Essays on Cooperative Behaviour and Collective Action

Anindya Bhattacharya

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Thesis Supervisor: Professor Bhaskar Dutta

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Chapter 1

Introduction

This dissertation explores some issues concerning the behaviour of coalitions of individuals in a game theoretic set-up and also studies one aspect of a society facing collective action of the masses. The common feature of the issues explored in the dissertation is that it investigates the properties of stable social states. Chapter 2 introduces a notion of a social state that is unlikely to be displaced by any coalition of agents endowed with a certain notion of rationality and a certain degree of farsightedness and explores the properties of such states. Chapter 3 is also concerned with coalitionally stable social states with a different social set-up and social norm. Chapter 4 is an analysis of a society where the prevailing social state is threatened with collective action by the masses.

1.1 Coalitional Stability and Credibility

The theme of Chapter 2 entitled "Coalitional Stability with a Credibility Constraint" is to study the properties of stable social states when the coalitions are restricted to deviate "credibly". Later we shall explain the precise meaning in which we use the term "credible deviations".

Many social systems are inherently unstable to coalitional deviations - whatever be the *status quo* social state, a coalition of agents has an incentive to enforce a different social state from it. The well-known example of "paradox of voting" is a clear illustration of that. Suppose there are three persons: 1, 2 and 3 and three social states: a, b and c. By $x \succ_i y$ we mean that person i strictly prefers state x to state y. Suppose the persons order the social states in the following manner:

$$b \succ_1 a \succ_1 c, \quad c \succ_2 b \succ_2 a, \quad a \succ_3 c \succ_3 b.$$

Suppose a majority coalition can enforce one state from another. Note that if a is the *status quo* social state, then 1 and 2 can enforce b from it; if b is the *status quo* social state, then 2 and 3 can enforce c from it and if c is the *status quo* social state, then 1 and 3 can enforce a from it. The key feature of this example is that for every social state there is a majority coalition that prefers a different social state. This somewhat disturbing aspect of a society has generated a substantial body of literature proposing different rules that identify outcomes which are immune to coalitional deviations. Of course, the assumptions on the behaviour of agents vary from one rule to another.

The society under consideration in this essay is represented by a proper simple game, a subclass of simple games.

DEFINITION 1.1.1 *A simple game Γ is the tuple $\ll N, Z, B, (\succ_i)_{i \in N} \gg$ where N is the finite set of players, Z is the set of outcomes, $\succ_i$ is the preference relation for $i \in N$ on Z and $B \subset 2^N$ is the set of winning coalitions which satisfies the following:*
$S \subset T$ and $S \in B \Longrightarrow T \in B$.

A simple game is said to be *proper* if the following condition is met:

$$S \in B \implies N \setminus S \notin B.$$

For each $i \in N$, $\succ_i$ is a total and transitive binary relation on Z.[1]

If for some $S \in B$ and any $a, b \in Z$, $a \succ_i b$ for all $i \in S$ then *a dominates*

[1] Throughout this dissertation, we have adopted the convention of Aliprantis and Border (1999) for naming the properties of a binary relation.

b via the coalition S which is denoted here as $a \succ_S b$. If there exists $S \in B$ such that $a \succ_S b$ then we shall denote that as $a \succ b$.

The core is the most well-studied solution for simple games.

DEFINITION 1.1.2 *The core of a simple game* Γ, $C(\Gamma) = \{a \in Z |$ *there is no* $b \in Z$ *such that* $b \succ a\}$.

However, it has been noted by several authors that although the core has an obvious interpretation and hence intuitive appeal, it has the following disconcerting features. First, the core as a solution notion presupposes that the agents are completely myopic: when they enforce an outcome from another they do not take into account the possibility of further moves by other coalitions. Secondly, there are many simple games for which the core is empty (Le Breton (1987) explores this issue for a large class of simple games). So, a number of less restrictive solution concepts have been introduced which presuppose different notions of rationality and different degrees of farsightedness on the part of the players (for example, Chakravorti (1999), Chwe (1994), Dutta *et al.* (1989), Li (1991), Ray and Vohra (1997), Rubinstein (1980), Xue (1998)).

The ideas behind many of these solution concepts have the following feature in common. If a coalition decides to enforce an outcome b from an outcome a, it anticipates that some other coalition may further deviate from b. The degree of this foresight differs from solution to solution - a coalition may foresee only one step further (as in Rubinstein (1980)) or it may foresee arbitrarily many steps (e.g. Chakravorti (1999), Chwe (1994)). However, whatever the degree of farsightedness, while making a move, a coalition is assumed to take into account the *resultant* outcomes of subsequent deviations – whether or not these will be beneficial for the players in the coalition.

The consistent set introduced by Chwe (1994) is an important solution concept in this genre. Apart from some nice properties of such a set, its intuitively appealing feature is that this concept incorporates the desirable properties of *consistency* (Thomson (1996) has provided a detailed survey of consistency in various economic situations) and farsightedness of an arbitrary order.[2]

In Chwe's set-up of a game in effectivity function form, the agents consider both *direct* and *indirect* domination of one outcome by another. We refer to his paper for the formal definitions of the above ideas and restrict ourselves to discussing the basic intuition behind a consistent set. A consistent set is a set of outcomes for which the following holds. Let there be a deviation from an outcome a in a consistent set Y to some outcome b via a coalition S. Then there is at least one outcome c in Y *itself* which is either the same outcome b or one that dominates b (directly or indirectly) but which is not strictly preferred by all the members of S to a. Moreover, *no* outcome outside Y satisfies this property. So, the stability of the outcome a rests on the *threat* that any deviation from it may generate further deviations leading to a stable outcome (i.e. an outcome in the consistent set) that may not be liked by the coalition which perpetrated the initial deviation. Chwe showed that there is a well-defined *largest* consistent set which is non-empty for all games with a finite number of outcomes.

However, this idea has a number of conceptual drawbacks which have been mentioned by Chwe himself. He noted that a consistent set may be too inclusive. His intention was

> ...to define a weak concept, one which eliminates with confidence... If Y is consistent and $a \in Y$, the interpretation is not that a *will be* stable but that it is *possible* for a to be stable. If an outcome b is not contained in any consistent Y, the interpretation is that b cannot possibly be stable: there is no consistent story in which b is stable. (Chwe (1994), italics in the original)

In this essay, we concentrate on *one* of the drawbacks. Very often an outcome is a member of a consistent set under the tacit assumption that a coalition

[2]Solutions (not necessarily on simple games) incorporating the idea of consistency are many (e.g. Dutta *et al.* (1989), Ray and Vohra (1997)). Greenberg (1990) devised a general framework called *situations* for studying consistent solutions. Li's (1991) notion of *farsighted core* is an early attempt to analyse coalitional deviations with farsighted agents.

would move to some outcome of the set even when a better outcome within the set is available to it.

> ...the largest consistent set does not incorporate any idea of "best response": coalitions will move to any, not just the best of the outcomes which are better than the status quo. (Chwe (1994))

So, the threat of further deviation to an outcome not liked by the perpetrator of the initial deviation which we have mentioned above is often an *incredible* one.

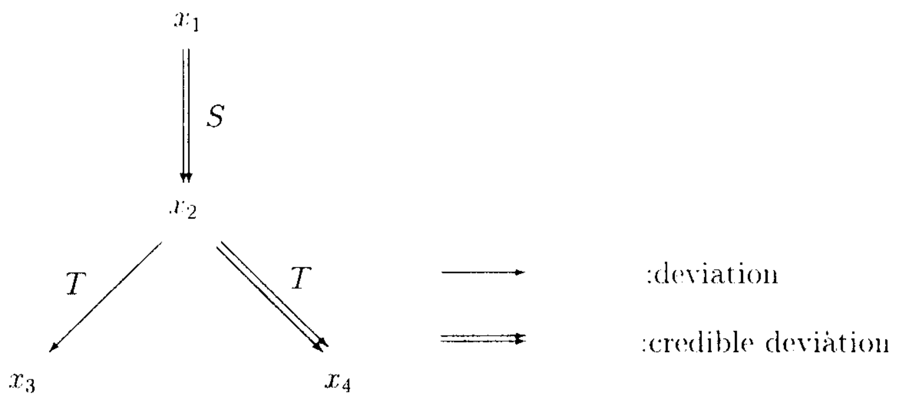

Figure 1: Stability owing to Incredible Deviation

Consider for example the following situation. Let the player set be N and the set of outcomes be $Z = \{x_1, \ldots, x_4\}$. Suppose there are two coalitions S and T such that S can enforce x_2 from x_1 and T can enforce x_4 and x_3 from x_2. No other outcome can be enforced from any other outcome. Let the individual preferences be such that all members of S prefer x_4 to x_1 but some members of S prefer x_1 to x_3. Moreover, all members of T prefer x_3 to x_2 and x_4 to x_3 and so prefer x_4 to x_2. For no other x, y in Z is it true that all players of S or T prefer x to y. Then, $\{x_1,\ x_3,\ x_4\}$ is consistent.[3] Let S move from x_1 to x_2. Then T can move to x_3 which is preferred by all its members. But this deviation is not liked by at least one player in S and hence the threat of further deviation to x_3 would deter the enforcement of x_2 from x_1 by S. Notice however, that as all players in T prefer the "stable" outcome x_4 to x_3, the move from x_2 to x_3 by T is not a *credible* one: T would rather move to x_4. As S prefers x_4 to x_1, if players in S anticipate that the further movements would be the credible ones they would surely move from x_1 to x_2. Thus the "stability" of x_1 rests on the threat of a deviation that is incredible. These deviations by coalitions are illustrated in Figure 1.

To get rid of this shortcoming we introduce a modification of the consistent set. We work with the class of proper simple games with a finite number of outcomes and the additional assumption that the individual preference orderings are irreflexive and asymmetric. Our modification of a consistent set turns out to be a refinement of the largest consistent set. The central idea is that if a coalition blocks an outcome by another, then the blocking outcome itself must not be a "dominated" one – it is not credible that a coalition would move to an outcome when another outcome is available to it which is strictly preferred by all its members. From any status quo outcome a if a coalition S conceives of blocking it by moving to another outcome b then it must take into account not *all* possible further deviations to "stable" outcomes from b but only the *credible* ones.[4] We define a solution which is similar in spirit to a consistent set but which takes into account the above constraint. We call it a *credibly consistent set.* Our choice of proper simple games is motivated by the fact that in this set up we can define credible coalitional deviations

[3] The outcome x_3 is in the consistent set because no outcome can be enforced from it.

[4] Wilson (1971) long ago analysed stable sets and bargaining sets of voting games with coalitions restricted to make credible moves. Another solution concept that incorporates both the ideas of credible coalitional deviations and consistency is the ***consistent bargaining set*** introduced by Dutta *et al.* (1989).

in a simple way.

We examine the issue of the existence and non-emptiness of such sets and also investigate their relation to some other solution concepts prevalent in the literature concerning coalitional stability. One somewhat startling result we get is that even in our restrictive set-up there are games for which no non-empty credibly consistent set exists even if the core is non-empty. Thus, credibility and consistency together seem to be too stringent a requirement if one wants a prediction of stable social states in every situation.

1.2 Equity and Coalitional Stability

The concepts of equity and fairness have generated a substantial body of literature in social sciences. However, an explicit consideration of coalitional stability in a society with equitable social norms is quite recent. In Chapter 3 entitled "On the Equal Division Core" we study a set of stable social states called the *Equal Division Core* (EDC) that is closely related to the norm of equity.

Our set-up is a transferable utility game in characteristic function form, a common representation of a society for studying coalitional behaviour.[5]

DEFINITION 1.2.1 *A Transferable Utility Cooperative Game (TU game) is a pair* (N, v) *where* N *is a finite set of players and* v *is a function that associates a real number* $v(S)$ *with each subset* S *of* N. *We assume* $v(\emptyset) = 0$.

Theoretical analysis of equitable pay-off vectors for TU games started with Shapley (1953). Equity-based or egalitarian solutions for the related area of cooperative bargaining problems have been extensively studied (Thomson (1994) provides an exhaustive survey). A novel but direct approach in this area of study was introduced by Dutta and Ray (1989, 1991) in a couple of papers. Later, we shall dwell on their contribution in greater detail.

[5]Myerson (1991) contains an account of the underlying logic of such a representation. Ray and Vohra (1997) provide a good critique of such a representation

Given a TU game (N, v), we introduce the following notation. The set of efficient pay-off vectors for the grand coalition, $X(N,v)=\{x \in \mathbf{R}^N | \sum_{i \in N} x_i = v(N)\}$. For $S \subseteq N$, $S \neq \emptyset$, $a(S,v) = v(S)/|S|$, the average worth of the coalition S. For $S \subseteq N$, $S \neq \emptyset$ the equal division allocation for S, $e(S,v)$ is the vector $x \in \mathbf{R}^S$ such that $x_i = a(S,v)$ for all $i \in S$. Given two vectors $x, y \in \mathbf{R}^n$, $x \gg y$ means that $x_i > y_i$ for $i = 1, \ldots n$. Given a vector $x \in \mathbf{R}^N$ we denote the restriction of x on a coalition S by x_S.

DEFINITION 1.2.2 *For a TU game (N, v), the equal division core (EDC) of (N, v), $L(N, v) = \{x \in X(N, v)|$ there is no coalition S for which $e(S, v) \gg x_S\}$.*

So, in other words, "a pay-off vector is in the equal division core, if no coalition can divide its value equally among its members and in this way give more to each of the members than they receive in the pay-off vector" (Selten (1972)).

