), we have

$$(L_N - a_N g)\frac{m}{g} = \frac{\alpha}{1-\alpha}a_N(\theta + g + m) - (L_N - a_N g). \tag{5.2.15}$$

[19] Here, $a_S = a_m + \lambda$; and λ takes a higher value when the IPR is stronger.

Since an increase in a_S leads to a decrease in both g and m, equation (5.2.15) shows that $(L_N - a_N g)\frac{m}{g}$ is also reduced in this case. Also equation (5.2.9) shows that $a_S g$ is reduced due to an increase in a_S. So the North South relative wage is increased in the new steady state equilibrium when a_S is increased[20]. So we can establish the following proposition.

Proposition 5.2.1. *The strengthening of IPR protection in the South lowers the rate of growth (innovation) in both the countries in the new steady state equilibrium. It also lowers the rate of imitation and the level of unemployment of the unskilled workers in the South in the new steady state equilibrium. However, it raises the North South relative wage.*

We can provide an intuitive explanation of the above mentioned result. Note that the definition of strengthening IPR protection in this model is similar to that in GH (1991b)[21]; and its effects on the rate of innovation and on the rate of imitation are also similar to those in GH (1991b). The profit rate of a typical Southern imitative firm is a negative function of a_S and g and the cost of its capital is a positive function of g[22]. The strengthening of IPR protection in the South raises the cost of imitation there. This lowers the profit rate but leaves the cost of capital unchanged. This results into a decrease in the rate of growth of the imitated products, g.

As the rate of growth falls in the South and as both North and South grow at the same rate in the steady state equilibrium, the profit rate of a typical Northern firm is increased and the cost of its capital is decreased[23]. So the rate of imitation has to fall to equilibrate the profit rate with the cost of capital in the North. This is so because the

[20]The wide gap property will not be disturbed in the new steady state equilibrium because increase in a_S raises the North South relative wage.

[21]GH (1991b) defined a tax (subsidy) in the Southern imitative R&D sector as the strengthening (lax) of protection of IPR protection there.

[22]The profit rate is defined as $\frac{\pi_S}{v_S} = \frac{(1-\alpha)\Omega}{(1-\delta)\alpha\, a_S}(H_S - a_S g)$. The cost of capital is $r = \theta + g$.

[23]The profit rate is defined as $\frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha\, a_N}(L_N - a_N g)\left(\frac{m+g}{g}\right)$ where v_N is the value of a typical Northern firm and it is equal to the cost of innovating a new blue print in the North in the equilibrium. The cost of capital is $r + m = \theta + g + m$.

profit rate of a typical Northern firm is a negative function of g and a positive function of m and the cost of its capital is a positive function of both g and m. Also the effect of a change in m on the profit rate is higher than its effect on the cost of capital. An increase in a_S also causes more firms to stay in the North in the new steady state equilibrium because the ratio $\frac{g}{m}$ $(= \frac{n_N}{n_S})$ is increased. This leads to an increase in the demand for labour in the production sector in the North. Hence the relative wage of the North is increased due to stronger protection of IPR in the South.

The size of the imitative R&D sector in the South is also reduced now because the increase in the cost of imitation (per unit skilled labour requirement) in the South causes a substantial reduction in the rate of growth. So more human capital is released to the production sector. The relative skilled unskilled wage in the South is given; and so the demand for skilled labour per unit of unskilled labour is fixed in the production sector of the South. Thus the increase in the skilled labour employment in the production sector is matched by an equal proportionate increase in the unskilled labour employment. So the level of unemployment in the unskilled labour market of the South is reduced. Note that the unemployment problem of skilled labour can never arise in our model because the linear production technology in the imitative R&D sector implies an infinite demand for skilled labour there.

The share of the unskilled workers in the national income of the South is given by

$$\frac{w_{SL}(L_S - U_S)}{p_S n_S x_S} = \frac{w_{SL}}{w_{SH}} \; \frac{\alpha(1-\delta)}{\Omega \; h(.) \; K^{\frac{1}{1+\rho}}}. \tag{5.2.16}$$

The right hand side of the above equation is independent of the change in a_S because $\frac{w_{SL}}{w_{SH}}$ is determined by equation (5.1.15). So the relative income share of the unskilled workers in the South remains unaffected in the wide gap equilibrium when IPR protection is strengthened there.

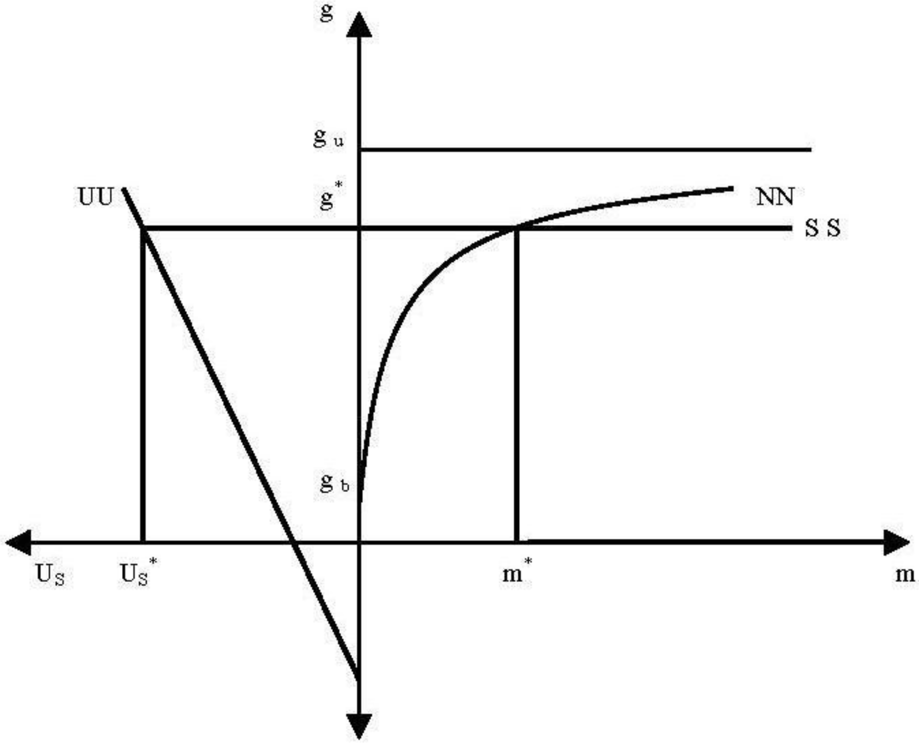

Figure 5.1 : wide gap equilibrium

5.2.1.2 Factor endowment change

An increase in H_S makes the horizontal SS curve shift upward in the figure 5.1 but leaves NN curve unaffected[24]. Then, in the new steady state equilibrium, both g and m are increased. Also $\frac{m}{g}$ and $(H_S - a_S g)$ are increased in the new steady state equilibrium[25]. Since $(H_S - a_S g)$ is increased, from equation (5.2.12), we find that $(L_S - U_S)$ also rises proportionately because the R.H.S. of equation (5.2.12) is independent of H_S. This implies that U_S falls when H_S is increased. From equation (5.2.14), we have

$$\frac{w_{SH}}{w_N} = \left(\frac{(L_N - a_N g)\frac{m}{g}}{H_S - a_S g} \right)^{1-\alpha} \frac{1-\delta}{\Omega^{1+\frac{\alpha}{\rho}}}. \tag{5.2.17}$$

Now, using equations (5.2.9) and (5.2.10), we have

$$\frac{(L_N - a_N g)\frac{m}{g}}{H_S - a_S g} = \left[\frac{\alpha}{1-\alpha} a_N \left(1 + \frac{m}{g+\theta} \right) - \left(\frac{L_N - a_N g}{g+\theta} \right) \right] \frac{(1-\alpha)\,\Omega}{(1-\delta)\,\alpha\, a_S}. \tag{5.2.18}$$

---

[24]See equations (5.2.9) and (5.2.10).

[25]From equation (5.2.9) we obtain $(H_S - a_S g)\Omega = a_S(g+\theta)\frac{\alpha(1-\delta)}{1-\alpha}$. Increase in H_S increases g. So $(H_S - a_S g)$ is increased.

Since an increase in H_S raises g, m and $\frac{m}{g}$, the R.H.S. of equation (5.2.18) is increased in this case. So equation (5.2.17) implies that $\frac{w_{SH}}{w_N}$ is increased. Since the skilled unskilled relative wage in the South is fixed by the efficiency wage hypothesis, $\frac{w_{SL}}{w_N}$ is also increased in this case.

An increase in the unskilled labour endowment in the South, L_S, does not affect g and m because equations (5.2.9) and (5.2.10) are independent of L_S. Equation (5.2.12) shows that this only raises the level of unemployment of unskilled labour in the South. Since g and m remain unaffected, $\frac{w_{SH}}{w_N}$ and $\frac{w_{SL}}{w_N}$ also remain unchanged.

Finally, an increase in the Northern labour endowment, L_N, causes NN curve in the figure 5.1 to shift upward and leaves SS curve unaffected. In the new steady state equilibrium, g remains unchanged and m is decreased. U_S also remains unchanged. Equations (5.2.17) and (5.2.18) together show that $\frac{w_{SH}}{w_N}$ and $\frac{w_{SL}}{w_N}$ are decreased.

We can summarize the results mentioned above in the following proposition.

Proposition 5.2.2. *(i) An expansion of the Southern skilled labour endowment raises the rate of innovation, the rate of imitation and the South North relative wage and lowers the level of unemployment in the South in the new steady state equilibrium. (ii) An expansion of the Northern labour endowment lowers the rate of imitation as well as the South North relative wage but keeps the rate of innovation and the level of unemployment unchanged. (iii) An expansion of the unskilled labour endowment in the South raises the unemployment level there but leaves the rate of innovation, the rate of imitation and the South North relative wage unaffected.*

We now explain the intuition behind these results. Given other things unchanged, an increase in the endowment of the skilled labour in the South allocates more labour to the Southern imitative R&D sector; and this raises the rate of imitation in the South. An increase in the rate of imitation raises both the cost of capital and the profit rate of a typical Northern firm. However, its effect on the profit rate dominates its effect on the cost of capital; and so the rate of innovation in the North is increased. An increase in the rate of innovation raises the demand for skilled labour in the Southern manufacturing sector. Given the relative wage rigidity in the South caused by the efficiency wage hypothesis, the demand for unskilled labour is also increased; and so their unemployment level is reduced.

An increase in the Northern labour endowment has no effect on the Southern equilibrium conditions and this raises the profit rate of a typical Northern firm. So the rate of innovation is not changed but the rate of imitation should be reduced to maintain the equality between the profit rate and the cost of capital in the North. Since the rate of innovation remains unchanged, the demand for each type of labour in the Southern manufacturing sector also remains unchanged. So the level of unemployment remains the same.

A change in the unskilled labour endowment in the South can not alter the allocation of skilled labour between the imitative R&D sector and manufacturing sector due to the rigidity of the efficiency wage above the market clearing level. So the expansion of the unskilled labour endowment does not affect the rate of innovation and the rate of imitation. It only raises the level of unemployment in the South.

Regarding the effects of the changes in the labour endowments (except for the unskilled labour endowment in the South) on the North South relative wage our results are similar to those obtained in GH(1991b). A change in the unskilled labour endowment in the South has no effect on the North South relative wage because it can not affect the rate of innovation and the rate of imitation and can not alter the skilled unskilled wage ratio in the South due to rigidity of the efficiency wage there. Our results are different from those in Lai(1995) who, in his full employment model, shows that an increase in the unskilled labour endowment in the South lowers its relative wage.

5.2.2 The narrow gap equilibrium case

In this case, our equational structure consists of equations (5.2.1), (5.2.2), (5.2.3), (5.2.4), (5.2.6) and (5.2.7). Since all the equations except (5.2.6) and (5.2.7) are same as those in the wide gap case, the shapes of NN curve and of UU curve remain same as they have in the wide gap case[26]. Since $\frac{n_S}{n_N} = \frac{m}{g}$, using equation (5.2.7), we obtain the equation corresponding to XX curve as given by

$$\frac{L_N - a_N g}{H_S - a_S g}.(\Omega)^{\frac{1}{\rho}}.\left(\frac{m}{g}\right) = \alpha^{\varepsilon}. \tag{5.2.19}$$

We assume that $\frac{L_N}{a_N} < \frac{H_S}{a_S}$. This gives a positive relationship between g and m for $0 < g < \frac{L_N}{a_N}$; and so XX curve in the narrow gap case slopes positively starting from the origin. The slope of XX curve exceeds that of NN curve at any common values of g and m[27]. Now equations (5.2.11), (5.2.12) and (5.2.19) solve for the equilibrium values of g, U_S and m simultaneously. Using equations (5.2.1), (5.2.2) and (5.2.6), we have

$$\frac{w_N}{w_{SH}} = \frac{a_S(\theta + g)}{H_S - a_S g}.(\Omega)^{\frac{1}{\rho}} + \frac{1}{1-\delta}(\Omega)^{\frac{1+\rho}{\rho}}. \tag{5.2.20}$$

Equation (5.2.20) solves for the equilibrium value of the North South relative wage[28]. A graphical presentation of the steady state equilibrium in the narrow gap case is shown in the figure 5.2. Since NN curve starts from a point on the g-axis and XX curve starts from the origin with a higher positive slope, XX curve must cut NN curve from below at their unique point of intersection.

[26]NN curve is obtained from equations (5.2.1) and (5.2.4). UU curve is obtained from the equation (5.2.12) derived from equations (5.2.2) and (5.2.3).

[27]It is shown in the Appendix 5.3.

[28]Determination of $\frac{w_N}{w_{SH}}$ also implies the determination of $\frac{w_N}{w_{SL}}$ because $\frac{w_{SL}}{w_{SH}}$ is uniquely solved from equation (5.1.15).

5.2.2.1 IPR protection

The strengthening of the IPR protection in the South, i.e., an increase in the value of the parameter, a_S, causes XX curve to shift leftward. However, NN curve does not shift. So, at the new point of intersection, both g and m take lower values. Using equations (5.2.10) and (5.2.19), we have

$$H_S - a_S g = \alpha^{-\varepsilon}(\Omega)^{\frac{1}{\rho}} \left[\frac{\alpha}{1-\alpha} a_N(\theta + g + m) - (L_N - a_N g) \right]. \tag{5.2.21}$$

Equation (5.2.21) shows that $a_S g$ is increased in this case. Equation (5.2.12) shows that U_S is increased when $a_S g$ rises[29]. Equation (5.2.20) shows that $(\frac{w_N}{w_{SH}})$ rises in this case[30]. Since $\frac{w_{SL}}{w_R}$ is uniquely determined by equation (5.1.15) and since w_R is proportional to w_{SH}, then $\frac{w_N}{w_{SH}}$ is also uniquely determined here. Hence $\frac{w_N}{w_{SL}}$ also rises at the same rate in this case. So we have the following proposition.

Proposition 5.2.3. *The strengthening of IPR in the South lowers the rate of innovation in the North and the rate of imitation in the South in the new steady state equilibrium. However, it raises the level of unemployment in the South and the North South relative wage.*

Effects of strengthening IPR protection on the growth (innovation) rate and on the imitation rate in this narrow gap case are similar to those in the wide gap case. An increase in a_S lowers the per firm availability of skilled labour for the production sector in the South when g is given. This leads to a decrease in the rate of imitation, given g. This is so because the relative sales volume of a Northern firm to a Southern firm is constant in the narrow gap case. As m is reduced, this lowers both the profit rate and the cost of capital of a typical Northern firm. The former effect dominates the later; and hence the rate of innovation is decreased. This increase in a_S causes more

[29] A sufficient condition for U_S to be positive is $K^{\frac{1}{1+\rho}} h(.) > \frac{H_S}{L_S}$.

[30] Note the potential danger of carrying out the comparative static of increasing a_S. Since the North South relative wage increases we may violate the condition for the narrow gap case. We assume that the exogenous changes are sufficiently small so that we don't violate the narrow gap condition.

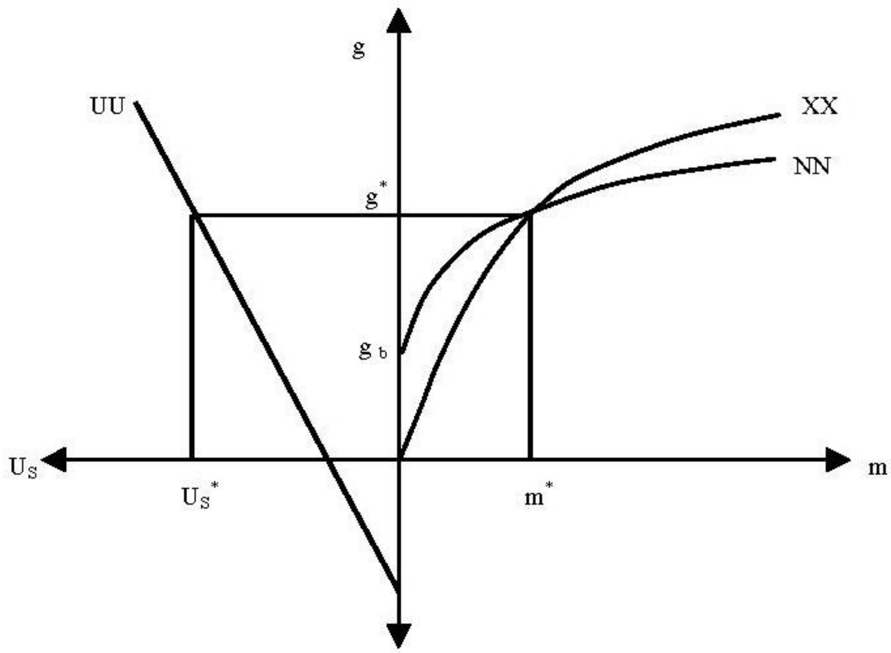

Figure 5.2 : narrow gap equilibrium

firms to stay in the North. This leads to an increase in the demand for production labour there; and hence the relative wage of the North is increased. An increase in the skilled labour requirement in the imitative R&D sector in the South lowers the rate product development (growth) there but this negative effect is very weak. So the total skilled labour employment in the imitative R&D sector, by which its size is measured, is increased when the IPR protection is strengthened. So the level of employment of skilled labour in the production sector in the South is reduced. Since the skilled unskilled employment ratio in the production sector in the South is fixed by the rigidity of the efficiency wage, the level of employment of unskilled labour in the production sector also falls at the same proportion. Since no other sector absorbs unskilled workers in the South, their unemployment level is increased.

The share of the unskilled workers in the national income of the South in this narrow gap equilibrium case is given by

$$\frac{w_{SL}(L_S - U_S)}{p_S n_S x_S} = \frac{w_{SL}}{w_N} \ \frac{\Omega^{\frac{1}{\rho}}}{h(.)\ K^{\frac{1}{1+\rho}}}. \tag{5.2.22}$$

Unlike in the wide gap case, the right hand side of the equation (5.2.22) is not independent of the change in a_S because $\frac{w_{SL}}{w_N}$ is decreased when a_S is increased. So the relative income share of the unskilled workers in the South is decreased in the narrow gap equilibrium when IPR protection is strengthened there.

5.2.2.2 Factor endowment change

An increase in H_S causes XX curve in the figure 5.2 to shift rightward and leaves NN curve unaffected. So, in the new steady state equilibrium, g, m and $\frac{m}{g}$ are increased. Equation (5.2.21) shows that $(H_S - a_S g)$ is also increased. Then equation (5.2.12) shows that U_S is decreased. We express equation (5.2.10) as

$$\frac{(L_N - a_N g)\frac{m}{g}}{g} = \frac{a_N}{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\, a_N\right)\, \frac{m}{g} - \left(\frac{L_N - a_N \frac{\alpha}{1-\alpha}\theta}{g}\right) ; \qquad (5.2.23)$$

and equation (5.2.19) as

$$\frac{\left(\frac{(L_N - a_N g)\frac{m}{g}}{g}\right)}{\left(\frac{H_S - a_S g}{g}\right)} .(\Omega)^{\frac{1}{\rho}} = \alpha^{\varepsilon}. \qquad (5.2.24)$$

Since g, m and $\frac{m}{g}$ are increased in this case, the L.H.S. of equation (5.2.23) is also increased because $L_N > a_N \frac{\alpha}{1-\alpha}\theta$. Hence equation (5.2.24) shows that $\frac{H_S - a_S g}{g}$ is also increased. Then, from equation (5.2.20), we find that $\frac{w_N}{w_{SH}}$ and $\frac{w_N}{w_{SL}}$ are decreased in this case.

An increase in L_S does not affect g, m and $\frac{w_{SH}}{w_N}$ because equations (5.2.10), (5.2.19) and (5.2.20) are independent of L_S. So this only raises the level of unemployment of the unskilled labour in the South; and it is shown by equation (5.2.12).

An increase L_N causes both NN curve and XX curve in figure 5.2 to shift leftward. As in GH(1991b), the extent of the leftward shift of NN curve must be larger at the initial g and that of the XX curve is greater at the initial value of m. In the new steady state growth equilibrium, g is increased and m is decreased. Thus, the long run rate of innovation (imitation) is directly (inversely) related to the size of the labour endowment in the North. Since g is increased, we find that $\frac{w_N}{w_{SH}}$, $\frac{w_N}{w_{SL}}$ and U_S are also increased[31]. We can summarize the results discussed above in the following proposition.

