EMPIRICAL DETERMINATION AND FORECASTABILITY OF FOREIGN EXCHANGE RATE OF INDIA

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Dedicated to

My Grandfather

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Contents

CHAPTER 1

Introduction

The first chapter of this thesis begins with a brief review of the existing literature on foreign exchange rate models and their forecasting performance. Thereafter it presents the motivation as well as the main aspects of this study. The format of this chapter is as follows. A brief review of the relevant literature is presented in the first section. This review includes the important theoretical / structural as well as time series models of exchange rate. The motivation of the thesis is discussed in Section 1.2. Section 1.3 presents a brief account of the Indian economic reforms since 1993 with special emphasis on those carried out in case of foreign exchange rate. A brief review of the empirical studies on India's foreign exchange rate is also given in this section. Finally, the focus and format of the thesis is described in Section 1.4.

1.1 A brief review of the models on foreign exchange rate

In economic debates, foreign exchange rate or exchange rate, in short, is always singled out as one of the most important economic and financial variables for an economy. Given the existence of separate national currencies, there is an evident need for the conversion of one currency into another when goods and services are traded internationally and international capital transactions across various countries occur. The foreign exchange rate is defined as the price of one country's money in terms of that of another country. Thus, it is a means of comparison of prices of goods and services produced in different countries. A basic justification of a foreign exchange market is, therefore, to permit the conversion and transfer of funds between nations in the most efficient way possible. Now, it is worth stating that while exchange rate is an important variable for all countries, it is all the more so for the developing as well as emerging ones. These countries, by virtue of their weak currency status, are often affected most in case of an external event, and hence a stable currency is very important for such countries to build confidence in the economy. In fact, some experts have argued that exchange rate policies pursued by some developing countries in the late 1970s were inappropriate, and this caused acute overvaluation of their currencies and ultimately contributed to their debt crises. Such overvaluation may reduce exports, harm agriculture and generate destabilizing capital outflows in the developing countries. Issues like energy crisis which became prevalent in the 1970s, stimulated a new interest in matters of exchange rate adjustment and behaviour to external shocks since oil was being imported on a large scale by most of the countries including the developing ones.

Prior to World War II, the 1930s saw a period of flexible exchange rates marked by high volatilities and competitive exchange rate policies. On December 27, 1945, the Bretton Woods conference of representatives from advanced countries agreed to begin a period of pegged, but adjustable exchange rates. It was believed that a more stable system of exchange rates would promote the growth of international trade. It was agreed that the par value of each member should be expressed in terms of gold as a common denominator or in terms of the US dollar. Furthermore, the maximum as well as the minimum rates should not differ from the parity by more than one per cent. The national banks were to maintain reserves to buy or sell dollars for their domestic currencies, thus making it i.e., the US dollar, the official intervention currency. Under the Bretton Woods system of adjustable par value, all countries signing the treaty were required to adhere to the declared par values of their currencies which could only be altered to correct a fundamental disequilibrium, and that too only after consultation with the International Monetary Fund (IMF). However, the shortcomings of such a system were soon felt and the member countries started withdrawing from such a system. This withdrawal was characterized by the desire to achieve greater independence of the domestic monetary policy and to reduce the impact of American economic policies on their economies.

Now, with the demise of Bretton Woods system, large industrialized countries floated their exchange rates. Such floating regimes provided economists with empirical data sets to resolve various academic debates which were related to suitable modelling of exchange rate variable. A comprehensive review of the literature which focuses primarily on exchange rate determination and prediction can be found in the existing surveys of MacDonald and Taylor (1989,1992,1993a), MacDonald (1990a,b), Grossman and Rogoff (1995), Taylor (1995) and Sarno and Taylor (2002). There are some other references too, like, for instance, Dornbusch (1987), Boughton (1988), Kenen (1988), Krugman (1993), Meese (1990) and Mussa (1990) which, however, concentrate on more selective perspectives.

1.1.1 Structural models and their forecasting performance

Economists have imputed a lot of importance on theoretical exchange rate models. Over the years, a large number of such models have been developed. These models are based primarily on the relationship between exchange rate and relevant macroeconomic variables, and usually referred to as structural models. The literature on this class of models being quite substantial, we are mentioning only the important ones in this survey.

The earliest models were based on the Keynsian approach and developed initially by Lerner (1936), Metzler (1942a, 1942b), Harberger (1950), Laursen and Metzler (1950) and Alexander (1952). These models involve the elasticity of demand for and supply of exports and imports as well as demand and supply of foreign currency. Other works such as those supporting the fixed exchange rate system (Nurkse (1944)) and the flexible exchange rate system (Friedman (1953)) also emerged during this time. During the same period, Meade (1951) introduced the Keynsian income-expenditure model which came to be considered as an important contribution to this literature. A major advancement in exchange rate modelling took place in the early 1960s, primarily due to Mundell (1961, 1962, 1963) and Fleming (1962). They extended the Keynsian model by introducing capital flows in the analysis.

During the 1970s, there was a shift in exchange rate modelling towards what is called the asset market approach. In this approach, the exchange rate is perceived as the relative price of two currencies and the price is determined by the relative demand of these currencies *vis a vis* other currencies. This demand is based on the currency’s utility as a medium of exchange, store of value and unit of account. There are some assumptions needed for the validity of such models, the main being that the capital is perfectly mobile internationally so that there are no regulations on international finance. The other assumption is that the interest rate parity condition, which evolves when the expected foreign exchange gain from holding one currency rather than another must be just offset by the opportunity cost of holding funds in this currency rather than the other, must hold. A central feature of the asset market approach is the notion of rational expectations, which implies that all relevant and currently available information are used by agents when making economic decisions.

The assumptions concerning the substitutability of domestic and foreign securities lead to the dichotomy of the asset models to the monetary as well as portfolio models. In monetary models, domestic and foreign securities are assumed to be perfect substitutes but, in contrast, the portfolio models treat domestic and foreign securities as imperfect substitutes.

The monetary model, considered as the workhorse of international finance, can also be viewed as an extension of the quantity theory of money (Cagan (1956)) in an open economy. The two important monetary models which have found prominence in the literature are the 'sticky-price monetary model', due to Dornbusch (1976,1983,1987) and Frankel (1979, 1981) and the 'flexible-price monetary model' (see Bilson (1978, 1979) and Frenkel (1976) and Frenkel and Johnson (1978), for details on such models). Some economists have tried to extend these models in several directions. The most relevant of these has been the one by Hooper and Morton (1978, 1982), where they have attempted at extending the Dornbusch-Frankel model by incorporating the effects of current account. Further, there is another important model, called the 'portfolio balance' model which was originally due to Tobin (1969). This model has been made popular by Kouri (1976), Branson (1977), Girton and Henderson (1977) and Allen and Kenen (1980), among others (see Branson and Henderson (1985), for details). These models consider that the domestic and foreign securities are imperfect substitutes and that changes in expected yields and risks associated with different securities lead to portfolio diversification and wealth redistribution, which, in turn, affect the exchange rate.

In a classic study by Meese and Rogoff (1983 a,b), the forecasting performance of a variety of structural as well as nonstructural exchange rate models has been examined. From the asset model literature, Meese and Rogoff (1983a) selected three models – the flexible-price and sticky-price monetary models and the Hooper-Morton model. They used the quasi reduced form specifications of all the three models and subsumed these into one general specification given by:

$$s_t = a_0 + a_1(m_t - m_t^*) + a_2(y_t - y_t^*) + a_3(r_{st} - r_{st}^*) + a_4(\pi_t^e - \pi_t^{e^*}) \\ + a_5\overline{TB_t} + a_6\overline{TB_t}^* + u_t. \tag{1.1}$$

Here s_t, the dependent variable, is the logarithm of the price of the foreign currency at time point t, $m_t - m_t^*$ is the logarithm of the ratio of the domestic money supply to the foreign money supply, $y_t - y_t^*$ is the logarithm of the ratio of domestic to foreign real income, $r_{st} - r_{st}^*$ is the short term interest rate differential and $\pi_t^e - \pi_t^{e^*}$ is the expected long-run inflation differential. $\overline{TB}_t$ and $\overline{TB_t}^*$ represent the cumulated domestic and foreign trade balances, respectively and u_t is the disturbance term. This model has been considered as the representative structural model and its parameters have been estimated and forecasts obtained. Boothe and Glassman (1987) also used a similar specification to test the performance of such a model empirically. However, the model in (1.1) has been criticized on the ground that variables like relative money supply, income and short-term interest differential have been treated as exogenous variables there, although these should be realistically thought of as endogenous variables.

Meese and Rogoff used the structural model in (1.1) and compared the forecasting performance of this model with several nonstructural models. These models include univariate time series models involving a variety of prefiltering techniques such as differencing, deseasonalizing and trend removing methods. Further, they used the random walk model with and without a drift parameter and also an unconstrained vector autoregression (VAR) model. The VAR, used by them, is composed of the variables in equation (1.1). They showed that the structural models, in particular, failed to improve on the random walk model. These models predicted much worse, especially at one month horizon, if serial correlation was not accounted for. After this startling result, a large number of studies emerged, and each of these has tried to either corroborate Meese and Rogoff's findings favoring the random walk model, or discarding it stressing the relevance of economic fundamentals such as money supply and real income in determining exchange rate behavior. Even after 20 years of hindsight, the Meese-Rogoff results have not been convincingly overturned (see, in this context, Neely and Sarno (2002) and Cheung *et al.* (2003)).

Some authors have pursued complex structural models as well as sophisticated econometric estimation techniques in their attempts to overturn these profound negative results on structural models. For instance, Meese and Rose (1991) made such an attempt where they used a variety of nonlinear and nonparametric techniques in the context of these structural models. However, they could neither improve upon nor explain the poor forecasting performance of these structural models. A recent work by Qi and Wu (2003), where a neural network model with market fundamentals has been used, has found that such a model cannot beat the random walk model in out-of-sample forecast accuracy. Abhyankar *et al.* (2005) have stressed on the economic value of predictability rather than the statistical measures which were used for comparing the forecasting performance of these models. Very recently, Hong *et al.* (2007) have used intra-day data to see whether random walk model can be outperformed or not.

Although the main finding of Meese and Rogoff is quite robust, some researchers have actually found models whose out-of-sample forecasting performance improves over the random walk model (see, for instance, MacDonald and Taylor (1993b, 1994), Finn (1986), Mark (1995) and MacDonald and Marsh (1997)). Further studies like those done by Hogan (1986) and Kim and Mo (1995) have shown that while time series models may be superior in short-run, structural models may perform quite well over long-run. Also, there have been evidences as well that if structural models are generalized to include lagged adjusted mechanisms (see, for instance, Somanath (1986) and Edison (1991)) or in case their parameters are allowed to vary over time i.e., by introducing equation dynamics, as in Wolff (1987), Schinasi and Swamy (1989), Koedijk and Schotman (1990), De Arcangelis (1992) and MacDonald and Taylor (1993, 1994), their forecasts can be somewhat improved.

While some of the inference procedures applied and some of the results on robustness in this wave of post Meese-Rogoff papers are questionable (see Kilian (1999), Berkowitz and Giorgianni (2001), Berben and Van Dijk (1998)), recent studies have shown that it is possible, *albeit* difficult, to beat the random walk model. Also, critical examinations of most of the studies which claim improvement over the random walk model in out-of-sample forecasting performance, have later shown them to be quite fragile.

One possible explanation given for the dismal performance of these structural models is that these models of exchange rate determination are essentially inadequate from consideration of economic theory. Such an interpretation, however, is against deeply held 'beliefs' among many economists. A more charitable interpretation is that the theory is fundamentally sound but its empirical implementation in the framework of a linear statistical model is flawed. From this perspective, structural models of exchange rate imply long-run equilibrium conditions only, toward which the economy may adjust in a nonlinear fashion. Indeed there have been recent studies which show that there are nonlinearities in adjustment from deviations of the exchange rate from the economic fundamentals (Balke and Fomby (1997), Taylor and Peel (2000), Taylor *et al*. (2001) and Kilian and Taylor (2003)). Due to the presence of nonlinear relationship, the use of linear models (to capture this relationship) results in poor forecasting performance. A theoretical model has been developed by Krugman (1991), called the target zone model, where the central bank enforces a known and credible band within which the exchange rate is allowed to move and intervention occurs to keep the exchange rate from reaching the edges of the band. This is assumed to deliver the nonlinear dynamics to the exchange rate. Hsieh (1992) has used this model and assumed that intervention takes place only when the change in exchange rate is large. Some other papers which highlight the importance of nonlinear adjustment of the exchange rate to the value implied by fundamentals include those by Michael *et al*. (1997), Obstfeld and Taylor (1997), Taylor *et al.* (2001) and Kilian and Taylor (2003). More recently, researchers have argued that many exchange rate models actually have the implication that exchange rates should follow a random walk. They have concluded that the criterion of whether or not the model is useful for forecasting out-of-sample is not always a valid basis for judging the models (see Engel and West (2005) and Engel *et al.* (2007), for further details).

1.1.2 Time series models for exchange rate

In this brief review on exchange rate models and their forecasting performance, we have, so far, summarized the important studies on structural models along with those which compared their forecasting performances against a particular time series model called the random walk model. In this section, we present a summary of the different time series models, including the nonlinear ones, which have been extensively applied in determination and prediction of foreign exchange rate.

Time series modelling is a rapidly evolving field and naturally it has found wide applications in case of economic and financial variables. In the particular case of exchange rate variable, time series model has been used extensively. In fact, the development of time series models and their subsequent use in exchange rate modelling has, over the last quarter of a century, taken a prominent place in the literature. It all began, as already stated in the previous section, with the classical work by Meese and Rogoff (1983 a,b) who showed that a simple random walk model performed better than complex structural models in terms of out-of-sample forecasting. This important finding motivated a large number of economists to use time series modelling for exchange rate variable.

An initial explanatory technique which takes precedence over more complex model building is to consider the univariate time series modelling. The most important model in this category is known as the autoregressive moving average (ARMA) model. Box and Jenkins (1970) and Harvey (1981), among others, have popularized the use of this model. This model requires the assumption of covariance (weak) stationarity of the time series. But many economic variables, including exchange rate, have been found to be nonstationary. Box and Jenkins (1970) recommended differencing the time series to achieve stationarity and then using the ARMA model for the stationary series thus obtained. It is, therefore, essential that a test for stationarity be carried out before ARMA modelling is done. The most widely used test for stationarity is known as the augmented Dickey Fuller (ADF) test (Fuller (1976), Dickey and Fuller (1979, 1981) and Said and Dickey (1984)). Other such tests, also known as the unit root tests, are due to Phillips (1987), Phillips and Perron (1988) and Schmidt and Phillips (1992).

One distinctive feature that has been observed in most economic and financial time series, including the foreign exchange rate series, is the presence of nonlinear dependences, especially the second order dependence. A few noted evidences of such dependences in the context of exchange rate series are: Hinich and Patterson (1985), Scheinkmann and LeBaron (1989), Hsieh (1989, 1991), Crato and de Lima (1994) and Brooks (1996). This empirical finding has led to the development of nonlinear time series models where the nonlinearity is in the conditional variance. In fact, over the years, several nonlinear time series models have been proposed to describe the dynamic behavior of many economic and financial variables, and this development has been primarily in two aspects, *viz*., nonlinearity in the conditional mean function and nonlinearity in the conditional variance specification. The model capturing nonlinearity in conditional variance is well known as the autoregressive conditional heteroscedastic (ARCH) model (Engle (1982)). It has been observed that large changes of the financial asset prices tend to be followed by large changes of either sign and likewise small changes tend to be followed by small changes of either sign. This behavior, called the 'volatility clustering', is described by the ARCH process. There have been generalizations and extensions of this basic ARCH model (see, for details of some such generalizations, Bollerslev (1986), Nelson (1991), Engle *et al*. (1987), Bera and Higgins (1993) and Sarkar (2000)).

While Bollerslev's (1986) generalization called the GARCH model, is similar to that of the AR process being generalized to the ARMA process, the most important of these extensions / generalizations is due to Nelson (1991) who proposed the exponential GARCH (EGARCH) model. This model takes care of what is known, especially in the context of stock market, as 'leverage effect' which essentially states that past returns and volatility are negatively correlated. Glosten *et al.* (1991) have also proposed another formulation to deal with this asymmetric behavior between volatility and past returns. Zakoian (1990) has proposed an extension which is called the threshold ARCH (TARCH) model. In most of the empirical studies concerning time series data on exchange rate of developed economies, the GARCH form of conditional heteroscedasticity has been found to be adequate (see, for example, Domowitz and Hakkio (1985), Engle and Bollerslev (1986), Milhoj (1987), Diebold (1988), Hsieh (1988,1989), McCurdy and Morgan (1988), Baillie and Bollerslev (1989), Bollerslev (1990) and Bekaert (1992) and for a survey see Bollerslev *et al.*(1992)). In this context, it is worthwhile to note that in some such studies (*cf*. Baillie and Bollerslev (1989), Diebold (1988) and Hsieh (1989)), the sum of estimates of all the parameters excluding the intercept of conditional variance has been found to be close to unity, suggesting that the (G)ARCH model may not be the most appropriate one for explaining the volatility of returns on exchange rates. In this context, it maybe noted that in case the sum of the parameters, excluding the constant, of any (G)ARCH model is exactly unity, the unconditional variance then becomes infinity and this is very much an empirical possibility, as noted by Mandelbrot (1963). In such a situation, the (G)ARCH model is called the integrated (G)ARCH (I(G)ARCH) model, and this model has been found to be appropriate for few exchange rate series as well.

While most of the (G)ARCH models applied to exchange rate series have used weekly or daily level data, some of the very recent works are based on intra-day data. High frequency data are now available for exchange rate series as well. Consequently, there has been a spurt in studies with, say, hourly or even higher frequency data. Some references on studies with intra-day exchange rate series are: Engle *et al.* (1990), Baillie and Bollerslev (1991), Andersen and Bollerslev (1998), Chang and Taylor (1998, 2003), Malik (2005), and Hua and Gau (2006). It is also worth mentioning that, in recent years, there have been some developments towards nonparametric volatility models as well (see Andersen *et al*. (2005) and Linton and Mammen (2004), for details on such models).

While the class of (G)ARCH models and its various extensions / generalizations describe the nonlinear behaviour of conditional variance of the series, the other class of nonlinear models is designed to capture the nonlinearity in the conditional mean specification in a very particular way. This class of time series models defines states of the world or regimes and allows for the possibility that the dynamic behaviour of economic variables depends on the regime that occurs at any given point in time. The so-called ‘state dependent dynamic behavior’ (Franses and Van Dijk (2000)) means that certain properties of the time series, such as mean, variance, autocorrelation are different for different regimes. These models, called the regime-switching models, differ in the way the regimes evolve over time. There are basically two kinds of models in this category of nonlinear time series models. The first kind assumes that the regimes can be characterized by an observable variable while the models under the second assume that the regimes cannot be actually observed but can be determined by an underlying unobservable stochastic process. The first type of models are called the threshold autoregressive (TAR) models (see, for instance, Tong and Lim (1980), Tong (1978, 1983, 1990), Chan and Tong (1986) and Tsay (1989)). When these regimes are determined by the variable itself then the models are called self-exciting TAR (SETAR) models. The SETAR model has found wide applications in modelling exchange rate series, particularly in the environment of what is called 'managed floats'. One of the earliest applications of this model is due to Kräger and Kugler (1993) who reported the results of application of the SETAR model to weekly exchange rates of five currencies of developed economies. Chappell *et al.* (1996) have used this model to explain the behaviour of exchange rates of some of the European countries. Some other notable references are: Peel and Speight (1994), Brooks (1996, 1997, 2001), Clements and Smith (1999, 2001), Dacco and Satchell (1999) and Boero and Marrocu (2002, 2004). Most of these works have compared the forecasting performance of a range of nonlinear models. For instance, Clements and Smith (1999) have compared the multi-period forecasting performance of a number of empirical SETAR models using time series data on exchange rate. Boero and Marrocu (2002) have studied the relative performance of nonlinear models like the SETAR and GARCH models as contrasted with other linear counterparts for returns on three most important exchange rates in terms of US dollar, namely, the French franc, the German mark and the Japanese yen. Some of these studies have produced evidence of forecasting gains from nonlinear models as compared to linear specification, although there is no clear evidence in favor of nonlinear models insofar as out-of-sample accuracy is concerned.

The SETAR model assumes that the border between the two regimes is given by a specific value of the threshold variable. Such a feature is characterized by an indicator function in the model. However, a more gradual transition between the different regimes can be obtained if the indicator function is replaced by a continuous function. The resultant model is called the smooth transition autoregressive (STAR) model. While the use of SETAR models in nominal exchange rate modelling is considerable, the number of studies using the STAR model is less. Boero and Marrocu (2002) is such a study where the out-of-sample forecasting performance of various nonlinear models, including SETAR and STAR have been compared. Medeiros *et al.* (2001) have used artificial neural network (ANN) model as well as neuro-coefficient smooth transition autoregression which nests the SETAR, STAR and ANN models and compared the different alternatives to model and forecast the monthly exchange rate series of some countries including India. Studies by Micheal *et al.* (1997), Taylor *et al.* (2001), Holmes (2004) and Baharumshah and Liew (2006) and Rapach and Wohar (2006b) have employed the STAR model to study the nonlinear dynamics.

The second type of regime-switching models implies that one can never be certain about the regime the variable is in at a particular point in time, but can only assign probabilities to the occurrence of different regimes. One important model which falls in this class is the Markov switching regression (MSR) model (Goldfeld and Quandt (1973) and Hamilton (1989)). MSR model has been used quite extensively for modelling foreign exchange rate. Regime switching in foreign exchange rate has been documented by Engel and Hamilton (1990), Bekaert and Hodrick (1993), Engel (1994), Engel and Hakkio (1996), Bollen *et al.* (2000), Marsh (2000) and Frömmel *et al.* (2005). Engel (1994) has fitted the MSR model to 18 exchange rates, including 11 non-U.S. dollar exchange rates, at quarterly frequencies and shown that the MSR model fits well in-sample for many exchange rates. However, using mean squared error criterion they have found that the MSR model does not generate forecasts which are superior to random walk model. Marsh (2000) has used a two-state MSR model for daily exchange rate data with interest rate differentials as the only fundamental and concluded that the approach does not provide superior forecasts compared to other time series models. Clarida *et al*. (2001) have applied the MSR multivariate model to weekly spot and forward rates and concluded that allowing regime switching in error correction framework provides forecasts which outperform linear model. Bollen *et al.*

(2000) have used an augmented form of the standard MSR model to allow two regimes for the mean and two regimes for the variance of log exchange rate changes and found that a model with independent mean and variance shifts provides tighter in-sample fit and more accurate variance forecasts.

There have been major developments in multivariate time series analysis as well. Starting from simple predictive regression models where all the variables are converted into stationary variables and analysis made thereafter, recent studies have used the vector autoregression (VAR) analysis where all the variables are studied in their level forms. In situations where all the variables are integrated of order 1, more generally, integrated of the same order, researchers and analysts have applied the methodology of cointegration introduced by Engle and Granger (1987), to obtain the long-run cointegrated relation(s) involving the variables as well as the short-run dynamics, as captured through the vector error correction model (VECM). There have been some, but not many, applications of this time series methodology involving foreign exchange rate of some developed economies and relevant economic and financial variables. A few references of such studies are Masih and Masih (1996), Kumah and Ibrahim (1996), Nagayasu (2004), Phylaktis and Ravazzolo (2005), and Kasman and Ayhan (2007).

The recent literature on time series modelling of economic and financial variables also involves the use of nonparametric approach. In this context, it is relevant to mention that the most popular of such models is the artificial neural network (ANN) model. Insofar as the application of ANN model in the case of foreign exchange rate is concerned, mention may be made of Diebold and Nason (1990) who were one of the earliest researchers to use nonparametric methods for estimating the conditional expectation of exchange rate. Since then there have been few other studies which have tried to use nonparametric methods for modelling and predicting exchange rates. Some recent references include those of Trippi and Turban (1993), Azoff (1994), Kuan and Liu (1995), Refenes (1995), Gately (1996), Brooks (1997), Franses and Van Griensven (1998), Franses and Van Homelen (1998) and Gencay (1999)(also see Qi (1996), for a survey). ANN models have become popular because these are able to approximate almost any nonlinear function arbitrarily close. However, the main drawback of such models is that the parameters of these models are difficult to interpret. Because of this difficulty in assigning meanings to the parameter values, these models are often considered as black box models, and constructed mainly for the purpose of pattern recognition and forecasting. Though in-sample fits have often been found to be superior, there is no guarantee that this class of models performs well in out-of-sample forecasting. Further, the possibility of overfitting is a serious drawback with such models.

1.2 Motivation

The literature on empirical modelling of the time series of foreign exchange rate is quite significant, but most of these involve the exchange rate of developed economies. It is somewhat surprising that with an explosion of research in this area, the number of studies on this topic concerning developing and emerging economies, not to talk of underdeveloped or poor economies, is very few. Since foreign exchange rate is one of the most important economic and financial variables for any economy, especially an emerging one, it is quite natural that detailed studies on different aspects of this series should be very useful from consideration of not only academic research but also policy decisions on the part of the government concerned. It is, therefore, only very natural that researchers would make attempts to formulate models on foreign exchange rate determination, which would be meaningful from consideration of economics and finance and also econometrically appropriate, and which would work well in estimation, forecasting and policy-making.

Now, one of the most important objectives of any study on modelling of time series is forecasting. In particular, for foreign exchange rate, there are several important purposes for forecasting. Some of these are the following: (i) to earn income from speculative activities, (ii) to determine optimal government policies, (iii) to base scientific judgments on outcomes of predictions, and (iv) to make business decisions. Financial decisions often involve long-run commitments of resources, the returns to which will depend on what happens in future, and hence accuracy of forecasts is extremely important for policy considerations. Since there are many international transactions that do not require immediate settlements, there are provisions of contractual arrangements for extension of credit and subsequent payments for the obligations involved. A prior knowledge on the behavior of exchange rate can actually help in such deals.

Before proceeding further, it may be relevant to state why studies on emerging market economies which are defined as economies with low-to-middle per capita income, should be very useful not only for the EMEs but also for the developed economies. Economies are usually considered emerging because of their developments and reforms, and such emerging economies constitute 80% of the world population and represent 20% of the world economy. Countries belonging to this category embark on economic development and reform programs, and open up their markets and "emerge" into the global scene. EMEs are considered to be fast growing economies and are characterized as transitional - meaning that these are in the process of moving from a closed to an open market economy while building accountability within the system. An EME embarks on an economic reform program that will lead it to stronger and more responsible economic performance levels as well as transparency and efficiency in all the important sectors, but most importantly from EME’s point of view, in the capital market as well as in the exchange rate market. An EME reforms its exchange rate system because a stable local currency builds confidence in the economy, especially when foreigners are considering investing. Exchange rate reforms also reduce the desire for local investors to send their capital abroad. One key characteristic of the EME is an increase in both local and foreign investment (portfolio and direct). A growth in investment in a country often indicates that the country has been able to build confidence in the local economy. Moreover, foreign investment is a signal that the world has begun to take notice of the emerging market, and when international capital flows are directed toward an EME, the injection of foreign currency into the local economy adds volume to the country's stock market and long-term investment in the infrastructure. For foreign investors or developed-economy businesses, an EME provides an outlet for expansion by serving, for example, as a new place for a new factory or for new sources of revenue. For the recipient country, employment levels rise, labor and managerial skills become more refined, and a sharing and transfer of technology occurs. In the long-run, the EME's overall production level should rise leading to increase in its gross domestic product and eventually lessening the gap between the emerged and emerging worlds.

Given this extremely important role of foreign exchange rate variable in shaping the future of an EME- from the emerging / developing status to the developed one – more and more studies concerning determination and predictability of exchange rate need to be undertaken. Such studies are all the more necessary because during the phase of transition from underdevelopment to development, it is likely that important economic and financial variables including exchange rate, would possess, at least to some extent, characteristics similar to those of developed economies while retaining at the same time some features of underdevelopment as well. And in that case, the data generating processes (DGP) of such variables would possibly become more complex reflecting features underlying the dynamics of this transition, and hence it would be useful as well as academically interesting to undertake such empirical studies.

This thesis has been basically motivated by the fact that comprehensive, detailed and methodologically sound empirical studies on modelling and forecastability of a very important economic and financial variable like foreign exchange rate is almost non-existent for important emerging market economies whose importance in world economy can no longer be ignored. The thesis is concerned with such a study for one of the most important emerging market economies (EME) with huge growth potential, called India. As reviewed in the next section, despite the growing importance of India as a major economic power, studies on exchange rate modelling for India are very few, and even those are very limited in their approaches and scopes. This relative dearth[1] of sound empirical work is indeed the motivation behind this thesis. In other words, the aim of this thesis is to make a comprehensive empirical study on modelling and forecastability of the Indian rupee/US dollar exchange rate series. Obviously, this calls for exchange rate determination using various linear and nonlinear time series models, and then compare their performances in terms of different forecasting criteria. Since 'good' forecasts requires, *inter alia*, that the underlying model is appropriately

[1] It is only very recently i.e., in 2006 and 2007, to be precise, that we find that some such studies have been undertaken, and this is certainly encouraging.

specified, and accordingly the thesis gives due importance to the issue of specification. It is also noteworthy, in this context, that incorrectly / inappropriately specified conditional mean might as well lead to misspecification of conditional variance. In fact, conditional variance specification would be correctly specified if there is no serial correlation. There are also other issues like choice of appropriate macroeconomic variables, and short-run and long-run predictability in such studies which require proper understanding from consideration of both economics and econometrics. All throughout the thesis, attempts have been made to take care of such issues and then use appropriate econometric techniques to deal with them, especially those relating to appropriate specification which includes issues like trend shift, selection of inappropriate lag length, parameter instability, residual autocorrelation and omitted variables.

1.3 India's foreign exchange rate scenario

As one of the most important emerging economies having a population size of around 1114 million in 2006, India is poised to be a major economic power in the near future. This turn-around began in the early 1990s when India had embarked on a series of structural and regulatory reforms in its economy to free itself from extremely fragile economic conditions arising primarily due to prevalence of mixed economy dominated by public sector, extreme bureaucratic red-tapeism, sluggish growth, foreign exchange crisis etc. India has eventually moved to the path of liberalization which has allowed bigger foreign participation as witnessed by the increase in foreign investment inflow from 103 million US dollars in 1990-91 to 20,243 million US dollars in 2005-06[2].

1.3.1 India's exchange rate regime

In the 1950s and 60s, i.e., during the early years after India's independence in 1947, Indian officials believed that trade was biased against the developing countries and

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[2] In terms of other important macroeconomic variables as well, the performance of Indian economy after liberalization is very positive although, in terms of social indicators, the achievement is still very moderate. The data concerning these variables and indictors are available, inter alia, in National Accounts Statistics as well as in the websites of the Central Bank of India, called the Reserve Bank of India and Central Statistical Organization, Government of India.

that prospects for exports were severely limited. Therefore, the government aimed at self-sufficiency in most products through import substitution, with exports covering the cost of residual import requirements. Foreign trade was subjected to strict government controls, which consisted of an all-inclusive system of foreign exchange and direct controls over imports and exports. Largely because of oil price increase in the 1970s, which contributed to balance of payments difficulties, the Indian government in the 1970s and 80s placed more emphasis on the promotion of exports. They hoped exports would provide foreign exchange needed for the import of oil and high-technology capital goods. Nevertheless, in the early 1990s, India's share of world trade stood at only 0.5 percent. Because foreign exchange transactions were so tightly controlled, Indian authorities were able to manage the exchange rate, and from 1975 to 1992 the Indian currency, called the Indian rupee, was tied to a trade-weighted basket of currencies. In February 1992, the government began its move to make the rupee convertible. In India, partial convertibility of rupee was introduced in March 1992 through a dual exchange rate system, known as the Liberalized Exchange Rate Management System (LERMS). In July 1995, Rs 31.81 were worth one unit of US dollar, compared with Rs 7.86 in 1980, Rs 12.37 in 1985, and Rs 17.50 in 1990. The stability imparted by LERMS resulted in a smooth change-over to a regime under which the day-to-day movements in exchange rates were market determined.

The movement to market determined exchange rate was accompanied by convertibility on current account and a cautious approach to capital account liberalization. In March 1993, a single floating exchange rate was implemented. Restrictions on current account convertibility were relaxed in a phased manner till August 20, 1994. With a view to promoting orderly development of foreign exchange markets and facilitating external payment in a liberalized regime, the Foreign Exchange Management Act (FEMA) was introduced from June 1, 2000 replacing the earlier Foreign Exchange Regulation Act (FERA). The FEMA is consistent with full current account convertibility and contains provisions for progressive liberalization of capital account. This liberalization experience of India has been studied by many economists (Kohli (2000) and Kohli and Kletzer (2001), to name a few). This experience has been compared to those of other countries, and it has often been termed as a success in terms of its policies. For instance, Williamson (2000) has compared the liberalization experience of India with that of New Zealand, and praised India’s success story.

Following liberalization, India today has, as in most of the countries, an ‘intermediate regime’, which lies between the two textbook versions of fixed and flexible regimes. The exchange rate is partly managed and a scrutiny of the exchange rate management strategy of the Reserve Bank of India (RBI), which is the Central Bank of this country, reveals a strong commitment to exchange rate stability. Kohli (2000) has argued that RBI keeps the exchange rate aligned to its fundamentals, the most important one being the price level. Ghosh (2002), has however, noted that even though the RBI has not deemed it feasible to pursue exchange rate targeting, there is indeed some definitive targeting by the RBI based on value of purchasing power parity (PPP). In May 1997, the ‘Tarapore committee report on capital account convertibility’ had recommended the RBI to have a ‘Monitoring Exchange Rate Band’ of +5/-5 percent around the neutral real effective exchange rate (REER) as part of transparent exchange rate policy. The committee suggested that the RBI should intervene when the REER is outside the band and that it should maintain transparency about its intervention. However, the RBI has been highly secretive in its intervention activities and, like most other countries, refuses to release data on intervention on a daily basis.

Other than intervention, the RBI also acts as the banker of last resort where it injects funds into the system to help participants tide over temporary mismatches of funds. This is implemented through the Liquidity Adjustment Facility (LAF) which was made effective from the 5th of June 2000. The system is being implemented in phases, and currently it is a daily exercise in which banks and primary dealers participate. Here the RBI conducts an auction system of repos (the rates at which RBI borrows from the banks) and reverse repos to suck-out and inject liquidity to the market. The exact quantum of liquidity to be absorbed or injected and the accompanying repo and reverse repo rates are determined by the Financial Markets Committee after taking into consideration the liquidity conditions in the market, the interest rate situation and the stance of monetary policy.

Other than these developments, we also note that India, in course of its liberalization, began a pragmatic monetary policy which reacted strongly only when inflation went above 10 percent. The capital account has been heavily controlled although there was some gradual liberalization, especially on the inflow side, during 1990s. Any meaningful study concerning exchange rate of India becomes very relevant since liberalization has allowed bigger foreign participation, as discussed earlier. In this process, the reserves were built up from US dollar 5834 million in the middle of 1991 to 151,622 million in 2005-2006. Thus we see that the liberalization process which was initiated in 1993 has led India to emerge as an important economy, and consequently it is now having its share of discussions and debates on issues relating to appropriate exchange rate systems, policies on intervention, capital control and many others.

1.3.2 Empirical studies on India's exchange rate series

To the best of our knowledge, the first available study on modelling of Indian exchange rate return, where application of GARCH model for volatility has been done, is by Unnikrishnan and Mohan (2001). While they have applied the GARCH model to the nominal effective exchange rate series, Singh (2002) has estimated this volatility model for a comprehensive set of both weighted (export and trade) as well as unweighted (official and black market) real exchange rate series for India.

Insofar as nonlinear time series models are concerned, Sundar (1997) was perhaps the first to undertake such a study, although somewhat sketchy, for India's exchange rate series. The next such study which is quite comprehensive is due to Medeiros *et al.* (2001) who have used several nonlinear time series models to model the Indian monthly exchange rate along with several other series to find out whether these nonlinear models perform better than the autoregressive and random walk models. They have used the artificial neural network model as well as the neuro-coefficient smooth transition autoregression which nests the SETAR, STAR and ANN models, and compared the different alternatives to determine and forecast the monthly exchange rate series. Panda and Narasimhan (2007) have used the ANN model to make one-step-ahead prediction of weekly Indian rupee / US dollar exchange rate, and compared the forecasting accuracy of this model with that of the linear autoregression model.

Holmes (2004) has used logistic as well as exponential STAR models to study the nonlinearities in the behavior of real exchange rates of eleven Asian economies (including India) and found that the extent of nonlinearities varied across the Asian countries, with India and Singapore exhibiting the sharpest transition between regimes. They have found that the logistic STAR model can successfully take care of the nonlinearities of the Indian rupee / US dollar real exchange rate series. In a recent paper, Baharumshah and Liew (2006) have used the STAR model for yen-based currencies of six major East Asian countries and discovered strong evidence of nonlinear mean reversion in deviation from purchasing power parity. They have also shown that the STAR model has outperformed the AR model for their data sets.

There have been some studies where the role of imporant macro variables in determining exchange rates, have been studied. Ghosh (2002) used a Tobit / logit model for studying the role of intervention on exchange rate using daily data. Rao (2000) undertook a study to assess the two-way interactions between business cycles and exchange rate, and the paper provides an analytical framework which, by formalizing the nature of relationships between key macro-economic variables, helps to forecast the exchange rate in the Indian context. In a somewhat different kind of a study, Hasan (2006a) has examined the issue of equilibrium and efficiency of exchange rate in a silver-based monetary system during nineteenth century India and Iran. The results, based on cointegration tests, indicate a reliable long-run relationship between the metallic value and the exchange value of currencies in a silver-based monetary standard. Thomakos and Bhattacharya (2005) have reported the results from a forecasting study for inflation, industrial output and exchange rate for India. They have used the ARIMA, bivariate transfer function model and restricted VAR model for data of different frequencies. Hasan (2006b) has used cointegration-VECM approach to examine the long-run relationship between the exchange rate of silver-based currencies and the intrinsic value of silver in India and Iran in a bivariate model set-up. The results, based on unit root and cointegration tests, indicate a reliable long-run relationship between the price of silver and the exchange rate of silver-based currencies. Vuyyuri (2005) has investigated the cointegrating relationship and the causality between the financial and real sectors of Indian economy using monthly observations of financial variables like interest rates, inflation rate, exchange rate, stock return and industrial productivity with the latter used as a proxy for the real sector.

Ghosh (1998) has used various cointegration tests to examine the validity of the monetary model as a theory of long-run equilibrium condition for the exchange rate of India. Their study offers no evidence of long-run equilibrium relationship among the variables of the monetary model.

Vayyuri and Seshaiah (2004) have studied, using data for the period 1970-2002, the interaction of budget deficit of India with other macroeconomic variables such as nominal effective exchange rate, GDP, consumer price index and money supply, by using cointegration approach and the VECM. The results reveal that the variables under study are cointegrated and there is a bi-directional causality between budget deficit and nominal effective exchange rate. Mishra (2004) has attempted to examine whether stock market and foreign exchange markets are related to each other or not by using the VECM framework on monthly stock return, exchange rate, interest rate and demand for money. He has found that there exists a unidirectional causality between exchange rate and interest rate and between exchange rate and demand for money. However, there is no Granger causality between exchange rate return and stock return. Damele *et al.* (2004) have analyzed the market integration involving the stock market, foreign exchange market and bullion market. Their study shows that stock index and exchange rate have inverse relationship.

1.4 Focus and format of the thesis

We discuss below the focus of this thesis along with the important aspects of its coverage. As already stated in Section 1.2, this thesis primarily aims at carrying out a systematic and comprehensive study on the empirical determination and forecastability of India's foreign exchange rate variable using linear as well as some nonlinear time series models. All throughout, the study tries to deal with all relevant econometric issues like appropriate specification, choice of independent variables and short-run / long-run forecasting in appropriate ways. This study is based on the daily / monthly spot foreign exchange rate (with respect to US dollar) series, covering the period November 1994 to March 2005. We now briefly discuss the broad aspects of this thesis.

(i) Mean and volatility dynamics in the framework of appropriate specification

It is sometimes found that a simple linear dynamic model with appropriate volatility specification performs quite well by standard criteria of model evaluation. Keeping this in mind, the first model considered in this thesis is a linear dynamic model. In this study, due emphasis has been given on appropriate specification of both the conditional first and second order moments so that the final inferences are free from any possible consequences of misspecification of the underlying model. While the issue of appropriate specification is always very important, this is all the more so when the data are at a frequency, the daily level for this study, at which data of most macroeconomic and financial variables are not available, leading to the possibility of omission of variables. While there are several aspects to the general understanding of specification, parameter instability or structural change, to use a broader terminology, is probably the most important one in the context of time series analysis and this affects modelling inferences, if not accounted for appropriately. Using the recent developments in testing for the presence of structural break(s) (Chow (1960) and Quandt (1960), followed by Andrews (1993, 2003), Andrews and Ploberger (1994), Bai (1994, 1997a, b), Chong (1995), Hansen (1997, 2001) and Bai and Perron (1998)), we have examined the existence of break(s) in the daily return on Indian rupee / US dollar spot exchange rate series and found the presence of structural breaks.

That the foreign exchange rate series of India is marked by instability is hardly a surprising result for an emerging economy, and hence in our study we have first determined the break point(s) in the time series, and then accordingly partitioned the entire time period into sub-periods of stable parameters each. Thereafter, we have tried to specify the conditional mean properly for each sub-period. In this context, it is also relevant to note that incorrectly specified conditional mean might as well lead to misspecification of conditional variance. Hence, we have carried out tests for misspecification (Lumsdaine and Ng (1999)) of conditional mean and consequently made the mean specification as adequate as possible before determining an appropriate specification for the conditional variance. As regards the form of the conditional variance, the GARCH form of conditional heteroscedasticity has often been found to be adequate for exchange rate returns of developed economies, (see, for example, Baillie and Bollerslev (1989), Bekaert (1992), Bollerslev (1990), Hsieh (1989) and Milhoj (1987), Domowitz and Hakkio (1985), Engle and Bollerslev (1986), McCurdy and Morgan (1988), Malik (2005), for a survey see Bollerslev *et al.* (1992)). However, for India's exchange rate series, the GARCH model was not found to be suitable; instead the EGARCH specification was found to be the 'best' both in terms of diagnostic tests and out-of-sample forecasting criteria.

(*ii) The threshold autoregressive model*

In nonlinear time series literature, the class of threshold autoregressive (TAR) models (see, for instance, Tong and Lim (1980), Tong (1978, 1983, 1990), Chan and Tong (1986) and Tsay (1989)) is considered to be very important. After linear dynamic model framework, this class of models has been considered for our study. The TAR model has been found to be a very important class of nonlinear time series models, and it has become an integral part of studies relating to time series modelling of exchange rate return. This class of nonlinear models allows a locally linear approximation over a number of states (regimes) so that globally the model is nonlinear. Clearly, these models are important when the observations may be drawn from one autoregressive model in one regime, but a different autoregressive model in another. Tong and Lim (1980) proposed a special case of TAR model where the state-determining variable is the past lags of the variable under study itself, and in that case the model is called the self-exciting TAR or SETAR model. The other special situation where instead of the regime change being taken care of through an indicator function, there is a more gradual transition between the different regimes. The latter is introduced through a continuous function, and the model is thus called the smooth transition autoregressive (STAR) model. There are two types of STAR models depending on the function used for model specification : an exponential STAR (ESTAR) and a logistic STAR (LSTAR). The idea of STAR which dates back to Bacon and Watts (1971), was introduced into the nonlinear time series literature by Chan and Tong (1986) and popularized by Granger and Teräsvirta (1993) and Teräsvirta (1994).

Until very recently, applied researchers had to choose the TAR (in particular, SETAR) model while incorporating nonlinearity in the analysis of financial and economic time series without any consideration to volatility. However, volatility being a very important characteristic of such series, some researchers have attempted to introduce volatility in the framework of TAR models. This has led Tong (1990) to suggest what is now called the SETAR-GARCH model, i.e., the threshold model with a single conditional variance specification. Later Li and Li (1996) have also generalized the threshold autoregressive model to a double threshold ARCH (DTARCH) model where threshold is considered in both the conditional mean and conditional variance.

In our study, we have considered both the SETAR and STAR models along with the SETAR-GARCH and DTGARCH models for the exchange rate return series of India. Out-of-sample forecasts for all these models have been obtained and the performance of these models compared by suitable criteria.

(iii) Markov switching regression model

The Markov switching regression (MSR) model has been popularized by Hamilton (1989), although the essence of these were introduced by Goldfeld and Quandt (1973). This class of models also allows us to take into account multiple structural breaks in a time series and help in explaining the nonlinearities in the data. Along with Engel and Hamilton (1990), regime switching of this kind in foreign exchange rate has been documented by Bekaert and Hodrick (1993), Engel (1994), Engel and Hakkio (1996), Bollen *et al.* (2000), Marsh (2000) and Frömmel *et al.* (2005). These have also been used to explain and date the turning points of the business cycle (Hamilton (1989)). Since one can never be certain about the regime the variable is in at a particular point in time, this class of switching regime models only assigns probabilities to the occurrence of different regimes.

The MSR model can be generalized to include autoregressive as well as autoregressive conditional heteroscedastic processes. The ARCH model, though successful in capturing the volatility of high frequency data, often imputes a lot of persistence to stock as well as exchange rate volatility. It is sometimes useful to allow the parameters of the ARCH process to come from one of several different regimes, with transition between regimes being governed by an unobserved Markov chain. There are two types of parameterization through which this can be accomplished. One of these refers to Brunner (1991) and Cai (1994) who have proposed a model which allows for the possibility of sudden discrete changes in the values of the parameters of an ARCH(q) process, as in the case of MSR model in Hamilton (1989). The other approach is due to Hamilton and Susmel (1994) who have proposed the parameterization of the switching ARCH (SWARCH) kind where changes in the regime are modelled as changes in the scale of the ARCH process.

In the thesis, we have fitted a simple two-state Markov switching regression model similar to the mixture of normal distributions and then considered the more general model where an autoregressive process has been introduced in the conditional mean specification. Later, we have also applied the SWARCH model using the parameterization of Hamilton and Susmel (1994) to the time series of returns on Indian rupee / US dollar exchange rate.

(iv) The role of macroeconomic variables in forecastability of exchange rate return

Here a simple predictive regression model is first applied to find the relevant macrovariables which have predictive ability for return on India's exchange rate at monthly frequency, and then these macrovariables are used to set-up a single equation dynamic regression model for determining the "best" such model from consideration of fitting and prediction. In this exercise, we have closely followed the approach of Rapach *et al.* (1995). In other words, we have used both in-sample and out-of-sample tests of return predictability. While the in-sample analysis employs what is known in statistics as predictive regression approach, the out-of-sample forecasts are analyzed using a pair of recently-developed-and potentially more powerful tests due to Clark and McCracken (2001) and McCracken (2004).

Another aspect to such a study is data mining. Since our interest is in testing the predictive ability of a large number of macro variables in turn, it is only natural that the issue of data mining would arise. The conventional wisdom holds that out-of-sample tests help guard against data mining. However, it has been recently argued that both the in-sample and out-of-sample tests are equally susceptible to data mining and the only way we can account for this data mining problem is by using an appropriate bootstrap procedure. We have followed the bootstrap procedure used by Rapach *et al.* (2005) and Rapach and Wohar (2005), which are originally due to Nelson and Kim (1993), Mark (1995), Kothari and Shanken (1997) and Kilian (1999), to find the macroeconomic variables which significantly explain India's exchange rate return series.

(v) Long run relationship between exchange rate and macroeconomic variables

The models so far discussed are concerned only with short-run perspective having consideration to both determination and forecastability of India's exchange rate return. However, there have also been some studies towards obtaining models in the long-run context for several asset prices. In this thesis, we have also taken up such a study for the exchange rate series of India. Such a study requires carrying out a cointegration exercise involving the exchange rate variable and other relevant macroeconomic variables all of which are $I(1)$ i.e., integrated of order 1 in our study.

Existence of cointegration suggests a long-run equilibrium relationship involving the concerned $I(1)$ variables. The associated vector error correction model (VECM) introduces past disequilibrium as explanatory variables in the dynamic behavior of current variables, and thus can be viewed as the corresponding short-run dynamics of the cointegrated variables.

Other than the use of cointegration analysis for studying the monetary model by McDonald and Taylor (1993b), Kim and Mo (1995), Diamandis *et al.* (1998) and a few others, there have been some studies on long run relations between exchange rate and other macro variables as well. Masih and Masih (1996) carried out such a work where they tried to discern the dynamic causal chain among the real output, money, interest rate, inflation and exchange rate. Kumah and Ibrahim (1996) have used real exchange rate for their cointegration analysis. They have adopted a multivariate data analysis approach to analyze the effects of domestic real (technological) and nominal shocks on the nominal exchange rate and current account balance.

There are also some studies where the effects of some specific variables on exchange rate has been studied using such a framework. For example, Phylaktis and

Ravazzolo (2005) have studied long-run and short-run dynamics between different stock prices and exchange rate. Pan *et al.* (2006) have examined the dynamic linkages between exchange rate and stock prices for seven east Asian countries and found significant causal relations from exchange rate to stock prices but not the other way round. Recently, there has also been some works studying the role of foreign exchange intervention on exchange rate (see, for instance, Nagayasu (2004)). Another recent study by Kasman and Ayhan (2007) has focussed on long-run relationship between exchange rate and foreign exchange reserves.

Using the current state of knowledge on cointegration, we have carried out the cointegration-VECM exercise to find the long-run relationship involving India's foreign exchange rate and relevant macrovariables, and thus understand the predictability aspect of India's exchange rate in the long-run sense.

The other chapters of the thesis have been organized keeping in mind that the thesis contains the analysis of India's foreign exchange rate series using data at two levels of frequency *viz*,. daily and monthly. Since comparison across models using the same data set i.e., the data having the same frequency and span, is meaningful, we have first arranged those chapters, i.e., Chapters 2 through 5 which present results based on linear and nonlinear models with daily-level data, and then in Chapters 6 and 7, we have presented our work relating to monthly-level data analysis.

The other chapters of this thesis have been organized as follows:

CHAPTER 2: Mean and Volatility Dynamics of Daily Exchange Rate Return in the Framework of Linear Model

This chapter of the thesis deals with the issues of determining the most appropriate model for India's exchange rate return at daily level frequency in the framework of linear dynamic model with volatility given by the GARCH specification. The framework of this analysis also involves appropriate specification of both the conditional mean and conditional variance so that the final inferences are free from any possible consequences of misspecification. Further, the out-of-sample forecasting performance of this model has also been studied by standard forecast evaluation criteria.

The chapter is formatted as follows. Section 2.1 gives the introduction to this chapter. In Section 2.2, we discuss the model and briefly describe the methodology applied. Data used in this study are described in Section 2.3. Empirical results are discussed in the next section. Finally, this chapter ends with some concluding remarks in Section 2.5.

CHAPTER 3: Forecastability of the SETAR, SETAR-GARCH and Double Threshold GARCH Models

In this chapter, we are concerned with the fitting of some nonlinear models belonging to the class of switching regime models such as SETAR, SETAR-GARCH and DTGARCH for return on exchange rate of Indian rupee in terms of US dollar. This empirical exercise is likely to throw some light on the extent to which these nonlinear models are able to capture volatility, persistence and regime shifts inherent in the exchange rate variable of an emerging / developing country like India. Empirical evidences based on likelihood ratio test, diagnostic checks as well as out-of-sample forecasting performance, clearly show that the performance of DTGARCH model with one threshold is the best amongst all such nonlinear time series models considered in this chapter.

The outline of this chapter is as follows. Section 3.1 gives a brief introduction along with the references of some relevant works in this literature. In Section 3.2, we discuss the methodology applied including the relevant tests and diagnostic checks for model adequacy. Empirical results are discussed in Section 3.3. Finally, some concluding observations are made in Section 3.4.

CHAPTER 4: Smooth Transition Autoregressive Model for Daily Exchange Rate Return

Smooth transition autorgressive (STAR) model is a variant of the SETAR model. Introduced by Chan and Tong (1986) and extensively explored by Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and Teräsvirta (1994). In this model, the parameters are allowed to change smoothly over time. In case of STAR models, the indicator function used in the SETAR model is replaced by a continuous function. The functions used for STAR model are usually the logistic and exponential functions. The use of STAR model has often been made to study the behaviour of real exchange rates and the number of studies on nominal exchange rate is rather limited. In this chapter, we have applied the STAR model to study the behaviour of India's exchange rate return at daily frequency. Our finding is that the second order logistic STAR model performs better than the other STAR models for the exchange rate return series of India. We also examine the forecasting performance of such a model using out-of-sample forecasts.

This chapter is formatted as follows. Section 4.1 gives the introduction to this chapter. In Section 4.2 we discuss the model and briefly describe the methodology used. Section 4.3 discusses the data and the empirical results. The paper ends with some concluding remarks in Section 4.4.

CHAPTER 5: The Markov Switching Regression Model for Exchange Rate Return at Daily Frequency

The Markov switching regression models are designed to capture discrete changes in the economic mechanism that generates the data. These models have been popularized by Hamilton (1989, 1990, 1994) and Engel and Hamilton (1990), although these were originally motivated by Goldfeld and Quandt (1973). In this chapter, we study the performance of this class of models for India's exchange rate return at daily frequency. Here we fit the simple two-state MSR model which is similar to the mixture of two normal distributions and then consider the more general model where Hamilton and Susmel (1994)'s SWARCH model along with an autoregressive process in the conditional mean specification is considered. We also study the performance of such a model using the out-of-sample forecasts, and compare this performance with those of the other models used so far, in terms of standard criteria for forecast evaluation.

The outline of the chapter is as follows. In Section 5.1, we discuss some of the studies which have used such models. We briefly discuss the two-state MSR model and its estimation procedure followed by the more complex MSWARCH model with an autoregressive process in the mean specification and finally its estimation, in Section 5.2. The empirical results including the forecasting performance of this model are reported in Section 5.3. A comparative performance of all the models used so far are made in Section 5.4. This chapter ends with some concluding remarks in Section 5.5.

CHAPTER 6: Modelling Monthly Exchange Rate Return with Macroeconomic Variables: A Predictive Regression Approach

In this chapter, we study the predictability of exchange rate return of India using macro variables such as money supply growth, stock price returns, inflation rate, foreign investment, trade balance, foreign exchange reserve etc., which have been found to be relevant in similar studies concerning other, mostly developed, economies and / or which are considered to be important in theoretical studies on exchange rate. The full set of macro variables used, to begin with, comprises 25 variables. Inferences on predictive ability of each of these variables are based on recently developed out-of-sample tests of predictive ability due to West (1996), Clark and McCracken (2001) and McCracken (2004). In this selection procedure, specific-to-general as well as general-to-specific approaches of model selection are used, and we also check our results using a data-mining-robust bootstrap procedure. Thereafter, we use the macro variables which are thus found to have significant predictive ability and obtain a final model for exchange rate return of India in linear dynamic regression framework, and then carry out all relevant diagnostic tests on the residuals of this model.

The chapter has been organized as follows. Section 6.1 gives the introduction to this chapter. The methodology applied in this study is briefly described in Section 6.2. Section 6.3 presents a brief description of the data used in our analysis. Empirical findings are discussed in Section 6.4. This chapter ends with some remarks in Section 6.5.

CHAPTER 7: Long-Run Relationship Between Exchange rate and Macroeconomic Variables

Exchange rate modelling and its predictability have so far been studied in terms of its return which unlike exchange rate series, is stationary. Hence prediction, in this context, refers to short-run periods only. While short-run forecasts are undoubtedly very important, financial decisions often involve long-run commitments and hence long-run relationship, if exists, also needs to be studied. Accordingly, in this chapter, we study such relations involving the foreign exchange rate of India and the relevant macroeconomic variables, applying the methodology of cointegration, introduced by Granger (1981), to study the common stochastic trend among all the variables concerned. To that end, we have applied the VAR based methodology developed by Johansen (1988,1991,1995) and Johansen and Juselius (1990). But, since cointegration relations do not appear explicitly in the VAR framework, a more convenient modelling set-up obtained by rewriting the VAR model, known as the vector error correction model (VECM), is used for cointegration analysis. The data used for this study is the monthly level data, as in Chapter 6.

The presentation in this chapter is as follows. The chapter begins with an introduction in Section 7.1. Section 7.2 presents the cointegration methodology very briefly. Empirical findings are discussed in Section 7.3. Concluding observations are made in Section 7.4.

CHAPTER 8: Conclusions

The last chapter of the thesis presents a brief introduction of the applied problem under study. Section 8.2 gives a summary of the major findings of the entire work done in this thesis. The concluding section contains few ideas for further studies on this topic.

CHAPTER 2

Mean and Volatility Dynamics of Daily Exchange Rate Return in the Framework of Linear Model*

2.1 Introduction

During the last decade, there has been a renewed interest in issues relating to trade and consequently exchange rate has become one of the most important economic and financial variables to be studied. As discussed in the preceding chapter, earlier the focus was on structural models i.e., models which are based on the relationship between exchange rate and relevant macroeconomic variables. Later, Meese and Rogoff (1983a,b) found that a simple martingale process forecasts better than the more complex structural models. This empirical success of time series models over complex structural models has propelled the use of various time series models (both linear and nonlinear) in exchange rate modelling.

However, it is now well established that return on exchange rate is not independently distributed over time primarily because of presence of volatility clustering. This observation has led to the use of autoregressive conditional heteroscedastic (ARCH) model which was introduced by Engle (1982) as well as the generalized ARCH (GARCH) model (Bollerslev (1986)). There have been several extensions and generalizations of the basic (G)ARCH model. The most important of these is due to Nelson (1991) who suggested that a symmetric conditional variance function may be inappropriate for modelling volatility because it cannot represent 'leverage effect', and then proposed what is known in the literature as the exponential GARCH (EGARCH) model. Further, other than volatility clustering, the deviation of exchange rate return from the random walk model may be due to the presence of its

* A paper (written jointly with Nityananda Sarkar) containing the materials of this chapter, entitled 'Mean and Volatility Dynamics of Indian Rupee/U.S. Dollar Exchange Rate Series: An Empirical Investigation' has been published in *Asia Pacific Financial Markets*, 2006, 13, 41-69.

own lags and some kind of calendar effects. Hsieh (1989) has, for instance, found that each day of the week may have a different distribution (see also Grilli and Kaminsky (1991) and Huizinga (1987)).

In this chapter, the focus of our study is on empirical determination of a ‘proper’ linear dynamic model for return on daily exchange rate, where due consideration to conditional heteroscedasticity is also given so that the dynamics of both mean and volatility are duly captured. Despite the supposed nature of nonlinear models like the self exciting threshold autoregressive (SETAR), smooth transiton autoregressive (STAR) and Markov switching regression (MSR) models being able to capture various complicated characteristics of the exchange rate data, the simple linear dynamic model with appropriate volatility specification may perform, as noted by De Gooijer and Kumar (1992), quite well by standard criteria of model evaluation. Further, due emphasis on appropriate specification of both the conditional first and second order moments is given so that the final inferences concerning predictability are free from any possible consequences of misspecification of the underlying model. We bring in this issue of appropriate specification of the first two conditional moments since it is now too well recognized that inferences based on models suffering from misspecification could be misleading and incorrect. While there are several aspects to the general understanding of specification, parameter instability or structural change, to use a broader terminology, is probably the most important one in the context of time series data and this affects modelling inferences, if not accounted for appropriately. There is also the issue of omission of variables which is also quite common. Now, as regards structural change, there exists an enormous literature on tests for structural change(s), and this can be traced back to Chow (1960) and Quandt (1960), followed by Andrews (1993, 2003), Andrews and Ploberger (1994), Bai and Perron (1998) and Hansen (1997, 2001). These researches have led to the development of a testing procedure to determine whether there exists one or more structural breaks in a given time series. Further, Bai (1994, 1997a, b) and Chong (1995) have found methods of estimating break points.

The time series of India’s foreign exchange rate has been found to be marked by instability which is, however, hardly a surprising result for an emerging economy, and hence we have determined the break point(s) in the time series, and then accordingly partitioned the entire time period into sub-periods of stable parameters each. Thereafter, we have tried to specify the conditional mean properly for each sub-period. In this context, it is also relevant to note that incorrectly specified conditional mean might as well lead to misspecification of conditional variance. Hence, we have carried out tests for misspecification of conditional mean and consequently made the mean specification as adequate as possible before determining an appropriate specification for the conditional variance. Finally, an out-of-sample forecasting exercise has been carried out to gauge the performance of such a model by using standard forecast evaluation criteria. At this stage we note that the forecasting performance of the chosen model can be judged meaningfully by considering a naïve forecast model such as the random walk model, and then comparing the values of out-of-sample forecasting criteria of the former with those of the latter. Thus, the random walk model is considered as the benchmark model in evaluating the forecasting performance of the model obtained by our approach. Following this procedure, we have found the existence of four breaks in the daily-level exchange rate series over the sample period ranging from 1, November 1994 to 13, February 2004. As regards the appropriate volatility model, we have found, unlike most other studies, EGARCH to be the most appropriate specification of conditional heteroscedasticity for India's exchange rate series for each of the five sub-periods.