Evidently, it is a solution concept related to the norm of equity. The EDC was proposed by Selten (1972) to explain outcomes of experimental cooperative games. It has been found that experimental "evidence clearly suggests that equity considerations have a strong influence on observed pay-off division" (Selten (1987)) in a laboratory set-up and the EDC has proved to be quite successful as a solution in this context. Selten (1972) reports that in 76% of the 207 experimental games he studied, the outcomes had a "strong tendency to be in the equal division core". He explains the intuitive motivation behind the solution-notions like the EDC thus:

> It is unreasonable to suppose that the experimental subjects perform complicated mathematical operations in an attempt to understand the strategic structure of the situation. It seems plausible to assume that they look for easily accessible cues, such as obvious ordinal power comparisons and equitable shares, in order to form aspiration levels for their payoffs. (Selten (1987))

Farell and Scotchmer (1988) also cite a number of real-life examples where a coalition is observed to share its worth equally.

The EDC has a theoretical justification as well from a quite different angle. It is obtained as one core-like solution of a game if the norm of egalitarianism is used consistently for the coalitions (Dutta and Ray (1991)). In a later section we shall explain this idea in greater detail.

In our second essay we have provided axiomatic characterization of the EDC as a solution on two classes of TU games.

DEFINITION 1.2.3 *Let Γ_0 be a set of TU games. A solution on Γ_0 is a mapping ψ which associates with each game $(N, v) \in \Gamma_0$ a subset $\psi(N, v)$ of $X(N, v)$.*

The literature on characterization of different solutions is vast (two surveys by Peleg (1992) and Moulin (1999) may be cited as examples of the power and scope of axiomatic method). The analysis of the egalitarian core-like solutions for TU games in an axiomatic framework began with Dutta (1990) who provided a characterization of the weak egalitarian solution (defined by Dutta and Ray (1989)) on the class of convex games. Recent contributions in this area include Hokari (2000), Hougaard *et al.* (1999), Klijn *et al.* (2000). Since the EDC was proposed as an ad-hoc solution to explain experimental results, a theoretical justification of it from an axiomatic standpoint is a worthwhile exercise.

1.3 Upsetting Stability: A Model of Collective Action

In the final chapter entitled "A Model of Collective Action" we study the phenomenon of revolt as a collective action. The analytical study of collective action possibly began with Olson (1971). He identified the problems of "free-riding" behaviour that the self-interested agents must pursue when faced with a choice to undertake collective action. This becomes all the more problematic for the collective action of revolt where the cost of participating in a revolt may be too high. "Revolutions have been viewed by social scientists and historians, for the most part, as largely inexplicable events" (Roemer (1985)). So, revolts were thought to be ideologically driven behaviour not amenable to rational analysis.

There are several related aspects of revolt that have come to be analysed albeit somewhat partially. The role of a government or the ruling class facing a revolt, the actual decision of the masses whether to revolt or not, the informational aspects of revolt, the role of a revolutionary party or agent in instigating the revolt etc. are some of the crucial issues.

Although Roemer (1985) and Grossman (1991) were amongst the first to provide formal analyses of revolt, their models assumed complete information on the part of the agents. However, there is pervasive asymmetry of information in a revolutionary situation. A single individual does not know whether another individual is willing to revolt, the masses as a whole do not know whether the government they would fight is strong or weak, the government does not know the revolutionary capability of the masses etc.. Some recent papers have begun to grapple with these problems, notable among them Acemoglu and Robinson (1999a, 1999b), Chwe (1999, 2000). Their analyses are also related to the questions addressed in the older literature, but in a more realistic framework of incomplete information.

In Chwe's (1999, 2000) model, the individuals in the society can be of two types: they can either be willing to revolt or unwilling. He recognizes the fact that a willing person would like to revolt only if he knows that sufficiently many persons are also willing. He models the relationship among the individuals as a *directed* network, if person 1 is connected to person 2, then he knows the type of person 2. He shows that certain minimal amount of information flow through establishing links between persons is necessary for a willing person to revolt in equilibrium. He calls such a structure of links a *minimal sufficient network.* He analyses the *structure* of such a network. He also shows that the role of an organization is quite crucial for revolt to take place.

Acemoglu and Robinson (1999a in particular) have studied another aspect of the informational problems of revolt. The decision of the masses to revolt depends on their information about the strength of the government: its capability of (*a*) suppressing a revolt in case it breaks out, and (*b*) the ease with which it can take preemptive measures that helps in thwarting the revolt. In their model, the government can be any of three types: strong, flexible and weak. The government knows its own type, but the type of a government can be only imperfectly observed by the masses. The government can signal about its type by taking actions like repressing or making some democratic concessions. The masses update their belief about the government and decide whether to revolt or not. Acemoglu and Robinson look at the *Perfect Bayesian Equilibria* of the above signalling game. They have two contentions: (*a*) revolt takes place *only* owing to incomplete information, and (*b*) a strong government always represses whereas a relatively weak government makes a relatively large concession as a small dose of concession may be construed as the sign of its weakness.

Our third essay takes Acemoglu and Robinson (1999a) as a point of departure. We also model a society facing a possibility of revolt as a signalling game. Although, revolt is too complicated a phenomenon for identifying a general behavioral pattern, we note an empirical feature of revolt that has been observed a number of times. If a government is relatively strong and confident that it can successfully quell a revolt, then often it does not unleash repression on the masses even under the threat of a revolt. It lets the prevailing situation to continue or at best makes some trifling political reforms. In contrast, facing a revolutionary situation, very often a relatively weak government takes the measure of repression. For example, during the nineteen twenties and thirties, there was a widespread tendency toward radicalism, almost every country in Europe was threatened with social revolution. However, countries like England did not unleash repression (even after the

General Strike of 1926) comparable to the scale of that in Germany or Italy where the ruling class was much weaker (Hobsbawm(1994)). Again, during the revolt of 1956 in Hungary, the immediate response of the then Soviet Union (which was effectively the ruling elite) was to put up Imre Nagy as the Prime Minister, it did not start repression immediately. However, when this step did not deter the masses from revolting, the Soviet authorities launched repression and could suppress the revolt successfully (Vadney (1987)). The examples of regimes where a palpably weak government has tried widespread repression in the face of a revolt and then collapsed are also numerous (e.g. Cuba under Batista, India under the Emergency regime of Indira Gandhi). This feature has also been generalised in the radical political theory of Fascism. The extreme measures of a Fascist regime is seen to be the response of a weak ruling class which has lost confidence in itself.

However, this feature is not explained by the model of Acemoglu and Robinson. In their set-up whenever one observes repression in a potentially revolutionary situation, one should unambiguously infer that the government is relatively stronger. But we show that in some situations, a stronger government would unambiguously make concessions whereas a weaker government would repress. We also identify certain situations for which this behavioral pattern is the only pattern that would be observed.

Chapter 2

Coalitional Stability with a Credibility Constraint

As we have mentioned in the introduction, the objective of this chapter is to study a notion of coalitional stability that has the following desirable features: (a) the agents are farsighted, (b) coalitions of agents are restricted to make only credible deviations to stable outcomes and (c) a property of consistency is built into the notion. Our framework of study is a proper simple game with a finite number of outcomes. We call the set of outcomes stable with respect to the above notion a *credibly consistent set*. In section 2.1 we give some preliminary definitions and notation in addition to those introduced in the introductory chapter. Section 2.2 deals with the issues of existence and non-emptiness of credibly consistent sets. We find that in games with veto players, every consistent set is credibly consistent as well but in games without a veto player there is no close relationship between a consistent set and a credibly consistent set. However, every credibly consistent set is in the *largest* consistent set. Next we show that a stable set is credibly consistent. Then we show that there are games for which there is no non-empty credibly consistent set. Moreover, for many games there is no *largest* credibly consistent set. In section 2.3, we investigate the relation of a credibly consistent set to some other solution concepts prevalent in the literature concerning coalitional stability. With respect to the core we find that for many games there may not exist any non-empty credibly consistent set even though the core is non-empty. The stability set of Rubinstein contains any credibly consistent set and the top cycle has a non-empty intersection with any non-empty credibly consistent set. There is no close relation between a credibly consistent set and a consistent stability set, a relatively new solution concept due to Chakravorti (1999). Finally, we give two examples to show that a credibly consistent set often refines the largest consistent set in interesting ways and it is able to pick up intuitively plausible outcomes as stable outcomes in situations where most other solution concepts have no predictive power at all.

2.1 Preliminary Definitions

Throughout the chapter we assume that for each $i \in N$, $\succ_i$ is an irreflexive, total, transitive and asymmetric binary relation on Z : i.e., the individual preferences are assumed to be strict.

The set of minimal winning coalitions will be denoted by W. So we can represent a proper simple game as another tuple $< N,\ Z,\ W,\ (\succ_i)_{i \in N} >$ as well. Throughout this chapter we consider only minimal winning coalitions. There is no loss of generality owing to this simplification.

Next we rephrase the concept of domination in terms of minimal winning coalitions. If for some $S \in W$ and some $a, b \in Z$, $a \succ_i b$ for all $i \in S$ then *a dominates b* via the coalition S which is denoted here as $a \succ_S b$. If there exists $S \in W$ such that $a \succ_S b$ then we shall denote that as $a \succ b$.

For $a,\ b \in Z$, b indirectly dominates a, denoted as $b \gg a$, if there exist $a_0,\ a_1, \ldots,\ a_m$ in Z where $a_0 = a$ and $a_m = b$ and minimal winning coalitions $S_0,\ S_1, \ldots,\ S_{m-1}$ such that for $j = 0, \ldots,\ m-1$, $a_m \succ_{S_j} a_j$. For general games representable by effectivity functions the definition of $\gg$, the indirect domination relation, is analogous. This was the approach of Chwe (1994).

Notice that the paths along which an outcome b can indirectly dominate an outcome a can be of ***arbitrary length.*** Therefore, the definition of a credible deviation from within such a set of paths may be conceptually difficult. However, it is easily seen that for a proper simple game an outcome b ***indirectly dominates*** an outcome a if and only if b ***directly dominates*** a. This fact considerably simplifies our analysis of credible deviations as we can confine our attention only to the direct dominations. We define the notion of credible domination *from within* a set as follows.

Given $Y \subseteq Z$ and $b \in Z$ if there is $S \in W$ and $a \in Y$ such that $a \succ_S b$ and for no other $c \in Y$ it is the case that $c \succ_S a$ then *a dominates b credibly from within Y.* This is denoted as $a \succ_S^c (Y) b$. If for some $Y \subseteq Z$ there exists $S \in W$ such that $a \succ_S^c (Y) b$ then we shall denote that as $a \succ^c (Y) b$.

In the rest of the chapter we shall often call a proper simple game simply a game with no possibility of confusion.

2.2 Consistent Sets and Credibly Consistent Sets

The definition of a consistent set for the class of proper simple games is the following:

DEFINITION 2.2.1 *A set* $Y \subseteq Z$ *is consistent if* $Y = \{a \in Z | \ \forall (S,\ d) \in (W \times Z),\ \exists\ e \in Y$ *such that* $e = d$ *or* $e \succ d$ *and* $e \not\succ_S a\}$.

Moreover,

DEFINITION 2.2.2 *The set* $\Lambda \subseteq Z$ *is the largest consistent set (LCS hereafter) of* Γ *if and only if it is consistent and it contains all the consistent sets.*

The key results regarding consistent sets relevant to our context are the following which we state without proof.[1]

[1] Xue (1997) has generalised Chwe's result concerning the existence of a non-empty consistent set.

PROPOSITION 2.2.1 (Chwe (1994)) (*i*) *For any game there exists a unique largest consistent set.*
(*ii*) *For every game with a finite number of outcomes there exists a non-empty consistent set and hence a non-empty largest consistent set.*

In the introduction we have illustrated that the concept of consistent sets suffers from the possibility of dominated and hence incredible moves by taking an arbitrary game. We show that the same is true in the case of proper simple games with the restrictions imposed by us. Consider for example the following game:

EXAMPLE 2.2.1 $N = \{1, \ldots, 6\}, Z = \{x_1, \ldots, x_4\}, W = \{S_1, \ldots, S_4\}$ where

$$S_1 = \{1,2,3\}, \quad S_2 = \{1,4,5\}, \quad S_3 = \{2,4,6\}, \quad S_4 = \{3,5,6\}.$$

The players' preferences over Z are the following:

1	2	3	4	5	6
x_4	x_2	x_3	x_4	x_3	x_2
x_1	x_4	x_4	x_1	x_4	x_1
x_2	x_1	x_1	x_3	x_1	x_3
x_3	x_3	x_2	x_2	x_2	x_4

This generates the following domination relations:

$$x_4 \succ_{S_1} x_1, \; x_4 \succ_{S_2} x_1, \; x_4 \succ_{S_2} x_2, \; x_1 \succ_{S_2} x_2, \; x_1 \succ_{S_3} x_3 \; x_3 \succ_{S_4} x_4.$$

For no other $a, b \in Z$ and $S \in W$ is it true that $a \succ_S b$.

One can check that $\{x_1, x_3, x_4\}$ is consistent. However, consider the pair (S_1, x_2) with respect to x_1. Then $x_1 \succ_{S_2} x_2$, but of course, $x_1 \not\succ_{S_1} x_1$. This renders x_1 to be a possibly stable outcome. Notice however, that since $x_4 \succ_{S_2} x_1$, S_2's movement from x_2 to x_1 cannot be a credible one. The coalition S_2

would rather move to x_4 which is the *unique* outcome that credibly dominates x_2 from within $\{x_1, x_3, x_4\}$. Since $x_4 \succ_{S_1} x_1$, the stability of x_1 rests on an incredible move.