Proposition 5.2.4. *(i) An expansion of the Southern skilled labour endowment raises the rate of innovation, the rate of imitation and the South North relative wage but lowers the level of unemployment in the South in the new steady state equilibrium. (ii) An expansion of the Northern labour endowment raises the rate of innovation, the North South relative wage and the level of unemployment but lowers the rate of imitation. (iii) An expansion of the unskilled labour endowment in the South raises the unemployment level there and leaves the rate of innovation, the rate of imitation and the South North relative wage unaffected.*

Effects of changes in the Northern and the Southern skilled labour endowments on the rate of innovation, on the rate of imitation and on the North South relative wage in our model are similar to those obtained from GH(1991b). Effects of these expansions on the unemployment level does not depend on the North South wage gap. In both the wide gap and the narrow gap cases, an increase in the skilled (unskilled) labour endowment in the South lowers (raises) the level of unemployment there.

However, the effect of a change in the Northern labour endowment on the Southern unemployment level crucially depend on the North South wage gap. This raises the level of unemployment in the narrow gap case and keeps it unchanged in the wide gap case. In the narrow gap case, an increase in the Northern labour endowment lowers the level of employment of the Southern skilled labour in the manufacturing sector. Since the skilled unskilled wage ratio is fixed by the rigidity of the efficiency wage, this lowers the demand for unskilled labour in the South which, in turn, raises the level of unemployment.

[31] See equations (5.2.12) and (5.2.20).

Chapter 6

Conclusions

In earlier chapters of this thesis, we have analysed a few theoretical problems related to North South trade and economic growth with special emphasis given to the policy of strengthening intellectual property rights protection in the South. In this chapter, we summarize the major results obtained in different chapters and mention the limitations of the existing work with a discussion on the scope for future research.

6.1 Summary of the thesis

In chapter 1 of this thesis, we have made a survey of the existing theoretical literature on endogenous growth and North South trade. We have paid special attention to review the works of Grossman and Helpman (1991) and of Helpman (1993) along with their extensions.

In chapter 2 of this thesis, the Helpman (1993) model is extended in three directions. In subsection 2.1, it is assumed that imitated products in the South do not contribute to the stock of knowledge capital in the Northern R&D sector. This is the case of localised knowledge spillover as emphasized by Jacobs (1969). The strengthening of IPR protection in the South in this case not only raises the rate of growth (product development) of the world economy but also may raise the levels of welfare in both the regions.

In subsection 2.2, perfect international mobility of labour between the North and the South is introduced in an otherwise identical Helpman (1993) model. It is shown that the strengthening of IPR protection policy in the South induces the Southern labour to migrate to the North. This raises the supply of labour in the North which, in turn, raises the balanced rate of growth of both the regions. The rate of growth depends on the size of the integrated world economy and not just on the size of the innovating region. In subsection 2.3, we introduce outsourcing of jobs from the North to the South. In the presence of North-South outsourcing of R&D jobs, the strengthening of IPR protection in the South may raise the rate of growth. However, in the case of outsourcing of production jobs, the strengthening of IPR protection in the South always lowers the rate of growth (innovation) at a higher rate than that obtained in the Helpman (1993) model. We also analyse the effects of the exogenous increase in the extent of outsourcing on the rate of growth (innovation). In the case of outsourcing of production jobs, this exogenous increase produces a positive effect on the rate of growth. However, this is not necessarily true in the case of outsourcing of R&D jobs. So our analysis done within the restricted framework of Helpman (1993), suggests the adoption of favourable policies towards North South production outsourcing. However, in case of R&D outsourcing, this should be accompanied by a policy of strengthening IPR protection in the South.

In chapter 3 of this thesis, we have analysed the local stability properties of the GH (1991b) model. In subsection 3.1, we have shown that the unique steady state equilibrium in the GH (1991b) model is saddle point stable; and this result is valid in the wide gap case as well as in the narrow gap case. In subsection 3.2, we have modified the GH (1991b) model with Jacobs (1969) type of localised knowledge spillover and have analysed the local stability properties of this modified model. Here the unique steady state equilibrium appears to be saddle point stable in the narrow gap case but is unstable in the wide gap case. We have analysed various transitional dynamic properties of the modified GH (1991b) model in the narrow gap case and have shown how the behaviour of the transitional path from one steady state equilibrium to the other is sensitive to the once for all parametric changes. We also analyse the welfare effects of strengthening IPR protection in the South in both the regions. This policy may lead to a welfare gain in both the countries (regions) because it has a positive comparative steady state effect on the rate of growth leading to a positive welfare effect through the availability of greater variety of products. This positive effect may outweigh the combined negative effect of inter regional allocation of production and of the intertemporal R&D expenditure.

In chapter 4 of the present thesis, we have considered multinationalisation as the channel of North-South production transfer and have extended the model of Lai (1998) in two directions. (i) The imitation activity in the South is assumed to be costly[1]. (ii) The South has two types of labour - skilled and unskilled. With these changes in assumptions, we have shown that the stronger protection of IPR in the South, defined as an increase in the cost of imitation there, would lower the balanced rate of growth (product development) of the world economy as well as the rate of multinational flow to the South but would raise the skilled-unskilled wage ratio in the South as well as the North-South relative wage. These results are completely opposite to those of Lai (1998) who has assumed imitation activity to be costless and has defined strengthening of IPR protection as an exogenous reduction in the imitation rate.

In chapter 5 of this thesis, we have analysed the problem of unemployment of unskilled labour in less developed countries introducing unemployment of unskilled labour in the South in the GH (1991b) model. The unemployment equilibrium has been explained by the efficiency wage hypothesis. We have analysed the impact of strengthening IPR protection in the South not only on the balanced rate of growth of the world economy

[1]Mansfield et al. (1981) have shown that the expenditure on imitative R&D is almost 60% to that of the innovative R&D.

and on the rate of imitation in the South but also on the level of unemployment in the South. We have shown that the strengthening of IPR protection in the South would lower both the equilibrium rate of growth and the rate of imitation. However, the level of unemployment of the unskilled labour in the South is increased in the narrow gap case and is decreased in the wide gap case. We have also analysed the effects of changes in factor endowments in this chapter. The level of unemployment of the unskilled labour in the South always varies inversely (directly) with the size of its skilled (unskilled) labour endowment. However, an increase in the size of the Northern labour endowment raises (does not affect) the unemployment level in the narrow (wide) gap case.

6.2 Limitations and scope for future research

In this section, we discuss some of the limitations of the present work and point out scopes for further research. First, we have adopted the Grossman and Helpman (1991) framework of an expanding product variety type R&D based North-South growth model. So our analysis suffers from the problem of 'scale effects' which receives very little empirical support as pointed out by Jones (1995a, 1995b). After Jones (1995a, 1995b) criticism, researchers have shown interest in developing 'scale free' endogenous growth models and a voluminous literature has been developed in this line. We have not considered any 'scale free' endogenous growth model in any chapters of this thesis.

Secondly, in all the models developed in different chapters of this thesis, we ignore the role of innovation activities in the South. However, Currie et al. (1999, p. 60) point out that many countries like Japan, South Korea, Taiwan etc., who were earlier engaged in product imitation, are now emerging as centers of R&D activity and innovation. Models developed by Arnold (2003), Chui et al. (2001), Currie et al. (1999) etc. analyse the endogenous switching of the South from imitation to innovation. However, our work does not take care of it.

Thirdly, our analysis of the stability properties of the GH (1991b) model and of its modified version developed in subsection 3.2 in chapter 3 have not progressed upto a global stability analysis. We have only performed local stability analysis of the models. For example, Arnold (2000a) has analysed the global stability properties of the Romer (1990) model of a closed economy. However, it must be understood that the GH (1991b) North-South model is far more complicated than the Romer (1990) model of a closed economy. Even if the steady state equilibrium satisfies the saddle point stability, one should look at the speed of convergence along the transition path. The economies remain on the transition path for a long time if the speed of convergence is very low. We should analyse the effect of strengthening IPR protection on the transition path in this case. Recently Tanaka et al. (2007) have done this in a quality ladder based R&D driven North-South endogenous growth model with licensing and technology transfer. However, some models developed in this thesis (for example, models developed in chapters 4 and 5 of this thesis) are more complicated than theirs; and so we can not examine the properties of transitional path analytically.

Fourthly, we have incorporated the role of public capital accumulation and human capital accumulation in the production function in none of the chapters of this thesis. Stock of knowledge defined as the number of products is the only externality considered here; and the rate of accumulation of the stock of knowledge determines the rate of growth. There exists a few works who combine endogenous product development and endogenous human capital accumulation in the same model; and the small literature includes the works of Caballe and Santos (1993), Arnold (1998), Funke and Strulik (2000) etc. The human capital accumulation is also influenced by the degree of tightening of IPR protection. Recently, Parello (2008) has examined the role of stronger intellectual property rights protection in the south on the processes of R&D investment, technology transfer and skill accumulation. If we consider human capital accumulation to be endogenous, then the labour endowments in the two regions become endogenous too and their rates of accumulation may vary from North to South.

Fifthly, we have not considered the problem of environmental pollution in the South resulting from the multinationalisation of the Southern firms. Generally, environmental protection laws in the South are weaker than those in the North; and so Northern firms have incentives to transfer the production of pollution intensive varieties to the South. This may have negative welfare impact in the South. So the effectiveness of strengthening IPR protection and environmental protection laws should be analysed simultaneously.

Sixthly, we have not considered the problem of international capital mobility in the present work. We have not considered physical capital as an additional input in the production function. Dollar (1986), in his product variety based endogenous growth model has analysed the problem of North-South capital mobility. It is interesting to incorporate physical capital into our models and to analyse the relationship between relative capital intensity and trade pattern.

Seventhly, we ignore the distributional aspect of unemployment in the model developed in chapter 5 of our thesis by assuming that all households have same proportion of unemployed members. So there is no problem of economic inequality resulting from unemployment. In reality unemployment is one of the important causes of economic inequality; and so we should integrate these two problems in future research.

Eighthly, we have explained unemployment of unskilled labour in the South using the efficiency wage hypothesis. If the efficiency wage hypothesis is not relevant, then the model developed in chapter 5 of this thesis is reduced to a full employment model. Many authors have explained unemployment of unskilled labour in less developed countries using the efficiency wage hypothesis. However, they have not considered our formulation in which the reference wage is proportional to some peer group's wage. This particular form applies mostly to those workers who are hard to monitor; and skilled workers are always harder to monitor than unskilled workers. A positive relationship between the worker's efficiency and the wage rate is necessary to explain the existence of an unemployment equilibrium; and this positive relationship exists in any form of the efficiency wage function including the form we consider here. Our results are conditional on the assumption that the reference wage is proportional to the wage of the skilled worker. Due to this assumption, the equilibrium condition of the efficiency wage gives us the unique equilibrium value of skilled unskilled wage ratio which is consistent with the assumption of steady state growth equilibrium where all types of wages are to grow at equal rates. If we drop this assumption we may not be able to prove the existence of a unique time independent skilled unskilled wage ratio. However, the empirical literature on efficiency wage theory has been unable to conclude so far on the most plausible form of efficiency wages and, in particular, on the reference wage level entering the worker's effort decision. In a partial equilibrium shirking model, the outside option is often treated as the reference wage. However, we can not do this in the present dynamic general equilibrium model. The assumption of perfect mobility of unskilled workers among all Southern firms and the competitive labour market assumption rule out the possibility of an equilibrium with the difference between the worker's actual wage inside the firm and his expected wage in case he is to leave the firm. Danthine and Kurmann (2006) assume firm's productivity as the reference wage. In the model considered in chapter 5, the profit maximising average productivity of the unskilled or skilled labour in the production sector is a function of the skilled unskilled wage ratio because the production function of each variety in the South satisfies constant returns to scale. So introducing firm's productivity as the reference wage, no additional gain is made in this model. However, future research should deal with the robustness of the results to different functional forms of the efficiency function and to different assumptions about the reference wage.

Lastly, leisure is not included as an argument in the utility function of the representative consumer in all the models; and so the household does not make any optimum labour-leisure allocation. So there is no growth induced income effect on the labour supply decision of the worker. The strengthening of the IPR protection does not cause any shift of the labour supply curve. However, leisure is an important argument in the utility function of the household. It is interesting to analyse the effectiveness of strengthening IPR protection into a framework with endogenous labour-leisure allocation; and we plan to do this in our future research.

Appendices

Chapter 2

Appendix 2.1

The problem of the representative agent is to maximise

$$W_N = \int_t^\infty e^{-\rho(\tau - t)} log(U_N(\tau)) d\tau \tag{A.1}$$

subject to the intertemporal budget constraint given by

$$\int_t^\infty e^{-r_N(\tau - t)} E_N(\tau) d\tau = \int_t^\infty e^{-r_N(\tau - t)} I_N(\tau) d\tau + A_N(t) \quad \forall t \; . \tag{A.2}$$

Here,

$$E_N = \int_0^n p(z) x_N(z) dz \; ; \tag{A.3}$$

and

$$U_N = \left(\int_0^n x_N(z)^\alpha dz \right)^{\frac{1}{\alpha}} \; . \tag{A.4}$$

The agent solves this dynamic optimisation problem in two stages. First, it chooses the composition of a given level of spending to maximise the instantaneous utility. Then it optimizes W_N through the time path of spending. In stage 1, the agent's static optimisation exercise is to maximise

$$U_N = \left(\int_0^n x_N(z)^\alpha dz \right)^{\frac{1}{\alpha}}$$

subject to

$$E_N = \int_0^n p(z) x_N(z) dz.$$

We obtain the following demand function for each variety given by

$$x_N(z) = E_N \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}} \qquad \forall z \in [0, n] \ . \qquad (A.5)$$

Using equations (A.4) and (A.5) we obtain the indirect instantaneous utility function given by

$$U_N = \left(n_N {x_N}^{\alpha} + n_S {x_S}^{\alpha}\right)^{\frac{1}{\alpha}} = \frac{E_N}{P} \ . \qquad (A.6)$$

Here,

$$P^{(1-\varepsilon)} = \int_0^n p(u)^{(1-\varepsilon)} du = n_N p_N^{(1-\varepsilon)} + n_S p_S^{(1-\varepsilon)}.$$

Using equations (A.1) and (A.6) we have

$$W_N = \int_t^{\infty} e^{-\rho(\tau - t)} log \left[\frac{E_N(\tau)}{P(\tau)}\right] d\tau.$$

Differentiating both sides of equation (A.2) with respect to time we have

$$\dot{A_N} = I_N - E_N + r_N A_N.$$

Now we write the current value Hamiltonian corresponding to this dynamic optimisation problem as

$$H = log(U_N) + j \ [I_N - E_N + r_N A_N]$$

$$or, \qquad H = [log(E_N) - log(P)] + j \ [I_N - E_N + r_N A_N]$$

where j is the costate variable. The first order optimality condition is given by

$$\frac{\partial H}{\partial E_N} = \frac{\delta U_N}{\delta E_N} - j = 0 \ ,$$

$$\text{or,} \qquad \frac{1}{E_N} = j \ . \qquad (A.7)$$

Hence, we have

$$\frac{\dot{j}}{j} = -\frac{\dot{E_N}}{E_N}. \qquad (A.8)$$

Optimum time path of j should satisfy the following equation of motion.

$$\dot{j} = \rho j - \frac{\partial H}{\partial A_N} = (\rho - r_N) \ j.$$

Now, using equation (A.8), we have

$$\frac{\dot{E_N}}{E_N} = r_N - \rho.$$

The instantaneous demand function for a representative Southern consumer is

$$x_S(z) = E_S \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}} \qquad \forall z \in [0, n] \ .$$

From the above equation we obtain the indirect utility function given by

$$U_S = \left(\int_0^n x_S(z)^{\alpha} dz \right)^{\frac{1}{\alpha}} = \frac{E_S}{P}. \tag{A.9}$$

Let e_i stands for the per capita spending of the consumer in the ith region for $i = N, S$.

Then from equations (A.6) and (A.9) we have

$$U_i = \frac{e_i}{P} \qquad \text{for } i = N, S.$$

This implies that

$$\frac{U_N}{U_S} = \frac{e_N}{e_S} \ .$$

Appendix 2.2

Derivation of equations (2.1.13) and (2.1.14):

Equation (2.1.13) can be derived as

$$\frac{\dot{\xi}}{\xi} = \frac{\dot{n_N}}{n_N} - g = (\frac{\dot{n}}{n_N} - \frac{\dot{n_S}}{n_N}) - (\frac{g}{\xi})\xi = (\theta - m) - \theta\xi \Rightarrow \dot{\xi} = \theta\xi - (\theta\xi + m)\xi$$

To derive equation (2.1.14) we proceed as follows:

We have

$$\frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \frac{g}{\xi}) \quad \textit{and} \quad \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - \frac{\dot{n_N}}{n_N} \Longrightarrow \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - (\frac{\dot{\xi}}{\xi} + g) \quad ; \ \textit{since} \ \ n_N = n\xi$$

Using this two in the no arbitrage condition (2.1.11) we solve for $\frac{\dot{p_N}}{p_N}$ as

$$\frac{\dot{p_N}}{p_N} = r_N + m + (\frac{\dot{\xi}}{\xi} + g) - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \frac{g}{\xi}) \tag{A1.1}$$

From equation (2.1.12) we have $E_N = p_N n_N x_N$. Also from the intertemporal utility maximisation of the representative consumer we get $\frac{\dot{E_N}}{E_N} = r_N - \rho$. Now $E_N = p_N n_N x_N$ imply $\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(L_N \dot{-} a_N \frac{g}{\xi})}{(L_N - a_N \frac{g}{\xi}}$. Then we have

$$r_N - \rho = \frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} - a_N \frac{(\dot{\frac{g}{\xi}})}{L_N - a_N \frac{g}{\xi}}$$

$$\Longrightarrow \frac{\dot{p_N}}{p_N} = (r_N - \rho) + a_N \frac{\dot{g}\xi - \dot{\xi}g}{(L_N\xi - a_N g)} \frac{1}{\xi} \tag{A1.2}$$

Equations (A1.1) and (A1.2) together implies

$$m + \frac{\dot{\xi}}{\xi} + g - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \frac{g}{\xi}) = a_N \frac{\dot{g}\xi - \dot{\xi}g}{(L_N\xi - a_N g)} \frac{1}{\xi} - \rho$$

Using equation (2.1.13) and the definition of θ, this last equation implies

$$\dot{\theta} = (\frac{L_N}{a_N} - \theta)[\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)].$$

Appendix 2.3

Derivation of the solution of the equations of motions:

Linearising (2.1.13) and (2.1.14) around their steady-state values we get

$$\begin{bmatrix} \dot{\theta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \frac{\partial\dot{\theta}}{\partial\theta}]_{(\theta^*,\xi^*)} & \frac{\partial\dot{\theta}}{\partial\xi}]_{(\theta^*,\xi^*)} \\ \frac{\partial\dot{\xi}}{\partial\theta}]_{(\theta^*,\xi^*)} & \frac{\partial\dot{\xi}}{\partial\xi}]_{(\theta^*,\xi^*)} \end{bmatrix} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \xi(t) - \xi^* \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} \dot{\theta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \frac{L_N}{a_N} + \rho & 0 \\ \frac{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha - m)m}{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)^2} & m - ((1-\alpha)\frac{L_N}{a_N} - \rho\alpha) \end{bmatrix} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \xi(t) - \xi^* \end{bmatrix}$$

We assume that $m < (1-\alpha)\frac{L_N}{a_N} - \rho\alpha = \theta^*$. Let us denote $a_{11} = \frac{L_N}{a_N} + \rho$, $a_{12} = 0$, $a_{21} = \frac{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha - m)m}{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)^2}$ and $a_{22} = m - ((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)$.

We have $a_{11} > 0$, $a_{21} > 0$ and $a_{22} < 0$. Since the trace of the matrix on the right hand side is positive and the determinant is negative , it has one positive root (a_{11}) and one negative root (a_{22}). This proves that the steady-state equilibrium point is a saddle point. We choose the eigenvector corresponding to a_{11}, the positive root, as $\begin{pmatrix} a_{11} - a_{22} \\ a_{21} \end{pmatrix}$ and the eigenvector corresponding to a_{22}, the negative root, as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. To ensure long run convergence we choose that at time zero $\theta(t)$ takes the value θ^*, i.e., $\theta(0) = \theta^*$. This procedure leads to the solution

$$\begin{aligned} &\theta(t) = \theta^* \\ &\xi(t) = \xi^* + [\xi(0) - \xi^*]e^{a_{22}t} \end{aligned}$$

Appendix 2.4

Feasibility restriction on m:

To find the feasibility restriction on m given in inequality (B) we proceed as follows.

$$\frac{w_N}{w_S} > 1 \Rightarrow \frac{p_N}{p_S} > \frac{1}{\alpha} \Rightarrow (\frac{x_N}{x_S})^{-\frac{1}{\varepsilon}} > \frac{1}{\alpha} \Rightarrow \frac{x_N}{x_S} > \alpha^\varepsilon \Rightarrow \frac{L_N - a_N\theta}{L_S}\frac{(1-\xi)}{\xi} > \alpha^\varepsilon$$

$$\Rightarrow \frac{1}{\xi} - 1 > \frac{L_S\alpha^\varepsilon}{L_N - a_N\theta} \Rightarrow \xi > \frac{L_N - a_N\theta}{L_S\alpha^\varepsilon + L_N - a_N\theta}$$

The values of (θ, ξ) satisfying above inequality would ensure $w_N > w_S$. This region in the (θ, ξ) axis is shown by the area under the WW curve (see Figure 2.1.1). Now in the steady-state we have $\xi = \frac{\theta - m}{\theta}$ and then using the above inequality we get

$$\frac{\theta - m}{\theta} > \frac{L_N - a_N\theta}{L_S\alpha^\varepsilon + L_N - a_N\theta} \Rightarrow m < \theta\frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta}$$

where θ takes its steady-state value.