The chapter is formatted as follows. In Section 2.2, we discuss the model and briefly describe the methodology used. Data used in this study are described in Section 2.3. Empirical results are discussed in the next section. Finally, the chapter ends with some concluding remarks in Section 2.5.

2.2 The model and methodology

In this section, we describe the methodology used for building the first model of this thesis. Here, we try to build an appropriate dynamic econometric model for India's exchange rate return at daily-level frequency in the framework of a single-equation linear dynamic model with other variables (treated as exogenous) for which daily-level data are available. Further, due consideration to conditional heteroscedasticity is also given while building this model. The model specified by us bears some resemblance to the univariate model by Baillie and Bollerslev (1989) and Hsieh (1989) and is similar to the one used by Sarkar and Mukhopadhyay (2005) for their study on stock returns of India. Recently**,** Malik (2005) has also used a similar model to explain European exchange rate volatility. As already stated, our study envisages a systematic approach towards determining an appropriate model for the daily foreign exchange rate return of India with due consideration to possible misspecification of both the conditional mean and conditional variance. To that end, we have tried to account for the existence of serial correlations by incorporating lags of exchange rate return and used dummies to capture the day-of-the-week effects in the conditional mean. A possible source of misspecification of the conditional mean is exclusion of contemporaneous variables. Now, intervention in the form of sale/purchase of foreign exchange is certainly an important variable. However, since our analysis is based on daily level exchange rate series, we could not use data on intervention because the Reserve Bank of India[1] (RBI) publishes only monthly intervention data, and like most central banks, keeps its daily intervention a closely guarded secret. Further, macro economic variables like inflation, money growth, balance of payments couldn't be included due to their non-availability at daily-level frequency. However, we have used the daily call money rate which is the rate at which the commercial banks borrow money from other banks to maintain a minimum cash reserve requirement. The call money market and foreign exchange market are closely linked as there exists arbitrage opportunities between the two markets. When call money rates increase, banks borrow dollars from their overseas branches, swap them for rupees and lend them in call money market.

Another important variable, which is closely associated with the foreign exchange market, is the stock market. There are several studies suggesting that stock market innovations may affect or, in turn, be affected by exchange rate dynamics. For details, see the studies by Aggarwal (1981), Ajayi and Mougoue (1996), Abdalla and Murinde (1997), Ajayi *et al*. (1998), Nieh *et al.* (2001), Ki –ho Kim (2003) and Phylaktis and Ravazzolo (2005). In our study, we have used the daily level Bombay stock exchange

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[1] Reserve Bank of India is India's central bank.

sensitive index (BSESENSEX) as a representative stock price index of India to study its role in determining the model for foreign exchange rate of India.[2]

Another probable variable that might affect exchange rate is the Federal funds rate or short term interest rate of the US. The effects of US interest rate shocks on the economies of other developed countries have been studied by Kim and Roubini (2000), and the general observation is that a rise in Federal funds rate is accompanied with devaluation of other world currencies. However, there are no such studies on the relationship between Federal funds rate and Indian exchange rate. Keeping this is mind, we have included Federal funds rate as an independent macroeconomic variable in determining the model for India's foreign exchange rate. Thus, the three contemporaneous[3] independent macro variables used in this study are call money rate, BSESENSEX and Federal funds rate.

Taking all these into consideration, we propose the following specification for the return on daily-level foreign exchange rate of India:

$$y_t = \sum_{k=1}^{p} \phi_k y_{t-k} + \sum_{j=1}^{d} \xi_j D_{jt} + \omega i_t + \eta b_t + \lambda f_t + \varepsilon_t, t = 1,2,......,T \qquad (2.1)$$

where y_t is the first difference of the logarithms of spot exchange rate, D_j's $(j = 1,2,...,d)$ denote the daily 0-1 dummies, i_t is the call money rate, b_t is the first difference of logarithms of BSESENSEX i.e., return on stock price index, f_t is the first difference of the logarithms of Federal funds rate and p is the appropriate lag value of y_t capturing its autocorrelations. The three variables *viz.*, y_t, b_t and f_t have been taken in their respective logarithmic difference values since these variables, as reported in the next section, were found to have unit roots at level values but their log-differences were obtained as stationary series. The specification in (2.1) may be conveniently written as

[2] It may be stated in this context that Bombay Stock Exchange which was established in 1875 is the premier stock exchange of India, and BSESENSEX is the most important and most widely used stock index.

[3] The specification in (2.1) should also include lagged values of i_t, b_t and f_t. However, in our empirical work, no lagged coefficient was found to be significant, and hence, for the sake of notational simplicity, these lagged terms are being excluded from this modelling

$$y_t = z_t'\gamma + \varepsilon_t \tag{2.2}$$

where $z_t' = (y_{t-1},..., y_{t-p}, D_{1t},..., D_{dt}, i_t, b_t, f_t)$ and $\gamma' = (\phi_1,...,\phi_m, \xi_1,..., \xi_d, \omega, \eta, \lambda)$.

Given this specification, we first test for the most important source of misspecification *viz.*, parameter instability. This is done by applying the Quandt-Andrews test.

2.2.1 Quandt-Andrews test

The first classical test of an exogenously given structural change in the econometric literature is due to Chow (1960). At the same time, Quandt (1960) also suggested testing the null hypothesis of constant coefficients against a more general alternative, where the break point is unknown and the error variance is also allowed to change. However, because of lack of a proper distribution theory, this test could not be applied. It was only after three decades that Andrews (1993) and Andrews and Ploberger (1994) derived the asymptotic distribution of the likelihood-ratio test as well as the analogous Wald and Lagrange multiplier / Rao's score test for a one-time unknown structural change. These distributions are valid for models with no deterministic or stochastic trend as well as for nonlinear models. Andrews (1993, 2003) also provided the asymptotic critical values. In this test, the test statistics are obtained as a function of all possible break dates. However, as noted by Hansen (2001), the break dates cannot be considered to be too close to the beginning or end of sample, because otherwise there are not enough observations to identify the sub-sample parameters. Conventionally, the search is confined to the range between 15 and 85 percent of the observations. This sequence of test statistic values is plotted against the candidate break points, and then checked to see if the sequence moves above Andrew's appropriate critical value. If it does, then we conclude that the time series has a structural break.

When the Quandt-Andrews test suggests a structural break, we then need to estimate the break point. Following Bai (1994,1997a,b), the sample is split at each possible break date, the parameters of the model are then estimated by the ordinary least squares (OLS) method and the sum of squared errors calculated. The least squares break date estimate is the date that minimizes the full-sample sum of squared errors.

Bai (1997b) and Chong (1995) have also discussed how to estimate multiple break dates sequentially. In presence of multiple structural breaks, the sum of squared errors can have a local minimum near each break date. Thus the global minimum can be used as a break date estimate and the other local minima can be viewed, after careful consideration, as candidate break dates. The sample is then split at the break date estimates, and analysis continues on the sub-samples (Bai (1997b)).

Once the break points are determined, we partition the whole period into sub-periods of stable parameters each, search for a proper mean specification of each of the sub-periods and then carry out tests for model misspecification. Once parametric stability has been achieved, model misspecification in mean primarily refers to omitted variables which may also be nonlinear in nature. To test for any remaining misspecification, we apply the approach advocated by Lumsdaine and Ng (1999).

2.2.2 Test for misspecification

In the context of our model, the approach used by Lumsdaine and Ng (1999) envisages approximation of any remaining misspecification of the conditional mean by functions of recursive residuals. The motivation is that any unobserved nonlinearity will be manifested in the recursive residuals. The method proposed by them involves estimation in two steps. The first step requires starting from the $(k+1)^{th}$ observation where $k= p+d+3$ for our model, and performing recursive estimation of the dependent variable y_t on the set of independent variables z_t over the remaining $T-k$ observations. This leads to a set of estimates $\hat{\gamma}_t$, $t = k+1,\ldots,T$, of γ and a set of recursive residuals $\hat{w}_t = y_t - z_t'\hat{\gamma}_{t-1}$, $t = k+2,\ldots,T$. These recursive residuals contain the information used to update $\hat{\gamma}_t$ from $\hat{\gamma}_{t-1}$ and cannot be predicted by the regression model given information at time $t-1$. These are serially uncorrelated by construction if the model is correctly specified. But when the model is misspecified in the conditional mean, $\hat{w}_t$ will contain information about true conditional mean not captured by the regression function.

In the second step, we estimate the equation**:**

$$y_t = z_t'\gamma + g(\hat{w}_{t-1}) + \nu_t \tag{2.3}$$

where $g(\hat{w}_{t-1})$ is a (possibly nonlinear) function of the recursive residuals, $\hat{w}_{t-1}$. The role of $g(\hat{w}_{t-1})$ is to orthogonalize ε_t in $y_t = z_t'\gamma + \varepsilon_t$ so that the conditional mean of the resulting regression error v_t shrinks towards zero. The use of recursive residuals are appealing as they are easy to compute and $\hat{w}_{t-1} \in I_{t-1}$ is in econometrician's information set at time *t*. Thus, we use $\hat{w}_{t-1}$ instead of $\hat{w}_t$ in (2.3). Given that the objective of the exercise is to guard against omitted variables or misspecification in functional form of the conditional mean and the conditioning information set, the natural candidate for $g(.)$ is a flexible function of the recursive residuals. As originally proposed by Ramsey (1969) and Ramsey and Schmidt (1976) and also used by Ng and Lumsdaine, a suitable candidate is a polynomial in the recursive residual of the form $g(\hat{w}_{t-1}) = \sum_{i=1}^{l} \delta_i \hat{w}_{t-1}^{i}$ for a series expansion of length *l* in $\hat{w}_{t-1}$. Further, significance of δ_i can be interpreted as a diagnostic check for the misspecification of the conditional mean.

Following the approach outlined above, we include nonlinear functions of recursive residuals in the conditional mean function in (2.3) for each sub-period and estimate the model along with appropriate specification for conditional heteroscedasticity, h_t. The original GARCH formulation (Engle (1982) and Bollerslev (1986)) for h_t, as given below in (2.4), has been found to be appropriate in many empirical studies:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_n \varepsilon_{t-n}^2 + \beta_1 h_{t-1} + ... + \beta_m h_{t-m} \tag{2.4}$$

where the stochastic error ε_t conditional on the realized values of the set of variables $\psi_{t-1} = \{y_{t-1}, z_{t-1}, y_{t-2}, z_{t-2}, ...\}$ is assumed to be normally distributed i.e., $\varepsilon_t \mid \psi_{t-1} \sim N(0, h_t)$. The inequality restrictions $\alpha_0 > 0$, $\alpha_i \geq 0$ for *i*=1,…., *n*, $\beta_i \geq 0$ for *i*=1,…, *m* are imposed to ensure that the conditional variance is strictly positive[4]. Bollerslev (1986) has the necessary and sufficient condition , $\sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{m} \beta_i < 1$, for

[4] Nelson and Cao (1992) have shown that other weaker conditions also ensure positivity of h_t.

the existence of unconditional variance. There are many other alternative models for volatility which have also found applications in the analysis of financial variables. One important alternative specification for volatility is known as the exponential GARCH (EGARCH) (Nelson (1991)) model. This is given as

$$\log(h_t) = \alpha_0 + \sum_{i=1}^{n} \alpha_i f(\tau_{t-i}) + \sum_{i=1}^{m} \beta_i \log(h_{t-i}) \tag{2.5}$$

where $f(\tau_t) = \theta\tau_t + \kappa[|\tau_t| - E|\tau_t|]$ and τ_t is independent and identically distributed with mean zero and variance one. It is easy to note that $f(\tau_t)$ is independent with mean zero and constant finite variance. It may be noted that unlike GARCH, the exponential GARCH model does not require any nonnegativity restrictions on the parameters involved in h_t.

2.2.3 Forecasting

In order to assess the performance of the model discussed in the preceding section, we have obtained out-of-sample forecasts and then compared these with the actual values by standard forecast evaluation criteria. One such well known criterion is the mean squared error (MSE) of the forecasts defined as

$$MSE = \frac{1}{T_1} \sum_{t=T+1}^{T+T_1} (y_{t+s} - f_{t,s})^2 \tag{2.6}$$

where $f_{t,s}$ is the *s*-step ahead forecast from time *t* and y_{t+s} is the actual value of the exchange rate return at time *t+s*, $T+T_1$ is the total sample size (in-sample plus out-of-sample), and $(T+1)$ *th* observation is the first out-of-sample forecast observation so that the total hold-out sample size is T_1. Another standard criterion for evaluating forecasting performance is the mean absolute forecast error which is given by

$$MAE = \frac{1}{T_1} \sum_{t=T+1}^{T+T_1} |y_{t+s} - f_{t,s}| \quad . \tag{2.7}$$

The third criterion used for this study is known as the adjusted mean absolute percentage error (AMAPE) defined as

$$AMAPE = \frac{\sum_{t=T+1}^{T+T_1} \left| \frac{y_{t+s} - f_{t,s}}{y_{t+s} + f_{t,s}} \right|}{T_1} \quad . \tag{2.8}$$

The last criterion is due to Gerlow *et al.* (1993) who have argued that the accuracy of forecasts according to traditional statistical criteria might provide less information regarding the potential profitability of using those forecasts in a market trading strategy. There are models which perform poorly on statistical grounds, but these may yield profits. Models that can accurately forecast the signs of future returns, or can predict turning points in a series have been found to be more profitable (Leitch and Tanner, 1991). A possible indicator of the ability of a model to predict direction changes irrespective of their magnitudes has been suggested by Pesaran and Timmerman (1992). It is defined as the percentage of correct sign predictions (PCSP) i.e.,

$$PCSP = \frac{100}{T_1} \sum_{t=T+1}^{T+T_1} z_{t+s} \tag{2.9}$$

where $z_{t+s} = 1$ if $(y_{t+s} . f_{t,s}) > 0$ and 0 otherwise. Insofar as generation of *s*-step ahead forecasts are concerned, we have used a recursive window where the series of forecasts is generated with the initial estimation date fixed and additional observations are added one at a time to the estimation period.

2.3 Data and software

We have applied the methodology stated in the previous section using the time series data on India's foreign exchange rate. We have taken daily level data of spot exchange rate (Reserve Bank of India reference rate) spanning from November 1, 1994 to February 13, 2004, a total of 2287 data points, for the in-sample analysis. For the purpose of out-of-sample forecasting, hold-out sample covering the period February 16, 2004 to July 14, 2004 – a total of 100 data points- has been taken. All the data have been collected from the RBI site (www.rbi.org.in). The spot exchange rate is the price of one unit of the US dollar in rupee terms. Though the floating regime started from March 1993, the period from March 1993 till November 1994 was a prolonged phase of near constant exchange rate. As regards the data on the other variables, the series on Bombay stock exchange sensitive index (BSESENSEX) was downloaded from site, www.bseindia.com and that of call money rate from the RBI site, www.rbi.org.in. Federal funds rate, also interpreted as short-term interest rate, is defined as the interest rate at which a depository institution lends immediately available funds (balances at Federal Reserve) to another depository institution overnight. The time series of this variable was downloaded from the site of Federal Reserve Bank of New York (www.newyorkfed.org). We have used E-VIEWS 3.1 and SAS 8.02 for carrying out the necessary computations.

2.4 Empirical analysis

In this section, we report and discuss the results of our analysis of the daily level exchange rate data. For the purpose of this study, the observations have been changed to their logarithmic values, and we denote this series by P_t say. Now, we first check whether the series of logarithmic values of spot exchange rate is stationary or not. The augmented Dickey Fuller (ADF) regression obtained for this purpose is given below:

$$\begin{aligned}\Delta\hat{P}_t &= \underset{(0.411)}{0.002} - \underset{(0.377)}{(9.5\times 10^{-8})}\,t - \underset{(0.328)}{0.0003}\,P_{t-1} + \underset{(4.709)^{***}}{0.099}\,\Delta P_{t-1} + \underset{(1.066)}{0.022}\,\Delta P_{t-2} \\ &- \underset{(3.303)^{***}}{0.069}\,\Delta P_{t-3} + \underset{(6.446)^{***}}{0.135}\,\Delta P_{t-4} - \underset{(2.375)^{**}}{0.05}\,\Delta P_{t-5}.\end{aligned} \tag{2.10}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Here the significance of the coefficient associated with P_{t-1} is tested. The computed (absolute) value, 0.328, is compared with the tabulated values *viz.*, 3.414 and 3.967 at 5 percent and 1 percent levels of significance, respectively, due originally to Fuller (1976) and later extended by Guilkey and Schmidt (1989) and MacKinnon (1994). Accordingly, we conclude that the underlying null hypothesis of presence of unit root cannot be rejected and the series has a unit root. The optimum lag of ΔP_t is determined using Hall's (1994) procedure and it has been found to be 5. The value of Ljung-Box $Q(k)$ test statistic is computed for lag k upto 36, and we find that the null of Gaussian white noise for errors cannot be rejected at 5% level of significance for all lags. Since the presence of unit root has been established, we have then carried out the ADF test on the values of the first difference i.e., $y_t = P_t - P_{t-1}$, and the computed (absolute) value of the test statistic is now obtained as 19.328, which is highly significant. We thus conclude that the first difference i.e., the exchange rate return series, is stationary.

The graphical representation of returns is given in Figure 2.1. A visual inspection of the series indicates that the return series is stationary around zero with no deterministic trend, and it exhibits significant volatility in the series. The usual

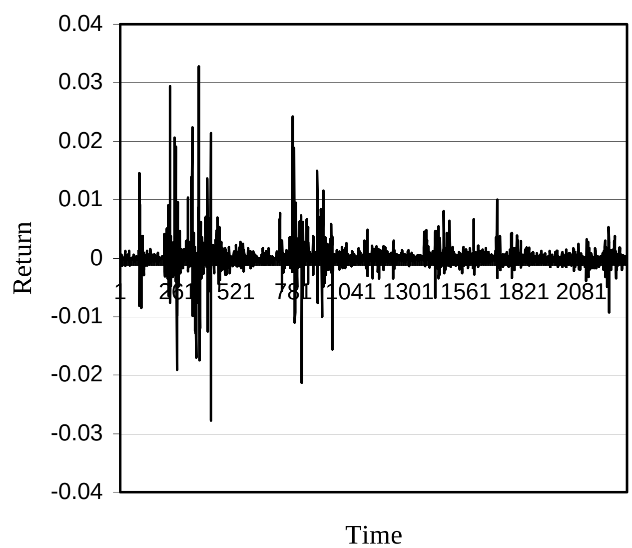


Fig 2.1 Plot of returns on daily foreign exchange rate of India from August 10, 1995 to February 13, 2004.

descriptive statistics of returns are as follows: mean = 0.00016, standard deviation = 0.002712 and measures of skewness and kurtosis are 1.487 and 42.037, respectively. The last two indicate that the underlying distribution is asymmetric and highly leptokurtic.

As a preliminary analysis we have fitted a random walk model to the exchange rate series (*cf.* equation (2.10 a)). Later, we have considered this random walk model to be the benchmark model to gauge the performance of our chosen model. We now present the estimated random walk model with a drift (equation (2.10a)). We have also computed the values of MSE, MAE, AMAPE and PCSP of this model to evaluate the out-of-sample forecasting performance of the other models relative to that of this benchmark model.

$$P_t = \underset{(2.949)}{0.000173} + P_{t-1} + \varepsilon_t \qquad (2.10a)$$

[*The values in parentheses indicate the absolute values of t-ratios*]

Here P_t is the logarithmic value of the Indian rupee per US dollar exchange rate and ε_t is the error. We note from equation (2.10a) that the drift parameter is significant.

The ADF test was also carried out for the three time series considered as independent variables in this study *viz*., call money rate, BSESENSEX and Federal funds rate. It was found that while call money rate was stationary, the other two were not. For the last two series, the first differences of their logarithmic values were taken, and the ADF test on these differenced series showed stationarity.

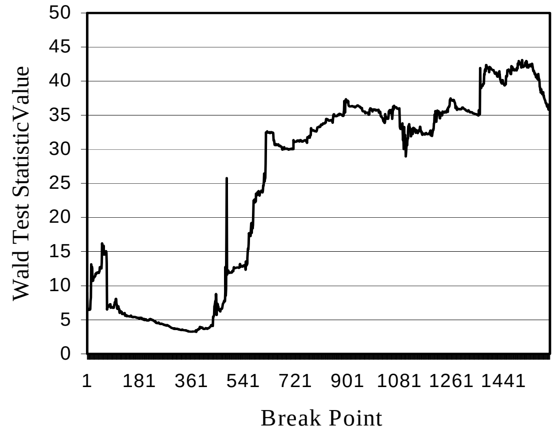
2.4.1 Testing for parameter stability

We have already mentioned that we have followed Quandt-Andrews procedure for testing structural change. We now report our findings on Andrews Sup Wald (W) test for testing parameter stability in the model specified in (2.1). It may be relevant to point out that in this testing procedure, the alternative is taken to be one where a structural change has occurred at some unknown time point and the error variance is allowed to change. The issue of break point being assumed to be known *a priori* or taken to be unknown and hence to be determined, still remains somewhat debatable. The fact is that while on one hand, some prior information often exists about the dates of major shocks and hence the likely location of the break point (s), on the other, the results can be highly sensitive, as pointed out by Hansen (2001), if the *a priori* choices are somewhat arbitrary. Hence, the latter can hardly be considered to be a sound scientific practice. Insofar as we are considered, we have, as already stated, taken the break points to be determined endogenously by Bai's (1994,1997a and b) least squares based procedure once Quandt-Andrews test has concluded that there exists a break in the return series.

For the purpose of carrying out this test for parameter stability, we have first taken p=1 in equation (2.1). Thereafter, we have considered some higher values of p as well. However, results of this test were found to have hardly changed with higher lags and hence we are reporting the computational figures for p=1 only. It may be noted, in this context, that the specification in (2.1) is being taken as the 'true specification' and parameter changes in the alternative are understood in respect of this specification.

For the purpose of computing a sequence of Wald statistics as a function of candidate break dates, we have eliminated the first and the last 15 percent of the data points. A plot where the candidate break dates are plotted on the x-axis and the values of the Wald statistics on the y-axis, is given in Figure 2.2. It is evident from this plot that the maximum value of the sequence of Wald statistics is 43.098, and that this clearly exceeds the Andrews' critical value of 30.42 at 1 percent level of significance[5]. Hence, we conclude that the null hypothesis of no structural break is rejected in favor of the existence of a structural break in the time series of India's daily foreign exchange rate. It now becomes essential to estimate the break date and then examine further if there are multiple breaks. Using Bai's procedure, as shown in Figure 2.3, we plot the residual sum of squares as a function of a single break date. The sample is split at each break date and regression parameters are estimated separately on each sub-sample. The sum of squared errors over the full sample is then calculated and plotted on the y-axis while the break dates are on the x-axis. In case the true parameters are constant, the sub-sample estimates and hence the sum of squared errors will vary randomly and erratically across candidate break dates. If, however, there is a structural break then the sub-sample estimates will vary systematically across candidate break dates, and the plot of sum of squared errors will show a well-defined minimum near the true break date.

[5] This value corresponds to degrees of freedom being 9 and trimming parameter 0.15.



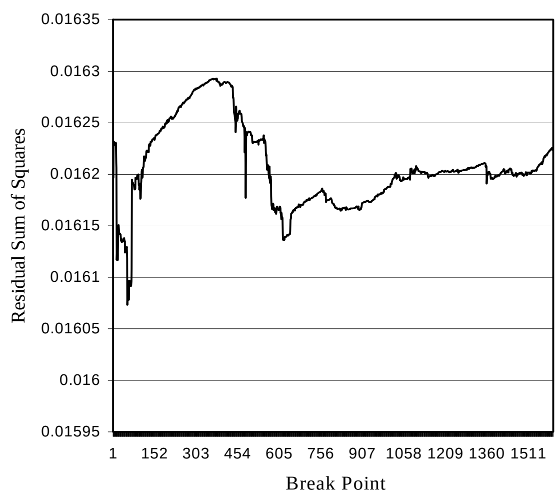


Fig 2.2 Wald test statistic value (February 28, 1996 to September 12,2002)

Fig 2.3 Plot of residual sum of squares (February 28, 1996 to September 12, 2002)

The estimate of the break date is taken to be the one for which the sum of squared errors for the full sample (as a function of break point) is minimum. By this procedure, the minimum is found at observation number 395 (May 10, 1996). We can thus conclude that the foreign exchange rate return series of India had a structural break at this point of time or more appropriately, in an interval around this time point.

Following the procedure suggested by Bai (1997b) and Chong (1995), we now briefly state the results concerning the test for multiple break dates. To this end, the full sample is split into two sub-samples [1,395] and [396,2287] and test for structural break is carried out on the two sub-samples. Figure 2.4a gives the plot of the values of the Wald statistic across the candidate break dates for the period [396,2287]. The maximum value is found to be 77.0734, which obviously exceeds the Andrews critical value at 1 percent level (30.42), and hence we reject the hypothesis of no structural break. Figure 2.4b presents the plot of residual sum of squares as a function of break date and the estimated break date is 720 (August 21, 1997). Using a sequential method similar to that in Hansen (2001), we further split the sample into two sub-samples [1,720] and [721,2287]. Both the periods show parameter instability and the estimated break points are 396 (May 13, 1996) and 962 (August 24, 1998) (*cf*. Figure 2.5b, for the latter) respectively.

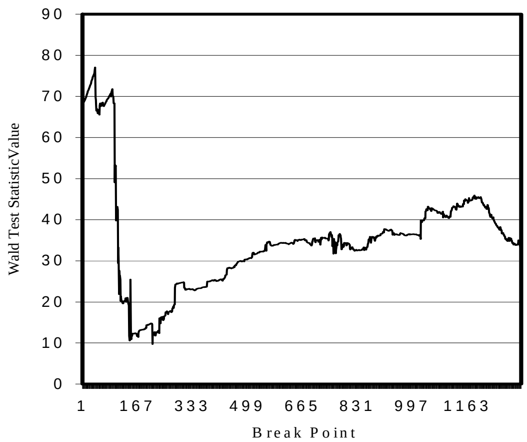


Fig 2.4a Wald test statistic value (June 24, 1997 to December 11,2002)

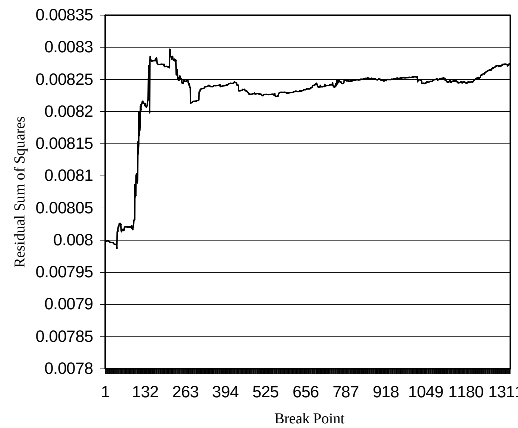


Fig 2.4b Plot of residual sum of squares (June 24, 1997 to December 11,2002)

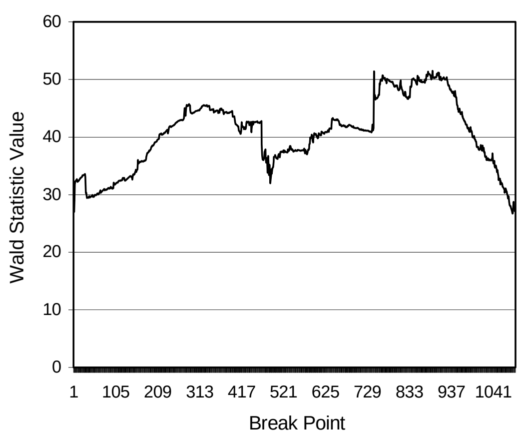


Fig 2.5a Wald test statistic value (August 20,1998 to February 20, 2003)

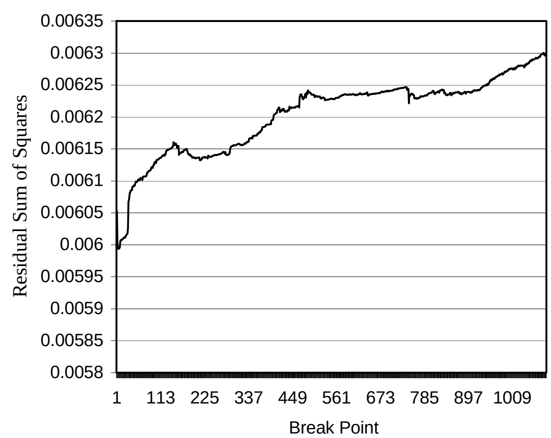


Fig 2.5b Plot of residual sum of squares (August 20,1998 to February 20, 2003)

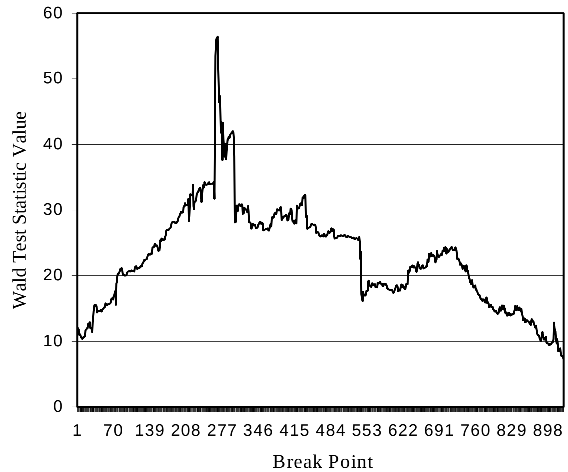


Fig 2.6a Wald test statistic value (June 30,1999 to April 23, 2003)

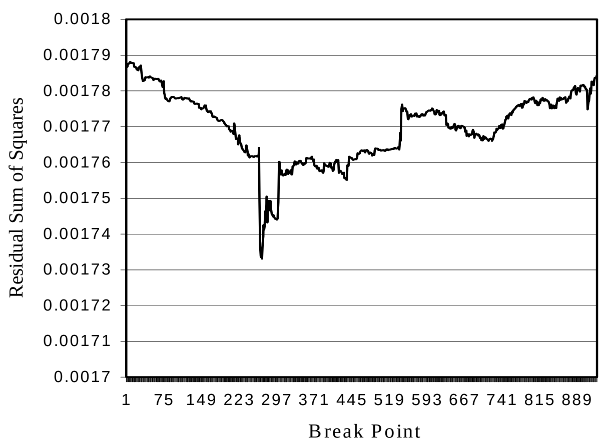


Fig 2.6b Plot of residual sum of squares (June 30,1999 to April 23, 2003)

Quandt- Andrews test finds evidence for parameter instability in the period [963,2287] and the estimated break point is 1429 (July 31, 2000), as shown in Figures 2.6a, b. Next we consider the sub sample [1430,2287] and the relevant test statistic value is now found to be much below the critical value indicating stability in this sub-period. So, we can conclude that there is no break in this sub-period.

Based on these evidences, the break points of the series (with reference to month) have been estimated as May 1996, August 1997, August 1998 and August 2000, and accordingly we have identified the five sub-periods to be [November 1994, May 1996], [May 1996, August 1997], [August 1997, August 1998], [August 1998, July 2000] and [August 2000, February 2004][6].

The period from March 1993 till November 1994 was a prolonged phase of near constant exchange rate. The exchange rate fluctuated little till August 1995. The Indian economy experienced surges of capital inflow during 1993-94, 1994-95 and first half of 1995-96, which coupled with robust export growth exerted an upward pressure on the exchange rate. At this point, the Reserve Bank of India (RBI) intervened to ensure the market correction of overvalued exchange rate and hence, as expectedly, our empirical exercise has identified a structural break in May 1996.

Because of certain international developments, a high level of activity was noted in the second quarter of 1997, supported by an accommodating monetary stance in the major economies and subdued inflation. However, currency and country risk factors were given greater consideration in the wake of financial turbulence observed in certain Eastern European and Asian countries during this period. Further, there was uncertainty surrounding the introduction of the single European currency around this time. Changes in market sentiments were reflected in a movement away from core continental European currencies and towards the US dollar, reflecting large interest differentials and renewed concerns with respect to the implementation of European economic and monetary union. All these factors possibly resulted in India experiencing a period of heightened volatility from August 1997 till January 1998. Described as a foreign exchange crisis, a look at the then financial news showed that the trigger point was an announcement by the RBI that the rupee would be allowed to move in a band. It is important to mention that the second break point has been

[6] To be specific, these sub periods are as follows: [November 1, 1994 - May 10, 1996]; [May 13, 1996 - August 21, 1997]; [August 22, 1997 – August 24,1998]; [August 25, 1998 – July 31, 2000] and [August 1, 2000 – February 13, 2004].

identified at around the beginning of this heightened volatile period. This crisis marked the beginning of innovative measures used by the RBI to ward off speculation (*cf.* Ghosh (2002)). The most important feature of the interventions used by the RBI is that these were indirect measures.

Following some easing of market tensions in Asia in the earlier part of 1998, some internal developments were marked which included the international economic sanctions in the aftermath of nuclear tests by India during May 1998. There was renewed financial turbulence in the second quarter of 1998. Nervousness about the economic and financial conditions of Japan accentuated the coolness toward yen-denominated assets. Problems of policy credibility in several Asian countries put downward pressures on currencies in the region, with markets paying increasing attention to the risk of a devaluation of the Chinese yuan. These events, together with political uncertainty, initiated a new wave of contagion to other emerging market economies, resulting in a flight towards the perceived safe markets of the United States and Europe. Indian financial markets were also plagued by turmoil in August 1998, the devaluation of Russian rouble and fears of devaluation of Chinese yuan being held responsible, these sentiments were further fueled by domestic political compulsions. A package of measures was announced on August 20, 1998. As we find, our empirical analysis shows a break during this time. Another internal development in 1999 was the border conflict between India and its neighbor Pakistan during May-June 1999. The year 1999 has also been marked by a subsequent increase in crude prices. In May 2000, renewed crisis emerged in the foreign exchange market - the possible reason being the announcement of rate hike by the Federal Reserve Bank. These may explain the finding of August 2000 as the fourth and final break date for the sample period considered in our study. Our findings on structural breaks in the time series of India’s foreign exchange rate thus seem consistent with major developments in the foreign exchange market of India as well as, to some extent, its neighbors and other important economies of the world during this period.

Based on the findings on structural breaks in the exchange rate series, the entire time period has been partitioned into sub-periods of stable parameters each, as already mentioned. Thereafter, we have attempted at obtaining appropriate specification of the conditional mean for each of the five sub-periods. In the first sub-period, the first 200 observations were found to be near constant, and hence we eliminated these initial observations and then estimated the conditional mean based on the rest of the observations in this sub-period. The conditional mean functions thus estimated for each sub-period are reported below.

Sub-Period I (August 1995-May 1996)[7]

$$y_t = \underset{(2.842)^{***}}{0.202}\, y_{t-1} + \hat{\varepsilon}_t \tag{2.11}$$

Sub-Period II(May 1996-August 1997)

$$y_t = -\underset{(7.027)^{***}}{0.383}\, y_{t-1} - \underset{(2.410)^{**}}{0.133}\, y_{t-2} + \hat{\varepsilon}_t \tag{2.12}$$

Sub-Period III (August 1997-August 1998)

$$y_t = \underset{(3.035)^{***}}{0.185}\, y_{t-1} + \underset{(3.759)^{***}}{0.237}\, y_{t-4} - \underset{(3.654)^{***}}{0.056}\, b_t + \hat{\varepsilon}_t \tag{2.13}$$

Sub-Period IV (August 1998-August 2000)

$$\begin{aligned} y_t = &- \underset{(3.378)^{***}}{0.156}\, y_{t-1} + \underset{(2.451)^{**}}{0.122}\, y_{t-7} + \underset{(2.82)^{***}}{0.144}\, y_{t-9} + \underset{(2.873)^{***}}{0.148}\, y_{t-10} - \underset{(2.474)^{**}}{0.126}\, y_{t-15} \\ &- \underset{(2.26)^{**}}{0.116}\, y_{t-16} + \underset{(2.041)^{**}}{0.0002}\, D_3 + \underset{(2.949)^{***}}{0.0003}\, D_4 + \hat{\varepsilon}_t \end{aligned} \tag{2.14}$$

Sub-Period V (August 2000-February 2004)

$$\begin{aligned} y_t = &\underset{(2.311)^{**}}{0.077}\, y_{t-1} - \underset{(2.074)^{**}}{0.068}\, y_{t-8} - \underset{(1.851)^{*}}{0.060}\, y_{t-13} + \underset{(2.434)^{**}}{0.078}\, y_{t-17} + \underset{(3.445)^{***}}{0.00003}\, i_t \\ &- \underset{(5.570)^{***}}{0.016}\, b_t - \underset{(3.754)^{***}}{0.0004}\, D_1 - \underset{(3.279)^{***}}{0.0003}\, D_3 - \underset{(2.188)^{**}}{0.0002}\, D_5 + \hat{\varepsilon}_t \end{aligned} \tag{2.15}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

[7] This sub-period, in effect, spans from August 9, 1995 to May 10, 1996.

We note from the above equations that returns on BSESENSEX is significant in Sub-periods III and V, while some day-of-the-week dummies are significant in Sub-periods IV and V although, in the latter, the significant days are different in the two sub-periods except for D_3, i.e., Wednesday. We can, therefore, conclude that apart from own lags, the model for exchange rate return depends on return on BSESENSEX and dummy variables representing the day-of-the-week effects in some sub-periods. Further, call money rate has been found to be significant in Sub-period V only.

Finally, the Ljung-Box Q (k) statistic values based on the residuals $\hat{\varepsilon}_t$ for each of the five sub-periods are reported in Table 2.1. We find from this table that the null hypothesis of no serial correlation in the residuals cannot be rejected for all the five sub-periods, and hence we conclude that there is no serial correlation remaining in the residuals of each of these models.

Table 2.1

Ljung –Box Test Statistic ($Q(k)$) Values for Residuals

Lag k	Sub-Period I		Sub-Period II		Sub-Period III		Sub-Period IV		Sub-Period V	
	Q-Stat	Prob.	Q-Stat	Prob.	Q-Stat	Prob.	Q-Stat	Prob.	Q-Stat	Prob.
1	0.116	0.734	0.002	0.969	0.062	0.804	0.065	0.799	0.190	0.663
2	2.346	0.309	0.273	0.872	5.024	0.081	0.208	0.901	1.592	0.451
3	4.788	0.188	2.082	0.556	7.095	0.069	0.301	0.96	1.700	0.637
4	8.243	0.083	2.607	0.626	7.221	0.125	2.527	0.64	1.756	0.780
5	9.529	0.09	2.667	0.751	9.346	0.096	5.157	0.524	2.518	0.774
6	10.777	0.096	2.667	0.849	10.032	0.123	5.158	0.524	2.519	0.866
7	10.902	0.143	2.883	0.896	10.307	0.172	5.419	0.609	3.017	0.883
8	11.536	0.173	4.578	0.802	11.052	0.199	5.945	0.653	3.093	0.928
9	11.567	0.239	5.048	0.83	11.252	0.259	5.968	0.743	3.506	0.941
18	17.252	0.506	7.625	0.984	17.332	0.5	9.761	0.939	7.189	0.988
36	43.598	0.18	31.023	0.704	30.086	0.745	30.187	0.741	18.374	0.994

Note: *All the test statistic values indicate that there are no significant autocorrelations in the residuals at 5% level of significance.*
'Prob.' stands for the p-value under the null hypothesis of no autocorrelation.

2.4.2 Testing for misspecification

We now report the results of the recursive residual based test of misspecification of the conditional mean. This test is carried out to find whether the conditional mean has been misspecified. After obtaining the recursive residuals, $\hat{w}_t$, as discussed in Section 2.2.2, and then including terms such as $\hat{w}_{t-1}, \hat{w}_{t-1}^2, \hat{w}_{t-1}^3, \hat{w}_{t-1}^4, \sum_{i=1}^{t-1}\hat{w}_i$ etc. , we obtain the following estimated models for the five sub-periods. It may, however, be noted that $\hat{w}_{t-1}$ was found to be insignificant in all the models and hence this term has been omitted.

Sub-Period I

$$y_t = \underset{(4.176)^{***}}{0.4495}\, y_{t-1} - \underset{(1.142)}{6.293}\, \hat{w}_{t-1}^2 - \underset{(1.914)^{*}}{569.157}\, \hat{w}_{t-1}^3 + \hat{\varepsilon}_t \tag{2.16}$$

Sub-Period II

$$y_t = \underset{(0.522)}{0.029}\, y_{t-1} - \underset{(0.013)}{0.0006}\, y_{t-2} + \underset{(1.887)^{*}}{25.25}\, \hat{w}_{t-1}^2 - \underset{(5.567)^{***}}{1481.386}\, \hat{w}_{t-1}^3 - \underset{(4.246)^{***}}{179183.1}\, \hat{w}_{t-1}^4 + \hat{\varepsilon}_t \tag{2.17}$$

Sub-Period III

$$y_t = \underset{(1.19)}{0.109}\, y_{t-1} + \underset{(3.947)^{***}}{0.253}\, y_{t-4} - \underset{(3.69)^{***}}{0.058}\, b_t - \underset{(1.137)}{6.046}\, \hat{w}_{t-1}^2 + \underset{(1.187)}{553.58}\, \hat{w}_{t-1}^3 + \hat{\varepsilon}_t \tag{2.18}$$

Sub-Period IV

$$\begin{aligned} y_t = {} & \underset{(2.541)^{**}}{0.161}\, y_{t-1} + \underset{(2.358)^{**}}{0.113}\, y_{t-7} + \underset{(2.85)^{***}}{0.139}\, y_{t-9} + \underset{(2.641)^{***}}{0.131}\, y_{t-10} - \underset{(2.395)^{**}}{0.117}\, y_{t-15} \\ & - \underset{(1.544)}{0.076}\, y_{t-16} + \underset{(2.423)^{**}}{0.0002}\, D_3 + \underset{(2.893)^{***}}{0.0003}\, D_4 - \underset{(2.504)^{**}}{38.91}\, \hat{w}_{t-1}^2 - \underset{(7.108)^{***}}{30315.73}\, \hat{w}_{t-1}^3 + \hat{\varepsilon}_t \end{aligned} \tag{2.19}$$

Sub-Period V

$$\begin{aligned} y_t = {} & \underset{(4.139)^{***}}{0.176}\, y_{t-1} - \underset{(1.965)^{**}}{0.065}\, y_{t-8} - \underset{(1.757)^{*}}{0.058}\, y_{t-13} + \underset{(2.467)^{**}}{0.082}\, y_{t-17} + \underset{(4.162)^{***}}{0.00004}\, i_t \\ & - \underset{(4.959)^{***}}{0.014}\, b_t - \underset{(3.916)^{***}}{0.0004}\, D_1 - \underset{(3.754)^{***}}{0.0004}\, D_3 - \underset{(2.768)^{***}}{0.0003}\, D_5 - \underset{(1.470)}{12.807}\, \hat{w}_{t-1}^2 \\ & - \underset{(3.692)^{***}}{5087.239}\, \hat{w}_{t-1}^3 + \hat{\varepsilon}_t \end{aligned} \tag{2.20}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

It is evident from these estimated equations that the conditional mean is properly specified for Sub-periods I and III since the coefficients associated with $\hat{w}_{t-1}^2$ and $\hat{w}_{t-1}^3$ are insignificant[8]. However, we note from the remaining three equations for Sub-periods II, IV and V that some of the nonlinear terms are significant in these equations. This indicates that the conditional means for these sub-periods are not properly specified. It may be noted, as stated in Section 2.2.2, that once parametric stability has been ensured, misspecification in conditional mean primarily refers to omitted variables which may also be nonlinear in nature (Lumsdaine and Ng (1999)), and our conclusions on appropriate specification of conditional mean or lack of it are from this consideration only. Before incorporating appropriate volatility specification into the models, we, therefore, include the relevant terms from $\hat{w}_{t-1}^2$, $\hat{w}_{t-1}^3$ and $\hat{w}_{t-1}^4$, as found significant, in the specifications of mean for Sub-periods II, IV and V, since estimation after due consideration to volatility specification will not be appropriate with misspecified conditional mean.

2.4.3 Estimation with appropriate volatility specification

Once the conditional mean of return on exchange rate has been properly specified, we have estimated the model along with GARCH assumption for the conditional heteroscedasticity, h_t, as specified in (2.4). It has been, however, found that the estimates of the parameters in h_t violate the condition, $\sum_{i=1}^{n}\alpha_i + \sum_{i=1}^{m}\beta_i < 1$, required for the existence of unconditional variance, for all the sub-periods. However, the extent to which the value of [9] $\sum_{i=1}^{n}\hat{\alpha}_i + \sum_{i=1}^{m}\hat{\beta}_i$ exceeds 1 is found to be substantially different for the five sub-periods. In fact, the difference is substantial for the first three sub-periods and very small for the last two. For instance, in case of Sub-period II, $\hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\beta}_1$ (GARCH(1,2) being found to be the best volatility model for this sub-period) is 2.9987, but the same for Sub-periods IV and V are 1.0929 and 1.0035, respectively.

[8] To draw these conclusions, we are considering, as are usually done, significance upto 5 percent level only.

[9] '^' stands for estimate of the parameter concerned.

Thus, we find that GARCH specification for volatility is untenable for modelling return on foreign exchange rate of India for all the five sub-periods.

Therefore, what we have done next is to apply alternative specifications for h_t *viz.*, the integrated GARCH (IGARCH) and EGARCH models. However, since for the first three sub-periods, the sum of estimated coefficients of GARCH conditional variance has exceeded 1 by a significant magnitude unlike the last two sub-periods, we have applied only the EGARCH model for capturing the volatility of returns for Sub-periods I , II and III, and both the EGARCH and IGARCH models for Sub-periods IV and V. The performance of the IGARCH model, however, was not found to be quite satisfactory in terms of Ljung-Box *Q(k)* test on residuals since presence of some significant serial correlation was found. This is evident from the relevant *p*-values *viz.*, 0.068 for $k = 1$ for Sub-period IV and 0.018 and 0.036 for $k = 1$ and 2, respectively, for Sub-period V. The performance of the EGARCH model, on the other hand, is found to be very satisfactory, as reported below, not only for Sub-periods IV and V, but also for the first three sub-periods. Thus, our empirical findings with EGARCH as the volatility specification suggest this to be the appropriate volatility model for conditional variance for return on foreign exchange rate of India for all the sub-periods. This is laid out in equations (2.21) through (2.25) and in Table 2.2.

As such, it is not easy to explain why the EGARCH model should be appropriate for modelling volatility of return on India's exchange rate series. Empirical evidences are also very few with other exchange rate series. The notable one is a study on Canadian dollar, Swiss franc and the Deutsche mark by Hsieh (1989), where EGARCH has been found to perform reasonably well. Some other studies which report significant volatility asymmetry in the GARCH family of estimations for exchange rates are Kim (1999), Kim *et al.* (2000) and McKenzie (2002). A plausible explanation could be that, like in equity markets, there is some sort of asymmetric response of volatility to positive and negative shocks in the foreign exchange market, especially in case of emerging market economies like India where foreign exchange market is still under partial government control. In a way, this has also been stated by Brooks (2002, p. 469) *viz.*, "…..there is equally no reason to suppose that such asymmetries only exist in equity markets." Bollen *et al.* (2000) have also noted that exchange rates can exhibit volatility given by EGARCH when the policy decisions are asymmetric in nature. Very recently, some Indian authors have tried to study the asymmetric nature of exchange rate intervention in the Indian foreign exchange market which is triggered, perhaps, by concerns about India's export competitiveness (Ramachandran and Srinivasan (2007)). The nature of the intervention is, perhaps, the reason behind this finding of EGARCH volatility specification being appropriate for the Indian exchange rate series. Also as noted by Kim and Sheen (2006), movements of exchange rates at times can be determined mostly by developments in one country rather than both. In addition if exchange rates are determined mostly by capital flows in the short run, asymmetric investment flows may lead to asymmetric volatility effects. We further note from these estimated models that none of the independent variables used has now been found to be significant, and return on India’s exchange rate is best explained by its own lags and EGARCH specification for volatility for all the sub-periods except the third one where return on BSESENSEX has been found to be significant.

It may be mentioned that time varying risk premia may also be a predictor of exchange rate return, as originally proposed by Engle *et al.* (1987) in the form of ARCH-M model, and accordingly we included a term, $g(h_t)$, in (2.2), where $g(h_t)$ is a monotonic function of the conditional variance h_t. Although risk may have a general representation like the Box-Cox transformation, as suggested by Das and Sarkar (2000), we have considered only three functional forms for $g(h_t)$, i.e., h_t, $\sqrt{h_t}$ and $\log h_t$. However, δ was found to be insignificant for all the five sub-periods, and hence the risk premia term has been omitted from all the equations.

<u>Sub-Period I</u>

$$y_t = \underset{(2.08)^{**}}{0.154}\, y_{t-1} + \hat{\hat{\varepsilon}}_t \tag{2.21}$$

$$\log \hat{g}(h_t) = -\underset{(5.2)^{***}}{3.132} + \underset{(3.98)^{***}}{0.584}\, g(\eta_{t-1}) + \underset{(3.59)^{***}}{0.52}\, g(\eta_{t-2}) + \underset{(12.42)^{***}}{0.696}\, \log(h_{t-1})$$

$$\hat{\theta} = \underset{(3.52)^{***}}{0.352}$$

Sub-Period II

$$y_t = \underset{(3.17)^{***}}{0.209}\, y_{t-1} - \underset{(4.35)^{***}}{0.182}\, y_{t-2} + \underset{(156.62)^{***}}{26.979}\, \hat{w}_{t-1}^{2} - \underset{(5203.8)^{***}}{1481}\, \hat{w}_{t-1}^{3} -$$

$$\underset{(781467)^{***}}{179183}\, \hat{w}_{t-1}^{4} + \hat{\hat{\varepsilon}}_t$$

$$\log\hat{g}(h_t) = -\underset{(5.91)^{***}}{3.386} + \underset{(13.07)^{***}}{1.268}\, g(\eta_{t-1}) + \underset{(17.75)^{***}}{0.739}\, \log(h_{t-1})\,;\, \hat{\theta} = -\underset{(2.00)^{**}}{0.127} \qquad (2.22)$$

Sub-Period III

$$y_t = -\underset{(2.75)^{***}}{0.158}\, y_{t-1} + \underset{(3.76)^{***}}{0.158}\, y_{t-4} - \underset{(7.62)^{***}}{0.064}\, b_t + \hat{\hat{\varepsilon}}_t$$

$$\log\hat{g}(h_t) = -\underset{(8.00)^{***}}{7.94} + \underset{(7.26)^{***}}{1.4}\, g(\eta_{t-1}) + \underset{(3.12)^{***}}{0.273}\, \log(h_{t-1})\,;\, \hat{\theta} = \underset{(2.97)^{***}}{0.261} \qquad (2.23)$$

Sub-Period IV

$$y_t = \underset{(2.11)^{**}}{0.083}\, y_{t-4} + \underset{(2.25)^{**}}{0.085}\, y_{t-5} + \underset{(3.89)^{***}}{0.138}\, y_{t-9} + \underset{(3.16)^{***}}{0.103}\, y_{t-10} - \underset{(2.09)^{**}}{0.063}\, y_{t-15} + \hat{\hat{\varepsilon}}_t$$

$$\log\hat{g}(h_t) = -\underset{(4.22)^{***}}{1.838} + \underset{(15.81)^{***}}{0.725}\, g(\eta_{t-1}) + \underset{(6.92)^{***}}{0.657}\, \log(h_{t-1}) + \underset{(2..31)^{**}}{0.213}\, \log(h_{t-2})\,;$$

$$\hat{\theta} = \underset{(3.72)^{***}}{0.287} \qquad (2.24)$$

Sub-Period V

$$y_t = \underset{(2.19)^{**}}{0.087}\, y_{t-1} - \underset{(2.49)^{**}}{0.051}\, y_{t-13} - \underset{(3035)^{***}}{4411}\, \hat{w}_{t-1}^{3} + \hat{\hat{\varepsilon}}_t$$

$$\log\hat{g}(h_t) = -\underset{(5.98)^{***}}{3.048} + \underset{(9.34)^{***}}{0.575}\, g(\eta_{t-1}) + \underset{(20.99)^{***}}{0.777}\, \log(h_{t-1})\,;\, \hat{\theta} = \underset{(6.34)^{***}}{0.451} \qquad (2.25)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

The values of the Ljung-Box $Q(k)$ and $Q^2(k)$ statistics have been provided in Table 2.2 and these indicate that there is no significant serial correlation present in the standardized residuals as well as in the squared standardized residuals at 5 percent level of significance.

Summarizing these results, we can conclude that for the first sub-period, the first lag of y_t alone appropriately models the conditional mean part while the conditional variance is properly captured by an EGARCH (1,2) model. For the second sub-period, only the first two lags have been found to be significant, but the conditional mean is not properly specified unless $\hat{w}_{t-1}^2, \hat{w}_{t-1}^3$ and $\hat{w}_{t-1}^4$ are included. The appropriate volatility specification for this sub-period is found to be an EGARCH (1,1) specification. It is only in the third sub-period that the coefficient of b_t, the return on stock price index (BSESENSEX), is significant and EGARCH (1,1) is adequate for volatility. Sub-period IV yields a mean specification with higher lags of exchange rate returns and an EGARCH (2,1) specification for conditional variance. For the last sub-period, we find that the conditional mean specification contains a higher order lag of the dependent variable along with a nonlinear (cubic) function of the recursive residuals and EGARCH (1,1) is the appropriate volatility specification. It may be noteworthy that a comparison between estimated models without consideration to volatility i.e., equations (2.16) through (2.20), and those with appropriate volatility specification i.e., equations (2.21) through (2.25), shows that the estimated conditional mean function is, as expectedly, somewhat different for the two situations. Earlier, most of the sub-periods showed misspecification of the conditional mean. But, with due consideration to volatility, the conditional mean is misspecified only for Sub-periods II and V.

2.4.4 Testing for the presence of higher-order dynamics

Finally, we carry out a simple exercise to check whether there is any remaining higher order, say 3rd or 4th order, dependences in the standardized residual, $\tilde{\varepsilon}_t$, defined as $\tilde{\varepsilon}_t = \hat{\hat{\varepsilon}}_t / \sqrt{\hat{h}_t}$, where $\hat{\hat{\varepsilon}}_t$ and $\hat{h}_t$ are given in equations (2.21) through (2.25). To this end, we consider dynamic relations involving $\tilde{\varepsilon}_t^3$ and $\tilde{\varepsilon}_t^4$ and then carry out tests of significance of the coefficients of the lag terms considered. These are given in equations (2.26) through (2.35) below. Looking at these, we observe that none of the lag values (of $\tilde{\varepsilon}_t^3$ and $\tilde{\varepsilon}_t^4$) considered is significant in any of the five sub-periods, showing thereby that there are no 3rd or 4th order dependences in the standardized

Table 2.2

Ljung-Box Test of Autocorrelation for Standardized Residuals (*Q*(*k*)) and Squared Standardized Residuals ($Q^2(k)$)

S. P	Lag *k*	1	2	3	4	5	6	7	8	9	18	36
1	$Q(k)$	1.381	3.8	3.83	5.563	5.799	5.973	5.977	10.51	10.51	20.99	33.23
	Prob.	0.24	0.15	0.28	0.234	0.326	0.426	0.542	0.231	0.311	0.28	0.601
	$Q^2(k)$	0.032	0.322	0.465	1.925	2.297	2.337	2.38	5.214	5.736	17.11	26.89
	Prob.	0.859	0.851	0.926	0.749	0.807	0.886	0.936	0.735	0.766	0.515	0.864
II	$Q(k)$	4.711	4.833	4.833	4.864	5.074	7.353	7.621	9.713	9.716	16.74	40.51
	Prob.	0.03*	0.089	0.184	0.302	0.407	0.289	0.367	0.286	0.374	0.541	0.278
	$Q^2(k)$	0.008	1.132	1.456	1.694	1.712	2.714	3.053	4.378	4.429	18.54	40.49
	Prob.	0.929	0.568	0.692	0.792	0.887	0.844	0.88	0.821	0.881	0.421	0.279
III	$Q(k)$	2.052	4.077	5.509	5.7	5.87	6.495	6.563	9.033	9.786	15.63	25.21
	Prob.	0.152	0.13	0.138	0.223	0.319	0.37	0.476	0.34	0.368	0.615	0.911
	$Q^2(k)$	0.463	0.588	0.613	2.629	2.959	3.215	3.249	3.74	4.161	13.01	27.21
	Prob.	0.495	0.745	0.893	0.622	0.706	0.781	0.861	0.88	0.9	0.791	0.854
IV	$Q(k)$	0.156	0.302	0.307	0.316	0.342	0.703	0.786	0.801	0.879	10.88	34.58
	Prob.	0.693	0.86	0.959	0.989	0.997	0.994	0.998	0.999	1.00	0.899	0.536
	$Q^2(k)$	0.17	0.494	0.495	1.265	2.104	2.919	3.284	3.293	3.528	9.538	17.33
	Prob.	0.68	0.781	0.92	0.867	0.835	0.819	0.858	0.915	0.94	0.946	0.996
V	$Q(k)$	1.531	2.484	2.485	2.957	3.428	4.290	4.336	4.753	5.130	18.93	42.18
	Prob.	0.216	0.289	0.478	0.565	0.634	0.637	0.740	0.784	0.823	0.396	0.221
	$Q^2(k)$	0.127	0.127	0.539	0.565	0.961	0.961	2.313	2.888	2.956	6.01	13.81
	Prob.	0.722	0.938	0.910	0.967	0.966	0.987	0.941	0.941	0.966	0.996	1.000

Note: *All the test statistic values indicate that there are no significant autocorrelations in the residuals at 5% level of significance.*
'Prob.' stands for the p-value under the null hypothesis that there is no autocorrelation.

residuals, and to that extent, the models obtained in (2.21) through (2.25) can be considered to be the 'best' models for the return on India's foreign exchange rate for the five sub-periods.