The idea of the credibly consistent set is motivated by the desire to rule out outcomes like x_1 in Example 2.2.1 from being stable. A set $Y \subseteq Z$ is a credibly consistent set if the following holds. Let an outcome a be a member of Y and suppose some coalition S moves from a to another outcome b. If $b \in Y$ then b is a plausibly stable outcome. If $b \not\succ_S a$, that implies this deviation to a stable outcome b is not liked by at least one member of S. If $b \notin Y$ or $b \in Y$ but $b \succ_S a$, then there should be some outcome c in the credibly consistent set itself which dominates b *credibly from within* Y and which is not strictly preferred to a by all the members of S. The stability of the outcome a rests on the *threat* that any deviation from it would generate a further *credible* deviation which will possibly lead to a stable outcome that is not liked by some member of the coalition which perpetrated the initial deviation. Moreover, for any outcome outside Y the above should not hold.

DEFINITION 2.2.3 *A set $Y \subseteq Z$ is said to be credibly consistent if $Y = \{a \in Z | \ \forall (S,\ d) \in (W \times Z),\ \exists\ e \in Y$ such that $e = d$ or $e \succ^c (Y) d$ and $e \not\succ_S a\}$.*

Notice that since $\emptyset$ satisfies this definition, a credibly consistent set exists for all games. However, for many games no non-empty credibly consistent set exists. Also, the example in the proof of part (*ii*) of Proposition 2.2.5 below suggests that there may exist multiple credibly consistent sets for a game.

Let us first examine to what extent credibly consistent sets are refinements of consistent sets. First, we look at the games with veto players. Recall that $i \in N$ is a *veto player* of Γ if i belongs to *every* winning coalition.

PROPOSITION 2.2.2 *If there is a veto player of Γ then every consistent set is a credibly consistent set.*

Proof: Let some $Y \subseteq Z$ be consistent but not credibly consistent. Then, there are $x_1 \in Y$ and $(T,\ x_2) \in (W \times Z)$ such that
(i) for all $e \in Y$, if $e = x_2$ or $e \succ^c (Y)x_2$ then $e \succ_T x_1$ and
(ii) there is $x_3 \in Y$ (x_3 may be x_1 itself) such that $x_3 \succ x_2$ and $x_3 \not\succ_T x_1$.
Clearly it is not the case that $x_3 \succ^c (Y)x_2$. So, there exists $x_4 \in Y$ such that $x_4 \succ^c (Y)x_3$, $x_4 \succ^c (Y)x_2$, and $x_4 \succ x_1$.

Let $i \in N$ be a veto player. Then, since for any $a,\ b \in Z$, $a \succ b$ implies $a \succ_i b$, we must have:

$$x_4 \succ_i x_3,\ x_4 \succ_i x_2,\ x_4 \succ_i x_1$$

Now consider the domination $x_4 \succ_{T'} x_3$, $T' \in W$. Since Y is consistent and $x_3 \in Y$, there must exist $x_5 \in Y$ such that $x_5 \succ x_4$ and $x_5 \not\succ_{T'} x_3$. Evidently, x_5 cannot be x_1, x_2 or x_3. Hence, we get that

$$x_5 \succ_i x_4,\ x_5 \succ_i x_3,\ x_5 \succ_i x_2,\ x_5 \succ_i x_1.$$

Now consider the domination $x_5 \succ_{T''} x_4$, $T'' \in W$. Proceeding in this manner, for any positive integer k we find that there must exist $x_k \in Y$ such that

$$x_k \succ_i x_{k-1},\ \ldots,\ x_k \succ_i x_2,\ x_k \succ_i x_1$$

which is impossible as Z is finite. ∎

Corollary 2.2.1 *If $|W| \leq 2$ for a game then every consistent set is credibly consistent.*

Proof: By the definition of a proper simple game, for any $S, T \in W$, $S \cap T \neq \emptyset$. So, if $|W| \leq 2$, then there must exist a veto player of the game. Hence, the corollary follows. ∎

REMARK 2.2.1 It can be shown that if $|W| \geq 3$, then there is a game for which no non-empty credibly consistent set exists.

Every credibly consistent set is contained in the LCS.

PROPOSITION 2.2.3 *If $Y \subseteq Z$ is credibly consistent then $Y \subseteq \Lambda$, where Λ is the LCS.*

Proof: Call $X \subseteq Z$ *internally consistent* if $a \in X \Longrightarrow \forall(S,\ d) \in (W \times Z)$, $\exists\ e \in X$ such that $e = d$ or $e \succ d$ and $e \not\succ_S a$. Define $\Lambda' = \cup\{X \subseteq Z |\ X$ is internally consistent$\}$.

We claim that Λ' is consistent. We need to show that $a \in Z \setminus \Lambda' \Longrightarrow \exists(S, d) \in (W \times Z)$ such that for all $e \in \Lambda'$, $[e = d$ or $e \succ d] \Longrightarrow e \succ_S a$. Suppose not, i.e. let there exist $a \in Z \setminus \Lambda'$ for which the following is true:

$$\forall(S,\ d) \in (W \times Z),\ \exists\ e \in \Lambda' \text{ such that } e = d \text{ or } e \succ d \text{ and } e \not\succ_S a.$$

Then clearly, $\Lambda' \cup \{a\}$ is internally consistent which violates the definition of Λ'. So, the claim is proved.

Now, since Y is credibly consistent, Y must be internally consistent. Thus $Y \subseteq \Lambda' \subseteq \Lambda$. ∎

Next we discuss the relationship of a credibly consistent set to a stable set. This will be useful in the subsequent discussion.

DEFINITION 2.2.4 *Given $\succ$, a set $Y \subseteq Z$ satisfies with respect to $\succ$*
(i) internal stability if for all $a, b \in Y$ it is not the case that $a \succ b$ or $b \succ a$,
(ii) external stability if for all $a \in Z \setminus Y$ there exists $b \in Y$ such that $b \succ a$.
A set $Y \subseteq Z$ is a stable set if it is both internally and externally stable.

For more general classes of games one can define a stable set with respect to $\gg$, the indirect domination relation, in an exactly analogous manner. Chwe shows that every stable set with respect to $\gg$ is in the LCS. Owing to the equivalence of $\gg$ and $\succ$ for a proper simple game, a straightforward translation of this result in the context of such a game is that every stable set is in the LCS. In fact, for proper simple games one can prove the stronger result that every stable set is consistent and moreover, a consistent set must be externally stable. The proof would be similar to that of Proposition 2.2.4 below. We have a similar set of results for credibly consistent sets:

PROPOSITION 2.2.4 (i) *If Y is a stable set then Y is credibly consistent.* (ii) *Moreover, every non-empty credibly consistent set is externally stable.*

Proof: (i) Let Y be a stable set and let $a \in Y$. Pick any $(S, d) \in (W \times Z)$. If $d \in Y$ then by internal stability of Y, $d \not\succ_S a$. If $d \notin Y$ then by external stability of Y there is $e \in Y$ such that $e \succ d$ and by internal stability of Y, $e \succ^c (Y)d$. Again by internal stability of Y, $e \not\succ_S a$.

Let $a \notin Y$. Then by external stability of Y there is $d \in Y$ such that $d \succ a$. By internal stability of Y, $\{e \in Y | \ e \succ^c (Y)d\} = \emptyset$. So, there exists $(S, d) \in (W \times Z)$ such that for all $e \in Y$, $e = d$ or $e \succ^c (Y)d$ implies $e \succ_S a$. Hence, Y is credibly consistent.

(ii) Suppose a non-empty $Y \subseteq Z$ is not an externally stable set. Then there is $d \in Z \setminus Y$ such that for all $e \in Y$, $e \not\succ d$. Take any $a \in Y$ and for any $S \in W$ consider the pair (S, d). Since $d \notin Y$ and $\{e \in Y | \ e \succ^c (Y)d\} = \emptyset$, Y cannot be credibly consistent. ■

We have noted above that a credibly consistent set is a refinement of *some* consistent set. However, we find that a stable set is both consistent and credibly consistent. So, the constraint of credible coalitional deviations can refine a consistent set only if the set is not stable.

Now we show that with the imposition of the credibility constraint the nice results of Chwe mentioned in Proposition 2.2.1 break down completely.

PROPOSITION 2.2.5 (i) *If $|Z| \geq 4$, then there exists a game for which there is no non-empty credibly consistent set.*
(ii) *If $|Z| \geq 5$, then there exist games for which no largest credibly consistent set exists.*

Proof: (i) Consider the game in Example 2.2.1. It is easily checked that the LCS for this game, Λ, is $\{x_1, x_3, x_4\}$. Let, if possible, $Y \subseteq Z, Y \neq \emptyset$ be a credibly consistent set. By Proposition 2.2.3, $Y \subseteq \Lambda$. However, Y cannot be Λ. To see this consider the pair (S_1, x_2) with respect to x_1. The set $\{x \in \Lambda | \ x \succ^c (\Lambda)x_2\} = \{x_4\}$. But $x_4 \succ_{S_1} x_1$, and thus $Y \neq \Lambda$. No singleton subset of Λ is externally stable and hence, by Proposition 2.2.4, Y cannot be singleton. Let $Y = \{a, b\}$, $a, b \in \Lambda$. Let, without loss of generality, $b \succ_S a$ for some $S \in W$. This must be true because in this game, for any two $a, b \in \Lambda$, either $a \succ b$ or $b \succ a$. Now consider the pair (S, b) with respect to a. The set $\{x \in Y | \ x \succ^c (Y)b\} = \emptyset$. Thus $Y \neq \{a, b\}$.

(*ii*) Consider the following game:
$N = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$, $Z = \{x_1, x_2, y_1, y_2, d\}$, $W = \{S_1, S_2, S_3, S_4\}$ where $S_1 = \{1, 2, 3\}$, $S_2 = \{1, 5, 6\}$, $S_3 = \{2, 4, 5\}$, $S_4 = \{3, 4, 6\}$.
The individual preferences are given by the following:

1	2	3	4	5	6
x_2	y_2	y_1	x_1	d	x_2
y_1	y_1	x_1	y_1	y_2	d
x_1	x_2	d	y_2	x_1	x_1
y_2	x_1	y_2	d	x_2	y_2
d	d	x_2	x_2	y_1	y_1

This implies the following domination relations:
$x_1 \succ_{S_1} d$, $y_1 \succ_{S_1} d$, $y_1 \succ_{S_1} x_1$, $x_2 \succ_{S_2} y_1$, $y_2 \succ_{S_3} x_2$, $x_1 \succ_{S_4} y_2$.
For no other $a, b \in Z$ and $S \in W$ is it true that $a \succ_S b$.

For this game both the sets $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ are stable sets and hence, by Proposition 2.2.4, these are credibly consistent.

However, neither $X \cup Y$ nor Z is credibly consistent. This is seen as follows.

In the case of $X \cup Y$ consider the pair (S_1, d) with respect to x_1. Then, $\{x \in X \cup Y | \ x \succ^c (X \cup Y)d\} = \{y_1\}$. Since, $y_1 \succ_{S_1} x_1$, $X \cup Y$ is not credibly consistent. Similarly, for Z, consider the pair (S_1, x_1) with respect to the outcome d. Then for both x_1 and y_1 (which is the only outcome that credibly dominates x_1 from within Z), $x_1 \succ_{S_1} d$ and $y_1 \succ_{S_1} d$. Therefore, Z is not credibly consistent. ■

REMARK 2.2.2 The possibility that a game may have no non-empty credibly consistent set may persist even if we add more restrictions. For example, it can be shown that if $|Z| \geq 5$, then there exist games for which no non-empty credibly consistent set exists even if the domination relation is *total.*

REMARK 2.2.3 One explanation of the fact that there may not exist any non-empty credibly consistent set for a game may be as follows. The LCS includes many elements that are stable owing to *only* incredible deviations. If these elements are eliminated then the set of remaining outcomes becomes too small to be internally consistent (an internally consistent set has been defined in the proof of Proposition 2.2.3). As internal consistency is a necessary condition for a set to be credibly consistent, any non-empty credibly consistent set fails to exist.

2.3 Relation to Some Other Solutions

Here we investigate the relationship of credibly consistent sets with some other solution concepts proposed in the literature concerning coalitional stability.

Let us begin with the core $C(\Gamma)$ of a proper simple game Γ.

PROPOSITION 2.3.1 *(i) If Y is a non-empty credibly consistent set of Γ then $C(\Gamma) \subseteq Y$.*
(ii) If $\succ$ is a total relation on Z, then $C(\Gamma) \neq \emptyset$, $\Longrightarrow C(\Gamma)$ is the unique non-empty credibly consistent set.
(iii) If $\succ$ is not total, then there may not exist any non-empty credibly consistent set even if $C(\Gamma) \neq \emptyset$.

Proof: *(i)* This follows from the fact that Y must be an externally stable set.
(ii) Let $\succ$ be total and $C(\Gamma) \neq \emptyset$. It is easily seen that $C(\Gamma)$ must be a

singleton set. Let $C(\Gamma)$ be $\{a\}$. Since $\succ$ is total, evidently $C(\Gamma)$ is a stable set. Hence, by Proposition 2.2.4 it is credibly consistent.

By part (i) of this proposition every non-empty credibly consistent set contains a. Let Y be such a set and let there exist $b \in Y$, $b \neq a$. Since $\succ$ is total, there exists $S \in W$ such that $a \succ_S b$. By considering the pair (S, a) with respect to b we find that Y cannot be credibly consistent.