Appendix 2.5

Derivation of instantaneous utility functions (2.1.20) and (2.1.21):

We can write the instantaneous demand function as $x_b(j) = p(j)^{-\varepsilon}\frac{E_b}{P^{1-\varepsilon}}$, where $P = [\int_0^n p(j)^{1-\varepsilon}dj]^{\frac{1}{1-\varepsilon}}$ and b= N, S. Substituting this demand function into the instantaneous utility function we obtain the indirect utility function

$$logU_b = logE_b - logP$$

Now,

$$P^{1-\varepsilon} = n_N p_N^{1-\varepsilon} + n_S p_S^{1-\varepsilon} \Longrightarrow P = p_S n^{\frac{1}{1-\varepsilon}}[\xi(\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]^{\frac{1}{1-\varepsilon}}$$

or,

$$P = p_N n^{\frac{1}{1-\varepsilon}}[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$$

In the South, per capita income is the wage rate $w_S = p_S$. Then $E_S = p_S$ and in North it is $E_N = p_N(1 - \frac{a_N\theta}{L_N})$. Therefore,

$$logU_S = \frac{1}{\varepsilon - 1}log(n) + \frac{1}{\varepsilon - 1}log[\xi(\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]$$

and

$$logU_N = \frac{1}{\varepsilon - 1}log(n) + \frac{1}{\varepsilon - 1}log[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}] + log(1 - \frac{a_N\theta}{L_N})$$

Appendix 2.6

To prove $\Delta_N > 0$:

$log(n(t)) = log(n(0)) + \int_0^t g(\tau)d\tau \Rightarrow \frac{dlog(n(t))}{d\mu} = \int_0^t \frac{dg(\tau)}{d\mu}d\tau = \int_0^t (1 - e^{a_{22}\tau})d\tau \qquad from\ (2.1.19)$

$= \int_0^t d\tau - \int_0^t e^{a_{22}\tau}d\tau = t - [\frac{1}{a_{22}}e^{a_{22}\tau}]_0^t = t + \frac{1}{a_{22}} - \frac{1}{a_{22}}e^{a_{22}t}$. Then,

$\Delta_N = \int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{d\mu}dt = \int_0^\infty e^{-\rho t}(t + \frac{1}{a_{22}} - \frac{1}{a_{22}}e^{a_{22}t})dt = \int_0^\infty e^{-\rho t}tdt$

$+ \frac{1}{a_{22}}\int_0^\infty e^{-\rho t}dt - \frac{1}{a_{22}}\int_0^\infty e^{-(\rho - a_{22})t}dt = \frac{1}{\rho^2} + \frac{1}{\rho a_{22}} - \frac{1}{(\rho - a_{22})a_{22}} = \frac{-a_{22}}{\rho^2(\rho - a_{22})} > 0.$

Appendix 2.7

To prove $\Delta_e^S < 0$:

$$\Delta_e^S = \int_0^\infty e^{-\rho t}\frac{dlog[\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)]}{d\mu}]dt = \int_0^\infty e^{-\rho t}\frac{1}{\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)}[(\frac{p_S}{p_N})^{\varepsilon-1}\frac{d\xi}{d\mu}-\frac{d\xi}{d\mu}+\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi];$$

First two terms in the third bracket of the last expression $[(\frac{p_S}{p_N})^{\varepsilon-1}\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}]$ captures the effect of interregional allocation of production on welfare (keeping the terms of trade unchanged). Since $\frac{p_S}{p_N} < 1$, $(\varepsilon - 1) > 0$ and $\frac{d\xi(t)}{d\mu} > 0$ welfare of the South decreases due to interregional allocation of production only. The last term in the third bracket of the last expression $[\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi]$ captures the welfare change due to terms of trade effect only. We have $(\frac{p_S}{p_N})^{\varepsilon-1} = (\frac{L_N-a_N\theta}{L_S}\frac{1-\xi}{\xi})^\alpha$ at the steady-state. Then $\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi = -\frac{d\xi(t)}{d\mu}\frac{\alpha}{1-\xi}(\frac{p_S}{p_N})^{\varepsilon-1}$. This is clearly negative. Hence welfare of the South decreases due to the change in the terms of trade only. The expression for Δ_e^S looks like

$$\Delta_e^S = \int_0^\infty e^{-\rho t}\frac{1}{\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)}[(\frac{p_S}{p_N})^{\varepsilon-1} - 1 - (\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}](\frac{d\xi(t)}{d\mu})dt$$

$$= \frac{(\frac{p_S}{p_N})^{\varepsilon-1}-1-(\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)}[\int_0^\infty e^{-\rho t}(\frac{d\xi(t)}{d\mu})dt] = \frac{(\frac{p_S}{p_N})^{\varepsilon-1}-1-(\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)}[\int_0^\infty e^{-\rho t}\frac{1}{\theta^*}(1-e^{a_{22}})dt]$$

$$= \frac{(\frac{p_S}{p_N})^{\varepsilon-1}-1-(\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1}+(1-\xi)}[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}] < 0;$$ where $\frac{p_S}{p_N}$ and ξ are measured at their steady-state.

Appendix 2.8

To prove $\frac{dW_S(0)}{d\mu} > 0$:

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon-1}(\Delta_N + \Delta_e^S) = \frac{1}{\varepsilon-1}\frac{-a_{22}}{\rho(\rho-a_{22})}[\frac{1}{\rho} + \frac{1}{\theta^*}\frac{(\frac{p_S}{p_N})^{\varepsilon-1}(1-\frac{\alpha}{1-\xi^*})-1}{\xi^*(\frac{p_N}{p_S})^{1-\varepsilon}+(1-\xi^*)}]$$

$$= \frac{1}{\varepsilon-1}\frac{-a_{22}}{\rho(\rho-a_{22})}[\frac{\theta^*\xi^*(\frac{p_S}{p_N})^{\varepsilon-1}+\theta^*(1-\xi^*)+\rho(1-\frac{\alpha}{1-\xi^*})(\frac{p_S}{p_N})^{\varepsilon-1}-\rho}{\rho\theta^*\{\xi^*(\frac{p_N}{p_S})^{1-\varepsilon}+(1-\xi^*\}}]$$

The numerator in the third bracket of the last expression is positive under the sufficient assumption that $(1-\frac{\alpha}{1-\xi^*}) \geq 0$ and $\theta^*(1-\xi^*) \geq \rho$. Now, $(1-\frac{\alpha}{1-\xi^*}) \geq 0$ if $m \geq \alpha\theta^*$, [note that $1-\xi^* = \frac{m}{\theta^*}$]. Also $\theta^*(1-\xi^*) \geq \rho$ is true if $m \geq \rho$. Therefore, for $m \geq max(\rho, \alpha\theta^*)$ we have $\frac{dW_S(0)}{d\mu} > 0$.

Appendix 2.9

To prove $\Delta_e^N > 0$:

$$\Delta_e^N = \int_0^\infty e^{-\rho t}\frac{dlog[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]}{d\mu}]dt$$

$$= \int_0^\infty e^{-\rho t}\frac{1}{\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}[\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}(\frac{p_S}{p_N})^{1-\varepsilon} + (1-\xi)\frac{d(\frac{p_S}{p_N})^{1-\varepsilon}}{d\mu}]dt$$

The first two terms in the third bracket of the last expression $[\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}(\frac{p_S}{p_N})^{1-\varepsilon}]$ captures the welfare effect due to changes in the interregional allocation of production only (keeping the terms of trade unchanged). This is clearly negative. The last term $[(1-\xi)\frac{d(\frac{p_S}{p_N})^{1-\varepsilon}}{d\mu}]$ captures the welfare change due to changes in the terms of trade only. We have got $(1-\xi)\frac{d(\frac{p_S}{p_N})^{1-\varepsilon}}{d\mu} = \frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}\frac{d\xi}{d\mu}$. This is clearly positive. Then the expression for Δ_e^N looks like,

$$\Delta_e^N = \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}+\frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}\int_0^\infty e^{-\rho t}[\frac{d\xi(t)}{d\mu}]dt$$

$$= \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}+\frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}\int_0^\infty e^{-\rho t}[\frac{1}{\theta^*}(1-e^{a_{22}})]dt$$

$$= \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}\{1-\frac{\alpha}{\xi}\}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}], \quad \text{where } \frac{p_S}{p_N} \text{ and } \xi \text{ take their steady-state value.}$$

The numerator of the first term in the last expression is positive if $\xi^* \leq \alpha \Rightarrow (1-\frac{m}{\theta^*}) \leq$

$\alpha \Rightarrow m \geq (1-\alpha)\theta^*$. The second term in the third bracket of the last expression is always positive. Hence $\Delta_e^N > 0$ under the condition $m \geq (1-\alpha)\theta^*$.

Appendix 2.10

Welfare effect of Southern labour endowment change:

From equations (2.1.15) and (2.1.16) we see that the change in L_S does not affect the time path of $\xi(t)$ and $\theta(t)$; and so, $g(t) = \xi(t)\theta(t)$ remains unchanged. Hence

$$\Delta_N^{L_i} = \int_0^\infty e^{-\rho t} \frac{dlog(n(t))}{dL_i} dt = 0 \quad \text{for i=S.}$$

However, change in L_S does affects the terms-of-trade. We have,

$$\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} = \left[\frac{L_N - a_N\theta}{L_S} \frac{1-\xi}{\xi}\right]^\alpha$$

Then,

$$\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{dL_S} < 0; \quad \text{since change in } L_S \text{ does not affect } \theta(t) \text{ and } \xi(t).$$

This implies that

$$\Delta_e^{SL_i} < 0 \quad \text{and} \quad \Delta_e^{NL_i} > 0 \quad \text{for i=S.}$$

Also,

$$\Delta_S^{NL_i} = 0; \quad \text{for i=S, since change in } L_S \text{ does not affect } \theta(t).$$

Welfare effect of Northern labour endowment change:

To find the welfare effect of Northern labour endowment change, we calculate the following effects separately. Note that the derivative of a variable with respect to L_N is evaluated at the steady state equilibrium point.

$$\begin{aligned}
\frac{dlog(n(t))}{dL_N} = \int_0^t \frac{d(g(\tau))}{dL_N} d\tau &= \int_0^t \frac{d(\theta(\tau)\xi(\tau))}{dL_N} d\tau \\
&= \int_0^t \left[\xi^* \frac{1-\alpha}{a_N} + \theta^* \frac{d\xi^*}{dL_N}(1-e^{a_{22}\tau})\right] d\tau \\
&= \xi^* \frac{1-\alpha}{a_N} t + \theta^* \frac{d\xi^*}{dL_N} \left[\int_0^t (1-e^{a_{22}\tau}) d\tau\right].
\end{aligned}$$

Then,

$$\Delta_N^{L_N} = \left(\xi^* \frac{1-\alpha}{a_N}\right) \int_0^\infty e^{-\rho t} t dt + \theta^* \frac{d\xi^*}{dL_N} \int_0^\infty e^{-\rho t} \left[\int_0^t (1 - e^{a_{22}\tau}) d\tau\right] dt.$$

Since $\frac{d\xi^*}{dL_N} > 0$ (see the second part of proposition 2.1.1), both terms in the R.H.S. of the above expression are positive (see Appendix 2.6 for the detail calculation). Hence $\Delta_N^{L_N} > 0$.

From equation (2.1.16), we obtain

$$\frac{d(\xi(t))}{dL_N} = \frac{d\xi^*}{dL_N}(1 - e^{a_{22}t}) > 0; \quad \text{for all } t > 0,$$

and

$$\frac{d\left(\frac{p_S}{p_N}\right)^{\varepsilon-1}}{dL_N} = \frac{d\left[\frac{L_N - a_N\theta}{L_S}\frac{1-\xi}{\xi}\right]^\alpha}{dL_N} < 0.$$

The above two expressions imply that $\Delta_e^{SL_N}$ and $\Delta_e^{NL_N}$ are indeterminate in sign. This is so because, an increase in L_N raises $\xi(t)$ but lowers $\left(\frac{p_S}{p_N}\right)^{\varepsilon-1}$ and so, the combined effect of the interregional allocation of production and terms-of-trade remain ambiguous. We also obtain

$$\frac{d(\frac{L_N - a_N\theta}{L_N})}{dL_N} = -\frac{\alpha a_N \rho}{L_N^2} < 0.$$

Then $\Delta_S^{NL_N} < 0$.

Appendix 2.11

Derivations of M_g and M_ξ

From equation (2.2.29) we have

$$\left(\frac{\bar{L}_N + M}{\bar{L}_N + M - a_N g}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\bar{L}_S - M}{\bar{L}_N + M - a_N g}\,\frac{\xi}{1-\xi}\right).$$

or,

$$\frac{log(\bar{L}_N + M)}{1-\alpha} - \frac{\alpha\, log(\bar{L}_N + M - a_N g)}{1-\alpha} - log(\bar{L}_S - M) = log(\xi) - log(1-\xi).$$

Its total differential is given by

$$\underbrace{\left[\frac{1}{\bar{L}_N + M} - \frac{\alpha}{\bar{L}_N + M - a_N g} + \frac{1}{\bar{L}_S - M}\right]}_{term\ 1} dM + \left[\frac{\alpha\, a_N}{\bar{L}_N + M - a_N g}\right] dg - \left[\frac{1-\alpha}{\xi} + \frac{1-\alpha}{1-\xi}\right] d\xi = 0$$

$term$ 1 is positive if

$$\bar{L}_N + M \geq \frac{a_N g}{1-\alpha}. \qquad (B1)$$

It is same as the condition (2.2.25) in the body of the paper. Then, using condition (B1), we obtain

$$M_g = \frac{\partial M}{\partial g} = -\frac{\frac{\alpha\, a_N}{\bar{L}_N + M - a_N g}}{\frac{1}{\bar{L}_N + M} - \frac{\alpha}{\bar{L}_N + M - a_N g} + \frac{1}{\bar{L}_S - M}} < 0;.$$

and

$$M_\xi = \frac{\partial M}{\partial \xi} = \frac{\frac{1-\alpha}{\xi} + \frac{1-\alpha}{1-\xi}}{\frac{1}{\bar{L}_N + M} - \frac{\alpha}{\bar{L}_N + M - a_N g} + \frac{1}{\bar{L}_S - M}} > 0.$$

Appendix 2.12

The derivation of equation (2.2.32)

Using equations (2.2.11), (2.2.13), (2.2.15), (2.2.16), (2.2.27) and (2.2.28), we have

$$\begin{aligned} \frac{\pi_N}{v_N} &= \left(\frac{1-\alpha}{\alpha} w_N x_N\right) \frac{1}{\frac{a_N}{n} w_N} \\ &= \frac{1-\alpha}{\alpha a_N} [\bar{L}_N + M - a_N g] \frac{1}{\xi}\ . \end{aligned} \qquad \text{(C.1)}$$

We also have

$$\frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - g\ . \qquad (C.2)$$

Also from the intertemporal utility maximisation exercise of the representative consumer we have

$$\frac{\dot{E_N}}{E_N} = r_N - \rho. \qquad (C.3)$$

Using equations (2.2.11), (2.2.13), (2.2.18) and (2.2.28) we have

$$\frac{E_N}{p_N} = (\bar{L}_N + M - a_N g). \qquad (C.4)$$

Differentiating both sides of equation (C.4) with respect to time we have

$$\frac{\dot{E_N}}{E_N} - \frac{\dot{p_N}}{p_N} = \frac{(M_g - a_N)\dot{g} + M_\xi \dot{\xi}}{\bar{L}_N + M - a_N g}. \tag{C.5}$$

Now, using equations (2.2.17) and (C.2), we have

$$\frac{\pi_N}{v_N} + \left[\frac{\dot{p_N}}{p_N} - g\right] = r_N + m\ .$$

Using equation (C.5), this can be written as

$$\frac{\pi_N}{v_N} + \left[\frac{\dot{E_N}}{E_N} - \frac{(M_g - a_N)\dot{g} + M_\xi \dot{\xi}}{\bar{L}_N + M - a_N g} - g\right] = r_N + m.$$

Then, using equation (C.3), this is written as

$$\frac{\pi_N}{v_N} + (r_N - \rho) - (r_N + m) - g = \frac{(M_g - a_N)\dot{g} + M_\xi \dot{\xi}}{\bar{L}_N + M - a_N g}\ .$$

Then, using equations (C.1) and (2.2.31), we obtain the following expression.

$$\dot{g}(M_g - a_N) + M_\xi[g - (m+g)\xi] = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right) - (\rho + m + g)\right]\ .$$

This is the equation (2.2.32).

Appendix 2.13

Stability of the steady state equilibrium

Here the two equations of motion are

$$\dot{\xi} = g - (g + m)\xi \tag{2.2.31}$$

and

$$\dot{g}(M_g - a_N) + M_\xi[g - (m+g)\xi] = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right) - (\rho + m + g)\right]. \tag{2.2.32}$$

The steady state equilibrium conditions are

$$\frac{1-\alpha}{\alpha a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right) = (\rho + m + g); \tag{2.2.33}$$

and

$$g - (g+m)\xi = 0. \tag{2.2.34}$$

Differentiating equation (2.2.32) with respect to g at the steady state equilibrium point $(g^*,\ \xi^*)$ we obtain

$$\left[\frac{\partial \dot{g}}{\partial g}\right]_{(g^*,\xi^*)} [M_g - a_N] + M_\xi(1-\xi) = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\left(\frac{M_g - a_N}{\xi}\right) - 1\right]$$

or,

$$\left[\frac{\partial \dot{g}}{\partial g}\right]_{(g^*,\xi^*)} = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\frac{1}{\xi} - \frac{1}{M_g - a_N}\right] - \frac{M_\xi(1-\xi)}{M_g - a_N}.$$

Again, differentiating equation (2.2.32) with respect to ξ at the steady state equilibrium point, $(g^*,\ \xi^*)$ we obtain

$$\left[\frac{\partial \dot{g}}{\partial \xi}\right]_{(g^*,\xi^*)} [M_g - a_N] + M_\xi(-m-g) = (\bar{L}_N + M - a_N g)\left[-\frac{1-\alpha}{\alpha a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi^2}\right)\right]$$

or,

$$\left[\frac{\partial \dot{g}}{\partial \xi}\right]_{(g^*,\xi^*)} = \frac{M_\xi(m+g)}{M_g - a_N} - \frac{1-\alpha}{\alpha a_N}\frac{1}{M_g - a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right)^2.$$

Similarly, differentiating equation (2.2.31) with respect to g and ξ at the steady state equilibrium point, we obtain

$$\left[\frac{\partial \dot{\xi}}{\partial g}\right]_{(g^*,\xi^*)} = 1 - \xi;$$

and

$$\left[\frac{\partial \dot{\xi}}{\partial \xi}\right]_{(g^*,\xi^*)} = -(m+g).$$

Here the Jacobian matrix is given by

$$J = \begin{bmatrix} (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\frac{1}{\xi} - \frac{1}{M_g - a_N}\right] - \frac{M_\xi(1-\xi)}{M_g - a_N} & \frac{M_\xi(m+g)}{M_g - a_N} - \frac{\frac{1-\alpha}{\alpha a_N}}{M_g - a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right)^2 \\ (1-\xi) & -(m+g) \end{bmatrix}.$$

The trace of the matrix J is given by

$$Tr(J) = (\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\frac{1}{\xi} - \frac{1}{M_g - a_N}\right] - \frac{M_\xi(1-\xi)}{M_g - a_N} - m - g. \tag{D.1}$$

From the steady state equilibrium condition (2.2.33) and equation (D.1) we have

$$Tr(J) = \rho - \frac{\bar{L}_N + M - a_N g}{M_g - a_N} - \frac{M_\xi(1-\xi)}{M_g - a_N}.$$

In Appendix 2.11, we have proved that $M_g < 0$ if

$$\bar{L}_N + M \geq \frac{a_N g}{1-\alpha}.$$

Hence

$$Tr(J) > 0.$$

In this case the determinant of the matrix J is given by

$$Det(J) = \underbrace{-(\bar{L}_N + M - a_N g)\left[\frac{1-\alpha}{\alpha a_N}\frac{1}{\xi} - \frac{1}{M_g - a_N}\right](m+g)}_{term\ 1} + \underbrace{(1-\xi)\frac{\frac{1-\alpha}{\alpha a_N}}{M_g - a_N}\left(\frac{\bar{L}_N + M - a_N g}{\xi}\right)^2}_{term\ 2}$$

Both *term* 1 and *term* 2 are negative because $M_g < 0$. So

$$Det(J) < 0.$$

So the latent roots of the Jacobian matrix are of opposite sign. Hence the steady state equilibrium satisfies saddle point stability.