<u>Sub-Period I</u>

$$\tilde{\varepsilon}_t^3 = \underset{(1.391)}{1.218} + \underset{(0.104)}{0.008}\,\tilde{\varepsilon}_{t-1}^3 - \underset{(0.0003)}{0.00002}\,\tilde{\varepsilon}_{t-2}^3 - \underset{(0.065)}{0.005}\,\tilde{\varepsilon}_{t-3}^3 \quad (2.26)$$

$$\tilde{\varepsilon}_t^4 = \underset{(2.12)}{8.867} - \underset{(0.243)}{0.018}\,\tilde{\varepsilon}_{t-1}^4 - \underset{(0.28)}{0.02}\,\tilde{\varepsilon}_{t-2}^4 - \underset{(0.288)}{0.021}\,\tilde{\varepsilon}_{t-3}^4 \quad (2.27)$$

<u>Sub-Period II</u>

$$\tilde{\varepsilon}_t^3 = \underset{(0.276)}{0.14} - \underset{(1.625)}{0.092}\,\tilde{\varepsilon}_{t-1}^3 - \underset{(0.02)}{0.001}\,\tilde{\varepsilon}_{t-2}^3 + \underset{(0.508)}{0.031}\,\tilde{\varepsilon}_{t-3}^3 \tag{2.28}$$

$$\tilde{\varepsilon}_t^4 = \underset{(3.505)}{6.871} + \underset{(0.768)}{0.043}\,\tilde{\varepsilon}_{t-1}^4 - \underset{(0.739)}{0.042}\,\tilde{\varepsilon}_{t-2}^4 + \underset{(0.038)}{0.002}\,\tilde{\varepsilon}_{t-3}^4 \tag{2.29}$$

<u>Sub-Period III</u>

$$\tilde{\varepsilon}_t^3 = \underset{(1.163)}{0.949} - \underset{(0.039)}{0.003}\,\tilde{\varepsilon}_{t-1}^3 - \underset{(0.228)}{0.015}\,\tilde{\varepsilon}_{t-2}^3 - \underset{(0.614)}{0.041}\,\tilde{\varepsilon}_{t-3}^3 \tag{2.30}$$

$$\tilde{\varepsilon}_t^4 = \underset{(2.843)}{10.201} - \underset{(0.495)}{0.033}\,\tilde{\varepsilon}_{t-1}^4 - \underset{(0.466)}{0.031}\,\tilde{\varepsilon}_{t-2}^4 - \underset{(0.27)}{0.018}\,\tilde{\varepsilon}_{t-3}^4 \tag{2.31}$$

<u>Sub-Period IV</u>

$$\tilde{\varepsilon}_t^3 = \underset{(1.83)}{0.782} + \underset{(0.712)}{0.034}\,\tilde{\varepsilon}_{t-1}^3 - \underset{(0.004)}{0.0002}\,\tilde{\varepsilon}_{t-2}^3 - \underset{(0.11)}{0.005}\,\tilde{\varepsilon}_{t-3}^3 \tag{2.32}$$

$$\tilde{\varepsilon}_t^4 = \underset{(3.377)}{6.179} - \underset{(0.122)}{0.006}\,\tilde{\varepsilon}_{t-1}^4 + \underset{(0.021)}{0.001}\,\tilde{\varepsilon}_{t-2}^4 - \underset{(0.243)}{0.011}\,\tilde{\varepsilon}_{t-3}^4 \tag{2.33}$$

<u>Sub-Period V</u>

$$\tilde{\varepsilon}_t^3 = \underset{(1.861)}{1.008} + \underset{(0.158)}{0.005}\,\tilde{\varepsilon}_{t-1}^3 - \underset{(0.107)}{0.004}\,\tilde{\varepsilon}_{t-2}^3 - \underset{(1.030)}{0.036}\,\tilde{\varepsilon}_{t-3}^3 \tag{2.34}$$

$$\tilde{\varepsilon}_t^4 = \underset{(2.324)}{8.373} - \underset{(0.121)}{0.004}\,\tilde{\varepsilon}_{t-1}^4 - \underset{(0.109)}{0.004}\,\tilde{\varepsilon}_{t-2}^4 + \underset{(0.226)}{0.008}\,\tilde{\varepsilon}_{t-3}^4 \tag{2.35}$$

[*The values in parentheses indicate corresponding absolute values of t-ratios.*]

2.4.5 Forecasting performance

In order to assess the performance of the models thus obtained for the five sub-periods, we have obtained out-of-sample forecasts and then compared these with the observations in hold-out sample (February 16, 2004 **–** July, 14, 2004) using the criteria stated in Section 2.2.3. Obviously, the model appropriate for computing these forecasts would be the one corresponding to the last sub-period, i.e., equation (2.25). However, forecasts based on equations having functions of recursive residuals as explanatory variables are not possible to be computed, and thus we have used cubic function of return in place of cubic function of recursive residuals for equation (2.25), and obtained the following models for conditional mean and conditional variance for Sub-Period V:

$$y_t = \underset{(2.16)^{**}}{0.086}\, y_{t-1} - \underset{(2.34)^{**}}{0.050}\, y_{t-13} - \underset{(24517)^{***}}{3609}\, y_{t-1}^{3}$$

$$\log\hat{g}(h_t) = -\underset{(6.14)^{***}}{3.063} + \underset{(9.46)^{***}}{0.574}\, g(\eta_{t-1}) + \underset{(21.43)^{***}}{0.775}\, \log(h_{t-1})\,;\ \hat{\theta} = \underset{(6.42)}{0.451}. \qquad (2.36)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

It may be noted that the resulting model in (2.36) is almost the same model as in (2.25) both for the conditional mean and conditional variance. This model for sub-period V, which has been estimated using the sample observations from August 1, 2000 to February 13, 2004, has been re-estimated following recursive window method by expanding the sample with one more observation each time over time points of hold-out sample. In Table 2.3 we have given the values of out-of-sample forecast criteria of both the random walk and the chosen AR-EGARCH models. In this table, we have reported only the 1-,5- and 10-step ahead forecast aggregates. We observe from this table that the values of MSE, MAE and AMAPE are very small for both the models and for all the three types ('type' refers to forecast period) of forecasts. Further, comparing the out-of-sample forecasting performance of the AR-EGARCH model with the random walk model, we find that the performance of the AR-EGARCH model is better for all the three types of forecasts by all the four criteria. Although the difference in the two values corresponding to the two models by any of the criteria is quite small, it is thus found that in case of India's exchange rate, the performance of the AR-EGARCH model is better than the random walk model with drift. It may be mentioned that in this case, four observations, i.e., the 2316th, 2317th, 2321st and 2335th observations in the hold-out sample have been excluded from the computations since these observations are found to be outliers as compared to the rest of the observations.

Table 2.3

Out-of-Sample Forecast Performance

Random walk model with drift (i.e., equation 2.10a)				
Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	1.710E-05	0.00264	1.352	53.5
5	1.747E-05	0.00274	1.193	56
10	1.751E-05	0.00276	1.155	59
Chosen model (i.e., equation 2.36)				
1	1.182E-05	0.0023	1.042	55.2
5	1.263E-05	0.002439	1.105	55.2
10	1.268E-05	0.002467	1.134	55.2

2.5 Conclusions

In this paper, we have carried out a study for empirical determination of the appropriate model for return on daily foreign exchange rate of India after taking into account the modelling aspect of appropriate specification of both the conditional mean and conditional variance. The consideration for appropriate specification is due to the fact that misspecification of these moments can lead to misleading inferences on the model and consequently on its performance. Probable sources of misspecification of conditional mean considered in this paper are parameter instability, serial correlation and omission of other explanatory variables and higher order dynamics of own lagged values. Using Quandt-Andrews test, we have concluded that there have been four structural breaks in the Indian exchange rate series, and the corresponding break points have been estimated by the least squares based procedure, as suggested by Bai. As regards volatility, it has been found that the GARCH specification is not appropriate for any of the five sub-periods mentioned above. However, Nelson's EGARCH formulation has turned out to be performing very well with Ljung-Box diagnostic test suggesting no significant autocorrelation in the standardized residuals as well as in their squared values. We have thus obtained the 'best' model for India's exchange rate return for each of the five sub-periods. We have also empirically checked if there are still any remaining nonlinear dependences in the EGARCH adjusted residuals, and found evidence of no such dynamics in higher order moments. Finally, we have generated out-of-sample forecasts using the estimated model for last sub-period and obtained the values of some standard forecasting criteria. These values suggest that the forecasting performance of the chosen model is quite good.

It may also be stated that essentially the same approach of single-equation linear dynamic model with other exogenous variables has been adopted in Chapter 6 where the data frequency is monthly. Since in India time series data at monthly frequency are available for all relevant macrovariables, we have considered all such variables for the study and applied the predictive regression approach to finally decide which of the variables are to be included.

CHAPTER 3

Forecastability of the SETAR, SETAR-GARCH and Double Threshold GARCH Models

3.1 Introduction

In the preceding chapter, we have considered modelling and forecastability of daily exchange rate return of India in a linear dynamic model set-up. In this chapter as well as in the subsequent two chapters, the framework of analysis is changed from linear to nonlinear time series models. Financial time series display typical nonlinear characteristics which have also been detected by various statistical tests by Hinich and Patterson (1985), Scheinkmann and LeBaron (1989), Hsieh (1989,1991), Crato and de Lima (1994), among others. Now, nonlinearity in exchange rate series, in particular, has been documented by many researchers (Hsieh (1992,1993) and Brooks(1996,2001), to cite a few). In fact, as we have discussed in Chapter 1, it is now widely accepted that foreign exchange rate series exhibits strong signs of nonlinearity. Given such evidences in favour of nonlinearities, researchers have proposed different types of models for incorporating nonlinearity in the specification and then fitted those nonlinear time series models to financial data. The empirical specification of nonlinear models for exchange rate has also been motivated by nonlinear solutions presented for such variables in a number of theoretical models, such as the target zone models (Krugman (1991)) and the rational expectations model with central bank stochastic intervention rules (Hsieh (1992)).

Many economic and financial time series seem to undergo episodes during which the behaviour of the series changes quite dramatically compared to that exhibited previously. The behaviour may change once and for all, usually known as a ‘structural break’, in a series. However, it may as well change for a period of time before reverting back to its original behaviour or switching to yet another type of behaviour. This i.e., the latter is typically termed as ‘regime shift’ or ‘regime switch’, and the models capturing this behaviour are called regime switching models. Such models should not only allow all the observations to be used for the purpose of estimation, but also provide sufficient flexibility for different types of behaviour at different points of time. Two classes of regime switching models that potentially allow this to occur are the threshold autoregressive models and Markov switching regression models.

In this chapter, we consider the first model in the class of threshold autoregressive models while the second one is taken up in Chapter 4. Threshold autoregressive (TAR) models are basically a class of autoregressive models which allows a locally linear approximation of the mean function over a number of states (regimes), but globally (i.e., when it is taken as a whole) the model is nonlinear, and in this sense this is a class of nonlinear time series models (see, for details, Tong (1978, 1983, 1990), Tong and Lim (1980), Chan and Tong (1986) and Tsay (1989)). Clearly, these models are important when the observations may be drawn from one autoregressive model in one regime, but a different autoregressive model in another. Tong and Lim (1980) proposed a special case of TAR model where the state-determining variable is the variable under study itself and in that case the model is called the self-exciting TAR or SETAR model. Here it is the lag of the variable itself, which determines the regime that the variable is currently in. SETAR model has found wide applications in modelling exchange rate series, particularly in the environment of what is called 'managed floats'.

One of the earliest applications of SETAR model is due to Kräger and Kugler (1993) who reported the results of application of this model to weekly exchange rates of five currencies of developed economies. They argued that intuitively monetary authorities may intervene in the foreign exchange market as a reaction to large depreciations and appreciations of a currency, which lead to different behaviours for moderate and large changes of the exchange rate. In fact, in the European Monetary System (EMS) as well in some other countries, there is a requirement that currencies have to remain within some prescribed band around central parity, and any movement away from this band forces the central banks to intervene in the markets. Yadav *et al.* (1994) have used the model to analyze the time series of future 'basis' (the future price minus the spot price) for a future contract. Chappell *et al.* (1996) have used this important nonlinear time series model to explain exchange rates of European countries. They have employed SETAR model to allow for different behaviors of the exchange rates depending on whether these were near the exchange rate mechanism boundary or not.

Till very recently, nonlinearity as represented by the SETAR model i.e., a special kind of regime switching models, used to be incorporated in the analysis of financial and economic variables without any consideration to volatility specification in such models. In other words, the aspect of volatility which is so very much prevalent in financial variables could not be considered in SETAR (or for that matter, in TAR) models because of the absence of any such work which combines both these aspects. But researchers like Lamoureux and Lastrapes (1990) have noted that presence of structural breaks in the variance could lead to GARCH type of conditional variances which are highly persistent. As we know, volatility specification of the GARCH type or its other generalizations represent nonlinearity through conditional variance. Since both types of nonlinearity i.e., nonlinearity in the conditional mean as well as in the conditional variance are important for time series modelling, some researchers have attempted to combine the SETAR model and the GARCH model. This has led Tong (1990) to suggest what is now called the SETAR-GARCH model i.e., the special threshold autoregressive model with a changing conditional variance. Although this model should perform better than the SETAR model (see Li and Lam (1995), in this context), it is constrained by the fact that it assumes a fixed description of the conditional variance for all the observations.

It is now fairly well known that in financial time series the behavior of the conditional variance is often asymmetric conditional on the previous information (see, for instance, Campbell and Hentschell (1990), Nelson (1990,1991), Rabemanjara and Zakoian (1993), for details). In fact, the threshold behaviour assumed for the conditional mean function can very well be exhibited by the conditional variance function also. If regime switching in the mean is a feature with the data, this may be accompanied by regime switching in the conditional variance function also. Keeping this in mind, Li and Li (1996) have generalized the TAR model to what they have called a double threshold ARCH (DTARCH) model. They have applied this model to

Hong Kong stock returns, and found that the proposed model is useful in capturing asymmetries in both return and its volatility which could not be found under one of the component models alone.

Although there have been several applications of SETAR model and only a few of its generalizations like the SETAR-GARCH and DTARCH models for studying exchange rate dynamics, studies focussing on forecasts generated by these models are comparatively fewer. Some of the notable references are: Peel and Speight (1994), Brooks (1996, 1997, 2001), Clements and Smith (1999, 2001), Dacco and Satchell (1999) and Boero and Marrocu (2002, 2004). In addition to obtaining forecasts, some of these studies have also compared the forecasting performances of these models. For instance, Clements and Smith (1999) have compared the multi-period forecast performance of a number of empirical SETAR models using time series data on exchange rate as well as some other variables. While Brooks (2001) has investigated the out-of-sample forecasting performance of DTGARCH model for French franc / Deutschmark exchange rate series between a single regime and a two-regime model, Boero and Marrocu (2002) have studied the relative performance of SETAR and GARCH models as contrasted with other linear counterparts for returns on three most important exchange rates in terms of US dollar, namely, French franc, German mark and Japanese yen. Some of these studies have produced evidences of forecasting gains from nonlinear models as compared to linear specification although there is no clear evidence in favor of these nonlinear models insofar as out-of-sample accuracy is concerned.

In this chapter, we are basically concerned with the fitting of SETAR as well as DTGARCH models for return on exchange rate of Indian rupee in terms of US dollar and then evaluating the forecasting performances of these two models. In this context, it may be pointed out that insofar as the exchange rate variable is concerned, what we observe in India today is, as in most of the countries, an ‘intermediate regime’, which lies between the two textbook versions of fixed and flexible regimes. The exchange rate is partly managed, and a scrutiny of the exchange rate management strategy of the Reserve Bank of India (RBI) reveals a strong commitment to exchange rate stability. Kohli (2000) has argued that RBI keeps the exchange rate aligned to its fundamentals, the most important one being the price level. Ghosh (2002), has however, noted that even though the RBI has not deemed it feasible to pursue exchange rate targeting, there is indeed some definitive targeting by the RBI based on the value of purchasing power parity (PPP). The targeting, according to him, is reinforced by the belief that there is strong correlation between over-valuation of rupee and simultaneous disturbances in the foreign exchange market that brings back rupee to its purchasing power parity (PPP) value. There is indeed some definitive nominal exchange rate targeting by the RBI based on some priority, which changes from time to time. The priority could be to maintain foreign exchange market stability or money market stability. In May 1997, the ‘Tarapore committee report on capital account convertibility’ had recommended the RBI to have a ‘Monitoring Exchange Rate Band’ of +5/-5 percent around the neutral real effective exchange rate (REER) as part of transparent exchange rate policy. The committee suggested that the RBI should intervene when the REER is outside the band and that it should maintain transparency about its intervention. However, the RBI has been highly secretive in its intervention activities and refuses to release data on intervention on a daily basis. This targeting is, therefore, not very clear, and hence a study using a SETAR model could provide some insight to this important issue.

To the best of our knowledge, hardly any such work with India’s exchange rate data has yet been carried out. In fact, there are very few studies on nonlinear time series modelling with exchange rate data of any emerging or, for that matter, developing economy. One such study, Medeiros *et al.* (2001), has used nonlinear models to model the Indian monthly exchange rate along with several other series to find out whether these nonlinear models perform better than the autoregressive (AR) and random walk (RW) models. They have used the artificial neural network (ANN) model as well as the neuro-coefficient smooth transition autoregression which nests the SETAR, smooth transition AR (STAR) and ANN models and compared the different alternatives to model as well as forecast the monthly exchange rate series.

To conclude this section, we may state that the findings of this chapter is likely to throw some light on the extent to which nonlinear models like the SETAR and DTGARCH models are able to capture volatility, persistence and regime shifts inherent in the exchange rate variable of an important emerging economy like India. Empirical evidences based on likelihood ratio test, diagnostic checks as well as out-of-sample forecasting performance have clearly shown that, insofar as return on India's exchange rate is concerned, the performance of DTGARCH model is the best amongst all the nonlinear time series models considered in this chapter. This finding, therefore, establishes the importance of this class of nonlinear models where thresholds are considered both for conditional mean and conditional variance.

The outline of this chapter is as follows. In the next section, the model and methodology including relevant tests and diagnostic checks for model adequacy are described. Empirical results are discussed in Section 3.3. Some concluding remarks are made in Section 3.4.

3.2 The model and methodology

In this section, we introduce the SETAR, SETAR-GARCH and DTGARCH models and briefly describe their estimation procedures. Some relevant issues on adequacy of a chosen model are also discussed. It was Tong (1978, 1983 and 1990) who first proposed the class of threshold autoregressive models. This model (see also, Tong and Lim (1980)) is a simple relaxation of a standard linear autoregression model where local linear approximation is allowed for a number of states (regimes). This model is globally nonlinear although each component is piece-wise linear. Following Tong (1990) and Brooks (2002), we specify a general threshold autoregressive model (TAR) for exchange rate return, y_t, as

$$y_t = \sum_{j=1}^{J} I_t^{(j)} \left(\phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} + \sigma_j \varepsilon_t \right), \; s_{j-1} < z_{t-d} \le s_j \;, t = 1,2,\ldots,T \tag{3.1}$$

where $I_t^{(j)}$ is an indicator function for the jth regime taking the value one if the underlying variable is in state j and zero otherwise, z_{t-d} is an observed variable determining the switching point after some delay, d, s_j's are the threshold values, ε_t is an independently and identically distributed (i.i.d.) error process with zero mean and unit variance and σ_j is the standard deviation of the errors for regime j. If the regime changes are driven by own lags of the underlying variable y_t i.e., $z_{t-d} \equiv y_{t-d}$, then the model is called the self-exciting TAR (SETAR) model. Standard conditions, as stated in Tong (1990), for stationarity of TAR / SETAR processes are assumed to hold. An interesting point about the SETAR model is that the stationarity of y_t does not necessarily require the model to be stationary at each regime.

Estimation of the parameters, $\theta = (\phi_i^{(j)}, s_j, d, p_j, \sigma_j)'$, of the SETAR model is, however, more difficult than that for the standard autoregressive model since these cannot be determined simultaneously, and the values chosen for one parameter are likely to influence estimates of the others. The estimation procedure to be followed for a SETAR model can be divided into four parts. First is the determination of the threshold values. Tong (1983, 1990) has suggested a complex nonparametric lag regression procedure to estimate these values. However, it is preferable to endogenously estimate the values of the threshold as part of the nonlinear least square (NLS) optimization procedure. But this is not feasible since the underlying relationship between the variables is discontinuous at the thresholds, and hence these cannot be estimated at the same time as the other parameters of the model. Insofar as the actual estimation of this model is concerned, we use a method where initially s_j is determined using a grid search procedure that seeks the minimal residual sum of squares over a range of probable values of threshold of an assumed model.

The second step in the estimation procedure is concerned with the determination of appropriate lag lengths, and this involves the use of an approach which is conditional upon the specified threshold values. In this method, an information criterion is employed to select across the lag lengths in each regime simultaneously. Franses and Van Dijk (2000), however, found that in practice the system would be resident in a regime for a considerably longer period of time than the others. As noted by Brooks (2002), the performance of information-based criteria will not be satisfactory in such situations. This is because in case the number of observations is small in any regime, the overall reduction in the residual sum of squares will be very small if more parameters are added, and this will lead the criteria to select very small model orders for states containing few observations. A solution, therefore, is to define an information criterion that does not penalize the whole model for additional parameters in one state. Tong (1990) proposed a modified version of Akaike's information criterion (AIC) that weighs the residual variance for each regime by the number of observations in that regime. For the two-regime case, the modified AIC can be written as

$$AIC(p_1, p_2) = \tilde{T} \ln \hat{\sigma}_1^2 + (T - \tilde{T}) \ln \hat{\sigma}_2^2 + 2(p_1 + 1) + 2(p_2 + 1) \tag{3.2}$$

where $\tilde{T}$ is the number of observations in regime 1 and T is the total number of observations in regimes 1 and 2 taken together, p_1 and p_2 are the lag lengths and $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ the residual variances of regimes 1 and 2, respectively.

The determination of delay parameter, d, is carried out in the third step. This can be done in a variety of ways one of which is the use of an information criterion. However, in many applications it is typically set to 1 on theoretical grounds. It has been argued (Kräger and Kugler (1993)) that in the context of financial markets, it is most likely that the most recent past value of the state-determining variable would be the one to determine the current state.

Finally, the autoregressive coefficients are estimated by using the nonlinear least squares estimation procedure. The details of this method for a two- as well as a three-regime SETAR model, each of which has, in fact, been fitted to India's exchange rate return series are described in Section 3.3.3.

As discussed in the preceding section, the SETAR model has been generalized by Tong (1990) himself by combining it with GARCH type of volatility specification so as to produce what he termed as the SETAR-GARCH model. This combined model is specified as below:

$$y_t = \sum_{j=1}^{J} I_t^{(j)} \left(\phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} \right) + \sqrt{h_t}\, \varepsilon_t \,, \quad s_{j-1} < y_{t-d} \le s_j \tag{3.3}$$

$$h_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i h_{t-i} \varepsilon_{t-i}^2 + \sum_{i=1}^{m} \beta_i h_{t-i} \tag{3.4}$$

where ε_t is i.i.d. N(0,1) and h_t is a function of the elements of the information set $\psi_{t-1} = \{y_{t-1}, y_{t-2}, ...\}$ at time t-1. Obviously, $\sqrt{h_t}\,\varepsilon_t (= \varepsilon_t^*,$ say) is conditionally normal with $E(\varepsilon_t^* \mid \psi_{t-1}) = \sqrt{h_t}\,E(\varepsilon_t \mid \psi_{t-1}) = 0$ and $V(\varepsilon_t^* \mid \psi_{t-1}) = h_t V(\varepsilon_t \mid \psi_{t-1}) = h_t$. However, this model is constrained by the fact that it assumes a fixed description of the conditional variance irrespective of the regime in which an observation falls. Obviously, this would not hold good, in general. The generalization by Li and Li (1996) makes allowance for regime shift to affect the conditional variance specification as well. Thus, they allowed for the possibility of threshold effects both in the conditional mean and conditional variance so that the characteristic of asymmetric volatility is captured in the model. This model, called the double-threshold autoregressive (DTARCH) or a more generalized version called the DTGARCH model, is specified as follows:

$$y_t = \sum_{j=1}^{J} I_t^{(j)} \left(\phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} + \varepsilon_t^* \right) \quad , \quad s_{j-1} < y_{t-d} \le s_j \tag{3.5}$$

$$h_t = \sum_{j=1}^{J} I_t^{(j)} \left(\alpha_0^{(j)} + \sum_{i=1}^{n_j} \alpha_i^{(j)} \varepsilon^{*2}_{t-i} + \sum_{i=1}^{m_j} \beta_i^{(j)} h_{t-i} \right) \quad . \tag{3.6}$$

Li and Li have proposed the ML method of estimation for the parameters of DTGARCH model. In fact, they have shown that the estimates can be obtained via a weighted least squares algorithm.

3.2.1 Testing the case for higher models

In this section, we briefly discuss some tests of relevance for models with regime-switching nonlinearity in conditional mean and / or conditional variance. Although estimation methods for such regime switching models including the DTGARCH model are now readily available, it is a good idea to explore the potential usefulness of these models having broader formulations as compared to those with simpler formulations. To that end, some of these tests could be the following: linearity *versus* SETAR, SETAR *versus* SETAR-GARCH, SETAR-GARCH *versus* DTGARCH, and SETAR *versus* DTGARCH. We first discuss the first one *viz*., linearity *versus*

SETAR. While considering the SETAR model instead of a single regime linear AR model, an important question that naturally arises is whether the additional regimes add significantly to explaining the dynamic behavior of y_t. A natural approach to answering this question empirically is to take the single regime linear model as the null hypothesis and the regime switching SETAR model as the alternative. Thus, for instance, in case of a two-regime SETAR model, the null (H_0) and alternative (H_1) hypotheses are specified, assuming, without any loss of generality, that $p_1 = p_2 = p$ (say), as

$$H_0 : \phi_0^{(1)} = \phi_0^{(2)}, \quad \phi_1^{(1)} = \phi_1^{(2)}, \quad .... \quad , \quad \phi_p^{(1)} = \phi_p^{(2)}$$

and $H_1 : \phi_i^{(1)} \neq \phi_i^{(2)}$ for at least one $i \in \{0,1,....., p\}$.

Now, the underlying statistical test for this testing problem suffers from the problem of unidentified nuisance parameters under the null hypothesis (see Chan (1990, 1993), Chan and Tong (1990), Hansen (1997, 2000) and Franses and Dijk (2000), for details on this problem). For the SETAR model, this parameter[1] is *s*. It is clear that *s* is the unidentified nuisance parameter since it is not restricted under the null hypothesis, and further it is not present in the linear model. The main problem in such cases is that the conventional statistical theory cannot be applied to obtain the (asymptotic) distribution of the test statistics (see Davies (1977) and Hansen (1996)), and consequently the test statistics have nonstandard distributions under H_0, the critical values of which are to be obtained by means of simulation and / or bootstrap method(s). A solution to this identification problem is to obtain a likelihood ratio (LR) or *F*-statistic which tests the restrictions as given by the null hypothesis. Defining the point-wise *F*-statistic at threshold level *s* as

$$F(s) = T\left(\frac{\tilde{\sigma}^2 - \hat{\sigma}^2(s)}{\hat{\sigma}^2(s)} \right) \tag{3.7}$$

[1] In case of a two-regime SETAR model, the subscript j associated with the threshold parameter s_j is obviously dropped, and the threshold conditions (rules) may be stated as $y_{t-d} \leq s$ and $y_{t-d} > s$.

where $\tilde{\sigma}^2$ is the estimate of the residual variance under the null hypothesis of linearity and $\hat{\sigma}^2(s)$ is the estimated residual variance for the SETAR model given threshold *s*, the relevant test statistic, $F(\hat{s})$, is equivalent to the supremum over the set S of possible thresholds of the point-wise test statistic $F(s)$ i.e., $F(\hat{s}) = \underset{s \in S}{Sup}\, F(s)$. As noted in Hansen (1997, 2000), the point-wise *F*- statistic can also be computed as TR^2 where R^2 is the coefficient of determination of an appropriately defined artificial regression, and it has an asymptotic χ^2 distribution with $p+1$ degrees of freedom. The test statistic $F(\hat{s})$ is, therefore, the supremum of a number of dependent statistics, each of which follows an asymptotic χ^2 distribution. This shows that the distribution of $F(\hat{s})$ itself is nonstandard. Chan (1991) has tabulated the approximate percentage points of the asymptotic null distribution of this nonstandard test statistic.

Once this test exercise is carried out and the conclusion is found to be in favor of the SETAR model, the next test, in order, should be the one between the SETAR and SETAR-GARCH models. Since returns on exchange rate are most likely to exhibit volatility, adequate representation of nonlinearity in conditional mean by means of a SETAR model without any consideration to volatility modelling is unlikely to yield the most appropriate nonlinear dynamic model for y_t. We, therefore, next seek to establish if a broader modelling set-up as given by the SETAR-GARCH model where a fixed GARCH specification for conditional variance without any threshold consideration is considered, can significantly explain the dynamic behavior of y_t better. To that end, the null hypothesis is specified as $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_n = \beta_1 = \beta_2 = ... = \beta_m = 0$ and the alternative as H_1: at least one inequality holds. Obviously, the problem of unidentified nuisance parameter under the null does not arise in this test since threshold *s* which defines the two regimes, is present in both the null and alternative hypotheses. Therefore, the usual likelihood-based tests like, for instance, the LR test can be used to carry out this testing exercise. If this test concludes that the SETAR-GARCH has a better explanatory power than the

SETAR model i.e., if a model having a piece-wise linear conditional mean and fixed GARCH innovation is better than just a piece-wise linear conditional mean model, the question that naturally arises is whether the DTGARCH model where in addition to piece-wise linear conditional mean, the conditional variance specification is also piece-wise linear given previous information, could further explain significantly the dynamic behavior of y_t. Another interesting rival to the DTGARCH model could be the original SETAR model in the sense that it may be of interest to test whether this simpler formulation, i.e., the SETAR model is adequate in describing the return on India's exchange rate. Given the threshold structure, the usual LR test statistic for testing a SETAR model against a more general DTGARCH model is valid in this situation. Another interesting hypothesis is the converse of the above i.e., whether a model with threshold both in conditional mean and conditional variance fits significantly better than a model with threshold in conditional variance only. The corresponding likelihood ratio test is again valid given the threshold structure.

3.2.2 Diagnostic checking for adequacy of the DTGARCH-model

In this section, we consider checking the adequacy of conditional mean and variance of the DTGARCH model. An important issue when using nonlinear time series model is whether the proposed model adequately captures all nonlinear features of the time series under investigation. One possible procedure to examine this is to apply a test for any remaining nonlinearity in an estimated model. If the model in question is a SETAR one, testing for remaining nonlinearity involves estimating the multiple-regime model or estimating a combined model like the SETAR-GARCH or DTGARCH, and then carrying out appropriate test(s) for the significance of the additional parameters in the more general model. Alternatively, residual autocorrelations or their higher ordered values may be used for checking the adequacy. Now, Eitrheim and Teräsvirta (1996) have opposed the use of customary portmanteau test of Ljung and Box (1978), and introduced Lagrange multiplier (LM) / Rao's score-type test for testing the remaining nonlinearity. Li and Li (1996), on the other hand, have asserted the use of overall test of fit based on the first *k* residual autocorrelations. Earlier, Li (1992) has obtained the asymptotic distribution of residual autocorrelations for a general nonlinear time series model which includes the TAR model as a special case, and Li and Mak (1994) have derived the asymptotic distribution of the squared residual autocorrelations for a general conditional heteroscedastic nonlinear time series model. Their results show that the standard errors of the first few residual autocorrelations and squared residual autocorrelations could be much less than $1/\sqrt{T}$, and hence following Li and Li (1996), the adequacy of DTGARCH model can be checked using the residuals of this i.e., DTGARCH model.

In fact, Li and Li (1996) have derived the asymptotic standard errors of the residual autocorrelation function (ACF) and squared residual ACF of the DTGARCH model and also introduced two chi-squared statistics, $Q_J(M)$ and $Q_{JJ}(M)$ which are defined later, to check the adequacy of the model based on first M residuals. For the DTGARCH model as specified in (3.5) and (3.6), the lag k autocorrelation of the standardized residual $\hat{\varepsilon}_t = \hat{\varepsilon}_t^* / \sqrt{h_t}$ is defined as

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^{T} (\hat{\varepsilon}_t^* / \sqrt{\hat{h}_t} - \gamma)(\hat{\varepsilon}_{t-k}^* / \sqrt{\hat{h}_{t-k}} - \gamma)}{\sum_{t=1}^{T} (\hat{\varepsilon}_t^* / \sqrt{\hat{h}_t} - \gamma)^2}, \text{ for } k = 1,2,3\ldots \tag{3.8}$$

$$\mu_t(\theta) = \sum_{j=1}^{J} I_t^{(j)} \left(\phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} \right) \tag{3.9}$$

where, $\gamma = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^* / \sqrt{\hat{h}_t}$, $\hat{\varepsilon}_t^* = y_t - \hat{\mu}_t(\theta)$, $\hat{\mu}_t(\theta)$ and $\hat{h}_t$ are the estimated values of $\mu_t(\theta)$ and h_t at $\theta = \hat{\theta}$, respectively, and $\hat{\theta}$ is the ML estimate of θ, the vector containing all the parameters in conditional mean and conditional variance. Assuming that y_t is an ergodic and stationary DTGARCH process, Li and Li (1996) have shown that

$$\sqrt{T}\hat{\rho} \sim N(0,U), \qquad U = I - HG^{-1}H'$$

where $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2, ..., \hat{\rho}_M)'$, I is the identity matrix, $H' = (H_1, H_2, ..., H_M)'$ with H_k defined as $H_k = -E\left(\frac{\varepsilon^*_{t-k}}{\sqrt{h_t h_{t-k}}} \frac{\partial \mu_t(\theta)}{\partial \theta} \right)$, G is the Fisher information matrix[2] i.e.,

$$G = -E\left(T \frac{\partial^2 L(\theta)}{\partial \theta \partial \theta'} \right) \quad , \quad L(\theta) = -\frac{1}{2T} \sum_{j=1}^{J} I_t^{(j)} \left(\sum_{t=l+1}^{T} (\ln h_t + \varepsilon_t^{*2} h_t^{-1}) \right) \quad \text{and}$$

l = max ($p_1, p_2, ..., p_J, m_1, m_2, ..., m_J, n_1, n_2, ..., n_J$) .

The overall goodness of fit statistic for the conditional mean specification is measured by $Q_J(M)$ defined as $Q_J(M) = T\hat{\rho}'\hat{U}^{-1}\hat{\rho}$. It has an asymptotic χ^2 distribution with M degrees of freedom under the null that the model is adequate.

In order to check for the adequacy of the conditional variance, Li and Li have used the usual lag k squared standardized residual autocorrelation defined as

$$\rho_k^* = \frac{\sum_{t=k+1}^{T} (\hat{\varepsilon}_t^{*2} / \hat{h}_t - \gamma^*)(\hat{\varepsilon}_{t-k}^{*2} / \hat{h}_{t-k} - \gamma^*)}{\sum_{t=1}^{T} (\hat{\varepsilon}_t^{*2} / \hat{h}_t - \gamma^*)^2} \quad \text{, for } k = 1,2,3,\ldots \qquad (3.10)$$

$$\gamma^* = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^{*2} / \hat{h}_t \quad .$$

If the model is adequate, γ^* converges to one in probability and ρ_k^* can be replaced by

[2] The log-likelihood function has been normalized by T.

$$\hat{\hat{\rho}}_k = \frac{\sum_{t=k+1}^{T} (\hat{\varepsilon}_t^{*2} / \hat{h}_t - 1)(\hat{\varepsilon}_{t-k}^{*2} / \hat{h}_{t-k} - 1)}{\sum_{t=1}^{T} (\hat{\varepsilon}_t^{*2} / \hat{h}_t - 1)^2} \quad \text{, for } k\text{=1,2,3,….} \tag{3.11}$$

assuming, as before, that y_t is an ergodic and stationary DTARCH process, Li and Li have obtained the asymptotic distribution of $\hat{\hat{\rho}}_k$, which is given below:

$$\sqrt{T}\hat{\hat{\rho}} \sim N(0,V), \qquad V = I - \frac{1}{4} D G^{-1} D'$$

where $D' = (D_1, .D_2 ....., D_M)$ and D_k is defined as

$$D_k = -E\left(\frac{1}{h_t} \frac{\partial h_t}{\partial \theta} \left(\frac{\varepsilon_{t-k}^{*2}}{h_{t-k}} - 1 \right) \right) \quad . \quad . \tag{3.12}$$

This result would produce more accurate asymptotic standard errors than $1/\sqrt{T}$ for the squared residual autocorrelations. Based on this distributional result, one can immediately note that $Q_{JJ}(M)$ defined as $Q_{JJ}(M) = T\hat{\hat{\rho}}' \hat{V} \hat{\hat{\rho}}$ can be used as a goodness of fit test. This statistic has an asymptotic χ^2 distribution with *M* degrees of freedom if the model is adequate.

Finally, the out-of-sample forecasting performance of these models are evaluated by using the standard forecast evaluation criteria like the MSE, MAE, AMAPE and PCSP, which have been defined in Section 2.2.3 of the preceding chapter.

3.3 Empirical results

In this section, we report and discuss the empirical findings on the estimation and testing of SETAR, SETAR-GARCH and DTGARCH models as well as their out-of-sample forecasting performances. The computations required for the two- and three-regime SETAR models have been carried out by using the software WinRats and codes for SETAR, as provided in Brooks (2002). Insofar as estimation and forecasts for the DTGARCH models are concerned, the codes made available by Chris Brooks have been used.

As stated in Chapter 2, the foreign exchange rate data used in this study is the Reserve Bank of India (RBI) reference rate of Indian rupee/US dollar, and these were downloaded from the RBI website www.rbi.org.in. Termed as the spot exchange rate, it is the price of one unit of the US dollar in rupee terms. This daily level data on the spot exchange rate (RBI reference rate) spanning from 1st November 1994 to 14th July 2004 - a total of 2388 data points- have been taken for this study. Out of these 2388 data points, 2287 points i.e., those from 1st November 1994 to 13th February 2004 have been used for the purpose of model estimation and testing, and the remaining observations have been kept as hold-out sample for obtaining the out-of-sample forecasts.

3.3.1 The two-regime SETAR model

We have fitted two SETAR models to our data set- one having two regimes and the other three. We first present the results of the two-regime (i.e., one threshold) SETAR model where the value of the delay parameter, *d*, is taken to be one. This choice of *d* has been done on theoretical grounds, as mentioned earlier. At first it is necessary to find a tentative threshold value, and for this we have taken, to start with, the same lag value for both the regimes. As described in Section 3.2, we initially started with a very large lag (15, to be specific) for both the regimes and used a grid search procedure to find the minimal residual sum of squares over a range of values for the threshold. All probable values of the threshold considered by us have included the values of the dependent variable, y_t, upon eliminating the first and the last 20 per cent of the observations when arranged in increasing or decreasing order. The threshold value thus obtained from this exercise is 0.000506971. Now, the 15th lag was found to be insignificant for both the regimes, and hence conditional on this threshold value, we reduced the lag length and selected it according to Hall's (1994) criterion based on the significance of the last lag value. In order to determine the appropriate number of lags for both the regimes i.e., p_1 and p_2, we have used the Akaike's Information criterion (AIC), as given in (3.2), for all possible combinations of lag lengths. The model with the minimum AIC value was found to have lag length of 5 and 4 for the two regimes, respectively. Using these lag values, we again carried out the exercise of finding the appropriate threshold value which corresponds to the minimum residual sum of squares. Now the threshold value was obtained as 0.000544662. Using this as the threshold, we once again determined the values of p_1 and p_2 by AIC and these came out to be the same as earlier i.e., p_1=5 and p_2=4. Finally, the autoregressive coefficients were estimated by the method of nonlinear least squares of the final model having lags 5 and 4, respectively for the two regimes and the estimated threshold value being 0.000544662.

The two-regime SETAR model thus obtained for return on Indian rupee / US dollar exchange rate is as follows:

$$\hat{y}_t = \underset{(2.602)^{***}}{0.00013} + \underset{(2.354)^{***}}{0.21985}\, y_{t-1} - \underset{(0.053)}{0.00334}\, y_{t-2} - \underset{(0.717)}{0.04114}\, y_{t-3} + \underset{(2.073)^{**}}{0.07562}\, y_{t-4} - \underset{(1.072)}{0.05383}\, y_{t-5}, \quad \text{for } y_{t-1} < 0.000544662 \tag{3.13}$$

$$\hat{y}_t = \underset{(3.839)^{***}}{0.00083} - \underset{(1.180)}{0.12251}\, y_{t-1} + \underset{(1.444)}{0.17300}\, y_{t-2} - \underset{(1.057)}{0.82321}\, y_{t-3} + \underset{(2.705)^{***}}{0.34768}\, y_{t-4}, \quad \text{for } y_{t-1} \geq 0.000544662 \tag{3.14}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

The sample size for the first regime is 1703 while that of the second is 583.

3.3.2 Testing for threshold autoregression

As discussed in Section 3.2.1, we now test the null hypothesis of a (single-regime) linear model against the alternative of a two-regime SETAR model. In the threshold model, the lag lengths have been kept uniform across the two regimes and selected according to Hall's criterion. Since in the linear model we had found lag 15 to be significant, we have taken 15 lags as the number of lags in our analysis. Also, we eliminated the first 20 % and the last 20% of the data (in ascending / descending order) as probable threshold values. The F statistic, as given in Section 3.2.1 i.e., $F(\hat{s}) = \underset{s \in S}{Sup}\, F(s)$ has been computed where S is the set of all possible threshold values and $F(s)$ is as defined in (3.7). We find that the value of $F(\hat{s})$ thus computed for our data is 148.785 and this exceeds the available critical value as tabulated in Chan (1991), and hence we can conclude that the null hypothesis of linearity is rejected indicating thereby that a nonlinear two-regime SETAR model can better explain the Indian rupee / US dollar exchange rate return than the (single-regime) linear model.

3.3.3 The three-regime SETAR model

In this section we report the results of a three-regime SETAR model fitted to the Indian rupee / US dollar exchange rate return series. Note that a three-regime SETAR model has been found to be appropriate for exchange rates within a European exchange rate mechanism (ERM) (Chappell *et al.* (1996)). Such an exercise would empirically establish if the behaviour of foreign exchange rate series of a major emerging economy like India is somewhat similar to those of the developed European economies. For the estimation of this model, we have proposed a procedure which is somewhat similar to the one discussed in the case of two-regime or equivalently, one-threshold, model. As a starting point, we have taken the value of the threshold obtained in the two-regime SETAR model i.e., 0.000544662, as one of the two threshold values and then used a grid search procedure to determine the other threshold. Once this threshold is determined, we set it as given and then carried out a grid search exercise to obtain the first threshold. This grid search is thus continued till convergence is attained. The initial values of thresholds thus obtained are –0.000236435 and 0.000288892. Using these two thresholds, we have used AIC to determine the appropriate lags. Based on these computations, the appropriate lag lengths for the three regimes have been obtained as 4, 2 and 4, respectively. Using these lag lengths for specifying the models for the three regimes, we have once again determined the threshold values and these have been found to be -0.000214754 and 0.000544662. Finally, the autoregressive coefficients have been estimated, as in the two-regime model, by applying the nonlinear least square method of estimation. The following is the estimated three-regime SETAR model for the Indian rupee / US dollar exchange rate return series:

$$\hat{y}_t = \underset{(1.902)^{*}}{0.00021} + \underset{(2.267)^{**}}{0.23705}\, y_{t-1} + \underset{(0.813)}{0.04819}\, y_{t-2} - \underset{(0.802)}{0.04626}\, y_{t-3} + \underset{(2.192)^{**}}{0.0765}\, y_{t-4} \quad ,$$

$$\text{for } y_{t-1} < -0.000214754 \tag{3.15}$$

$$\hat{y}_t = \underset{(0.953)}{0.00005} + \underset{(1.654)^{*}}{0.4197}\, y_{t-1} - \underset{(1.668)^{*}}{0.24008}\, y_{t-2} \quad ,$$

$$\text{for } -0.000214754 \le y_{t-1} < 0.000544662 \tag{3.16}$$

$$\hat{y}_t = \underset{(3.839)^{***}}{0.00083} - \underset{(1.181)}{0.12251}\, y_{t-1} + \underset{(1.444)}{0.17300}\, y_{t-2} - \underset{(1.057)}{0.08232}\, y_{t-3} + \underset{(2.705)^{***}}{0.34768}\, y_{t-4} \quad ,$$

$$\text{for } y_{t-1} \ge 0.000544662 \tag{3.17}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

The sample sizes for the first, second and third regimes are 839, 864 and 583, respectively. Hence it is clear that the sample sizes for both the two-regime and three-regime SETAR models are large enough for all the regimes and hence the estimates of the parameters in each regime are likely to be reliable. Comparing between the two-regime and three-regime models, we observe that apart from the constant, lags 1 and 4 are significant in the first regime of the two-regime model and the same is the case with the first regime of the three-regime model as well. Exactly similar is the finding with respect to the last regime i.e., second regime in the two-regime model and third regime in the three-regime model. The middle regime in the three-regime model, however, is found to have lags 1 and 2 significant only at 10% level of significance. The model obtained for the middle regime is barely statistically significant. Since the coefficients do not have strong statistical significance, the chosen model, in effect, becomes a random walk model with drift. Thus, it might as well be stated that a random walk model is the best description of the Indian exchange rate series for the middle regime.

The maximum log-likelihood values of the two-regime and three-regime SETAR models have been found as 10330.5 and 10339.3, respectively. This shows that there is some improvement in the three-regime model by the criterion of maximum log-likelihood value although at the cost of two additional parameters, and to that end, the more general three-regime SETAR model can be used to model the return series of India’s exchange rate. However, it may be noted that a statistical conclusion between these two models is found analogous to the situation of testing a linear model against a two-regime SETAR model. In any case, since the middle regime in the three-regime SETAR model is barely significant, we may conclude that the aspect of two thresholds in the Indian exchange rate market is rather absent. In India, it is rather the case of one threshold that is found to be empirically valid. Thus, insofar as the behaviour of India’s foreign exchange rate is concerned, one can say that it is different from the developed European countries where the three-regime SETAR model is quite empirically established for their exchange rates. Of course, given India’s exchange rate policies during the sample period, such a finding is only likely.

3.3.4 The SETAR-GARCH model

We now briefly report the results of the SETAR model (both two-regime and three-regime) where volatility has been duly considered but with a single specification i.e., without any consideration to threshold in the conditional variance. This model, denoted as the SETAR-GARCH model in Section 3.2, is expected to explain the dynamics of y_t better because of volatility being included, although in a limited way. Since GARCH (1,1) model has often been found to be adequate for capturing observed volatilities, we have considered such a model i.e., a SETAR-GARCH (1,1) model, for India’s exchange rate return series. It may be noted that in empirical works, GARCH models with orders (1,2) as well as (2,1) have sometimes been found to be appropriate; GARCH (2,2) model has also been found, although very rarely, to explain volatility. Keeping all these in mind, we have also fitted the SETAR-GARCH(1,2), SETAR-GARCH(2,1) and SETAR-GARCH (2,2) models to India’s exchange rate return series. But these models produced either non-converging iterative procedures for the parameter estimates or converging estimates which violated Nelson and Cao’s (1992) weaker restrictions on parameters for positivity of conditional variance.

Since the EGARCH model was found to be the appropriate volatility model for this series in the linear dynamic model set-up (*cf.* Chapter 2), we also considered the EGARCH form of volatility to examine the usefulness of this volatility specification which explicitly takes into account the leverage effect, in the framework of SETAR model for the conditional mean. The empirical findings, however, suggest that the SETAR-EGARCH model is not an appropriate model for return on India's foreign exchange rate. This is because for the three-regime SETAR-EGARCH model (the same volatility specification for all the observations) considered by us, all the coefficients in the conditional mean function for all the three regimes were found to be insignificant while the coefficients of EGARCH (1,1) specification were found to be significant. This, of course, reconfirms, in a way, the finding that the EGARCH model is the appropriate volatility specification in the usual i.e., non-TAR / SETAR model, as found in Chapter 2. But this also empirically shows, at the same time, that the combination of SETAR model for the conditional mean and EGARCH specification for the conditional variance does not explain the dynamics of India's exchange rate return any better than EGARCH model alone.

Now, insofar as the performance of SETAR-GARCH (1,1) model is concerned, we note from Table 3.1 that for both the two-regime as well as the three-regime models, all the coefficients of GARCH(1,1) model are significant at 1% level of significance, and also that all the three coefficients have estimates which are positive in sign, indicating thereby that these conditional variance parameters satisfy the nonnegativity restrictions so that $\hat{h}_t$ is positive. We also find from this table that unlike the two-regime SETAR model (*cf.* equations (3.13) and (3.14), only the estimate of ϕ_5 which is the coefficient associated with y_{t-5}, is now significant at 10 % level of significance, in the first regime. This means that in the event of volatility being incorporated into the framework of analysis, the statistical significance of the first regime weakens to some extent, which is, however, not unlikely since SETAR model without any consideration to volatility may be looked upon as a misspecified model. This is, however, not the case with the three-regime model, as evidenced from equation (3.15) and column 4 of Table 3.1 *viz.*, the coefficients of both y_{t-1} and y_{t-4} are significant in the first regime for both the SETAR and SETAR-GARCH (1,1) models.

We now compare the performance of the SETAR-GARCH (1,1) model with that of SETAR using likelihood ratio test in order to find if inclusion of GARCH model to take care of volatility in the series leads to any statistically significant improvement. Noting that the maximum log-likelihood values of the three-regime SETAR-GARCH (1,1) and SETAR models have been obtained as 13587.50 and 10339.3, respectively so that the LR test statistic value under the null of three-regime SETAR turns out to be 6513.99. Since this computed value is obviously much higher than the corresponding tabulated value of $\chi^2_{(2)}$, we can conclude that the statistical gain from incorporating the GARCH (1,1) conditional variance specification in SETAR framework is significant, and hence the combined model of SETAR-GARCH (1,1) is a better model for India's exchange rate return series.

3.3.5 The DTGARCH model

We finally take up for discussion the findings on fitting the DTGARCH model to this return series. This model is characterized by thresholds in both the conditional mean and conditional variance, and such a model is expected to perform better than the SETAR-GARCH model provided, of course, the underlying DGP of India's exchange rate return indeed has such nonlinear characteristics. In this exercise, we have considered both the two-regime and three-regime DTGARCH models. As regards the choice of the orders of GARCH model for the different regimes, we have considered all possible combinations of GARCH(1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2) models, and finally chosen the one which explained the data 'best' in the DTGARCH framework. Now, for some of these models, convergence could not be achieved even after several iterations. For some others, the estimates were found to violate nonnegativity restrictions or the weaker restrictions (Nelson and Cao (1992)) for positivity of conditional variance. In this case also, we tried with EGARCH models of different orders as volatility specifications for the regimes, but no meaningful results could be empirically obtained for the exchange rate return. Similar was the outcome with combinations of EGARCH and GARCH models as the volatility specifications for the three regime DTGARCH models. Discarding, therefore, all those models, we finally report, insofar as the two-regime DTGARCH model is concerned, the estimates of the parameters of that model only where GARCH(1,1) is considered for both the regimes. As for the three-regime model, we report three models which have been found to give appropriate results. These are: (i) GARCH(1,1) specification for all the three regimes, (ii) GARCH(1,1) for the first and middle regimes and GARCH(2,1) for the last regime, and (iii) GARCH (2,1) for the first and last regimes and GARCH(1,1) for the middle regime.

We first report in Table 3.2, the estimates of the coefficients of the DTGARCH model with one threshold in the conditional mean and one in the conditional variance. The estimated value of the threshold parameter for this two-regime DTGARCH model has been obtained as 0.000544662 and the values of maximum lag in the conditional mean specification as 5 and 4, respectively for the two regimes. Both the regimes have GARCH (1,1) as the conditional variance. All the coefficients in the variance specification are significant except α_1 in the first regime, and further all these estimates have positive values so that the positivity of conditional variance is ensured.
In Table 3.3, the empirical findings of DTGARCH model with two thresholds both in the conditional mean and conditional variance are presented. The two threshold values have been obtained as -0.000214754 and 0.000544662 and the values of maximum lag of returns in the mean specification for the three regimes as 4, 2 and 4, respectively. Further, the parameters in the conditional variance specification of GARCH(1,1) for each of the three regimes are significant except the parameter α_0 in the first regime and α_1 in the second regime. We also note from this table that although most of the lags in the mean specification for the first and last regimes are significant, none of the lags in the middle regime are so. As in the three-regime models reported earlier, this finding once again shows that the model for the conditional mean for the middle regime is statistically insignificant. It can thus be concluded at 5% level of significance that the middle regime follows a random walk.

In Table 3.4, we report the results of fitting DTGARCH model with two thresholds in both the conditional mean and variance, but the conditional variance specifications for the regimes are not the same. To be specific, each of the first two regimes follow GARCH (1,1) process and the last regime has GARCH (2,1) process. The maximized log likelihood value for this model is 13868.731. Here also most of the lags in the mean specification are significant in the first as well as the last regimes, but, as before, not a single coefficient is significant for the middle regime. However, all the coefficients of GARCH model are significant for all the three regimes except for β_1, the coefficient of h_{t-1}, in the last regime.

Table 3.1

Estimated parameters of the two-regime and three-regime SETAR-GARCH(1,1) models with no threshold in the conditional variance specification

Par	Two-regime SETAR-GARCH(1,1) model		Three-regime SETAR-GARCH(1,1) model		
	Below threshold i.e., $y_{t-1} < 0.0005446$	Above threshold i.e., $y_{t-1} \geq 0.00054466$	Below lower threshold i.e., $y_{t-1} < -0.00021475$	Between thresholds i.e., $-0.000214754 \leq y_{t-}$ < 0.000544662	Above upper threshold i.e., $y_{t-1} \geq 0.00054466$
ϕ_0	-0.000024 (0.480)	-0.00021 (3.366)***	0.000082 (1.364)	-0.000028 (0.632)	-0.00004 (0.562)
ϕ_1	0.00769 (0.0906)	-0.2403 (4.546)***	0.1190 (1.802)*	0.0589 (0.309)	$-$ 0.3476 (4.444)***
ϕ_2	-0.0136 (0.187)	-0.2252 (6.979)***	-0.0527 (1.388)	0.1113 (2.090)**	-0.2008 (1.785)*
ϕ_3	0.0312 (0.501)	-0.1416 (3.936)***	0.0586 (1.637)		-0.1823 (3.904)***
ϕ_4	0.0184 (0.375)	0.1830 (4.362)***	0.0797 (3.569)***		0.1435 (3.341)***
ϕ_5	0.0616 (1.870)*				
α_0	0.0000002 (4.622)***		0.000003 (3.273)***		
α_1	0.3929 (7.565)***		0.3944 (12.516)***		
β_1	0.5971 (11.494)***		0.506 (16.049)***		

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Table 3.2

Estimate parameters of the two-regime DTGARCH (1,1) model with one threshold both in the conditional mean and conditional variance specifications

Parameter	Below threshold i.e., $y_{t-1} < 0.000544662$	Above threshold i.e., $y_{t-1} \geq 0.000544662$
ϕ_0	0.000071 (2.596)***	0.000106 (1.136)
ϕ_1	0.0241 (0.517)	-0.1823 (3.264)***
ϕ_2	0.0543 (1.337)	-0.1182 (1.435)
ϕ_3	0.0677 (1.780)*	-0.0393 (0.806)
ϕ_4	0.0488 (1.925)*	0.2321 (6.230)***
ϕ_5	0.0609 (2.093)**	
α_0	0.000001 (13.907)***	0.000001 (2.432)**
α_1	0.2021 (0.556)	0.5063 (2.913)***
β_1	0.6503 (60.511)***	0.4837 (2.783)***

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Table 3.3

Estimated parameters of the three-regime DTGARCH (1,1) model with two thresholds both in the conditional mean and conditional variance specifications

Parameter	Below lower threshold i.e., $y_{t-1} < -0.000214754$	Between thresholds i.e., $-0.000214754 \le y_{t-1} <$ 0.000544662	Above upper threshold i.e., $y_{t-1} \geq 0.000544662$
ϕ_0	0.000143 (3.318)***	0.000031 (1.225)	0.00002 (0.297)
ϕ_1	0.1008 (1.379)	0.0413 (0.364)	-0.1416 (2.344)**
ϕ_2	-0.0348 (1.251)	0.0422 (0.848)	-0.0787 (2.278)**
ϕ_3	0.0500 (0.764)		-0.1604 (5.374)***
ϕ_4	0.0896 (3.292)***		0.1811 (5.637)***
α_0	0.000001 (1.226)	0.000001 (3.192)***	0.000002 (5.337)***
α_1	0.5380 (5.541)***	0.1332 (1.613)	0.5698 (24.925)***
β_1	0.2998 (7.500)***	0.8568 (10.371)***	0.4202 (18.384)***

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Table 3.4

Estimated parameters of the three-regime DTGARCH model (having GARCH orders (1,1), (1,1) and (2,1)) with two thresholds both in the conditional mean and conditional variance specification

Parameter	Below lower Threshold, i.e. $y_{t-1} < -0.000214754$	Between thresholds, i.e. $-0.000214754 \le y_{t-1} <$ 0.000544662	Above upper threshold, i.e. $y_{t-1} \geq 0.000544662$
ϕ_0	0.000128 (3.600)***	0.000029 (1.186)	0.000129 (1.361)
ϕ_1	0.083 (1.479)	0.013 (0.118)	−0.3024 (3.850)***
ϕ_2	−0.024 (0.891)	0.0183 (0.377)	−0.00253 (0.069)
ϕ_3	0.053 (2.236)**		−0.2809 (7.727)***
ϕ_4	0.1022 (5.306)***		0.1027 (3.043)***
α_0	0.000001 (2.087)**	0.000001 (2.365)**	0.000002 (2.906)***
α_1	0.6235 (21.065)***	0.0133 (0.174)	0.6266 (17.174)***
β_1	0.2431 (5.086)***	0.8867 (11.588)***	0.0796 (0.9977)
β_2	–	–	0.2838 (4.841)***

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Table 3.5

Estimated parameters of the three-regime DTGARCH model (having GARCH orders (2,1), (1,1) and (2,1)) with two thresholds both in the conditional mean and conditional variance specifications

Parameter	Below lower threshold i.e., $y_{t-1} < -0.000214754$	Between thresholds i.e., $-0.000214754 \leq y_{t-1} <$ 0.000544662	Above upper threshold i.e., $y_{t-1} \geq 0.000544662$
ϕ_0	0.000136 (3.833)***	0.0000356 (1.391)	0.000142 (1.268)
ϕ_1	0.1014 (1.913)*	−0.00356 (0.0317)	−0.3226 (6.761)***
ϕ_2	−0.0232 (0.824)	0.0121 (0.254)	−0.0208 (0.535)
ϕ_3	0.0478 (1.710)*		−0.2874 (5.391)***
ϕ_4	0.0957 (3.912)***		0.1229 (2.451)**
α_0	0.000001 (1.665)*	0.000001 (5.357)***	0.000002 (2.871)***
α_1	0.5863 (4.694)***	0.000521 (0.108)	0.6302 (20.571)***
β_1	0.1984 (7.420)***	0.8948 (18.516)***	0.0707 (1.443)
β_2	0.0593 (1.731)*	–	0.2892 (5.910)***

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Finally, in Table 3.5, we report the model which has been found to perform the best in terms of maximum log likelihood value (13871.837) amongst all the models considered. It has two thresholds in both the conditional mean and conditional variance. The conditional variance specification for the first and last regimes are GARCH (2,1) while for the middle regime it is GARCH (1,1). Barring two, i.e., α_1 in the second regime and β_1 in the third regime, all other coefficients in the three GARCH specifications are found to be significant. Also, most of the coefficients for the conditional mean specification for the first and last regimes are found to be significant. However, as expectedly, none of the coefficients in the mean specification for the middle regime is found to be significant. Since this volatility combination for the three regimes has been found to be the best in terms of maximum log likelihood value, we have taken this model for checking adequacy of the fitted model as well as for the purpose of out-of-sample forecasting.

3.3.6 Checking model adequacy

We have closely followed Li and Li (1996) in finding if the chosen DTGARCH model (in terms of maximum log likelihood value) i.e., the three-regime SETAR model with GARCH (2,1) volatility specification for the first and the last regimes and GARCH (1,1) specification for the middle regime (*cf.* Table 3.5) is indeed adequate for our data. To that end, we have first checked for the significance of residual autocorrelation of the three-regime DTGARCH model upto lag 5. As discussed in Section 3.2.2, we have obtained the asymptotic standard errors of $\hat{\rho}_k$ and $\hat{\hat{\rho}}_k$, the ACFs of residuals and squared residuals, respectively and then computed the statistic (*t*-ratio) values which are presented along with the values of $\hat{\rho}_k$ and $\hat{\hat{\rho}}_k$ for the first five lags in Table 3.6. Since the asymptotic distribution of $\hat{\rho}_k$ and $\hat{\hat{\rho}}_k$ are both normal, we conclude, by comparing the computed values of the *t*-ratios with the corresponding critical values of the standard normal distribution, that the null hypotheses of no autocorrelation in the residuals is rejected at 1% level of significance for the first three lags, although the null of no autocorrelation in the squared residuals cannot be rejected for all the five lags even at 10% level of significance.

Now, in terms of overall goodness-of-fit of this DTGARCH model, which is measured by $Q_J(M)$ and $Q_{JJ}(M)$, we have found that $Q_J(5) = 57.725$ and $Q_{JJ}(5) = 1.332$. Since the critical value of χ_5^2 at 1% level of significance is 15.086, we conclude that the overall fit from consideration of conditional mean specification is not adequate since the underlying null hypothesis is rejected. However, the value of $Q_{JJ}(5) = 1.332$ obviously suggests the adequacy of conditional variance specification. On the whole, therefore, we may conclude that insofar as India's exchange rate return series is concerned, this DTGARCH model is quite appropriate from the standpoint of adequately capturing nonlinearity in both the conditional mean and variance specifications, although the underlying autocorrelation cannot be fully described by this model. Thus, we note that when we introduce a threshold in the conditional variance, in addition to threshold being present in the conditional mean, the goodness of fit test suggests inadequacy of the conditional mean. However, without any threshold in the conditional variance, the same conditional mean specification is found to be adequate. A possible explanation may be that India's exchange rate return series has over-riding regime-specific volatility characteristic, and hence its introduction leads to the conclusion of inadequacy in the conditional mean suggesting thereby change in the regime specific linear dependence as well.