(iii) Consider the following game which is a slight modification of the game cited in Example 2.2.1.
$N = \{1, \ldots, 6\}$, $Z = \{x_1, \ldots, x_5\}$, $W = \{S_1, \ldots, S_4\}$ where

$$S_1 = \{1, 2, 3\}, \quad S_2 = \{1, 4, 5\}, \quad S_3 = \{2, 4, 6\}, \quad S_4 = \{3, 5, 6\}.$$

The players' preferences over Z are the following:

1	2	3	4	5	6
x_5	x_2	x_3	x_4	x_3	x_5
x_4	x_4	x_4	x_1	x_4	x_2
x_1	x_1	x_1	x_3	x_1	x_1
x_2	x_3	x_2	x_2	x_2	x_3
x_3	x_5	x_5	x_5	x_5	x_4

This generates the following domination relations:

$$x_4 \succ_{S_1} x_1, \; x_4 \succ_{S_2} x_1, \; x_4 \succ_{S_2} x_2, \; x_1 \succ_{S_2} x_2, \; x_1 \succ_{S_3} x_3 \; x_3 \succ_{S_4} x_4.$$

For no other $a, b \in Z$ and $S \in W$ is it true that $a \succ_S b$.

Notice that x_5 is in the core of this game but x_5 dominates no outcome. This implies $\{x_5\}$ cannot be a credibly consistent set as it is not externally stable. Notice that in spite of the inclusion of x_5 the domination relations remain the same as in the game in Example 2.2.1. So, analogous arguments would show that there is no non-empty credibly consistent set for this game.∎

Part (iii) of the result is driven by the fact that the core, even if non-empty, may not be externally stable.

The stability set, introduced by Rubinstein (1980), was one of the early attempts to introduce a limited amount of farsightedness in the behaviour of the coalitions. The intuition behind the stability set is as follows. Suppose an outcome a is the status quo. Let it not be in the core. So there is a minimal winning coalition S that can move to another outcome b such that $b \succ_S a$. However, the members of S must consider the fact that there might exist another minimal winning coalition T which would then move from b to some other outcome c such that $c \succ_T b$ but which may not be desirable to some member in S in comparison to a. If such a possibility exists then S would desist from making the move from a to b. Hence, although there may exist a profitable deviation from a the *threat* that further deviations can be harmful for the coalition which makes the initial move is likely to deter the deviation. Hence, a is a stable outcome in a certain sense and it is a member of the stability set. Formally,

DEFINITION 2.3.1 (Rubinstein (1980)) *The stability set,* $R = \{a \in Z | \ \forall\ (S,d) \in (W \times Z),\ d \succ_S a \implies \exists\ (e,i) \in (Z \times S)$ *such that* $e \succ d$ *and* $a \succ_i e\}$.

The top cycle is the smallest subset of Z such that every outcome outside it is dominated by at least one outcome in it.

DEFINITION 2.3.2 *The top cycle, T is the smallest non-empty subset of Z with respect to set inclusion satisfying* $[\forall a \in Z \setminus T,\ \forall b \in T,\ b \succ a]$.

PROPOSITION 2.3.2 (i) *If $Y \subseteq Z$ is credibly consistent then $Y \subseteq R$.* (ii) *If $Y \subseteq Z$ is credibly consistent and $Y \neq \emptyset$ then $Y \cap T \neq \emptyset$.*

Proof: (i) Since the players' preferences are assumed to be strict, for any $S \in W$ and $a,\ b\ \in Z$, $a \not\succ_S b \implies \exists i \in S$ such that $b \succ_i a$. Hence the result is straightforward from the definition of a credibly consistent set.[2]

[2]Similarly, it is also straightforward from the definitions of the LCS and the stability set that for a proper simple game with strict individual preferences the LCS is contained

(*ii*) If $Y \neq \emptyset$ and $Y \cap T$ is $\emptyset$ then Y cannot be externally stable. Then Y cannot be credibly consistent. ■

REMARK 2.3.1 One recent significant addition to the solution concepts for coalitional stability is a *consistent stability set* (Chakravorti (1999)) which is an attempt to blend the ideas of the stability set and a consistent set. We find that there is no close relation between a consistent stability set and a credibly consistent set. There exists a game for which no consistent stability set exists but Z is credibly consistent. The game cited in Section 1.1 illustrating the "paradox of voting" is an example in this regard. On the other hand, a game may have a consistent stability set but yet may not have any non-empty credibly consistent set. The game cited in the proof of part (*iii*) of Proposition 2.3.1 has this feature.

Next we give some examples to show that when a credibly consistent set is non-empty then often it performs better than many of the solution concepts in deterring unreasonable outcomes from being stable. In the examples that follow, the LCS has no predictive power at all: it is the entire set of outcomes. However, for each of these games there exists a unique non-empty credibly consistent set that refines the LCS in interesting ways.

First we introduce two other solution concepts for the purpose of comparison with a credibly consistent set. These have been defined in the literature for games for which $\succ$ is a total relation on Z. The first one is the uncovered set introduced by Miller (1980).

DEFINITION 2.3.3 *Given $X \subseteq Z$ and $a, b \in Z$, b is said to cover a in X if and only if $b \succ a$ and for all $c \in X$, $a \succ c$ implies $b \succ c$. The uncovered set of $X \subseteq Z$, denoted $uc(X)$, is the set: $\{a \in X \mid$ for no $b \in X$, b covers a in $X\}$.*

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in the stability set. Chwe (1994) has shown this result for majority rule voting games with odd number of players and strict individual preferences.

The uncovered set of Z is simply called the uncovered set.

A covering set (Dutta (1988)) is defined as follows:

DEFINITION 2.3.4 *Given $X \subseteq Z$, $X_1 \subseteq X$ is a covering set of X if and only if (i) $uc(X_1) = X_1$ and (ii) if $b \in X \setminus X_1$ then $b \notin uc(X_1 \cup \{b\})$.*

Dutta (1988) has shown that there exists a unique minimal covering set of Z for every game for which the domination relation is total.

EXAMPLE 2.3.1 Consider the following game:

$$N = \{1, \ \ldots, \ 6\}, \ Z = \{x_1, \ \ldots, \ x_6\}, \ W = \{S_1, \ \ldots, \ S_4\} \text{ where}$$

$$S_1 = \{1,2,3\}, \quad S_2 = \{1,4,5\}, \quad S_3 = \{2,4,6\}, \quad S_4 = \{3,5,6\},$$

The players' preferences over Z are the following:

1	2	3	4	5	6
x_3	x_1	x_2	x_1	x_3	x_2
x_1	x_2	x_3	x_6	x_5	x_4
x_5	x_6	x_4	x_4	x_6	x_5
x_2	x_3	x_1	x_3	x_2	x_1
x_6	x_4	x_5	x_5	x_4	x_6
x_4	x_5	x_6	x_2	x_1	x_3

This generates the following domination relations:

$x_1 \succ_{S_1} x_6$, $x_2 \succ_{S_1} x_6$, $x_3 \succ_{S_1} x_4$, $x_2 \succ_{S_1} x_4$; $x_1 \succ_{S_1} x_5$, $x_3 \succ_{S_1} x_5$,

$x_6 \succ_{S_2} x_4$, $x_5 \succ_{S_2} x_2$, $x_3 \succ_{S_2} x_5$, $x_3 \succ_{S_2} x_2$,

$x_4 \succ_{S_3} x_5$, $x_1 \succ_{S_3} x_6$, $x_1 \succ_{S_3} x_3$, $x_6 \succ_{S_3} x_3$,

$x_5 \succ_{S_4} x_6$, $x_2 \succ_{S_4} x_4$, $x_4 \succ_{S_4} x_1$, $x_2 \succ_{S_4} x_1$.

For no other $a, b \in Z$ and $S \in W$ is it true that $a \succ_S b$. Note that for this game, for every $a, b \in Z$, either $a \succ b$ or $b \succ a$.

This example is less complicated than it seems. There are some underlying symmetries within Z and W captured by the following permutations.

Define the permutation $\xi : Z \mapsto Z$ such that $\xi(x_1) = x_2$, $\xi(x_2) = x_3$, $\xi(x_3) = x_1, \xi(x_4) = x_5$, $\xi(x_5) = x_6$, $\xi(x_6) = x_4$, and the permutation $\eta : W \mapsto W$ such that $\eta(S_1) = S_1, \eta(S_2) = S_3, \eta(S_3) = S_4, \eta(S_4) = S_2$. We shall use these symmetries in proving results later in the example.

Now let us partition Z into two sets: $Z_1 = \{x_1, x_2, x_3\}$ and $Z_2 = \{x_4, x_5, x_6\}$. Notice that each outcome in Z_1 is dominated by two other outcomes whereas each outcome in Z_2 is dominated by three other outcomes. So, intuitively, elements in Z_1 are more suitable as stable outcomes.

The LCS for this game is Z. We can check this as follows. First, take x_1. Consider the pairs (S_4, x_4) and (S_4, x_2) with respect to x_1. Then $x_3 \succ x_4$ and $x_3 \succ x_2$ but $x_3 \not\succ x_1$. For no other $(S, d) \in (W \times Z)$ it is the case that $d \succ_S x_1$. Then take x_4. Consider the pairs (S_1, x_2) and (S_4, x_2) with respect to x_4. Then $x_5 \succ x_2$ but $x_5 \not\succ x_4$. Again, consider the pairs (S_1, x_3) and (S_2, x_6) with respect to x_4. Then $x_1 \succ x_3$, $x_1 \succ x_6$ but $x_1 \not\succ x_4$. For no other $(S, d) \in (W \times Z)$ it is the case that $d \succ_S x_4$. Thus, for $x \in \{x_1, x_4\}$ it is true that $\forall (S,\ d) \in (W \times Z)$, $\exists\ e \in Z$ such that $e = d$ or $e \succ d$ and $e \not\succ_S x$. Now invoke the permutations ξ and η and carry on applying the argument given above for outcome x to $\xi(x)$ using $\xi(y)$ in place of each $y \in Z$ and $\eta(S)$ in place of each $S \in W$. Thus we can check that Z is indeed the LCS.

Therefore, the stability set as well is Z (see footnote 4). The core of this game is empty and there is no stable set. One can also check that Z is the top cycle and the uncovered set. So, a number of solutions quite common in the literature have little predictive value for this game.

However, we can show that Z_1 is the *unique* credibly consistent set for this game. First we show that Z_1 is credibly consistent. Take x_1. Consider the pairs (S, x_4), $(S \in W)$ and (S_4, x_2) with respect to x_1. Then $x_3 \succ^c (Z_1) x_4$ and $x_3 \succ^c (Z_1) x_2$ but $x_3 \not\succ x_1$. For any pair of the type (S, x_5) or (S, x_6), $(S \in W)$, $x_1 \succ^c (Z_1) x_5$ and $x_1 \succ^c (Z_1) x_6$. But, of course, $x_1 \not\succ x_1$. For no pair $(S, d) \in (W \times Z)$ other than (S_4, x_2) and (S_4, x_4), it is the case that $d \succ_S x_1$. So, for x_1, it is true that $\forall (S,\ d) \in (W \times Z)$, $\exists\ e \in Z_1$ such that $e = d$ or $e \succ^c (Z_1) d$ and $e \not\succ_S x_1$. Once again invoking the permutations ξ

and η and replacing every outcome x by $\xi(x)$ and every coalition S by $\eta(S)$ we can check that the same is true for x_2 and x_3.

Now we consider outcomes in Z_2. Take x_4 and consider the pair (S_1, x_2) with respect to it. Then $x_2 \succ_{S_1} x_4$. The only element in Z_1 that credibly dominates x_2 from within Z_1 is x_3 and $x_3 \succ_{S_1} x_4$. Similarly, by considering (S_1, x_3) with respect to x_5 and (S_1, x_1) with respect to x_6 we find that for each $a \in Z_2$, it is true that $\exists(S, d) \in (W \times Z)$ such that for all $e \in Z_1$, $[e = d$ or $e \succ^c (Z_1)d] \implies e \succ_S a$. So, Z_1 is credibly consistent.

Next we show that no other $Y \subseteq Z$ can be credibly consistent. Let $Y \subseteq Z$, $Y \neq Z_1$ be credibly consistent. First we claim that if $x_3 \in Y$, then $x_4 \notin Y$. This is seen by considering the pair (S_1, x_2) with respect to x_4. If $x_3 \in Y$ then the only outcome that credibly dominates x_2 from within Y is x_3. And both $x_2 \succ_{S_1} x_4$ and $x_3 \succ_{S_1} x_4$. So, the claim is proved. Similarly, by considering (S_1, x_3) with respect to x_5 and (S_1, x_1) with respect to x_6 we find that if $x_1 \in Y$, then $x_5 \notin Y$ and if $x_2 \in Y$, then $x_6 \notin Y$. This implies $|Y| \leq 3$. Now, no singleton subset of Z is externally stable and so, by Proposition 2.2.4, cannot be credibly consistent. Hence, $|Y| \neq 1$. Since this game is strong, no doubleton subset can also be credibly consistent. So, $|Y| = 3$.

Now let, if possible, $Y = Z_2$. Consider (S_2, x_3) with respect to x_4. Then, only x_6 credibly dominates x_3 from within Z_2. But $x_6 \succ_{S_2} x_4$. So, Z_2 is not credibly consistent. Now let Y have two elements from Z_1 and one from Z_2. Let, without loss of generality, $x_1, x_2 \in Y$. Then, as we have shown above, the element from Z_2 must be x_4. But then considering the pair (S_1, x_2) with respect to x_4, we find that $\{x_1, x_2, x_4\}$ is not credibly consistent. Using the permutations ξ and η in the same way as we have done above, we can easily check that no such set with two elements from Z_1 and one from Z_2 can be credibly consistent.