Appendix 2.14

Derivations of comparative steady state effects

IPR tightening

Using equation (2.2.35) we obtain

$$log(\bar{L}_N + M - a_N g) + log(g+m) - log(g) - log(\rho + m + g) = log(\frac{a_N \alpha}{1-\alpha}); \quad (E.1)$$

and using equation (2.2.36), we obtain

$$\frac{1}{1-\alpha}log(\bar{L}_N + M) - \frac{\alpha}{1-\alpha}\,log(\bar{L}_N + M - a_N g) - log(\bar{L}_S - M) = log(g) - log(m). \quad (E.2)$$

Differentiating both sides of these two equations with respect to m and arranging the terms we obtain

$$\begin{bmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g}+\frac{1}{g+m}-\frac{1}{g}-\frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\frac{\alpha}{1-\alpha}a_N}{\bar{L}_N+M-a_N g}-\frac{1}{g} & \frac{\frac{1}{1-\alpha}}{\bar{L}_N+M}+\frac{1}{\bar{L}_S-M}-\frac{\frac{\alpha}{1-\alpha}}{\bar{L}_N+M-a_N g} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial g}{\partial m} \\ \frac{\partial M}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{-\rho}{(\rho+g+m)(g+m)} \\ -\frac{1}{m} \end{bmatrix}.$$

Solving them by Cramer's rule we obtain

$$\frac{\partial g}{\partial m} = \frac{\begin{vmatrix} \frac{-\rho}{(\rho+g+m)(g+m)} & \frac{1}{\bar{L}_N+M-a_N g} \\ -\frac{1}{m} & \frac{\frac{1}{1-\alpha}}{\bar{L}_N+M}+\frac{1}{\bar{L}_S-M}-\frac{\frac{\alpha}{1-\alpha}}{\bar{L}_N+M-a_N g} \end{vmatrix}}{|D|}; \tag{E.3}$$

and

$$\frac{\partial M}{\partial m} = \frac{\begin{vmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g}+\frac{1}{g+m}-\frac{1}{g}-\frac{1}{\rho+g+m} & \frac{-\rho}{(\rho+g+m)(g+m)} \\ \frac{\frac{\alpha}{1-\alpha}a_N}{\bar{L}_N+M-a_N g}-\frac{1}{g} & -\frac{1}{m} \end{vmatrix}}{|D|}. \tag{E.4}$$

Here

$$|D| = \begin{vmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g}+\frac{1}{g+m}-\frac{1}{g}-\frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\frac{\alpha}{1-\alpha}a_N}{\bar{L}_N+M-a_N g}-\frac{1}{g} & \frac{\frac{1}{1-\alpha}}{\bar{L}_N+M}+\frac{1}{\bar{L}_S-M}-\frac{\frac{\alpha}{1-\alpha}}{\bar{L}_N+M-a_N g} \end{vmatrix}$$

$$= \left\{ \underbrace{\frac{-(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}}_{term\ 1} + \frac{\rho}{(\rho+g+m)(g+m)} \right\} \cdot \left\{ \underbrace{\frac{\bar{L}_N+M-\frac{a_N g}{1-\alpha}}{(\bar{L}_N+M-a_N g)(\bar{L}_N+M)}}_{term\ 2} + \frac{1}{\bar{L}_S-M} \right\}$$

$$- \underbrace{\frac{1}{\bar{L}_N+M-a_N g} \cdot \left\{ \frac{\frac{a_N g}{1-\alpha}-(\bar{L}_N+M)}{g(\bar{L}_N+M-a_N g)} \right\}}_{term\ 3}.$$

Here we have

$$term\ 1 \times term\ 2 = term\ 3.$$

Hence we have

$$|D| = \left(\frac{-(\bar{L}_N + M)}{(\bar{L}_N + M - a_N g)g} \frac{1}{\bar{L}_S - M} \right) + \frac{\rho}{(\rho + g + m)(g + m)} \left\{ \frac{\bar{L}_N + M - \frac{a_N g}{1-\alpha}}{(\bar{L}_N + M - a_N g)(\bar{L}_N + M)} + \frac{1}{\bar{L}_S - M} \right. \tag{E.5}$$

Also the numerator for the $\frac{\partial g}{\partial m}$ expression of equation (E.3) can be written as

$$\frac{-\rho}{(\rho + g + m)(g + m)} \left\{ \frac{\bar{L}_N + M - \frac{a_N g}{1-\alpha}}{(\bar{L}_N + M - a_N g)(\bar{L}_N + M)} + \frac{1}{\bar{L}_S - M} \right\} + \frac{1}{m} \frac{1}{\bar{L}_N + M - a_N g}. \tag{E.6}$$

We define Δ such that

$$\Delta = \frac{\rho}{(\rho + g + m)(g + m)} \left\{ \frac{\bar{L}_N + M - \frac{a_N g}{1-\alpha}}{(\bar{L}_N + M - a_N g)(\bar{L}_N + M)} + \frac{1}{\bar{L}_S - M} \right\} > 0. \tag{E.7}$$

Then, using equations (E.3), (E.5), (E.6) and (E.7), we have

$$\frac{\partial g}{\partial m} = \frac{-\left(\Delta - \frac{1}{m} \frac{1}{\bar{L}_N + M - a_N g} \right)}{\Delta - \frac{(\bar{L}_N + M)}{(\bar{L}_N + M - a_N g)g} \frac{1}{\bar{L}_S - M}}. \tag{E.8}$$

Using equation (E.7) it can be shown that

$$\lim_{\rho \to 0} \Delta = 0. \tag{E.9}$$

Then, using equations (E.8) and (E.9), we have

$$\begin{aligned} \lim_{\rho \to 0} \left(\frac{\partial g}{\partial m} \right) &= \frac{\frac{1}{m} \frac{1}{\bar{L}_N + M - a_N g}}{-\frac{(\bar{L}_N + M)}{(\bar{L}_N + M - a_N g)g} \frac{1}{\bar{L}_S - M}} \\ &= -\frac{\bar{L}_S - M}{m(\bar{L}_N + M)} < 0. \end{aligned} \tag{E.10}$$

Again the numerator for the $\frac{\partial M}{\partial m}$ expression of equation (E.4) can be written as

$$\begin{aligned} &\left(\frac{-(\bar{L}_N + M)}{g(\bar{L}_N + M - a_N g)} + \frac{\rho}{(\rho + g + m)(g + m)} \right) \left(-\frac{1}{m} \right) + \frac{\rho}{(\rho + g + m)(g + m)} \frac{\frac{a_N g}{1-\alpha} - (\bar{L}_N + M)}{(\bar{L}_N + M - a_N g)g} \\ &= \frac{(\bar{L}_N + M)}{g(\bar{L}_N + M - a_N g)} \frac{1}{m} - \frac{\rho}{(\rho + g + m)(g + m)} \left(\frac{1}{m} + \frac{\bar{L}_N + M - \frac{a_N g}{1-\alpha}}{(\bar{L}_N + M - a_N g)g} \right). \end{aligned} \tag{E.11}$$

So, using equations (E.4), (E.5), (E.7) and (E.11), we have

$$\frac{\partial M}{\partial m} = \frac{\frac{(\bar{L}_N + M)}{g(\bar{L}_N + M - a_N g)} \frac{1}{m} - \frac{\rho}{(\rho + g + m)(g + m)} \left(\frac{1}{m} + \frac{\bar{L}_N + M - \frac{a_N g}{1-\alpha}}{(\bar{L}_N + M - a_N g)g} \right)}{\Delta - \frac{(\bar{L}_N + M)}{(\bar{L}_N + M - a_N g)g} \frac{1}{\bar{L}_S - M}}. \tag{E.12}$$

Using equations (E.9) and (E.12) we have

$$\lim_{\rho\to 0}\left(\frac{\partial M}{\partial m}\right) = \frac{\frac{(\bar{L}_N+M)}{g(\bar{L}_N+M-a_N g)}\frac{1}{m}}{-\frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}}$$
$$= -\frac{\bar{L}_S - M}{m} < 0. \tag{E.13}$$

Labour endowment change

Differentiating both sides of equations (E.1) and (E.2) with respect to $\bar{L}_N$ and arranging the terms we obtain

$$\begin{bmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g} + \frac{1}{g+m} - \frac{1}{g} - \frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\alpha\, a_N}{\bar{L}_N+M-a_N g} - \frac{1-\alpha}{g} & \frac{1}{\bar{L}_N+M} + \frac{1-\alpha}{\bar{L}_S-M} - \frac{\alpha}{\bar{L}_N+M-a_N g} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial g}{\partial \bar{L}_N} \\ \frac{\partial M}{\partial \bar{L}_N} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-1}{\bar{L}_N+M-a_N g} \\ \frac{\alpha}{\bar{L}_N+M-a_N g} - \frac{1}{\bar{L}_N+M} \end{bmatrix}.$$

Solving by Cramer's rule we obtain

$$\frac{\partial g}{\partial \bar{L}_N} = \frac{\begin{vmatrix} \frac{-1}{\bar{L}_N+M-a_N g} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\alpha}{\bar{L}_N+M-a_N g} - \frac{1}{\bar{L}_N+M} & \frac{1}{\bar{L}_N+M} + \frac{1-\alpha}{\bar{L}_S-M} - \frac{\alpha}{\bar{L}_N+M-a_N g} \end{vmatrix}}{|D_1|}; \tag{E.14}$$

and

$$\frac{\partial M}{\partial \bar{L}_N} = \frac{\begin{vmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g} + \frac{1}{g+m} - \frac{1}{g} - \frac{1}{\rho+g+m} & \frac{-1}{\bar{L}_N+M-a_N g} \\ \frac{\alpha\, a_N}{\bar{L}_N+M-a_N g} - \frac{1-\alpha}{g} & \frac{\alpha}{\bar{L}_N+M-a_N g} - \frac{1}{\bar{L}_N+M} \end{vmatrix}}{|D_1|}. \tag{E.15}$$

Here

$$|D_1| = \begin{vmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g} + \frac{1}{g+m} - \frac{1}{g} - \frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\alpha\, a_N}{\bar{L}_N+M-a_N g} - \frac{1-\alpha}{g} & \frac{1}{\bar{L}_N+M} + \frac{1-\alpha}{\bar{L}_S-M} - \frac{\alpha}{\bar{L}_N+M-a_N g} \end{vmatrix}$$

$$= (1-\alpha) \begin{vmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g} + \frac{1}{g+m} - \frac{1}{g} - \frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \frac{\frac{\alpha}{1-\alpha} a_N}{\bar{L}_N+M-a_N g} - \frac{1}{g} & \frac{\frac{1}{1-\alpha}}{\bar{L}_N+M} + \frac{1}{\bar{L}_S-M} - \frac{\frac{\alpha}{1-\alpha}}{\bar{L}_N+M-a_N g} \end{vmatrix}$$

$$= (1-\alpha)\,|D|\,.$$

So we obtain

$$|D_1| = (1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right) \tag{E.16}$$

where Δ is defined earlier (see equation (E.7) in Appendix 2.14).

Then, using equations (E.14) and (E.16), we have

$$\frac{\partial g}{\partial \bar{L}_N} = \frac{-\left(\frac{1}{\bar{L}_N+M-a_N g}\right)\left(\frac{1-\alpha}{\bar{L}_S-M}\right)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)}\,. \tag{E.17}$$

Again, using equations (E.15) and (E.16), we have

$$\frac{\partial M}{\partial \bar{L}_N} = \frac{\left(\frac{1}{m+g} - \frac{1}{\rho+m+g}\right)(-\Delta_1)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)}; \tag{E.18}$$

where

$$\Delta_1 = \frac{(1-\alpha)(\bar{L}_N+M) - a_N g}{(\bar{L}_N+M)(\bar{L}_N+M-a_N g)} > 0.$$

Again, differentiating both sides of equations (E.1) and (E.2) with respect to $\bar{L}_S$ and arranging the terms, we obtain

$$\begin{bmatrix} \frac{-a_N}{\bar{L}_N+M-a_N g} + \frac{1}{g+m} - \frac{1}{g} - \frac{1}{\rho+g+m} & \frac{1}{\bar{L}_N+M-a_N g} \\ \\ \frac{\alpha\, a_N}{\bar{L}_N+M-a_N g} - \frac{1-\alpha}{g} & \frac{1}{\bar{L}_N+M} + \frac{1-\alpha}{\bar{L}_S-M} - \frac{\alpha}{\bar{L}_N+M-a_N g} \end{bmatrix} . \begin{bmatrix} \frac{\partial g}{\partial \bar{L}_S} \\ \\ \frac{\partial M}{\partial \bar{L}_S} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \frac{1-\alpha}{\bar{L}_S-M} \end{bmatrix}.$$

Solving by Cramer's rule and using equation (E.16) we obtain

$$\frac{\partial g}{\partial \bar{L}_S} = \frac{-\left(\frac{1}{\bar{L}_N+M-a_N g}\right)\left(\frac{1-\alpha}{\bar{L}_S-M}\right)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)}; \qquad (E.19)$$

and

$$\frac{\partial M}{\partial \bar{L}_S} = \frac{\frac{1-\alpha}{\bar{L}_S-M}(\Delta_2)}{(1-\alpha)\left(\Delta - \frac{(\bar{L}_N+M)}{(\bar{L}_N+M-a_N g)g}\frac{1}{\bar{L}_S-M}\right)}; \qquad (E.20)$$

where

$$\Delta_2 = \left(\frac{1}{m+g} - \frac{1}{\rho+m+g} - \frac{a_N}{\bar{L}_N+M)-a_N g} - \frac{1}{g}\right) < 0.$$

Appendix 2.15

We write the L.H.S. of equation (2.3.17) as

$$L.H.S._{(17)} = (1-\alpha)g^{1-\alpha}$$

and the R.H.S. of equation (2.3.17) as

$$R.H.S._{(17)} = f_1(g).f_2(g) + f_1(g).f_3(g)$$

where

$$\begin{aligned} f_1(g) &= g, \\ f_2(g) &= \frac{(r+m)}{(g+m)}\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g}, \end{aligned}$$

and

$$f_3(g) = \frac{(r+m)}{(g+m)}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\left(\frac{m}{L_S-(1-\beta)a_S g}\right)^{1-\alpha}.$$

Then differentiating both the $L.H.S._{(17)}$ and $R.H.S._{(17)}$ with respaect to g we get

$$\frac{dL.H.S._{(17)}}{dg} = (1-\alpha)^2 g^{-\alpha} \tag{2E.1}$$

and

$$\frac{dR.H.S._{(17)}}{dg} = [f_1'(g).f_2(g) + f_1(g).f_2'(g)] + [f_1'(g).f_3(g) + f_1(g).f_3'(g)]. \tag{2E.2}$$

Note that $R.H.S._{(17)}$ is an increasing function of g because

$$f_1(g).f_2(g) = (\frac{r+m}{\frac{g+m}{g}}).(\frac{a_N \alpha \beta g^{1-\alpha}}{L_N - \beta a_N g})$$

is an increasing function of g and

$$f_1(g).f_3(g) = [\frac{r+m}{\frac{g+m}{g}}].[\frac{(1-\beta)a_S}{(L_N - \beta a_N g)^\alpha}(\frac{m}{L_S - (1-\beta)a_S g})^{1-\alpha}]$$

is also an increasing function of g.

We have $f_1'(g) = 1$. Also, as $g \to 0$, we find that $f_1(g) \to 0$, $f_2(g) \to 0$, and $f_3(g) \to \frac{\rho+m}{m}\frac{(1-\beta)a_S}{L_N^\alpha}(\frac{m}{L_S})^{1-\alpha}$. Thus from equation (2E.2) we get

$$\lim_{g\to 0^+} \frac{dR.H.S._{(17)}}{dg} = \frac{\rho+m}{m}\frac{(1-\beta)a_S}{L_N^\alpha}(\frac{m}{L_S})^{1-\alpha}. \tag{2E.3}$$

From equation (2E.1) it is clear that

$$\lim_{g\to 0^+} \frac{dL.H.S._{(17)}}{dg} = \infty. \tag{2E.4}$$

Comparing equations (2E.3) and (2E.4) we obtain

$$\lim_{g\to 0^+} \frac{d(L.H.S._{(17)})}{dg} > \lim_{g\to 0^+} \frac{d(R.H.S._{(17)})}{dg}.$$

Curvature

We are to prove that the R.H.S. of equation (2.3.17) is convex to the origin. We proceed as follows

$$R.H.S._{(17)} = f_1(g).[f_2(g) + f_3(g)],$$

or,

$$\frac{R.H.S._{(17)}}{g} = f_2(g)+f_3(g) = \frac{(r+m)}{(g+m)}\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g}+\frac{(r+m)}{(g+m)}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}(\frac{m}{L_S-(1-\beta)a_S g})^{1-\alpha},$$

or,

$$\frac{R.H.S._{(17)}}{g} = \left[g.\frac{(r+m)}{(g+m)}\right]\left[\frac{(a_N\alpha\beta)g^{-\alpha}}{L_N-\beta a_N g}+\frac{1}{(L_S-(1-\beta)a_S g)g}(\frac{L_S-(1-\beta)a_S g}{L_N-\beta a_N g})^{\alpha}\frac{(1-\beta)a_S m}{m^{\alpha}}\right]. \quad (2E.5)$$

$R.H.S._{(17)}$ is an increasing function of g; and $R.H.S._{(17)} = 0$ when $g = 0$. So the $R.H.S._{(17)}$ is a convex function of g if $\frac{R.H.S._{(17)}}{g}$ is an increasing function of g.

The term in the first third bracket in the R.H.S. of equation (2E.5) is $g.\frac{(r+m)}{g+m}$; and this is clearly an increasing function of g.

The first term in the second third bracket in the R.H.S. of equation (2E.5) is $\frac{(a_N\alpha\beta)g^{-\alpha}}{L_N-\beta a_N g}$, and this is an increasing function of g if $1 < \frac{L_N}{\beta a_N g} \leq 2$.

The second term in the second third bracket in the R.H.S. of equation (2E.5) is $\frac{1}{(L_S-(1-\beta)a_S g)g}(\frac{L_S-(1-\beta}{L_N-\beta a}$ The first term of this expression is $\frac{1}{(L_S-(1-\beta)a_S g)g}$, and this is an increasing function of g if $1 < \frac{L_S}{(1-\beta)a_S g} < 2$. The second term of this expression is $(\frac{L_S-(1-\beta)a_S g}{L_N-\beta a_N g})^{\alpha}$, and this is clearly an increasing function of g if $\frac{L_S}{(1-\beta)a_S} > \frac{L_N}{\beta a_N}$. The last term, $\frac{(1-\beta)a_S m}{m^{\alpha}}$, does not depend on g.

So, $\frac{R.H.S._{(17)}}{g}$ is an increasing function of g if $1 < \frac{L_N}{\beta a_N g} < \frac{L_S}{(1-\beta)a_S g} < 2$. Hence the R.H.S. of equation (2.3.17) is a convex function of g in this case.

Appendix 2.16

comparative static with respect to m

The R.H.S. of equation (2.3.17) is

$$R.H.S._{(17)} = g\frac{(r+m)}{(g+m)}\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g} + g\frac{(r+m)}{(g+m)}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\left(\frac{m}{L_S-(1-\beta)a_S g}\right)^{1-\alpha}$$

$$= g\frac{(r+m)}{(g+m)}\left[\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g} + \frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\frac{m^{1-\alpha}}{(L_S-(1-\beta)a_S g)^{1-\alpha}}\right]. \tag{3E.1}$$

Differentiating this with respect to m we have

$$\frac{\partial R.H.S._{(17)}}{\partial m} = g\frac{-\rho}{(g+m)^2}\left[\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g} + \frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\frac{m^{1-\alpha}}{(L_S-(1-\beta)a_S g)^{1-\alpha}}\right]$$

$$+ g\frac{(r+m)}{(g+m)}\left[(1-\alpha)m^{-\alpha}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\frac{1}{(L_S-(1-\beta)a_S g)^{1-\alpha}}\right]. \tag{3E.2}$$

For $\alpha \to 0$ we get from equation (3E.2),

$$\lim_{\alpha\to 0}\frac{\partial R.H.S._{(17)}}{\partial m} = \frac{-\rho g}{(g+m)^2}\left[\frac{(1-\beta)a_S m}{L_S-(1-\beta)a_S g}\right] + g\frac{(r+m)}{(g+m)}\left[\frac{(1-\beta)a_S}{L_S-(1-\beta)a_S g}\right]$$

$$= \frac{g}{g+m}\ \frac{(1-\beta)a_S}{L_S-(1-\beta)a_S g}\ \left[\frac{-\rho m}{g+m} + (r+m)\right]$$

$$= \frac{g}{g+m}\ \frac{(1-\beta)a_S}{L_S-(1-\beta)a_S g}\ \left[\frac{\rho g + (g+m)^2}{g+m}\right] > 0. \qquad [since\ \ r = \rho + g]$$

So, a decrease in m causes the R.H.S. curve of equation (2.3.17) to shift downward when $\alpha \to 0$.

For $\rho \to 0$ we get from equation (3E.2),

$$\lim_{\rho\to 0}\frac{\partial R.H.S._{(17)}}{\partial m} = g\left[(1-\alpha)m^{-\alpha}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^{\alpha}}\frac{1}{(L_S-(1-\beta)a_S g)^{1-\alpha}}\right] > 0. \ \ [since\ \ r = \rho + g]$$

So, a decrease in m causes the R.H.S. curve of equation (2.3.17) to shift downward when $\rho \to 0$.