Table 3.6

Checking adequacy of the chosen three-regime DTGARCH model

Lags	$\hat{\rho}_k$	Test statistic	$\hat{\hat{\rho}}_k$	Test statistic
1	0.115754	5.535640***	0.004169	0.199363
2	0.062525	2.990110***	0.005817	0.278168
3	0.072280	3.456615***	-0.000779	-0.037264
4	0.042587	2.036614**	0.013368	0.639306
5	0.029955	1.432516	-0.018760	-0.897170

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

3.3.7 Forecasting performance

In Section 2.2.3 of Chapter 2, we have discussed the process of obtaining out-of-sample forecasts and then comparing these with the actual values by standard forecast evaluation criteria *viz.*, the MSE, MAE, AMAPE and PCSP. The forecasts of exchange rate returns for the SETAR and DTGARCH models have been calculated recursively for the daily series for one, five and ten steps ahead, respectively. For the purpose of obtaining these forecasts, the model has been re-estimated by expanding the sample with one observation at each successive stage over the hold-out period of 100 time points covering the period 16th February 2004 to 14th July 2004.

Table 3.7

Forecasts for the two-regime and three-regime SETAR models

SETAR with one threshold				
Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	$1.785E^{-5}$	0.00275	1.618	50.5
5	$1.819E^{-5}$	0.00286	2.191	50.0
10	$1.825E^{-5}$	0.00288	2.131	52.0
SETAR with two thresholds				
1	$1.801E^{-5}$	0.00277	1.997	50.5
5	$1.822E^{-5}$	0.00287	2.208	51.0
10	$1.828E^{-5}$	0.00290	2.152	51.0

Table 3.8

Forecasts for the two-regime and three-regime DTGARCH models

DTGARCH with one threshold				
Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	$1.699E^{-5}$	0.00274	1.535	53
5	$1.757E^{-5}$	0.00286	1.441	52
10	$1.758E^{-5}$	0.00289	1.459	49
DTGARCH with two thresholds				
1	$1.757E^{-5}$	0.00270	1.471	49
5	$1.829E^{-5}$	0.00285	2.025	48
10	$1.818E^{-5}$	0.00288	1.868	47

While table 3.7 presents the values of MSE, MAE, AMAPE and PCSP for the two-regime as well as the three-regime SETAR models, the forecasting performance of the two-regime and three-regime DTGARCH models, are reported in Table 3.8. In the latter, we have generated the forecasts for the DTGARCH model with one threshold and GARCH (1,1) specification for the conditional variance (*cf*. Table 3.2), and also for the DTGARCH model with two thresholds in both the conditional mean and conditional variance having GARCH (2,1) specification for the first and last regimes and GARCH (1,1) for the second regime (*cf.* Table 3.5). In other words, we have generated forecasts for the models which have been found to have performed 'best' in terms of in-sample performance. It is clearly seen that the values of the MSE, MAE and AMAPE are reduced for the DTGARCH model when compared with the SETAR model. For instance, the value of MAE for SETAR with one threshold (two thresholds) for 1-step ahead forecast is 0.00275 (0.00277) while the same for the DTGARCH model is 0.00274 (0.00270) indicating thereby that the inclusion of GARCH formulation has indeed improved the latter model in terms of a forecasting criterion. The conclusion is the same in terms of MSE criterion as well. The one-threshold DTGARCH model is found to be a better model than SETAR in terms of PCSP also. The last observation means that there are more cases in the DTGARCH model having the same sign of forecasts as that of the return than that in the corresponding SETAR model. We can, on the whole, conclude that the performance of DTGARCH model is better than the SETAR model in terms of the usual out-of-sample forecasting criteria.

Now, it is worth noting that between two- and three-regime SETAR models, the latter performs marginally worse than the former. For instance, the value of the 1-step ahead forecast is 0.00275 by the MAE criterion, and it increases to 0.00277 for the three-regime SETAR model. As it is, this finding is rather surprising, but it would not appear to be so if we recall the estimated two and three-regime SETAR models. We have noted in the preceding sections that the second regime of the three-regime SETAR model did not have any significant coefficients. Thus, introduction of an additional regime in the specification was not found to be statistically rewarding. This is also reflected by the forecasting exercise as well. As regards comparing between the two-regime and three-regime DTGARCH models, we find that the performance of DTGARCH model with three regimes is marginally worse than that of the two-regime model, in terms of MSE. However, the conclusion is different if it is based on the MAE criterion, which suggests the DTGARCH model with three-regimes to be marginally better than the two-regime one. The PCSP criterion indicates that the two regime DTGARCH model is slightly better than the three-regime model. Combining the performances by all these criteria, we can conclude that the performances of the two-regime and three-regime DTGARCH models may be considered to be more or less the same. However it may be stated that the two-regime DTGARCH model has a slight edge over the three regime DTGARCH model.

The out-of-sample forecasts of the models obtained in this chapter have been compared with those of the random walk with drift (*cf.* equation (2.10a)) and the findings have been reported in Chapter 5 (Section 5.4) along with those from other nonlinear models considered in this thesis.

3.4 Conclusions

Several studies with the time series of foreign exchange rates of developed economies have suggested that the nonlinear time series models considered in this chapter are often more appropriate than the linear model. Considering that no such comprehensive study concerning any emerging economy has yet been done, we have carried out such an analysis with the foreign exchange rate data of one of the most important emerging economies, namely, India.

To this end, we have first fitted the SETAR model - both two-regime and three-regime. Then we have also considered the SETAR model generalized by the inclusion of volatility model – both without and with consideration to threshold in volatility given by the GARCH model. Thus, models like the SETAR-GARCH and DTGARCH, which differ from consideration of the fact that the former considers only one single specification for the conditional variance whereas the latter allows for threshold in the conditional variance as well, have then been used for analyzing the return on India's exchange rate series. These models have been estimated and their forecasting performances have been evaluated using criteria such as the MSE, MAE, AMAPE and PCSP. It has been found that the introduction of threshold in volatility has indeed improved the forecasting performance of SETAR models. In fact, the DTGARCH model has been found to have performed the best in terms of maximum log-likelihood value. Further, adequacy of the DTGARCH model has been evaluated using appropriate diagnostic tests.

We can thus conclude that the nonlinear time series model, called the double threshold GARCH model, has been found to adequately determine and predict the exchange rate return of India. Thus, the importance of the class of SETAR models, the use of which has remained confined mostly to economic and financial variables of the developed economies only, has been established for an emerging economy like India as well. Further, we have found that the DTGARCH model which has not been applied extensively as yet, is, in fact, a better model than SETAR for explaining the dynamics of Indian rupee / US dollar exchange rate in the nonlinear framework, as described by such models. These findings, therefore, lend strong empirical support for wider applicability of the DTGARCH model for the time series of foreign exchange rate return.

CHAPTER 4

Smooth Transition Autoregressive Model for Daily Exchange Rate Return

4.1 Introduction

Empirical studies on exchange rate have shown the presence of strong nonlinearity, and consequently a number of nonlinear time series models have been proposed, which have subsequently been found suitable for modelling exchange rate, mostly of developed economies. As discussed earlier the specifications of such models for exchange rate have also been motivated by nonlinear solutions presented for such asset variables in a number of theoretical models, such as the target zone model (Krugman (1991)) and rational expectations model with central bank stochastic intervention rules (Hsieh (1992)).

Like the preceding chapter, in this chapter as well as in the succeeding one we are basically concerned with univariate nonlinear time series modelling of daily-level exchange rate return series of India. The univariate modelling set-up is used as it is plausible to assume that all the relevant information are embodied in the most recent exchange rate return, so that it is unnecessary to include economic fundamentals in the set of explanatory variables. As discussed in Chapter 1, the literature on nonlinear time series modelling of exchange rate indicates that popular nonlinear models to be used in case of exchange rate modelling are the TAR models, and in the preceding chapter, we have considered a particular TAR model called the SETAR model to explain the behaviour of India's exchange rate return.

A variant of the SETAR model, introduced by Chan and Tong (1986) and extensively explored by Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and Teräsvirta (1994), can be obtained if, unlike the SETAR model where an indicator function is used to incorporate regime-switching of on-off kind, the parameters are allowed to change smoothly over time. The resulting model is called the smooth transition autoregressive (STAR) model. The idea of smooth transition between regimes dates back to Bacon and Watts (1971), and a comprehensive review of the STAR model along with extensions allowing for exogenous variables as regressors can be found in Teräsvirta (1998).

While there are quite a few studies, as already mentioned in the preceding chapter, where the performance of SETAR model in modelling exchange rate have been studied, there are relatively fewer studies on the use of STAR models in exchange rate modelling. Boero and Marrocu (2002) is one such study where the relative performance of nonlinear models of the SETAR and STAR types has been contrasted with their linear counterparts. The forecasting performance of these models used on three most traded exchange rates *viz*., the French franc, the German mark and the Japanese yen with respect to the US dollar, has been compared. Medeiros *et al.* (2001) have used nonlinear models to model India's monthly exchange rate series along with several other series to find whether these nonlinear models perform better than the autoregressive and random walk models. They have used the artificial neural network (ANN) model as well as the neuro-coefficient smooth transition autoregression which nest the SETAR, STAR and ANN models and compared the different alternatives for the purposes of modelling and forecasting the monthly exchange rate series. They have found that the monthly Indian rupee / US dollar exchange rate is nonlinear, but this nonlinearity is relevant only for some periods of the series and not spread uniformly.

The class of STAR models is being increasingly used to explain the real exchange rate dynamics. This has been motivated by theoretical models which state that the real exchange rate becomes increasingly mean reverting with the size of deviation from the equilibrium level. Some works in this direction are due to Obstfeld and Taylor (1997), Micheal *et al.* (1997) and Taylor *et al.* (2001). In a more recent work, Rapach and Wohar (2006b) have analyzed the out-of-sample forecasting performance of some nonlinear models including the STAR in the context of studying the behaviour of US dollar real exchange rate series. Some recent studies have also used nonlinear models for the real exchange rate series of India. For instance, Holmes (2004) has used logistic as well as exponential STAR models to study the nonlinearities in the behavior of real exchange rates of eleven Asian economies including India and found that the extent of nonlinearities varied across the Asian countries with India and Singapore exhibiting the sharpest transition between regimes. They have also used this model to find evidence of purchasing power parity. They have also found that an LSTAR model can successfully take care of the nonlinearities of the Indian rupee / US dollar real exchange rate series. Baharumshah and Liew (2006) have used STAR model for yen-based currencies of six major East Asian countries and discovered strong evidence of nonlinear mean reversion in deviation from purchasing power parity. They have also shown that the STAR model outperforms the AR model.

The number of studies using STAR model to nominal exchange rate is thus limited, and to the best of our knowledge, no such work has been done for the daily (nominal) exchange rate series of India. Thus, in this chapter, we study the STAR model to explain the nonlinear dynamics of this series.

This chapter is organized as follows. In Section 4.2, we discuss the model and briefly describe the methodology used. Section 4.3 discusses the empirical findings. Brief concluding remarks are made in the last section.

4.2 The model and methodology

The basic smooth transition autoregressive (STAR) model, originally proposed by Chan and Tong (1986), for a univariate time series y_t is given by

$$\begin{aligned} y_t &= (\phi_{1,0} + \phi_{1,1} y_{t-1} + ... + \phi_{1,p} y_{t-p})(1 - G(s_t;\gamma,c)) \\ &+ (\phi_{2,0} + \phi_{2,1} y_{t-1} + ... + \phi_{2,p} y_{t-p}) G(s_t;\gamma,c) + \varepsilon_t, \quad t = 1,2,...,T \end{aligned} \quad . \qquad (4.1)$$

This may be conveniently expressed as

$$y_t = \phi_1{}' \tilde{y}_t (1 - G(s_t;\gamma,c)) + \phi_2{}' \tilde{y}_t G(s_t;\gamma,c) + \varepsilon_t \qquad (4.2)$$

where $\tilde{y}_t = (1, y_{t-1}, ..., y_{t-p})'$ and $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, $i = 1,2$. This model can be easily extended to include exogenous variables $x_t = (x_{1t}, x_{2t}, ..., x_{kt})'$ as additional regressors, and in that case $\tilde{y}_t$ is to be replaced by $y_t{}^* = (\tilde{y}_t', x_t')'$ and ϕ_i's to be suitably augmented by the parameters attached to the k exogenous variables. A discussion on the resultant smooth transition autoregression (STAR) has been done in Teräsvirta (1998). The assumptions about ε_t are that $E(\varepsilon_t \mid \psi_{t-1}) = 0$ and

$E(\varepsilon_t^2 \mid \psi_{t-1}) = \sigma^2$, where $\psi_{t-1} = \{y_{t-1}, y_{t-2},....\}$ is the information set upto the time *t*-1. It may be noted that the second assumption on ε_t is being made for the sake of simplicity; an extension of the STAR model which allows for (possibly asymmetric) autoregressive conditional heteroscedasticity has been done by Lundbergh and Teräsvirta (1998).

The transition function $G(s_t;\gamma,c)$ in the STAR model is a continuous function that is bounded between 0 and 1. It is worth noting that unlike the SETAR model, where it is assumed that the border between the two regimes is given by a specific value of the threshold variable y_{t-d}, the STAR model allows for a gradual transition between the different regimes. The transition variable s_t is often assumed to be a lagged endogenous variable i.e., $s_t = y_{t-d}$ for certain integer $d > 0$. However, other assumptions like the transition variable being an exogenous variable i.e., $s_t = z_t$, or a (possibly nonlinear) function of lagged endogenous variables can also be made. Further, the transition variable can even be a linear time trend, say $s_t = t$, which gives rise to a model with smoothly changing parameters (see, in this context, Lin and Teräsvirta (1994)).

Following the 'two regime' interpretation, which is very common in the STAR literature, we may state that different choices for the transition function $G(s_t;\gamma,c)$ give rise to different types of regime-switching behaviour. A popular choice for $G(s_t;\gamma,c)$ is the first-order logistic function

$$G(s_t;\gamma,c) = \frac{1}{1+\exp\{-\gamma(s_t - c)\}} \quad , \quad \gamma > 0 \tag{4.3}$$

and the resultant model is called the logistic STAR (LSTAR) model. The parameter *c* in (4.3) can be interpreted as the threshold between the two regimes corresponding to $G(s_t;\gamma,c) = 0$ and $G(s_t;\gamma,c) = 1$ in the sense that the logistic function changes monotonically from 0 to 1 as s_t increases, while $G(c;\gamma,c) = 0.5$. The parameter γ determines the smoothness of the change in the value of the logistic function, and thus the transition from one regime to the other. As γ becomes very large, the change of

$G(s_t;\gamma,c)$ from 0 to 1 becomes almost instantaneous at $s_t = c$ and, consequently, the logistic function $G(s_t;\gamma,c)$ approaches the indicator function $I[s_t > c]$. Hence the LSTAR model represented by (4.2) and (4.3) nests a two-regime threshold autoregressive (TAR) model as a special case. In fact, if $s_t = y_{t-d}$, the STAR model reduces to the two-regime SETAR model. When $\gamma \to 0$, the logistic function tends to the constant, 0.5 and when $\gamma = 0$, the STAR model reduces to a linear model.

An alternative choice for the transition function $G(s_t;\gamma,c)$ is the exponential function

$$G(s_t;\gamma,c) = 1 - \exp\{-\gamma(s_t - c)^2\}, \quad \gamma > 0 \quad . \tag{4.4}$$

The exponential function has the property that $G(s_t;\gamma,c) \to 1$ as $s_t \to -\infty$ or $s_t \to \infty$ where as $G(s_t;\gamma,c) = 0$ for $s_t = c$. The resultant STAR model is called the exponential STAR (ESTAR) model.

A drawback of the exponential function (4.4) is that for either $\gamma \to 0$ or $\gamma \to \infty$, the function collapses to a constant (equal to 0 or 1, respectively). Hence, the model becomes linear in both the cases. Further, the ESTAR model does not nest the SETAR model as a special case. If such a nesting is considered to be desirable, one can instead use the second-order logistic function

$$G(s_t;\gamma,c) = \frac{1}{1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)\}}, \quad c_1 \le c_2 \text{ and } \gamma > 0 \tag{4.5}$$

where c is now $c = (c_1, c_2)'$. In this case, the model becomes linear if $\gamma \to 0$. However, if $\gamma \to \infty$ and $c_1 \ne c_2$, the function $G(s_t;\gamma,c)$ is equal to 1 for $s_t < c_1$ and $s_t > c_2$ and equal to 0 for $c_1 \le s_t \le c_2$. Hence, the STAR model with this particular transition function nests a restricted three-regime TAR model, where the restriction is that the outer regimes are identical. In case, $s_t = y_{t-d}$, then the transition function nests, in particular, the three-regime SETAR model. It may be noted that for moderate values of γ, the minimum value of the second-order logistic function, attained for $s_t = (c_1 + c_2)/2$, remains between 0 and 1/2 , unless $\gamma \to \infty$. In the latter case, the minimum value equals zero.

Finally, although the first-order and second-order logistic functions as specified in (4.3) and (4.5), respectively are most often used in LSTAR models, it is only natural that the general *nth* order logistic function defined as

$$G(s_t;\gamma,c)=\frac{1}{1+\exp\{-\gamma\prod_{j=1}^{n}(s_t-c_j)\}}\ ,\ \ c_1\leq c_2\leq .....\leq c_n\ \text{ and }\ \gamma>0 \tag{4.6}$$

can be used to obtain multiple switches between the two regimes.

The representation of the STAR model thus far cannot obviously accommodate more than two regimes, irrespective of what form the transition function takes. Even though two regimes are sufficient in many applications, it maybe desirable to allow for multiple regimes. To that end, under the assumption that the prevailing regimes can be determined by a single transition variable s_t, as before, one can start with the LSTAR model specified in (4.2) and (4.3), which can be rewritten as

$$y_t=\phi_1'\tilde{y}_t+(\phi_2-\phi_1)'\tilde{y}_tG_1(s_t;\gamma_1,c_1)+\varepsilon_t \tag{4.7}$$

where a subscript 1 has now been added to the logistic function as well as the parameters contained therein, for the purpose of notational distinction. Likewise, a three-regime model can be obtained by adding a second nonlinear component as follows:

$$y_t=\phi_1'\tilde{y}_t+(\phi_2-\phi_1)'\tilde{y}_tG_1(s_t;\gamma_1,c_1)+(\phi_3-\phi_2)'\tilde{y}_tG_2(s_t;\gamma_2,c_2)+\varepsilon_t\ . \tag{4.8}$$

Now, if it is assumed that $c_1<c_2$, the autoregressive parameter changes smoothly from ϕ_1 via ϕ_2 to ϕ_3 for increasing values of s_t, as the function G_1 first changes from 0 to 1, followed by a similar change of G_2. In this way, one can arrive at a STAR model with J regimes as

$$\begin{aligned}y_t=\phi_1'\tilde{y}_t&+(\phi_2-\phi_1)'\tilde{y}_tG_1(s_t;\gamma_1,c_1)+(\phi_3-\phi_2)'\tilde{y}_tG_2(s_t;\gamma_2,c_2)+.......+\\&(\phi_J-\phi_{J-1})'\tilde{y}_tG_{J-1}(s_t;\gamma_{J-1},c_{J-1})\end{aligned}\ . \tag{4.9}$$

In case all smoothness parameters become very large, such a STAR model effectively becomes a SETAR model with *J* regimes.

4.2.1 Hypothesis testing in STAR framework

The first step towards building a STAR model is to test linearity against STAR. In terms of model (4.2), the null hypothesis is then $H_0 : \phi_1 = \phi_2$ and the alternative hypothesis is $H_1 : \phi_{1,j} \neq \phi_{2,j}$ for at least one $j \in \{0,1,...,p\}$. This testing problem is complicated by the presence of unidentified nuisance parameters, γ and *c*, under the null hypothesis. The solution to this problem was suggested by Luukkonen *et al.* (1988). They proposed replacing the transition function $G(s_t;\gamma,c)$ by a suitable Taylor series approximation. In the reparameterized model, the identification problem is no longer present, and the null of linearity can be tested using a Lagrange Multiplier (LM) / Rao's score (RS) test with a standard asymptotic χ^2 distribution under the null hypothesis .

Tests against LSTAR model: We consider the LSTAR model given in (4.2) and (4.3) and rewritten as $y_t = \phi_1' \tilde{y}_t + (\phi_2 - \phi_1)' \tilde{y}_t G(s_t;\gamma,c) + \varepsilon_t$ (*cf.* (4.7)). Further, we assume that $\varepsilon_t \sim$ i.i.d $N(0,\sigma^2)$. In case $G(s_t;\gamma,c)$ is taken to be the logistic function given as in (4.3), a first-order Taylor series approximation around $\gamma = 0$ leads to the following auxiliary regression

$$y_t = \beta_0' \tilde{y}_t + \beta_1' \tilde{y}_t s_t + v_t \tag{4.10}$$

where $\beta_i = (\beta_{i,0}, \beta_{i,1}, ..., \beta_{i,p})'$, i = 0, 1, are appropriately defined functions of ϕ_1, ϕ_2, γ and c , and $v_t = \varepsilon_t + (\phi_2 - \phi_1)' \tilde{y}_t R_1(s_t;\gamma,c)$, where $R_1(s_t;\gamma,c)$ is the remainder term from the Taylor series expansion. Under the null hypothesis, $R_1(s_t;\gamma,c) = 0$ and $v_t = \varepsilon_t$. Consequently, this remainder term does not affect the properties of the errors under the null hypothesis and hence the asymptotic distribution theory. The parameters β_i , i = 0,1, in the auxiliary regression (4.10) are functions of the parameters in the STAR model (4.7) such that the restriction $\gamma = 0$ implies $\beta_{0,j} \neq 0$ and $\beta_{1,j} = 0$ for j = 0,1,…., p. Hence, testing the null hypothesis $H_0' : \gamma = 0$ (or $H_0 : \phi_1 = \phi_2$) in (4.7) is equivalent to testing the null hypothesis $H_0'' : \beta_1 = 0$ in

(4.10). The relevant test statistic obtained, denoted as LM_1, has an asymptotic χ^2 distribution with $p+1$ degrees of freedom under the null hypothesis of linearity. It may be noted that in case $s_t = y_{t-d}$, for certain integer $1 \le d \le p$, $\beta_{1,0}s_t$ should be dropped from (4.10) to avoid multicollinearity.

Now, as pointed out by Luukkonen *et al*. (1988), the LM_1 statistic does not have power in situations where only the intercept differs across regimes. The problem can be solved by approximating the transition function $G(s_t, \gamma, c)$ by a third-order Taylor series approximation, which leads to the auxiliary regression

$$y_t = \beta_0{}' \tilde{y}_t + \beta_1{}' \tilde{y}_t s_t + \beta_2{}' \tilde{y}_t s_t^2 + \beta_3{}' \tilde{y}_t s_t^3 + e_t \tag{4.11}$$

where e_t is now $e_t = \varepsilon_t + (\phi_2 - \phi_1)' \tilde{y}_t R_3(s_t; \gamma, c)$ and β_i, $i = 0,1,2,3$, are functions of the parameters ϕ_1, ϕ_2, γ and c. In this case, the null hypothesis of linearity $H_0{}' : \gamma = 0$ now corresponds to $H_0{}'' : \beta_1 = \beta_2 = \beta_3 = 0$, which can be tested by a standard LM / RS type test. Here, the test statistic, say LM_3, has an asymptotic χ^2 distribution with 3(p+1) degrees of freedom under the null hypothesis of linearity. Again, if $s_t = y_{t-d}$, for certain integer $1 \le d \le p$, $\beta_{i,0}s_t^i$, i =1,2,3, should be dropped from the auxiliary regression (4.11). In small samples, it is a good strategy to use *F*-versions of the LM test statistics because these have better size properties than χ^2 variants. This involves computation of the following steps. Suppose that we are considering the LM / RS test statistic based on (4.11). Then

Step (i) We first estimate the model under the null hypothesis of linearity (i.e., $\beta_1 = \beta_2 = \beta_3 = 0$) by regressing y_t on $\tilde{y}_t$, and obtain the residuals $\hat{e}_t$ and the sum of squared residuals $RSS_0 = \sum_{t=1}^{T} \hat{e}_t^2$.

Step (ii) We estimate the auxiliary regression of y_t on $\tilde{y}_t$ and $\tilde{y}_t s_t^1$, $\tilde{y}_t s_t^2$ and $\tilde{y}_t s_t^3$, and obtain the residuals $\tilde{e}_t$ and the sum of squared residuals $RSS_1 = \sum_{t=1}^{T} \tilde{e}_t^2$.

Step (iii) χ^2 - version of the LM_3 statistic is then obtained as

$$LM_3 = \frac{T(RSS_0 - RSS_1)}{RSS_0} \tag{4.12}$$

whereas the *F*-version can be computed as

$$F = \frac{(RSS_0 - RSS_1)/3(p+1)}{RSS_1/(T-4(p+1))} \quad . \tag{4.13}$$

Under the null hypothesis of linearity, the *F*-version of the test approximately follows a F distribution with 3(*p*+1) and *T*- 4(*p*+1) degrees of freedom.

Tests against ESTAR model : Testing of linearity against the other assumptions about the transition function *viz.*, exponential, has been suggested by Saikkonen and Luukkonen (1988) . This requires using the following auxiliary regression based on, as before, Taylor series expansion of (4.4):

$$y_t = \beta_0{}'\tilde{y}_t + \beta_1{}'\tilde{y}_t s_t + \beta_2{}'\tilde{y}_t s_t^2 + e_t \tag{4.14}$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)'\tilde{y}_t R_2(s_t;\gamma,c)$. The expression for $\beta_i, i = 0,1,2$, show that the restriction $\gamma = 0$ corresponds to $\beta_1 = \beta_2 = 0$ in (4.14). The corresponding test statistic, denoted as LM_2 , follows an asymptotic χ^2 distribution with $2(p+1)$ degrees of freedom. Escribano and Jörda (1999) have, however claimed that the following auxiliary regression

$$y_t = \beta_0{}'\tilde{y}_t + \beta_1{}'\tilde{y}_t s_t + \beta_2{}'\tilde{y}_t s_t^2 + \beta_3{}'\tilde{y}_t s_t^3 + \beta_4{}'\tilde{y}_t s_t^4 + e_t \tag{4.15}$$

is better than (4.14) from the point of view of performance of the test. The resultant LM-type test statistic, LM_4, has an asymptotic χ^2 distribution with $4(p+1)$ degrees of freedom under the null hypothesis.

4.2.2 The STAR modelling procedure

Once the null of linearity is rejected, the question as to how the STAR model should be developed arises. To that end, Granger (1993) has recommended a specific-to-general strategy. This implies starting with a restricted model and proceeding to more complicated ones only if diagnostic tests indicate that the maintained model is inadequate. The data based modelling procedure for the STAR model put forward by Teräsvirta (1994) first requires specifying a linear AR model of order *p* where the order of the AR model is selected by conventional methods such as the AIC, BIC or Ljung-Box test of no autocorrelations in residuals. The null hypothesis of linearity is then tested against the alternative of STAR nonlinearity. If linearity is rejected, the appropriate transition variable s_t and the form of transition function $G(s_t;\gamma,c)$ are then selected.

The test has already been discussed in the previous section. The appropriate transition variable in the STAR model can be determined first without specifying the form of the transition function. This is done by computing the LM_3 statistic in (4.12) for various candidate transition variables $s_{1t}, s_{2t}, \ldots\ldots, s_{mt}$ and then selecting the one for which the *p*-value of the test is smallest. In a similar way, we can find out the appropriate transition function. The candidate functions are LSTAR1, LSTAR2 and ESTAR given in (4.3), (4.5) and (4.4), respectively.

Insofar as estimation of the STAR model is concerned, it is done by applying the nonlinear least squares (NLS) method of estimation to any particular model, say (4.2) in this case. In other words, the $(l \times 1)$ parameter vector[1] $\theta = (\phi_1', \phi_2', \gamma, c)'$ is estimated as $\hat{\theta} = \arg\min_{\theta} \sum_{t=1}^{T} (y_t - F(\tilde{y}_t, \theta))^2$, where $F(\tilde{y}_t, \theta) = \phi_1' \tilde{y}_t (1 - G(s_t;\gamma,c)) + \phi_2' \tilde{y}_t G(s_t;\gamma,c)$. Under the additional assumption that the errors ε_t are normally distributed, NLS method is equivalent to maximum likelihood method; otherwise NLS estimates can be interpreted as quasi maximum likelihood estimates. This estimation can be performed using any conventional nonlinear optimization procedure (see Quandt (1983), Hamilton (1994) and Hendry

(1995), for a survey of such procedures). Obviously, in such procedures, the choice of starting values for parameters is an important issue. It may be noted that when the parameters γ and c are known and fixed, the STAR model is linear in the autoregressive parameters ϕ_1 and ϕ_2. Thus, conditional upon γ and c, the estimates $\phi = (\phi_1', \phi_2')$ can be obtained by OLS procedure. It, therefore, obviously follows that sensible starting values for the nonlinear optimization algorithm can be found by a two-dimensional grid search over γ and c. In the estimation procedure, the transition function in, say (4.3), is scaled in the sense that $(s_t - c)$ is replaced by $(s_t - c)/\hat{\sigma}_{st}$ where $\hat{\sigma}_{st}$ is the sample standard deviation of s_t. This makes γ approximately scale free. A meaningful set of values for c maybe defined as sample percentiles of the transition variable s_t. It is noteworthy that obtaining precise estimate of the smoothness parameter is rather difficult when it is large. This is due to the fact that for large values of γ, the transition function comes close to a step function and the shape of logistic function changes only little. However, this should not be interpreted as evidence for weak nonlinearity as the underlying *t*-statistic, as already discussed, does not follow the usual *t*-distribution under the null of $\gamma = 0$. In this case, the large standard error is purely numerical. However, high accuracy in estimation of γ is not necessary since large changes in γ have only small effect on the transition function.

4.2.3 Tests for adequacy of the STAR model

Eitrheim and Teräsvirta (1996) have developed an LM test to detect the presence of serial correlation in the residuals of STAR model. Considering the general nonlinear autoregressive model for order *p*, as specified in (4.2), an LM test for the q^{th} order serial dependence in ε_t can be obtained as TR^2 where R^2 is the coefficient of determination from the regression of $\hat{\varepsilon}_t$ on $\hat{k}_t$ and *q* lagged residuals $\hat{\varepsilon}_{t-1}, \ldots., \hat{\varepsilon}_{t-q}$ where $\hat{k}_t = \partial F(\tilde{y}_t; \hat{\theta}) / \partial \theta$. The resulting test statistic has a χ^2 distribution with q

[1] '*l*' stands for the number of parameters in θ for any general model; obviously, $l=2p+4$ for the particular model (4.2) considered.

degrees of freedom asymptotically under the null of no residual autocorrelation. In the *F*-version of this test , the test statistic is given by

$$F_{LM} = \{(RSS_0 - RSS_1)/q\}/\{RSS_1/T - l - q\} \tag{4.16}$$

where RSS_0 is the sum of squared residuals from the STAR model, RSS_1 is the sum of squared residual from the auxiliary regression. The statistic has an approximate *F*-distribution with *q* and *T-l-q* degrees of freedom under the null hypothesis, the *F*-version of the test, based on the asymptotic distribution theory (Lütkepohl and Krätzig (2004)), is found to be preferable to the χ^2-statistic.

4.3 Empirical findings

The first step in fitting the STAR model to the time series of return, y_t , on India's exchange rate involves the choice of appropriate value for lag p. The lag order should be such that the corresponding residuals are approximately white noise. As already noted, the order of the autoregressive model can be selected using any of the conventional methods such as the AIC, BIC or Ljung-Box test for no autocorrelation. It should be kept in mind that if linearity is rejected while testing for it against the alternative of STAR model, the lag order used in the AR model is not necessarily the appropriate lag order in the alternative STAR model although it provides a reasonable first guess. Using the BIC, we have found 4 to be the appropriate lag order[2] i.e., p = 4.

4.3.1 Testing linearity against STAR

We have discussed earlier that the null of linearity is rejected if the *p* value of the LM_3 statistic in (4.12) or *F*-statistic in (4.13) is low. It may be pointed out at this stage that although the LM_3 test statistic was developed as a test against the class of LSTAR alternatives, it has power against the ESTAR alternative as well. As the χ^2-statistic can be severely size distorted in small and even moderate samples, the corresponding *F*-statistic is often used. Now, to build the STAR model, a set of

[2] All computations in this chapter has been carried out using JmulTi software (Krätzig (2004)).

potential transition variables are required to be selected. This set may contain the various lag values of the endogenous variable. If the null hypothesis for (4.13) is rejected for all these transition variables chosen, then the transition variable that needs to be selected should be the one for which the *p*-value of this test is minimum. The rationale behind this suggestion is that the rejection of the null hypothesis is stronger against the correct alternative than the alternative models. However, if several small *p*-values are close to each other, it may be useful to proceed by estimating the corresponding STAR models and leaving the choice between them at the evaluation stage.

Keeping this in mind, our computations suggest that there are two transition variables y_{t-1} and y_{t-4} which are highly significant and their corresponding *p*-values are very close to each other being 1.58×10^{-30} and 1.19×10^{-30}. Next we need to decide on the type of transition function. As discussed, we are considering only three such functions *viz.*, LSTAR1, LSTAR2 and ESTAR which are specified in (4.3), (4.5) and (4.4), respectively. The choice of transition function is based on the auxiliary regression (4.11). The coefficient vectors β_j, $j = 1,2,3$, in (4.11) are functions of the parameters ϕ_1, ϕ_2, γ and *c*. In the special case $c = 0$, it can be shown that $\beta_2 = 0$ when the model is an LSTAR1 model, whereas $\beta_1 = \beta_3 = 0$ when the model is either LSTAR2 or ESTAR (see Teräsvirta (1994) for details). When $c \neq 0$, β_2 is closer to the null vector than β_1 or β_3 when the model is LSTAR1, and *vice versa* for the LSTAR2 model. Thus, the following short test sequence is useful in determining the transition function:

(i) Test the null hypothesis $H_{04} : \beta_3 = 0$ in (4.11) ;

(ii) Test $H_{03} : \beta_2 = 0 \mid \beta_3 = 0$;

(iii) Test $H_{02} : \beta_1 = 0 \mid \beta_2 = \beta_3 = 0$.

If the test of H_{03} yields the strongest rejection measured in terms of the *p*-value (smallest), then we choose either the LSTAR2 or ESTAR model[3]; otherwise we select

[3] While carrying out the estimation exercise, we have used only the LSTAR2 model simply because the software JMulTi contained computations relating to this model only.

the LSTAR1 model. All the three hypotheses stated above can simultaneously be rejected at conventional significance level and that is the reason we are interested in the strongest rejection. For India’s exchange rate return series, the transition variable y_{t-4} has yielded the minimum *p*-value (1.68×10^{-16}) for the *F*-statistic given in (4.13), which suggests the strongest rejection and we can conclude that LSTAR2 or ESTAR is the most appropriate transition function for our series. However, as stated earlier, the *p*-value of the *F*-statistic in (4.13) for the transition variable y_{t-1} is very close to that of y_{t-4}, and hence it will be useful for us to decide on the transition variable at the evaluation stage. For that purpose, we are interested in the appropriate transition function when the transition variable is chosen to be y_{t-1}. With this choice, computations were done and the *p*-value of H_{03} turned out to be the maximum, being 3.0147×10^{-4} and thus suggesting LSTAR1 to be the appropriate model for the transition variable y_{t-1}. Sometimes it is useful to fit the LSTAR1 as well as LSTAR2 models to the series and make the choice between the two at the evaluation stage.

4.3.2 Estimation

After the selection of the transition variable and function, estimation of the model is done by using appropriate starting values for γ, c_1 and c_2. The parameters of the STAR model are estimated using conditional maximum likelihood. The log-likelihood is maximized numerically, and JMulTi software does that by using the iterative BFGS algorithm. Finding good starting values is thus important. When the values of γ and *c* of the transition function are fixed then the model becomes linear in parameters. This suggests constructing a grid. The remaining parameters of the model are obtained conditional on the values of γ and *c*, and the sum of squared residuals are computed and the process is repeated for all probable combinations of these parameters. Finally, those parameter values are selected for which the sum of squared residuals is minimized. Because the grid is only two- or three- dimensional (in case of LSTAR2) , the restriction $c_1 \le c_2$ constrains the size of the grid further to make the procedure computationally manageable.

The starting values of γ, c_1 and c_2 obtained through grid search, for the LSTAR2 model with transition variable y_{t-4}, are $\gamma = 0.615$, $c_1 = 0.0035$ and $c_2 = 0.0202$. The estimation of this LSTAR2 model yields the following:

$$
\begin{aligned}
y_t =& -\underset{(1.305)}{0.00076} + \underset{(10.340)^{***}}{0.49159}\, y_{t-1} - \underset{(1.518)}{0.05951}\, y_{t-2} - \underset{(3.919)^{***}}{0.16075}\, y_{t-3} + \underset{(4.894)^{***}}{0.37124}\, y_{t-4} \\
& - \underset{(3.753)^{***}}{0.22118}\, y_{t-5} - \underset{(0.1877)}{0.00876}\, y_{t-6} + \underset{(3.567)^{***}}{0.24257}\, y_{t-7} - \underset{(3.020)^{***}}{0.19101}\, y_{t-8} + ((\underset{(1.502)}{0.00088} \\
& - \underset{(9.389)^{***}}{0.49476}\, y_{t-1} + \underset{(2.718)^{***}}{0.12511}\, y_{t-2} + \underset{(3.683)^{***}}{0.17525}\, y_{t-3} - \underset{(3.8558)^{***}}{0.30781}\, y_{t-4} + \underset{(3.425)^{***}}{0.21532}\, y_{t-5} \\
& - \underset{(0.819)}{0.04257}\, y_{t-6} - \underset{(3.194)^{***}}{0.22769}\, y_{t-7} - \underset{(3.146)^{***}}{0.21021}\, y_{t-8})\,(1 + \exp\{-(39.237)(y_{t-4} - 0.0036) \\
& (y_{t-4} - 0.0209)\})^{-1} + \hat{\varepsilon}_t
\end{aligned}
\tag{4.17}
$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

For this model, $\hat{\sigma} = 0.0026$ and $R^2 = 0.1029$, where $\hat{\sigma}$ is the residual standard deviation. It can be seen from (4.17) that most of the parameters for the lags are significant. Therefore we do not make any attempt to reduce the size of the model. Estimating the unrestricted STAR model might pose some problem owing to small sample sizes. However, in our case since we have a reasonable sample size, such a problem does not arise. As discussed earlier, for an LSTAR2 model the parameters change symmetrically around $(c_1 + c_2)/2$. Sometimes a nonlinear equilibrium correction in which the strength of attraction varies nonlinearly as a function of the size of the deviation from the equilibrium can be characterized by an LSTAR2 model.

The value of γ, for this estimated model, is very high indicating that the change from one regime to another is almost instantaneous. This also indicates that the STAR model for India's exchange rate converges to the SETAR model. A specific numerical problem, however, exists in the estimation of the STAR models when the value of γ is very large. When determining the curvature of the transition function a large number of observations in the neighbourhood of c_1 and c_2 is required. This lack of observations, and hence information, may manifest itself in the standard deviation estimate of $\hat{\gamma}$ becoming very large. The ensuing small value of *t*-ratio does not, however, suggest redundancy of the nonlinear component. Besides, quite apart from the numerical problem, the identification problem invalidates the standard interpretation of the *t*-ratio as a test of the hypothesis $\gamma = 0$.

We now consider the other model where the transition variable is chosen to be y_{t-1} and the appropriate transition function is LSTAR1. Since the results in the specification stage were found to be very similar for these two models, it is useful to find the performance of such a model in studying the behaviour of return on India's exchange rate. As before, we start with the grid search procedure and the starting values are obtained as $\gamma = 1.1425$ and $c = 0.0098$. The LSTAR1 model thus obtained is,

$$
\begin{aligned}
y_t &= \underset{(0.1814)}{0.00003} + \underset{(6.0123)^{***}}{0.22502}\, y_{t-1} + \underset{(1.1911)}{0.02602}\, y_{t-2} - \underset{(2.8009)^{***}}{0.06608}\, y_{t-3} + \underset{(4.3144)^{***}}{0.11313}\, y_{t-4} \\
&- \underset{(1.803)^{*}}{0.4056}\, y_{t-5} + (\underset{(3.451)^{***}}{0.00712} - \underset{(7.2392)^{***}}{0.86116}\, y_{t-1} + \underset{(0.2775)}{0.05349}\, y_{t-2} + \underset{(1.0619)}{0.24846}\, y_{t-3} \\
&- \underset{(2.7484)^{***}}{0.75023}\, y_{t-4} + \underset{(1.8205)^{*}}{0.22170}\, y_{t-5})(1 + \exp\{-(\underset{(2.4644)^{**}}{1.11221})(y_{t-1} - \underset{(5.55)^{***}}{0.00999})\})^{-1} + \hat{\varepsilon}_t
\end{aligned} \tag{4.18}
$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

For this model, $\hat{\sigma} = 0.0026$, $R^2 = 0.0851$ and the appropriate lag is found to be 5. Further, the estimated value of the threshold is 0.00999. It may be noted that the estimate of γ for LSTAR1 is very small compared to that obtained for the LSTAR2 model.

4.3.3 Diagnostics

In this section, we report the findings on diagnostic tests carried out with the residuals of the estimated STAR models.

No error autocorrelation: The procedure here is to regress the estimated residuals (from the nonlinear estimation) on lagged residuals and the partial derivatives of the log likelihood function with respect to the parameters of the model. From the above discussions, it is evident that it is relevant to determine which of the two models in

(4.17) and (4.18) can explain the behaviour of India's exchange rate return series more appropriately. This decision relies on the evaluation stage. The first step towards this is to see whether the models can adequately take care of the autocorrelation component. For this, the test statistic in (4.16) is computed and the values are reported in Table 4.1. It is seen from this table that for LSTAR2 model, the *F*-values obtained for the 8 lags indicate that the null hypothesis of no autocorrelation cannot be rejected. However, for the LSTAR1 model the values of *F*-statistic show that the null is rejected at 1 percent level of significance for all the 8 lags considered. One can, however, say that such a result may be due to the selection of lesser lags for the LSTAR1 model. In any case, such a dismal performance of the LSTAR1 model indeed suggests that this particular STAR model is not appropriate for modelling the exchange rate return series for India. In other words, the DGP of this time series is not explained well by the LSTAR1 model. It is, in fact the LSTAR2 model which is seen, in terms of this diagnostic test, to perform quite well.

Table 4.1

Test for no error autocorrelation for LSTAR1 and LSTAR2 models

LSTAR2					LSTAR1				
Lag	*F*-value	df1	df2	*p*-value	Lag	*F*-value	df1	df2	*p*-value
1	0.0133	1	2255	0.9082	1	10.8380	1	2266	0.0100
2	0.0188	2	2253	0.9814	2	10.7960	2	2264	0.0000
3	0.0227	3	2251	0.9954	3	9.6878	3	2262	0.0000
4	0.5277	4	2249	0.7154	4	7.3330	4	2260	0.0000
5	0.9590	5	2247	0.4416	5	6.3114	5	2258	0.0000
6	0.8893	6	2245	0.5017	6	6.0287	6	2256	0.0000
7	0.9857	7	2243	0.4397	7	8.4071	7	2254	0.0000
8	0.8684	8	2241	0.5425	8	7.3523	8	2252	0.0000

Test of no additive nonlinearity: Eitrheim and Teräsvirta (1996) developed an LM-statistic to test the LSTAR model represented by (4.7) against the alternative of an additive STAR model given by (4.8). If it is assumed that $c_1 < c_2$, the autoregressive parameters of the model change smoothly from ϕ_1 to ϕ_3 *via* ϕ_2 for increasing values of s_t as first the function G_1 changes from 0 to 1 followed by a similar change in G_2. This way we can generalize the model for *J* regimes. The null hypothesis of a 2-regime model can thus be expressed as $H_0 : \gamma_2 = 0$. As this testing problem suffers from similar identification problem, as discussed earlier, we replace the transition function $G(s_t; \gamma_2, c_2)$ by a Taylor series approximation around $\gamma_2 = 0$. If a third order expansion is assumed, the resultant auxiliary model is obtained as

$$y_t = \beta_0' \tilde{y}_t + (\phi_2 - \phi_1)' \tilde{y}_t G_1(s_t; \gamma_2, c_2) + \beta_1' \tilde{y}_t s_t + \beta_2' \tilde{y}_t s_t^2 + \beta_3' \tilde{y}_t s_t^3 + e_t \quad (4.19)$$

where β_i, $i = 0,1,2,3$ are functions of the parameters $\phi_1, \phi_2, \phi_3, \gamma_2, c_2$.

It is obvious that the null hypothesis $H_0 : \gamma_2 = 0$ translates into $H_0' : \beta_1 = \beta_2 = \beta_3 = 0$. The test statistic for this test can be computed as TR^2 from the auxiliary regression of the residuals obtained from estimating the model under the null hypothesis on the partial derivatives of the regression function with respect to the parameters in the two-regime (one transition function) model evaluated under the null hypothesis and the auxiliary regressors $\tilde{y}_t s_t^i$, $i = 1,2,3$. The resultant LM test statistic has an asymptotic χ^2 distribution with $3(p+1)$ degrees of freedom. Again in the *F*-version of this test, the *p*-values have been obtained to be very low (9.0056×10^{-14} for the LSTAR2 model and 4.5882×10^{-22} for the LSTAR1 model) which indicate that the null hypothesis is rejected.

Testing parameter constancy: The representation of a time varying STAR model is made in the following way :

$$y_t = \phi_1(t)' \tilde{y}_t (1 - G_1(s_t; \gamma_1, c_1)) + \phi_2(t)' \tilde{y}_t G_1(s_t; \gamma_1, c_1) + u_t$$

with

$$\phi_1(t) = \phi_1[1 - G_2(t; \gamma_2, c_2)] + \phi_3 G_2(t; \gamma_2, c_2)$$

$$\phi_2(t) = \phi_2[1 - G_2(t; \gamma_2, c_2)] + \phi_4 G_2(t; \gamma_2, c_2) .$$

By testing the hypothesis $\gamma_2 = 0$, one can test for parameter constancy in the two-regime STAR model against the alternative of smoothly changing parameters. The appropriate LM-type test statistic based on a third-order Taylor series approximation of $G_2(t;\gamma_2,c_2)$ is identical to the LM statistic obtained in the previous test only with $s_{2t}=t$. The asymptotic distribution theory remains the same even if the transition variable is non-stationary. The *p*-values obtained in our model, being ≤ 0.001, it indicates that the null hypothesis is rejected. Though these misspecification tests seem straightforward, often it becomes difficult to decide what to do when some of these tests result in rejection of null hypothesis. Error autocorrelation indicates misspecification but its specific nature is not defined. The test may not only have power against misspecified dynamics but also against omitted variables. Rejecting the null of no additive nonlinearity may suggest adding another STAR component to the model. But then, since a rejection as such does not say anything definite about the cause, the idea of extending the model further has to be weighted against other considerations such as the risk of overfitting. Some protection against overfitting may be obtained by applying low significance levels. This is important because the number of tests typically carried out at the evaluation stage can be large.

Parameter constancy tests are also indicative of general misspecification, and there is no unique way of responding to a rejection. Carrying out the tests for subsets of parameters, however, may provide important information about the shortcomings of the model. In some cases, it is found to be reasonable to respond to rejection by extending an estimated STAR model with a time varying STAR model, as recently done by Van Dijk *et al.* (2003) and Teräsvirta *et al.* (2003) on time varying seasonal patterns in quarterly industrial production series.

4.3.4 Out-of-sample forecasting performance

It is evident from the diagnostics, that the LSTAR2 model has emerged as the most appropriate model amongst the STAR models used for India's exchange rate return series. It is thus necessary for us to see the performance of such a model in general by using out-of-sample forecasting techniques. We have discussed in Section 2.2.3 the various forecast evaluation criteria used by us. We have obtained the out-of-sample forecasts and then compared these with the actual values by standard forecast evaluation criteria, like the MSE, MAE, AMAPE and PCSP for the LSTAR2 model, and these are reported in Table 4.2. We have made a detailed discussion on the out-of-sample forecast performance of this model relative to the naïve forecast model, *viz.* the random walk model in Chapter 5, Section 5.4.

Table 4.2

Out-of-Sample forecast performance

LSTAR2 model				
Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	2.418.E-05	0.00350	_	45.0
5	2.576E-05	0.003791	_	45.36
10	2.985E-05	0.004140	_	44.44

After the evaluation stage, we can conclude that LSTAR2 model is more appropriate than LSTAR1 for the return on daily nominal Indian rupee / US dollar exchange rate series. We come to this conclusion since the diagnostics are better for the LSTAR2 model in terms of absence of the error autocorrelation for the lags. However, there is also the scope for improvement in terms of using other advanced models like the TVSTAR model (*cf.* Lundbergh *et al.* (1999), and more recently Van Dijk *et al.* (2003) and Teräsvirta *et al.* (2003)). Also, the daily exchange rate series is marked by high volatility which has not been taken into account in our model primarily because of two reasons. First, there is hardly any such study available. We have come across only few studies, the notable one being by Lündbergh and Teräsvirta (1998). The fact that there is practically no other study where SETAR and a volatility model have been combined, it may be concluded that either such a model is infeasible or the computations are too difficult and time consuming. The second reason is related to the fact that in this model we have found the smoothness parameter to have very high estimated value, indicating that the STAR model is then reduced to the SETAR model. Since in Chapter 3, we have already studied the out-of-sample forecasting performance of this model, such an exercise, therefore, seems to be not really necessary in this chapter.

4.4 Conclusions

We have used the STAR model to study the nonlinearity of the Indian rupee / US dollar exchange rate data, and found that such a model can indeed be used for capturing the nonlinearities of the time series. At the specification stage we have found that both an LSTAR2 model with fourth lag of the exchange rate variable as the transition variable and LSTAR1 model with first lag as the transition variable are candidate models. We have estimated both these models. For the LSTAR2 model, the appropriate lag length of the model was found to be 8, while for the LSTAR1 model it was found to be 5. At the evaluation stage, however, we have noted that the performance of the LSTAR2 model is better considering the diagnostics based on no error autocorrelation. Thus, we can say that the LSTAR2 model performs better than the LSTAR1 model for India's exchange rate. This second order logistic function nests a restricted three-regime SETAR model, if the transition variable equals the value of return with some lags, where the restriction is that the outer regimes are identical. The estimated LSTAR2 model was found to have very high value of γ which indicates that the switching from one regime to the other is almost instantaneous. This means the LSTAR2 model, thus found, tends to nest a restricted three-regime SETAR model. We have also computed the out-of-sample forecasts for the LSTAR2 model to find its performance in terms of standard forecast evaluation criteria.

CHAPTER 5

The Markov Switching Regression Model for Exchange Rate Return at Daily Frequency

5.1 Introduction

Empirical research in international finance has taken a prominent role in the last two decades. Although many of the models commonly used in empirical finance are linear, the nature of financial data suggests that nonlinear models are likely to be more appropriate for forecasting and accurately describing return and volatility. Several nonlinear time series models which are appropriate for modelling and forecasting economic and financial time series have been proposed in this literature. In fact, the enormous number of such models makes choosing the best model for a particular application quite a daunting task (for some relevant references, see Tong (1990), Peel and Speight (1994), Brooks (1996, 1997, 2001), Clements and Smith (1999, 2001), Dacco and Satchell (1999) and Boero and Marrocu (2002, 2004)). However, a class of recently-developed nonlinear time series models, called the regime switching models, that permits sufficient flexibility to allow different types of behavior at different points in time, has been found to be potentially useful. An important member of this class of nonlinear models is known as the Markov switching regression (MSR) model. These models are designed to capture discrete changes in the economic mechanism that generates the data. This class of models implies that one can never be certain about the regime the variable is in a particular point of time, but can only assign probabilities to the occurrence of different regimes. These models have been popularized by Hamilton (1989, 1990, 1994) and Engel and Hamilton (1990), although these were originally motivated by Goldfeld and Quandt (1973).

The MSR model has been used quite extensively for studying foreign exchange rate. Along with Engel and Hamilton (1990), regime switching in foreign exchange rate has been documented by Bekaert and Hodrick (1993), Engel (1994), Engel and

Hakkio (1996), Bollen *et al.* (2000), Marsh (2000) and Frömmel *et. al.* (2005). While Engel and Hamilton (1990) concentrated on the dollar value, Engel (1994) fitted the MSR model to 18 exchange rates, including 11 non-U.S. dollar exchange rates, at quarterly frequencies and showed that the model fits well in-sample for many exchange rates. However, using mean squared error criterion, they showed that the MSR model does not generate superior forecasts to the random walk model. Marsh (2000) estimated a two-state MSR model for high frequency exchange rate series and investigated the profitability of following the generated forecasts. The two-state MSR model used by him, which is similar to the model proposed by Engel and Hamilton (1990), assumes that the variable is drawn from either of the two states which are characterized by different means and variances. In other words, the underlying distribution for the data resembles a mixture of normal distributions with the only difference that the value of the variable at a certain time point depends on its values at some previous time points. Bollen *et al*. (2000) have used an augmented form of this standard model to allow two regimes for the mean and two regimes for the variance, and found that a model with independent mean and variance shift provides tighter in-sample fit and more accurate variance forecasts of the return on exchange rate.

The MSR model has also been generalized to incorporate an autoregressive component in the conditional mean and an autoregressive conditional heteroscedastic (ARCH) process in the conditional variance, and this generalization has been found to be especially useful for high frequency exchange rate data. Such a model is called the switching-regime ARCH (SWARCH) model. The ARCH model, though successful in capturing the volatility of high frequency data, often imputes a lot of persistence to exchange rate volatility. This has, in fact, been found to be true for stock prices as well. It has been argued that it is sometimes useful to allow one or more of the parameters of ARCH model to have different values for different regimes, with transition between regimes being governed by an unobserved Markov chain. Brunner (1991) and Cai (1994) have used such a model which allows for the possibility of sudden discrete changes in the values of the parameters of an ARCH process, as in the MSR model by Hamilton (1989). Cai (1994) has modelled the dynamics of variance in the switching regime ARCH as following a first order Markov process. In such a model - denoted as the Markov SWARCH (MSWARCH) - the conditional variance is no longer determined by an exact linear combination of past conditional variances and past shocks, as in the standard ARCH process. The intercept in the conditional variance does not change every period but only in response to occasional discrete events. Thus, the model retains the volatility clustering feature of the ARCH model and, in addition, captures the discrete shift in the intercept of the conditional variance that may cause spurious apparent persistence in the variance process. Hamilton and Susmel (1994) have proposed another parameterization of the MSWARCH model where they have modelled changes in the regime as changes in the scale of the ARCH process. Their estimates attribute most of the persistence in the financial variable volatility to the persistence of the low, moderate and high volatility regimes. Dueker (1997) has used both these parameterizations along with two more to examine their multi-period stock-market volatility forecasts as predictions of options-implied volatilities.

In this chapter, we use the MSR as well as the MSWARCH models for modelling the daily-level time series data on the foreign exchange rate of India. In other words, here we first fit the simple two-state MSR model which is similar to fitting the mixture of two normal distributions, and then consider the more general model where Hamilton and Susmel (1994)'s MSWARCH model along with an autoregressive process in the conditional mean specification is considered. As in the case of previous chapters, the performance of the relevant model is studied by obtaining the out-of-sample forecasts and then comparing these with the actual values by standard forecast evaluation criteria. In fact, in this chapter we also make a comparison among the four different time series models considered so far, in terms of their out-of-sample forecasting performances. These four models considered in Chapters 2 through 5 have been fitted to the same data set *viz*., the time series of return on India's foreign exchange rate at daily-level frequency covering the period from November 1, 1994 to February 13, 2004 and hence such an exercise would enable us to conclude on the relative rankings of these models for this time series, in terms of out-of-sample forecasting performance.

The outline of the paper is as follows. In the next section we briefly describe the two-state MSR model and its estimation procedure followed by the same for the more complex MSWARCH model with an autoregressive process in the mean specification. The empirical results for these two models are reported and discussed in Section 5.3. The comparative performance of the four different time series models considered so far in the thesis, in terms of standard out-of-sample forecasting criteria, are presented in Section 5.4. The paper ends with some comments in Section 5.5.

5.2 The models and their estimations

In this section, we first present the two models in the class of MSR models, which we have fitted to returns on the time series of foreign exchange rate of India, and then discuss their estimation methods briefly. The first model is due to Engel and Hamilton (1990), which resembles a 'mixture of normal distributions' and the other is the MSWARCH model with the parameterization, as discussed in Hamilton and Susmel (1994).

5.2.1 The MSR model resembling mixture distribution

The Engel and Hamilton (1990) model is similar to the mixture distribution of two normal variables, with the sole difference that the probability of a particular observation y_t coming from one distribution depends on the realization of y_t at some other previous time points, where y_t is the variable under study i.e., the exchange rate return, in our case. We describe the model for the general case of *J* regimes. Let the regime that a given process is in on date t be indexed by an unobserved random variable, s_t, which takes on integer values only. When the process is in regime 1, the observed variable y_t is presumed to be drawn from a $N(\mu_1, \sigma_1^2)$ distribution. If the process is in regime 2, then y_t is presumed to be drawn from $N(\mu_2, \sigma_2^2)$ and so on. The state variable, s_t, is assumed to evolve according to a Markov chain such that the probability of being in state 1 at a time *t* given that state 1 existed at time *t*-1, equals

p_{11}. Obviously, $p_{jj}, j = 2,3,..., J$, are similarly defined. The density of y_t conditional on the random variable s_t taking on the value *j* is given by

$$f(y_t \mid s_t = j;\theta) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left\{ \frac{-(y_t - \mu_j)^2}{2\sigma_j^2} \right\} \tag{5.1}$$

for $j = 1,2,..., J$. Here θ is the vector of population parameters that include $\mu_1, \mu_2, ..., \mu_J$ and $\sigma_1^2, \sigma_2^2, ..., \sigma_J^2$. The unobserved regime $\{s_t\}$ is presumed to have been generated by some probability distribution, for which the unconditional probability that s_t takes on the value *j* is denoted by π_j i.e., $P\{s_t = j;\theta\} = \pi_j$ for $j = 1,2,...J$. The probabilities $\pi_1, \pi_2, ..., \pi_J$ are also included in θ so that $\theta \equiv (\mu_1, \mu_2, ..., \mu_J, \sigma_1^2, \sigma_2^2, ..., \sigma_J^2, \pi_1, \pi_2, ..., \pi_J)'$.

Now, if we are interested in the probability of the joint event that $s_t = j$ and that y_t falls within some interval [*c*,*d*], this could be found by integrating the joint density function of y_t and s_t i.e.,

$$p(y_t, s_t = j;\theta) = f(y_t \mid s_t = j;\theta).P\{s_t = j;\theta\} \tag{5.2}$$

over all the values of y_t between *c* and *d*. Obviously,

$$p(y_t, s_t = j;\theta) = const \cdot \frac{\pi_j}{\sigma_j} \exp\left\{ \frac{-(y_t - \mu_j)^2}{2\sigma_j^2} \right\} \tag{5.3}$$

where $const = 1/\sqrt{2\pi}$. The unconditional density of y_t can be found by summing (5.3) over all possible values of j i.e.,

$$f(y_t;\theta) = \sum_{j=1}^{J} p(y_t, s_t = j;\theta) \quad . \tag{5.4}$$

Assuming that the unobserved regime variable s_t is distributed i.i.d. across different dates *t*, the log likelihood for the observed data $y_1, y_2, ..., y_T$ can be obtained as

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t;\theta) \quad . \tag{5.5}$$

The maximum likelihood estimate of θ is obtained by maximizing (5.5) subject to the constraints that $\pi_1 + \pi_2 + ... + \pi_J = 1$ and $\pi_j \geq 0$ for j = 1,2,…,J. This can be achieved by using the expected maximization (EM) algorithm (see Dempster *et al.* (1997), for details on this algorithm), as used by Hamilton (1994). Consequently, the maximum likelihood estimators $\hat{\theta}$ of the parameters in θ are obtained as follows:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} y_t P\{s_t = j \mid y_t; \hat{\theta}\}}{\sum_{t=1}^{T} P\{s_t = j \mid y_t; \hat{\theta}\}} \tag{5.6}$$

$$\hat{\sigma}_j^2 = \frac{\sum_{t=1}^{T} (y_t - \hat{\mu}_j)^2 P\{s_t = j \mid y_t; \hat{\theta}\}}{\sum_{t=1}^{T} P\{s_t = j \mid y_t; \hat{\theta})} \tag{5.7}$$

and

$$\hat{\pi}_j = T^{-1} \sum_{t=1}^{T} P\{s_t = j \mid y_t; \hat{\theta}\} \tag{5.8}$$

for j = 1,2,…, J. Since $P\{s_t = j \mid y_t; \hat{\theta}\}$ involved in (5.6) through (5.8) above is a probability value, it always lies between 0 and 1, and hence the estimate $\hat{\mu}_j$ of μ_j is a weighted average of all the observations in the sample, where the weight for observation y_t is proportional to the probability that observation for t^{th} time point was generated by regime j. Similarly, $\hat{\sigma}_j^2$ is a weighted average of the squared deviations of y_t from $\hat{\mu}_j$ while $\hat{\pi}_j$ is the fraction of observations that appear to have come from regime j. One can essentially calculate the transition probabilities from (5.8) (see Marsh (2000), for further details). Because the equations from (5.6) to (5.8) are nonlinear, it is not possible to solve them analytically for $\hat{\theta}$ as a function of $\{y_1, y_2, ..., y_T\}$. However, these equations suggest an appealing iterative algorithm for finding the maximum likelihood estimates. Starting from an arbitrary initial guess for the value of θ, denoted as $\theta^{(0)}$, one can calculate $P\{s_t = j \mid y_t; \theta^{(0)}\}$ using the following relation

$$P\{s_t = j \mid y_t;\theta\} = \frac{p(y_t, s_t = j;\theta)}{f(y_t;\theta)} = \frac{\pi_j f(y_t \mid s_t = j;\theta)}{f(y_t;\theta)} \tag{5.9}$$

which follows from the definition of conditional probability. Thereafter $\hat{\mu}_j$, $\hat{\sigma}_j^2$ and $\hat{\pi}_j$, j = 1,2,…,J are obtained from (5.7), (5.8) and (5.9) with $\theta^{(0)}$ being substituted for $\hat{\theta}$. The new estimates, say $\hat{\theta}^{(1)}$, are thus obtained. The process is then repeated with $\hat{\theta}^{(1)}$ and it continues until convergence.