Now let Y have two elements from Z_2 and one from Z_1. Let, without loss of generality, $x_4, x_5 \in Y$. Then, as we have shown above, the element from Z_1 must be x_2. Then Y fails to be externally stable as no outcome in $\{x_4, x_5, x_2\}$ dominates x_3. So, $\{x_4, x_5, x_2\}$ is not credibly consistent. Once again, using the permutations ξ and η, we can easily check that no such set with two elements from Z_2 and one from Z_1 can be credibly consistent. So, Z_1 is the unique credibly consistent set.

We remark here that Z_1 is Dutta's minimal covering set as well.

EXAMPLE 2.3.2 Consider the following game:
$N = \{1, \ldots, 8\}$, $Z = \{x_1, x_2, y_1, y_2, z_1, z_2, a, b\}$, $W = \{S_1, \ldots, S_5\}$ where

$S_1 = \{1,4,5\}$, $S_2 = \{2,4,6,8\}$, $S_3 = \{3,5,6\}$, $S_4 = \{1,2,3\}$, $S_5 = \{1,5,6,7\}$.

The players' preferences over Z are the following:

1	2	3	4	5	6	7	8
z_2	x_2	y_2	a	b	b	x_1	b
z_1	x_1	y_1	z_1	y_2	x_2	y_1	z_2
b	a	a	y_1	x_2	z_2	z_1	y_2
a	z_1	x_1	x_1	z_2	a	x_2	x_2
y_1	y_1	z_1	b	a	x_1	y_2	a
x_1	b	b	z_2	y_1	z_1	z_2	x_1
y_2	z_2	x_2	y_2	x_1	y_1	a	z_1
x_2	y_2	z_2	x_2	z_1	y_2	b	y_1

This generates the following domination relations:
$b \succ_{S_1} y_2$, $b \succ_{S_1} x_2$, $y_2 \succ_{S_1} x_2$, $a \succ_{S_1} y_1$, $a \succ_{S_1} x_1$, $y_1 \succ_{S_1} x_1$,
$b \succ_{S_2} z_2$, $b \succ_{S_2} y_2$, $z_2 \succ_{S_2} y_2$, $a \succ_{S_2} z_1$, $a \succ_{S_2} y_1$, $z_1 \succ_{S_2} y_1$,
$b \succ_{S_3} x_2$, $b \succ_{S_3} z_2$, $x_2 \succ_{S_3} z_2$, $a \succ_{S_3} x_1$, $a \succ_{S_3} z_1$, $x_1 \succ_{S_3} z_1$,
$z_1 \succ_{S_4} b$,
$z_2 \succ_{S_5} a$.
For no other $a, b \in Z$ and $S \in W$ is it true that $a \succ_S b$. Like in the game in Example 2.3.1, there are certain symmetries within Z and W in this game as well which we shall indicate later.

Notice that the outcomes a and b are dominated by the least number of outcomes - only one outcome dominates each of them. Moreover, these two outcomes also dominate the maximum number of outcomes, both of them dominate three other different outcomes. Therefore, intuitively a and b have relatively superior claims to be counted as stable outcomes for this game.

We show that the LCS for this game is Z. This is seen as follows. Take a. Consider the pair (S_5, z_2) with respect to it. Then $b \succ z_2$ but $b \not\succ a$. For no other pair $(S, d) \in (W \times Z)$ is it the case that $d \succ_S a$. For b, consider the pair (S_4, z_1) with respect to it. Then $a \succ z_1$ but $a \not\succ b$. For no other pair $(S, d) \in (W \times Z)$ is it the case that $d \succ_S b$. Now take x_1. Consider the pairs (S_1, y_1), (S_1, a) and (S_3, a) with respect to it. Then $z_1 \succ y_1$ but $z_1 \not\succ x_1$, and $z_2 \succ a$ but $z_2 \not\succ x_1$. For no other pair $(S, d) \in W \times Z$ is it the case that $d \succ_S x_1$. So, for each $x \in \{a, b, x_1\}$, we find that $\forall (S,\ d) \in (W \times Z)$, $\exists\ e \in Z$ such that $e = d$ or $e \succ d$ and $e \not\succ_S x$. Now define the permutation $\xi : Z \mapsto Z$ such that $\xi(x_1) = y_1$, $\xi(y_1) = z_1$, $\xi(z_1) = x_1, \xi(x_2) = x_2$, $\xi(y_2) = y_2$, $\xi(z_2) = z_2, \xi(a) = a$, $\xi(b) = b$ and the permutation $\eta : W \mapsto W$ such that $\eta(S_1) = S_2, \eta(S_2) = S_3, \eta(S_3) = S_1, \eta(S_4) = S_4, \eta(S_5) = S_5$. Then carry on applying the argument given above for x to $\xi(x)$ using $\xi(y)$ in place of each $y \in Z$ and $\eta(S)$ in place of each $S \in W$. This will verify that each $x \in \{x_1, y_1, z_1\}$ satisfies the condition of consistency. Now apply a different pair of permutations ξ' and η' as follows. Define $\xi' : Z \mapsto Z$ such that $\xi'(x_1) = x_2$, $\xi'(y_1) = y_2$, $\xi'(z_1) = z_2, \xi'(x_2) = x_1$, $\xi'(y_2) = y_1$, $\xi'(z_2) = z_1, \xi'(a) = b$, $\xi'(b) = a$ and the permutation $\eta' : W \mapsto W$ such that $\eta'(S_1) = S_1, \eta'(S_2) = S_2, \eta'(S_3) = S_3, \eta'(S_4) = S_5, \eta'(S_5) = S_4$. Then apply the argument given above for x to $\xi'(x)$ using $\xi'(y)$ in place of each $y \in Z$ and $\eta'(S)$ in place of each $S \in W$. Then we find that for every $x \in \{x_2, y_2, z_2\}$ also, it is true that $\forall (S,\ d) \in (W \times Z)$, $\exists\ e \in Z$ such that $e = d$ or $e \succ d$ and $e \not\succ_S x$. Thus, Z is consistent.

Hence, the stability set for this game is also Z (see footnote 4). It can also be checked that the top cycle is Z. So, like in Example 2.3.1, a number of solutions have little predictive value for this game. However, $\{a, b\}$ is the *unique* credibly consistent set for this game.

First note that $\{a, b\}$ is a stable set for this game and therefore, by Proposition 2.2.4, it is credibly consistent. Let $Y \subseteq Z$ be any other credibly consistent set. We claim that if $a \in Y$, then neither of x_1, y_1 or z_1 is in Y. Take x_1 and consider the pair (S_1, y_1) with respect to it. If $a \in Y$ then the only outcome that credibly dominates y_1 from within Y is a. But $a \succ_{S_1} x_1$ and also $y_1 \succ_{S_1} x_1$. So, $x_1 \notin Y$. Similarly, considering (S_2, z_1) with respect to y_1 and (S_3, x_1) with respect to z_1 we find the claim to be true. Using the permutations ξ' and η' in the same manner as we have done above, we can show that if $b \in Y$, then neither of x_2, y_2 or z_2 is in Y. Now notice that neither of the sets $\{a, x_2, y_2, z_2\}$ and $\{b, x_1, y_1, z_1\}$ is externally stable. So, $Y \subseteq \{x_1, y_1, z_1, x_2, y_2, z_2\}$.

Notice that for Y to be externally stable, it must be that $\{z_1, z_2\} \subseteq Y$. But then neither of y_1 and y_2 is in Y. To see this consider the pair (S_2, b) with respect to y_1 and the pair (S_2, a) with respect to y_2. Then only $z_1 \succ^c (Y)b$ and $z_2 \succ^c (Y)a$. However, $z_1 \succ_{S_2} y_1$ and $z_2 \succ_{S_2} y_2$. So, $Y \subseteq \{x_1, z_1, x_2, z_2\}$. However, this is not possible. Consider the pair (S_3, x_1) with respect to z_1. Then $x_1 \succ_{S_3} z_1$ and no outcome in Y dominates x_1. So, $Y \neq \{x_1, z_1, x_2, z_2\}$. But no proper subset of $\{x_1, z_1, x_2, z_2\}$ is externally stable. Hence, $\{a, b\}$ is the unique credibly consistent set for this game.

2.4 Conclusion

In this chapter we have analysed *one* aspect of coalitional stability under the constraint of credible coalitional deviations. Our analysis suggests that the imposition of such a constraint can change the properties of a solution very drastically. An analogous constraint with respect to the stability set yields results somewhat similar to those relating to the credibly consistent sets. However, the issue of coalitional stability with credible coalitional deviations is largely an unexplored one. The analysis of such deviations is fraught with conceptual difficulties for the general class of games representable by effectivity functions.

Chapter 3

On the Equal Division Core

The main theme of the present chapter is an axiomatic characterization of the equal division core (EDC) on two classes of TU games. In section 3.1 we briefly mention an alternative interpretation of the EDC owing to Dutta and Ray (1991). In section 3.2 we describe a set of properties of a solution on the class of TU games for which the EDC is non-empty and show that EDC is the unique solution on that class which satisfies all the properties in the set. In section 3.3 we show that almost a similar set of properties characterize the EDC on the class of *all* TU games. In this chapter we shall often simply use *game* for a TU game with no possibility of confusion.

3.1 The EDC and the Lorenz Core

We pointed out in the introductory chapter that the idea of EDC was conceived to explain results of experimental TU games where it was observed that the agents seem to follow an ad-hoc norm of fairness. However, it was later found out that the concept of the EDC is quite crucial for the analysis of egalitarian core-like solutions on TU games. Dutta and Ray (1991) arrived at the notion of EDC starting with an idea quite different from Selten's. To get to their interpretation of the EDC let us recall the notion of Lorenz-domination. Let $A \subset \mathbf{R}^n$ be a set of n-person allocations of a given total. For any vector $x \in A$, we denote by $\tilde{x}$ the vector in $\mathbf{R}^n$ obtained by permuting the components of x such that $\tilde{x}_1 \leq \tilde{x}_2 \leq \ldots \leq \tilde{x}_n$.

For two vectors $x, y \in A$, x Lorenz-dominates y if for every integer $k, 1 \leq k \leq n-1$, $\sum_{i=1}^{k} \tilde{x}_i \geq \sum_{i=1}^{k} \tilde{y}_i$ with at least one strict inequality.

By $E(A)$ we denote the set $\{x \in A|$ there is no $y \in A$ such that y Lorenz-dominates $x\}$

Dutta and Ray defined the Lorenz core as follows:

DEFINITION 3.1.1 (Dutta and Ray) *Given a game (N, v), the Lorenz core of a singleton coalition $\{i\} \subset N$, $\lambda(\{i\}) = v(\{i\})$. Suppose we have defined the Lorenz core for all coalitions of cardinality less than or equal to k. Then the Lorenz core of a coalition S of size $k+1$ is given by: $\lambda(S) = \{x \in \mathbf{R}^S | \sum_{i \in S} x_i = v(S)$ and there do not exist $T \subset S$ and $y \in E(\lambda(T))$ such that $y \gg x_T\}$.*

So, the Lorenz core is the set of imputations for the grand coalition that remain unblocked when the following are true. All the subcoalitions of N are commited to a norm of egalitarianism represented by Lorenz domination. Moreover, an allocation for a coalition can be objected to by a subcoalition only with an allocation which is egalitarian (in the sense of Lorenz ordering) within its Lorenz core.

They proved the following result connecting the Lorenz core and the EDC:

PROPOSITION 3.1.1 (Dutta and Ray) *For any game (N, v), $\lambda(N) = L(N, v)$.*

This new interpretation of the EDC enhances its importance as an equity-related solution on TU games.

3.2 Characterization of the EDC on Γ_L

Throughout this chapter, the set of games for which the EDC is non-empty is denoted by Γ_L and the set of all TU games is denoted by $\bar{\Gamma}$.

REMARK 3.2.1 The class Γ_L is quite large: many games derived from economic situations are in it. Γ_L contains the class of weakly superadditive games (see Dutta and Ray (1991)).

Let us recall the definition of a solution.

DEFINITION 3.2.1 *Let $\Gamma_0 \subseteq \bar{\Gamma}$. A solution on Γ_0 is a mapping ψ which associates with each game $(N, v) \in \Gamma_0$ a subset $\psi(N, v)$ of $X(N, v)$.*

Now, let us introduce the following axioms. Take $(N, v) \in \Gamma_L$.
1. **Non-emptiness** (NE):
The set $\psi(N, v) \neq \emptyset$.
2. **Weak Upper Hemicontinuity** (WUHC):
Let $\{(N, v^k)\}$ be a sequence of games in Γ_L such that for all k, $v^k(S) = v(S)$ for $S \subset N$ and $v^k(N) \longrightarrow v(N)$. Let $\{x^k\}$ be a sequence such that $x^k \in \psi(N, v^k)$ for all k and $x^k \longrightarrow x$. Then $x \in \psi(N, v)$.

This axiom states that if two games in Γ_L, (N, v^1) and (N, v^2), are close enough - the differences in the worth of the grand coalitions being small and the worth of the other coalitions being equal - then an allocation in the solution of (N, v^1) will be close enough to one allocation in the solution of (N, v^2). Similar axioms are quite prevalent in the literature (e.g. Lahiri (1998), Thomson (1983)).
3. **Antimonotonicity** (AM):
Let $(N, v') \in \Gamma_L$ be such that $v'(S) \leq v(S)$ for all $S \subset N$ and $v'(N)=v(N)$. Then $\psi(N, v) \subseteq \psi(N, v')$.