For $m \to 0$ to start with, we get from equation (3E.2),

$$\lim_{m\to 0}\frac{\partial R.H.S._{(17)}}{\partial m} = \lim_{m\to 0}[\frac{-\rho g}{(g+m)^2}\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g}]+\lim_{m\to 0}[\frac{-\rho g}{(g+m)^2}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^\alpha}\frac{m^{1-\alpha}}{(L_S-(1-\beta)a_S g)^{1-\alpha}}$$

$$+\lim_{m\to 0}[g\frac{(r+m)}{(g+m)}[(1-\alpha)m^{-\alpha}\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^\alpha}\frac{1}{(L_S-(1-\beta)a_S g)^{1-\alpha}}],$$

or,

$$\lim_{m\to 0}\frac{\partial R.H.S._{(17)}}{\partial m} = [\frac{-\rho g}{g^2}\frac{a_N\alpha\beta g^{1-\alpha}}{L_N-\beta a_N g}]+\frac{r}{g}[(1-\alpha)\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^\alpha}\frac{1}{(L_S-(1-\beta)a_S g)^{1-\alpha}}]\{\lim_{m\to 0}(gm^{-\alpha})\}$$

We have

$$\lim_{m\to 0}(gm^{-\alpha}) = \lim_{m\to 0}(\frac{g}{m}.m^{1-\alpha}) = \lim_{m\to 0}(\frac{n_N}{n_S}.m^{1-\alpha}) = \frac{n_N}{n_S}.\lim_{m\to 0}(m^{1-\alpha}) = 0.$$

This holds because in the steady-state $\frac{n_N}{n_S}$ is constant. So we get

$$\lim_{m\to 0}\frac{\partial R.H.S._{(17)}}{\partial m} = \frac{-\rho g}{g^2}\frac{a_N\alpha\beta g^{1-\alpha}}{L_N-\beta a_N g} < 0.$$

So, when the rate of imitation is close to zero, a decrease in m causes the R.H.S. curve of equation (2.3.17) to shift upward.

comparative static with respect to β

We write equation (3E.1) as follows

$$R.H.S._{(17)} = g\frac{(r+m)}{(g+m)}[\frac{(a_N\alpha\beta)g^{1-\alpha}}{L_N-\beta a_N g}+\frac{(1-\beta)a_S}{(L_N-\beta a_N g)^\alpha}\frac{m^{1-\alpha}}{(L_S-(1-\beta)a_S g)^{1-\alpha}}]$$

$$= g\frac{(r+m)}{(g+m)}[\alpha f_1(\beta)+m^{1-\alpha}f_2(\beta)\{f_3(\beta)\}^\alpha] \qquad (3E.3)$$

where

$$f_1(\beta) = \frac{(a_N\beta)g^{1-\alpha}}{L_N-\beta a_N g},$$

$$f_2(\beta) = \frac{(1-\beta)a_S}{(L_S-(1-\beta)a_S g)},$$

and

$$f_3(\beta) = \frac{L_S - (1-\beta)a_S g}{L_N - \beta a_N g}.$$

Clearly $f_1'(\beta) > 0$, $f_2'(\beta) < 0$ and $f_3'(\beta) > 0$.

Differentiating equation (3E.3) with respect to β we get,

$$\frac{\partial R.H.S._{(17)}}{\partial \beta} = g\frac{(r+m)}{(g+m)}[\alpha f_1'(\beta) + m^{1-\alpha} f_2'(\beta)\{f_3(\beta)\}^{\alpha} + m^{1-\alpha} f_2(\beta)\alpha\{f_3(\beta)\}^{\alpha-1} f_3'(\beta)] \tag{3E.4}$$

So we have,

$$\lim_{\alpha\to 0} \frac{\partial R.H.S._{(17)}}{\partial \beta} = g\frac{(r+m)}{(g+m)}[m f_2'(\beta)] < 0$$

and

$$\lim_{m\to 0} \frac{\partial R.H.S._{(17)}}{\partial \beta} = r[\alpha f_1'(\beta)] > 0.$$

So, a decrease in β leads to an upward shift of the R.H.S. curve of equation (2.3.17) when $\alpha \to 0$ and a downward shift of the R.H.S. curve of equation (2.3.17) when $m \to 0$.

Appendix 2.17

The R.H.S. of equation (2.3.18) is

$$R.H.S._{(18)} = \frac{(r+m)g}{(g+m)} \frac{\alpha}{1-\alpha} \frac{a_N}{L_N - a_N g}. \tag{4E.1}$$

This is an increasing function of g. To prove that it is convex to the origin we need to show that

$$\frac{d(\frac{R.H.S._{(18)}}{g})}{dg} > 0.$$

From equation (4E.1) we have

$$\frac{R.H.S._{(18)}}{g} = \frac{(r+m)g}{(g+m)} \frac{a_N}{g(L_N - a_N g)} \frac{\alpha}{1-\alpha}. \tag{4E.2}$$

Note that $\frac{(r+m)g}{(g+m)}$ is an increasing function of g. $\frac{a_N}{g(L_N - a_N g)}$ is also an increasing function of g if $1 < \frac{L_N}{a_N g} < 2$. So, the R.H.S. curve of equation (2.3.18) is convex to the origin if

$1 < \frac{L_N}{a_N g} < 2$ is satisfied. When $L_N = a_N g$, $R.H.S._{(18)}$ is infinitely large. So this curve is asymptotic to the $g = \frac{L_N}{a_N}$ vertical straight line in the figure 2.3.2.

Appendix 2.18

Stability analysis in the case of R&D outsoutrcing.

Using $\gamma = 1$ and equations (2.3.5), (2.3.7) we have

$$L_N = n_N x_N + \beta a_N g \ . \tag{5E.1}$$

Similarly using equations (2.3.6), (2.3.8) we have

$$L_S = n_S x_S + (1-\beta) a_S g \ . \tag{5E.2}$$

Then using equations (2.3.3) and (2.3.10) and $\gamma = 1$ we have

$$p_N = \frac{w_N}{\alpha} \ , \tag{5E.3}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N . \tag{5E.4}$$

We express equation (2.3.11) as follows

$$\frac{n \ v_N}{w_N} = \beta a_N + (1-\beta) a_S \left(\frac{w_S}{w_N}\right) . \tag{5E.5}$$

We define

$$k = \frac{w_S}{w_N} \ , \tag{5E.6}$$

$$\xi = \frac{n_N}{n} \ , \tag{5E.7}$$

and

$$g = \frac{\dot{n}}{n} \ . \tag{5E.8}$$

Then using equations (5E.3), (5E.6) and equation (5E.5) we have

$$\frac{n \ v_N}{\alpha p_N} = \beta a_N + (1-\beta) a_S k \ ;$$

and differentiating both sides with respect to time, we have

$$\frac{\dot{n}}{n} + \frac{\dot{v_N}}{v_N} - \frac{\dot{p_N}}{p_N} = \frac{(1-\beta)a_S\,\dot{k}}{\beta a_N + (1-\beta)a_S k}.$$

Using equations (2.3.9), (5E.8) and the above mentioned equation, we have

$$\frac{\dot{p_N}}{p_N} = r_N + m + g - \frac{\pi_N}{v_N} - \frac{(1-\beta)a_S\,\dot{k}}{\beta a_N + (1-\beta)a_S k}.$$

Using equations (2.3.11), (5E.4), (5E.1) and (5E.7) we can modify the above equation as follows

$$\frac{\dot{p_N}}{p_N} = r_N + m + g - \frac{1-\alpha}{\alpha}\frac{L_N - \beta a_N g}{\beta a_N + (1-\beta)a_S k}\,\frac{1}{\xi} - \frac{(1-\beta)a_S\,\dot{k}}{\beta a_N + (1-\beta)a_S k}. \qquad (5E.9)$$

Using equation (2.3.2) we get

$$\frac{x_N}{x_S} = \left(\frac{p_N}{p_S}\right)^{-\varepsilon}$$

and then using equations (5E.3), (2.3.4) and (5E.6) we have

$$\frac{x_N}{x_S} = (\alpha\;k)^{\varepsilon}\;.$$

Using equations (5E.1), (5E.2) and (5E.7) the above equation can be written as

$$(\alpha\;k)^{\varepsilon} = \frac{L_N - \beta a_N g}{L_S - (1-\beta)a_S g}\,\frac{1-\xi}{\xi}. \qquad (5E.9.1)$$

Taking time derivative on both sides of the above equation we have

$$\dot{k} = \;-\frac{\beta a_N \dot{g}}{L_N - \beta a_N g}\,\frac{k}{\varepsilon} \;+\frac{(1-\beta)a_S \dot{g}}{L_S - (1-\beta)a_S g}\,\frac{k}{\varepsilon} \;-\frac{\dot{\xi}}{\xi(1-\xi)}\,\frac{k}{\varepsilon}. \qquad (5E.10)$$

The balanced budget condition for the North is

$$E_N = p_N n_N x_N.$$

Using equation (5E.1) the above equation can be written as

$$\frac{E_N}{p_N} = L_N - \beta a_N g\;.$$

Taking time derivative on both sides of the above equation we have

$$\frac{\dot{E_N}}{E_N} - \frac{\dot{p_N}}{p_N} \;=\; -\frac{\beta a_N \dot{g}}{L_N - \beta a_N g} \;.$$

Using equations (2.3.1) and (5E.9), the above equation implies that

$$r_N - \rho - \left(r_N + m + g - \frac{1-\alpha}{\alpha} \frac{L_N - \beta a_N g}{\beta a_N + (1-\beta) a_S k} \; \frac{1}{\xi} - \frac{(1-\beta) a_S \; \dot{k}}{\beta a_N + (1-\beta) a_S k} \right) \;=\; -\frac{\beta a_N \dot{g}}{L_N - \beta a_N g} \;.$$

This implies that

$$\frac{\beta a_N \dot{g}}{L_N - \beta a_N g} + \frac{(1-\beta) a_S \; \dot{k}}{\beta a_N + (1-\beta) a_S k} = \rho + m + g - \frac{1-\alpha}{\alpha} \frac{L_N - \beta a_N g}{\beta a_N + (1-\beta) a_S k} \; \frac{1}{\xi} \;. \quad (5E.11)$$

Again from equation (5E.7) we obtain the following

$$\begin{aligned} \frac{\dot{\xi}}{\xi} &= \frac{\dot{n_N}}{n_N} - \frac{\dot{n}}{n} \\ &= \frac{\dot{n} - \dot{n_S}}{n_N} - \frac{\dot{n}}{n} \quad \text{[using the fact that } n = n_N + n_S\text{]} \\ &= \frac{\dot{n}}{n} \frac{n}{n_N} - \frac{\dot{n_S}}{n_N} - \frac{\dot{n}}{n} \\ &= \frac{g}{\xi} - m - g \quad \text{[using equations (5E.7), (5E.8) and the fact that } m = \tfrac{\dot{n_S}}{n_N}\text{]} \end{aligned}$$

This implies that

$$\dot{\xi} = g - (g+m)\xi \;. \qquad (5E.12)$$

Using equations (5E.10) and (5E.12), we write equation (5E.11) as follows

$$\dot{g} \left[\frac{\beta a_N}{L_N - \beta a_N g} - \frac{\beta a_N (1-\beta) a_S}{\{\beta a_N + (1-\beta) a_S k\}\{L_S - (1-\beta) a_S g\}} \; \frac{k}{\varepsilon} + \frac{(1-\beta)^2 a_S^2}{\{\beta a_N + (1-\beta) a_S k\}\{L_S - (1-\beta) a_S} \right.$$

$$= \rho + m + g + \frac{(1-\beta) a_S}{\beta a_N + (1-\beta) a_S k} \; \left(\frac{g - (g+m)\xi}{\xi(1-\xi)} \right) \; \frac{k}{\varepsilon} - \frac{1-\alpha}{\alpha} \frac{L_N - \beta a_N g}{\beta a_N + (1-\beta) a_S k} \; \frac{1}{\xi} \;.$$

(5E.13)

Equations (5E.12) and (5E.13) are two dynamic equations to be used to analyse the local stability of the steady-state growth equilibrium of the model. Linearising equations (5E.12) and (5E.13) around their steady-state equilibrium values we get

$$\begin{bmatrix} \dot{\xi} \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*, g^*)} & \frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*, g^*)} \\ \frac{\partial \dot{g}}{\partial \xi}]_{(\xi^*, g^*)} & \frac{\partial \dot{g}}{\partial g}]_{(\xi^*, g^*)} \end{bmatrix} . \begin{bmatrix} \xi(t) - \xi^* \\ g(t) - g^* \end{bmatrix}$$

We have

$$\frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)} = -(g+m) < 0 \ , \tag{5E.14}$$

and

$$\begin{aligned}\frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*,g^*)} &= 1-\xi \\ &= \frac{m}{g+m} \ . \end{aligned} \tag{5E.15}$$

Then from equations (5E.14) and (5E.15) we obtain

$$\lim_{m\to 0}\left(\frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)}\right) = -g < 0 \ ;$$

and

$$\lim_{m\to 0}\left(\frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*,g^*)}\right) = 0 \ .$$

Also,

$$\frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)}\left[\frac{\beta a_N}{L_N-\beta a_N g} - \frac{\beta a_N(1-\beta)a_S}{\{\beta a_N+(1-\beta)a_S k\}\{L_S-(1-\beta)a_S g\}}\ \frac{k}{\varepsilon} + \frac{(1-\beta)^2 a_S^2}{\{\beta a_N+(1-\beta)a_S k\}\{L_S-(1-\beta)a_S g\}}\ \frac{k}{\varepsilon}\right]$$

$$= \quad 1+\frac{(1-\beta)a_S}{\beta a_N+(1-\beta)a_S k}\ \left(\frac{1}{\xi}\right)\ \frac{k}{\varepsilon}\ -$$

$$\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left[\frac{(-\beta a_N)\{\beta a_N+(1-\beta)a_S k\}-(L_N-\beta a_N g)(1-\beta)a_S\left(\frac{\partial k}{\partial g}\right)}{\{\beta a_N+(1-\beta)a_S k\}^2}\right] \ .$$

From equation (5E.9.1) we find that as $m\to 0$, $k\to 0$. Also $\frac{\partial k}{\partial g}\to 0$ as $m\to 0$. Then the above equation implies that

$$\lim_{m\to 0}\left(\frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)}\right)\left[\frac{\beta a_N}{L_N-\beta a_N g}\right] = 1+\frac{1-\alpha}{\alpha} = \frac{1}{\alpha}$$

or,

$$\lim_{m\to 0}\left(\frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)}\right) = \frac{\frac{1}{\alpha}}{\frac{\beta a_N}{L_N-\beta a_N g}} \ > 0 \ .$$

So the Jacobian is a 2×2 triangular matrix with one positive diagonal term and one negative diagonal term. So its one latent root is positive and the other is negative. This implies that the equilibrium is a saddle point when m is very small. Helpman(1993) also analyses the comparative stady-state properties in his model in the limiting case of $m \to 0$.

Appendix 2.19

Stability analysis in the case of production outsourcing.

In this case we have $\beta = 1$. Then from equations (2.3.5) and (2.3.7) we have

$$L_N = \gamma n_N x_N + a_N g \; ; \qquad (6E.1)$$

and from equation (2.3.6) we have

$$L_S = (1-\gamma) n_N x_N + n_S x_S \; . \qquad (6E.2)$$

Similarly from equation (2.3.11), we have

$$v_N = \frac{a_N}{n} w_N \; . \qquad (6E.3)$$

Using equations (6E.1) and (6E.2) we obtain

$$\frac{n_S x_S}{n_N x_N} = \frac{\gamma L_S - (1-\gamma)(L_N - a_N g)}{L_N - a_N g} \; .$$

Using equations (2.3.2) and (5E.7) we can express the above equation as follows

$$\frac{1-\xi}{\xi} \left(\frac{p_N}{p_S} \right)^{\varepsilon} = \frac{\gamma L_S - (1-\gamma)(L_N - a_N g)}{L_N - a_N g} \; .$$

Then using equations (2.3.3), (2.3.4) and (5E.6) we write the above equation as follows

$$\left(\frac{\gamma + (1-\gamma)k}{\alpha k} \right)^{\varepsilon} = \frac{\xi}{1-\xi} \; \frac{\gamma L_S - (1-\gamma)(L_N - a_N g)}{L_N - a_N g} \; ; \qquad (6E.4)$$

and differentiating both sides with respect to time, we have

$$\frac{\dot{k}}{\gamma + (1-\gamma)k} = -\frac{k}{\gamma \xi} \; \frac{\dot{\xi}}{\xi(1-\xi)} - \frac{k}{\gamma \xi} \; \frac{(a_N \dot{g}) \;\; \gamma L_S}{\{\gamma L_S - (1-\gamma)(L_N - a_N g)\}(L_N - a_N g)} \; . \quad (6E.5)$$

From equations (2.3.3), (6E.3) and (5E.6) we have

$$\frac{p_N\ \alpha\ a_N}{n\ v_N} = \gamma + (1-\gamma)k\ .$$

Taking time derivative on both sides of the above equation we have

$$\frac{\dot{p_N}}{p_N} - \frac{\dot{n}}{n} - \frac{\dot{v_N}}{v_N} \ = \ \frac{(1-\gamma)\dot{k}}{\gamma + (1-\gamma)k}\ .$$

Then using equations (5E.8) and (2.3.9) we write the above equation as follows

$$\frac{\dot{p_N}}{p_N} = g + r_N + m - \frac{\pi_N}{v_N} + \frac{(1-\gamma)\dot{k}}{\gamma + (1-\gamma)k}\ . \tag{6E.6}$$

The balanced budget equation for the North is given by

$$E_N = p_N n_N x_N\ .$$

Using equation (6E.1), we write the above equation as follows

$$\frac{E_N}{p_N} = \frac{L_N - a_n g}{\gamma}\ .$$

and then differentiating both sides with respect to time we have

$$\frac{\dot{E_N}}{E_N} - \frac{\dot{p_N}}{p_N} = \frac{-a_N \dot{g}}{L_N - a_N g}\ .$$

Using equations (2.3.1) and (6E.6) the above equation can be expressed as follows

$$r_N - \rho - \left(g + r_N + m - \frac{\pi_N}{v_N} + \frac{(1-\gamma)\dot{k}}{\gamma + (1-\gamma)k} \right) = \frac{-a_N \dot{g}}{L_N - a_N g}\ ;$$

and then using equations (2.3.10), (6E.3), (6E.1), (5E.6), (5E.7) and (6E.5), the above equation can finally be expressed as

$$\dot{g} \left(\frac{-a_N}{L_N - a_N g} \right) \left[1 + \frac{(1-\alpha)(1-\gamma)kL_S}{\gamma L_S - (1-\gamma)(L_N - a_N g)} \right] = \frac{k(1-\alpha)(1-\gamma)}{\gamma} \frac{\dot{\xi}}{\xi(1-\xi)}$$

$$+ \frac{1-\alpha}{\alpha\ a_N} \left(\frac{\gamma + (1-\gamma)k}{\gamma} \right) \frac{L_N - a_N g}{\xi} - (\rho + m + g)\ . \tag{6E.7}$$

The other dynamic equation obtained from equation (5E.7) is

$$\dot{\xi} = g - (g+m)\xi \ . \tag{6E.8}$$

Equations (6E.7) and (6E.8) are two dynamic equations to be used to analyse the local stability of the steady-state growth equilibrium of the model. Linearising equations (6E.7) and (6E.8) around their steady-state equilibrium values we get

$$\begin{bmatrix} \dot{\xi} \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)} & \frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*,g^*)} \\ \frac{\partial \dot{g}}{\partial \xi}]_{(\xi^*,g^*)} & \frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)} \end{bmatrix} . \begin{bmatrix} \xi(t) - \xi^* \\ g(t) - g^* \end{bmatrix}$$

We have

$$\frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)} = -(g+m) < 0 \ , \tag{6E.9}$$

and

$$\begin{aligned} \frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*,g^*)} &= 1 - \xi \\ &= \frac{m}{g+m} \ . \end{aligned} \tag{6E.10}$$

Then from equations (6E.9) and (6E.10) we obtain

$$\lim_{m\to 0} \left(\frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)} \right) = -g < 0 \ ;$$

and

$$\lim_{m\to 0} \left(\frac{\partial \dot{\xi}}{\partial g}]_{(\xi^*,g^*)} \right) = 0 \ .$$

Also we obtain,

$$\begin{aligned} \frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)} \left(\frac{-a_N}{L_N - a_N g} \right) \left[1 + \frac{(1-\alpha)(1-\gamma)kL_S}{\gamma L_S - (1-\gamma)(L_N - a_N g)} \right] &= \frac{k(1-\alpha)(1-\gamma)}{\gamma} \frac{[\frac{\partial \dot{\xi}}{\partial \xi}]_{(\xi^*,g^*)}}{\xi(1-\xi)} \\ + \left[\frac{1-\alpha}{\alpha \ a_N} \ \frac{1-\gamma}{\gamma} \ \left(\frac{\partial k}{\partial g} \right) \ \frac{L_N - a_N g}{\xi} + \frac{1-\alpha}{\alpha \ a_N} \ \frac{\gamma + (1-\gamma)k}{\gamma} \left(\frac{-a_N}{\xi} \right) \right] - 1 \ . \end{aligned}$$

From equation (6E.4) we find that as $m \to 0$, $k \to 0$. Also $\frac{\partial k}{\partial g} \to 0$ as $m \to 0$. Then the above equation implies that

$$\begin{aligned} \lim_{m\to 0} \left(\frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)} \right) \left(\frac{-a_N}{L_N - a_N g} \right) &= -\frac{1-\alpha}{\alpha} - 1 \\ &= -\frac{1}{\alpha} \end{aligned}$$

or,

$$\lim_{m\to 0}\left(\frac{\partial \dot{g}}{\partial g}]_{(\xi^*,g^*)}\right) = \frac{L_N - a_N g}{\alpha a_N} \;\; > 0 \; .$$

So the Jacobian is a 2×2 triangular matrix with one positive diagonal term and one negative diagonal term. So its one latent root is positive and the other is negative. This implies that the equilibrium is a saddle point when m is very small. Helpman(1993) also analyses the comparative stady-state properties in his model in the limiting case of $m \to 0$.