5.2.2 The MSWARCH model

The other model considered in this paper is one where conditional variance in the form of autoregressive conditional heteroscedasticity (ARCH) is incorporated in the basic MSR model set-up. Since volatility is an important property observed in most financial time series, more so in case of high-frequency data, such an extension of the MSR model is considered to be very useful. Given the framework of MSR model, it is only natural that we explore a specification in which the parameters of an ARCH process can occasionally change. Following Engle's (1982) ARCH specification, we write $\varepsilon_t = \sigma_t . v_t$ where ε_t is the error term associated with an autoregressive process

$$y_t = \phi_0 + \phi_1\ y_{t-1} + \varepsilon_t \tag{5.10}$$

and v_t, $t = 1,2,...,T$, are i.i.d. normal with zero mean and unit variance. The conditional variance of ε_t is specified to be a function of its finite past realizations. While Engle considered the conditional variance specification to depend linearly on the past squared realizations of ε_t i.e.,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2 \quad , \tag{5.11}$$

Bollerslev (1986) generalized the form by including $\sum_{i=1}^{m} \beta_i \sigma_{t-i}^2$ in (5.11), denoted as GARCH(m,n), so that σ_t^2 now has the specification $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{m} \beta_i \sigma_{t-i}^2$. We can further allow for the possibility of sudden discrete changes in the values of the parameters of the ARCH (n) process[1]. The resulting model, termed as the Markov-switching ARCH (MSWARCH) model, has been successfully used by Brunner (1991), Cai (1994), Hamilton and Susmel (1994) and others in modelling volatility of some important financial variables.

Hamilton and Susmel (1994) employed the corresponding parameterization in their MSWARCH model by considering $\varepsilon_t = \sqrt{g_{s_t}} \cdot \tilde{\varepsilon}_t$, where $\tilde{\varepsilon}_t = \sqrt{h_t}.v_t$. Given that $\varepsilon_t = \sigma_t.v_t$ and $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2$ (*c.f.* (5.11)) , $\tilde{\varepsilon}_t$ follows a standard ARCH(n) process given by

$$h_t = \alpha_0 + \alpha_1 \tilde{\varepsilon}_{t-1}^2 + ... + \alpha_n \tilde{\varepsilon}_{t-n}^2 \quad . \tag{5.12}$$

The underlying ARCH variable, $\tilde{\varepsilon}_t$, is multiplied by the constant $\sqrt{g_1}$ when the process is in regime represented by $s_t = 1$ and by $\sqrt{g_2}$ when $s_t = 2$ and so on. The factor for the first state i.e., g_1 , is normalized to unity with $g_j \geq 1$ for j=2,3,…, J . The idea here is to model changes in regime as changes in the scale of the process. Conditional on knowing the current and past regimes, the variance implied for the residual ε_t is

$$\begin{aligned} &E(\varepsilon_t^2 \mid s_t, s_{t-1}, ..., s_{t-n}, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-n}) \\ &= g_{s_t} \{\alpha_0 + \alpha_1.(\varepsilon_{t-1}^2 / g_{s_{t-1}}) + \alpha_2.(\varepsilon_{t-2}^2 / g_{s_{t-2}}) + ... + \alpha_n.(\varepsilon_{t-n}^2 / g_{s_{t-n}})\} \end{aligned} \tag{5.13}$$

$$\equiv \sigma_t^2 (s_t, s_{t-1}, ..., s_{t-n}) .$$

We thus say that ε_t follows a J-state, n th-order Markov-switching ARCH process, denoted as $\varepsilon_t \sim MSWARCH(J, n)$. Now, as in the case of MSR model, s_t is described by a Markov chain so that the transition probability given by $Prob(s_t = j \mid s_{t-1} = i, s_{t-2} = k, ..., y_{t-1}, y_{t-2}, ...; \theta) = Prob(s_t = j \mid s_{t-1} = i; \theta) = p_{ij}$ (say) for i , j = 1,2,…, J. It is convenient to collect the transition probabilities in a $(J \times J)$ matrix P as:

---

[1] Without any loss of generality, we are considering the ARCH (n) process for representing volatility.

$$P = \begin{bmatrix} p_{11} & p_{21} & ..... & p_{J1} \\ p_{12} & p_{22} & ..... & p_{J2} \\ ..... & ..... & ..... & ..... \\ p_{1J} & p_{2J} & ..... & p_{JJ} \end{bmatrix} \tag{5.14}$$

where each column of the matrix P sums to unity.

If the density of y_t conditional on its own lagged values as well as on current and previous n values for the state is of a known form, then methods developed in Hamilton (1989, 1994) can be used to evaluate the likelihood function for the observed data and make inferences about the unobserved regimes. In our case, the density of y_t is given as:

$$f(y_t \mid s_t, s_{t-1}, ..., s_{t-n}, y_{t-1}, ..., y_{t-n}; \theta) = \frac{1}{\sqrt{2\pi}\sigma_t(s_t, s_{t-1}, ..., s_{t-n})} \exp\left\{ \frac{-(y_t - \phi_0 - \phi_1 y_{t-1})^2}{2\sigma_t^2(s_t, s_{t-1}, ..., s_{t-n})} \right\} . \tag{5.15}$$

Under the assumptions made, the sample log-likelihood function

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t \mid y_{t-1}, y_{t-2} ....; \theta) \tag{5.16}$$

is maximized numerically with respect to the population parameters $\theta = (\phi_0, \phi_1, \alpha_0, ..., \alpha_n, p_{11}, p_{12}, ..., p_{JJ}, g_1, g_2, ..., g_J)'$ subject to the constraints that $g_1 = 1$, $\sum_{j=1}^{J} p_{ij} = 1$ and $0 \le p_{ij} \le 1$ for i , j = 1,2,…, J. $L(\theta)$ is evaluated using an algorithm in Hamilton and Susmel (1994). At step t of the iteration for calculating the log-likelihood function, the input *viz*., the filter probability is given by

$$p(s_t, s_{t-1}, ..., s_{t-n} \mid y_t, y_{t-1,} .....; \theta) \tag{5.17}$$

This probability is called the filter probability since this inference is based on information observed through date t. It denotes the conditional probability of the state values $s_t, s_{t-1}, ..., s_{t-n}$ on dates $t, t-1, ..., t-n$, respectively given observations $y_t, y_{t-1}, ..., y_{t-n}$. Since there are J^{n+1} possible configurations for $(s_t, s_{t-1}, ... s_{t-n})$, there are J^{n+1} separate numbers of the form of (5.17), which add up to unity by construction. Each of the J^{n+1} numbers represented by (5.17) is multiplied by $p_{s_t, s_{t+1}}$ and by $f(y_{t+1} | s_{t+1}, s_t, ..., s_{t-n+1}, y_t, y_{t-1}, ..., y_{t-n+1}; \theta)$ to yield the J^{n+2} separate numbers

$$p(s_{t+1}, s_t, s_{t-1}, ..., s_{t-n+1}, y_{t+1} | y_t, y_{t-1}, ...; \theta). \tag{5.18}$$

These numbers in (5.18) are summed to obtain the conditional density of y_{t+1} i.e.,

$$\begin{aligned} & f(y_{t+1} | y_t, y_{t-1}, ...; \theta) = \\ & \sum_{s_{t+1}=1}^{J} \sum_{s_t=1}^{J} ... \sum_{s_{t-n}=1}^{J} p(s_{t+1}, s_t, s_{t-1}, ..., s_{t-n}, y_{t+1} | y_t, y_{t-1}, ...; \theta) \end{aligned} \tag{5.19}$$

from which the sample log-likelihood (5.16) can be calculated. If for any given $s_{t+1}, s_t, ..., s_{t-n+1}$ the numbers in (5.18) are summed over J possible values for s_{t-n} and the result is then divided by (5.19), one obtains $p(s_{t+1}, s_t, ..., s_{t-n+1} | y_{t+1}, y_t, ...; \theta)$ which is the input for step t+1 of the iteration. The iteration is started with $p(s_0, s_{-1}, ...s_{-n} | y_0, y_{-1}, ...; \theta)$ set equal to the ergodic probabilities implied by the Markov chain as described in Hamilton (1994). Kim's (1993) algorithm for calculating the smoothed probabilities $p(s_t | y_T, y_{T-1}, ..., \theta)$ is described in Hamilton (1994). In this case, the full sample of observations is used to construct the 'smoothed probability'.

5.3 Empirical results

We now discuss the computational findings of the application of the two nonlinear time series models belonging to the MSR class, mentioned in the preceding section, to India's daily exchange rate return series. The details of the exchange rate series have been mentioned in Section 2.3 of Chapter 2. However, here the stationary series (of returns) has been multiplied by 100. This is being done so that the errors of approximation arising out of complex computations with very small numbers could be minimized. All computations in this chapter have been done with the GAUSS package as well as with the codes provided by Hamilton and Susmel (1994).

Initially, we have applied the model proposed by Engel and Hamilton (1990) which has been described in the preceding section. As already noted, this model is similar to a mixture model of two normal distributions. In our case where J = 2, this distribution is the superposition of two simple normal distributions. The only difference lies in the fact that unlike a mixture normal distribution, the observations here are not independent. So, the parameters of our two-state model are the mean and variance of each of the simple normal distributions and the two unconditional probabilities that s_t takes on the value j where j = 1,2. We estimate the transition probabilities which is essentially the number of times transition was made from state j to j, as a fraction of the number of times the process had been in state j in the previous period.

Using the EM algorithm as described in the preceding section, we have obtained the following estimates for the parameters of this model: $\underset{(0.004)}{\hat{\mu}_1 = 0.007614}$, $\underset{(0.069)}{\hat{\mu}_2 = 0.094315}$, $\underset{(0.002)}{\hat{p}_{11} = 0.989538}$, $\underset{(0.033)}{\hat{p}_{22} = 0.862211}$, $\underset{(0.001)}{\hat{\sigma}_1^2 = 0.037193}$ and $\underset{(0.875)}{\hat{\sigma}_1^2 = 0.875224}$ where the values in parentheses are the standard errors. The values of $\hat{\pi}_1$ and $\hat{\pi}_2$ are 0.929 and 0.071 respectively. We have also plotted the figures for the smooth probabilities $p(s_t = 1 \mid y_T, y_{T-1},....;\hat{\theta})$ and $p(s_t = 2 \mid y_T, y_{T-1},....;\hat{\theta})$. The dates at which it can be concluded that the process has switched between regimes are based on whether $p(s_t = 2 \mid y_T, y_{T-1},...;\hat{\theta}) >< 0.5$. From the plots given in Figure 5.1, we find that there is evidence of considerable switching over across the two regimes. From the figure, we can roughly say that the switching over has taken place around the data points 200, 400, 700, 1400, 1700 and 2200. Looking at the estimated transition probabilities we note that these are very high (nearly 1), implying high persistence in both the regimes.

Engel and Hamilton (1990), in their study, have documented a property of exchange rate, called ‘long swings’. ‘Long swings’ are characterized by high transition probabilities and opposite signs of the mean. This implies that there are two distinct states given by the depreciation and appreciation of exchange rates, but there is very high persistence in these two states. In our results, however, no evidence of ‘long swings’ is found as the means of the two states are of the same sign. However, the persistence among the two states is very high i.e., when the variable is in a particular state it will tend to be in that state. We observe that the values of the means are very close to each other, making it essential for us to check whether the null hypothesis that the means of both the regimes are same is rejected or not. We thus test the hypothesis $H_0 : \mu_1 = \mu_2$ using the Wald statistic, $\frac{(\hat{\mu}_1 - \hat{\mu}_2)}{\hat{\mathrm{var}}(\hat{\mu}_1) + \hat{\mathrm{var}}(\hat{\mu}_2) - 2\,\hat{\mathrm{cov}}(\hat{\mu}_1, \hat{\mu}_2)} \sim \chi^2(1)$ under H_0.

The value of this statistic obtained by us is 1.59149 which is less than the critical value (3.84) of $\chi^2(1)$ even at 5% level of significance, indicating that the relevant null hypothesis cannot be rejected, and hence we can conclude that the mean values for the two regimes are statistically the same. Thus we find that this simple MSR model does not perform well for the time series return on India’s daily exchange rate.

We now report and discuss the empirical findings of the more general MSWARCH model. Based on the conclusion on the means of the previous model, we assume that regime shifting phenomenon is not present in the mean specification. We thus use only one mean specification given by an AR(1) process, and the change in regime is introduced through the conditional variance following the ARCH model as well as a formulation in the line of GJR (Glosten *et al.* (1993)) model which is a simple extension of the GARCH model with an additional term added to account for possible asymmetry which is typically attributed to ‘leverage effect’. The consideration to GJR model is made keeping in mind the particular empirical finding in Chapter 2, that in the framework of a linear dynamic model allowing for structural breaks, the volatility specification appropriate for India’s foreign exchange return is the EGARCH (Nelson (1991)) model which explicitly takes into consideration the ‘leverage effect’ although in a way different from the GJR model considered here (*cf.* (5.21) below). As already discussed, the MSWARCH model as proposed by Hamilton and Susmel (1994), considers model changes in regimes as changes in the scale of the assumed ARCH process. In our study, we have used a two-state as well as a three-state MSWARCH model and the choice of order n in ARCH (n) specification has been made upto a maximum of 3.

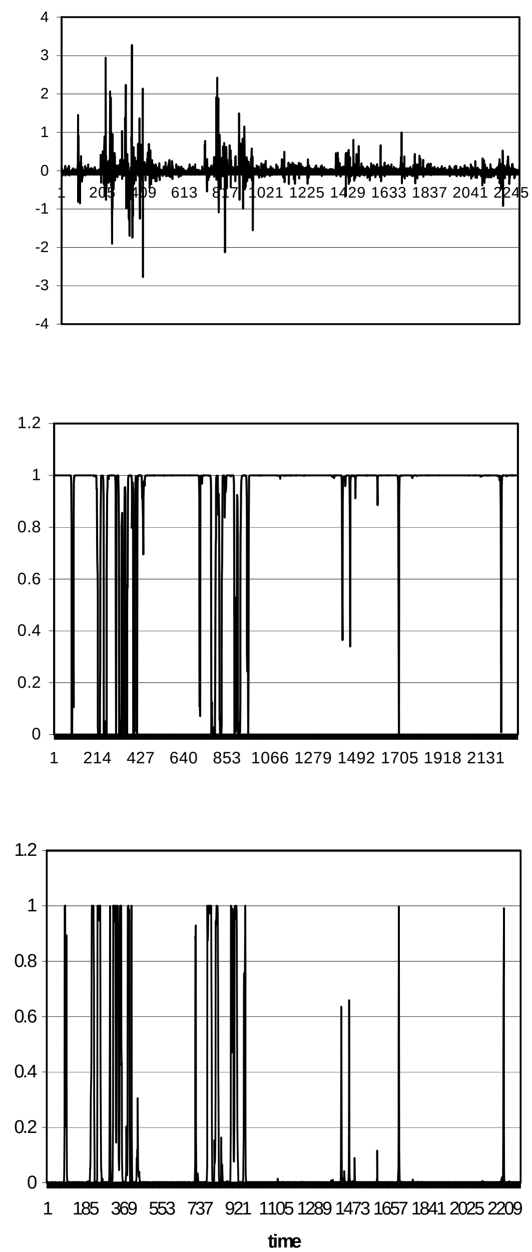


Figure 5.1 The top most panel gives the plot of daily exchange rate return against time. The next 2 panels are plots of $p(s_t = 1 | y_T, y_{T-1},....)$ and $p(s_t = 2 | y_T, y_{T-1}....)$, respectively for the 2-state Markov switching regression model .

Hamilton and Susmel (1994), in their paper, have taken v_t to follow the Student's *t* distribution. The implied conditional density of $\tilde{\varepsilon}_t$ is then given by:

$$f(\tilde{\varepsilon}_t \mid \tilde{\varepsilon}_{t-1}, \tilde{\varepsilon}_{t-2}, ...) = \frac{\Gamma\{(\tau+1)/2\}}{\sqrt{\pi}\Gamma(\tau/2)}(\tau-2)^{-1/2}\sigma_t^{-1} \times \left[1 + \frac{\tilde{\varepsilon}_t^2}{\sigma_t^2(\tau-2)}\right]^{-(\tau+1)/2} \tag{5.20}$$

where τ is the degrees of freedom associated with the Student's *t* distribution which has been normalized to have a unit variance. The other variant of MSWARCH model used by them, which incorporates the 'leverage effect' has the following volatility specification:

$$\begin{aligned} &E(\varepsilon_t^2 \mid s_t, s_{t-1}, ..., s_{t-n}, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-n}) \\ &= g_{s_t}\{\alpha_0 + \alpha_1.(\varepsilon_{t-1}^2 / g_{s_{t-1}}) + \alpha_2.(\varepsilon_{t-2}^2 / g_{s_{t-2}}) + ... + \alpha_n.(\varepsilon_{t-n}^2 / g_{s_{t-n}}) \\ &\qquad + \xi.d_{t-1}.(\varepsilon_{t-1}^2 / g_{s_{t-1}})\} \end{aligned} \tag{5.21}$$

where $d_{t-1} = 1$ for $\varepsilon_{t-1} \le 0$ and $d_{t-1} = 0$ for $\varepsilon_{t-1} > 0$. This can obviously be considered to be a generalization of the volatility specification given in (5.13). In the absence of leverage effect i.e., when $\xi = 0$, we say that ε_t follows a *J*-state, *n* th-order Markov-switching ARCH process, denoted as ε_t ~ MSWARCH(J, n). In the presence of leverage effect i.e., when $\xi \ne 0$, it may be designated as an MSWARCH –L*(J, n)* specification. Thus, the different kinds of MSWARCH models considered in our study include this generalization along with assumption of both the standard Gaussian and Student's *t* distribution for v_t. At the outset, it may be stated that the 'leverage effect' was found to be insignificant under both the distributional assumptions and hence these are being removed subsequently. Further, the results under the assumption of Student's *t* distribution are more or less similar to those under N(0,1), and hence we are only reporting the empirical findings pertaining to the case where the error is standard normal and there is no 'leverage effect'.

Before discussing the empirical findings on the MSWARCH model, we first report the findings for a simple model with heteroscedasticity *viz.*, an AR(1)-ARCH(2) model to check for the persistence of volatility present in returns on India's foreign exchange rate. The estimated conditional mean and variance models are

$$\begin{aligned} y_t &= -\underset{(3.2)^{***}}{0.00320} - \underset{(34.533)^{***}}{0.20720}\, y_{t-1} + \hat{\varepsilon}_t \\ \sigma_t^2 &= \underset{(12.2)^{***}}{0.01220} + \underset{(67.2)^{***}}{0.60480}\, \hat{\varepsilon}_{t-1}^2 + \underset{(39.1)^{***}}{0.39100}\, \hat{\varepsilon}_{t-2}^2 \quad . \end{aligned} \tag{5.22}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

We note from (5.22) that the coefficients of $\hat{\varepsilon}_{t-1}^2$ and $\hat{\varepsilon}_{t-2}^2$ are significant at 1 per cent level of significance. The maximum log-likelihood value has been obtained as 724.49. Since the sum of the estimated coefficients associated with $\hat{\varepsilon}_{t-1}^2$ and $\hat{\varepsilon}_{t-2}^2$ is 0.9958, it is clear that in this simple model where regime switching has not been considered, there is persistence in volatility. As discussed in Hamilton and Susmel, one way of modelling persistence is considering Markov switching regime model with conditional heteroscedasticity. And it is expected that an application of MSWARCH model would reduce persistence prevalent in the return series.

We have first considered the simplest MSWARCH model *viz.*, an MSWARCH model with 2 states and 2 lagged values in the ARCH specification with errors being standard Gaussian. There are no constraints imposed on any of the transition probabilities, p_{ij}, other than the usual conditions that $0 \le p_{ij} \le 1$ and $\sum_{j=1}^{2} p_{ij} = 1$, i = 1,2. For this model, the maximum log-likelihood value has been obtained as 1639.83. The modelling improvement brought about by consideration of Markov switching regimes is evident by the fact that the maximum log-likelihood value corresponding to the non-switching regime AR(1)-ARCH(2) model, as already reported, came out to be only 724.49. The estimated MSWARCH (2, 2) model with standard Gaussian errors for ν_t has been obtained as follows:

$$\hat{y}_t = \underset{(1.4)}{-0.0014} - \underset{(2.65)^{***}}{0.0424}\, y_{t-1}$$

$$\hat{h}_t = \underset{(1.9)^{*}}{0.0019} + \underset{(8.529)^{***}}{0.435}\, \tilde{\varepsilon}_{t-1}^{2} + \underset{(7.232)^{***}}{0.405}\, \tilde{\varepsilon}_{t-2}^{2}$$

$$g_1 = 1,\ \hat{g}_2 = \underset{(8.535)^{***}}{28.833} \tag{5.23}$$

$$\text{and} \quad \hat{\mathbf{P}} = \begin{bmatrix} \underset{(2.246)^{***}}{0.9300} & \underset{(1.353)}{0.2976} \\ \underset{(0.169)}{0.0701} & \underset{(3.194)^{***}}{0.7024} \end{bmatrix}.$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Looking at the values of g_1 (whose value is normalized to 1) and $\hat{g}_2$, we note that variance in the high volatility state is about 29 times as large as in the low volatility state. Though the transition probabilities are very high, state 1 is expected to last on an average for $(1-\hat{p}_{11})^{-1} = 14$ days while for state 2 the number of days is $(1-\hat{p}_{22})^{-1}$, i.e., on an average 3 days only. These figures indicate high switching across the two regimes. The two plots of smoothed probabilities for the two regimes, as indicated in the last two panels of Figure 5.2, also suggest that for both the states, at low volatility and high volatility, the switching over has taken place continuously. We may note that the persistence of volatility has reduced to (0.435+0.405) =0.84 from the previous value of 0.9958 for the ARCH model without any regime switching.

Next we report the findings of the MSWARCH (3,3) model where there are three states and n = 3. The estimated equations under the MSWARCH (3,3) model with Gaussian errors have been obtained as below:

$$\hat{y}_t = \underset{(0.96)}{-0.00096} - \underset{(0.545)}{0.00927}\, y_{t-1}$$

$$\hat{h}_t = \underset{(8.89)^{***}}{0.001778} + \underset{(8.566)^{***}}{0.424}\, \tilde{\varepsilon}^2_{t-1} + \underset{(5.844)^{***}}{0.271}\, \tilde{\varepsilon}^2_{t-2} + \underset{(3.506)^{***}}{0.121}\, \tilde{\varepsilon}^2_{t-3}$$

$$g_1 = 1,\ \hat{g}_2 = \underset{(3.603)^{***}}{2.0344},\ \hat{g}_3 = \underset{(5.095)^{***}}{42.049} \tag{5.24}$$

$$\hat{\mathbf{P}} = \begin{bmatrix} \underset{(0.727)}{0.987} & 0.0 & \underset{(0.094)}{0.056} \\ \underset{(0.0096)}{0.013} & \underset{(2.845)^{***}}{0.674} & \underset{(3.772)^{***}}{0.430} \\ 0.0 & \underset{(1.376)}{0.326} & \underset{(0.608)}{0.513} \end{bmatrix}.$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

The maximum log likelihood value of this model has been obtained as 1712.5740, which shows a clear improvement over the 2-state MSWARCH model. It may be noted that unlike the 2-state MSWARCH model, in this case the coefficient of y_{t-1} is insignificant, indicating that even the first lag is not significant in the single mean specification for the 3-state MSWARCH model. As before, to start with, we impose no constraints on any of the transition probabilities, p_{ij}, other than $0 \le p_{ij} \le 1$ and $\sum_{j=1}^{3} p_{ij} = 1$, i = 1,2,3. Several of these unrestricted MLEs were found to fall on the boundary of $p_{ij} = 0$, which is a violation of the regularity conditions. To calculate the standard errors, we then imposed zero restrictions on those $p_{ij}'s$ and treated those as known constants for the purpose of calculating the second derivatives of the log-likelihood function. For the MSWARCH (3, 3) model, we kept p_{12} and p_{31} as zero and then carried out the required computations.

It can be seen from the estimates that the variance of the medium volatility state is two times that of the first, while the variance of the high volatility state is 42 times the volatility of the first state. The same is also exhibited in the last two plots of Figure

5.3. It is evident from these plots that the switching over is very high amongst the medium volatility and high volatility states. However, the observations have come, most of the time, from the low volatility state. The value of $\hat{p}_{11}$ which is very close to one, indicates a very high persistence for state 1. From the estimates of p_{11}, p_{22} and p_{33}, we can note that state 1 is expected to last, on an average, for 77 days while for medium and high volatility states, these are only 3 and 2 days, respectively. The persistence of the variance has however decreased with the increase in the number of states. It is found to be only (0.424+0.271+0.121) = 0.816 as against the values of 0.84 for the 2-state MSWARCH model and 0.9958 for no regime-switching ARCH model.

Finally, we have obtained the out-of-sample forecasts for the MSWARCH (3,3) model, to evaluate the forecasting performance of this model. Following procedures similar to those done for the previous models, we have obtained the out-of-sample forecasts for 1-, 5-, 10- steps ahead. The values of the four standard forecast evaluation criteria have been presented in Table 5.1 . These values with those obtained for the other models reported in Chapter 2 through 4, are now being taken together to make a comparative forecasting performance among the four different time series models applied so far for the purpose of modelling the return series of India's foreign exchange rate.

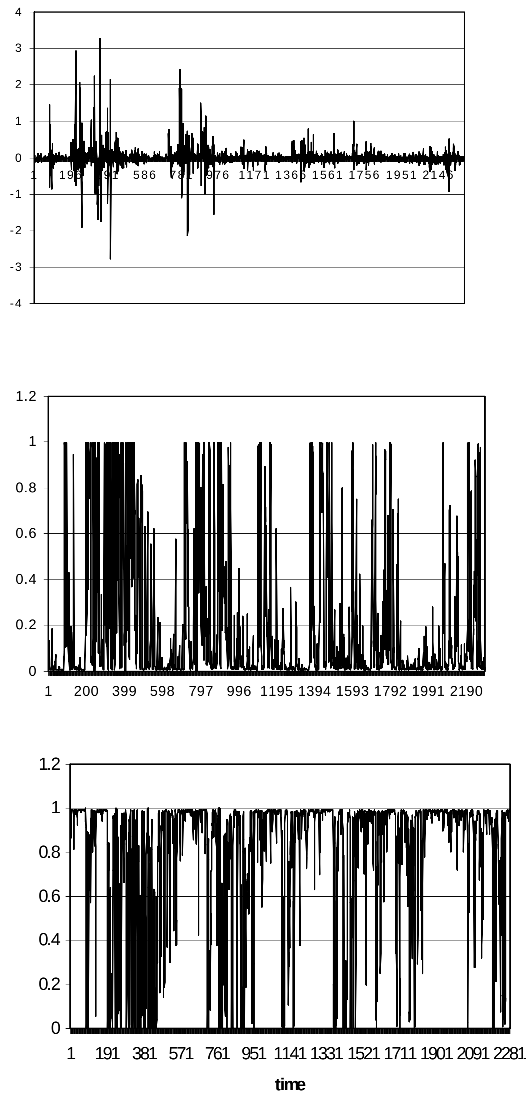


Figure 5.2 The top most panel gives the plot of exchange rate return against time. The next 2 panels are plots of $p(s_t = 1 \mid y_T, y_{T-1} \ldots .)$ and $p(s_t = 2 \mid y_T, y_{T-1} \ldots .)$, respectively for the MSWARCH(2,2) model

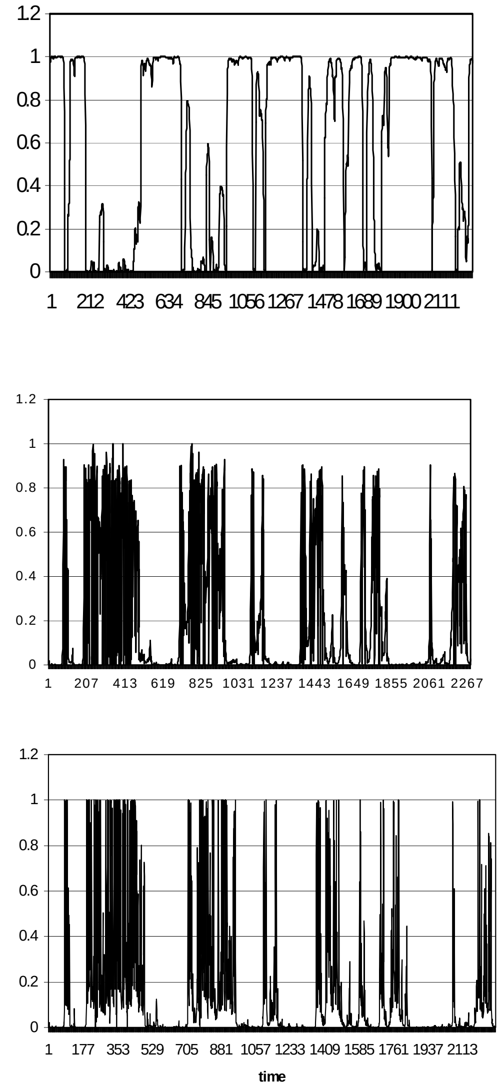


Figure 5.3 The 3 panels are plots of $p(s_t = 1 \mid y_T, y_{T-1} \ldots .)$, $p(s_t = 2 \mid y_T, y_{T-1} \ldots .)$ and $p(s_t = 3 \mid y_T, y_{T-1} \ldots .)$, respectively for the MSWARCH (3,3) model

Table 5.1

Out-of-sample forecasts of India's exchange rate return based on MSWARCH(3,3) model

Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	0.170142	0.263525	0.99978	47.0
5	0.174910	0.274189	1.00073	44.0
10	0.175433	0.276777	1.00116	42.0

5.4 Out-of-sample forecasting comparison of all the four models

We have so far considered, basically, four different time series models in Chapters 2 through 5 *viz.*, the linear dynamic model with volatility, and three nonlinear models *viz.*, the SETAR, STAR and MSR models. Of course, there are other combined models as well, which have also been studied. The time series under study i.e., daily return on foreign exchange rate of India, has been fitted to all these models and their out-of-sample forecasting performances have been evaluated by four standard criteria *viz.*, the MSE, MAE, AMAPE and PCSP. In this section, we make a comparative analysis among these models based on these four criteria. The purpose of such comparisons are to rank these models so that we are able to conclude which of these models performs the best and so on, in the forecasting sense, for India's exchange rate return. To that end, values of all the four criteria and for all the models are presented for returns which are now converted to percentage figures so that comparisons among all these models are meaningful.

Table 5.2

Out-of-Sample Forecasting Comparison of all the Models

Random walk model with drift				
Number of steps ahead	MSE	MAE	AMAPE	PCSP
1	0.1710	0.264	1.352	53.5
5	0.1747	0.274	1.119	56
10	0.1751	0.276	1.155	59
Chosen AR-EGARCH model				
1	0.1182	0.230	1.042	55.2
5	0.1263	0.244	1.105	55.2
10	0.1268	0.247	1.134	55.2
SETAR model with one threshold				
1	0.1785	0.275	1.618	50.5
5	0.1819	0.286	2.191	50.0
10	0.1825	0.288	2.131	52.0

Table 5.2

Out-of-Sample Forecasting Comparison of all the Models (Continued)

SETAR model with two thresholds				
1	0.1801	0.277	1.997	50.5
5	0.1822	0.287	2.208	51.0
10	0.1828	0.290	2.152	51.0
DTGARCH model with one threshold				
1	0.1699	0.274	1.535	53
5	0.1757	0.286	1.441	52
10	0.1758	0.289	1.459	49
DTGARCH model with two thresholds				
1	0.1757	0.270	1.471	49
5	0.1829	0.285	2.025	48
10	0.1818	0.288	1.868	47

Table 5.2

Out-of-Sample Forecasting Comparison of all the Models (Continued)

LSTAR2 model				
1	0.2418	0.350	_	45.0
5	0.2576	0.380	_	45.36
10	0.2985	0.414	_	44.44
MSWARCH(3,3) Model				
1	0.1701	0.263	0.99978	47.0
5	0.1749	0.274	1.00073	44.0
10	0.1754	0.277	1.00116	42.0

* *For the sake of comparison across all the models the values of all the four criteria for all the models are presented for the exchange rate data in percentages*

It is evident from Table 5.2 that the linear dynamic regression model with appropriate specification and having EGARCH volatility specification performs the best among all these time series models in terms of out-of-sample forecasting criteria. The MSE and MAE values for this model are the smallest among all the models considered for all the 1-, 5-, and 10- step ahead forecasts. In terms of the other two criteria as well, the main finding is almost the same. While such a finding seems a bit surprising, but the fact is that there are some evidences (Franses and van Dijk (2000)) to the effect that such linear models have performed better than the complex nonlinear models for exchange rate return series even for some developed economies. Since the linear dynamic model considered in Chapter 2 incorporates all the important characteristics of the data in due process of modelling, it is expected to perform quite well. But its supremacy over the nonlinear models which have, in general, been found to perform better than the linear dynamic model for exchange rate series of advanced economies, shows that the data generating process of India's exchange rate return has not yet become that complex or nonlinear in nature. Given the history of India's exchange rate regime since liberalization in 1993, such an empirical finding *viz*., the linear dynamic model with due consideration to volatility and the modelling aspect of appropriate specification is the best, is therefore, somewhat likely.

While the 'best' model for the daily-level India's exchange rate return series, in terms of out-of-sample forecasting performance, is the appropriately specified linear dynamic model with EGARCH volatility specification, the model that has performed the worst among all these models is the STAR model. As already discussed in Chapter 4, LSTAR2 was found to be the best in-sample performing model among the different STAR models considered in that chapter. Now, we find that in terms of out-of-sample forecasting performance, LSTAR2 model has the worst performance for India's exchange rate return series. In terms of both the MSE and MAE criteria, its values are the highest. This empirical finding is probably due to the fact that volatility has not been considered in the context of this class of models. The reason for not doing so is that there is hardly any such volatility combined STAR model available in the literature (in fact, to the best of our knowledge, there are very few papers, the most relevant being Lundbergh and Teräsvirta (1998)). Obviously, there is no such software available and our own efforts suggest that there are huge computational difficulties involved. Added to this is the fact that insofar as our data set is concerned, we have found that the smoothness parameter γ of the LSTAR 2 model, has a very high value of 39.237 , and at such a high value of γ , the LSTAR2 model tends to nest a three-regime SETAR model if the transition variable equals the value of return with some lag. And for the latter, volatility combined models have already been considered in Chapter 3.

Now, looking at the performance of the four models in the class of SETAR models *viz*., SETAR and DTGARCH models – each with one as well as two-thresholds- we find that the performance of the DTGARCH model with one-threshold is the best. For instance, its MSE values for the 1-, 5-, 10-step ahead forecasts are 0.1699, 0.1757 and 0.1758 and these are the lowest as compared to the corresponding values of the other three models. In terms of MAE criteria, DTGARCH two-threshold model has slight edge over the one-threshold model while in terms of AMAPE, the latter performs somewhat better. Combining all these, we can conclude that the performance of DTGARCH model with one threshold is the best in the SETAR class of models in terms of out-of-sample forecasting performance. It may be recalled that, as stated in Chapter 3, the in-sample modelling performance also showed that the middle regime of the two-threshold model is statistically insignificant and hence the DTGARCH one-threshold (two-regime) model is effectively the better of the two DTGARCH models for our time series. It may be stated that the orders (1,1) of the underlying GARCH process was found to perform the best in this class of DTGARCH (both two-regime and three-regime) models.

Finally, we compare between the DTGARCH one-threshold model with the MSWARCH (3,3) model of MSR class of models. Both are members of the class of regime switching models. In terms of MSE criterion, we find that leaving the 1-step ahead forecast, the performance of the MSWARCH model is better than the DTGARCH model. Insofar as the MAE criterion is concerned, we note that its values are 0.263, 0.274 and 0.277 corresponding to 1-, 5- and 10-step ahead forecasts for the MSWARCH (3,3) model whereas the same for the DTGARCH one-threshold model are 0.274, 0.286 and 0.289, respectively. In terms of AMAPE criterion also, MSWARCH (3,3) models performs better than its competitor.

Finally, we make out-of-sample forecasting comparison of all these models relative to a naïve random walk model with drift which has been considered as the benchmark model in the thesis. As can be noted from Table 5.2, the AR-EGARCH, DTGARCH model with one threshold and the MSWARCH(3,3,) model perform better than the random walk model with drift by MSE criterion with respect to 1-step ahead forecast. The MSE of 1-step ahead forecast of the random walk model is 0.1710 while that of the AR-EGARCH, DTGARCH with one threshold and MSWARCH(3,3) are 0.1182, 0.1699 and 0.1701, respectively. It is only in terms of the PCSP that the random walk model performs better than the rest of the models. The PCSP of 1-step ahead forecast for the AR-EGARCH model is, however, better than the naïve model. As discussed earlier, the AR-EGARCH model performs the best across all the models developed by us. The DTGARCH with one threshold and the MSWARCH(3,3) models are the two nonlinear models whose out-of-sample forecasts are comparable to the AR-EGARCH model. In terms of the forecasting criteria, we find that the random walk model is better than the DTGARCH model with one threshold by all the four criteria in all cases except for the MSE of 1-step ahead forecast. But for the MSWARCH(3,3) model, the 1-step ahead forecast aggregates are smaller than the random walk. For instance, the MSE, MAE and AMAPE values for the MSWARCH(3,3) model are 0.1701, 0.263 and 0.99978 while that for the random walk model are 0.1710, 0.264 and 1.352, respectively. However, for the rest of the step-ahead aggregates, *viz.*, the 5-step and the 10-step ahead forecasts, we find that the random walk model performs marginally better. Also, in terms of PCSP criterion the random walk model performs much better.

It is to be noted that for the PCSP criterion the normal cut-off is 50%. The models which have PCSP value greater than 50% are considered to have satisfactory performance. However, since the rupee per dollar is, in general, trending upward, in the sample period, it will be appropriate to compute the average percent correct forecasts of direction of change of pure linear trend and use this value instead of 50% to base our conclusions. This value has come out to be 53.5%. We observe that the performance of the first model of our thesis i.e., the AR-EGARCH, is the best since its PCSP is exceeding this value. However, the other models do not have satisfactory performances in terms of PCSP.

Clark and McCracken (2001) developed a test for comparing the out-of-sample forecast performance of two models, one labeled as the unrestricted and the other, the restricted model. This test is called the *MSE-F* test. The *MSE-F* statistic is used to test the null hypothesis that the unrestricted model forecast mean squared error (MSE) is equal to the restricted model forecast MSE against the one-sided (upper tail) alternative hypothesis that the unrestricted model forecast MSE is less than the restricted model forecast MSE. A significant *MSE-F* statistic indicates that the unrestricted model forecasts are statistically superior to those of the restricted model.

In order to make inferences, bootstrapped critical values are generated by performing repeated simulations under the null hypothesis. The bootstrapped statistics are derived by first estimating the model under the null hypothesis and then performing bootstrap simulations of the data by drawing randomly (with replacement) from the errors of the appropriate model. Data are obtained by iterating forward using these randomly chosen errors, and forecasts of returns are computed using the simulated data for both the models. The values of the test statistic are then computed and this process is repeated. In our case, we have repeated the process 1000 times.

For our comparison, the unrestricted model is the AR-EGARCH model which has been found to perform best, based on the values of MSE, MAE, AMAPE and PCSP, amongst all the other models considered by us, and the restricted model is the random walk model with drift.

The *MSE-F* statistic for the one-step ahead forecast has been computed as -30.877 with the AR-EGARCH model being the unrestricted model and the random walk with drift as the restricted model. The empirical distribution of the *MSE-F* statistic was obtained using the bootstrapping method. The *p*-value which is the proportion of the bootstrapped statistics that are greater than the statistic computed using the original sample, has been found to be 0.99, suggesting that the statistic is not significant at all and hence we infer that in terms of out-of-sample forecast performance, as measured by MSE, the chosen AR-EGARCH model does not beat the benchmark model of random walk with drift, by any statistically significant margin.

Thus we find that even though in terms of out-of-sample MSE value the AR-EGARCH model seems to be superior to the benchmark random walk model, this superiority is not statistically significant, as found by the *MSE-F* test.

Combining all these empirical findings we, therefore, finally conclude that in terms of out-of-sample forecasting performance as measured by the usual criteria of MSE, MAE, AMAPE and PCSP, the best model for the series of daily return on exchange rate of India, is the appropriately specified linear dynamic model with EGARCH volatility specification. As regards comparing among the nonlinear time series models, we can conclude that, the performance of DTGARCH with one-threshold and MSWARCH (3,3) models are about the same, with the latter having slight edge over the former.

5.5 Conclusions

In this chapter, we have applied the Markov switching regression model to study India’s exchange rate return series. These models are designed to capture discrete changes in the economic mechanism that generates the data. Two specific models have been considered in this study. The first MSR model studied in this chapter resembles the mixture distribution model of two normal variables with the only difference that the value of the variable at a certain time point depends on its values at some previous time points. However, we have found that such a model could not successfully explain India’s exchange rate return series. The other model is a more general model where ARCH process is introduced in the MSR framework along with an autoregressive process in the conditional mean specification. We have found that the performance of such a model, called the MSWARCH model, is satisfactory for our data set. We have considered both MSWARCH (2,2) as well as MSWARCH (3,3) models with AR (1) specification in the mean, and found the performance of the MSWARCH (3,3) model better in terms of maximum log likelihood value. Finally, we have obtained the out-of-sample forecasts for the MSWARCH (3,3) model and compared its performance with the other time series models considered so far in this thesis. This comparative performance in terms of out-of-sample forecasts involving all the models suggest that, insofar as India’s daily exchange rate return series is concerned, the linear dynamic regression model with EGARCH volatility specification performs the best among all the models considered from Chapters 2 through 5. The STAR model’s performance is the worst among all the models. Between the two DTGARCH models considered – two-regime and three-regime models the first one performs better than the latter. Further, the DTGARCH (1,1) with one threshold model and the MSWARCH model are very close to each other in terms of forecasting performance, although a closer look suggests that the MSWARCH (3,3) performs better than the DTGARCH (1,1) with one threshold model.

CHAPTER 6

Modelling Monthly Exchange Rate Return with Macroeconomic Variables: A Predictive Regression Approach

6.1 Introduction

In this thesis, we have so far considered several time series models - linear as well as nonlinear - and fitted the same to the time series of return on India's foreign exchange rate at daily-level frequency. The forecasting performances of these models have also been studied so as to be able to conclude which of these models fits the return series best. For this chapter as well as the next chapter, we have shifted our focus from the daily level frequency to the monthly level frequency. Now, the exchange rate data used for these models were at daily level frequency and consequently, as mentioned in Chapter 2, hardly any other relevant macroeconomic variables could be used to determine and predict India's exchange rate simply because the data for these variables are not available at daily-level but at best at monthly-level frequency. In this chapter, we would be concerned with the same study as in Chapter 2, but the modelling and predictability aspects of India's exchange rate variable would be studied in association with other relevant macroeconomic variables - with all time series being at monthly-level frequency.

The framework of analysis remains the same i.e., the linear dynamic model in single equation set-up, but it now involves several variables and hence the issue of structural versus non–structural model becomes relevant. The other important issue for such a study is the role of the macroeconomic variables in the determination and prediction of India's exchange rate. As discussed in Chapter 1, after the publication of the two seminal papers by Meese and Rogoff (1983a,b), where they observed that a simple random walk model forecasts better than the more complex structural models, several alternative models have been developed. There have been evidences as well that if structural models are generalized to include lagged adjusted mechanisms (see, for instance, Somanath (1986) and Edison (1991)), or in case their parameters are allowed to vary over time as in Schinasi and Swamy (1989) and De Arcangelis (1992), their forecasts can be somewhat improved. Further, Hogan (1986), Chinn and Meese (1995) and Kim and Mo (1995) have shown that while time series models may be superior in short-run, structural models may perform quite well over long-run. Others have stressed the relevance of economic fundamentals such as money supply and real income in determining exchange rate behaviour, and reaffirmed the superiority of structural models over the random walk model - at least for medium and long-run horizons. Another recent work in support of empirical evidence that structural models of exchange rate are valid is due to Engel *et al.* (2007) which emphasizes the point that beating a random walk in forecasting may be too strong a criterion for accepting an exchange rate model. They also proposed alternative ways to evaluate models. Further, they emphasized the importance of monetary policy rule and its effect on expectations in determining exchange rates.

In an earlier paper, Engel and West (2005) tried to explain the reason why fundamentals provide little help in predicting changes in floating exchange rates. They showed analytically that in a rational expectations present-value model, an asset price manifests near random walk behavior if fundamentals are I (1) and the factor for discounting future fundamentals is near one. They however, concluded that the exchange rate and fundamentals are linked in a way that is broadly consistent with the asset-pricing model.

In this chapter, we are interested in empirical determination and forecastability of India's monthly exchange rate return using various macroeconomic variables. Now, one of the most important issues in such a study is the identification of the macroeconomic variables (henceforth to be referred to as macro variables) which are likely to be relevant in predicting exchange rate return. More so because all such studies which have been carried out mostly for the developed economies, have not found, as expectedly, the same set of macro variables to be relevant. To that end, the mixed results in the extant literature make it difficult, on the whole, to determine which particular macro variables are reliable indicators of exchange rate return.

To deal with this problem, we have considered, in this chapter, both in-sample and out-of-sample tests of return predictability. While the in-sample analysis employs what is known in statistical / econometric literature as predictive regression framework, the out-of-sample forecasts are analyzed using a pair of recently-developed-and potentially more powerful tests due to Clark and McCracken (2001) and McCracken (2004). The test statistics of these two tests are due to Diebold and Mariano (1995) and West (1996) and Harvey *et al.* (1998) (see also Rapach *et al.* (2005), for some relevant details). Another aspect to such a study is data mining. Since our interest is in testing the predictive ability of a large number of macro variables in turn, it is only natural that the issue of data mining would arise. The conventional wisdom holds that out-of-sample tests help guard against data mining. However, it has been recently argued that both the in-sample and out-of-sample tests are equally susceptible to data mining and the only way we can account for this data mining problem is by using an appropriate bootstrap procedure. We have followed the bootstrap procedure used by Rapach *et al.* (2005) and Rapach and Wohar (2006a), which are originally due to Nelson and Kim (1993), Mark (1995), Kothari and Shanken (1997) and Kilian (1999), to find the macro variables which significantly explain India's exchange rate return series.

As regards the set of macro variables to start with, we consider what may be taken to be a set of 'standard' relevant macro variables. Based on the extant empirical literature, the set comprises Bombay stock exchange sensitivity index (BSESENSEX), call money rate (CMR), M0 (this variable is a component of the stock of money, basically defined as the reserve money), M1 defined as the *narrow money,* M3 (*Broad money),* consumer price index (CPI), wholesale price index (WPI), foreign currency asset (FCA), total reserve of foreign exchange (TR), industrial production (IP), domestic petrol price (DPP), export (EX), import (IM), trade balance (TB), gross fiscal deficit (GFD), sale/purchase of US dollar (SPUSD), open market operations (OMO), Federal funds rate (FFR), six-month treasury bill rate of US (TBRU6), three-month treasury bill rate of US (TBRU3), NASDAQ, world gold price (WGP), foreign direct investment (FDI), foreign institutional investment (FII), total foreign investment (FINV).

While the effects of a rise in domestic and foreign money supply, interest rate and inflation rate, industrial production (a proxy of output) and trade balances on exchange rate are pretty straightforward due to the various theories which have developed over time, the effects of the other variables might not be easy to explain, especially because the number of studies in the latter category is very limited even in developed economies. For example, the effect of an important macro variable, the budget deficit on exchange rate, has been studied by Nyahoho (2006) where he has shown that there is no relationship between the two using statistical and empirical analyses based on data from the OECD countries. He carried out a regression of first difference of exchange rate and budget deficit in order to reach to this conclusion.

Recently some studies have tried to deal with the relation between exchange rate and stock prices. According to the monetarist models of exchange rate determination, equities, being part of wealth, may affect the behaviour of exchange rates through demand for money (see Gavin (1989)). Similar links can be traced through portfolio balance models as well (Branson (1983) and Frankel (1983)). However, a recent study by Ki-Ho-Kim (2003) states that the domestic stock prices have a positive effect on exchange rate since higher stock prices indicate better performance of the economy and this attracts foreign funds which lead to appreciation of the domestic exchange rate. Some other studies in this direction are due to Aggarwal (1981), Soenen and Hennigar (1988), Ma and Koa (1990), Roll (1992) Abdalla and Murinde (1997), Chow *et al.* (1997), Ajayi *et al.* (1998), Nieh and Lee (2001), Phylaktis and Ravazzolo (2005) and Pan *et al.* (2007). These studies have found different results concerning the two markets. While Aggarwal (1981) showed that the revaluation of the US dollar is positively related to stock market returns, Soenen and Hennigar (1988) found a significant negative relation. Roll (1992) found a positive relationship between the two markets while Chow *et al*. (1997) found no relationship between the two. Insofar India is concerned, the relationship between stock index and exchange rates has been studied by Mishra (2004) and Damele *et al.* (2004).

The role of foreign direct investment growth of an economy has also been studied in great detail by Alfaro *et al.* (2004). This is an important variable for study on foreign exchange rate as this macro variable is often assumed to influence the return on foreign exchange rate. An increase in foreign investment or its components should obviously lead to an appreciation of domestic currency due to inflow of foreign funds.

As regards the relationship between interest rate market and foreign exchange market, it is known that these are closely linked as there exists arbitrage opportunities between the two markets. However, monetarists assert that an increase in domestic interest rate (essentially increasing the interest rate differential) will decrease the real demand for money, and given a fixed nominal money supply, this will be achieved by a rise in domestic price level and hence a depreciation of exchange rate. Hence, this effect is opposite to the standard Keynsian model with incorporated capital mobility, as described by Mundell (1968) and Fleming (1962) where a rise in interest rate leads to an appreciation of the domestic currency. This latter result is often viewed as a short-run result where the prices are considered to be sticky. There have been some works which have tried to study the relationship between exchange rate and interest rate, but these have been mostly in terms of testing uncovered interest parity.

Ramachandran (2006) has made a comprehensive study of foreign exchange reserves of India. He has found that the asymmetric control over capital inflows and asymmetric intervention in favour of strengthening export competitiveness in an era of persistent capital inflows seem to be responsible for the stockpile of reserves in India. The study by Kasman and Ayhan (2007) is another recent one where the long run relationship between exchange rate and reserves has been studied.

Tarhan (1995) has empirically investigated the effect of Federal Reserve open market operations (OMO) on both short-term and long-term interest rates along with the influence of OMO on the stock markets and exchange rate markets. Another probable variable that might affect exchange rate is the Federal funds rate or short term interest rate of the US. The effects of US interest rate shocks on the economies of other developed countries have been studied by Kim and Roubini (2000), and the general observation is that a rise in Federal funds rate is accompanied with devaluation of other world currencies. However, there are no such studies on the relationship between Federal funds rate and Indian exchange rate. Keeping this is mind, we have included Federal funds rate as an independent macro variable in determining the model for India's monthly exchange rate.

It is well known that quite often the central banks of the countries have to intervene in the exchange rate market to influence its movement towards some desired direction. In case of India, the most important instrument of the Reserve Bank of India (RBI) which is the central bank of India, is to directly intervene in this market by means of sale/purchase of US dollars. A purchase of US dollars is done to depreciate the domestic currency while it is sold when a depreciation of the domestic currency is to be countered. Another proxy of central bank intervention often used in studies is the change in foreign exchange reserves. Such macro variables are likely to affect modelling of exchange rate and its predictability. Further, sometimes the government might choose to sterilize the intervention made by them in the foreign exchange market. This may be done using the open market operations. Hence, this variable may also play some role in predictability of exchange rate return. Some important works on intervention are due to Bonser-Neal (1996), Baillie and Osterberg (1997), Chang and Taylor (1998), Dominguez (1998) and Nagayasu (2004). The effectiveness of Japanese intervention policies on the Japanese yen / US dollar exchange rate, has been analyzed by Nagayasu (2004), and he has suggested that Japanese intervention were effective in influencing exchange rate movements when such operations were coordinated with the Federal Reserve. Most of these studies using data of Japan generally concluded that Japanese intervention were ineffective and did not influence exchange rate changes (Baillie and Osterberg (1997)). Rather, they increased uncertainty and volatility of exchange rates (Bonser-Neal (1996), Chang and Taylor (1998) and Dominguez (1998)). Kim and Sheen (2006) has carried out a recent study which tests the effectiveness of Bank of Japan's foreign exchange intervention on the conditional first and second moments of exchange rate return and traded volumes using a bivariate EGARCH model of the Japanese yen / US dollar market. For a comprehensive survey of theoretical and empirical literature on foreign exchange rate intervention, see Edison (1993), Almekinders (1995), Sarno and Taylor (2001) and Frankel *et al.* (2004).

In India, there have been some empirical studies on the effect of intervention on foreign exchange rate. Bhaumik and Mukhopadhyay (2000) have considered a specification to link central bank's direct interventions in the foreign exchange market with changes in the country's exchange rate using the Mundell-Fleming model. Ghosh (2002) has used a Tobit and logit model for studying the role of intervention on exchange rate using daily data. Baig *et al.* (2003) have formulated and estimated a small open economy where a measure of exchange market pressure and an index of intervention activity have been constructed. The analysis of these two parameters highlights the fact that the RBI prefers to accommodate rupee[1] depreciation, while aggressively preventing appreciation. The net sale of foreign exchange is only resorted to in times of crises. The large amount of foreign exchange reserves that the RBI has built up bears ample testimony to its intervention in the foreign exchange market.

Other than intervention, the RBI also acts as the banker of last resort where it injects funds into the system to help participants tide over temporary mismatches of funds. This was implemented through the Liquidity Adjustment Facility (LAF) which was made effective on the 5th of June 2000. The system is being implemented in phases and currently is a daily exercise in which banks and primary dealers (PD) participate. Here the RBI conducts an auction system of repos (the rates at which RBI borrows from the banks) and reverse repos to suck-out and inject liquidity to the market. The exact quantum of liquidity to be absorbed or injected and the accompanying repo and reverse repo rates are determined by the Financial Markets Committee after taking into consideration the liquidity conditions in the market, the interest rate situation and the stance of monetary policy. Thus, the values of repos and reverse repos can help in explaining exchange rate. However, we could not use this variable in our analysis since this time series is available only from 2000 while our study uses all the data sets starting from 1994.

In addition to analyzing the predictive ability of each macro variable in turn, we also apply a procedure that combines general-to-specific model selection with out-of-sample tests of forecasting ability. The findings of these two procedures are combined for the purpose of identifying the set of appropriate macro variables for predicting the foreign exchange rate for India and a model using these variables is constructed.

Once the macro variables have been identified, we check if the conditional mean thus assumed is correctly specified. As already mentioned in Chapter 2, this is so because it is now well-known that inferences based on models suffering from misspecification could be misleading and incorrect. For linear dynamic models,

[1] Rupee is the name of India’s currency.

notable cases of such misspecifications include failing to take account for parameter instability, residual autocorrelations, misspecification of functional forms and omitted variables. It is worthwhile to note that an incorrectly specified conditional mean might as well lead to misspecification of conditional variance, provided, of course, volatility is found to be significant in the monthly exchange rate data.

Thus, the focus in the latter part of the chapter - after including adequate lags to take care of autocorrelation in the return series – is on the aspect of specification, and to that end, we carry out appropriate tests for detecting parameter stability as well as functional form misspecification and omission of other relevant variables which might not have been included in the mean function by both the specific-to-general and general-to-specific approaches for selection of macro variables, and then take appropriate steps to guard against misspecification in the mean function in case the test rejects the null hypothesis of no misspecification of conditional mean. Thereafter, standard residual-based diagnostic tests including the BDS test (Brock *et al.* (1996)) are performed to detect the presence of second as well as other higher order dependences in the errors of the chosen model.

The chapter is organized as follows. The methodology applied in this study is briefly described in the next section. Section 6.3 presents a brief description of the data sets used in our analysis. Empirical findings are discussed in Section 6.4. The paper ends with some remarks in Section 6.5.

6.2 Methodology and the final model

A number of econometric tools have been used in this study to determine the relevant macro variables which have predictive ability for exchange rate return, and also to test for misspecification of the final model thus obtained. While the details concerning the latter has been discussed in Chapter 2, in the Section 2.2, we first discuss the details regarding the former. To that end, we first describe the predictive regression approach and the tests of predictability based on out-of-sample forecasting performance of the predictive regressions.

6.2.1 Predictive regression and out-of-sample tests of predictability

As stated in the preceding section, the selection of the macro variables is done by analyzing the predictive ability of each macro variable in turn, using predictive regression and then combining these findings with those obtained by the general-to-specific model selection procedure with out-of-sample tests of forecasting ability. In predictive regression, the predictive ability of a stationary variable is studied with a regression model having one regressor at a time. This model takes the form,

$$y_{t+1}^{k} = \alpha + \beta z_t + \gamma y_t + u_{t+1}^{k} \qquad (6.1)$$

where y_t is the return on exchange rate from period t-1 to period t, $y_{t+1}^{k} = y_{t+1} + ... + y_{t+k}$ is the return from period t to $t + k$, k is the forecast horizon, z_t is a stationary macro variable believed to potentially predict future returns on exchange rate, and u_{t+1}^{k} is the disturbance term. It maybe noted that a lagged return term has been included in (6.1) as a control variable since it is often found that the first lag is significant and quite adequate to describe the autocorrelations in foreign exchange return. The return on foreign exchange rate can be perceived as return that agents get from holding foreign currency. Under the null hypothesis $\beta = 0$, this variable does not have any predictive power for future returns while under the alternative hypothesis $\beta \neq 0$, z_t has predictive power for future returns. We have T observations on y_t and z_t of which T-k observations are usable and these are used to estimate the in-sample predictive regression model as well as for out-of-sample forecasting.

The predictive ability of z_t in the predictive regression framework is assessed by means of the t-statistic corresponding to $\hat{\beta}$, the ordinary least squares (OLS) estimate of β, as well as the goodness-of-fit measure R^2. The problems associated with estimating a predictive regression model like (6.1) are small sample bias and overlapping observations. The latter problem is often dealt with by using the standard errors proposed by Newey and West (1987), as these are robust to heteroskedasticity and serial correlation in the disturbance term. In spite of using robust standard errors to compute *t*-statistics, there can be serious size distortions when basing inferences on standard asymptotic distribution theory. To guard against size distortions, we base inferences on the concerning β in (6.1) on bootstrap procedures similar to Nelson and Kim (1993), Mark (1995), Kothari and Shanken (1997) and Kilian (1999).

