A NOTE ON CONCORDANCE

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If there are k items of work paid at the rate of c_i per hour for the i-th work, usually the rates are fixed on subjective estimation of the hardness of the work. Sometimes, an objective estimation of the rates is also made by setting up normal outputs. That is, number of units of a particular work which can be done by a normal worker is obtained experimentally and a unit is paid less when a large number of units can be finished in an hour. But here also the value of the hour is determined *a priori*. In this note I suggest that the money equivalent of the hour may be fixed with reference to the preference of the individual workers.

Let the ranks of the k items of work according to the first worker be $a_{11}, a_{21}, a_{31}, \ldots a_{k1}$ where a's are permutations of the integers 1, 2, . . k. Ultimately we get the rank matrix as

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{k1} & a_{k2} & \cdots & \cdots & a_{km} \end{vmatrix} \qquad \ldots (1)$$

Provided there is concordance between the rankings of the different workers, it will be logical to make the money equivalent of the hour depend on $\bar{a}_i = \frac{1}{m}\sum_{j=1}^{m} a_{ij}$. That is we will adopt the principle of paying less for the most preferred work.

For example if $\bar{a}_1 > \bar{a}_2 > \ldots > \bar{a}_k$ we can make $c_1 > c_2 > \ldots > c_k$.

Given a rank matrix in the above form, divergence from a purely random structure can be tested by χ^2 test where

$$\chi^2 = \frac{12m}{k(k+1)} \sum_{i=1}^{k} \left(\bar{a}_i - \frac{k+1}{2} \right)^2 \qquad \ldots (2)$$

In an actual experiment concerning 10 items of work of 87 workers the value of χ^2 was found to be as large as 184·88 for 9 degrees of freedom. This clearly shows that the pattern in the matrix is by no means random.

Co-efficient of concordance is defined as $w = 12s/m^2(k^3-k)$ where $s = m^2\Sigma(\bar{a}_i - \bar{a})^2$, $\bar{a}$ being the general mean. The value of the coefficient lies between 0 and 1. If ρ_{av} is the average value of the Spearman's rank correlation between any two possible pairs of columns in (1), then it can be shown that $\rho_{av} = (mw-1)/(m-1)$. In our experiment the value of w has come out to be 0.2493 which in itself does not indicate high concordance.

A rigorous test of significance of w has not been arrived at for large values of m and k, but in such case, $z = \frac{1}{2}\log_e[(m-1)w/(1-w)]$ with $\nu_1 = (k-1) - 2/m$ and $\nu_2 = (m-1)[(k-1) - 2/m]$ degrees of freedom can be shown to follow the z distribution. Our value of z comes to be 2.3519 with 9 and 772 degrees of freedom. From z tables this is clearly seen to be significant. This conclusively proves that the rank matrix as a whole shows concordance.

The actual mean values $\bar{a}_i$ are given below :

$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$\bar{a}_4$	$\bar{a}_5$	$\bar{a}_6$	$\bar{a}_7$	$\bar{a}_8$	$\bar{a}_9$	$\bar{a}_{10}$
8·3	6·0	3·1	5·0	5.5	3.8	5·2	4·3	5·8	6·0

In continuation of what has been stated earlier it may be pointed out that the money equivalent of the hour can be made to depend on these values. Some method of testing significance of the difference $\bar{a}_9 - \bar{a}_4$ will add immensely to the value of the method. As it is, there is very little force in the assertion that the ninth work is more preferred than the fourth.

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