Chapter 3

Appendix 3.1

Differentiating equation (3.1.15) with respect to g we have,

$$\frac{\partial \dot{g}}{\partial g} = (-1)\underbrace{\left[\rho + m + g - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi}\right]}_{term\ 1} + \underbrace{\left(\frac{L_N}{a_N} - g\right)\left[1 + \frac{1-\alpha}{\alpha}\frac{1}{\xi}\right]}_{term\ 2}.$$

Using equation (3.1.15.1) we find that *term* 1 is equal to zero at the steady state equilibrium point. Again, using equation (3.1.15.1), it can be shown that

$$\left[\frac{\partial \dot{g}}{\partial g}\right]_{(m^*,\xi^*,g^*)} = \ term\ 2 = \left(\frac{L_N}{a_N} - g\right) + \rho + m + g. \qquad (A1)$$

Differentiating equation (3.1.15) with respect to m we have,

$$\left[\frac{\partial \dot{g}}{\partial m}\right]_{(m^*,\xi^*,g^*)} = \left(\frac{L_N}{a_N} - g\right)\ . \qquad (A2)$$

Differentiating equation (15) with respect to ξ and using equations (3.1.15.1) and (3.1.28.1) we have, at the steady state equilibrium point,

$$\left[\frac{\partial \dot{g}}{\partial \xi}\right]_{(m^*,\xi^*,g^*)} = \left(\frac{L_N}{a_N} - g\right)[\rho + m + g]\ \frac{g+m}{g}. \qquad (A3)$$

Next, differentiating equation (3.1.29) with respect to m, we have

$$\frac{\partial \dot{m}}{\partial m} = \frac{1-\xi}{\xi}\left(-\frac{\xi}{1-\xi}\right)\underbrace{\left[\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right]}_{term\ 1} + \frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)$$
$$\left[\frac{\xi}{1-\xi} + \frac{1-\alpha}{\alpha}\frac{\xi}{1-\xi}\right] - \frac{1}{\xi(1-\xi)}\underbrace{(g - (g+m)\xi)}_{term\ 2} - \frac{m}{\xi(1-\xi)}(-\xi).$$

Using equation (3.1.29.1) we find that *term* 1 is equal to zero; and using equation (3.1.28.1) we find *term* 2 to be equal to zero. So we have

$$\begin{aligned}\left[\frac{\partial \dot{m}}{\partial m}\right]_{(m^*,\xi^*,g^*)} &= \frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\left[\frac{\xi}{1-\xi} + \frac{1-\alpha}{\alpha}\frac{\xi}{1-\xi}\right] + \frac{m}{(1-\xi)} \\ &= \left(\frac{L_S}{a_S} - g\right)\frac{1}{\alpha} + g + m\ .\ \text{[Using equation (3.1.28.1)]}\end{aligned}$$

Using equations (3.1.28.1) and (3.1.29.1) we have:

$$\rho + g = \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - g\right) .$$

Then we have

$$\begin{aligned}\left[\frac{\partial \dot{m}}{\partial m}\right]_{(m^*,\xi^*,g^*)} &= \left(\frac{L_S}{a_S} - g\right)\frac{1}{\alpha} + g + m \\ &= \left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m - \rho. \qquad (A4)\end{aligned}$$

Differentiating equation (3.1.29) with respect to ξ, we have

$$\frac{\partial \dot{m}}{\partial \xi} = \frac{\partial\left\{\frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right\}}{\partial \xi}.\underbrace{\left[\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right]}_{term\ 1} + \frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)$$

$$\underbrace{\frac{\partial\left[\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right]}{\partial \xi}}_{term\ 2} - \frac{\partial\left\{\frac{m}{\xi(1-\xi)}\right\}}{\partial \xi}.\underbrace{\{g-(g+m)\xi\}}_{term\ 3} - \frac{m}{\xi(1-\xi)}.\underbrace{\frac{\partial\{g-(g+m)\xi\}}{\partial \xi}}_{term\ 4}.$$

From equation (3.1.29.1) we find *term* 1 to be zero and from equation (3.1.28.1) we find *term* 3 to be zero. *term* 2 can be written as follows:

$$\frac{\partial\left[\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\right]}{\partial \xi} = \frac{1}{\alpha}\frac{m}{(1-\xi)^2}.$$

term 4 can be written as follows:

$$\frac{\partial\{g-(g+m)\xi\}}{\partial \xi} = -(g+m).$$

So, we have

$$\begin{aligned}\left[\frac{\partial \dot{m}}{\partial \xi}\right]_{(m^*,\xi^*,g^*)} &= \frac{1-\xi}{\xi}\left(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}\right)\frac{1}{\alpha}\frac{m}{(1-\xi)^2} + \frac{m}{\xi(1-\xi)}(g+m) \\ &= \left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m - \rho\right]\frac{(g+m)^2}{g} \text{ .[Using equation (3.1.28.1)]}\end{aligned}$$

(A5)

Differentiating equation (3.1.29) with respect to g and using equation (3.1.28.1) we have

$$\left[\frac{\partial \dot{m}}{\partial g}\right]_{(m^*,\xi^*,g^*)} = -\frac{m(g+m)}{g}. \tag{A6}$$

Next, differentiating equation (3.1.28) with respect to g, ξ and m respectively and using equation (3.1.28.1) we have

$$\left[\frac{\partial \dot{\xi}}{\partial g}\right]_{(m^*,\xi^*,g^*)} = \frac{m}{g+m}\ ; \tag{A7}$$

$$\left[\frac{\partial \dot{\xi}}{\partial \xi}\right]_{(m^*,\xi^*,g^*)} = -(g+m)\ ; \tag{A8}$$

and

$$\left[\frac{\partial \dot{\xi}}{\partial m}\right]_{(m^*,\xi^*,g^*)} = -\frac{g}{g+m}\ . \tag{A9}$$

$$\begin{aligned}
Tr(A) &= \left[\frac{\partial \dot{m}}{\partial m}\right]_{(m^*,\xi^*,g^*)} + \left[\frac{\partial \dot{\xi}}{\partial \xi}\right]_{(m^*,\xi^*,g^*)} + \left[\frac{\partial \dot{g}}{\partial g}\right]_{(m^*,\xi^*,g^*)} \\
&= \left[\left(\frac{L_S}{a_S}-g\right)\left(1+2\frac{1-\alpha}{\alpha}\right)+m-\rho\right] + [-(g+m)] + \left[\left(\frac{L_N}{a_N}-g\right)+\rho+m+g\right] \\
&\qquad \text{[Using equations (A1), (A4) and (A8).]} \\
&= \left(\frac{L_S}{a_S}-g\right)\left(1+2\frac{1-\alpha}{\alpha}\right)+m+\left(\frac{L_N}{a_N}-g\right) \qquad \text{(A10)}
\end{aligned}$$

$$Det(A) = \frac{\partial \dot{m}}{\partial m}\left[\underbrace{\frac{\partial \dot{\xi}}{\partial \xi}.\frac{\partial \dot{g}}{\partial g}}_{term\ 1} - \underbrace{\frac{\partial \dot{\xi}}{\partial g}.\frac{\partial \dot{g}}{\partial \xi}}_{term\ 2}\right] - \frac{\partial \dot{m}}{\partial \xi}\left[\underbrace{\frac{\partial \dot{\xi}}{\partial m}.\frac{\partial \dot{g}}{\partial g}}_{term\ 3} - \underbrace{\frac{\partial \dot{\xi}}{\partial g}.\frac{\partial \dot{g}}{\partial m}}_{term\ 4}\right] + \frac{\partial \dot{m}}{\partial g}\left[\underbrace{\frac{\partial \dot{\xi}}{\partial m}.\frac{\partial \dot{g}}{\partial \xi}}_{term\ 5} - \underbrace{\frac{\partial \dot{g}}{\partial m}.\frac{\partial \dot{\xi}}{\partial \xi}}_{term\ 6}\right]$$

where all the above derivatives are evaluated at the steady state equilibrium point $(m^*,\ \xi^*,\ g^*)$. Multiplying *term* 1 with $\frac{\partial \dot{m}}{\partial m}$ and using equations (A1), (A4) and (A8) we have

$$\frac{\partial \dot{m}}{\partial m}.\left\{\frac{\partial \dot{\xi}}{\partial \xi}.\frac{\partial \dot{g}}{\partial g}\right\} = \left[\left(\frac{L_S}{a_S}-g\right)\left(1+2\frac{1-\alpha}{\alpha}\right)+m-\rho\right].\left[-(g+m).\left\{\left(\frac{L_N}{a_N}-g\right)+\rho+m+g\right\}\right]. \tag{A11}$$

Again, multiplying *term* 3 with $\frac{\partial \dot{m}}{\partial \xi}$ and using equations (A1), (A5) and (A9 we have

$$\frac{\partial \dot{m}}{\partial \xi}.\left\{\frac{\partial \dot{\xi}}{\partial m}.\frac{\partial \dot{g}}{\partial g}\right\}$$
$$= \left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m - \rho\right].\left[-(g+m).\left\{\left(\frac{L_N}{a_N} - g\right) + \rho + m + g\right\}\right]. \tag{A12}$$

Comparing equations (A11) and (A12), we find that

$$\frac{\partial \dot{m}}{\partial m}.\underbrace{\left\{\frac{\partial \dot{\xi}}{\partial \xi}.\frac{\partial \dot{g}}{\partial g}\right\}}_{term\ 1} = \frac{\partial \dot{m}}{\partial \xi}.\underbrace{\left\{\frac{\partial \dot{\xi}}{\partial m}.\frac{\partial \dot{g}}{\partial g}\right\}}_{term\ 3}. \tag{$A13$}$$

Using equations (A3) and (A7), we have

$$term\ 2 = \frac{\partial \dot{\xi}}{\partial g}.\frac{\partial \dot{g}}{\partial \xi} = \frac{m}{g}\left(\frac{L_N}{a_N} - g\right)(\rho + m + g). \tag{A14}$$

Using equations (A2) and (A7), we have

$$term\ 4 = \frac{\partial \dot{\xi}}{\partial g}.\frac{\partial \dot{g}}{\partial m} = \frac{m}{g+m}\left(\frac{L_N}{a_N} - g\right). \tag{$A15$}$$

Using equations (A3) and (A9), we have

$$term\ 5 = \frac{\partial \dot{\xi}}{\partial m}.\frac{\partial \dot{g}}{\partial \xi} = -\left(\frac{L_N}{a_N} - g\right)(\rho + m + g). \tag{A16}$$

Using equations (A2) and (A8), we have

$$term\ 6 = \frac{\partial \dot{g}}{\partial m}.\frac{\partial \dot{\xi}}{\partial \xi} = -\left(\frac{L_N}{a_N} - g\right)(g + m). \tag{$A17$}$$

Then using equations (A4), (A5), (A6), (A13), (A14), (A15), (A16) and (A17) we have

$$\begin{aligned} Det(A) \quad = \quad & \left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m - \rho\right]\left[-\frac{m}{g}\left(\frac{L_N}{a_N} - g\right)(\rho + m + g)\right] + \\ & \left[\left\{\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) + m - \rho\right\}\frac{(g+m)^2}{g}\right]\frac{m}{g+m}\left(\frac{L_N}{a_N} - g\right) - \\ & \frac{m(g+m)}{g}\left[-\left(\frac{L_N}{a_N} - g\right)(\rho + m + g) + \left(\frac{L_N}{a_N} - g\right)(g+m)\right] \\ = \quad & -\rho\,\frac{m}{g}\left(\frac{L_N}{a_N} - g\right)\left[\left(\frac{L_S}{a_S} - g\right)\left(1 + 2\frac{1-\alpha}{\alpha}\right) - \rho - g\right]. \end{aligned}$$

Appendix 3.2

From equation (3.1.39.1) we have

$$\left(\frac{L_N}{a_N}-g\right)\left[\frac{1-\alpha}{\alpha}\frac{1}{\xi}+\left(\frac{1-\xi}{\xi}\right)^2\frac{a_N\alpha^{-\varepsilon}}{a_S}\right]=\rho+g+\frac{L_S}{a_S}\frac{1-\xi}{\xi} \qquad (B1)$$

Also, using equations (3.1.39.1) and (3.1.40.1) we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N}-g\right)=\rho\xi+g \qquad \text{(B2)}$$

Differentiating equation (3.1.39) with respect to g we get,

$$\begin{aligned}\frac{\partial\dot{g}}{\partial g} &= (-1)\left[\underbrace{\rho+g-\frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N}-g\right)\frac{1}{\xi}+\frac{1-\xi}{\xi}\left\{\frac{L_S}{a_S}-\frac{1-\xi}{\xi}\left(\frac{L_N}{a_N}-g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}\right\}}_{term\ 1}\right]+\Big(\frac{L_N}{a_N}-g \\ &\quad \left[1+\underbrace{\frac{1-\alpha}{\alpha}\frac{1}{\xi}+\left(\frac{1-\xi}{\xi}\right)^2\frac{a_N\alpha^{-\varepsilon}}{a_S}}_{term\ 2}\right].\end{aligned}$$

From equation (3.1.39.1) we get *term* 1 of the above expression to be zero at the steady state equilibrium point. Also, from equation (B1), we find that

$$term\ 2\times\left(\frac{L_N}{a_N}-g\right)=\left(\rho+g+\frac{L_S}{a_S}\frac{1-\xi}{\xi}\right).$$

So, using this equation, we have

$$\left[\frac{\partial\dot{g}}{\partial g}\right]_{(k^*,\,\xi^*,\,g^*)}=\frac{L_N}{a_N}+\rho+\frac{L_S}{a_S}\frac{1-\xi}{\xi}. \qquad \text{(B3)}$$

Differentiating equation (3.1.39) with respect to ξ we get,

$$\frac{\partial\dot{g}}{\partial\xi}=\left(\frac{L_N}{a_N}-g\right)\frac{1}{\xi^2}\left[\underbrace{\frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N}-g\right)}_{term\ 1}-\frac{L_S}{a_S}+2\underbrace{\frac{1-\xi}{\xi}\left(\frac{L_N}{a_N}-g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}}_{term\ 2}\right].$$

Using equation (B2), *term* 1 of the above expression can be reduced to $(\rho\xi+g)$. Also using equation (3.1.40.1), *term* 2 can be reduced to $2\left(\frac{L_S}{a_S}-g\right)$. Replacing these in the above expression we get, in the steady state equilibrium,

$$\left[\frac{\partial \dot{g}}{\partial \xi}\right]_{(k^*,\, \xi^*,\, g^*)} = \left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi^2}\left[\rho\xi - g + \frac{L_S}{a_S}\right]. \tag{B4}$$

Differentiating equation (3.1.40) with respect to g we have,

$$\frac{\partial \dot{\xi}}{\partial g} = (1-\xi)\left[1 - \frac{1-\xi}{\xi}\frac{a_N\alpha^{-\varepsilon}}{a_S}\right].$$

Equation (3.1.40.1) shows that

$$\frac{1-\xi}{\xi}\frac{a_N\alpha^{-\varepsilon}}{a_S} = \frac{\frac{L_S}{a_S} - g}{\frac{L_N}{a_N} - g}\,.$$

Then, using this equation, we have

$$\left[\frac{\partial \dot{\xi}}{\partial g}\right]_{(k^*,\, \xi^*,\, g^*)} = (1-\xi)\left[\frac{\frac{L_N}{a_N} - \frac{L_S}{a_S}}{\frac{L_N}{a_N} - g}\right]. \tag{B5}$$

Differentiating equation (3.1.40) with respect to ξ we have,

$$\frac{\partial \dot{\xi}}{\partial \xi} = -\left[g + \frac{1-\xi}{\xi}\underbrace{\left\{\frac{L_S}{a_S} - \frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}\right\}}_{term\ 1}\right] - \xi . \frac{1}{\xi^2}\left[-\frac{L_S}{a_S} + 2\underbrace{\frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N\alpha^{-\varepsilon}}{a_S}}_{term\ 2}\right]$$

Equation (3.1.40.1) shows that $term\ 1 = g$ and $term\ 2 = \ 2\left(\frac{L_S}{a_S} - g\right)$. Using these we have

$$\left[\frac{\partial \dot{\xi}}{\partial \xi}\right]_{(k^*,\, \xi^*,\, g^*)} = -\frac{1}{\xi}\left[\frac{L_S}{a_S} - g\right]. \tag{B6}$$

From equations (3.1.39) and (3.1.40), it is clear that

$$\left[\frac{\partial \dot{g}}{\partial k}\right]_{(k^*,\, \xi^*,\, g^*)} = \left[\frac{\partial \dot{\xi}}{\partial k}\right]_{(k^*,\, \xi^*,\, g^*)} = 0\,.$$

Differentiating equation (3.1.35) with respect to k we have

$$\frac{\partial \dot{k}}{\partial k} = \underbrace{2k\frac{a_N\alpha^{-\varepsilon}}{a_S}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi}}_{term\ 1} - \left(\frac{L_S}{a_S} + \rho\right) + \underbrace{\frac{\frac{\partial(k\dot{g})}{\partial k}}{\frac{L_N}{a_N} - g}}_{term\ 2} + \underbrace{\frac{\frac{\partial(k\dot{\xi})}{\partial k}}{\xi(1-\xi)}}_{term\ 3}.$$

From equation (3.1.35.1) we find that the $term\ 1 \;=\; 2\left(\rho + \frac{L_S}{a_S}\right)$.

At the steady state equilibrium $\dot{g} = \dot{\xi} = 0$, and equations (3.1.39) and (3.1.40) show that both $\dot{g}$ and $\dot{\xi}$ do not depend on k. Hence, Both $term$ 2 and $term$ 3 of the above equation are equal to zero at the steady state equilibrium point. So we obtain the following

$$\left[\frac{\partial \dot{k}}{\partial k}\right]_{(k^*,\, \xi^*,\, g^*)} = \left(\frac{L_S}{a_S} + \rho\right). \tag{B7}$$

$$\begin{aligned}
Tr(B) &= \left[\frac{\partial \dot{k}}{\partial k}\right]_{(k^*,\, \xi^*,\, g^*)} + \left[\frac{\partial \dot{\xi}}{\partial \xi}\right]_{(k^*,\, \xi^*,\, g^*)} + \left[\frac{\partial \dot{g}}{\partial g}\right]_{(k^*,\, \xi^*,\, g^*)} \\
&= \left(\frac{L_S}{a_S} + \rho\right) - \frac{1}{\xi}\left[\frac{L_S}{a_S} - g\right] + \frac{L_N}{a_N} + \rho + \frac{L_S}{a_S}\frac{1-\xi}{\xi} \qquad \text{[Using equations (B7), (B6) and (B3} \\
&= 2\rho + \frac{g}{\xi} + \frac{L_N}{a_N} > 0.
\end{aligned}$$

$$Det(B) = \frac{\partial \dot{k}}{\partial k}\left[\frac{\partial \dot{\xi}}{\partial \xi}\frac{\partial \dot{g}}{\partial g} - \frac{\partial \dot{\xi}}{\partial g}\frac{\partial \dot{g}}{\partial \xi}\right]. \qquad \left[\text{since } \frac{\partial \dot{g}}{\partial k} = \frac{\partial \dot{\xi}}{\partial k} = 0 \text{ at the steady-state.}\right]$$

Using equations (B3), (B4), (B5), (B6) and (B7) we have

$$\begin{aligned}
&Det(B) \\
&= \left(\frac{L_S}{a_S} + \rho\right)\left[-\frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\left(\frac{L_N}{a_N} + \rho + \frac{L_S}{a_S}\frac{1-\xi}{\xi}\right) - (1-\xi)\left(\frac{\frac{L_N}{a_N} - \frac{L_S}{a_S}}{\frac{L_N}{a_N} - g}\right)\left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi^2}\left(\rho\xi - g\right.\right. \\
&= \left(\frac{L_S}{a_S} + \rho\right)\left[\left(\frac{L_S}{a_S} - \frac{L_N}{a_N}\right)\left\{\frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right) + \frac{1-\xi}{\xi^2}\left(\rho\xi + \frac{L_S}{a_S} - g\right)\right\} - \frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\left(\rho + \frac{L_S}{a_S}\frac{1}{\xi}\right)\right] \\
&= \left(\frac{L_S}{a_S} + \rho\right)\left[\left(\frac{L_S}{a_S} - \frac{L_N}{a_N}\right)\left(\frac{1-\xi}{\xi}\rho\right) + \frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\left(\frac{1}{\xi}\frac{L_S}{a_S} - \frac{1}{\xi}\frac{L_N}{a_N} - \rho - \frac{L_S}{a_S}\frac{1}{\xi}\right)\right] \\
&= \left(\frac{L_S}{a_S} + \rho\right)\left[\left(\frac{L_S}{a_S} - \frac{L_N}{a_N}\right)\left(\frac{\rho}{\xi} - \rho\right) - \frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\frac{1}{\xi}\frac{L_N}{a_N} - \frac{\rho}{\xi}\left(\frac{L_S}{a_S} - g\right)\right] \\
&= \left(\frac{L_S}{a_S} + \rho\right)\left[-\frac{\rho}{\xi}\left(\frac{L_N}{a_N} - g\right) - \rho\left(\frac{L_S}{a_S} - \frac{L_N}{a_N}\right) - \frac{1}{\xi}\left(\frac{L_S}{a_S} - g\right)\frac{1}{\xi}\frac{L_N}{a_N}\right].
\end{aligned}$$

Appendix 3.3

Using equations (3.1.4), (3.1.25), (3.1.30), equation (3.1.34) can be written as

$$r_S - \rho = \left(\frac{\dot{v_S}}{v_S} + \frac{\dot{n_S}}{n_S} + \frac{\dot{k}}{k}\right) - \left(a_S \frac{(m\dot{\frac{\xi}{1-\xi}})}{L_S - m a_s \frac{\xi}{1-\xi}}\right) .$$

Using equations (3.1.11), (3.1.23) and (3.1.33), we express the above equation as

$$r_S - \rho = \left(r_S - \frac{\pi_S}{v_S} + m\frac{\xi}{1-\xi} + \frac{\dot{k}}{k}\right) - \left(a_N \frac{\dot{g}}{L_N - a_N g} + \frac{\dot{\xi}}{\xi(1-\xi)}\right) . \qquad (C1)$$

Now, using equations (3.1.31) and (3.1.32), we obtain,

$$\frac{\pi_S}{v_S} = \frac{k-1}{a_S}\alpha^{-\varepsilon}(L_N - a_N g)\frac{1-\xi}{\xi} ;$$

and, from equation (3.1.32), we obtain

$$m\frac{\xi}{1-\xi} = \frac{L_S}{a_S} - \frac{1-\xi}{\xi}\left(\frac{L_N}{a_N} - g\right)\frac{a_N \alpha^{-\varepsilon}}{a_S} .$$

Using the above mentioned two equations, equation (C1) can be written as

$$\dot{k} = \left[k^2 \frac{a_N}{a_S}\alpha^{-\varepsilon}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi}\right] - \frac{L_S}{a_S}k - \rho k + a_N \frac{k\dot{g}}{L_N - a_N g} + \frac{k\dot{\xi}}{\xi(1-\xi)} .$$

This is equation (3.1.35) in our model.