As regards out-of-sample tests of predictability, we first need to have the out-of-sample forecasts, and these are obtained based on recursive scheme where the total sample of observations is divided into in-sample (say, the first *R* observations for y_t and z_t) and out-sample portions (the remaining ones). The first out-of-sample forecast for the unrestricted model (i.e., where $\beta \neq 0$) is generated in the following way. The unrestricted predictive regression model is first estimated by the OLS method using data available through *R*. Let these estimates be denoted as $\hat{\alpha}_{1,R}$, $\hat{\beta}_{1,R}$ and $\hat{\gamma}_{1,R}$. Using these estimates, forecast is generated for the next i.e., (*R*+1)*th* observation and hence the forecast error, denoted as $\hat{u}^k_{1,R+1}$. Similarly, the initial forecast for the restricted model (i.e., where $\beta = 0$) is generated and denoted as $\hat{u}^k_{0,R+1}$. A second set of forecasts is generated by updating the above procedure one period by using data available through period *R*+1 and using the estimates obtained from the restricted and unrestricted predictive regression models. The forecast errors thus obtained are $\hat{u}^k_{1,R+2}$ for the unrestricted model and $\hat{u}^k_{0,R+2}$ for the restricted model. This process is repeated through the available sample, and thus are obtained two sets of *T*-*R*-*k*+1 recursive forecast errors- one each for the unrestricted and restricted regression models ($\{\hat{u}_{1,t+1}^{k}\}_{t=R}^{T-k}$ and $\{\hat{u}_{0,t+1}^{k}\}_{t=R}^{T-k}$).

Now, in order to be able to infer on the predictive ability of z_t, we need to compare between the out-of-sample forecasts from the unrestricted and restricted predictive regression models. If the unrestricted model forecasts are superior to the restricted model forecasts, then the variable z_t improves the out-of-sample forecasts of y_{t+1}^{k} relative to the first order autoregressive (AR) benchmark model where z_t is excluded. To this end, Theil's U , the ratio of the unrestricted model forecast root-mean-squared error (RMSE) to the restricted model forecast RMSE is used as a descriptive measure; $U < 1$ implies that the unrestricted model forecast RMSE is less than the restricted model forecast RMSE and hence performance of unrestricted model in terms of forecasting is better. A more formal test to find out whether the unrestricted regression model forecasts are significantly superior to the restricted regression model forecasts involves using the McCracken (2004) *MSE-F* and Clark and McCracken (2001) *ENC-NEW* test statistics. Of the two, the first test statistic is a variant of the test statistics proposed by Diebold and Mariano (1995) and West (1996) to test for equal predictive ability, and the second is a variant of Harvey *et al.* (1998) test statistic for testing forecast encompassing.

MSE-F statistic: The *MSE-F* statistic is used to test the null hypothesis that the unrestricted model forecast mean squared error (MSE) is equal to the restricted model forecast MSE against the one-sided (upper-tail) alternative hypothesis that the unrestricted model forecast MSE is less than the restricted model forecast MSE. The *MSE-F* statistic is based on the loss differential, $\hat{d}_{t+1}^{k} = (\hat{u}_{0,t+1}^{k})^2 - (\hat{u}_{1,t+1}^{k})^2$. Letting

$\overline{d} = (T - R - k + 1)^{-1} \sum_{t=R}^{T-k} \hat{d}_{t+1}^{k} = M\hat{S}E_0 - M\hat{S}E_1$

where $M\hat{S}E_i = (T - R - k + 1)^{-1} \sum_{t=R}^{T-k} (\hat{u}_{i,t+1}^k)^2$, $i = 0,1$, the McCracken (2004) *MSE-F* statistic is given by

$$MSE - F = (T - R - k + 1)\,\bar{d} \,/\, M\hat{S}E_1 . \tag{6.2}$$

A significant *MSE-F* statistic indicates that the unrestricted model forecasts are statistically superior to those of the restricted model. McCracken (2004) has shown that when comparing forecasts from nested models and for $k = 1$, the *MSE-F* statistic has a non-standard limiting distribution. Further, Clark and McCracken (2004) have demonstrated that the *MSE-F* statistic has a non-standard and non-pivotal limiting distribution in the case of nested models and for $k > 1$, and accordingly they have recommended basing inference on bootstrap procedure along the lines of Kilian (1999).

ENC-NEW statistic: The other out-of-sample statistic, *ENC-NEW*, relates to the concept of forecast encompassing. The *ENC-NEW* statistic due to Clark and McCracken (2001) takes the form,

$$ENC - NEW = (T - R - k + 1)\,\bar{c} \,/\, M\hat{S}E_1 \tag{6.3}$$

where $\bar{c} = (T - R - k + 1)^{-1} \sum_{t=R}^{T-k} \hat{c}_{t+1}^k$ and $\hat{c}_{t+1}^k = \hat{u}_{0,t+1}^k (\hat{u}_{0,t+1}^k - \hat{u}_{1,t+1}^k)$.

Under the null hypothesis, the weight attached to the unrestricted model forecast in the optimal composite forecast is zero and the restricted model forecasts encompass the unrestricted model forecasts. Under the one-sided (upper-tail) alternative hypothesis, the weight attached to the unrestricted model forecast in the optimal composite forecast is greater than zero, so that the restricted model forecasts do not encompass the unrestricted model forecasts. Similar to the *MSE-F* statistic, the limiting distribution of the *ENC-NEW* statistic is non-standard and pivotal for $k = 1$ and is non-standard and non-pivotal for $k > 1$ (Clark and McCracken (2004)) when comparing forecasts from nested models. As suggested by Clark and McCracken (2004), here again we base our inferences on a bootstrap procedure.

The bootstrap procedure: Following Rapach *et al.* (2005), we now describe the bootstrap procedure which is similar to those by Nelson and Kim (1993), Mark (1995), Kothari and Shanken (1997) and Kilian (1999). We postulate that the data are generated by the following system under the null hypothesis of no predictability:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_{1,t} \tag{6.4}$$

$$z_t = b_0 + b_1 z_{t-1} + \ldots + b_q z_{t-q} + \varepsilon_{2,t} \tag{6.5}$$

where the disturbance vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ is independently and identically distributed with covariance matrix Σ. First, (6.4) and (6.5) are estimated by the OLS procedure with lag order q in (6.5) selected using the Akaike's information criterion (AIC), and the OLS residuals $\{\hat{\varepsilon}_t = (\hat{\varepsilon}_{1,t}, \hat{\varepsilon}_{2,t})'\}_{t=1}^{T-q}$ are computed. In order to generate a series of disturbances for our pseudo-sample, we randomly draw (with replacement) T+100 times from the OLS residuals $\{\hat{\varepsilon}_t\}_{t=1}^{T-q}$, giving us a pseudo-series of disturbance terms $\{\hat{\varepsilon}_t *\}_{t=1}^{T+100}$. Drawings of the OLS residuals are made in tandem and the contemporaneous correlation between the disturbances of the original sample is maintained. Using the OLS estimates of the parameters in equations (6.4) and (6.5) and $\{\hat{\varepsilon}_t *\}_{t=1}^{T+100}$ and setting the initial observations of y_{t-1} and $z_{t-1}, z_{t-2}, \ldots, z_{t-q}$ equal to zero in equations (6.4) and (6.5), we can build up a pseudo-sample of T+100 observations for y_t and z_t, $\{y_t *, z_t *\}_{t=1}^{T+100}$. The first 100 transient start-up observations are dropped in order to randomize the initial observations. For this pseudo-sample, we calculate the *t*-statistic corresponding to β in the in-sample predictive regression model given in (6.1) and the two out-of-sample statistics given in (6.2) and (6.3). This process is repeated 1000 times, giving us empirical distribution for the in-sample *t*-statistic and the out-of-sample statistics. For each statistic, the *p*-value is the proportion of the bootstrapped statistics that are greater than the statistic computed using the original sample. As both the out-of-sample tests are one sided (upper-tail), an out-of-sample statistic is significant at, say, 10% level, if the *p*-value is less than or equal to 0.10 while for the in-sample *t*-test which is two-sided, the statistic is significant at 10 % if the *p*-value is less than or equal to 0.05 or greater than or equal to 0.95.

6.2.2 Data mining

It is now well-recognized that data-mining becomes a concern while testing the predictive ability of multiple variables. Lo and MacKinlay (1990) and Foster *et al.* (1997) have pointed out this with respect to in-sample tests of predictability. Data mining is considered to be a serious problem for in-sample tests of predictability, and the conventional wisdom holds that out-of-sample tests are better able to guard against data mining. In our study, we have used the same data-mining environment as considered by Inoue and Kilian (2004). Suppose there are *M* different macro variables $z_{j,t}, j = 1,....M$, in turn as candidate predictors in the predictive regression model (6.1). Inoue and Kilian (2004) have specified the null hypothesis as $H_0 : \beta_j = 0 \ \forall j$ and the alternative hypothesis as $H_1 : \beta_j \neq 0$ for some *j*, where β_j is the coefficient corresponding to $z_{j,t}$ in (6.1). For an in-sample test statistic, we use $\max_{j \in \{1,...M\}} | t_{\hat{\beta}_j} |$ where $t_{\hat{\beta}_j}$ is the *t*-statistic corresponding to β_j. For the out-of-sample test statistic, we use the maximal *MSE-F* and maximal *MSE-NEW* statistics. Inoue and Kilian (2004) have derived the asymptotic distribution for the maximal in-sample and out-of-sample statistics under the null hypothesis of no predictability as well as under the local alternatives in this data mining environment. Since the limiting distributions are generally data dependent, Inoue and Kilian (2004) have recommended bootstrap procedures.

The bootstrap procedure discussed earlier is modified a little to take account for data mining problem. For *M* different macro variables $z_{j,t}, j = 1,....,M$, serving as candidate predictors for the candidate predictive regression model (6.1), equation (6.5) is augmented as follows to consider all the M candidate predictors

$$\begin{aligned} z_{1,t} &= b_{1,0} + b_{1,1} z_{1,t-1} + ... + b_{1,q_1} z_{1,t-q_1} + \varepsilon_{1,2,t} \\ &. \\ &. \\ z_{M,t} &= b_{M,0} + b_{M,1} z_{M,t-1} + ... + b_{M,q_M} z_{M,t-q_M} + \varepsilon_{M,2,t} \end{aligned} \tag{6.6}$$

where the disturbance vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{1,2,t}, ..., \varepsilon_{M,2,t})'$ is independently and identically distributed with covariance matrix Σ . Using the system defined by (6.4) and (6.6), we proceed in a way which is similar to the bootstrap procedure described earlier to generate 1000 pseudo-samples of observations for y_t and $z_{1,t}, z_{2,t}, ..., z_{M,t}$ under the null hypothesis of no predictability, with each pseudo-sample matching the original sample-size. For each pseudo-sample, we calculate the *t*-statistic corresponding to β_j in the in-sample predictive regression model and the two out-of-sample statistics for each of the $z_{j,t}*$ variables (*j*=1, …, *M*) in turn. We then compute and store the largest and the smallest *t*-statistics as well as the maximal *MSE-F* and *ENC-NEW* statistics. After ordering the empirical distribution for each maximal out-of-sample statistics, the 900th , 950th and 970th values serve as the 10%, 5% and 1% critical values for each maximal out-of-sample statistics, respectively. For the in-sample *t*-statistic, the 950th, 975th and 995th values of the empirical distribution for the largest *t*-statistic serve as the 10%, 5% and 1% upper tail critical values, respectively for $\max_{j \in \{1,...,M\}} \left| t_{\hat{\beta}_j} \right|$ statistic.

6.2.3 General-to-specific approach

Along with analyzing each macro variable in turn, we have also employed the general-to-specific approach of model selection, as used by Clark (2004), to identify the relevant predictor macro variables. In this, we again use the predictive regression model defined in (6.1) but including all the variables,

$$y_{t+1}^{k} = \alpha + \beta_1 z_{1,t} + ... + \beta_M z_{M,t} + \gamma y_t + u_{t+1}^{k} \quad . \tag{6.7}$$

This model is estimated using data from the in-sample portion of the total sample. Each of the *t*-statistics corresponding to the $z_{j,t}$, *t*=1,…,*M*, variables in (6.7) are examined and if the smallest *t*-statistic (in absolute value) is greater than or equal to 1.645, we select the model that includes all *M* of the $z_{j,t}$ variables. If the smallest *t*-statistic is less than 1.645, we exclude the $z_{j,t}$ variable corresponding to the smallest *t*-statistic in the next model we consider. We proceed in this way and include only those values of $z_{j,t}$ variables which have significant *t*-statistics. If this exercise based on data from the in-sample period includes at least one of the $z_{j,t}$ variables, we then compare the out-of-sample return forecasts generated by the selected model to the out-of-sample forecasts generated by the benchmark model. We again form out-of-sample forecasts recursively and compare out-of-sample forecasts from the competing models using the *MSE-F* and *ENC-NEW* statistics. We generate *p*-values for the out-of-sample statistics by slightly modifying the bootstrap procedure described earlier. Here we generate a pseudo-sample of data for y_t and all of the $z_{j,t}$ variables under the null hypothesis that none of the $z_{j,t}$ variables is useful in predicting return. Using the pseudo-sample, we use the general-to-specific model selection procedure over the in-sample period in order to select the 'best' forecasting model, and if the selected model includes any of the $z_{j,t}$ variables, we calculate the out-of-sample *MSE-F* and *ENC-NEW* statistics. We repeat this process until we have empirical distributions of 1000 bootstrap statistics for both the out-of-sample statistics. For each out-of-sample statistic, the *p*-value is the proportion of bootstrapped statistics that are greater than the statistic computed using the original sample.

To sum up this methodology, what we do first is to use the above two methods *viz.*, the one based on each macro variable in turn- called the specific-to-general and the general-to-specific method, then determine the set of variables which have significant roles in the predictability of exchange rate, and finally check for the data mining problem to decide on the variables which appear to be important in modelling exchange rate return.

6.2.4 The final model

Now, it is not just enough from the point of view of modelling that we have been able to choose a set of relevant macro variables which have significant predictive ability for exchange rate return, and hence we need to check whether the macro variables thus obtained are adequate from the point of view of appropriate specification of the underlying relationship involving exchange rate return and the chosen macro variables. To that end, we need to account for serial correlation by considering appropriate lags of exchange rate return and also for any seasonal behavior in the series by including appropriate dummy variables. Taking all these into consideration, we finally propose the following specification, in the framework of a single-equation linear dynamic model, for the return on India's monthly exchange rate series:

$$y_t = \sum_{k=1}^{p} \phi_k y_{t-k} + \sum_{j=1}^{d} \xi_j D_{j,t} + \sum_{j=1}^{\widetilde{M}} \sum_{k=0}^{l} \beta_{jk} z_{j,t-k} + \varepsilon_t, \quad t = 1,2,......,T \tag{6.8}$$

where y_t is the difference of log of exchange rate, D_j's $(j = 1,2,...,d)$ denote the seasonal 0-1 dummies, p is the appropriate lag value of y_t capturing its autocorrelations and $z_{j,t-k} (j = 1,...,\widetilde{M}; k = 0,...,l)$ are the $\widetilde{M}, \widetilde{M} \le M$, independent macro variables having the current value as well as lags upto *l*, which have been identified to play significant roles in the prediction of exchange rate. We can write the equation compactly, in matrix notation, as

$$y_t = x_t'\gamma + \varepsilon_t \tag{6.9}$$

where $x_t' = (y_{t-1},..., y_{t-p}, D_{1t},..., D_{dt}, z_{1,t},..., z_{1,t-l},..., z_{\widetilde{M},t,},..., z_{\widetilde{M},t-l})$ and

$\gamma' = (\phi_1,...,\phi_p, \xi_1,...,\xi_d, \beta_{10},...,\beta_{1l},...,\beta_{\widetilde{M}0},...,\beta_{\widetilde{M}l})$.

Once the model has thus been specified, we carry out test for parameter instability or structural break, as it is often called, in the conditional mean function. This is done by following the approach described in Section 2.2.1 of Chapter 2. If the findings of this test suggest presence of one or more structural breaks, the sample is then split at the break date estimate(s) (*cf.* Bai (1994,1997a), and further analysis continues on the subsamples, provided the number of observations in each subsample is adequate; otherwise, dummy variables representing breaks are included in (6.9) and the analysis continues with this model.

To ensure that the conditional mean is appropriately specified, we next test, based on recursive residuals, for any remaining misspecification in the conditional mean. It is noteworthy that apart from omission of variables, any remaining misspecification of the conditional mean may be because of nonlinear dependence and this nonlinearity may be approximated by functions of the recursive residuals. As demonstrated by Kianifard and Swallow (1996), Lumsdaine and Ng (1999) and others, the use of recursive residuals, rather than the standard least squares residuals, increases the power of the tests for model misspecification. The test of misspecification applied here refers to in Lumsdaine and Ng (1999). This test (described in detail in Section 2.2.2) envisages augmenting the specification in (6.9) as $y_t = x_t{}'\gamma + g(\hat{w}_{t-1)} + v_t$, where $g(\hat{w}_{t-1})$ is a (possibly nonlinear) function of the recursive residuals $\hat{w}_{t-1}$. The role of $g(\hat{w}_{t-1})$ is to orthogonalize ε_t in (6.9) so that the conditional mean of the resulting regression error v_t shrinks to zero. Insofar as the choice of $g(\hat{w}_{t-1})$ is concerned, a suitable candidate is $g(\hat{w}_{t-1}) = \sum_{i=1}^{s} \delta_i \hat{w}_{t-1}^i$ for a series expansion of length s in $\hat{w}_{t-1}$. If one or more of the δ -coefficients turn out to be statistically significant, we retain the corresponding terms in the conditional mean specification of y_t so that there is no inadequacy in specification.

Finally, we perform the Lagrange multiplier / Rao's Score test for detecting second-order dependence in the residuals, as specified by the (G)ARCH model for the errors, and the BDS test (see Brock *et al.* (1996), for details) for detecting other higher-order dependences. In the set-up of BDS test, the null hypothesis states that the underlying random variables (here the errors) are independently and identically distributed (*i.i.d.*) and the alternative includes serial correlation, higher-order dependence specified by GARCH, and other unspecified nonlinear dependences. The BDS test statistic measures the statistical significance of the correlation dimension calculations, and its computation involves choosing values of two parameters, $\tilde{\xi}$ and $\tilde{m}$, where $\tilde{\xi}$ is the radius of the hypersphere, which determines whether two points are 'close' or not and $\tilde{m}$ represents the value of the embedding dimension. As suggested by Hsieh (1991), Sewell *et al.* (1993) and Brock *et al.* (1996), in most cases, the values of $\tilde{\xi}$ used are 0.5σ and σ, where σ represents the standard deviation of the linearly filtered data, and the value of $\tilde{m}$ is set in line with the number of observations (e.g. , using only $\tilde{m} \leq 5$ if $T \leq 500$).

6.3 The data

This study has been carried out with data at the level of monthly frequency. The choice of this frequency has been dictated by the fact that, in India, data on macrovariables are not available at any other higher frequency. The time series of exchange rate here refers to the time series of spot Indian rupee / US dollar exchange rate, and the return on exchange rate, as defined in the preceding sections, is the first difference of logarithmic values of the spot exchange rate series. The time period considered for this study covers the period from November, 1994 to March, 2005. Thus, there are a total of 124 observations in the sample. While in all relevant computations all the 124 observations have been used, in case of computations involving out-of-sample forecasting and *MSE-F* and *ENC-NEW* test statistics, the first 74 observations have been used as in-sample observations and the rest kept as hold-out sample. Beginning with November, 1994, the in-sample period, therefore, ends in January, 2001 and the out-of-sample period begins in February, 2001 and ends in March, 2005. The usual descriptive statistics like the mean, standard deviation, skewness and kurtosis values of return as well as of all the macro variables (at stationary values) along with the values of ADF test statistic (at level) values for unit root tests on these variables are given in Table 6.1.

In order to analyze the ability of each macro variable, in turn, in predicting Indian exchange rate return, we need to have, to start with, a set of relevant macro variables. To that end, we consider the following set of 25 macro variables which have been found to influence exchange rate prediction in studies concerning developed countries and which are also mentioned in theories on exchange rate.

From the definitions of these variables, it is evident that some of these variables are broadly similar in nature. The characterizations of these variables in terms of stationarity[2] and seasonality are stated below.

- Bombay Stock Exchange Sensitivity Index (BSESENSEX): The Bombay Stock Exchange is the oldest stock market not only in the country but also in Asia. Established in 1857, it obtained a permanent recognition from Government of India under the Securities Contracts Act, 1956[3]. Its most important and widely-

[2] As noted below (and also evident from Table 6.1) that except for three macro variables *viz*., GFD, SPUSD and OMO, all other series have unit roots and their first difference / logarithmic difference values are stationary. For the sake of convenience, while discussing the results, we may not always mention 'growth / change' in respect of these latter variables; we may merely state the names of the variables although these would refer to their growths or changes, as the case may be.

[3] Earlier it was an Association of Persons (AOP), but now it is a demutualised and corporatised entity according to Companies Act, 1956, pursuant to BSE (Corporatisation and Demutualisation) Scheme, 2005, notified by the Securities and Exchange Board of India (SEBI).

used index, called the BSESENSEX, is recognized worldwide. Since the monthly BSESENSEX series exhibits seasonality, we have applied Proc-X11 to deseasonalize this series. Thereafter, the ADF unit root test has been performed and the conclusion is that the deseasonalized series has a unit root. We have then taken the first difference in logarithm values, which is called the return on BSESENSEX, and then carried out the ADF test once again to conclude that the return series is now stationary. (Data source:www.bseindia.com)

- Call Money Rate (CMR): We use the call money rate which is the rate at which the commercial banks borrow money from other banks. This variable can be viewed as the short-term interest rate in India. The series exhibits no seasonality. However, application of the ADF test showed that it has a unit root. Accordingly, the differenced series which is found to be stationary, has been considered for the analysis. (Data source: www.rbi.org.in)
- M0: This variable is a component of the stock of money, basically defined as the reserve money. This series shows no seasonality and hence no seasonal adjustment is done. The ADF test for unit root showed that it is nonstationary and hence the first difference of its logarithmic values has been used. The series thus obtained may be called the reserve money growth, and this series has been found to be stationary. (Data source: www.rbi.org.in)
- M1: Defined as the *narrow money,* this important variable has been used in many similar works to study the relationship between exchange rate and money supply. Since the series shows seasonality, we have adjusted this series for seasonality and then used the stationary series of the first difference of its logarithmic values for analysis. The variable thus may be called the narrow money growth. (Data source: www.rbi.org.in)
- M3: The *broad money* series needed seasonal adjustment. Thereafter, the first difference of the logarithmic values of this deseasonalized series has been

considered to make it stationary. This variable thus may be called the broad money growth. (Data source: www.rbi.org.in)

- Consumer price index (CPI): The price level with base 1984-85=100 has been found to be nonstationary; so we have taken the first difference in logarithmic values of the series. This differenced series is usually known as inflation rate. (Data source: www.rbi.org.in)
- Wholesale price index (WPI): The price level with base 1984-85=100 is nonstationary while its first difference in logarithmic values is stationary. (Data source: www.rbi.org.in)
- Foreign currency asset (FCA): The foreign currency asset comprises foreign securities held in the issue department and balances held abroad along with investments in foreign securities held in the banking department. It is, in fact, a component of foreign exchange reserve. Since it has been found to be nonstationary, we have carried out our analysis with the stationary series obtained as first difference in logarithmic values. (Data source: www.rbi.org.in)
- Total reserve of foreign exchange (TR): This series has been found to be seasonal and hence it has been seasonally adjusted. The adjusted series has shown the presence of a unit root and accordingly its first difference at log-level has been taken for the purpose of our analysis. As shown in Table 6.1, the resulting series is stationary. (Data source: www.rbi.org.in)
- Industrial production (IP): The industrial production index with base 1993-1994=100 has been found to be highly seasonal and hence it has been adjusted for seasonality. Thereafter, we have taken the first difference in the log values of this index and this has been found to be stationary. This adjusted series may be called the growth in industrial production. (Data source: www.rbi.org.in)
- Domestic petrol price (DPP): We have taken the wholesale price index of fuel with base 1981-82=100. Since this series is nonstationary, we have considered the first

- Export (EX): This variable is an important component of trade. This variable includes transfer of the ownership of goods from residents of a country to non-residents and services provided by resident producers of the country to non-residents. Since this series was found to be nonstationary, we have considered the first difference of the log values to achieve stationarity. (Data source: www.rbi.org.in)
- Import (IM): We have considered the first difference of the log-levels of India's import so as to obtain a stationary series, and the resulting variable is import growth. Being a component of trade this variable is expected to be important for exchange rate predictability. (Data source: www.rbi.org.in)
- Trade balance (TB): This macro variable being the difference between exports and imports, is important for studying predictability of exchange rate. However, it is nonstationary and hence the first difference of the level values has been taken to achieve stationarity. The variable thus obtained is called the change in trade balance. (Data source: www.rbi.org.in)
- Gross fiscal deficit (GFD): This series was seasonally adjusted and the adjusted series has been found to be stationary. Thus, no differencing was required to be done to achieve stationarity for this series. (Data source: www.rbi.org.in)
- Sale/Purchase of US dollar (SPUSD): We have used the series without any seasonal adjustment as well as differencing, since it has been found to be stationary in the level values having no significant seasonality. (Data source: www.rbi.org.in)
- Open market operations (OMO): Open market operations by the Reserve Bank of India are confined to the purchase and sale of Government securities and treasury bills. The government might resort to this to sterilize the effects of intervention. We have considered the unadjusted level values of this macro variable for our

- Federal funds rate (FFR): The series has been considered at the first difference of its level values and this ensures stationarity. This, in fact, is the short term US interest rate. (Data source: www.federalreserve.gov)
- Six-month treasury bill rate of US (TBRU6): We have taken the first difference of this rate for our study as the series was found to be nonstationary. (Data source: www.federalreserve.gov)
- Three-month treasury bill rate of US (TBRU3): For this rate also, we have considered the first difference of its level values and thus achieved stationarity. (Data source: www.federalreserve.gov)
- NASDAQ: We have taken the monthly closing values of the NASDAQ composite index which is an important stock price index of the USA. This series was, however, found to be nonstationary and hence we have taken the first difference of the logarithms of this series to make it stationary. (Data source: www.finance.yahoo.com)
- World gold price (WGP): We have considered the A.M. fix of the London Gold Market, i.e., the price of gold in US dollar per troy oz fixed at 10:30 A.M. London local time by a group of select commercial banks constituting the London Gold Market Fixing Limited. The US dollar per troy oz is converted into rupees per troy oz of gold using the nominal exchange rate. Since the series was found to be nonstationary, we have used the first difference of its logarithmic values for our analysis. (Data source: thebulliondesk.com)
- Foreign direct investment (FDI): Foreign direct investment in India includes direct investment by non-residents and disinvestments of equity capital. The series is nonstationary; so we have taken the difference of log-level values for this variable. (Data source: www.rbi.org.in)

- Foreign institutional investment (FII): This represents the inflow of funds by foreign institutional investors. Since the ADF test suggests that this variable has a unit root, we have considered its first difference to achieve a stationary series. (Data source: www.rbi.org.in)
- Total foreign investment (FINV): This variable is, by definition, the sum of foreign direct investment and portfolio investment. As already mentioned, foreign investment in India include direct investment by non-residents and disinvestments of equity capital. Portfolio investment relates to purchase and sale of equity and debt securities usually traded in financial market. Major components of such investment include FIIs' investment, funds raised through GDRs /ADRs by Indian companies and through offshore funds. This macro variable might have an important role in the predictability of exchange rate. The series was found to be nonstationary and hence we have taken the first difference of this series to make it stationary (Data source: www.rbi.org.in).

Table 6.1

Descriptive statistics of the macroeconomic variables and results of unit root test

Variable	Mean	Standard deviation	Skewness	Kurtosis	ADF test statistic value	Critical value
EXRATE	0.002843	0.012865	1.455054	10.87957	-2.798771	-3.4839
BSE	0.002725	0.065032	-0.159991	2.955874	-1.804911	-4.0355
CMR	-0.029435	4.175531	-0.473210	15.47537	-3.211494	-4.0361
M0	0.008855	0.015595	0.187930	4.050973	-2.538960	-4.0348
M1	0.010398	0.010071	0.189329	4.441182	-1.728017	-4.0355
M3	0.012318	0.006207	1.005136	7.774536	-1.249168	-3.4843
CPI	0.004925	0.006749	1.513375	8.997051	-2.776162	-3.4847
WPI	0.004174	0.004398	0.779577	3.906735	-3.639175	-4.0348
FCA	0.018200	0.024129	0.129937	5.337466	-2.493902	-4.0342
TR	0.017037	0.021772	0.297738	5.299159	-1.982519	-4.0342
IP	0.005268	0.021772	0.297738	5.299159	-2.518701	-4.0348
DPP	0.007878	0.018136	2.363940	9.099963	-3.321858	-4.0361
EX	0.012177	0.074991	0.432297	4.355001	-2.649396	-4.0355

Note: Descriptive statistics are given for the stationary series of macroeconomic variables (including return on India's foreign exchange rate, denoted as EXRATE) used in the analysis.
** indicates that the concerned time series is stationary at level values. The ADF test statistic is obtained for the level values of all the variables. The estimating equation for the ADF test has both an intercept and linear trend term.*
The last column shows MacKinnon 1% critical values for rejection of hypothesis of a unit root.

Table 6.1 (Contd.)

Variables	Mean	Standard deviation	Skewness	Kurtosis	ADF test statistic value	Critical value
IM	0.013949	0.077014	-0.123651	2.718241	-2.545445	-4.0355
TB	-67.41158	1539.164	-0.045104	3.344025	-1.963985	-3.4843
GFD*	8960.139	5457.359	2.823618	18.85321	-7.344202*	-4.0355
SPUSD*	3137.686	5702.051	1.666698	7.969149	-4.176175*	-3.4843
OMO*	-1858.859	3146.882	-1.715684	5.566018	-3.755472*	-3.4852
FFR	-0.021452	0.176836	-1.178854	5.118306	-1.093513	-2.5825
TBUS6	-0.021935	0.190846	-0.847477	5.218031	-1.128894	-2.5827
TBUS3	-0.020565	0.186016	-1.221140	5.860363	-1.327914	-2.5824
NASDAQ	0.007903	0.083512	-0.683903	3.924829	-2.094944	-3.4839
WGP	0.003447	0.030464	0.836628	7.560381	1.629390	-2.5825
DI	0.004942	0.542500	0.126978	3.776005	-1.027511	-2.5827
FII	11.42742	467.4723	1.126644	15.42623	-3.332489	-3.4852
FINV	13.87903	518.5149	0.980891	11.94189	-2.969336	-3.4852

Note: Descriptive statistics are given for the stationary series of macroeconomic variables used in the analysis.
** indicates that the concerned time series is stationary at level values. The ADF test statistic is obtained for the level values of all the variables. The estimating equation for the ADF test has both an intercept and linear trend term.*
The last column shows MacKinnon 1% critical values for rejection of hypothesis of a unit root.

Other than these variables, there are two other relevant variables *viz.*, treasury bill rate of India and repo rates (as discussed in Section 6.1) which could not be included in our analysis, since the time series of these two variables are available from a much later period than considered by us in this study i.e., from the years 1999 and 2000, respectively. All the computations were done using GAUSS package and codes provided by Rapach and Wohar (2005) (http://pages.slu.edu/faculty/rapachde/Research.htm).

6.4 Empirical Results

6.4.1 Selection of macro variables

In this section, we first report the results of specific-to-general approach to macro variable selection using predictive regression. Now, it is quite evident from the description of the macro variables in the preceding section, that some of the variables are similar in nature. Some others are sum of two or more variables. Since this approach uses one variable at a time, it is quite meaningful if the initial choice is done from a larger set. Hence in this approach we have tried with all these variables- one at a time, and finally identified only those macro variables which have significant roles in predicting the return on India's exchange rate. On the other hand, while applying the general-to-specific approach, we have eliminated some such similar variables based on the *p*-values of in-sample predictive regression models obtained in the first approach.

Table 6.2 presents the in-sample regression results for the predictive regression in (6.1) for each of the macro variables in turn. This table also reports the values of Theil's *U* and the *MSE-F* and *ENC-NEW* statistics for the out-of-sample forecasts. For our computations, we have considered the horizons of 1, 3, 12 and 24 months.

We now describe briefly the results reported in Table 6.2 to examine the role of each variable in predictability of exchange rate return. Looking at the results for the first macro variable in our set *viz*., BSESENSEX, we find that none of the criteria- be it in-sample *t*-statistic value or *MSE-F* and *ENC-NEW* test statistics based on out-of-sample forecasting values- shows that this macro variable has any predicting ability for return on exchange rate since none of the test statistic value is significant for any of the four horizons. Even the value of Theil's *U* which is a descriptive measure, has a value greater than 1 for all the horizons indicating that the restricted model forecast

RMSE has a smaller value than that of the unrestricted one. As regards call money rate (CMR), the in-sample *t*-statistic value is significant for the 1- and 12-month horizons. But none of the out-of-sample statistics is significant for this variable. Also, the Theil's *U* value is less than 1 for $k = 3$ and 12. Thus, we may infer that CMR has some significant role in predicting the return on India's exchange rate. For reserve money or M0 as it is called, we find that the in-sample *t*-statistic is significant at 5 per cent level of significance for k=1 only and none of the out-of-sample statistics is significant. Thus, the statistical evidence for predictive ability of M0 is not very strong. None of the other money supply variables *viz*., M1 and M3 exhibit significance in terms of either in-sample *t*-statistic or out-of-sample *MSE-F* and *ENC-NEW* statistics. The results are similarly surprising for the price indices, CPI and WPI, which also show no significance in terms of any of the test statistics considered in this study. The in-sample *t*-statistic as well as the out-of-sample *MSE-F* and *ENC-NEW* statistics are significant for foreign currency asset (FCA) at 3-month horizon. The results are similar for total reserve (TR) where the in-sample *t* as well as the out-of-sample *MSE-F* and *ENC-NEW* statistics are significant at 3-month horizon while only the *MSE-F* statistic is significant at 1-month horizon. Also the Theil's *U*-measure yields a value which is less than 1 at horizons 1, 3 and 12. Note that FCA is a component of TR and hence we should include only one of them in our full model. Comparing the findings on these two macro variables, it is quite evident that TR has somewhat better predictive ability for return than FCA, and accordingly between these two variables, we choose TR for further analysis. As regards the last three macro variables which pertain to foreign investment *viz*., foreign direct investment (FDI), foreign institutional investment (FII) and total foreign investment (FINV), we find that while FDI has no predictive ability, FII and FINV seem to have some significant roles since the in-sample *t*-statistic value has been found to be significant for both these macro variables. However, these two macro variables are obviously of similar nature, and hence as in the case of choice between TR and FCA, we have chosen FINV instead of FII primarily because the *p*-value corresponding to the *t*-statistic is much smaller as compared to that for FII, and also for the fact that FINV is more representative of the foreign investment in a country while FII is a component of FINV.

Insofar as the findings on predictive regression for each of industrial production, domestic oil price, export, import and trade balance are concerned, we can conclude from the values of both the in-sample and out-of-sample forecasting test statistics that none of these have any significant predictive ability for any of the four horizons. We observe from Table 6.2 that the macro variable GFD has many significant test statistic values. While the in-sample *t*-statistic and the out-of-sample *MSE-F* statistic for this variable are significant for the 12 as well as 24- month horizons, the *ENC-NEW* statistic is significant for the 3, 12 and 24- month horizons. These results clearly establish the importance of this variable in predicting exchange rate return. Sale/purchase of US dollars (SPUSD) as well as open market operations (OMO) are found to have some of their test statistic values significant. For SPUSD, the in-sample *t*-statistic is significant for the 12-month horizon and the *ENC-NEW* statistic is significant for the 3-month horizon at 6 per cent level of significance only. As for OMO, the in-sample *t* and out-of-sample *MSE-F* and *ENC-NEW* statistics are found to be significant for the 12 and 24-month horizons. Although each of these two variables has been found to have significant predictive ability for return on exchange rate, it may be noted that both are essentially in the nature of effects of intervention by the RBI in the foreign exchange market. As expectedly, they have also been found to be highly correlated. Hence, both should not be included in the final model for returns on exchange rate, and accordingly we have considered SPUSD only for the subsequent analysis. All the three interest rates of the US *viz.*, Federal funds rate (FFR), six month US treasury bill rate (TBRU6) and three month US treasury bill rate (TBRU3) have been found to have some significant in-sample *t*-statistic values. While the FFR has significant 24-month horizon *t*-statistic, the 3-month and 6-month US treasury bill rates have significant in-sample *t*-statistics for the 12 as well as 24 –month horizons.

Table 6.2

In-sample and out-of-sample predictability test results

Horizon (month)	1	3	12	24	1	3	12	24
	BSESENSEX				Call Money Rate (CMR)			
$\hat{\beta}$	0.000935	0.000545	-0.000202	0.005979	-0.001569	0.000696	0.003228	-0.000790
t-statistic	0.773212 [0.208]	0.286482 [0.396]	-0.036301 [0.520]	1.151283 [0.220]	**-1.356089 [0.087]**	0.372293 [0.359]	**1.838157 [0.042]**	-0.300087 [0.533]
R^2	0.030955	0.020937	0.018084	0.000624	0.040826	0.021288	0.021632	0.001627
Theil's U	1.017502	1.017055	1.001503	1.0000757	1.004854	0.999321	0.997979	1.001551
MSE-F	-1.671170 [0.871]	-1.563083 [0.840]	-0.114004 [0.410]	-0.003935 [0.426]	-0.472292 [0.546]	0.063898 [0.238]	0.154079 [0.133]	-0.080486 [0.624]
ENC-NEW	-0.510228 [0.856]	-0.596818 [0.873]	-0.055035 [0.526]	-0.001268 [0.520]	-0.224068 [0.684]	0.032207 [0.360]	0.077323 [0.207]	-0.040152 [0.697]
	M0				M1			
$\hat{\beta}$	-0.001991	-0.002843	0.0008825	-0.001201	0.000594	-0.000729	0.000372	0.004051
t-statistic	**-1.722309 [0.045]**	-1.357130 [0.902]	0.227650 [0.397]	-0.397368 [0.590]	0.505872 [0.274]	-0.403508 [0.637]	0.107476 [0.428]	0.969883 [0.162]
R^2	0.0496198	0.033877	0.018326	0.001742	0.028199	0.021351	0.018104	0.003063
Theil's U	1.014914	0.996439	1.002269	1.002127	1.022462	1.00360	1.003266	1.001468
MSE-F	-1.429521 [0.834]	0.336437 [0.146]	-0.171839 [0.558]	-0.110232 [0.566]	-2.129303 [0.915]	-0.336915 [0.591]	-0.247012 [0.709]	-0.076203 [0.547]
ENC-NEW	0.186367 [0.270]	0.384658 [0.150]	-0.063243 [0.621]	-0.048708 [0.633]	-0.745720 [0.923]	-0.151423 [0.688]	-0.099846 [0.763]	-0.036541 [0.618]
	M3				Consumer Price Index (CPI)			
$\hat{\beta}$	0.000757	0.002602	0.002067	0.005022	0.000307	0.001323	0.006731	0.012601
t-statistic	0.654876 [0.228]	1.300604 [0.127]	0.428082 [0.363]	0.885690 [0.282]	0.264215 [0.395]	0.633327 [0.327]	1.093769 [0.193]	1.075454 [0.239]
R^2	0.029595	0.031876	0.019430	0.005243	0.026693	0.023384	0.033125	0.026760
Theil's U	1.012283	0.991505	1.005321	1.007548	1.001052	0.997992	0.992734	1.003059
MSE-F	-1.176996 [0.793]	0.808797 [0.077]	-0.401195 [0.613]	-0.388101 [0.680]	-0.102955 [0.317]	0.189363 [0.235]	0.558285 [0.184]	-0.158340 [0.503]
ENC-NEW	-0.400089 [0.811]	0.444650 [0.156]	-0.173093 [0.683]	-0.18786 [0.762]	-0.047127 [0.447]	0.109720 [0.368]	0.290219 [0.277]	-0..078313 [0.600]

Note: Bold entries indicate statistical significance. Figures in parentheses show the p-values.

Table 6.2 (Contd.)

Horizon	1	3	12	24	1	3	12	24
	Wholesale Price Index (WPI)				Foreign Currency Asset (FCA)			
$\hat{\beta}$	-0.000974	-0.002436	0.003419	0.0091119	-0.000776	-0.005507	-0.009003	-0.011389
t-statistic	-0.837116 [0.805]	-1.276392 [0.891]	0.470163 [0.368]	1.030508 [0.256]	-0.666015 [0.751]	**-2.162986 [0.032]**	-1.169428 [0.833]	-0.861398 [0.736]
R^2	0.031781	0.030289	0.021727	0.013000	0.0297135	0.069880	0.043240	0.020054
Theil's U	0.994889	0.997901	1.004141	0.998073	0.998014	0.968889	1.003351	1.029875
MSE-F	0.504734 [0.130]	0.1979310 [0.212]	-0.312740 [0.493]	0.100493 [0.343]	0.195218 [0.194]	**3.066709 [0.015]**	-0.253376 [0.493]	-1.486547 [0.840]
ENC-NEW	0.3114821 [0.199]	0.137634 [0.306]	-0.064035 [0.515]	0.0507521 [0.430]	0.181410 [0.257]	**2.273341 [0.025]**	-0.093537 [0.579]	-0.717124 [0.904]
	Total Reserve (TR)				Industrial Production (IP)			
$\hat{\beta}$	-0.001244	-0.006455	-0.012166	-0.018222	-0.000188	-0.001797	0.001984	0.002700
t-statistic	-1.068408 [0.856]	**-2.599236 [0.008]**	-1.644187 [0.888]	-1.292894 [0.813]	-0.157249 [0.562]	-1.046036 [0.864]	0.920186 [0.247]	1.108117 [0.220]
R^2	0.035303	0.087602	0.063215	0.046827	0.026328	0.0255538	0.019239	0.0025547
Theil's U	0.988272	0.953714	0.995197	1.032066	1.003155	0.9960045	1.002489	1.000407
MSE-F	**1.169878 [0.062]**	**4.672767 [0.003]**	0.367641 [0.192]	-1.59055 [0.875]	-0.307715 [0.423]	0.377838 [0.106]	-0.188478 [0.629]	-0.021144 [0.432]
ENC-NEW	0.801040 [0.104]	**3.574235 [0.003]**	0.256217 [0.255]	-0.770604 [0.934]	-0.143759 [0.553]	0.204079 [0.190]	-0.080930 [0.708]	-0.010509 [0.528]
	Domestic Petrol Price (DPP)				Export (EX)			
$\hat{\beta}$	-0.000853	-0.001274	0.003186	0.007749	-0.000882	0.000070	0.000259	0.000577
t-statistic	-0.737409 [0.768]	-0.618052 [0.700]	1.022920 [0.206]	2.134098 [0.076]	-0.755939 [0.782]	0.0531199 [0.471]	0.132297 [0.393]	0.176111 [0.402]
R^2	0.030520	0.023222	0.021466	0.011078	0.030742	0.020484	0.018092	0.001563
Theil's U	1.0042854	1.002456	0.998154	0.994923	1.003039	1.004692	1.00003	1.006709
MSE-F	-0.417285 [0.523]	-0.230010 [0.392]	0.140657 [0.247]	0.266024 [0.242]	-0.296483 [0.380]	-0.438002 [0.813]	-0.002194 [0.375]	-0.345366 [0.871]
ENC-NEW	-0.149673 [0.613]	-0.976841 [0.491]	0.073431 [0.361]	0.138199 [0.327]	-0.013273 [0.377]	0.0243760 [0.529]	0.0041032 [0.441]	-0.134122 [0.878]

Note: Bold entries indicate statistical significance. Figures in parentheses show the p-values.

Table 6.2 (Contd.)

Horizon	1	3	12	24	1	3	12	24
	Import (IM)				Trade Balance (TB)			
$\hat{\beta}$	-0.000233	-0.000715	-0.001481	0.000376	-0.000989	0.000955	0.001752	0.000654
t-statistic	-0.200472 [0.556]	-0.605533 [0.764]	-0.74143 [0.740]	0.170796 [0.415]	-0.840305 [0.788]	0.796551 [0.206]	0.612189 [0.279]	0.256193 [0.391]
R^2	0.026453	0.021338	0.018715	0.001540	0.031824	0.021888	0.018834	0.001565
Theil's U	1.011275	0.999502	1.000835	1.000853	1.006571	1.000896	1.002451	1.003388
MSE-F	-1.086542 [0.736]	0.046865 [0.223]	-0.063375 [0.494]	-0.044315 [0.511]	-0.637650 [0.608]	-0.084092 [0.410]	-0.185592 [0.681]	-0.175268 [0.692]
ENC-NEW	-0.478653 [0.817]	0.0287776 [0.330]	-0.030650 [0.581]	-0.021285 [0.601]	-0.091346 [0.482]	0.061694 [0.320]	-0.089194 [0.778]	-0.083116 [0.762]
	Gross Fiscal Deficit (GFD)				Sale/Purchase of US dollars (SPUSD)			
$\hat{\beta}$	-0.000870	-0.002217	-0.018881	-0.068832	-0.001329	-0.003303	-0.028036	-0.045936
t-statistic	-0.744061 [0.793]	-1.095640 [0.840]	**-2.147789 [0.070]**	**-4.972827 [0.006]**	-1.091797 [0.858]	-0.831508 [0.783]	**-3.625015 [0.009]**	-1.894119 [0.859]
R^2	0.030599	0.028610	0.120965	0.348166	0.035705	0.037015	0.170643	0.121563
Theil's U	1.001047	1.019187	0.964736	0.880456	1.037694	1.061187	0.978286	1.068959
MSE-F	-0.102533 [0.276]	-0.278647 [0.415]	**2.828793 [0.044]**	**7.539640 [0.011]**	-3.495167 [0.969]	-5.263673 [0.933]	1.705625 [0.177]	-3.246327 [0.737]
ENC-NEW	0.455598 [0.160]	**1.372943 [0.050]**	**5.893898 [0.001]**	**4.228756 [0.020]**	0.790506 [0.118]	**2.621505 [0.060]**	1.671682 [0.181]	-1.493358 [0.815]
	Open Market Operations (OMO)				Federal Funds Rate (FFR)			
$\hat{\beta}$	-0.000037	0.001853	0.013548	0.028774	-0.000885	-0.002346	0.009018	0.028680
t-statistic	-0.031034 [0.494]	0.953984 [0.201]	**2.241930 [0.049]**	**2.211037 [0.096]**	-0.762443 [0.752]	-1.102336 [0.812]	1.845685 [0.119]	**6.560040 [0.003]**
R^2	0.026135	0.026086	0.077412	0.103780	0.030821	0.029533	0.042896	0.125509
Theil's U	1.012052	1.011197	0.958962	0.945911	1.006652	1.010899	0.986116	0.956095
MSE-F	-1.160062 [0.771]	-1.035061 [0.732]	**3.321946 [0.042]**	**3.058480 [0.047]**	-0.645455 [0.592]	-1.007966 [0.535]	1.077548 [0.232]	2.442703 [0.155]
ENC-NEW	-0.377621 [0.786]	0.119916 [0.333]	**2.857213 [0.039]**	**1.871340 [0.076]**	-0.113824 [0.507]	-0.358485 [0.606]	1.216078 [0.253]	1.369916 [0.225]

Note: Bold entries indicate statistical significance. Figures in parentheses show the p-values.

Table 6.2 (Contd.)

Horizon	1	3	12	24	1	3	12	24
	Six-month US Treasury Bill Rate (TBRU6)				Three-month US Treasury Bill Rate (TBRU3)			
$\hat{\beta}$	0.000534	-0.000076	0.0051864	0.0215734	0.0001700	-0.000884	0.0069318	0.0232380
t-statistic	0.457087 [0.331]	-0.044338 [0.503]	**1.929268 [0.079]**	**4.680175 [0.012]**	0.145846 [0.435]	-0.494679 [0.659]	**1.850865 [0.098]**	**4.367199 [0.019]**
R^2	0.027819	0.020486	0.026206	0.071188	0.026299	0.021760	0.0328200	0.0839392
Theil's U	1.008953	1.008696	0.998929	0.982207	1.008505	1.007428	0.9918420	0.978891
MSE-F	-0.865712 [0.678]	-0.806892 [0.550]	0.0814643 [0.341]	0.950510 [0.260]	-0.822985 [0.685]	-0.690514 [0.491]	0.627679 [0.259]	1.133430 [0.221]
ENC-NEW	-0.127477 [0.534]	-0.286449 [0.597]	0.064437 [0.437]	0.500377 [0.260]	-0.158301 [0.594]	-0.294949 [0.598]	0.427109 [0.345]	0.606808 [0.285]
	NASDAQ				World Gold Price (WGP)			
$\hat{\beta}$	-0.000451	0.002135	0.007929	0.016958	-0.002039	-0.002796	-0.010711	-0.023140
t-statistic	-0.38763 [0.654]	1.460971 [0.106]	**1.912645 [0.059]**	**3.254108 [0.019]**	**-1.778890 [0.040]**	**-1.525339 [0.077]**	**-1.646804 [0.087]**	**-1.805563 [0.099]**
R^2	0.027345	0.028095	0.0391473	0.0487248	0.051149	0.0334102	0.054802	0.074420
Theil's U	1.005749	1.004238	0.990772	1.000725	0.977228	0.988114	0.977755	0.976318
MSE-F	-0.558600 [0.578]	-0.075460 [0.285]	0.711159 [0.132]	-0.088720 [0.486]	**2.310227 [0.022]**	**1.137521 [0.065]**	**1.748738 [0.017]**	**1.276631 [0.034]**
ENC-NEW	-0.178050 [0.600]	0.236426 [0.267]	0.479706 [0.177]	-0.040773 [0.566]	**1.823196 [0.035]**	0.640658 [0.117]	**1.049020 [0.033]**	**0.669944 [0.071]**
	Foreign Direct Investment (FDI)				Foreign Institutional Investment (FII)			
$\hat{\beta}$	-0.000924	-0.000130	0.000529	0.0002972	-0.000696	-0.002319	0.001043	0.000796
t-statistic	-0.798628 [0.818]	-0.097456 [0.550]	0.261895 [0.386]	0.148528 [0.466]	-0.576337 [0.737]	**-1.084165 [0.096]**	0.192453 [0.455]	0.084527 [0.466]
R^2	0.031276	0.020505	0.018157	0.001531	0.028815	0.0262282	0.018151	0.001532
Theil's U	1.007950	1.000527	1.000373	0.998080	1.040956	1.005912	1.003355	1.007745
MSE-F	-0.769914 [0.636]	-0.495408 [0.383]	-0.028321 [0.476]	0.100111 [0.170]	-3.779966 [0.118]	-0.550819 [0.271]	-0.253710 [0.259]	-0.398094 [0.046]
ENC-NEW	-0.167774 [0.540]	-0.019167 [0.502]	-0.011547 [0.547]	0.050530 [0.237]	-1.122203 [0.673]	-0.220754 [0.620]	-0.129483 [0.511]	-0.174495 [0.568]

Note: Bold entries indicate statistical significance. Figures in parentheses show the p-values.

Table 6.2 (Contd.)

	FINV			
$\hat{\beta}$	-0.000920	-0.00208	0.000103	0.000789
t-statistic	-0.774443 [0.786]	**-1.294212 [0.029]**	0.022957 [0.453]	0.157105 [0.397]
R^2	0.030970	0.025642	0.018073	0.001545
Theil's U	1.023941	0.999830	1.001489	1.002557
MSE-F	-2.264612 [0.681]	0.016009 [0.298]	-0.112940 [0.460]	-0.132472 [0.484]
ENC-NEW	-0.533029 [0.616]	0.054075 [0.437]	-0.036108 [0.546]	-0.059364 [0.560]

Note: Bold entries indicate statistical significance. Figures in parentheses show the p-values.

However, none of *MSE-F* and *ENC-NEW* forecasting test statistics has been found to be significant for any of these three variables. Our empirical findings on NASDAQ suggest that the US stock market does have some influence on the Indian exchange rate return as exhibited by the in-sample *t*-statistic value which is found to be significant for the 12 as well as 24- month. The findings corresponding to the next macro variable *viz*., world gold price (WGP) has been found to be very significant. All the in-sample and out-of-sample test statistic values at all the four horizons except the *ENC-NEW* for 3-month horizon have been found to be significant for this macro variable. This shows that the predictability of return on exchange rate is strongly influenced by world gold price.

In order to apply the in-sample general-to-specific model selection criterion combined with tests of out-of-sample forecasting ability, we cannot obviously begin with a large model having as many as 25 macro variables. To reduce this initial set, we may note that, as already discussed, there are some variables which are similar in nature while some are components of other variables. Accordingly, we have dropped FCA, six-month and three-month US treasury bill rates, FII and OMO from the set of macro variables which were found to have significant predictive ability by the specific-to-general model selection approach, and consequently we are left with a set of nine macro variables comprising CMR, M0, TR, GFD, SPUSD, FFR, NASDAQ, WGP and FINV. Other than these, we have also included some variables which are normally argued, in economics and finance, to have important roles in determining exchange rate but which have not been found to be significant by the first approach. These, to our understanding, are BSESENSEX, M1, CPI, IP and TB. Thus, we have the following 14 macro variables for the general-to-specific approach: BSESENSEX, CMR, M0, M1, CPI, TR, IP, GFD, SPUSD, TB, FFR, NASDAQ, WGP and FINV. The empirical findings by this approach are reported in Table 6.3. It may be noted that the critical values for $\max_{j \in \{1,\ldots,14\}} \left| t_{\hat{\beta}_j} \right|$, maximal *MSE-F* and maximal *ENC-NEW* for all the horizons have been generated using data-mining-robust bootstrap procedure discussed earlier. The critical value computed using the bootstrap procedure for $\max_{j \in \{1,\ldots,14\}} \left| t_{\hat{\beta}_j} \right|$ for the 24- month horizon is 5.52. This is obviously less than 6.56, which is the maximum (amongst these 14 variables) value of the *t*-statistic for the 24-month horizon. Also, the critical value of *MSE-F* test statistic is 3.985, which is less than the value of 4.673 obtained for the 3-month horizon. The same is the finding with respect to the *ENC-NEW* test statistic. Thus, for all these three tests, the null hypothesis of no predictability is rejected. We can, therefore, conclude that the best evidence for in-sample and out-of-sample predictive ability, which is reflected in the maximum (amongst these 14 variables) values of the *t*-statistic as well as the *MSE-F* and *ENC-NEW* statistics, is free from any data mining problem.

Now, analysing the results presented in Table 6.3, we note that for the 1-month horizon, the only variable which has been found to have significant explanatory power is M0 or the reserve money growth. For the 3-month horizon also, there is only one significant macro variable, but now the variable is total reserve (TR). For 12-month horizon, the number of explanatory variables has increased to five and these are GFD, SPUSD, FFR, NASDAQ and FINV. As for the 24-month horizon, eight macro variables *viz.*, CMR, M0, M1, GFD, SPUSD, FFR, WGP and FINV have been found to have predictive ability for return on India's foreign exchange rate at monthly-level frequency. Combining the findings for the four horizons, we note that the only macro variable which has been found to have significant predictive ability by this approach, but not by the earlier one, is M1, the narrow money; the other significant variables are the same by the two approaches.

Table 6.3

General-to-specific model selection results

Horizon (month)	1	3	12	24
Included variables	M0	TR	GFD,SPUSD,FFR, NASDAQ, FINV	CMR, M0, M1, GFD, SPUSD, FFR, WGP,FINV
Theil's U	1.014914	0.953714	0.979230	0.921188
MSE-F	-1.429521 [0.222]	4.672767 [0.023]	1.629134 [0.132]	4.639183 [0.091]
ENC-NEW	0.186367 [0.404]	3.574235 [0.085]	4.842013 [0.118]	2.772104 [0.203]

Note: Figures in parentheses indicate the p-values

Thus, based on both the methods of selection of macro variables and considering all the four horizons together, we can conclude that the relevant macroeconomic variables which have been found to have significant role in predicting India's monthly exchange rate return are ten in number and these are reserve money growth (M0), narrow money growth (M1), change in foreign exchange reserve (TR), gross fiscal deficit (GFD), sale/purchase of US dollar (SPUSD), change in Federal funds rate (FFR), US stock price return (NASDAQ), change in call money rate (CMR), rate of change in gold price (WGP) and change in total foreign investment (FINV).

6.4.2 The final estimated model

Once the significant macro economic variables have been chosen, we consider the dynamic linear regression model specified in (6.9) where we now use all the ten macro variables to obtain the 'best' model for India's monthly exchange rate return. Before we actually estimate the model, it is essential to check whether there is any structural break in the monthly exchange rate return series. As stated in Section 6.2.3, we have carried out the Quandt-Andrews test for parameter stability and the relevant statistic was found to be 9.91, which is lower than the tabulated value of 10.00- thus indicating that the null hypothesis of no structural break cannot be rejected for the monthly series. The final model is, therefore, obtained using all the sample observations. For estimating this model, the number of lagged values of return, *p*, was initially taken to be a moderate value of 10 so that the autocorrelation could be entirely captured by the model, and the value of *l*, the lag value for the independent macro variables, was fixed at 2. Further, the number of dummy variables (*d*) was obviously taken to be 12 since the data is at monthly level. This model was estimated by using the OLS method of estimation and the estimated model is presented in equation (6.10) below. The estimated model having significant variables only has been obtained as follows:

$$\hat{y}_t = \underset{(1.792)^{*}}{0.153}\, y_{t-3} - \underset{(2.332)^{**}}{0.185}\, y_{t-5} - \underset{(3.537)^{***}}{(7.50\times 10^{-6})}\, FINV_t - \underset{(4.997)^{***}}{(1.08\times 10^{-6})}\, SPUSD_t$$

$$+ \underset{(4.092)^{***}}{0.209}\, TR_t - \underset{(2.827)^{***}}{0.0007}\, CMR_{t-1} - \underset{(2.056)^{**}}{(4.44\times 10^{-6})}\, FINV_{t-1} + \underset{(2.887)^{***}}{0.262}\, M1_{t-1} \qquad (6.10)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

We note that only the third and fifth lags of exchange rate return are found to be significant. None of the monthly dummies is significant and hence we can conclude that there is no systematic month effect in foreign exchange return. The macro variables which are found to have contemporaneous dependence with exchange rate return are the change in total foreign investment (FINV), sale / purchase of US dollar (SPUSD) and the change in total reserve of foreign exchange (TR). The first lag of change in call money rate (CMR), change in FINV and narrow money growth (M1) are also found to influence exchange rate return. It is noteworthy that the macro variable, change in FINV has both contemporaneous and lagged effects on return. This shows the importance of this macro variable i.e., total foreign investment, in the determination and predictability of India's exchange rate return.

Once the model has been estimated, the usual diagnostic tests on the residuals of this estimated model were carried out. The Ljung-Box test suggested a few significant values. To be specific, the *p*-values for the first three lags were found to be 0.026, 0.083 and 0.086, indicating that while the first lag is quite highly significant , the other two *viz*., the second and third are significant only at 9 per cent level of significance. This implies that there is still some serial correlation remaining in the residuals which could not be captured by this model. However, the Ljung-Box statistic values for the squared residuals indicate that there is no squared dependence in the series. Thus, we can say that there is no time-dependent volatility in the monthly-level return data. In other words, the finding suggests that the conditional variance is constant at this data frequency, and that, in particular, it does not follow an ARCH / GARCH process. Based on these diagnostics, we can, therefore, conclude that while there is further scope of an improved model where the remaining autocorrelations could be taken care of, no modelling consideration for volatility is needed for the monthly return on India's foreign exchange rate since at this frequency the series exhibits no time-dependent volatility at all. The finding of no time-dependent volatility in the monthly series is quite likely since volatility is usually manifested in financial time series of high frequency like, for instance, the daily and hourly levels. We have also carried out the test for misspecification as discussed in Section 6.2. By augmenting the model in (6.10) by including suitable polynomial functions of the recursive residuals at t-1 (i.e., $\hat{w}_{t-1}$) and then estimating it by OLS procedure, we have obtained the following estimated model:

$$\hat{y}_t = \underset{(1.200)}{0.104}\, y_{t-3} - \underset{(1.775)^{*}}{0.134}\, y_{t-5} - \underset{(3.648)^{***}}{7.11\times 10^{-6}}\, FINV_t - \underset{(4.421)^{***}}{8.94\times 10^{-7}}\, SPUSD_t + \underset{(3.686)^{***}}{0.180}\, TR_t$$
$$- \underset{(1.197)}{0.0003}\, CMR_{t-1} - \underset{(2.489)^{**}}{4.94\times 10^{-6}}\, FINV_{t-1} + \underset{(2.430)^{**}}{0.224}\, M1_{t-1} + \underset{(1.272)}{0.204}\, \hat{w}_{t-1} + \underset{(0.340)}{4.128}\, \hat{w}_{t-1}^{2}$$
$$+ \underset{(0.359)}{166.140}\, \hat{w}_{t-1}^{3} - \underset{(0.270)}{5124.961}\, \hat{w}_{t-1}^{4} \qquad (6.11)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

From this regression, it is evident that there is no misspecification in the mean part since all the coefficients associated with $\hat{w}_{t-1}$, $\hat{w}_{t-1}^{2}$, $\hat{w}_{t-1}^{3}$ and $\hat{w}_{t-1}^{4}$ are insignificant. Thus, the performance of the model in (6.10) is quite satisfactory except for the diagnostics which suggest presence of some remaining autocorrelation in the residual series.