The intuition is that if the coalitions get impoverished then the pay-off vectors in the solution of the original game remain in the solution of the new game and additionally some more pay-off vectors feasible for the grand coalition may qualify as solution vectors. Keiding (1986) introduced this axiom in the literature.

The next axiom is one of consistency. Usually the consistency properties of a solution on the class of TU games is defined in terms of reduced games (see Thomson(1996)). Given a game (N, v) and a pay-off vector $x \in X(N, v)$ a reduced game on a coalition S describes the pay-offs available to the different subcoalitions of S. Different assumptions on the behaviour of the players in S and the possibility of their cooperating with the players outside S generate different reduced games.

Let us define the following reduced game.

DEFINITION 3.2.2 *Let $x \in X(N, v)$. The secession reduced game on $S \subset N$, $(S \neq \emptyset)$ with respect to x, (S, v_S^x) is given by:*

$$v_S^x(S) = v(N) - \sum_{i \in N \setminus S} x_i,$$

$$v_S^x(T) = v(T) \text{ for } T \subset S.$$

The intuition is that if the players in $N \setminus S$ leave, no cooperation with them is possible any more. However, the commitment to their pay-offs has to be honored by the grand coalition S in the reduced game. Moreover, since no cooperation with the players in $N \setminus S$ is possible, the worth of each $T \subset S$ in the reduced game remains what it was in the original game.

Nagahisa and Yamato (1992) have used a somewhat similar reduced game in their characterization of the core.

4. **Secession Consistency** (SC):
If $x \in \psi(N, v)$ then for any coalition S, (S, v_S^x), the secession reduced game on S with respect to x, is in Γ_L and $x_S \in \psi(S, v_S^x)$.

The next axiom is a condition on subgames.

DEFINITION 3.2.3 *Given a game (N, v), the subgame on a coalition S, (S, v_S) is given by:*

$$v_S(T) = v(T) \text{ for all } T \subseteq S.$$

5. **Weak Internal Stability for Proximal Coalitions:** (WISPC)
Let $x \in \psi(N, v)$ and let for some $S \subset N$ with $|S| = |N| - 1$, (S, v_S) be in Γ_L. Then for all $y \in \psi(S, v_S)$,

$$max_{j \in S}\ x_j \geq min_{j \in S}\ y_j.$$

Suppose for a coalition S proximal to N (obtained by dropping only one player) even the worst-paid player in a pay-off vector y in the solution of the subgame on S gets more than that is given to *any* player of S in an allocation x for the grand coalition. Then, this axiom specifies that if x is *so bad* for such a *large fraction of the players* then x should not be in the solution of the whole game.

PROPOSITION 3.2.1 *There is a unique solution on Γ_L that satisfies NE, WUHC, AM, SC and WISPC and it is the EDC.*

It is obvious that EDC satisfies NE, AM, SC and WISPC on Γ_L. We prove the rest of the proposition with the help of the following lemmata. Before proceeding onto the proof of the theorem we define the notion of an equity coalition that we shall use in the proof.

DEFINITION 3.2.4 *Given a game (N, v) we call a coalition S an equity coalition of (N, v) if*

$$a(S, v) \geq a(T, v) \textit{ for all } T \subset S.$$

LEMMA 3.2.1 *For any $(N, v) \in \Gamma_L$, $L(.)$ satisfies WUHC.*

Proof: Let $\{(N, v^k)\}$ be a sequence of games in Γ_L such that for all k, $v^k(S) = v(S)$ for $S \subset N$ and $v^k(N) \longrightarrow v(N)$. Let $\{x^k\}$ be a sequence such that $x^k \in L(N, v^k)$ for all k and $x^k \longrightarrow x$.

First we show that $\sum_{i \in N} x_i = v(N)$. Let $s(.) : \mathbf{R}^N \mapsto \mathbf{R}$ be the continuous map such that for $x \in \mathbf{R}^N$, $s(x) = \sum_{i \in N} x_i$. Since $x^k \longrightarrow x$, $s(x^k) \longrightarrow s(x)$. Now, for all k, $s(x^k) = v^k(N)$. As $v^k(N) \longrightarrow v(N)$, $s(x) = v(N)$.

Now let $x \notin L(N, v)$. Then there is $S \subset N$ such that $e(S, v) \gg x_S$. Choose $\epsilon > 0$ such that $\epsilon < a(S, v) - max_{j \in S} x_j$. Since $x^k \longrightarrow x$, there is $(N, v^l) \in \{(N, v^k)\}$ such that $x^l \in L(N, v^l)$ and $max_{i \in S}|x_i^l - x_i| < \epsilon$ (here $|.|$ stands for absolute value). But then clearly $e(S, v) \gg x_S^l$ and therefore,

$x^l \notin L(N, v^l)$. ■

LEMMA 3.2.2 *If a solution $\psi(.)$ satisfies NE, SC and WISPC on Γ_L then for any $(N,v) \in \Gamma_L$, $x \in \psi(N,v)$ implies that for all $i \in N$, $x_i \geq v(\{i\})$.*

Proof: Note that every one-player game is in Γ_L and hence by NE, for all $i \in N$, $\psi(\{i\}, v_{\{i\}}) \neq \emptyset$. So, by the definition of a solution (in particular, the property that the pay-off vectors in a solution must be efficient), $\psi(\{i\}, v_{\{i\}}) = v(\{i\})$. Now, let $|N| > 1$ and $x_i < v(\{i\})$ for some $i \in N$. If $|N| = 2$, then $\psi(.)$ clearly violates WISPC. If $|N| > 2$, then pick $j \in N \setminus \{i\}$ and construct $(\{i,j\}, v^x_{\{i,j\}})$, the secession reduced game on $\{i,j\}$ with respect to x. Then by SC, $(x_i, x_j) \in \psi(\{i,j\}, v^x_{\{i,j\}})$. But then again, $\psi(.)$ violates WISPC. ■

LEMMA 3.2.3 *If a solution $\psi(.)$ satisfies NE, WUHC, AM, SC and WISPC on Γ_L then for any $(N,v) \in \Gamma_L$, $L(N,v) \subseteq \psi(N,v)$.*

Proof: Take $(N,v) \in \Gamma_L$ and let $x \in L(N,v)$. Fix $\epsilon > 0$ and construct the game (N, v^ϵ) as follows:

$$v^\epsilon(N) = v(N) + \epsilon,$$

and for $S \subset N$,

$$v^\epsilon(S) = v(S).$$

Clearly, $(N, v^\epsilon) \in \Gamma_L$ and the vector x^ϵ, given by $x^\epsilon_i = x_i + \epsilon/|N| \; \forall i \in N$, is in $L(N, v^\epsilon)$.

Now, further construct the game $(N, v^{\epsilon,x})$ for which $v^{\epsilon,x}(S) = v^\epsilon(S)$ for every non-singleton coalition S and for all $i \in N$, $v^{\epsilon,x}(\{i\}) = x^\epsilon_i$. Then again, $(N, v^{\epsilon,x}) \in \Gamma_L$ and hence, by NE, $\psi(N, v^{\epsilon,x}) \neq \emptyset$. Then, by the definition of a solution and Lemma 3.2.2, $\psi(N, v^{\epsilon,x}) = \{x^\epsilon\}$. Therefore, by AM, $x^\epsilon \in \psi(N, v^\epsilon)$. Now, take a decreasing sequence of positive numbers $\{\epsilon^k\}$ such that $\epsilon^1 = \epsilon$ and $\epsilon^k \longrightarrow 0$. For each k, construct a game (N, v^{ϵ^k}) such that:

$$v^{\epsilon^k}(N) = v(N) + \epsilon^k,$$

and for $S \subset N$,

$$v^{\epsilon^k}(S) = v(S).$$

Let x^{ϵ^k} be the vector given by $x_i^{\epsilon^k} = x_i + \epsilon^k/|N|\ \forall i \in N$. By our argument above, for each k, x^{ϵ^k} is in $\psi(N, v^{\epsilon^k})$. Then for the sequence $\{(N, v^{\epsilon^k})\}$, $v^{\epsilon^k}(N) \longrightarrow v(N)$ and $x^{\epsilon^k} \longrightarrow x$. Then by WUHC, $x \in \psi(N, v)$. ■

LEMMA **3.2.4** *If a solution $\psi(.)$ satisfies NE, WUHC, AM, SC and WISPC on Γ_L then for any $(N, v) \in \Gamma_L$, $\psi(N, v) \subseteq L(N, v)$.*

Proof: Take $(N, v) \in \Gamma_L$ and let $x \in \psi(N, v) \setminus L(N, v)$. Then there is an equity coalition S for which $e(S, v) \gg x_S$. Notice that by Lemma 3.2.3, $e(S, v) \in \psi(S, v_S)$. Therefore, if $|S| = |N| - 1$, then $\psi(.)$ violates WISPC. Suppose $|S| < |N| - 1$. Then pick $j \in N \setminus S$ and let T be $S \cup \{j\}$. By SC, $x_T \in \psi(T, v_T^x)$. But then once again $\psi(.)$ violates WISPC. ■

This completes the proof of the proposition.

Below we show that every axiom is independent of the others. For each of the above axioms we show that there is a solution on Γ_L which satisfies the other four but fails to satisfy it.

NE: Let $\psi(N, v)$ be the core of (N, v)[1]. Then $\psi(.)$ satisfies WUHC, AM, SC and WISPC on Γ_L but not NE.

[1]Recall that the core of a game (N,v), $C(N, v) = \{x \in X(N, v) |$ for every coalition S, $\sum_{i \in S} x_i \geq v(S)\}$.

WUHC: Fix $(N', v') \in \Gamma_L$ such that $v'(N')/|N'| > v'(S)/|S|$ for all $S \subset N'$, $S \neq \emptyset$. Let $\psi(.)$ on Γ_L be such that $\psi(N, v)$=$L(N, v)$ if $(N, v) \in \Gamma_L \backslash \{(N', v')\}$ and $e(N', v')$ otherwise. Then $\psi(.)$ satisfies NE, AM, SC and WISPC on Γ_L but not WUHC.

AM: Fix $G' = (N', v') \in \Gamma_L$ and $i_0 \in N'$. Define $\psi(.)$ on Γ_L as follows. For $(N, v) \in \Gamma_L$ such that $N = N'$, $\psi(N, v) =\{x \in L(N, v)|\ x_{i_0} = v(\{i_0\})\}$ and $\psi(N, v) = L(N, v)$ otherwise. Then $\psi(.)$ satisfies NE, WUHC, SC and WISPC on Γ_L but not AM.

SC: Define $\psi(.)$ on Γ_L as:
$\psi(N, v) = \{x \in X(N, v)|$ there does not exist $S \subset N$ such that $|S| =|N| - 1$ and $e(S, v) \gg x_S\}$.
Then $\psi(.)$ satisfies NE, WUHC, AM and WISPC on Γ_L but not SC.

WISPC: Fix a player set $N' = \{1, 2, 3\}$. Let $\Gamma_{N'}$ be the family of games such that if $(N, v) \in \Gamma_{N'}$ then $N = N'$ and $v(\{1, 2, 3\}) = 9$, $v(\{1, 2\}) \leq 8$, and $v(S) \leq 0$ for any other coalition S.

Let $\psi(.)$ on Γ_L be as follows: $\psi(N, v) = L(N, v) \cup \{(3, 3, 3)\}$ for $(N, v) \in \Gamma_{N'}$ and $\psi(N, v) = L(N, v)$ otherwise. Then $\psi(.)$ satisfies NE, WUHC, AM and SC on Γ_L but not WISPC.

3.3 Characterization of the EDC on $\bar{\Gamma}$

Although Γ_L is quite large, there are games arising from real-life situations for which the EDC is empty. We cite one such example below.

EXAMPLE 3.3.1 (A modification of the Garbage game of Shapley-Shubik (1969))
Suppose n persons reside in a locality. They may stay in different houses or

any group of them may build a house for themselves and live together. There is an economy of scale in living together: if s persons move in together then this economy is measured by an increasing function $\phi(s)$. 1 unit of garbage is generated owing to each person and this is to be dumped in the yard of one of the houses. As garbage is a public bad, each person suffers 1 unit of disutility if 1 unit of garbage is dumped in the yard of his residence. Like Shapley and Shubik we assume that there is no free disposal outside the yards of the houses.

The characteristic function representation of the situation is the following:

$$v(N) = \phi(n) - n^2,$$

$$v(S) = \phi(|S|) - |S|(n - |S|) \text{ for } S \subset N,\ v(\emptyset) = 0.$$

Let $N = \{1, 2, 3\}$ and $\phi(|S|) = |S|$ for every non-singleton coalition while $\phi(1) = 0$. Then it is easily checked that $L(N, v) = \emptyset$.

The existence of such games motivates us to characterize the EDC on the set $\bar{\Gamma}$. We find that the EDC can be characterized by a set of almost similar axioms as in the case of Γ_L.

DEFINITION 3.3.1 *Call $U \subseteq \mathbf{R}^n$ weakly symmetric if there exists $x \in U$ such that any vector obtained by permuting the components of x is also in U.*

We introduce another axiom:

6. **Irrelevance of ψ-asymmetric Coalitions** (IRAC)

Suppose for no non-singleton and non-empty $S \subset N$ it holds that $\psi(S, v_S)$ is weakly symmetric. In that case, if there exists $x \in X(N, v)$ such that $x_i \geq v(\{i\})$ for all $i \in N$, then $\psi(N, v) \neq \emptyset$.

The explanation of this axiom is that if a coalition is to affect the solution set for the grand coalition, then the solution set for itself should have *some* symmetry.