Appendix 3.4

From equation (3.1.32), we have

$$m = \frac{1-\xi}{\xi}\left[\frac{L_S}{a_S} - \frac{\alpha^{-\varepsilon}a_N}{a_S}\left(\frac{L_N}{a_N} - g\right)\frac{1-\xi}{\xi}\right]. \qquad (D1)$$

Then, using equation (D1), we can express equation (3.1.39.1) as follows:

$$\rho + g - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - g\right)\frac{1}{\xi} + m = 0. \qquad (D2)$$

Equation (3.1.40.1) can be written as

$$m\frac{\xi}{1-\xi} = g . \qquad (D3)$$

Using equation (D2) and (D3) we have

$$\rho + g - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - g\right)\frac{g+m}{g} + m = 0 \; ;$$

which shows that g and m are positively related[2]. Also, at $m = 0$,

$$g = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha \; .$$

So for $m > 0$, we must have

$$g > (1-\alpha)\frac{L_N}{a_N} - \rho\alpha \; . \tag{$D4$}$$

Now from equations (3.1.35.1) and (3.1.40.1) we have

$$k\left(\frac{L_S}{a_S} - g\right) = \rho + \frac{L_S}{a_S} \; . \tag{$D5$}$$

In the *narrow gap* case we have $k(=\frac{w_N}{w_S}) \; < \frac{1}{\alpha}$. Then equation (D5) implies that

$$g < (1-\alpha)\frac{L_S}{a_S} - \rho\alpha \; . \tag{D6}$$

Inequalities (D4) and (D6) are satisfied together only if

$$\frac{L_S}{a_S} > \frac{L_N}{a_N} \; .$$

Appendix 3.5

We have

$$r_S - \rho = \frac{\dot{v_S}}{v_S} + \frac{\dot{n_S}}{n_S} + \frac{\dot{k}}{k} - a_S\frac{(m\dot{\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}};$$

Then using equations (3.2.21), the above equation implies

$$r_S - \rho = r_S - \frac{\pi_S}{v_S} + \frac{\dot{n_S}}{n_S} + \frac{\dot{k}}{k} - a_S\frac{(m\dot{\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}}$$

Differentiating equation (3.2.25) with respect to time we obtain

$$a_S\frac{(m\dot{\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}} = a_N\frac{\dot{\theta}}{L_N - a_N\theta} - \frac{\dot{\xi}}{\xi(1-\xi)}.$$

[2]This is the equation behind the NN curve in Grossman-Helpman(1991c), page 290, equation no (11.16).

Then, using these last two equations we obtain,

$$r_S - \rho = r_S - \frac{\pi_S}{v_S} + m\frac{\xi}{1-\xi} + \frac{\dot{k}}{k} - a_N\frac{\dot{\theta}}{L_N - a_N\theta} - \frac{\dot{\xi}}{\xi(1-\xi)}$$

$$\text{or}, -\rho = -[\frac{k-1}{a_S}\alpha^{-\varepsilon}(L_N - a_N\theta)\frac{1-\xi}{\xi}] + [\frac{L_S}{a_S} - \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}] + \frac{\dot{k}}{k} - [a_N\frac{\dot{\theta}}{L_N - a_N\theta} + \frac{\dot{\xi}}{\xi(1-\xi)}]$$

$$\Longrightarrow \dot{k} = k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - k[\frac{L_S}{a_S} + \rho] + [\frac{k\dot{\theta}}{\frac{L_N}{a_N} - \theta} + \frac{k\dot{\xi}}{\xi(1-\xi)}].$$

Appendix 3.6

We have

$$\frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - \frac{\dot{n_N}}{n_N} \Longrightarrow \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - (\theta - m).$$

This we get from the fact that $n_N = n\xi$ and equations (3.2.13) and (3.2.10). The no-arbitrage condition (3.2.6) then implies that

$$\frac{\dot{p_N}}{p_N} = r_N + \theta - \frac{1-\alpha}{\alpha a_N}(L_N - a_N\theta). \qquad (C1)$$

Again from

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(n_N\dot{x}_N)}{n_N x_N}$$

and using equations (3.2.11) and (3.1.4), we have

$$r_N - \rho = \frac{\dot{p_N}}{p_N} - a_N\frac{\dot{\theta}}{L_N - a_N\theta},$$

$$\Longrightarrow \frac{\dot{p_N}}{p_N} = r_N - \rho + a_N\frac{\dot{\theta}}{L_N - a_N\theta}. \qquad (C2)$$

Equations (C1) and (C2) together imply that

$$a_N\frac{\dot{\theta}}{L_N - a_N\theta} = \rho + \theta - \frac{1-\alpha}{\alpha a_N}(L_N - a_N\theta),$$

$$\Longrightarrow \dot{\theta} = [\rho + \theta - \frac{1-\alpha}{\alpha a_N}(\frac{L_N}{a_N} - \theta)](\frac{L_N}{a_N} - \theta).$$

Appendix 3.7

Here,

$$W_N = \int_0^\infty e^{-\rho t} log(U_N(t))dt = \int_0^\infty e^{-\rho t}[log(1-\frac{a_N\theta}{L_N})+\frac{1}{\varepsilon-1}log(n(t))+\frac{1}{\varepsilon-1}log\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}]dt$$

Then differentiating W_N with respect to a_S and evaluating the derivative at the steady-state, we have

$$\begin{aligned}\frac{dW_N}{da_S} = \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{da_S}]dt \quad &+ \quad \frac{1}{\varepsilon-1}\int_0^\infty e^{-\rho t}[\frac{dlog(n(t))}{da_S}]dt \\ &+ \quad \frac{1}{\varepsilon-1}\int_0^\infty e^{-\rho t}[\frac{dlog\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}}{da_S}]dt. \qquad (D1)\end{aligned}$$

At steady-state we have $\xi = \xi^*$ and $\theta = \theta^*$, and hence

$$\frac{dlog(1-\frac{a_N\theta}{L_N})}{da_S}|_{\theta=\theta^*} = 0 \Rightarrow \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{da_S}]dt = 0.$$

Note that $log(n(t)) = log(n(0)) + \int_0^t g(\tau)d\tau$. Differentiating this with respect to a_S we have

$$\frac{dlog(n(t))}{da_S} = \int_0^t \frac{dg(\tau)}{da_S}d\tau = \theta^*\frac{d\xi^*}{da_S}\int_0^t(1-e^{a_{22}\tau})d\tau = \theta^*\frac{d\xi^*}{da_S}[\int_0^t d\tau - \int_0^t e^{a_{22}\tau}d\tau]$$

$$= \theta^*\frac{d\xi^*}{da_S}[t - [\frac{1}{a_{22}}e^{a_{22}\tau}]_0^t] = \theta^*\frac{d\xi^*}{da_S}[t+\frac{1}{a_{22}}-\frac{1}{a_{22}}e^{a_{22}t}]$$

Then,

$$\int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{da_S}dt = \theta^*\frac{d\xi^*}{da_S}[\int_0^\infty e^{-\rho t}(t+\frac{1}{a_{22}}-\frac{1}{a_{22}}e^{a_{22}t})dt] = \theta^*\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho-a_{22})} > 0.$$

Also it can be shown that

$\frac{dlog\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}}{da_S}|_{\xi=\xi^*} = \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}[1-e^{a_{22}t}]\frac{d\xi^*}{da_S}$.

So we have

$$\begin{aligned}\int_0^\infty e^{-\rho t}[\frac{dlog\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}}{da_S}]dt &= \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\int_0^\infty e^{-\rho t}[1-e^{a_{22}t}]dt \\ &= \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}.\end{aligned}$$

Then from equation (D1) we have

$$\frac{dW_N}{da_S} = \frac{1}{\varepsilon - 1}[\theta^* \frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho - a_{22})} + \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})}].$$

Now using equation (3.2.58) we obtain

$$\begin{aligned}
\frac{dW_S}{da_S} &= \frac{dW_N}{da_S} + \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{1-\xi}{\xi}\frac{L_N}{L_S})}{da_S}]dt \\
&= \frac{dW_N}{da_S} - \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\int_0^\infty e^{-\rho t}[1 - e^{a_{22}t}]dt \\
&= \frac{dW_N}{da_S} - \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})}.
\end{aligned}$$

So, we have,

$$\frac{dW_S}{da_S} = -\frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})} + \frac{1}{\varepsilon - 1}[\theta^*\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho - a_{22})} + \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})}].$$

Appendix 3.8

All the following derivatives are evaluated at the steady-state equilibrium values of ξ and θ.

$$\Delta_N^{L_j} = \int_0^\infty e^{-\rho t}[\frac{dlog(n(t))}{dL_j}]dt = \theta^*\frac{d\xi^*}{dL_j}[\int_0^\infty e^{-\rho t}(t + \frac{1}{a_{22}} - \frac{1}{a_{22}}e^{a_{22}t})dt] = \theta^*\frac{d\xi^*}{dL_j}\frac{-a_{22}}{\rho^2(\rho - a_{22})}.$$

Since $\frac{d\xi^*}{dL_N} > 0$ and $\frac{d\xi^*}{dL_S} < 0$, we have $\Delta_N^{L_j} > 0$ for j=N and $\Delta_N^{L_j} < 0$ for j=S.

$$\Delta_e^{L_j} = \int_0^\infty e^{-\rho t}[\frac{dlog\{\xi + (1-\xi)\alpha^{1-\varepsilon}\}}{dL_j}]dt = \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{dL_j}\frac{-a_{22}}{\rho(\rho - a_{22})}.$$

Since $\alpha < 1$ and $\varepsilon > 1$ we have $\alpha^{1-\varepsilon} > 1$. Hence $\Delta_e^{L_j} < 0$ for j=N and $\Delta_e^{L_j} > 0$ for j=S.

At the steady-state equilibrium we have

$$\theta = \theta^* = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha$$

and this implies that at $\theta = \theta^*$,

$$\frac{d\theta}{dL_S} = 0 \text{ and } \frac{d\theta}{dL_N} = \frac{1-\alpha}{a_N}$$

Then,

$$\Delta_s^{NL_j} = \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{dL_j}]dt = 0 \qquad \text{for j=S; and}$$
$$= \int_0^\infty e^{-\rho t}[\frac{dlog(\alpha+\frac{\rho\alpha a_N}{L_N})}{dL_j}]dt = \frac{1}{\rho}\frac{1}{\alpha+\frac{\rho\alpha a_N}{L_N}}(-\frac{\rho\alpha a_N}{L_N^2}) < 0 \qquad \text{for j=N}$$

Again

$$\Delta_s^{SL_j} = \int_0^\infty e^{-\rho t}[\frac{dlog\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N}-\theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\}}{dL_j}]dt = \int_0^\infty e^{-\rho t}[\frac{dlog\{\frac{1-\xi}{\xi}(\frac{L_N}{L_S}+\rho\frac{a_N}{L_S})\alpha^{1-\varepsilon}\}}{dL_j}]dt$$
$$= \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{1-\xi}{\xi})}{dL_j}]dt + \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{L_N}{L_S}+\rho\frac{a_N}{L_S})}{dL_j}]dt.$$

Here the first term is negative and the second term is positive for j=N. So the net effect is ambiguous. Similarly, for j=S, the first term is positive and the second term is negative. So the net effect is ambiguous again.

Appendix 3.9

In the narrow gap case we have

$$\left(\frac{w_N}{w_S}\right)^* = k^* < \frac{1}{\alpha}.$$

Then equation (3.2.40) implies that

$$\frac{L_S+\rho a_S}{L_S-\theta^*\xi^* a_S} < \frac{1}{\alpha}.$$

The above inequality can be written as

$$\frac{L_S}{a_S} > \frac{\alpha\rho}{1-\alpha} + \frac{\theta^*\xi^*}{1-\alpha}.$$

Since the maximum value of ξ^* is 1; a sufficient condition for the above inequality to hold is

$$\frac{L_S}{a_S} > \frac{\alpha\rho}{1-\alpha} + \frac{\theta^*}{1-\alpha}.$$

Then replacing the value of θ^* from equation (3.2.36), a sufficient condition for the above inequality to hold is

$$\frac{L_S}{a_S} > \frac{L_N}{a_N}.$$

Chapter 4

Appendix 4.1

Derivation of equation (4.1.25):

Using equations (4.1.7) and (4.1.8), we express equation (4.1.17) as

$$\frac{(1-\alpha)p_N x_N}{(1-\alpha)p_M x_M} = \frac{r}{r+\iota} \; .$$

Using equations (4.1.2) and (4.1.3), the equation written above can be written as

$$\left(\frac{x_N}{x_M}\right)^{\alpha} = \frac{r}{r+\iota} \; .$$

Using equations (4.1.18) and (4.1.20), the equation written above can be written as

$$\left(\frac{L_N - a_N g}{H_S - a_I g} \; \frac{n_M}{n_N}\right)^{\alpha} = \frac{r}{r+\iota} \; . \tag{A1}$$

We write $\frac{n_M}{n_N}$ as follows:

$$\begin{aligned} \frac{n_M}{n_N} &= \left(\frac{n_S}{n_N}\right) \left(\frac{n_M}{n_S}\right) \\ &= \left(\frac{\dot{n_S}}{n_N} \; \frac{n_S}{\dot{n_S}}\right) \left(\frac{1}{1+\frac{n_I}{n_M}}\right) \qquad [\text{since} \;\; n_S = n_M + n_I] \\ &= \left(\frac{\omega}{g}\right) \left(\frac{1}{1+\frac{\dot{n_I}}{n_M}\frac{n_I}{\dot{n_I}}}\right) \\ &= \left(\frac{\omega}{g}\right) \left(\frac{1}{1+\frac{\iota}{g}}\right) . \end{aligned} \tag{A2}$$

Using equation (A2), equation (A1) can be written as

$$\left(\frac{L_N - a_N g}{H_S - a_I g} \; \frac{\omega}{g} \; \frac{1}{1+\frac{\iota}{g}}\right)^{\alpha} = \frac{r}{r+\iota} \; . \tag{A3}$$

From equation (4.1.21) we obtain

$$(L_N - a_N g)\frac{\omega}{g} = \frac{\alpha}{1-\alpha} a_N(\rho + g) - L_N + a_N g \; . \tag{A4}$$

Using equations (A4), (4.1.1) and (4.1.14), equation (A3) can be written as

$$\left(\frac{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}{H_S-a_Ig}\ \frac{g}{\iota+g}\right)^\alpha=\frac{\rho+g}{\rho+g+\iota}\ . \tag{A5}$$

This is our equation (4.1.25) in the text which represents the AA curve.

Slope of AA curve:

Taking logarithm in both sides of equation (A5) and then differentiating with respect to ι we get

$$\alpha\frac{\partial log\left\{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng\right\}}{\partial\iota}-\alpha\frac{\partial log(H_S-a_Ig)}{\partial\iota}+\alpha\frac{\partial log(g)}{\partial\iota}-\alpha\frac{\partial log(\iota+g)}{\partial\iota}$$
$$=\ \frac{\partial log(\rho+g)}{\partial\iota}-\frac{\partial log(\rho+g+\iota)}{\partial\iota}\ .$$

This implies that

$$\frac{\partial g}{\partial\iota}\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}+\frac{a_I\alpha}{H_S-a_Ig}+\frac{\alpha}{g}-\frac{\alpha}{\iota+g}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}\right]$$
$$=\frac{\alpha}{\iota+g}-\frac{1}{\rho+g+\iota}\ .$$

So we have

$$\frac{\partial g}{\partial\iota}=\frac{\frac{\alpha}{\iota+g}-\frac{1}{\rho+g+\iota}}{\underbrace{\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}-\frac{1}{\rho+g}}_{term\ 1}+\frac{a_I\alpha}{H_S-a_Ig}+\underbrace{\frac{\alpha}{g}-\frac{\alpha}{\iota+g}}_{term\ 2}+\frac{1}{\rho+g+\iota}}\ . \tag{A6}$$

Both *term 1* and *term 2* of the denominator of the RHS of equation (A6) are positive. The numerator can be written as

$$\begin{aligned}\frac{\alpha}{\iota+g}-\frac{1}{\rho+g+\iota}&=\frac{\alpha\rho-(1-\alpha)(g+\iota)}{(\iota+g)(\rho+g+\iota)}\\&=\frac{-\ (1-\alpha)\left(g+\iota-\frac{\alpha}{1-\alpha}\rho\right)}{(\iota+g)(\rho+g+\iota)}\ .\end{aligned}$$

We assume ρ to be sufficiently small so that

$$g+\iota>\frac{\alpha}{1-\alpha}\rho \tag{A7}$$

is satisfied. Then, under the assumption (A7), equation (A6) implies that

$$\frac{\partial g}{\partial \iota} < 0 \ .$$

Also, from equation (A5), we find that the LHS of this equation is positive if

$$\frac{H_S}{a_I} > g > \frac{L_N - \frac{\alpha}{1-\alpha} a_N \rho}{\frac{a_N}{1-\alpha}} \ . \tag{A8}$$

AA curve slopes negatively if the inequalities (A7) and (A8) are satisfied. Using the lower bound of g from inequality (A8) and taking $\iota = 0$, the sufficient condition for inequality (A7) to be satisfied is

$$\frac{L_N}{a_N} \geq \frac{\rho\alpha(2-\alpha)}{(1-\alpha)^2} \ .$$

Appendix 4.2

Comparative static exercises (IPR protection)

(i) Narrow gap equilibrium case:

Equations (4.1.24) and (4.1.25) solve for g and ι. Taking logarithms on both sides of equations (4.1.24) and (4.1.25) we get

$$log(L_S) - log(H_S - a_I g) + log(g) - log(\iota) = -\varepsilon \ log(\alpha) \ . \tag{B1}$$

and

$$\alpha \ log\left\{\frac{\alpha}{1-\alpha} a_N(\rho + g) - L_N + a_N g\right\} - \alpha \ log(H_S - a_I g) + \alpha \ log(g) - \alpha \ log(\iota + g)$$
$$= log(\rho + g) - log(\rho + g + \iota). \tag{B2}$$

Now differentiating the equations (B1) and (B2) with respect to a_I and arranging the terms we have

$$\frac{\partial g}{\partial a_I}\left[-\frac{a_I}{H_S - a_I g} - \frac{1}{g}\right] + \frac{\partial \iota}{\partial a_I}\left[\frac{1}{\iota}\right] = \frac{g}{H_S - a_I g} \ ; \tag{B3}$$

and

$$\frac{\partial g}{\partial a_I}\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}+\frac{a_I\alpha}{H_S-a_Ig}+\frac{\alpha}{g}-\frac{\alpha}{g+\iota}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}\right]$$
$$+\frac{\partial \iota}{\partial a_I}\left[\frac{1}{\rho+g+\iota}-\frac{\alpha}{g+\iota}\right]=-\frac{\alpha g}{H_S-a_Ig}\,. \tag{B4}$$

Here equation (B3) is derived from equation (4.1.24) and equation (B4) is derived from equation (4.1.25). We arrange equations (B3) and (B4) in the following way.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.\begin{bmatrix} \frac{\partial g}{\partial a_I} \\ \frac{\partial \iota}{\partial a_I} \end{bmatrix}=\begin{bmatrix} \frac{g}{H_S-a_Ig} \\ \frac{-\alpha g}{H_S-a_Ig} \end{bmatrix}. \tag{$B5$}$$

Here

$$a_{11}=-\frac{a_I}{H_S-a_Ig}-\frac{1}{g}\ <0\,;$$
$$a_{12}=\frac{1}{\iota}\ >0\,;$$
$$a_{21}=\underbrace{\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}}_{term\ 1}+\frac{a_I\alpha}{H_S-a_Ig}+\frac{\alpha\iota}{g(g+\iota)}-\underbrace{\frac{1}{\rho+g}}_{term\ 2}+\frac{1}{\rho+g+\iota}\ >0\,;$$

[since, (*term 1* - *term 2*) > 0]

and

$$a_{22}=\frac{1}{\rho+g+\iota}-\frac{\alpha}{g+\iota}\ >0\,;\qquad\text{[under assumption (A7)]}$$

Hence we have

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}<0.$$

We solve the system of equations (B5) using the Cramer rule and obtain the comparative static expressions given by

$$\frac{\partial g}{\partial a_I}=\frac{\begin{vmatrix} \frac{g}{H_S-a_Ig} & a_{12} \\ \frac{-\alpha g}{H_S-a_Ig} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}\,; \tag{B6}$$

and

$$\frac{\partial \iota}{\partial a_I} = \frac{\begin{vmatrix} a_{11} & \frac{g}{H_S - a_I g} \\ a_{21} & \frac{-\alpha g}{H_S - a_I g} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}. \tag{B7}$$

The numerator of the right hand side of equation (B6) is always positive because $H_S > a_I g$ in equilibrium and a_{12} and a_{22} are positive. Its denominator takes a negative sign. So $\frac{\partial g}{\partial a_I} < 0$.