Since no further lagged value of y_t was found to be significant, this finding motivated us to include a few more macro variables in our model, which were not found to have significant predictability for return on India's exchange rate by both the specific-to-general and general-to-specific approaches of variable selection. Since some studies suggest that domestic stock price (BSESENSEX in our case), inflation rate (CPI) and industrial production growth (IP) are likely to have significant effects on exchange rate return and some empirical evidences also support this, what we have done next is to include these three macro variables in our analysis and check whether these are significant or not. In case these are indeed significant, then inclusion of these should lead to an improvement in the residual diagnostics of the model. When the original full model in (6.9) is re-estimated with the inclusion of these three variables, it is found that BSESENSEX has a significant effect while the other two variables do not. Further, return on NASDAQ is now found to be significant at lag 2 at 10 per cent level of significance, as is clear from the estimated model (6.12) given below:

$$\hat{y}_t = \underset{(1.842)}{0.154}\, y_{t-3} - \underset{(2.099)^{**}}{0.163}\, y_{t-5} - \underset{(3.358)^{***}}{(7.08\times 10^{-6})}\, FINV_t - \underset{(4.670)^{***}}{(1.01\times 10^{-6})}\, SPUSD_t$$
$$+ \underset{(4.179)^{***}}{0.209}\, TR_t - \underset{(2.059)^{**}}{0.0332}\, BSESENSEX_t - \underset{(2.623)^{***}}{0.000676}\, CMR_{t-1} - \underset{(1.921)^{*}}{(4.07\times 10^{-6})}\, FINV_{t-1}$$
$$+ \underset{(2.735)^{***}}{0.244}\, M1_{t-1} + \underset{(1.683)^{*}}{0.0195}\, NASDAQ_{t-2} \qquad (6.12)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

From equation (6.12) we may note that the relationship between exchange rate and money supply and call money rate is in accordance to the standard economic models. One can view this equation similar to equation (1.1) in Chapter 1, which is the reduced form used for validating structural models. The rise in money supply is accompanied with a depreciation of the Indian currency while a rise in call money rate results in an appreciation. However, the US federal funds rate does not seem to have any effect on India's exchange rate. Usually, in theory, interest rate differential is considered as a variable (uncovered / covered interest rate parity) related to exchange rate. In our model we considered both the domestic and foreign interest rates. Such a specification can, therefore, be viewed, in a way, as a more general form of the Meese-Rogoff model in (1.1) which considers the interest rate differential, instead of the individual interest rates separately. Obviously, therefore, our modeling approach allows for the possibility of inclusion of this aspect of macro model as a special case. The contemporaneous effect of returns of BSESENSEX on the exchange rate returns is found to be significant. The coefficient is found to be negative implying that an increase in domestic stock returns should lead to an appreciation of rupee. This result has also been documented by some researchers (Ki-Ho-Kim (2003)) who have stated that an increase in domestic stock returns results in an appreciation of foreign exchange rate since this reflects the good performance of the economy concerned and hence it attracts more of foreign capital. Similarly, the direction of relationship between exchange rate return and return on NASDAQ is also desirable since an increase in this US stock price return is expected to lead to an outflow of funds from the domestic capital market to the foreign market and hence a depreciation of the domestic currency. However note that this dynamic effect requires a lag of two months to be effective. A rise in foreign investment must lead to an appreciation of the domestic currency due to inflow of funds. The only concern seems to be the change in total reserves and SPUSD. The change in total reserves can also be viewed as a proxy of the intervention activities of the government. As mentioned in the beginning of this chapter, the asymmetric nature of the intervention results in large stockpile of reserves. We have obtained a positive relation between the two, i.e., a rise in reserves leading to exchange rate depreciation which is different from the long-run situation in which more reserves indicate better performance of the economy and hence strengthening of the domestic currency. This result could be because of the intervention activities which are undertaken by the RBI to depreciate the Indian rupee. When RBI intervenes, total reserves rises and exchange rate depreciates and hence this could be the justification for this contemporaneous behaviour of change in reserves and exchange rate return.

The contemporaneous effect of intervention on exchange rate returns is picked up by the coefficient of SPUSD which has been found to be significant. Effectiveness of the intervention requires a positive sign, implying net market purchases of foreign exchange leading to rupee depreciation. However, in our case it was found to have a negative sign. As noted by Kim and Sheen (2006), who also found a similar result, this might suggest counter-productive intervention. Also the presence of simultaneity bias cannot be ruled out. One can however argue that such a purchase of dollar takes place only when RBI wants to intervene in the event of some capital inflow. If this purchase is not successful in mopping the excess foreign exchange from the markets then it results in an appreciation of the domestic currency.

From the chosen model, we also get some insights for policy formulations. As pointed out earlier, the government has, at its disposal various instruments for regulating foreign exchange rate. But the model obtained here suggests that intervention in the foreign exchange market by buying foreign exchange results in large stockpile of reserves without actually affecting the exchange rate. Thus, it makes the intervention counter-productive. Instead the instrument that can actually be used is the interest rate which has a significant effect on the exchange rate.

Once again, we performed the standard diagnostic tests on the residuals of this model and found those to be favorable since the *p*-values of the Ljung-Box *Q(k)* statistic are now 0.053, 0.142 and 0.129 for the first, second and third lags, respectively. These values which are reported in Table 6.4 clearly show that there is no evidence of autcorrelation in the residuals of this model at 5 per cent level of significance. Further, the recursive residual-based test for misspecification was done for the chosen model in (6.12) and the results, as before, showed that none of the coefficients associated with $\hat{w}_{t-1}$, $\hat{w}_{t-1}^2$, $\hat{w}_{t-1}^3$ and $\hat{w}_{t-1}^4$ is significant even at 10 per cent level of significance, indicating that there is no misspecification in the mean specification. Also, we tested for second order dependence in the residuals using ARCH -LM test and found the test statistic value to be 2.248, showing that the null

Table 6.4
Ljung-Box *Q(k)* test statistic values for both residuals and squared residuals of the final model

Lags	Residuals		Squared residuals	
	$Q(k)$	p-value	$Q(k)$	p-value
1	3.753	0.053	2.322	0.128
2	3.899	0.142	2.776	0.250
3	5.672	0.129	2.803	0.423
4	5.949	0.203	5.094	0.278
5	6.255	0.282	5.252	0.386
6	6.309	0.390	5.934	0.431
12	9.518	0.733	6.504	0.889
24	14.848	0.925	10.153	0.994
36	33.299	0.598	31.087	0.701

hypothesis of no (G)ARCH cannot be rejected. Finally, although no second order dependence in the residuals has been found, we applied the BDS test due to Brock *et al*. (1996) to detect the existence of any higher order dependence in the residuals. As stated in Section 6.2, the BDS test is a test where the null hypothesis of independently and identically distributed (*i.i.d.*) errors is tested against the alternative which include serial correlation, higher order dependencies specified by the GARCH model and other unspecified nonlinear forms. Thus, rejection of the null would, in our case where the serial correlation has been duly incorporated in the model, imply that there are other higher order dependences in the residuals of the model. However, a look at the BDS test statistic values in Table 6.5 makes it quite clear that for all the $(\tilde{\xi}/\sigma, \tilde{m})$ combinations considered, the null cannot be rejected.

Table 6.5

BDS test statistic values for the residuals of the final model

$\tilde{\xi}/\sigma$	$\tilde{m}$	Value
0.5	2	-0.6604
0.5	3	-2.9380
0.5	4	-1.2664
0.5	5	-0.6220
1	2	-1.0108
1	3	0.1993
1	4	-1.9913
1	5	-1.1209

Note: The values of BDS test statistic, based on residuals of (6.12), are compared with the simulated values given in Brock et al. (1991). All the test statistic values are insignificant at 5% level of significance. $\tilde{\xi}$, $\tilde{m}$ *and* σ *stand for distance, embedding dimension and the standard deviation of the linearly filtered data, respectively.*

For instance, the value of the BDS statistic for $\tilde{\xi}/\sigma = 1$ and $\tilde{m} = 2$ has been obtained as -1.0108, and the corresponding critical value (*cf.* Brock *et al.* (1991)) at 5 per cent level of significance is a number between -2.58 (for T=100) and -2.15 (for T=250). Obviously, the (absolute) computed value is smaller than the (absolute) critical value at 5 per cent level of significance, and hence the null hypothesis of *i.i.d.* errors cannot be rejected for this combination of $\tilde{\xi}/\sigma$ and $\tilde{m}$ values. In fact, there are no cases when the null hypothesis is rejected and we can, therefore, infer that the BDS test suggests that there is no further nonlinear dependence in the residuals. Thus, we can conclude that in terms of standard diagnostic tests on the residuals, the estimated model in (6.12) is the 'best' linear dynamic single-equation model for determination and predictability of return on India's monthly foreign exchange rate involving relevant macro variables.

6.5 Conclusions

In this paper, we have first studied the predictability aspect of India's monthly exchange rate return in terms of relevant macroeconomic variables and then obtained the 'best' linear dynamic single-equation model for this time series. Beginning with a set of 25 macro variables which have been found to be relevant in similar studies, mostly on developed economies, and which are also known to have some roles in theoretical studies on exchange rate, we have analyzed the predictive ability –both in-sample and out-of-sample- of each of these macro variables in turn, using specific-to-general as well as general-to-specific model selection criteria. Combining the empirical findings of these two approaches, we have found a set of 10 macro variables which have significant predictive ability for India's exchange rate return. These variables are: reserve money growth, narrow money growth, change in foreign exchange reserve, gross fiscal deficit, sale/purchase of US dollar, change in Federal funds rate, return on US stock index NASDAQ, change in call money rate, rate of change in gold price and change in total foreign investment. Using these macro variables along with the lagged values of these variables as well as of return itself and dummy variables representing month effects, as independent variables, we have estimated a linear dynamic model in single equation framework for India's exchange rate return. In addition to few lag values of return, only five macro variables *viz*., change in foreign exchange reserve, sale / purchase of US dollars, change in call money rate, narrow money growth and change in total foreign investment, were found to be significant.

The model thus obtained was then checked for misspecification in mean and also for presence of any remaining autocorrelation as well as higher order dependences. Though misspecification in mean was not found, residual diagnostics showed that autocorrelation was not completely captured by this model. To take care of this, some additional macro variables which are considered to be relevant, were included. Out of these variables, only BSESENSEX was found to be significant and we included this variable in the set of 10 macro variables considered for obtaining the final model. In the estimated final model, BSESENSEX and NASDAQ in addition to the already found five significant macro variables came out to be significant. This final model also satisfied all the standard diagnostic tests of model performance.

Thus, ten macrovariables have been found to have significant effect on India's exchange rate. These variables are reserve money growth, narrow money growth, change in foreign exchange reserve, gross fiscal deficit, sale / purchase of USD, change in US federal funds rate, return on NASDAQ, change in call money rate, rate of change in gold price and change in total foreign investment.

The contemporaneous effect of returns of BSESENSEX on the exchange rate returns was found to be negative implying that an increase in domestic stock returns leads to an appreciation of rupee. This result has also been documented by some researchers as well. Similarly, the direction of relationship between exchange rate return and return on NASDAQ was also found to be desirable since an increase in this US stock price return is expected to lead to an outflow of funds from the domestic capital market to the foreign market and hence a depreciation of the domestic currency. However the dynamic effect, in our model requires a lag of two months to be effective. A rise in foreign investment has been found to lead to an appreciation of the domestic currency due to inflow of funds. The directions of the CMR and M1 are also in accordance to the standard economic theories. The call money market and foreign exchange market are closely linked as there exists arbitrage opportunities between the two markets. When call money rates increase, banks borrow dollars from their overseas branches, swap them for rupees and lend them in call money market. This results in an appreciation of domestic currency. Growth in money supply leads to depreciation of domestic currency.

We have obtained a positive relation between the reserves and exchange rate, i.e., a rise in reserves leads to exchange rate depreciation. One explanation for this finding could be that intervention activities which are undertaken by the RBI to depreciate the Indian rupee results in large stocks of reserves (Ramachandran (2006)). Hence this contemporaneous relation between the two is observed.

Finally in case of intervention, the relation between the two is found to be negative, implying that net market purchases of foreign exchange leads to rupee appreciation. As noted by Kim and Sheen (2006), who also found a similar result, this might suggest counter-productive intervention. Also the presence of simultaneity bias cannot be ruled out. One can however argue that such a purchase of dollar takes place only when RBI wants to intervene in the event of some capital inflow. If this purchase is not successful in mopping the excess foreign exchange from the markets then it results in an appreciation of the domestic currency. From policy perspective, it can be concluded that the government can have more control over the exchange rate if it uses the interest rate as a policy variable rather than the sale / purchase of US dollar since it is found to be counter-productive for India's foreign exchange rate.

CHAPTER 7

Long-Run Relationship Between Exchange Rate and Macroeconomic Variables

7.1 Introduction

In the preceding chapter, we have investigated the relationship between return on foreign exchange rate and growths / changes in relevant macroeconomic variables in India. However, there we have analyzed the roles of variables in determining and predicting India's exchange rate return by taking all the macro variables including exchange rate in their stationary values. Obviously, forecasts based on such a model would be meaningful for short-run periods only. Our objective in this chapter is to study the relationship, if any, at the level (nonstationary) values of the variables concerned i.e., between the nonstationary exchange rate and nonstationary macro variables, by using the cointegration methodology so that existence of such relation(s) would yield predictability of India's exchange rate in the long-run sense.

Use of cointegration analysis for checking structural models for exchange rate has been done by a large number of economists including MacDonald and Taylor (1994), MacDonald and Marsh (1997), Diamandis *et al.* (1998), , and Van Aarle *et al.* (2000) and Mark and Sul (2001). Kim and Mo (1995) used multivariate cointegration to generate forecasts of exchange rate and showed that the monetary models outperform the random walk model based on the vector error correction model. Other studies which have used cointegration methodology on exchange rate series are mainly related to studying the relationship between spot and forward exchange rates and to check market efficiency. Some relevant works on these lines are due to Baillie (1989) and Copeland (1991). Others such as Masih and Masih (1996) have tried to discern the dynamic causal chain among real output, money, interest rate, inflation and exchange rate. Kumah and Ibrahim (1996) have used real exchange rate for their cointegration analysis. They have adopted a multivariate data analysis approach to analyze the effects of domestic real (technological) and nominal shocks on the nominal exchange rate and the current account balance.

It may be worthwhile to note that the number of studies on cointegration involving India's foreign exchange rate with respect to US dollar and the related macro variables is few, and most of those are confined to studying cointegration in a bivariate set-up where relationship between exchange rate and a relevant macro variable has been studied. For instance, Pattanaik and Mitra (2001) used impulse response to study the relation between interest rate and exchange rate. Their study suggests that a one-standard deviation shock first results in an exchange rate appreciation to be followed subsequently by a depreciation of exchange rate. Hasan (2006a,b) has used cointegration-VECM approach to examine the long-run relationship between exchange rate of silver-based currencies and the intrinsic value of silver in India and Iran in a bivariate model set-up. While such bivariate studies may be useful, although only to a limited extent, from the point of view of economics and financial understanding, these would be constrained by the omission of other relevant variables from such relations and, to that extent, the cointegration results may be limited in their usefulness since there may very well be other variables having similar comovements.

Our approach, therefore, is to study the long run relation(s) involving exchange rate and all other relevant macro variables using a cointegration framework so that the similar comovements of all the variables including exchange rate could be captured in the multivariate set-up of cointegration. To that extent, the long-run relationship, if it exists, as well as the short-run dynamics given by the VECM would be more appropriate. Recently, there have been some more studies where the long-run relationship between stock price and exchange rate has been examined, but most of these are concerned with the developed economies. The major ones are due to Ajayi *et al.* (1998), Nieh and Lee (2001), Ki-Ho-Kim (2003), Phylaktis and Ravazzolo (2005). Insofar India is concerned, the relationship between stock market and exchange rate has been studied by Mishra (2004) and Damele *et al.* (2004). Mishra (2004) has attempted to examine whether stock market and foreign exchange market are related to each other or not by using the VECM framework in which interest rate and demand for money are also involved. He has found that there exists a unidirectional causality between exchange rate and interest rate and exchange rate and demand for money, but no Granger causality between exchange rate return and stock return. Damele *et al.* (2004) have analyzed the market integration involving stock market, foreign exchange market and bullion market. Their study shows that stock index and exchange rate have inverse relationship.

There have also been some recent studies on foreign exchange reserves and exchange rate such as that by Kasman and Ayhan (2007) where they have empirically shown, using data from Turkey, that there indeed exists a long-run relation between the two. Very recently, Choi and Park (2007) studied the causal relation between interest rate and exchange rate for some south-east Asian countries during the 1997 Asian currency crisis period to investigate the appropriateness of tight monetary policy in stabilizing exchange rates. Their study did not find evidence of any such relationship barring those for a few countries. Caporale *et al.* (2005) have examined the effects of increase in domestic interest rate on exchange rate during the Asian financial crisis. They have used a VECM to study the effects of monetary policy tightening on nominal exchange rate.

Ghosh (1998) has used various cointegration tests to examine the validity of the monetary model as a theory of long-run equilibrium condition for the exchange rate of India. Their study offers no evidence of long-run equilibrium relationship among the variables of the monetary model. Rao (2000) undertook a study to assess the two-way interactions between business cycles and exchange rate, and their paper provides an analytical framework which, by formalizing the nature of relationships between key macro-economic variables, helps to forecast the exchange rate. Vayyuri and Seshaiah (2004) have studied, using data for the period 1970-2002, the interaction of budget deficit of India with other macroeconomic variables such as nominal effective exchange rate, GDP, consumer price index and money supply, by using cointegration approach and the VECM. The results reveal that the variables under study are cointegrated, and that there is a bi-directional causality between budget deficit and nominal effective exchange rate. Vuyyuri (2005) has investigated the cointegrating relationship and the causality between the financial and real sectors of Indian economy using monthly observations of financial variables like interest rates, inflation rate, exchange rate, stock return and industrial productivity with the latter used as a proxy for the real sector. Thomakos and Bhattacharya (2005) have reported the results from a forecasting study for inflation, industrial output and exchange rate for India. They have used the ARIMA, bivariate transfer function and restricted VAR models in their study where data of different frequencies have been used.

As already stated, in this chapter, we study the long-run relationship(s), if any, between foreign exchange rate and the relevant macro variables for India, and this is done by applying the VAR based methodology developed by Johansen (1988, 1991,1995) and Johansen and Juselius (1990). But, since cointegrating relations do not appear explicitly in the VAR framework, a more convenient modelling set-up obtained by rewriting the VAR model, known as the vector error correction model (VECM), is used for cointegration analysis. Since selection of macro variables is important in this study, we have primarily chosen an initial set of core macro variables on the basis of our findings in Chapter 6 as well as on the importance of different macro variables from consideration of economics and finance. Thereafter, meaningful additions and / or deletions, as discussed in detail in Section 7.3, in the set of core macro variables have been made and computations done once again with those variables. Finally, the data used for this study are at monthly level frequency and the series is the same as the one in the preceding chapter *viz.*, India's exchange rate covering the period from November 1994 to March 2005. The presentation in this chapter is as follows. The next section presents the cointegration methodology very briefly. Empirical findings are discussed in Section 7.3. The chapter concludes with some observations in Section 7.4.

7.2 Cointegration Methodology

Introduced by Granger (1981), cointegration arises if several variables which are at least integrated of order 1, i.e., I (1) are driven by a common stochastic trend. The concept uses an important property of *I* (1) variables *viz*., there can be linear combination(s) of these variables that are *I* (0) and such variables are then said to be cointegrated. We associate linear combination(s) with cointegration even though theoretically it is quite possible that nonlinear long-run relationships exist among a set of integrated variables.

A requirement for cointegration is that all the variables must be integrated of the same order. The literature which has developed over the years primarily focusses on variables which are *I*(1) since there are evidences suggesting that there are very few economic variables which are integrated of higher orders. The time paths of cointegrated variables are influenced by the extent of any deviation from the long run equilibrium. Some variables may respond to the magnitude of disequilibrium in order for the system to revert back on the equilibrium. Thus, the short-run dynamics must be influenced by the deviation from the long-run relationship. This dynamic model is known as the vector error correction model (VECM).

Following Lütkepohl and Krätzig (2004), we introduce the basic vector autoregressive (VAR) model and vector error correction model (VECM) without including deterministic terms and exogenous variables. The basic vector autoregressive model of order *p* (VAR(*p*)) has the form

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \quad , t = 1,2,\dots, T \tag{7.1}$$

for a set of *K* time series variables where $y_t = (y_{1t}, y_{2t}, ..., y_{Kt})$ is the $(K \times 1)$ vector of observations on the *K* variables at time *t*, A_i, i = 1,2,…, p , are $(K \times K)$ coefficient matrices and $u_t = (u_{1t}, u_{2t}, ..., u_{Kt})'$ is the unobservable error term. It is assumed to be a zero-mean independent white noise process with time-invariant, positive definite covariance matrix $E(u_t u_t') = \Sigma_u$. In other words, the u_t's are independent stochastic vectors with $u_t \sim (0, \Sigma_u)$. Although the model (7.1) accommodates variables with stochastic trend, it is not suitable for studying cointegration relations. The convenient modelling set-up for cointegration analysis is the VECM form given by

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{p-1} \Delta y_{t-p+1} + u_t \tag{7.2}$$

where $\Pi = -(I_k - A_1 - ... - A_p)$ and $\Gamma_i = -(A_{i+1} + ... + A_p)$ for $i = 1,..., p-1$. The VECM is obtained from the levels VAR form in (6.1) by subtracting y_{t-1} from both sides and then rearranging the terms. Note that Δy_t does not contain stochastic trend by our assumption that all variables must be *I*(1). Thus the term Πy_{t-1} must also be *I*(0), in order that the system of equations is consistent, and this implies that it contains the cointegrating relations. This model is abbreviated as the VECM (*p*-1) and the Γ_j 's $(j = 1, 2,..., p-1)$ are often referred to as the short term parameters while the Πy_{t-1} as the long-term part of the VECM. Now, for Πy_{t-1} to be *I*(0), Π must not be of full rank. Let $rank(\Pi) = r(< K)$, then Π can be written as a product of $(K \times r)$ matrices α and β such that $rank(\alpha) = rank(\beta) = r$ i.e., $\Pi = \alpha\beta'$. Premultiplying $\Pi y_{t-1} = \alpha\beta' y_{t-1}$ with $(\alpha'\alpha)^{-1}\alpha'$ yields $\beta' y_{t-1}$ which is an *I*(0) process since premultiplying an *I*(0) vector by some matrix results in an *I*(0) process. Hence, $\beta' y_{t-1}$ contains the cointegrating relations. It follows from here that there are r linearly independent cointegrating relations among the components of y_t , and thus the rank of Π is referred to as the cointegrating rank and β the cointegration matrix. The matrix α is called the loading matrix[1] as it contains the weights attached to the cointegrating relations in the individual equations of the model. The matrices α and β are not unique and there can be many possible matrices containing the cointegrating relations. Using any nonsingular $(r \times r)$ matrix *B*, we can obtain new loading matrix αB and new cointegration matrix $\beta B'^{-1}$, which satisfy $\Pi = \alpha B(\beta B'^{-1})'$. Thus we can conclude that cointegrating relations with economic content cannot be extracted from observed series alone. Some nonsample information is also required to identify them uniquely.

[1] In the particular case where *K* =2, the scalar α is called the parameter representing the speed of adjustment to long-run.

7.2.1 Tests for Cointegration

Since cointegration necessitates that all variables be integrated of order 1[2], the first step in testing for cointegration requires testing for the order of integration of all the variables involved in this study. This is done by applying the augmented Dickey-Fuller (ADF) test or the Phillips-Perron (PP) test. Thereafter the test for cointegration is most often carried out by following Johansen's (1988,1991) approach. The two tests developed by him are the trace test and the maximum eigenvalue test. Sequential testing procedures based on likelihood ratio (LR)- type tests are made because Gaussian ML estimates for the reduced-form VECM are easy to compute for a given cointegrating rank and thus the LR test statistics are readily available. The sequence of hypotheses which is considered while testing are:

$$H_0(0): rank(\Pi) = 0 \quad \text{versus} \quad H_1(0): rank(\Pi) > 0 \tag{7.3}$$

$$H_0(1): rank(\Pi) = 1 \quad \text{versus} \quad H_1(0): rank(\Pi) > 1$$

$$\vdots$$

$$H_0(K-1): rank(\Pi) = K-1 \quad \text{versus} \quad H_1(K-1): rank(\Pi) = K.$$

The testing sequence terminates and the corresponding cointegrating rank is selected when the null hypothesis cannot be rejected for the first time. If the first null hypothesis in the sequence cannot be rejected, then it can be concluded that there is no cointegrating relation amongst the variables.

Similar to the results of unit root testing, under the Gaussian assumptions, the LR statistic under $H_0(r_0)$ is nonstandard. It depends on the difference $K - r_0$ and on the deterministic terms included in the DGP. The deterministic terms and shift dummy variables in the DGP have an impact on the null distributions of the LR tests. Therefore, the LR-type tests have been derived under different assumptions regarding the deterministic term. To that end, let us consider the model

[2] As already stated at the beginning of this section that although a requirement for cointegration is that all the variables must be integrated of the same order, the literature has essentially developed with I(1) variables.

$$y_t = \mu_0 + \mu_1 t + \tilde{y}_t \tag{7.4}$$

where $\tilde{y}_t$ is a VAR(p) process. Then there are three cases of interest and these are:

(i) μ_0 arbitrary and $\mu_1 = 0$, i.e., there is just a constant mean and no deterministic trend term. Here the mean adjusted y_t is seen to have the VECM form

$$\Delta y_t = \Pi(y_{t-1} - \mu_0) + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t \quad . \tag{7.5}$$

On the other hand, if the intercept term is retained, the VECM form turns out to be

$$\Delta y_t = \nu_0^* + \Pi y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t \tag{7.6}$$

$$= \Pi^* \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

where $\Pi^* = [\Pi : \nu_0^*]$ is $(K \times (K+1))$ with $\nu_0^* = -\Pi \mu_0$. Due to the absence of a deterministic trend term, the intercept is absorbed in the cointegrating relations, and thus $\Pi^* = \alpha \beta^{*\prime}$ has rank *r*. Johansen (1995) has considered the intercept version (7.6) and provided critical values for the LR test statistic which has a nonstandard limiting distribution under H_0. This test, known as the LR test, is given by

$$LR(r_0) = -T \sum_{j=r_0+1}^{K} \log(1 - \hat{\lambda}_j) \tag{7.7}$$

where T is the total number of sample observations and $\hat{\lambda}_j$'s are the eigenvalues obtained by applying the reduced rank (RR) regression technique. Säikkonen and Luukkonen (1997) and Säikkonen and Lütkepohl (2000b) have used two-step procedures where the mean term is estimated by a feasible GLS procedure and then substituted for μ_0 in (7.5). Then an LR-type test based on the RR regression of (7.5) yields the test statistic which has an asymptotic distribution that is different from the one obtained for the intercept version in (7.6)

(ii) In presence of a linear deterministic trend in the DGP i.e., $\mu_1 \neq 0$, if the trend is confined to individual variables but is absent from the cointegration relations i.e., $\beta'\mu_1 = 0$, the VECM takes the form[3]

$$\Delta y_t - \mu_1 = \Pi(y_{t-1} - \mu_0) + \sum_{j=1}^{p-1} \Gamma_j (\Delta y_{t-j} - \mu_1) + u_t \quad . \tag{7.8}$$

Rearranging the constant term, we have

$$\Delta y_t = \nu_0 + \Pi y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t \tag{7.9}$$

where $\nu_0 = -\Pi\mu_0 + (\sum_{j=1}^{p} jA_j)\mu_1$. Saikkonen and Lütkepohl (2000a) have proposed a test based on the trend adjusted version (7.8). In this test, the mean and the trend parameters are estimated by a feasible GLS procedure and then the trend is subtracted from y_t. The test statistic is computed using the RR regression technique on equation (7.8). The critical values for the null distributions are tabulated in Siakkonen and Lütkepohl (2000). Johansen (1995), on the other hand, has proposed the LR test based on the intercept version given in (7.9).

(iii) This case arises if a fully unrestricted linear trend term is considered. This again gives rise to two VECMs.

$$\Delta y_t - \mu_1 = \Pi(y_{t-1} - \mu_0 - \mu_1(t-1)) + \sum_{j=1}^{p-1} \Gamma_j (\Delta y_{t-j} - \mu_1) + u_t \quad . \tag{7.10}$$

The other one is

$$\Delta y_t = \nu + \tilde{\Pi} \begin{bmatrix} y_{t-1} \\ t-1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t \tag{7.11}$$

where $\tilde{\Pi} = \alpha[\beta' : \eta]$ is a $(K \times (K+1))$ matrix of rank r with $\eta = -\beta'\mu_1$ and $\nu = -\Pi\mu_0 + (I_k - \Gamma_1 - ... - \Gamma_{p-1})\mu_1$. The latter VECM in (7.11) is obtained by rearranging the deterministic term in (7.10). Here again the test statistic can be

[3] See Lütkepohl (2004, p. 113) for details

obtained using the RR regression technique. For (7.10), the trend parameters are again estimated using GLS [see Saikkonen and Lütkepohl (2000b) for details, and Lütkepohl and Siakkonen (2000) for the critical values].

Instead of the pair of hypotheses in (7.3), one may alternatively test $H_0(r_0): rank(\Pi) = r_0$ versus $H_1(r_0): rank(\Pi) = r_0 + 1$. LR test for this pair of hypotheses was proposed by Johansen (1988, 1991), and it is known as the maximum eigen value test. This test is based on the statistic

$$LR_{\max}(r_0) = -T \ln(1 - \lambda_{r_0+1}). \tag{7.12}$$

This can be applied to the various cases discussed earlier. The limiting distribution of the statistics under the null hypothesis for the three cases are also nonstandard, and the critical values can be obtained from Osterwald-Lenum (1992) and Lütkepohl *et al.* (2001).

7.2.2 The VECM estimation

The two most important aspects of model specification in case of VECMs are the determination of the lag order and the cointegrating rank. Once these two are determined the model can be estimated. We have described the latter in the preceding section. As regards the former i.e., determination of the lag order, suffice it to say it is similar to the ones used for the univariate models. One popular approach is to start from a model with some prespecified maximum lag length and then applying tests sequentially to determine a suitable model order. Instead of sequential tests one may choose lag lengths by model selection procedures as well. The general approach here is to fit a VAR(m) model with all possible model orders and choose that order which minimizes the preferred criteria. The general form of the criterion in use for determining the lag length is

$$Cr(m) = \ln \det(\tilde{\Sigma}_u(m)) + \xi\phi(m) \tag{7.13}$$

where det(.) is the determinant, $\tilde{\Sigma}_u(m)$ is the residual covariance matrix estimator for model of order m, ξ is a sequence that depends on the sample size, and $\phi(m)$ is a function that penalizes large VAR orders. For Akaike's information criterion and

Schwarz's Bayesian criterion, the second component in (7.13), i.e., $\xi\phi(m)$, is $2mK^2/T$ and $mK^2 \ln T/T$, respectively.

The parameters of the VECM specified in (7.2) are obtained using a method called the reduced rank ML estimation. Following Lütkepohl and Krätzig (2004), we compactly write the VECM form (7.2) as

$$\Delta Y = \Pi Y_{-1} + \Gamma X + U \tag{7.14}$$

for a sample having T observations and p presample values, where $Y_{-1} = [y_0,..., y_{T-1}]$, $U = [u_1,...,u_T]$, $\Gamma = [\Gamma_1 : ... : \Gamma_{p-1}]$ and $X = [X_0,..., X_{T-1}]$ with

$$X_{t-1} = \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \end{bmatrix} .$$

Given a specific matrix Π, the equation-wise OLS estimator of Γ is thus

$$\hat{\Gamma} = (\Delta Y - \Pi Y_{-1}) X'(XX')^{-1} \quad . \tag{7.15}$$

Substituting $\hat{\Gamma}$ in (7.14), we get

$$\Delta YM = \Pi Y_{-1} M + \hat{U} \tag{7.16}$$

where $M = I - X'(XX')^{-1} X$. For a given integer r, 0< r < K, an estimator $\hat{\Pi}$ of Π with $rank(\hat{\Pi}) = r$ can be obtained by a method known as canonical correlation analysis or equivalently, reduced rank (RR) regression. Following Johansen (1995), we define

$$S_{00} = T^{-1} \Delta YM \Delta Y' \; , S_{01} = T^{-1} \Delta YMY'_{-1}, \; S_{11} = T^{-1} Y_{-1} MY'_{-1}$$

and then solve the generalized eigenvalue problem $\det(\lambda S_{11} - S'_{01} S_{00}^{-1} S_{01}) = 0$ to obtain the estimates.

Let the ordered eigenvalues be $\lambda_1 \geq ... \geq \lambda_K$ with corresponding matrix of eigenvectors $V = [b_1,...,b_K]$ satisfying $\lambda_i S_{11} b_i = S'_{01} S_{00}^{-1} S_{01} b_i$ and normalized such that $V' S_{11} V = I_k$. The reduced-rank estimator of $\Pi = \alpha\beta'$ is then obtained by choosing $\hat{\beta} = [b_1,...,b_r]$ and $\hat{\alpha} = \Delta YMY'_{-1}\hat{\beta}(\hat{\beta}'Y_{-1}MY'_{-1}\hat{\beta})^{-1}$. $\hat{\alpha}$ may be viewed as the OLS estimator of α from the model

$$\Delta YM = \alpha\hat{\beta}'Y_{-1}M + \tilde{U} \ .$$

The estimator of Π and Γ are $\hat{\Pi} = \hat{\alpha}\hat{\beta}'$ and $\hat{\Gamma} = (\Delta Y - \hat{\Pi}Y_{-1})X'(XX')^{-1}$, respectively. Under Gaussian assumption these estimators are the ML estimators conditional on the pre-sample values (Johansen (1988,1991,1995)).

The parameter estimate $\hat{\beta}$ is made unique here by normalization of the eigenvectors, and $\hat{\alpha}$ is adjusted accordingly. Since the identification restrictions are not econometric in nature only the cointegration space is estimated consistently. In order to estimate α and β, it is necessary to impose identifying restrictions. This is usually done by assuming the first part of β to be an identity matrix. Often this restriction amounts to normalizing the coefficient of the first variable to be 1. This normalization, however, requires some care in choosing the order of the variables. If the order of the variables are inappropriate, then it will lead to major distortions in the inferences from the model. Ideally, one chooses such an ordering of the variables that the cointegrating relations are economically interpretable when the normalization restrictions are imposed. Different orderings need to be checked and the one which leads to the most sensible set of cointegrating relations is finally chosen.

Another simple method for estimating the cointegration matrix is a simple two-step estimator. Briefly, this procedure is as follows. The matrix $\Pi = \alpha\beta'$ is partitioned as $\Pi = [\Pi_1 : \Pi_2]$, where Π_1 and Π_2 are of order $(K \times r)$ and $(K \times (K-r))$, respectively. In the first step, the short-term parameters, $\hat{\Gamma}$, are eliminated by substituting them with their OLS estimators, given Π, as in (7.15), and then the concentrated model (7.16) is considered. Using this model, Π is estimated by the OLS method. Denoting this estimator of Π by $\Pi = [\tilde{\Pi}_1 : \tilde{\Pi}_2]$ and using the estimator of α as $\tilde{\alpha} = \tilde{\Pi}_1$, the cointegration matrix β is estimated, in the second-step, by OLS from a suitable equation (see Lütkepohl and Krätzig (2004) for details). The usefulness of this method is not quite obvious since computation of the ML estimator seems quite simple. However, such a procedure of estimation is found to have advantages when restrictions are to be imposed on the cointegrating vectors.

7.3 Empirical results

Before proceeding to report the results of the cointegration exercise, we first state the variables which are to be included in this study. The set of variables, as stated earlier, is quite large and obviously it is not feasible to carry out the cointegration exercise with so many variables, particularly when the total number of observations is just 125. As discussed in Lütkepohl (2007), the dominant approach to analyzing systems of cointegrated variables is not well described by general-to-specific approach. He has advocated that for multivariate modelling and especially cointegration analysis, the leading approach should be specific to general. This is primarily because of the size of the general model. If all the variables and lags of potential interest were included in a VAR model from which to start the reduction procedure, there may be degrees of freedom problem in estimating the model. Therefore, in multivariate dynamic modelling, the strategy should be to limit the variables to be included in the analysis as much as possible and not to build a large overall model, but to build small models which capture only specific features of interest. This may also sometimes lead to large initial models. So what should be done is to have some ordering of the variables in mind according to their importance for the problem at hand. Then the exercise should start from a small set of core variables and the variables of lesser importance to be included only later to see whether these in groups or individually change the main conclusions of the small core model.

In our case, the core model includes some of those variables which were found to be significant in predicting exchange rate return in Chapter 6 and which are considered to be very important in exchange rate market from consideration of economics and finance. We take those variables at their level values and see whether all the variables are $I(1)$. Our understanding behind such a choice is that since these variables at their transformed $I(0)$ level were found to have significant effects in short term prediction in the single equation model set-up, these variables at their $I(1)$ level values may be found to be relevant in obtaining the long-run i.e., cointegrating relation(s) as well. We thus take the combination of these core variables out of the full set and check for the existence of cointegrating relation(s) among these variables, and then consider all possible combinations of those of the remaining variables of the set which are less important from consideration of determining a model for India's exchange rate, for choosing the final cointegrating variable(s), provided, of course, computations without too many identifying restrictions are possible[4].

We note from the preceding chapter that the macro variables (at stationary level) which have been found to have significant effects in predicting return on exchange rate are return on BSESENSEX , SPUSD, change in TR, change in CMR, change in FINV, growth in M1 and return on NASDAQ. Importance of these macro variables in foreign exchange market has also been discussed in the preceding chapter. Thus, our cointegration exercise begins with this set of macro variables at their level values.

At the very beginning of any cointegration exercise, it is required to check that the variables under study are $I(1)$. We recall from Table 6.1 in the preceding chapter that except SPUSD all the other six macro variables at their level values are $I(1)$. Likewise, in the set of all other remaining macro variables, all but GFD are $I(1)$ in their level values. In Table 7.1, we present the usual descriptive statistics of the macro variables along with the ADF test-statistic values, but this time these computations refer to their level values without any seasonal adjustment. Note that the values of the ADF test is slightly different from those presented in Table 6.1 of the previous chapter, since in Chapter 6 seasonal adjustments had been done, for some of the variables. In Table 7.1, the first column indicates whether the variables have been taken in their level or log-level values.

Figures 7.1 to 7.18 give the plots of the macroeconomic variables of interest *viz*., exchange rate (EXRATE), Indian stock prices (BSESENSEX), call money rate (CMR), reserve money (M0), narrow money (M1), consumer price index (CPI), foreign currency asset (FCA), total reserve (TR), industrial production (IP), gross fiscal deficit (GFD), sale/ purchase of USD (SPUSD), federal fund rate (FFR), six

[4] All computations on cointegration have been done by using the JMulTi software developed by Krätzig (2004).

month treasury bill rate of US (TBUS6), three month treasury bill rate (TBUS3), US stock price index (NASDAQ), world gold price (WGP), foreign institutional investment (FII), total foreign investment (FINV).

Since SPUSD has been found to be $I(0)$ at level values along with GFD, we have dropped both the macro variables from the purview of our cointegration exercise. Thus, the set of core macro variables is : EXRATE, TR, BSESENSEX (or BSE in short), CMR, FINV, M1 and NASDAQ. Now, what we have done is that we have considered these variables and run cointegration exercise involving these variables along with other deterministic components, as already discussed. Thereafter, many other combinations of the remaining variables which are supposedly less important in explaining the long-run behaviour of foreign exchange rate of India have been considered for cointegration subject to computations being possible without too many identifying restrictions. Based on the computations, we have found that there are only three cointegrating relations existing and all the three are economically meaningful and plausible. It may be worthwhile to note that in all these computations, the role of M1 has been found to be rather erratic and often insignificant. Hence, the results are presented dropping M1 from the set of core variables.

Table 7.1

Descriptive statistics of the macroeconomic variables and results of unit root test

Variable	Mean	Standard deviation	Skewness coefficient	Kurtosis coefficient	ADF test statistic value	Critical value
EXRATE (log-level)	3.729	0.136	-0.731	2.218	-2.356	-3.484
BSE (log-level)	8.253	0.215	0.812	2.813	-2.289	-4.035
CMR (level)	8.234	4.726	2.976	14.767	-3.211	-4.036
M0 (log-level)	12.495	0.307	0.099	1.897	-2.539	-4.038
M1 (log-level)	12.703	0.365	0.063	1.888	-4.023	-4.036
CPI (log-level)	6.029	0.175	-0.547	2.088	-2.776	-3.485
FCA (log-level)	11.956	0.719	0.303	1.887	-2.494	-4.034
TR (log-level)	12.067	0.664	0.404	1.924	-1.982	-4.034
IP (log-level)	5.046	0.170	0.038	2.277	-2.471	-4.040
GFD* (level)	9001.224	7408.532	0.845	4.446	-7127	-4.037
SPUSD* (level)	3121.841	5681.775	1.679	8.037	-4.176*	-3.484
FFR (level)	4.099	1.936	-0.557	1.602	-1.093	-2.582
TBUS6 (level)	3.900	1.784	-0.519	1.640	-1.129	-2.583
TBUS3 (level)	3.812	1.783	-0.538	1.620	-1.328	-2.582
NASDAQ (log-level)	7.462	0.404	0.236	2.950	-2.095	-3.484
WGP (log-level)	9.524	0.154	0.867	2.475	1.630	-2.582
FII (level)	242.696	484.800	2.756	12.826	-3.332	-3.485
FINV (level)	526.624	508.959	2.463	11.380	-2.969	-3.485

*Note: Descriptive statistics are given for the time series at the level values of the macroeconomic variables used in the analysis. * indicates that the concerned time series is stationary at level values. The ADF test statistic is obtained for the level values of all the variables, and the estimating equations contain constant and a linear trend.*
The last column shows MacKinnon 1% critical values for rejection of hypothesis of a unit root.

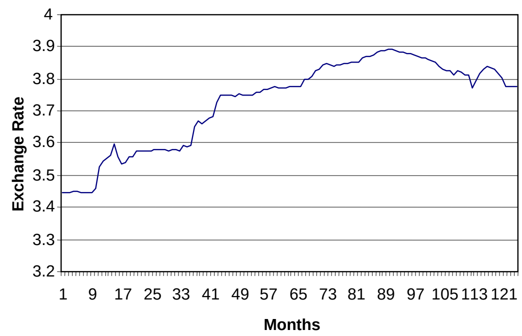


Fig 7.1 Time series of exchange rate

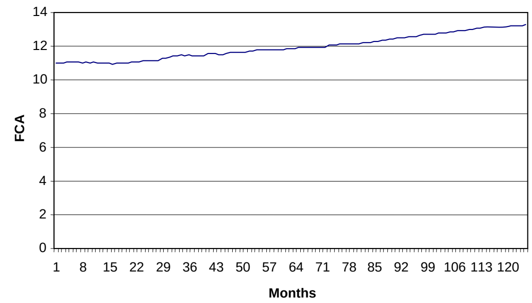


Fig 7. 2 Time series of foreign currency asset

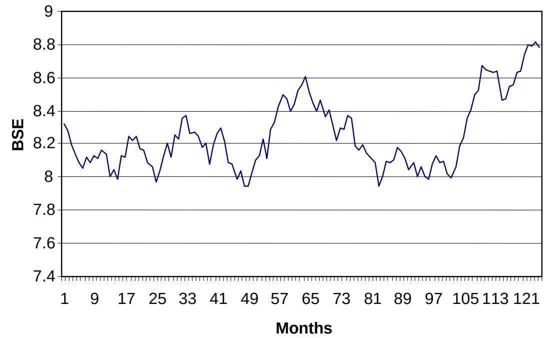


Fig 7. 3 Time series of BSE

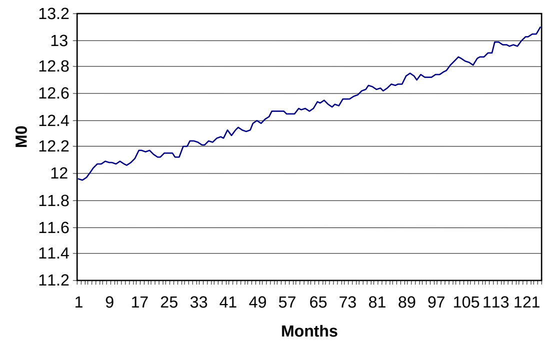


Fig 7. 4 Time series of M0

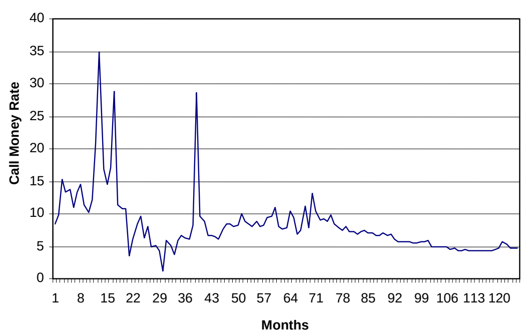


Fig 7. 5 Time series of call money rate (CMR)

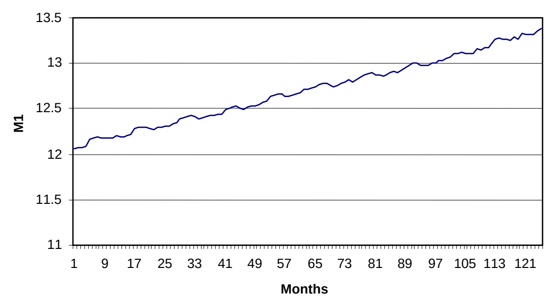


Fig 7. 6 Time series of M1

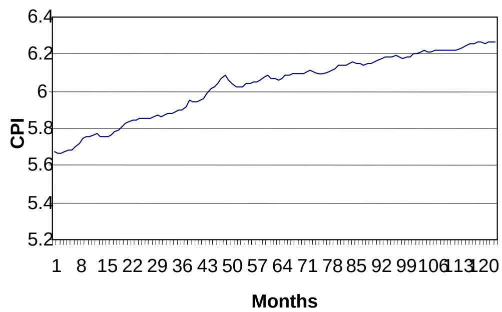


Fig 7.7 Time series of consumer price index (CPI)

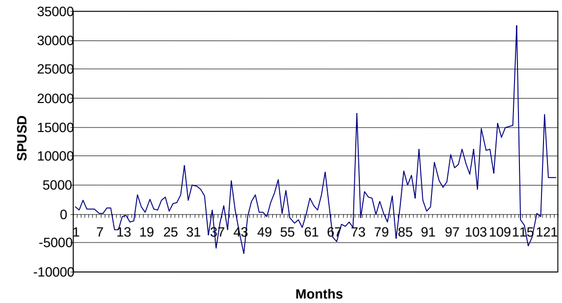


Fig 7.8 Time series of sale / purchase of USD

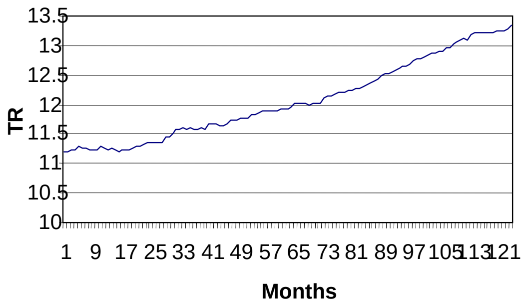


Fig 7.9 Time series of total reserve

Fig 7.10 Time series of world gold price

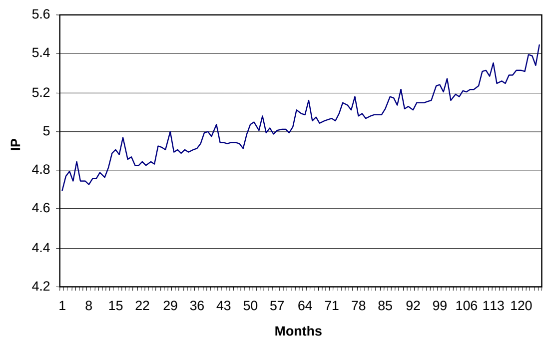


Fig 7.11 Time series of industrial production

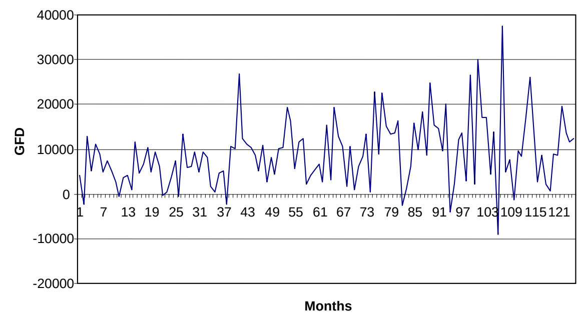


Fig 7.12 Time series of gross fiscal deficit

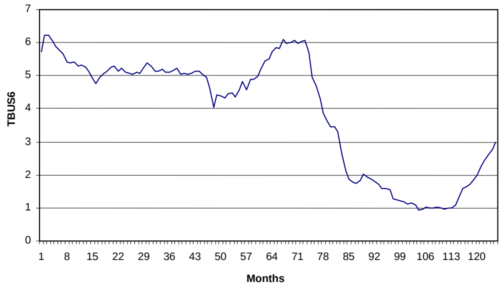


Fig 7.13 Time series of treasury bill US for 6 months

Fig 7.14 Time series of treasury bill US for 3 months

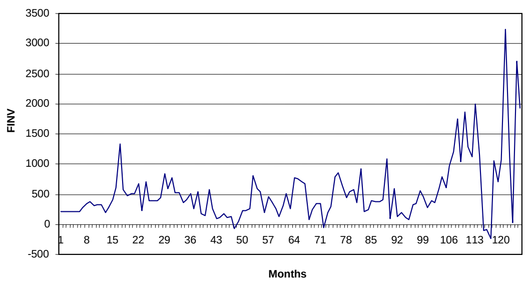


Fig 7. 15 Time series of foreign investment (FINV)

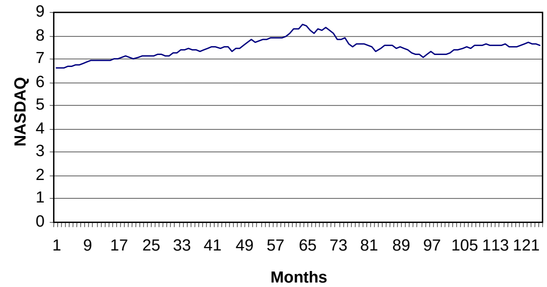


Fig 7.16 Time series of NASDAQ

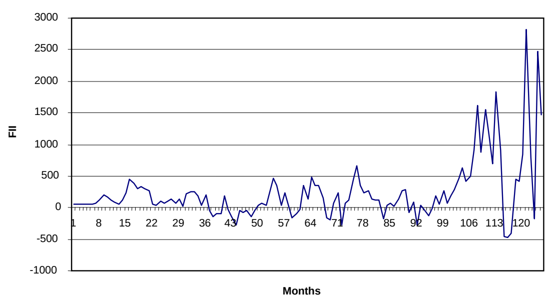


Fig 7.17 Time series of foreign institutional investment (FII)

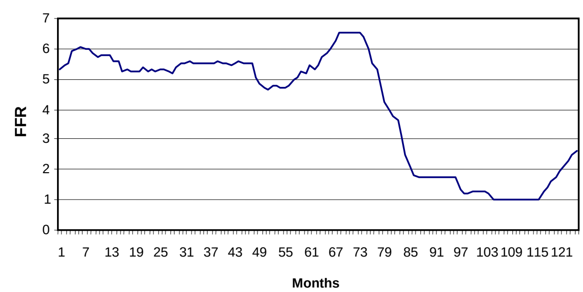


Fig 7.18 Time series of federal funds rate (FFR)

We now discuss the results for the first set of variables comprising exchange rate (EXRATE), CMR, BSE, NASDAQ, TR and FINV. Now, as we discussed in Section 7.2, it is first necessary to determine the cointegrating rank along with the endogenous lags. These are essential for model specification, and the estimation is to be done thereafter only. We have used both the trace test and maximum eigenvalue test for determining the cointegrating rank. In all the computations required for estimating the parameters involved in this cointegrating exercise, we have used Johansen's intercept version. Further, in the model specification, we have included deterministic terms comprising a constant term, linear trend / orthogonal trend and also some seasonal dummies- the latter from consideration of the fact that exchange rate series may exhibit some kind of monthly effects.

As regards the choice of lag value, we restricted it upto a maximum of 4. Since our number of observations is very moderate *viz*., 125 only, it is not computationally feasible to run cointegration tests with any higher lag values. Using the usual criteria for selecting the proper lag length, it was found that inclusion of lag value upto 2 was adequate. Table 7.2 gives the test statistic values for this set as well as for the other two sets for which cointegration has been found to exist. With this choice of the lag value of 2, the trace statistic value, obtained under the model having constant, trend (but the trend is not orthogonal) and seasonal dummies, for the null hypothesis that the cointegrating rank is zero i.e., $H_0 : r_0 = 0$, is 174.69, which is much larger than the 1% critical value of 124.75 (*cf*. Osterwald-Lenum (1992)). Thus, H_0 is rejected in favour of the alternative of $r_0 > 0$. Again for $H_0 : r_0 = 1$, the test statistic has the value 113.84, but the corresponding tabulated value at 1 % level of significance is 96.58, and hence this null hypothesis is again rejected in favour of the alternative hypothesis that the rank of Π is greater than 1. Finally, the null of $H_0 : r_0 = 2$ cannot be rejected as the value of the test statistic obtained is 57.32, which is much smaller than even the 5% critical value of 62.99. Thus, based on the trace test, we conclude that the rank of Π is 2 i.e., there are two cointegrating relations for this combination of core macro variables.

Table 7.2

Johansen's trace and maximum eigenvalue test statistics for cointegration

EXRATE, CMR, BSE, NASDAQ, TR and FINV					
Lags	r_0	Constant, trend, seasonal dummies		Orthogonal trend, seasonal dummies	
		Trace	Maximum eigenvalue	Trace	Maximum eigenvalue
2	0	174.69***	60.85***	146.35***	56.84***
	1	113.84***	56.52***	89.51***	47.39***
	2	57.32	24.16	42.12	24.07
EXRATE, NASDAQ,BSE,TR,CMR and CPI					
3	0	146.82***	52.23***	132.08***	51.31***
	1	94.59**	36.83*	80.77***	35.14**
	2	57.76	25.76	45.63	22.56
EXRATE, FFR,BSE,NASDAQ and CMR					
2	0	119.46***	50.52***	102.24***	50.52***
	1	68.94**	34.91**	51.72**	27.21**
	2	34.03	18.21	24.51	13.40

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

We also performed the maximum eigenvalue test to find the rank of Π. Comparing the values of this test statistic which are also presented in 7.2 with the corresponding critical values as obtained from Ostewald-Lenum (1992), we find the rank of Π to be the same as obtained for the trace test i.e., 2. Further, this finding on rank of Π remains the same irrespective of whether the assumed linear trend is orthogonal or not. As the cointegration rank and lag order have been determined, we present below the estimated VECM as well as the two cointegrated relations for the system of variables comprising EXRATE, CMR, BSESENSEX, NASDAQ, TR, FINV, in equations (7.17) and (7.18), respectively.

$$
\begin{bmatrix} \Delta EXRATE_t \\ \Delta CMR_t \\ \Delta BSE_t \\ \Delta NASDAQ_t \\ \Delta TR_t \\ \Delta FINV_t \end{bmatrix}
=
\begin{bmatrix}
\underset{(1.025)}{-0.034} & \underset{(0.869)}{0.000} \\
\underset{(4.028)^{***}}{28.923} & \underset{(7.167)^{***}}{-0.554} \\
\underset{(4.586)^{***}}{-0.814} & \underset{(0.341)}{0.001} \\
\underset{(0.794)}{-0.182} & \underset{(0.554)}{-0.001} \\
\underset{(2.878)^{***}}{-0.174} & \underset{(2.043)^{**}}{-0.001} \\
\underset{(3.539)^{***}}{-3682.531} & \underset{(0.642)}{7.193}
\end{bmatrix}
\left[
\begin{bmatrix}
1 & - & \underset{(3.091)^{***}}{0.192} & \underset{(2.107)^{**}}{-0.076} & \underset{(2.209)^{**}}{0.222} & \underset{(3.412)^{***}}{0.000} \\
- & 1 & \underset{(2.138)^{**}}{11.861} & \underset{(1.306)}{-4.210} & \underset{(1.562)}{-14.056} & \underset{(1.010)}{-0.002}
\end{bmatrix}
\right.
$$

$$
\left.
\begin{bmatrix} EXRATE_{t-1} \\ CMR_{t-1} \\ BSE_{t-1} \\ NASDAQ_{t-1} \\ TR_{t-1} \\ FINV_{t-1} \end{bmatrix}
+
\begin{bmatrix}
\underset{(7.197)^{***}}{-7.053} & \underset{(4.050)^{***}}{-0.008} \\
\underset{(0.817)}{71.715} & \underset{(1.669)^{*}}{0.286}
\end{bmatrix}
\begin{bmatrix} Const \\ Trend \end{bmatrix}
\right]
+
\begin{bmatrix}
\underset{(2.642)^{***}}{0.257} & \underset{(0.749)}{0.000} & \underset{(0.692)}{0.013} \\
\underset{(1.296)}{27.589} & \underset{(1.989)}{-0.153} & \underset{(2.583)^{***}}{10.361} \\
\underset{(0.805)}{-0.424} & \underset{(0.594)}{-0.001} & \underset{(0.580)}{-0.057} \\
\underset{(0.540)}{0.366} & \underset{(0.982)}{0.002} & \underset{(0.335)}{0.043} \\
\underset{(2.624)^{***}}{-0.469} & \underset{(0.605)}{0.000} & \underset{(1.285)}{0.043} \\
\underset{(0.191)}{-588.377} & \underset{(0.866)}{-9.672} & \underset{(2.526)^{**}}{1468.648}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\underset{(1.094)}{-0.016} & \underset{(1.008)}{-0.052} & \underset{(0.321)}{0.000} \\
\underset{(1.222)}{-3.794} & \underset{(3.822)^{***}}{-43.142} & \underset{(3.416)^{**}}{-0.002} \\
\underset{(0.977)}{0.075} & \underset{(0.090)}{-0.025} & \underset{(3.030)^{***}}{0.000} \\
\underset{(0.372)}{0.037} & \underset{(1.049)}{-0.377} & \underset{(0.283)}{0.000} \\
\underset{(0.842)}{-0.022} & \underset{(0.393)}{0.037} & \underset{(0.236)}{0.000} \\
\underset{(1.426)}{-641.988} & \underset{(1.019)}{-1666.119} & \underset{(4.309)^{***}}{-0.415}
\end{bmatrix}
\begin{bmatrix}
\Delta EXRATE_{t-1} \\ \Delta CMR_{t-1} \\ \Delta BSE_{t-1} \\ \Delta NASDAQ_{t-1} \\ \Delta TR_{t-1} \\ \Delta FINV_{t-1}
\end{bmatrix}
+
\begin{bmatrix}
\underset{(0.224)}{0.023} & \underset{(2.708)^{***}}{0.001} & \underset{(0.947)}{0.018} \\
\underset{(6.956)^{***}}{157.844} & \underset{(0.987)}{-0.067} & \underset{(1.778)^{*}}{7.322} \\
\underset{(1.719)}{-0.965} & \underset{(0.881)}{-0.001} & \underset{(2.344)^{**}}{0.239} \\
\underset{(1.339)}{-0.968} & \underset{(1.028)}{0.002} & \underset{(1.026)}{0.135} \\
\underset{(0.074)}{0.014} & \underset{(1.002)}{0.001} & \underset{(2.069)^{**}}{0.072} \\
\underset{(0.094)}{307.671} & \underset{(0.907)}{-8.956} & \underset{(1.996)^{**}}{1191.356}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\underset{(0.725)}{0.011} & \underset{(0.583)}{-0.032} & \underset{(0.089)}{0.000} \\
\underset{(0.581)}{-1.843} & \underset{(2.178)^{**}}{-26.255} & \underset{(1.447)}{-0.001} \\
\underset{(1.379)}{-0.108} & \underset{(0.238)}{0.071} & \underset{(0.448)}{0.000} \\
\underset{(0.762)}{-0.077} & \underset{(0.088)}{0.034} & \underset{(0.664)}{0.000} \\
\underset{(1.713)^{*}}{-0.046} & \underset{(0.197)}{-0.020} & \underset{(1.505)}{0.000} \\
\underset{(0.165)}{75.727} & \underset{(0.055)}{96.827} & \underset{(5.599)^{***}}{-0.517}
\end{bmatrix}
\begin{bmatrix}
\Delta EXRATE_{t-2} \\ \Delta CMR_{t-2} \\ \Delta BSE_{t-2} \\ \Delta NASDAQ_{t-2} \\ \Delta TR_{t-2} \\ \Delta FINV_{t-2}
\end{bmatrix}
+
\begin{bmatrix}
\underset{(0.385)}{0.002} & \underset{(0.479)}{-0.002} & \underset{(1.761)}{-0.008} \\
\underset{(0.193)}{0.197} & \underset{(0.625)}{-0.601} & \underset{(0.038)}{0.036} \\
\underset{(1.618)}{-0.041} & \underset{(1.279)}{-0.030} & \underset{(3.672)^{***}}{-0.086} \\
\underset{(0.703)}{0.023} & \underset{(1.334)}{-0.041} & \underset{(0.818)}{-0.025} \\
\underset{(1.247)}{-0.011} & \underset{(1.555)}{-0.013} & \underset{(3.249)^{***}}{0.026} \\
\underset{(2.608)^{**}}{-386.029} & \underset{(0.592)}{-82.397} & \underset{(0.303)}{41.661}
\end{bmatrix}
$$

$$
\begin{matrix}
\underset{(1.863)}{0.009} & \underset{(2.333)^{**}}{0.012} & \underset{(0.909)}{0.004} & \underset{(0.153)}{0.001} & \underset{(0.373)}{0.002} \\
\underset{(0.968)}{-1.060} & \underset{(0.907)}{0.978} & \underset{(1.446)}{-1.457} & \underset{(3.535)^{***}}{-3.692} & \underset{(0.816)}{-0.873} \\
\underset{(2.900)^{***}}{-0.079} & \underset{(2.152)^{**}}{-0.057} & \underset{(1.153)}{-0.029} & \underset{(2.163)^{**}}{-0.056} & \underset{(2.415)^{**}}{-0.064} \\
\underset{(0.225)}{0.008} & \underset{(0.268)}{-0.009} & \underset{(0.537)}{0.017} & \underset{(1.078)}{-0.036} & \underset{(0.913)}{-0.031} \\
\underset{(0.803)}{-0.007} & \underset{(0.431)}{0.004} & \underset{(1.616)}{-0.014} & \underset{(1.459)}{-0.013} & \underset{(1.798)}{-0.016} \\
\underset{(1.768)^{*}}{-280.582} & \underset{(1.385)}{-216.318} & \underset{(3.780)^{***}}{-551.975} & \underset{(2.385)^{**}}{-361.085} & \underset{(1.226)}{-190.001}
\end{matrix}
$$

$$
\left.\begin{matrix}
\underset{(1.778)^{*}}{0.008} & \underset{(0.347)}{-0.002} & \underset{(1.649)}{0.008} \\
\underset{(0.440)}{-0.453} & \underset{(0.319)}{0.331} & \underset{(0.528)}{0.540} \\
\underset{(3.337)^{***}}{-0.085} & \underset{(2.842)^{***}}{-0.073} & \underset{(2.123)^{**}}{-0.054} \\
\underset{(1.039)}{-0.034} & \underset{(0.497)}{0.016} & \underset{(0.484)}{0.016} \\
\underset{(0.256)}{-0.002} & \underset{(1.329)}{-0.012} & \underset{(0.555)}{0.005} \\
\underset{(1.930)^{*}}{-288.573} & \underset{(0.443)}{-66.681} & \underset{(0.529)}{-78.306}
\end{matrix}\right]
\begin{bmatrix} S_{1t} \\ S_{2t} \\ S_{3t} \\ S_{4t} \\ S_{5t} \\ S_{6t} \\ S_{7t} \\ S_{8t} \\ S_{9t} \\ S_{10t} \\ S_{11t} \end{bmatrix}
+
\begin{bmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \\ \hat{u}_{3t} \\ \hat{u}_{4t} \\ \hat{u}_{5t} \\ \hat{u}_{6t} \end{bmatrix}
\qquad (7.17)
$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

We note that (7.17) contains two cointegrating relations, and a constant and a trend term in the long run relationship. Since exchange rate is chosen as the first variable in this model, the coefficient associated with this variable has been automatically normalized to 1, and that of CMR to 0 in the first cointegrated relation. And, in the second cointegrating relation, the variable CMR has coefficient value 1 and EXRATE 0. However, since our primary interest is to explain and model the long-run behaviour of exchange rate, we do not discuss the second cointegrating relation. As discussed earlier, the loading matrix contains the weights attached to the cointegrating relations in the individual equations of the model, and these can be used for assessing whether the cointegration relations resulting from our normalization enter a specific equation significantly. Now, insofar as our empirical results are concerned, we find from equation (7.17), that some of the loading coefficients are significant. In particular, the loading coefficients associated with call money rate, BSE, total reserve and foreign investment i.e., 28.923, -0.814, -0.174 and -3682.531 are all significant. Thus, we can conclude that the cointegrating relation is an important component in this equation of VECM. It is also to be noted that both the constant and trend are significant in the cointegrating relation. This is quite an important empirical finding. This suggests that at the level of relationships involving $I(1)$ variables, the time series of exchange rate contains a deterministic trend term, apart from a constant. We also note that some of the lagged differences of the variables, such as that of CMR, TR, FINV and BSE are significant[5]. Obviously, these statistical significances establish the causal effect of these variables on exchange rate. The seasonal dummies are also found to be significant in some of the cases. For instance, the third seasonal dummy is found to be significant in the error correction part. Thus, the presence of a long-run cointegrating relation between foreign exchange rate of India and four macro variables called the BSE (domestic stock prices), NASDAQ (foreign stock prices), total reserve and foreign investment, has been found. This cointegrating relation obtained from the ML estimation with the coefficient of exchange rate normalized to one and CMR to zero is given by

$$\begin{aligned} EXRATE_t = \underset{(7.197)^{***}}{7.053} + \underset{(4.050)^{***}}{0.008}\ trend - \underset{(3.091)^{***}}{0.192}\ BSE_t + \underset{(2.107)^{**}}{0.076}\ NASDAQ_t \\ - \underset{(2.209)^{**}}{0.222}\ TR_t - \underset{(3.412)^{***}}{0.001}\ FINV + EC_t \end{aligned} \tag{7.18}$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

where EC_t is the error correction term. As discussed toward the end of this section, the chosen contegrating relationship (7.18) appears quite meaningful from consideration of theories of economics and finance involving foreign exchange rate on the one hand and the stated macro variables on the other.