Notice that the axioms WUHC, AM, SC and WISPC have been stated for games in Γ_L. If we replace Γ_L in the statements of these axioms by $\bar{\Gamma}$ or drop Γ_L (as appropriate), then we get a set of exactly similar axioms. Let us call them WUHC', AM', SC' and WISPC'. We state them below.

Take $(N, v) \in \bar{\Gamma}$.

Weak Upper Hemicontinuity (WUHC'):
Let $\{(N, v^k)\}$ be a sequence of games such that $\forall k$, $v^k(S) = v(S)$ for $S \subset N$ and $v^k(N) \longrightarrow v(N)$. Let $\{x^k\}$ be a sequence such that $x^k \in \psi(N, v^k)$ for all k and $x^k \longrightarrow x$. Then $x \in \psi(N, v)$.
Antimonotonicity (AM'):
Let (N, v') be such that $v'(S) \leq v(S)$ for all $S \subset N$ and $v'(N)=v(N)$. Then $\psi(N, v) \subseteq \psi(N, v')$.

Secession Consistency (SC'):
If $x \in \psi(N, v)$ then for any coalition S, $x_S \in \psi(S, v_S^x)$.

Weak Internal Stability for Proximal Coalitions: (WISPC')
Let $x \in \psi(N, v)$. Consider any $S \subset N$ such that $|S| = |N| - 1$. Then for all $y \in \psi(S, v_S)$,

$$max_{j \in S}\ x_j \geq min_{j \in S}\ y_j.$$

PROPOSITION 3.3.1 *There is a unique solution on $\bar{\Gamma}$ that satisfies WUHC', AM', SC', WISPC' and IRAC and it is the EDC.*

The fact that the EDC satisfies all these properties on $\bar{\Gamma}$ is quite easy to see. We prove the rest of the proposition with the help of the following lemmata.

LEMMA 3.3.1 *If a solution $\psi(.)$ satisfies IRAC, SC' and WISPC' on $\bar{\Gamma}$ then $x \in \psi(N, v)$ implies that for all $i \in N$, $x_i \geq v(\{i\})$.*

Proof: Note that by IRAC, for any single-player game (N, v), $\psi(N, v) \neq \emptyset$. The rest of the proof is exactly analogous to that of Lemma 3.2.2. ■

LEMMA 3.3.2 *If a solution $\psi(.)$ satisfies IRAC, WUHC', AM', SC' and WISPC' on $\bar{\Gamma}$ then for any $(N,v) \in \bar{\Gamma}$, $L(N,v) \subseteq \psi(N,v)$.*

Proof: Take $(N,v) \in \bar{\Gamma}$ and let $x \in L(N,v)$. Fix $\epsilon > 0$ and construct the game (N, v^ϵ) as follows:

$$v^\epsilon(N) = v(N) + \epsilon,$$

and for $S \subset N$,

$$v^\epsilon(S) = v(S).$$

Construct the vector x^ϵ, given by $x_i^\epsilon = x_i + \epsilon/|N| \; \forall i \in N$.

Now, further construct $(N, v^{\epsilon,x})$ for which $v^{\epsilon,x}(S) = v^\epsilon(S)$ for every non-singleton coalition S and for $i \in N$, $v^{\epsilon,x}(\{i\}) = x_i^\epsilon$.

We claim that for any $S \subset N$ such that $|S| > 1$, $\psi(S, v_S^{\epsilon,x})$ cannot be weakly symmetric. Suppose otherwise. Fix $S \subset N$, $|S| > 1$, such that $\psi(S, v_S^{\epsilon,x})$ is weakly symmetric. Let $y \in \psi(S, v_S^{\epsilon,x})$ be such that any vector obtained by permuting the components of y is also in $\psi(S, v_S^{\epsilon,x})$. Since $x \in L(N,v)$ it must be true that there exist $i,j \in S$ such that $x_i \geq y_j$. Suppose not. Then, $max_{k\in S}\; x_k < min_{k\in S}\; y_k$. Then, by the construction of $(N, v^{\epsilon,x})$ and the definition of a solution (in particular the fact that a vector in the solution must be efficient), the following string of inequalities is true:

$$max_{k\in S}\; x_k < min_{k\in S}\; y_k \leq a(S, v_S^{\epsilon,x}) = a(S,v).$$

But this contradicts the fact that $x \in L(N,v)$. Thus, there exist $i,j \in S$ such that $x_i \geq y_j$. This implies that $x_i^\epsilon > y_j$. Since $v^{\epsilon,x}(\{i\}) = x_i^\epsilon$ by construction, Lemma 3.3.1 is violated. Hence, the claim is true.

By the claim and using IRAC we find that $\psi(N, v^{\epsilon,x}) \neq \emptyset$ and by the definition of a solution and Lemma 3.3.1, $\psi(N, v^{\epsilon,x}) = \{x^\epsilon\}$. Therefore, by AM', $x^\epsilon \in \psi(N, v^\epsilon)$. Now, take a decreasing sequence of positive numbers $\{\epsilon^k\}$ such that $\epsilon^1 = \epsilon$ and $\epsilon^k \longrightarrow 0$. For each k, construct a game (N, v^{ϵ^k}) such that:

$$v^{\epsilon^k}(N) = v(N) + \epsilon^k,$$

and for $S \subset N$,

$$v^{\epsilon^k}(S) = v(S).$$

Let x^{ϵ^k} be the vector given by $x_i^{\epsilon^k} = x_i + \epsilon^k/|N|\ \forall i \in N$. By our argument above, for each k, x^{ϵ^k} is in $\psi(N, v^{\epsilon^k})$. Then for the sequence $\{(N, v^{\epsilon^k})\}$, $v^{\epsilon^k}(N) \longrightarrow v(N)$ and $x^{\epsilon^k} \longrightarrow x$. Then by WUHC', $x \in \psi(N, v)$. ∎

LEMMA 3.3.3 *If a solution $\psi(.)$ satisfies IRAC, WUHC', AM', SC' and WISPC' on $\bar{\Gamma}$ then for any $(N, v) \in \bar{\Gamma}$, $\psi(N, v) \subseteq L(N, v)$.*

Proof: The proof is identical to that of Lemma 3.2.4. ∎

Once again, we can show that the axioms are independent of each other.

IRAC: Let $\psi(N, v)$ be the core of (N, v). Then $\psi(.)$ satisfies WUHC', AM', SC' and WISPC' on $\bar{\Gamma}$ but not IRAC as the following example shows.

Let $N = \{1, 2, 3\}, v(\{1, 2, 3\}) = 5, v(\{1, 2\}) = 5, v(\{2, 3\}) = 4, v(\{1, 3\}) = 1$, $v(\{1\}) = v(\{3\}) = 1$ and $v(\{2\}) = 3$. In this game the core of none of the subgames on the doubleton coalitions is weakly symmetric. However, the core of this game is empty even though the vector (1,3,1) is individually rational.

The examples that we had provided for showing the independence of the axioms in case of Γ_L are sufficient for showing the independence of the respective axioms in this case as well.

Chapter 4

A Model of Collective Action

In this chapter we analyse a society facing the possibility of revolt by a section of its population. We study a simple model of a society comprising of two entities: a government and the masses. We model the actions of the agents in the society as a signalling game. In particular, we assume that the government can be either weak or strong, while the masses have incomplete information about the type of the government. The government is threatened with a possibility of revolt of the masses against it. Our principal objective in this chapter is to identify situations where faced with this possibility of revolt, a weak government would choose to repress whereas a strong government would let the prevailing set-up to continue or make some minor concessions. As we have mentioned in Chapter 1, this seems to fit a large number of historical experiences.

4.1 The Society

There are two agents in the society, a government ruled by the elite of the society (G), and the poor masses (M). There is a relatively liberal political system prevailing in the society. However, the elite is faced with a *potential threat of revolution*: below we shall make precise what we mean by this potential threat of revolution.

The government, G, can be of 2 types: strong ($\mathcal{S}$) and weak ($\mathcal{W}$). Its type is determined by its capability of (a) suppressing a revolt in case it breaks out, and (b) the ease with which it can take preemptive measures that helps in thwarting the revolt. Once again, we shall make the notion of the strength of a government precise below in terms of pay-off. The visible features of the strength of a government are administrative coordination, the level of competence of the military, certainty of intervention by friendly nations in time of need etc.. These are observable by the government but are only imperfectly observed by the masses. Thus, the government's type is known to itself, but is unknown to M. The prior belief of M about the type of G is given by the probability distribution ρ^G over $T_G = \{\mathcal{S}, \mathcal{W}\}$. The support of ρ^G is T_G.

The game proceeds in 2 stages:

- In stage 1, G declares a constitution and takes some preemptive policing measures: it can be tough (t) or be lenient (l). So, the action space of G, $A_G = \{t, l\}$. Playing t or acting tough implies increasing the government's military strength, purchasing sophisticated equipment, deploying intelligence agents, crackdown on media and protest activities etc.. So, if G plays t, then the regime is repressive. The action l implies that the government makes some minor liberal concessions (like some cosmetic constitutional amendments which have little impact on the agents' pay-offs) or may be it lets the prevailing liberal set-up continue. There is a cost associated with playing t. This cost differs across the two types. For $\mathcal{S}$, we denote this cost by $C_{\mathcal{S}}$ and for $\mathcal{W}$ by $C_{\mathcal{W}}$. We normalize the cost of playing l to 0 for either type of G.

- In stage 2, M observes G's action and updates its belief about G. Then M either revolts (r) or stays at home (x). Thus, the action space of M, $A_M = \{r, x\}$.

The pay-offs are affected in the following way. If the masses revolt, the probability of success of the revolt, denoted by $\pi(.)$, is dependent on:
(*a*) the type of G,
(*b*) whether G has played t or l.
So, $\pi: (T_G \times A_G) \mapsto [0,1]$.
If no revolt takes place then the masses get 0 and the elite get $\bar{V}$: $\bar{V}$ can be thought of as the surplus available to the society. If there is a successful revolt, the masses get a pay-off $V_M > 0$ and the elite in the government get 0. Note that V_M is not necessarily the same as $\bar{V}$: we allow for a change in production structure in the post-revolutionary regime. If the revolt fails, then M gets 0 and suffers a punishment of the amount Γ_M and the elite continue to get $\bar{V}$. However, in the event of a revolt, there is a net expense for the elite in fighting the civil war given by $\bar{C}$. We assume that $\bar{V}$ is sufficiently higher than $\bar{C}$ so that facing a revolt, G will *choose* to fight.[1] If the masses are in a repressive regime, then the act of revolt is costly. We denote this cost by C_r : this is the cost associated with acts like organisation, propaganda, agitation, collection of arms etc. under a repressive regime. We assume that this cost is normalized to 0 in a liberal regime.

We denote this game as $\mathcal{G}$.

4.2 The Pay-off Structure

We make a number of assumptions restricting the players' pay-offs. We retain these assumptions throughout this chapter.

First, we make the following assumption.

A1. Given any $a \in A_G$, $\pi(\mathcal{S}, a) < \pi(\mathcal{W}, a)$ and for any $\theta \in T_G$, $\pi(\theta, t) < \pi(\theta, l)$.

So, given any action of the government, the probability of success of revolt is higher with a weaker government. Also, substantial preemptive measures

[1]The following pair of conditions is sufficient for this: $(1 - \pi(\mathcal{W}, l))\bar{V} > \bar{C}$ and $(1 - \pi(\mathcal{W}, t))\bar{V} > C_{\mathcal{W}} + \bar{C}$.

lowers the probability of success of revolt for both types of government.

Now, we make precise what we mean by a potentially revolutionary situation. We assume that if no finer information is revealed about the government (i.e., if the masses hold the prior belief about the government), then in a lenient regime it is uniquely optimal for the masses to revolt with probability 1. Note that we identified a lenient regime to be the one where the prevailing political set-up continues or where at best some insignificant political changes have been made which have negligible impact on pay-off. So, this assumption implies that if the government does not act, it is sure to face revolt. Formally this implies:

A2. $\rho^G(\mathcal{W})[\pi(\mathcal{W},l)V_M-(1-\pi(\mathcal{W},l))\Gamma_M]+\rho^G(\mathcal{S})[\pi(\mathcal{S},l)V_M-(1-\pi(\mathcal{S},l))\Gamma_M] > 0$.

Next we assume:

A3. $\bar{C} > C_{\mathcal{W}} > C_{\mathcal{S}} > 0$.

The intuition behind this assumption is that if a government is strong to begin with, taking preemptive measures is relatively less costly for it than for a weak government. And the cost of a civil war is quite high in comparison to the cost of taking the preemptive measures.

Again, if a government is strong to begin with, it does not make any appreciable difference whether it has taken preemptive measures before or not. Recall that we listed items like administrative coordination, competence of the military etc. as some of the visible features of the strength of a government. Thus, if the military is strong enough and the administration is quite efficient to begin with, it is of little *additional* help to increase the strength of the military further or to deploy more intelligence agents. Therefore, we assume:

A4. $0 < \pi(\mathcal{S},l) - \pi(\mathcal{S},t) < \epsilon$, where $\epsilon > 0$ is infinitesimally small.

We maintain the assumptions A1 to A4 throughout this chapter.