The numerator of the R.H.S. of equation (B7) can be written as

$$\begin{aligned}
&\frac{-\alpha g}{H_S - a_I g}(a_{11}) - \frac{g}{H_S - a_I g}(a_{21}) \\
&= \frac{-\alpha g}{H_S - a_I g}\left(-\frac{a_I}{H_S - a_I g} - \frac{1}{g}\right) - \frac{g}{H_S - a_I g}\left(\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} + \right. \\
&\qquad\qquad \left. \frac{a_I \alpha}{H_S - a_I g} + \frac{\alpha \iota}{g(g+\iota)} - \frac{1}{\rho+g} + \frac{1}{\rho+g+\iota}\right) \\
&= \underbrace{\frac{\alpha g}{H_S - a_I g}}_{term\ 1}\left(\underbrace{\frac{a_I}{H_S - a_I g}}_{term\ 2} + \underbrace{\frac{1}{g}}_{term\ 3}\right) - \underbrace{\frac{g}{H_S - a_I g}}_{term\ 4}\left(\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} + \right. \\
&\qquad\qquad \left. \underbrace{\frac{a_I \alpha}{H_S - a_I g}}_{term\ 5} + \underbrace{\frac{\alpha}{g}}_{term\ 6} - \frac{\alpha}{\iota+g} - \frac{1}{\rho+g} + \frac{1}{\rho+g+\iota}\right)
\end{aligned}$$

From the above mentioned expression we have

$$(term\ 1)(term\ 2 + term\ 3) = (term\ 4)(term\ 5 + term\ 6).$$

Also we have

$$\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} > \frac{1}{\rho+g};$$

and

$$\frac{1}{\rho+g+\iota} > \frac{\alpha}{\iota+g} \qquad \text{if} \quad (g+\iota) > \tfrac{\alpha}{1-\alpha}\rho.$$

So the numerator of equation (B7) is negative under the sufficient condition given by assumption (A7). Then we have $\frac{\partial \iota}{\partial a_I} > 0$.

(ii) Wide gap equilibrium case:

Equations (4.1.28) and (4.1.25) solve for g and ι. Taking logarithms on the both sides of equation (4.1.28) we have

$$\begin{aligned}(1-\alpha)\ log(H_S - a_I g) + (1-\alpha)\ log(\iota) - (1-\alpha)\ log(g) - (1-\alpha)\ log(L_S) = log(a_I)\\ + log(\rho+g) - log(\frac{\alpha}{1-\alpha}\frac{1}{L_S})\ .\end{aligned} \tag{B8}$$

Differentiating equation (B8) with respect to a_I and arranging terms we obtain

$$\frac{\partial g}{\partial a_I}\left[-\frac{(1-\alpha)a_I}{H_S - a_I g} - \frac{(1-\alpha)}{g} - \frac{1}{\rho+g}\right] + \frac{\partial \iota}{\partial a_I}\left[\frac{1-\alpha}{\iota}\right] = \frac{1}{a_I} + \frac{g(1-\alpha)}{H_S - a_I g}. \tag{$B9$}$$

Equation (B4) remains unchanged here since it is derived from equation (4.1.25) which is also valid in the wide gap equilibrium case. Then, from equations (B4) and (B9), we have

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial g}{\partial a_I} \\ \frac{\partial \iota}{\partial a_I} \end{bmatrix} = \begin{bmatrix} \frac{-\alpha g}{H_S - a_I g} \\ \\ \frac{1}{a_I} + \frac{g(1-\alpha)}{H_S - a_I g} \end{bmatrix}.$$

Here

$$b_{11} = \underbrace{\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g}}_{term\ 1} + \frac{a_I\alpha}{H_S - a_I g} + \frac{\alpha}{g} - \underbrace{\frac{\alpha}{g+\iota}}_{term\ 2} - \underbrace{\frac{1}{\rho+g}}_{term\ 3} + \underbrace{\frac{1}{\rho+g+\iota}}_{term\ 4} > 0\ ;$$

[since, ($term\ 1 - term\ 3$) > 0 and ($term\ 4 - term\ 2$) > 0.]

$$b_{12} = \frac{1}{\rho+g+\iota} - \frac{\alpha}{g+\iota} > 0\ ; \qquad \text{[under assumption (A7)]}\ .$$

$$b_{21} = -\frac{(1-\alpha)a_I}{H_S - a_I g} - \frac{(1-\alpha)}{g} - \frac{1}{\rho+g} < 0\ ;$$

and

$$b_{22} = \frac{1-\alpha}{\iota} > 0.$$

Hence,

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} > 0.$$

Using Cramer rule, we obtain

$$\frac{\partial g}{\partial a_I} = \frac{\begin{vmatrix} \frac{-\alpha g}{H_S - a_I g} & b_{12} \\ \frac{1}{a_I} + \frac{g(1-\alpha)}{H_S - a_I g} & b_{22} \end{vmatrix}}{\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}}; \tag{$B10$}$$

and

$$\frac{\partial \iota}{\partial a_I} = \frac{\begin{vmatrix} b_{11} & \frac{-\alpha g}{H_S - a_I g} \\ b_{21} & \frac{1}{a_I} + \frac{g(1-\alpha)}{H_S - a_I g} \end{vmatrix}}{\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}}. \tag{$B11$}$$

It is very simple to show that the numerator of the R.H.S. of equation (B10) takes a negative sign. So $\frac{\partial g}{\partial a_I} < 0$.

We can write the numerator of the R.H.S. of equation (B11) as

$$b_{11}\left[\frac{1}{a_I} + \frac{g(1-\alpha)}{H_S - a_I g}\right] - b_{21}\left[\frac{-\alpha g}{H_S - a_I g}\right] =$$
$$\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} + \underbrace{\frac{a_I\alpha}{H_S - a_I g}}_{term\ 1} + \underbrace{\frac{\alpha}{g}}_{term\ 2} - \frac{\alpha}{g+\iota} - \frac{1}{\rho+g} + \frac{1}{\rho+g+\iota}\right]$$
$$\left[\underbrace{\frac{1}{a_I}}_{term\ 3} + \underbrace{\frac{g(1-\alpha)}{H_S - a_I g}}_{term\ 4}\right] - \left[\underbrace{-\frac{(1-\alpha)a_I}{H_S - a_I g} - \frac{(1-\alpha)}{g}}_{term\ 5} \underbrace{-\frac{1}{\rho+g}}_{term\ 6}\right]\left[\underbrace{\frac{-\alpha g}{H_S - a_I g}}_{term\ 7}\right]. \tag{B12}$$

We have

$(term\ 1 \times term\ 4) + (term\ 2 \times term\ 4) = (term\ 5 \times term\ 7)$

and

$(term\ 1 \times term\ 3) > (term\ 6 \times term\ 7)$ [since $\frac{g}{\rho+g} < 1$].

Also,

$(term\ 2 \times term\ 3) > 0.$

We also find that

$$\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}-\frac{\alpha}{g+\iota}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}$$
$$=\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}-\frac{1}{\rho+g}\right]+\left[\frac{1}{\rho+g+\iota}-\frac{\alpha}{g+\iota}\right]>0$$

since $L_N > a_N g$ and $(g+\iota) > \frac{\alpha}{1-\alpha}\rho$ (by assumption (A7)). So the R.H.S. of (B11) is positive; and hence $\frac{\partial\iota}{\partial a_I} > 0$.

Appendix 4.3

Comparative static exercises (Factor Endowment Change)

(i) Narrow gap equilibrium case:

Taking logarithms on both sides of equations (4.1.24) and (4.1.25) we get

$$log(L_S) - log(H_S - a_I g) + log(g) - log(\iota) = -\varepsilon\ log(\alpha)\ . \qquad (C1)$$

and

$$\alpha\ log\left\{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng\right\}-\alpha\ log(H_S-a_Ig)+\alpha\ log(g)-\alpha\ log(\iota+g)$$
$$= log(\rho+g) - log(\rho+g+\iota). \qquad \text{(C2)}$$

Differentiating equations (C1) and (C2) with respect to H_S and arranging the terms we obtain

$$\frac{\partial g}{\partial H_S}\left[\frac{a_I}{H_S-a_Ig}+\frac{1}{g}\right]+\frac{\partial\iota}{\partial H_S}\left[-\frac{1}{\iota}\right]=\frac{1}{H_S-a_Ig}\ ; \qquad (C3)$$

and

$$\frac{\partial g}{\partial H_S}\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}+\frac{a_I\alpha}{H_S-a_Ig}+\frac{\alpha}{g}-\frac{\alpha}{g+\iota}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}\right]$$
$$+\frac{\partial\iota}{\partial H_S}\left[\frac{1}{\rho+g+\iota}-\frac{\alpha}{g+\iota}\right]=\frac{\alpha}{H_S-a_Ig}\ . \qquad \text{(C4)}$$

We arrange equations (C3) and (C4) in the following way.

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} . \begin{bmatrix} \frac{\partial g}{\partial H_S} \\ \frac{\partial \iota}{\partial H_S} \end{bmatrix} = \begin{bmatrix} \frac{1}{H_S - a_I g} \\ \frac{\alpha}{H_S - a_I g} \end{bmatrix} . \tag{$C5$}$$

Here

$$c_{11} = \frac{a_I}{H_S - a_I g} + \frac{1}{g} \ > 0 \ ;$$

$$c_{12} = -\frac{1}{\iota} \ < 0 \ ;$$

$$c_{21} = \underbrace{\frac{\frac{\alpha}{1-\alpha} a_N}{\frac{\alpha}{1-\alpha} a_N(\rho + g) - L_N + a_N g}}_{term\ 1} + \frac{a_I \alpha}{H_S - a_I g} + \frac{\alpha \iota}{g(g+\iota)} - \underbrace{\frac{1}{\rho + g}}_{term\ 2} + \frac{1}{\rho + g + \iota} \ > 0 \ ;$$

[since, (*term 1* - *term 2*) > 0]

and

$$c_{22} = \frac{1}{\rho + g + \iota} - \frac{\alpha}{g + \iota} \ > 0 \ ; \qquad \text{[under assumption (A7)]}$$

Hence we have

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} > 0.$$

We solve the system of equations (C5) using the Cramer rule and obtain the comparative static expressions given by

$$\frac{\partial g}{\partial H_S} = \frac{\begin{vmatrix} \frac{1}{H_S - a_I g} & c_{12} \\ \frac{\alpha}{H_S - a_I g} & c_{22} \end{vmatrix}}{\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}} \ ; \tag{C6}$$

and

$$\frac{\partial \iota}{\partial H_S} = \frac{\begin{vmatrix} c_{11} & \frac{1}{H_S - a_I g} \\ c_{21} & \frac{\alpha}{H_S - a_I g} \end{vmatrix}}{\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}} . \tag{C7}$$

The numerator of the right hand side of equation (C6) is positive. Its denominator takes a positive sign. So $\frac{\partial g}{\partial H_S} > 0$.

The numerator of the R.H.S. of equation (C7) can be written as

$$\frac{\alpha}{H_S - a_I g}(c_{11}) - \frac{1}{H_S - a_I g}(c_{21})$$

$$= \frac{\alpha}{H_S - a_I g}\left(\frac{a_I}{H_S - a_I g} + \frac{1}{g}\right) - \frac{1}{H_S - a_I g}\left(\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} + \right.$$
$$\left. \frac{a_I \alpha}{H_S - a_I g} + \frac{\alpha}{g} - \frac{\alpha}{\iota + g} - \frac{1}{\rho + g} + \frac{1}{\rho + g + \iota}\right)$$

Since,

$$\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g) - L_N + a_N g} > \frac{1}{\rho + g};$$

and

$$\frac{1}{\rho + g + \iota} > \frac{\alpha}{\iota + g}\ ;\ \text{ by assumption (A7).}$$

So the numerator of equation (B7) is negative. Then we have $\frac{\partial \iota}{\partial H_S} < 0$.

(ii) Wide gap equilibrium case:

In the wide gap equilibrium case, equations (4.1.25) and (4.1.28) simultaneously solve for g and ι. Taking logarithms on the both sides of equation (4.1.28) we have

$$(1-\alpha)\ log(H_S - a_I g) + (1-\alpha)\ log(\iota) - (1-\alpha)\ log(g) - (1-\alpha)\ log(L_S) = log(a_I)$$
$$+ log(\rho + g) - log(\frac{\alpha}{1-\alpha}\frac{1}{L_S})\ . \tag{C8}$$

Differentiating equation (C8) with respect to H_S and arranging terms we obtain

$$\frac{\partial g}{\partial H_S}\left[-\frac{(1-\alpha)a_I}{H_S - a_I g} - \frac{1-\alpha}{g} - \frac{1}{g}\right] + \frac{\partial \iota}{\partial H_S}\left[\frac{1-\alpha}{\iota}\right] = -\frac{1-\alpha}{H_S - a_I g}. \tag{$C9$}$$

Equation (C4) remains unchanged here since it is derived from equation (4.1.25) which is also valid in the wide gap equilibrium case. Then, from equations (C4) and (C9), we have

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial g}{\partial H_S} \\ \frac{\partial \iota}{\partial H_S} \end{bmatrix} = \begin{bmatrix} \frac{-(1-\alpha)}{H_S - a_I g} \\ \\ \frac{\alpha}{H_S - a_I g} \end{bmatrix}.$$

Here

$$d_{11} = -\frac{(1-\alpha)a_I}{H_S - a_I g} - \frac{1-\alpha}{g} - \frac{1}{g} < 0.$$

$$d_{12} = \frac{1-\alpha}{\iota} > 0.$$

$$d_{21} = c_{21} > 0$$

$$d_{22} = c_{22} > 0.$$

Where we have already defined c_{21} and c_{22} in the narrow gap equilibrium case. Hence,

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} < 0.$$

Using Cramer rule, we obtain

$$\frac{\partial g}{\partial H_S} = \frac{\begin{vmatrix} \frac{-(1-\alpha)}{H_S - a_I g} & d_{12} \\ \frac{\alpha}{H_S - a_I g} & d_{22} \end{vmatrix}}{\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix}}; \tag{C10}$$

and

$$\frac{\partial \iota}{\partial H_S} = \frac{\begin{vmatrix} d_{11} & \frac{-(1-\alpha)}{H_S - a_I g} \\ d_{21} & \frac{\alpha}{H_S - a_I g} \end{vmatrix}}{\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix}}. \tag{C11}$$

It is very simple to show that the numerator of the R.H.S. of equation (C10) takes a negative sign. So $\frac{\partial g}{\partial H_S} > 0$.

We can write the numerator of the R.H.S. of equation (C11) as

$$d_{21}\left[\frac{(1-\alpha)}{H_S-a_Ig}\right]+d_{11}\left[\frac{\alpha}{H_S-a_Ig}\right] \;=\;$$
$$\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}+\underbrace{\frac{a_I\alpha}{H_S-a_Ig}}_{term\ 1}+\underbrace{\frac{\alpha}{g}}_{term\ 2}-\frac{\alpha}{g+\iota}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}\right]$$
$$\left[\underbrace{\frac{1-\alpha}{H_S-a_Ig}}_{term\ 3}\right]-\left[\underbrace{\frac{(1-\alpha)a_I}{H_S-a_Ig}+\frac{1-\alpha}{g}}_{term\ 4}+\frac{1}{g}\right]\left[\underbrace{\frac{\alpha}{H_S-a_Ig}}_{term\ 5}\right]. \tag{C12}$$

We have

$(term\ 1+term\ 2)\times term\ 3 \;=\; term\ 4\times term\ 5$

So, we can write the RHS of equation (C12) as follows.

$$\left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}a_N(\rho+g)-L_N+a_Ng}-\frac{\alpha}{g+\iota}-\frac{1}{\rho+g}+\frac{1}{\rho+g+\iota}\right]\left[\frac{1-\alpha}{H_S-a_Ig}\right]-\frac{1}{g}\frac{\alpha}{H_S-a_Ig}.$$

The sign of the above expression is not known. So, we have $\frac{\partial\iota}{\partial H_S}$ is ambiguous in sign.

Chapter 5

Appendix 5.1

The problem of the representative firm in the South is to maximise

$$\pi_S = p_S x_S - \left(w_{SH} H_p + w_{SL} \frac{L_D}{h\left(\frac{w_{SL}}{w_R}\right)} \right)$$

with respect to H_p, L_D and w_{SL}. The first order optimality conditions are the followings.

$$p_S \frac{\partial x_S}{\partial H_p} = \frac{w_{SH}}{\alpha}; \tag{5A.1}$$

$$p_S \frac{\partial x_S}{\partial L_D} = \frac{w_{SL}}{\alpha\, h(.)}; \tag{5A.2}$$

and

$$\frac{w_{SL}}{w_{SH}} \frac{h'(.)}{h(.)} = 1. \tag{5A.3}$$

From the production function of x_S, we have

$$\frac{\partial x_S}{\partial H_p} = \frac{x_S}{\delta {L_D}^{-\rho} + (1-\delta){H_p}^{-\rho}} (1-\delta)(H_p)^{-\rho-1}; \tag{5A.4}$$

and

$$\frac{\partial x_S}{\partial L_D} = \frac{x_S}{\delta {L_D}^{-\rho} + (1-\delta){H_p}^{-\rho}} \delta (L_D)^{-\rho-1}. \tag{5A.5}$$

Using these equations, we derive equations (5.1.12)-(5.1.15) in the text.

Appendix 5.2

Existence of the wide gap equilibrium

Equation (5.2.9) represents SS curve. Also from NN curve given by equation (5.2.11), we have

$$\frac{(1-\alpha)L_N}{a_N} > g \geq \frac{(1-\alpha)L_N}{a_N} - \theta\alpha.$$

The inequality mentioned above and equation (5.2.9) together imply that the existence of the wide gap equilibrium (with both g and m positive) is ensured if

$$\frac{(1-\alpha)L_N}{a_N} > \frac{H_S.(\Omega) - \frac{\alpha(1-\delta)}{1-\alpha}a_S\theta}{a_S(\frac{\alpha(1-\delta)}{1-\alpha}+\Omega)} > \frac{(1-\alpha)L_N}{a_N} - \theta\alpha. \qquad (5B.1)$$

If the inequality mentioned above is satisfied then SS curve and NN curve must have a unique point of intersection in the figure 5.1.

Appendix 5.3

Slope of NN curve and XX curve

Equation (5.2.19) represents XX curve. From this equation, we have

$$\frac{L_N - a_N g}{H_S - a_S g}(\frac{m}{g}) = \alpha^{\varepsilon}.(\Omega)^{-\frac{1}{\rho}}.$$

Differentiating its both sides with respect to g we have

$$\frac{-a_N H_S + a_S L_N}{(H_S - A_S g)^2}.\frac{m}{g} + \frac{L_N - a_N g}{H_S - a_S g}(\frac{dm}{dg}\frac{1}{g} - \frac{m}{g^2}) = 0,$$

or,

$$\frac{dm}{dg}\bigg]_{XX} = \frac{m}{g} + \frac{\frac{H_S}{a_S} - \frac{L_N}{a_N}}{\frac{H_S}{a_S} - g}\frac{m}{\frac{L_N}{a_N} - g},$$

or,

$$\frac{dm}{dg}\bigg]_{XX} = \frac{m}{g} + \frac{m}{\frac{L_N}{a_N} - g} - \frac{m}{\frac{H_S}{a_S} - g}. \qquad (5C.1)$$

Equation (5.2.11) represents NN curve. From this equation, we have

$$(L_N - a_N g)(1 + \frac{m}{g}) = (\theta + m + g)a_N\frac{\alpha}{1-\alpha}. \qquad (5C.2)$$

Differentiating its both sides with respect to g, we have

$$-a_N(1 + \frac{m}{g}) + (L_N - a_N g)(\frac{\frac{dm}{dg}g - m}{g^2}) = (1 + \frac{dm}{dg})a_N\frac{\alpha}{1-\alpha},$$

or,

$$-a_N(1 + \frac{m}{g}) + \frac{L_N - a_N g}{g}(\frac{dm}{dg} - \frac{m}{g}) = (1 + \frac{dm}{dg})a_N\frac{\alpha}{1-\alpha}.$$

From equation (5C.2), we have $\frac{L_N - a_N g}{g} = \frac{\theta+g+m}{g+m}\frac{\alpha}{1-\alpha}a_N$. Using this in the equation mentioned above we have

$$\frac{dm}{dg}(\frac{\theta+g+m}{g+m}-1) = \frac{g+m}{g\alpha} + \frac{m}{g}(\frac{\theta+g+m}{g+m}-1),$$

or,

$$\frac{dm}{dg} = \frac{m}{g} + \frac{g+m}{g\alpha}.\frac{g+m}{\theta}.$$

From equation (5C.2), we have $\frac{g+m}{g\alpha} = \frac{\theta+g+m}{(1-\alpha)(\frac{L_N}{a_N}-g)}$. Using this in the equation mentioned above we have

$$\frac{dm}{dg} = \frac{m}{g} + \frac{1+\frac{g+m}{\theta}}{(1-\alpha)(\frac{L_N}{a_N}-g)}.(g+m),$$

or,

$$\left.\frac{dm}{dg}\right]_{NN} = \frac{m}{g} + \frac{m}{(1-\alpha)(\frac{L_N}{a_N}-g)} + \frac{1}{1-\alpha}\frac{g+\frac{(g+m)^2}{\rho}}{\frac{L_N}{a_N}-g}. \tag{5C.3}$$

From equations (5C.1) and (5C.3), it is now clear that

$$\left.\frac{dm}{dg}\right]_{NN} > \left.\frac{dm}{dg}\right]_{XX}$$

for any given common values of m and g. Hence the slope of XX curve drawn in the in the figure 5.2 exceeds that of NN curve. This ensures that their intersection point is unique when it exists.

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