The second set of macro variables for which cointegration has been found to exist is EXRATE, NASDAQ, BSE, TR, CMR and CPI. It is to be noted that as compared to the first set, there is now only one change *viz.*, CPI instead of FINV, in this set of variables. Needless to mention that CPI is an important variable, much as like FINV is.

As in the case of first set of macro variables, the choice of lag value was restricted upto a maximum of 4, but unlike the first case, the lag length of 3 was now found to be appropriate. We find from the entries in Table 7.2 corresponding to this set of variables that for this choice of lag value of 3 and constant, linear trend and seasonal dummies in the deterministic term, the null hypothesis $H_0 : r_0 = 0$ is rejected in favour of $H_1 : r_0 > 0$ since the trace test and maximum eigenvalue test statistics

[5] The stationary variable, return on NASDAQ at lag 2 is barely significant, that too at 10 per cent level, in one of the short-run (VECM) equations only.

yield values of 146.82 and 52.23 which are greater than the corresponding critical values of 124.75 and 49.51, respectively at 1 per cent level of significance. The next test of $H_0 : r_0 = 1$ against $H_1 : r_0 > 1$ shows that the trace test statistic value is 94.59 which is, once again higher than the corresponding critical value of 87.31 at 5 per cent (as well as 91.06 at 7.5 per cent) level of significance. Hence $H_0 : r_0 = 1$ is rejected in favour of $H_1 : r_0 > 1$. In terms of the maximum eigenvalue test also, the same conclusion holds. The next test, however, suggests that $H_1 : r_0 = 2$ cannot be rejected in favour of even 10 per cent level by both the test statistics, the computed values being 57.76 and 25.76 as against the corresponding critical values of 59.14 and 29.12, respectively. The conclusion, therefore, is that rank of Π is 2 for this set of variables also. As evident from Table 7.2, the same conclusion on rank holds when the deterministic specification is changed to the one having orthogonal trend i.e., linear trend parameter is orthogonal to cointegration matrix, along with seasonal dummies. The VECM for EXRATE, NASDAQ, BSE, TR, CMR and CPI are presented in 7.19.

$$\begin{bmatrix} \Delta EXRATE_t \\ \Delta NASDAQ_t \\ \Delta BSE_t \\ \Delta TR_t \\ \Delta CMR_t \\ \Delta CPI_t \end{bmatrix} = \begin{bmatrix} \underset{(2.464)^{**}}{-0.128} & \underset{(0.084)}{-0.011} \\ \underset{(1.269)}{0.467} & \underset{(0.653)}{-0.049} \\ \underset{(2.852)^{***}}{-0.782} & \underset{(4.790)^{***}}{0.270} \\ \underset{(0.119)}{0.012} & \underset{(0.586)}{-0.012} \\ \underset{(3.633)^{***}}{41.034} & \underset{(1.124)}{2.607} \\ \underset{(3.603)^{***}}{0.087} & \underset{(0.935)}{-0.005} \end{bmatrix} \begin{bmatrix} 1 & - & \underset{(3.814)^{***}}{0.081} & \underset{(5.480)^{***}}{0.319} & \underset{(6.406)^{***}}{-0.010} & \underset{(4.622)^{***}}{-0.769} \\ - & 1 & \underset{(3.348)^{***}}{-1.392} & \underset{(6.380)^{***}}{1.818} & \underset{(3.938)^{***}}{-0.030} & \underset{(5.798)^{***}}{-4.726} \end{bmatrix}$$

$$
\begin{bmatrix} EXRATE_{t-1} \\ NASDAQ_{t-1} \\ BSE_{t-1} \\ TR_{t-1} \\ CMR_{t-1} \\ CPI_{t-1} \end{bmatrix}
+
\begin{bmatrix}
0.382 & -0.012 & 0.020 & 0.009 & -0.001 & 0.049 \\
(3.554)^{***} & (0.801) & (1.005) & (0.174) & (2.135)^{***} & (0.259) \\
-0.287 & 0.027 & -0.016 & -0.402 & 0.006 & 0.323 \\
(0.378) & (0.247) & (0.112) & (1.113) & (1.802)^{*} & (0.239) \\
-0.905 & -0.033 & 0.006 & -0.112 & -0.003 & -1.728 \\
(1.602) & (0.402) & (0.058) & (0.418) & (1.105) & (1.719)^{*} \\
-0.441 & 0.006 & -0.005 & 0.002 & -0.001 & -0.083 \\
(2.062)^{**} & (0.193) & (0.131) & (0.020) & (0.961) & (0.219) \\
25.961 & -5.176 & 10.563 & -40.040 & -0.087 & 14.728 \\
(1.115) & (1.535) & (2.484)^{**} & (3.618)^{***} & (0.874) & (0.355) \\
0.053 & 0.002 & -0.010 & -0.052 & 0.001 & 0.560 \\
(1.071) & (0.221) & (1.133) & (2.208)^{**} & (1.276) & (6.340)^{***}
\end{bmatrix}
$$

$$
\begin{bmatrix} \Delta EXRATE_{t-1} \\ \Delta NASDAQ_{t-1} \\ \Delta BSE_{t-1} \\ \Delta TR_{t-1} \\ \Delta CMR_{t-1} \\ \Delta CPI_{t-1} \end{bmatrix}
+
\begin{bmatrix}
0.035 & 0.013 & 0.006 & 0.042 & 0.001 & -0.008 \\
(0.334) & (0.896) & (0.311) & (0.762) & (1.259) & (0.039) \\
-1.380 & -0.087 & 0.103 & 0.004 & 0.004 & 0.119 \\
(1.885) & (0.818) & (0.734) & (0.010) & (1.488) & (0.081) \\
-0.704 & -0.213 & 0.331 & -0.038 & -0.005 & 2.115 \\
(1.293) & (2.703)^{***} & (3.162)^{***} & (0.132) & (2.348)^{**} & (1.930)^{*} \\
-0.010 & -0.017 & 0.029 & 0.049 & 0.001 & 0.169 \\
(0.048) & (0.567) & (0.734) & (0.447) & (0.328) & (0.408) \\
133.796 & 2.213 & 8.366 & -25.520 & -0.012 & 11.561 \\
(5.961)^{***} & (0.682) & (1.940)^{*} & (2.138)^{**} & (0.146) & (0.256) \\
-0.019 & -0.009 & 0.015 & -0.065 & 0.001 & -0.269 \\
(0.398) & (1.344) & (1.677)^{*} & (2.544)^{**} & (0.747) & (2.792)^{***}
\end{bmatrix}
$$

$$
\begin{bmatrix} \Delta EXRATE_{t-2} \\ \Delta NASDAQ_{t-2} \\ \Delta BSE_{t-2} \\ \Delta TR_{t-2} \\ \Delta CMR_{t-2} \\ \Delta CPI_{t-2} \end{bmatrix}
+
\begin{bmatrix}
-0.069 & 0.022 & -0.036 & 0.111 & 0.001 & -0.175 \\
(0.600) & (1.465) & (1.773)^{*} & (2.056)^{**} & (2.015)^{**} & (0.934) \\
-0.865 & 0.014 & -0.017 & -0.551 & 0.001 & 1.679 \\
(1.063) & (0.128) & (0.120) & (1.439) & (0.483) & (1.264) \\
1.451 & -0.141 & 0.318 & -0.408 & -0.001 & 1.435 \\
(2.397)^{**} & (1.787)^{*} & (2.987)^{***} & (1.432) & (0.450) & (1.452) \\
0.177 & 0.023 & 0.012 & 0.047 & 0.001 & 0.162 \\
(0.772) & (0.756) & (0.310) & (0.435) & (0.017) & (0.433) \\
-97.338 & -0.841 & -7.356 & 15.617 & -0.146 & 26.853 \\
(3.901)^{***} & (0.258) & (1.675)^{*} & (1.331) & (2.138)^{**} & (0.659) \\
-0.055 & -0.004 & -0.011 & -0.035 & 0.001 & 0.086 \\
(1.026) & (0.561) & (1.200) & (1.398) & (0.858) & (0.996)
\end{bmatrix}
$$

$$
\begin{bmatrix} \Delta EXRATE_{t-3} \\ \Delta NASDAQ_{t-3} \\ \Delta BSE_{t-3} \\ \Delta TR_{t-3} \\ \Delta CMR_{t-3} \\ \Delta CPI_{t-3} \end{bmatrix}
+ \begin{bmatrix}
\underset{(1.587)}{0.386} & \underset{(1.211)}{0.007} & \underset{(0.335)}{0.002} & \underset{(0.598)}{-0.003} & \underset{(2.455)^{**}}{0.014} \\
\underset{(1.143)}{-1.968} & \underset{(0.109)}{-0.004} & \underset{(1.221)}{-0.052} & \underset{(0.601)}{-0.023} & \underset{(0.688)}{-0.029} \\
\underset{(4.416)^{***}}{5.654} & \underset{(2.234)^{**}}{-0.065} & \underset{(0.467)}{-0.015} & \underset{(2.316)^{**}}{-0.067} & \underset{(2.594)^{***}}{-0.080} \\
\underset{(0.355)}{-0.172} & \underset{(1.635)}{-0.018} & \underset{(1.451)}{-0.017} & \underset{(2.082)^{**}}{0.023} & \underset{(0.033)}{-0.011} \\
\underset{(1.835)^{*}}{-96.852} & \underset{(0.332)}{0.397} & \underset{(0.439)}{-0.568} & \underset{(0.274)}{-0.328} & \underset{(0.660)}{0.843} \\
\underset{(2.896)^{***}}{-0.326} & \underset{(4.413)^{***}}{0.011} & \underset{(0.317)}{0.001} & \underset{(3.777)^{***}}{0.010} & \underset{(3.812)^{***}}{0.010}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\underset{(2.455)^{**}}{0.014} & \underset{(0.893)}{0.005} & \underset{(1.014)}{0.006} & \underset{(1.604)}{0.009} & \underset{(2.985)^{***}}{0.017} & \underset{(0.955)}{0.006} & \underset{(2.478)^{**}}{0.013} & \underset{(2.520)^{**}}{0.001} \\
\underset{(0.688)}{-0.029} & \underset{(0.462)}{0.018} & \underset{(1.378)}{-0.054} & \underset{(1.024)}{-0.042} & \underset{(1.758)^{*}}{-0.071} & \underset{(0.530)}{-0.022} & \underset{(0.021)}{0.001} & \underset{(1.233)}{-0.003} \\
\underset{(2.594)^{**}}{-0.080} & \underset{(0.471)}{-0.014} & \underset{(1.929)^{*}}{-0.056} & \underset{(2.061)^{**}}{-0.063} & \underset{(3.620)^{***}}{-0.108} & \underset{(2.484)^{**}}{-0.076} & \underset{(1.153)}{-0.033} & \underset{(0.801)}{0.001} \\
\underset{(0.033)}{0.001} & \underset{(1.045)}{-0.011} & \underset{(1.067)}{-0.012} & \underset{(1.349)}{-0.016} & \underset{(0.577)}{-0.007} & \underset{(1.138)}{-0.013} & \underset{(0.372)}{0.004} & \underset{(0.370)}{0.001} \\
\underset{(0.660)}{0.843} & \underset{(2.540)^{**}}{-3.011} & \underset{(3.747)^{***}}{-4.506} & \underset{(0.342)}{-0.434} & \underset{(0.320)}{-0.393} & \underset{(0.181)}{-0.227} & \underset{(1.095)}{1.284} & \underset{(4.444)^{***}}{-0.291} \\
\underset{(3.812)^{***}}{0.010} & \underset{(4.880)^{***}}{0.012} & \underset{(4.876)^{***}}{0.012} & \underset{(1.035)}{0.003} & \underset{(2.998)^{***}}{0.008} & \underset{(4.276)^{***}}{0.011} & \underset{(2.177)^{**}}{0.005} & \underset{(3.615)^{***}}{-0.001}
\end{bmatrix}
$$

$$
\begin{bmatrix} Const \\ S_{1t} \\ S_{2t} \\ S_{3t} \\ S_{4t} \\ S_{5t} \\ S_{6t} \\ S_{7t} \\ S_{8t} \\ S_{9t} \\ S_{10t} \\ S_{11t} \\ trend \end{bmatrix}
+ \begin{bmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \\ \hat{u}_{3t} \\ \hat{u}_{4t} \\ \hat{u}_{5t} \\ \hat{u}_{6t} \end{bmatrix}
\qquad (7.19)
$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Looking at the full system of estimated VECM for this second set of variables, we find that, in this cointegration, the constant and trend terms are not significant in the long-run cointegrating relation, unlike in the earlier case. (7.19) contains two cointegrated relations and similar to the earlier model, the coefficient associated with exchange rate has been normalized to one and the coefficient associated with NASDAQ set, to zero. Here most of the coefficients in the first column are found to be significant. The loading coefficients associated with BSE, CMR and CPI are -0.782, 41.034 and 0.087, respectively. Since all these values are significant at 5 per cent level of significance, we can conclude that the cointegrating relation is an important component in this equation. Some values of the constant, trend and seasonal dummies have also found to be significant. The cointegrating relation, given in (7.20), shows the long-run relation involving EXRATE, BSE, TR, CMR and CPI.

$$EXRATE_t = -\underset{(3.814)^{***}}{0.081} BSE_t - \underset{(5.480)^{***}}{0.319} TR_t + \underset{(6.406)^{***}}{0.010} CMR_t + \underset{(4.622)^{***}}{0.769} CPI_t + EC_t \quad (7.20)$$

.

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

The third and final set of variables which has been found to have significant as well as meaningful cointegrating relationship is EXRATE, FFR, BSE, NASDAQ and CMR. In this case also, the appropriate lag was found to be 2. The results of the Johansen trace test as well as the maximum eigenvalue test show that, as before, the cointegrating rank is 2, and accordingly we find two cointegrating relations involving EXRATE, FFR, BSE, NASDAQ and CMR. The estimated VECM and cointegration relations for the last set of variables are given below in (7.21).

$$\begin{bmatrix} \Delta EXRATE_t \\ \Delta FFR_t \\ \Delta BSE_t \\ \Delta NASDAQ_t \\ \Delta CMR_t \end{bmatrix} = \begin{bmatrix} \underset{(1.998)^{**}}{-0.086} & \underset{(2.244)^{**}}{-0.002} \\ \underset{(0.815)}{-0.339} & \underset{(2.725)^{***}}{-0.018} \\ \underset{(4.632)^{***}}{-1.054} & \underset{(1.805)^{*}}{-0.006} \\ \underset{(0.308)}{-0.093} & \underset{(0.342)}{-0.002} \\ \underset{(0.199)}{-2.081} & \underset{(3.094)^{***}}{-0.508} \end{bmatrix} \left[\begin{bmatrix} 1 & - & \underset{(9.799)^{***}}{0.406} & \underset{(10.407)^{***}}{-0.183} & \underset{(4.352)^{***}}{-0.008} \\ - & 1 & \underset{(3.276)^{***}}{-9.325} & \underset{(0.541)}{-0.655} & \underset{(5.210)^{***}}{0.667} \end{bmatrix} \right.$$

$$\left. \begin{bmatrix} EXRATE_{t-1} \\ FFR_{t-1} \\ BSE_{t-1} \\ NASDAQ_{t-1} \\ CMR_{t-1} \end{bmatrix} + \begin{bmatrix} \underset{(17.838)^{***}}{-5.457} & \underset{(15.213)^{***}}{-0.004} \\ \underset{(3.241)^{***}}{68.146} & \underset{(6.329)^{***}}{0.110} \end{bmatrix} \begin{bmatrix} Const \\ Trend \end{bmatrix} \right] + \left[\begin{matrix} \underset{(2.947)^{***}}{0.271} & \underset{(0.854)}{-0.007} \\ \underset{(0.092)}{-0.081} & \underset{(4.836)^{***}}{0.379} \\ \underset{(1.037)}{-0.503} & \underset{(1.212)}{0.052} \\ \underset{(0.134)}{0.086} & \underset{(1.327)}{0.076} \\ \underset{(1.154)}{25.754} & \underset{(1.280)}{-2.527} \end{matrix} \right.$$

$$\left. \begin{matrix} \underset{(0.606)}{0.011} & \underset{(1.625)}{-0.025} & \underset{(0.260)}{0.000} \\ \underset{(0.872)}{0.151} & \underset{(1.674)^{*}}{-0.251} & \underset{(1.725)^{*}}{0.006} \\ \underset{(0.318)}{0.030} & \underset{(0.009)}{0.001} & \underset{(2.210)^{**}}{-0.004} \\ \underset{(0.133)}{-0.017} & \underset{(0.749)}{0.082} & \underset{(0.407)}{0.001} \\ \underset{(1.379)}{6.027} & \underset{(1.032)}{-3.900} & \underset{(2.702)^{***}}{-0.230} \end{matrix} \right] \begin{bmatrix} \Delta EXRATE_{t-1} \\ \Delta FFR_{t-1} \\ \Delta BSE_{t-1} \\ \Delta NASDAQ_{t-1} \\ \Delta CMR_{t-1} \end{bmatrix} + \left[\begin{matrix} \underset{(0.898)}{0.085} & \underset{(0.423)}{-0.003} & \underset{(0.879)}{0.016} \\ \underset{(1.748)^{*}}{1.597} & \underset{(3.213)^{***}}{0.245} & \underset{(0.577)}{0.099} \\ \underset{(1.141)}{-0.569} & \underset{(0.806)}{-0.034} & \underset{(2.474)^{**}}{0.231} \\ \underset{(0.955)}{-0.633} & \underset{(0.972)}{-0.054} & \underset{(0.583)}{0.072} \\ \underset{(6.842)^{***}}{157.288} & \underset{(0.048)}{-0.092} & \underset{(1.052)}{4.531} \end{matrix} \right.$$

$$\left. \begin{matrix} \underset{(0.010)}{0.000} & \underset{(3.559)^{***}}{0.001} \\ \underset{(4.105)^{***}}{0.607} & \underset{(1.459)}{0.004} \\ \underset{(2.279)^{**}}{-0.184} & \underset{(2.064)^{**}}{-0.003} \\ \underset{(0.645)}{-0.069} & \underset{(0.742)}{0.002} \\ \underset{(0.882)}{-3.283} & \underset{(1.571)}{-0.119} \end{matrix} \right] \begin{bmatrix} \Delta EXRATE_{t-2} \\ \Delta FFR_{t-2} \\ \Delta BSE_{t-2} \\ \Delta NASDAQ_{t-2} \\ \Delta CMR_{t-2} \end{bmatrix} + \left[\begin{matrix} \underset{(0.121)}{-0.001} & \underset{(0.525)}{-0.002} & \underset{(1.943)^{*}}{-0.007} & \underset{(1.539)}{0.007} \\ \underset{(0.312)}{0.013} & \underset{(1.673)^{*}}{0.063} & \underset{(1.105)}{0.041} & \underset{(0.395)}{0.017} \\ \underset{(1.378)}{-0.030} & \underset{(0.584)}{-0.012} & \underset{(2.995)^{***}}{-0.061} & \underset{(3.370)^{***}}{-0.077} \\ \underset{(1.227)}{0.036} & \underset{(0.798)}{-0.022} & \underset{(0.240)}{-0.006} & \underset{(0.186)}{0.006} \\ \underset{(1.227)}{1.243} & \underset{(0.512)}{0.483} & \underset{(1.540)}{1.434} & \underset{(0.695)}{-0.733} \end{matrix} \right.$$

$$\left.\begin{matrix} \underset{(2.026)^{**}}{0.008} & \underset{(0.508)}{0.002} & \underset{(0.184)}{-0.001} & \underset{(0.311)}{0.001} & \underset{(1.857)^{*}}{0.007} & \underset{(0.844)}{-0.003} & \underset{(1.392)}{0.006} \\ \underset{(0.528)}{0.021} & \underset{(1.583)}{0.062} & \underset{(0.148)}{0.006} & \underset{(0.628)}{-0.024} & \underset{(0.655)}{0.025} & \underset{(1.776)}{-0.067} & \underset{(0.823)}{0.032} \\ \underset{(2.488)^{**}}{-0.054} & \underset{(0.806)}{-0.017} & \underset{(1.697)^{*}}{-0.036} & \underset{(1.455)}{-0.030} & \underset{(2.518)^{***}}{-0.052} & \underset{(2.147)^{**}}{-0.044} & \underset{(1.203)}{-0.026} \\ \underset{(0.162)}{0.005} & \underset{(1.353)}{0.039} & \underset{(-0.662)}{-0.019} & \underset{(0.227)}{-0.006} & \underset{(0.397)}{-0.011} & \underset{(1.208)}{0.033} & \underset{(1.465)}{0.042} \\ \underset{(2.083)^{**}}{2.066} & \underset{(0.311)}{0.308} & \underset{(1.962)^{*}}{-1.921} & \underset{(0.961)}{0.926} & \underset{(1.022)}{0.971} & \underset{(1.822)}{1.725} & \underset{(1.600)}{1.580} \end{matrix}\right] \begin{bmatrix} S_{1t} \\ S_{2t} \\ S_{3t} \\ S_{4t} \\ S_{5t} \\ S_{6t} \\ S_{7t} \\ S_{8t} \\ S_{9t} \\ S_{10t} \\ S_{11t} \end{bmatrix} + \begin{bmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \\ \hat{u}_{3t} \\ \hat{u}_{4t} \\ \hat{u}_{5t} \end{bmatrix} \qquad (7.21)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

In this equation we find that most of the loading coefficient values associated with the second column are significant. Here the values of the loading coefficients are significant for EXRATE, FFR, BSE and CMR variables. As in the earlier cases, here also we can conclude that the coinetgrating relation is of considerable importance in the overall equation. This finding confirms our understanding on the importance of cointegrating relations involving exchange rate and these macroeconomic variables for the Indian economy.

Here we note that, as in the first cointegrating relation, both the constant as well as the trend terms are significant in the long-run relation. Also, the second lag term of CMR as well as some of the monthly dummies are found to have significant effect in the short-run. The cointegrating relation obtained with the coefficient of exchange rate normalized to one and that of FFR to zero is given by,

$$EXRATE_t = \underset{(17.838)^{***}}{5.457} + \underset{(15.213)^{***}}{0.004}\ trend - \underset{(9.799)^{***}}{0.406}\ \mathrm{BSE}_t + \underset{(10.407)^{***}}{0.183}\ \mathrm{NASDAQ}_t + \underset{(4.352)^{***}}{0.008}\ CMR_t + EC_t \qquad (7.22)$$

*[The values in parentheses indicate corresponding absolute values of t-ratios; *, **, *** indicate significance at 10%, 5% and 1% levels of significance, respectively.]*

Now, from the three cointegrating relations given in (7.18), (7.20) and (7.22), it is clear that there exist long-run relations between exchange rate of India and its other macroeconomic variables such as BSE, NASDAQ, call money rate, total reserve, foreign investment and consumer price index, although not all of them together in one single cointegration. However, other supposedly relevant variables such as money supply, industrial production were not found to have any significant as well as meaningful relationship with exchange rate.

In the context of any cointegration study, in addition to emphasis being given to the cointegrating relationships and to the adjustment coefficients with which these enter each equation of the VECM, attention is also given to Granger- causality test to understand the direction of causation, if any, and then formulate policies accordingly. We have now presented the findings on causality in Table 7.4. We have carried out Granger causality test to determine causality involving the three sets of variables (in the VECM) considered in this study in two respects. In the first, we have singled out exchange rate to study its role in causing or being caused by other relevant macrovariables, and in the second, to study causal relation between two variables at a time with exchange rate being one of the two. We note from Table 7.4 that for all the three sets of variables in the VECMs considered by us, the null hypothesis of 'no causality' could not be accepted in all the cases. The *p*-values in almost all the cases are less than 0.01 indicating that the null hypothesis of 'no causality' is getting rejected at 1% level of significance. In fact, looking at the entries, it can be stated that there is evidence of bi-directional causality between these variables and exchange rate.

Table 7.4

Results of Granger-causality test

Effect	Cause	Statistic	*p*-values	Causal relation
EXRATE	CMR, BSE, NASDAQ, TR,FINV	50.515	0.000	Yes
CMR, BSE, NASDAQ, TR,FINV	EXRATE	8.463	0.000	Yes
EXRATE	NASDAQ, BSE,TR, CMR,CPI	2.470	0.000	Yes
NASDAQ, BSE,TR, CMR,CPI	EXRATE	7.9343	0.000	Yes
EXRATE	FFR,BSE, NASDAQ, CMR	2.218	0.010	Yes
FFR,BSE, NASDAQ, CMR	EXRATE	6.8433	0.000	Yes
EXRATE	CMR	4.367	0.014	Yes
CMR	EXRATE	27.700	0.000	Yes
EXRATE	FFR	0.780	0.460	No
FFR	EXRATE	0.359	0.699	No
EXRATE	FINV	0.154	0.857	No
FINV	EXRATE	2.301	0.103	No
EXRATE	M1	0.716	0.490	No
M1	EXRATE	0.959	0.385	No
EXRATE	TR	1.1887	0.307	No
TR	EXRATE	4.993	0.008	Yes

However looking at the findings on the existence of causal relationship between two variables, one of which is the exchange rate, we note that there exists bi-directional causality between exchange rate and call money rate. Most of the other variables either are not cointegrated or do not show evidence of causality with the exchange rate. It is only for TR (total reserve) that we find the existence of causal relation from exchange rate to TR. But there does not exist any causality from TR to exchange rate.

It may be stated that the existence of causality from the chosen group of variables to exchange rate is primarily due to existence of causality from CMR to exchange rate. As regards, the reverse causality i.e., causality from exchange rate to these groups of variables, the variables involved are CMR and TR. This suggests that insofar as roles of these macrovariables in terms of causality for the Indian exchange rate is concerned, call money rate (CMR) and total reserve (TR) appear to be the most important ones.

The observed cointegrating relations between exchange rate of India and its macro variables are quite in accordance with the standard economic theories. For instance, studies have shown that there exist long run relations between exchange rate and stock market index where a rise in the domestic stock price index indicates the strong performance of the economy, and hence it attracts foreign investments, essentially leading to an appreciation of the domestic currency. This is indicated by a negative coefficient value associated with BSE in equations (7.18), (7.19) and (7.20), since an appreciation of domestic currency would mean a fall in EXRATE value as it is representing the units of India’s rupee per US dollar. A similar but opposite interpretation can be put forward for the relation between the domestic currency and the foreign stock prices. A rise in the foreign investment definitely leads to an appreciation, and so will a rise in the total reserve. Since foreign exchange essentially means the ratio of prices of two countries, a rise in the prices of the domestic country (CPI) should lead to depreciation of domestic currency. A rise in the total reserves indicate that the performance of the economy is improving and this injects confidence in both the local and foreign investors resulting in increased capital inflow in the economy. The observed relation between call money rate (CMR) and money supply is in accordance to the theoretical models. Since there are only a few economic models on exchange rate which involve the other variables, we have tried to justify inclusion of these variables by mentioning various empirical studies where these variables have been used. Our test for causality, however yields one meaningful relation *viz.*, causality from CMR to exchange rate. This observation points to the importance of this variable CMR, for India's foreign exchange rate. In the long-run, a rise in interest rate results in depreciation of the exchange rate which is in accordance to the monetarist theory. This occurs since an increase in the domestic rate of interest will decrease the real demand for money ; and given a fixed nominal money supply, this will be achieved by a rise in the domestic price level and hence a depreciation of exchange rate. As regards, the foreign interest rate variable, we have taken the US interest rates (FFR and US treasury bill rates for the latter) in our study. There is however no evidence of causality from FFR to exchange rate. Thus, we can conclude that the domestic interest rate is the most important variable for India's exchange rate and can be used as a policy variable. Depending on certain long-term / short-term targets, the government can actually use this variable to align the exchange rate.

7.4 Conclusions

In this chapter, we have studied the cointegrating relation(s) involving the monthly foreign exchange rate of India and the relevant macroeconomic variables all of which are cointegrated of order 1. The selection of macro variables has been made based on their relevance in theoretical (structural) models of exchange rate as well as on their statistical significance in the empirical findings of similar studies, primarily of advanced economies. From this larger set, some core variables have been chosen for the first cointegration exercise. Thereafter, other less important variables have also been considered in finding the cointegrated relation(s). Applying Johansen's procedure, we have estimated the cointegrated relations where provisions for deterministic terms like constant, linear trend, monthly dummies have also been considered.

We have thus obtained three cointegrating relations (or long-run relations) involving India’s monthly exchange rate and its macro economic variables such as BSE, NASDAQ, call money rate, total reserve, foreign investment and consumer price index. The corresponding VECMs describing the short-run dynamics of the variables concerned have also been obtained. We have also tested for causality and found that the existence of causality from a chosen group of variables to exchange rate is primarily due to existence of causality from CMR to exchange rate. For variables like money supply and interest rate the relations between these variables and India’s exchange rate appear to be in accordance to the standard economic and financial theories. For the other variables for which similar economic theories do not exist, such relations and direction of causality appear to be similar to those obtained in studies with other exchange rates, mostly of developed economies. However, it may be concluded that the most important variable, from this consideration, turns out to be interest rate which can be viewed as a policy variable to be regulated according to long-run / short-run targets.

CHAPTER 8

Conclusions

8.1 Introduction

In this thesis, we have carried out a comprehensive and detailed empirical study on modelling and forecastability of the foreign exchange (spot) rate of India using linear as well as nonlinear time series models. All throughout this study, we have tried to deal with the relevant econometric issues involved in such a study, like, for instance, appropriate specification, choice of independent variables and short-run / long-run forecasting, in appropriate ways. This study is based on the daily / monthly foreign exchange (spot) rate (with respect to US dollar) series. In Chapters 2 through 5, we have used daily data spanning from 1 November 1994 to 13 February 2004 for our in-sample analysis while the data covering the period 16 February 2004 to 14 July 2004 have been used for the purpose of out-of-sample forecasting. The studies done in Chapters 6 and 7 involve modelling with monthly data and the span of the monthly time series used in these two chapters is from November 1994 to March 2005.

The first study carried out in this thesis involves fitting a linear dynamic model with appropriate volatility specification. In this study, due emphasis has been given on appropriate specification of both the conditional first and second order moments so that the final inferences are free from any possible consequences of misspecification of the underlying model. Since the nonlinear time series literature is quite developed, and a large number of studies – though almost all of them are for advanced economies – have shown that the time series of foreign exchange rate exhibits strong signs of nonlinearity, we have next moved from linear model to nonlinear ones. To that end, we have considered nonlinear models which are basically regime-switching models. In particular, we have considered the SETAR and STAR models which belong to the class of TAR models along with the MSR models. Thereafter, some models which combine these nonlinear models with volatility models like the GARCH model, have also been studied. For all such models including the linear one, out-of-sample forecasts have been obtained and a comparative performance across these models has also been made by using the standard forecasting criteria.

Since the empirical literature on studies on exchange rate often uses the relevant macroeconomic variables which have been found to play significant role in exchange rate predictability in the theoretical exchange rate models, we have undertaken such a study for India's exchange rate as well. For this purpose, we have applied the predictive regression technique to identify the relevant set of macro variable which have predictive ability for return on exchange rate. Beginning with a set of 25 variables, we have analyzed the predictive ability of each macro variable in turn, by employing a procedure that combines general-to-specific model selection with out-of-sample tests of forecasting ability. The findings of these two procedures were combined for the purpose of identifying the set of appropriate macro variables for predicting the foreign exchange rate return for India. Thereafter, the final model for India's monthly exchange rate return has been found by using the chosen macro variables along with other terms from considerations of autocorrelation, appropriate specification of the model etc. Finally, the time series methodology of cointegration analysis has been used to study the long-run relationship involving exchange rate and relevant macro variables at their nonstationary level values.

The last chapter of this thesis has been organized as follows. Major findings are summarized in the next section. Limitations of this study are also mentioned here. This chapter and of course, the thesis as well, ends with some discussions on a few ideas for further works on this topic.

8.2 Major findings

The thesis starts with a brief review of the models on foreign exchange rate, which includes both structural as well as time series models. A brief account of the various reforms in foreign exchange management, which have been undertaken by the

Government of India since India's liberalization in 1993, has also been given in this chapter i.e., Chapter 1. Obviously, the motivation as well as the format and focus of the thesis have also been presented here.

From the second chapter onwards, our study on the determination of India's exchange rate and its predictability has begun. In Chapter 2, we have considered a linear dynamic model for the conditional mean of return and GARCH and EGARCH models for capturing the volatility dynamics. We have first used the Quandt-Andrews test to examine the instability prevalent in India's daily exchange rate return series, and then applied Bai's least squares based procedure to estimate the break point(s), if any, in the time series. Then the entire time period has been accordingly partitioned into sub-periods of stable parameters each. Thereafter, we have tried to specify the conditional mean properly for each sub-period. To that end, we have carried out tests for misspecification of conditional mean and consequently made the mean specification as appropriate as possible before determining an appropriate specification for the conditional variance. Finally, an out-of-sample forecasting exercise has been carried out to gauge the performance of such a model by using standard forecast evaluation criteria.

Our major finding in this chapter is that there exists four structural breaks in this daily-level Indian rupee / US dollar exchange rate return series. These break dates or more appropriately, the interval estimates of the break dates were estimated / obtained, and these breaks were found to have occurred, more or less, in accordance with the major events in the recent past in India's exchange rate market. Return on exchange rate has been found to be predictable with past lags contributing most in determining the variations at time; however, in one sub-period we have found BSESENSEX to have a significant role in predictability and in another it is the call money rate which is significant. However, after incorporating the GARCH form of volatility, call money rate was found to be insignificant. Similarly in two sub-periods, a few daily dummies were found to be significant, indicating a daily deterministic behaviour affecting return. Some misspecifications were also detected to be significant, and these were appropriately taken care of by including appropriate functions of recursive residuals.

As regards the appropriate volatility model, we have found, unlike most other studies, EGARCH to be the most appropriate specification of conditional heteroscedasticity for India's exchange rate series for each of the five sub-periods. As noted by few researchers, such a finding could be because of the asymmetric nature of the intervention activities by the Reserve Bank of India. As discussed in the preceding chapters, the RBI intervenes actively to avoid appreciation of the rupee. This is corroborated by a recent study by Ramachandran (2006), where he has noted that the asymmetric control over capital flows and the asymmetric nature of the intervention has led to a large a stock pile of reserves in case of India. Our findings of an EGARCH model could be because of this heavy asymmetric intervention by the RBI, and this is quite likely for an emerging economy where some control on exchange rate, in the form of intervention, is inevitable for the overall growth of the economy. As observed by Bollen *et al.* (2000, p 246) "Foreign exchange rates may also exhibit asymmetric dependence on prior innovations, perhaps as a result of asymmetric policy decisions." Also as noted by Kim and Sheen (2006), movements of exchange rates at times can be determined mostly by developments in one country rather than both. In addition if exchange rates are determined mostly by capital flows in the short run, asymmetric investment flows may lead to asymmetric volatility effects.

In this study, we could not include many macroeconomic variables which have been found to be significant in exchange rate literature, due to their non-availability at daily frequency. We have however, taken up this issue in Chapter 6 where the exchange rate series is considered at monthly-level frequency.

In the next three chapters i.e., Chapters 3 , 4 and 5, we are interested in finding how nonlinear time series models perform for return on India's exchange rate. The nonlinear models considered in this thesis belong to the class of regime switching models. These consist of models from the class of TAR models where, in some cases, heteroscedasticity has also been considered. These models are the SETAR, SETAR-GARCH, DTGARCH and STAR models, and the simple MSR of the mixture model kind and the MSWARCH model. The first three models have been applied to our return data set and the findings are reported in Chapter 3. As we know, SETAR is a special kind of TAR model where the state (regime) – determining variable is the lag of the variable i.e., return in our case. The other two models allow for the consideration of GARCH volatility specification as well, and hence these models combine nonlinearity in conditional mean as well as in conditional variance in the framework of regime switching models. The findings of Chapter 3 suggest that introduction of threshold in volatility has improved the modelling performance. In fact, the performance of DTGARCH models has been found to be the best in terms of diagnostics and out-of-sample forecasts, in the class of TAR models considered in this study. As regards comparing between the two-regime and three-regime DTGARCH models, we have found that the introduction of an additional regime was not statistically useful. Considering the out-of-sample forecasting performance also, the two-regime model is found to be better than the three regime model. The finding of DTGARCH being satisfactory for India's exchange rate return is quite understandable since this model considers thresholds in both the conditional mean and conditional variance. Intervention by the RBI being a normal feature in India, it is quite natural that effects of threshold would be better captured if both conditional mean and conditional variance are allowed to respond to regime switching behaviour of the data.

The other TAR model considered in this thesis is the STAR model. This particular nonlinear model is characterized by the fact that unlike the SETAR model where an indicator function is used to incorporate regime switching, the parameters of this model are allowed to change smoothly over time. Between the two STAR models – LSTAR1 and LSTAR2, the latter as expectedly, has been found to perform better than the former. That LSTAR2 performs better than LSTAR1 is sensible since it assumes a more general smooth transition function *viz.*, a second-order logistic function. It may also be stated that the estimated value of the smoothness parameter was found to be quite large, suggesting that the model, in effect, nests a three-regime SETAR model if the transition variable, s_t, equals r_{t-d}, the value of return with lag d. In other words, while LSTAR2 can very well fit the time series of return on India's exchange rate, the empirical model essentially turns out to being close to the three-regime SETAR model. One limitation of this work is that we could not extend this study by including some volatility specification in the framework of this model. This is primarily because hardly any literature has developed along this line, and there is also the fact of non-availability of computational software.

In Chapter 5, we have studied the MSR model. Unlike the SETAR and STAR class of regime switching models, which have been considered in Chapters 3 and 4, respectively, in this class of models, one can never be certain about the regime the variable is in at a particular point of time, but can only assign probabilities to the occurrence of different regimes. Here we have considered two models – a simple two-state Markov-switching model and the MSWARCH model with a first-order autoregressive conditional mean specification. The latter model is a generalized MSR model where an ARCH process is assumed for the conditional variance and the dynamics of conditional variance across regimes is governed by a first-order Markov process.

Now, the first model *viz.*, simple two-state MSR model was not found to be appropriate since the mean values of the two regimes were found to be statistically the same. The strength of this model lies in its flexibility *viz.*, its capability of capturing changes in the variance between state processes, as well as changes in the mean. And we find that this flexibility is not empirically valid for our data set. The performance of the MSWARCH model, on the other hand, was found to be good in terms of both in-sample model fitting and out-of-sample forecasting performance. Both the two-regime and the three-regime MSWARCH models were fitted to our data set and the three-regime MSWARCH (3,3) model produced the 'best' model.

In Chapter 5, we have also made a comparative study of all the models so far considered in terms of their out-of-sample forecasting performances. All the empirical works presented in Chapters 2 through 5 use the daily level return on India's foreign exchange rate, and hence it is only meaningful to find which of these models performs the 'best' and so on insofar as this particular time series is concerned. In making these comparisons, we have used standard criteria like the MSE, MAE, AMAPE and PCSP of which the first two are obviously more important in making statistical conclusions. Based on the values of these criteria obtained for all the models considered by us, the first major conclusion is that the linear dynamic model with EGARCH as the conditional variance specification, where due consideration has been given to appropriate specification of both the conditional mean and conditional variance, performs the 'best' amongst all the models. This empirical finding appears quite in tune with the fact that India's foreign exchange regime as well as its management are not yet the same as in the developed economies for which the nonlinear models for exchange rate return have often been found to perform better than the linear model. As regards the worst model in terms of out-of-sample forecasting performance, we have found the STAR model to be that model and this, as already discussed in Chapter 4, is expected. The out-of-sample forecasting performance of both the DTGARCH(1,1) models - two-regime and three-regime - have been found to be superior to that of SETAR models. As regards comparing between these two DTGARCH models, we have found that the DTGARCH (1,1) model with two regimes (or, one threshold) performs better than the corresponding three-regimes (or, two thresholds) model. The explanation lies in the fact that the middle regime in the three-regime model has been found to be statistically insignificant insofar as the conditional mean is concerned. The best performance among the nonlinear models is by MSWARCH (3,3) model although DTGARCH (1,1) model with two-regime is very close to this model. Thus, as far as return on India's exchange rate is concerned, in terms of out-of-sample forecasting performance, the linear dynamic model with EGARCH volatility is the best followed by the MSWARCH (3,3) model.

We have, so far, carried out our study with daily-level returns on India's exchange rate. Our next study uses monthly level data, and the framework of analysis is a linear dynamic model where other macro variables are used as independent variables. Obviously, like in Chapter 1, issues of proper specification of both the conditional mean and variance are also given due consideration in the study. It is relevant to note that at monthly level frequency, data for all major macro variables are available for India. However, the existing studies between exchange rate return and such macro variables, mostly concerning developed economies, show a lack of uniformity in identifying the set of macro variables as predictors of return. In other words, available empirical evidences do not identify, most often, the same set of macro variables across different such studies; rather, these studies suggest sets where some variables are common. To deal with this problem of choosing the appropriate set of macro variables for determining and predicting the monthly return on exchange rate of India, we have applied what is known as the predictive regression approach. In this approach, the predictive ability for return of each of these variables is examined separately by using in-sample forecasts and out-of-sample tests of return predictability. In this selection procedure, specific-to-general as well as general-to-specific approaches of model selection have been used, and the results have also been checked using a data-mining-robust bootstrap procedure Combining the empirical findings of these two approaches, we have found a set of 10 macro variables, out of a set of 25, which have significant predictive ability for India's exchange rate return. These variables are: reserve money growth, narrow money growth, change in foreign exchange reserve, gross fiscal deficit, sale/purchase of US dollar, change in Federal funds rate, US stock price return (NASDAQ), change in call money rate, rate of change in gold price and change in total foreign investment.

Once these macro variables have been identified, we have used them to obtain the final model for exchange rate return of India in the linear dynamic regression framework. In this exercise of obtaining the final model, we have also considered lags of monthly return as well of the chosen macro variables along with dummies to represent monthly effects on return. Based on the performance of diagnostic tests of residuals of the estimated model, we attempted to improve on the model further by including one or two very important macro variables including return on India's stock index (BSESENSEX) the kind of which have been found to have very significant effects in most other studies. The final set of macroeconomic variables which were thus found to have significant roles in exchange rate return predictability are the following set of macro variables : change in foreign exchange reserve, sale / purchase of US dollars, change in call money rate, narrow money growth and change in total foreign investment, return on NASDAQ and return on BSESENSEX. Thereafter, we checked if the conditional mean thus obtained was correctly specified. For linear dynamic models, notable cases of such misspecifications include failing to take account for parameter instability, residual autocorrelations, misspecification of functional forms and omitted variables. Tests for parameter instability revealed that there is no break in the monthly data. Also, no seasonal component was found to be significant. Tests of misspecification revealed that the conditional mean was specified correctly. The volatility was not found to be significant in monthly return. Finally, standard residual-based diagnostic tests including the BDS test (Brock *et al.* (1996)) were performed to detect the presence of other higher order dependences in the errors of the chosen model, but no such dependences were detected. Thus, the final model chosen for determination and prediction of monthly exchange rate return of India involves the seven macro variables already mentioned, and their relevance from consideration of economics and finance seems meaningful. The effect of returns of BSESENSEX on the exchange rate returns is found to be negative implying that an increase in domestic stock returns leads to an appreciation of rupee. The possible reason being that an increase in domestic stock returns results in an appreciation of foreign exchange rate since this reflects the good performance of the economy concerned, thus attracting foreign capital. Similarly, the direction of relationship between exchange rate return and return on NASDAQ is also desirable since an increase in this US stock price return is expected to lead to an outflow of funds from the domestic capital market to the foreign market and hence a depreciation of the domestic currency. However the dynamic effect requires a lag of two months to be effective. A rise in foreign investment leads to an appreciation of the domestic currency due to inflow of funds. The directions of the CMR and M1 are also in accordance to the standard economic theories.

We have obtained a positive relation between the reserves and exchange rate, i.e., a rise in reserves leads to exchange rate depreciation. One explanation for this finding could be that intervention activities which are undertaken by the RBI to depreciate the Indian rupee results in large stocks of reserves (Ramachandran (2006)). Hence this contemporaneous relation between the two is observed.

Finally in case of intervention, the relation between the two is found to be negative, implying that net market purchases of foreign exchange leads to rupee appreciation. As noted by Kim and Sheen (2006), who also found a similar result, this might suggest counter-productive intervention. Also the presence of simultaneity bias cannot be ruled out. One can however argue that such a purchase of dollar takes place only when RBI wants to intervene in the event of some capital inflow. If this purchase is not successful in mopping the excess foreign exchange from the markets then it results in an appreciation of the domestic currency.

In the preceding chapters till Chapter 6, we have studied the aspect of predictability of return on India’s exchange rate from consideration of short-run periods only. While it is true that short-run predictions are very important for all concerned studies associated with exchange rate market, yet a study on predictability of exchange rate should also envisage studying the relationship, if any, between foreign exchange rate and other relevant macro variables – all at their nonstationary level values – so that it can be empirically examined if there exists any long-run relation involving exchange rate and these variables. In the event of such relation(s) indeed existing, it can be inferred that exchange rate has a comovement with these variables over time and hence the exchange rate is predictable in the long-run sense.

To examine this long-run relationship, we have applied the VAR based methodology developed by Johansen (1988, 1991,1995) and Johansen and Juselius (1990). But, since cointegration relations do not appear explicitly in the VAR framework, a more convenient modelling set-up obtained by rewriting the VAR model, known as the vector error correction model (VECM), has been used for cointegration analysis.

Now, our cointegration analysis suggests the existence of three cointegrating relations, and hence it is clear that there exists long-run relations between exchange rate and other macroeconomic variables such as BSESENSEX, NASDAQ, call money rate, total reserve, foreign investment and consumer price index, although not all of them together in one cointegration. However, other relevant variables such as money supply, industrial production were not found to have any significant relation with exchange rate in the long-run sense. The observed cointegrating relations between exchange rate and the macro variables having a significant role, are in accordance with the standard economic theories. For instance, studies have shown that there exists long run relations between the exchange rate and stock market index where a rise in the domestic stock price indices indicate the strong performance of the economy, and hence attracts foreign investments and this essentially leads to an appreciation of the domestic currency. A similar but opposite interpretation can be put forward for the relation between the domestic currency and the foreign stock prices. A rise in the foreign investment definitely leads to an appreciation. Since foreign exchange essentially means the ratio of prices of two countries, a rise in the prices of the domestic country (CPI) should lead to depreciation. A rise in the total reserves indicate that the performance of the economy is improving and builds confidence for both the local and foreign investors and increases capital inflow in the economy. The cointegrating relation between exchange rate and domestic interest rate follows the results of the monetarists' theory *viz.*, a rise in the interest rate leads to a depreciation of domestic currency.

To sum up, one of the most important objectives of any study on time series modelling is forecasting. In particular, for foreign exchange rate, there are several important purposes for forecasting. Some of these are the following: (i) to earn income from speculative activities, (ii) to determine optimal government policies, (iii) to base scientific judgments on outcomes of predictions, and (iv) to make business decisions. Financial decisions often involve long-run commitments of resources, the returns to which will depend on what happens in future, and hence accuracy of forecasts is extremely important for policy considerations. Since there are many international transactions that do not require immediate settlements, there are provisions of contractual arrangements for extension of credit and subsequent payments for the obligations involved. A prior knowledge on the behavior of exchange rate can actually help in such deals. Keeping this in mind, in this thesis, we have carried out a systematic and comprehensive study on determination and predictability of foreign exchange rate of an important emerging country like India. In this study, we have considered both linear and nonlinear models like the SETAR, STAR and MSR models. Further, due emphasis has been given to various econometric as well as economic issues such as volatility, structural break, appropriate specification of the first two conditional moments, roles of macroeconomic variables and short-run as well as long-run predictability. In order to deal with these issues appropriately, we have used various up-to-date econometric techniques like structural break analysis, tests for misspecification, predictive regression approach, out-of-sample tests of predictive ability, cointegration, and out-of-sample forecasting performance using criteria like the

MSE, MAE, AMAPE and PCSP. Applying all these tools, appropriate and meaningful models – both linear and nonlinear – have been obtained for return on foreign exchange rate of India. In terms of in-sample modelling performance including diagnostic tests as well as out-of-sample forecasting performance criteria, the appropriately specified linear dynamic model with EGARCH volatility specification performs the best among these models followed by the three-regime Markov switching-regime ARCH (3) model i.e., MSWARCH (3,3) model. Further the performance of the double-threshold GARCH (1,1) model with two regimes i.e., DTGARCH (1,1) with two-regimes (or, one threshold) is almost as good as the MSWARCH (3,3) model for the time series of return on India's daily exchange rate.

We have also found a model for predictability of India's exchange rate return based on monthly level exchange rate series, involving altogether seven relevant macro variables. From the point of view of long-run forecastability, we have found three cointegrating relations all of which are quite meaningful in the sense of economics and finance.

8.3 Ideas for future works

The thesis has been concerned with the study on modelling and predictability of foreign exchange rate of one of the most important emerging market economies with huge growth potential, called India. Since emerging economies like India are likely to grow more important in future, more such studies concerning important variables in the area of empirical finance should be undertaken for such countries, especially because the number of such studies till date are very few although the very recent trend looks somewhat optimistic with a growing awareness on the importance of such emerging economies. Obviously, scope for such studies are quite extensive. In the context of the study done in this thesis, we present some ideas for future works concerning foreign exchange rate of India or for that matter, foreign exchange market in India, to make the topic broader.

In our work all throughout except in Chapter 7, we have considered the time series of returns on exchange rate. While the exchange rate series is $I(1)$, the return series is $I(0)$ i.e., stationary. There have been some recent univariate analysis of exchange rates which have used the econometric technique of fractional integration to determine the order of integration of the exchange rate. Such studies may as well be done for India's exchange rate series.

Though we have used regime-switching nonlinear models for studying the behaviour of India's exchange rate, these studies have been in the framework of univariate analysis. Of late, some studies have been done where some of these models have been generalized to include other exogenous variables as well. Such studies can be undertaken to find to what extent the performance of the univariate nonlinear models found in this study can be improved upon. In our study with SETAR and MSR models, we have considered other models where regime-switching behaviour has also been extended to the conditional variance of the series. But, the same could not be done for the STAR model. In fact, there have been relatively fewer works (*cf.* Lundbergh and Teräsvirta (1998)) using such a model and hence this is one aspect which can be further explored for India's exchange rate series. In case of a STAR model, parameter constancy tests are indicative of general misspecification, and there is no unique way of responding to a rejection. However, carrying out the tests for subsets of parameters, may provide important information about the shortcomings of the model. In some cases, it is found to be reasonable to respond to the rejection by extending an estimated STAR model with a time varying-STAR (TVSTAR) model, as recently done by Van Dijk *et al.* (2003) and Teräsvirta *et al.* (2003) on time varying seasonal patterns in quarterly industrial production series. This model is discussed in detail in Lundbergh *et al.* (1999). Similarly for the MSR model also, there have been some studies which have used time varying transition probabilities for modelling exchange rate for some countries. Such models may be fitted to India's exchange rate series and this may lead to improvement in their performances.

Another area could be the use of nonparametric techniques in time series analysis of financial data, in particular for foreign exchange rate. In this context, it is relevant to mention that the most popular of such models is the artificial neural network (ANN) model. Though there have been some studies which have tried to use nonparametric methods for modelling and predicting exchange rates, the number of such studies are very few leaving a huge scope for studies using nonparametric techniques. It is also worth mentioning that, in recent years, there have been some developments towards nonparametric volatility models as well (see Anderson *et al*. (2000) and Linton and Mammen (2004), for details on such models), and such approaches may also be applied in dealing with volatility in India's exchange rate series. It would be interesting to make comparisons based on such approaches as against those based on parametric models.

Since our work has a considerable content on forecasting, it is worth mentioning about density forecasting which can be used for furthering this work. A density forecast of the realization of a random variable at some future time is an estimate of the probability distribution of the possible future values of that variable. It provides a complete description of the uncertainty associated with a prediction. Density forecasting is rapidly attaining importance in the area of research in analysis of economic and financial time series. It is now becoming important to have the knowledge of the full predictive density of a time series, rather than its conditional mean or variance. However, it is yet to become a routine exercise in the frequently encountered cases where a closed form does not exist for the predictive distribution or where the parameters of this distribution are complicated non-linear functions of the data (see Tay and Wallis (2000), for an excellent survey on density forecasts).

Along with the strand of density forecasting emerges another called modeling quantiles of the predictive distribution. Modelling quantiles rather than predictive density has the advantage of not requiring the specification of the functional form of the density.

Obviously, both density forecasting and modeling quantiles for India's foreign exchange rate return would be important topics for future research.

We conclude by mentioning another topic for future works. In the context of not-to-impressive performance of theoretical models using various macroeconomic variables, it has been argued that even though the theory is fundamentally sound, its empirical implementation in the framework of a linear statistical model is flawed.

From this perspective, it is argued that theoretical models of exchange rate imply long-run equilibrium conditions only, toward which the economy may adjust in a nonlinear fashion. Indeed there have been recent studies which show that there are nonlinearities in adjustment from deviations of the exchange rate from the economic fundamentals (Balke and Fomby (1997), Taylor and Peel (2000), Taylor *et al*. (2001) and Kilian and Taylor (2003)). Due to the presence of nonlinear relationship, the use of linear models (to capture this relationship) results in poor forecasting performance. A theoretical model has been developed by Krugman (1991), called the target zone model, where the central bank enforces a known and credible band within which the exchange rate is allowed to move and intervention occurs to keep the exchange rate from reaching the edges of the band. This is assumed to deliver the nonlinear dynamics to the exchange rate. Some other papers which highlight the importance of nonlinear adjustment of the exchange rate to the value implied by fundamentals include those by Michael *et al*. (1997), Obstfeld and Taylor (1997), Taylor *et al.* (2001) and Kilian and Taylor (2003). All these studies indicate that there may be some better way of modelling in terms of capturing the nonlinear relationships between exchange rate and these macroeconomic variables. A linear cointegration approach used by us in the thesis may not adequately model this nonlinear relation. Recent theoretical developments in the method of cointegration indicate that nonlinear cointegration approach can be applied, and hence this latest econometric methodology can be used for finding models involving exchange rate and the other variables.

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