4.3 The Equilibria

We look at the Perfect Bayesian Equilibria (PBE) of $\mathcal{G}$. A *strategy* σ_G for G is a map that specifies a possibly mixed action in A_G for every $\theta \in T_G$ and a strategy σ_M for M is a map that specifies a possibly mixed action in A_M for every $m \in A_G$. Thus, $\sigma_G(.) : T_G \mapsto \Delta(A_G)$ and $\sigma_M(.) : A_G \mapsto \Delta(A_M)$, where Δ stands for the unit simplex. For $a \in A_G$, $\sigma_G(a|\theta)$ denotes the probability with which G plays a if it is of type θ according to the strategy σ_G. For $a \in A_M$, $\sigma_M(a|m)$ denotes the probability with which M plays a according to the strategy σ_M after it has observed $m \in A_G$. The strategy set of player $i \in \{G, M\}$ is denoted by S_i. Given a strategy tuple $\sigma = (\sigma_G(.), \sigma_M(.))$, the conditional expected pay-off of G if these strategies are played, (conditional on G being of type θ) is denoted by $U_G(\sigma|\theta)$. Similarly, given belief $\mu \in \Delta(T_G)$, the expected pay-off of M from a strategy σ_M after an action $m \in A_G$ has been played is given by $U_M(m, \sigma_M, \mu)$.

DEFINITION 4.3.1 *A PBE of $\mathcal{G}$ is a pair (σ, μ) where*
(i) $\sigma = (\sigma_G, \sigma_M) \in S_G \times S_M$,
(ii) $\mu = (\mu_G(.|a))_{a \in A_G}$, such that for each $a \in A_G$, $\mu_G(.|a) \in \Delta(T_G)$
satisfying the following:

- Sequential Rationality: *(i) For each $\theta \in T_G$, $U_G(\sigma|\theta) \geq U_G(\sigma'_G, \sigma_M|\theta)$ for every $\sigma'_G \in S_G$.*
  (ii) For each $m \in A_G$, $U_M(m, \sigma_M, \mu_G(.|m)) \geq U_M(m, \sigma'_M, \mu_G(.|m))$ for every $\sigma'_M \in S_M$.

- Bayesian Updating: *If $\sigma_G(a|\theta) > 0$ for some $a \in A_G$ and $\theta \in T_G$, then,*

$$\mu_G(\theta|a) = \frac{\sigma_G(a|\theta).\rho^G(\theta)}{\sum\limits_{\theta' \in T_G} \sigma_G(a|\theta').\rho^G(\theta')}.$$

We shall denote a PBE of $\mathcal{G}$ generically by a pair (σ, μ) as described above.

Now, we define a refinement of PBE which we shall use as a *reasonable* equilibrium notion. This is the notion of Perfect Sequential Equilibrium (PSE), introduced by Grossman and Perry (1986). Below, we adapt the definition of a PSE to our context. Our choice of PSE as a reasonable equilibrium notion is motivated by its well-known power of refinement.

For $m \in A_G$ and $q \in \Delta(T_G)$, the set of best replies by M to m with belief q, $BR(m,q) = \{\tau \in S_M | \ \tau \text{ maximises } U_M(m, \sigma_M, q)\}$. Let (σ, μ) be a PBE of $\mathcal{G}$. For an action $\alpha \in A_G$, denote by $\tilde{\alpha}$ the strategy $\sigma'_G \in S_G$ such that $\sigma'_G(\alpha|\theta) = 1$ for all $\theta \in T_G$. For a $\tau \in BR(m,q)$, let $K^s(m,q,\tau) \subseteq T_G$ be such that

$$\theta \in K^s(m,q,\tau) \Longrightarrow U_G(\sigma|\theta) < U_G(\tilde{m},\tau|\theta)$$

and let $K^w(m,q,\tau) \subseteq T_G$ be such that

$$\theta \in K^w(m,q,\tau) \Longrightarrow U_G(\sigma|\theta) = U_G(\tilde{m},\tau|\theta)$$

Denote $K^s(m,q,\tau) \cup K^w(m,q,\tau)$ by K. Let $h : T_G \mapsto [0,1]$ be such that

$$h(\theta) \begin{cases} = 1 & \text{if } \theta \in K^s(m,q,\tau) \\ \in [0,1] & \text{if } \theta \in K^w(m,q,\tau) \\ = 0 & \text{if } \theta \notin K \end{cases}$$

and $\sum_{\theta \in T_G} h(\theta) > 0$. Define $c : T_G \mapsto [0,1]$ as follows:

$$c(\theta) = \begin{cases} \dfrac{h(\theta).\rho^G(\theta)}{\sum\limits_{\theta' \in K} h(\theta').\rho^G(\theta')} & \text{if } \theta \in K \\ 0 & \text{if } \theta \notin K \end{cases}$$

The belief $p \in \Delta(T_G)$ is *consistent* for $m \in A_G$ if there exist a $K \subseteq T_G$, $K \neq \emptyset$, a $\tau \in BR(m,p)$, and an $h(.)$ as defined above such that $c(\theta) = p(\theta)$ for all $\theta \in T_G$. For a PBE (σ, μ), μ is said to be *credible* with respect to σ if for any $m \in A_G$ such that $\sigma_G(m|\theta) = 0$ for every $\theta \in T_G$, $\mu_G(.|m)$ is equal to *some* belief $p \in \Delta(T_G)$ consistent for m whenever there exist some belief consistent for m. The PBE (σ, μ) is a PSE if μ is *credible* with respect to σ.

4.4 The Results

We are mainly interested in finding out the conditions under which we obtain a separating PBE in which $\mathcal{S}$ plays l and $\mathcal{W}$ plays t. Note that such a separating PBE is trivially a PSE.

We begin by identifying a condition for which such an equilibrium never exists.

PROPOSITION 4.4.1 *Suppose* $[\pi(\mathcal{S},l)V_M - (1-\pi(\mathcal{S},l))\Gamma_M] < 0$. *Then there is no PBE* (σ,μ) *for which* $\sigma_G(l|\mathcal{S}) = 1$, $\sigma_G(t|\mathcal{W}) = 1$.

Proof: For any such PBE, $\sigma_M(r|l) = 0$. So, whatever be the value of $\sigma_M(r|t)$, $\mathcal{W}$ will deviate to play l surely and gain at least $C_{\mathcal{W}} > 0$. ∎

Next we give a set of conditions which is sufficient for obtaining a separating PBE in which $\mathcal{S}$ plays l and $\mathcal{W}$ plays t.

C1. $\pi(\mathcal{S},l)V_M - (1-\pi(\mathcal{S},l))\Gamma_M \geq 0$,
C2. $\pi(\mathcal{W},t)V_M - (1-\pi(\mathcal{W},t))\Gamma_M - C_r \geq 0$,
C3. $\pi(\mathcal{W},l) - \pi(\mathcal{W},t) > \frac{C_{\mathcal{W}}}{V}$.[2]

PROPOSITION 4.4.2 *Suppose C1, C2 and C3 hold. Then there exists a PBE* (σ,μ) *given by:*
$\sigma_G(l|\mathcal{S}) = 1$, $\sigma_G(t|\mathcal{W}) = 1, \sigma_M(r|l) = \sigma_M(r|t) = 1$, *and* $\mu_G(\mathcal{S}|l) = 1$, $\mu_G(\mathcal{W}|t) = 1$.

Proof: To verify that (σ,μ) is a PBE we have to check that it satisfies the condition of sequential rationality in Definition 4.3.1.

[2]Condition C1 implies that in a lenient regime revolt is always at least a weakly optimal action. Condition C2 implies that if the government is revealed to be weak, then revolt is always at least a weakly optimal action. We point out the implication of C3 in Lemma 4.4.1.

First, take G of type $\mathcal{S}$.
$U_G(\sigma|\mathcal{S}) = (1 - \pi(\mathcal{S}, l))\bar{V} - \bar{C}$.
Consider $\sigma'_G \in S_G$ such that $0 < \sigma'_G(t|\mathcal{S}) = p_{\mathcal{S}} \leq 1$.
Then, $U_G(\sigma'_G, \sigma_M|\mathcal{S})$
$= p_{\mathcal{S}}[(1 - \pi(\mathcal{S}, t))\bar{V} - C_{\mathcal{S}}] + (1 - p_{\mathcal{S}})[(1 - \pi(\mathcal{S}, l))\bar{V}] - \bar{C}$
So, $U_G(\sigma|\mathcal{S}) - U_G(\sigma'_G, \sigma_M|\mathcal{S})$
$= p_{\mathcal{S}}[C_{\mathcal{S}} - (\pi(\mathcal{S}, l) - \pi(\mathcal{S}, t))\bar{V}]$
> 0 (by A3 and A4).
Similarly, for G of type $\mathcal{W}$, consider $\sigma'_G \in S_G$ such that $0 < \sigma'_G(l|\mathcal{W}) = p_{\mathcal{W}} \leq 1$.
Then, $U_G(\sigma'_G, \sigma_M|\mathcal{W})$
$= p_{\mathcal{W}}[(1 - \pi(\mathcal{W}, l))\bar{V}] + (1 - p_{\mathcal{W}})[(1 - \pi(\mathcal{W}, t))\bar{V} - C_{\mathcal{W}}] - \bar{C}$,
whereas, $U_G(\sigma|\mathcal{S}) = (1 - \pi(\mathcal{W}, t))\bar{V} - C_{\mathcal{W}} - \bar{C}$.
So, $U_G(\sigma|\mathcal{S}) - U_G(\sigma'_G, \sigma_M|\mathcal{S})$
$= p_{\mathcal{W}}[(\pi(\mathcal{W}, l) - \pi(\mathcal{W}, t))\bar{V} - C_{\mathcal{W}}]$
> 0 (by C3).

By C1 and C2, it is clear that for each $m \in A_G$, $U_M(m, \sigma_M, \mu_G(.|m)) \geq U_M(m, \sigma'_M, \mu_G(.|m))$ for every $\sigma'_M \in S_M$. ∎

Now we show that if C$\tilde{1}$ and C3 hold, then for a condition stricter than C2, for any PBE (σ, μ) it is true that $\sigma_G(l|\mathcal{S}) = 1$ and $\sigma_G(t|\mathcal{W}) = 1$. First, we prove the following lemma that would be useful for getting this result.

LEMMA 4.4.1 *Suppose C3 holds. Then for any PBE (σ, μ) such that $\sigma_M(r|l) = 1$, it must be that $\sigma_G(t|\mathcal{W}) = 1$.*

Proof: Suppose, C3 holds, but for a PBE (σ, μ), $\sigma_M(r|l) = 1$ and $\sigma_G(.|\mathcal{W}) \neq 1$. Let $\sigma_G(l|\mathcal{W}) = p_{\mathcal{W}} > 0$. We denote $\sigma_M(r|t)$ by $p_r \geq 0$.
Then,
$U_G(\sigma_G, \sigma_M|\mathcal{W}) = p_{\mathcal{W}}[(1 - \pi(\mathcal{W}, l))\bar{V} - \bar{C}] + (1 - p_{\mathcal{W}})[p_r((1 - \pi(\mathcal{W}, t))\bar{V} - \bar{C}) + (1 - p_r)\bar{V} - C_{\mathcal{W}}]$
Consider the strategy $\sigma'_G \in S_G$ such that $\sigma'_G(t|\mathcal{W}) = 1$, $\sigma'_G(.|\mathcal{S}) = \sigma_G(.|\mathcal{S})$.

Then,
$U_G(\sigma'_G, \sigma_M|\mathcal{W}) = p_r((1-\pi(\mathcal{W},t))\bar{V} - \bar{C}) + (1-p_r)\bar{V} - C_{\mathcal{W}}$.
So,
$U_G(\sigma'_G, \sigma_M|\mathcal{W}) - U_G(\sigma_G, \sigma_M|\mathcal{W})$
$= p_{\mathcal{W}}[p_r((1-\pi(\mathcal{W},t))\bar{V} - \bar{C}) + (1-p_r)\bar{V} - C_{\mathcal{W}} - (1-\pi(\mathcal{W},l))\bar{V} - \bar{C}]$.
By rearranging terms, this expression is simplified to:
$p_{\mathcal{W}}[p_r[(\pi(\mathcal{W},l) - \pi(\mathcal{W},t))\bar{V} - C_{\mathcal{W}}] + (1-p_r)[(\pi(\mathcal{W},l)\bar{V} + \bar{C} - C_{\mathcal{W}}]$
> 0 (By C3 and A3). Thus, (σ, μ) cannot be a PBE. ■

Now, we identify situations where in any PBE (σ, μ) it is true that $\sigma_G(l|\mathcal{S}) = 1$ and $\sigma_G(t|\mathcal{W}) = 1$.

PROPOSITION 4.4.3 *Suppose the following holds:*
(i) $\rho^G(\mathcal{W})[\pi(\mathcal{W},t)V_M - (1-\pi(\mathcal{W},t))\Gamma_M] + \rho^G(\mathcal{S})[\pi(\mathcal{S},t)V_M - (1-\pi(\mathcal{S},t))\Gamma_M] - C_r > 0$.[3]
Then under C1 and C3, for any PBE (σ, μ) *it is true that* $\sigma_G(l|\mathcal{S}) = 1$ *and* $\sigma_G(t|\mathcal{W}) = 1$.

Proof: We consider the following two cases.
Case 1: $\pi(\mathcal{S},l)V_M - (1-\pi(\mathcal{S},l))\Gamma_M > 0$.
In this case, we show that the ***unique*** PBE $(\bar{\sigma}, \bar{\mu})$ is given by
$\bar{\sigma}_G(l|\mathcal{S}) = 1,\ \bar{\sigma}_G(t|\mathcal{W}) = 1,\ \bar{\sigma}_M(r|l) = \bar{\sigma}_M(r|t) = 1$,
$\bar{\mu}_G(\mathcal{S}|l) = 1,\ \bar{\mu}_G(\mathcal{W}|t) = 1$.

Since condition (i) in this proposition implies C2, by Proposition 4.4.2, the strategies and the belief specified by $(\bar{\sigma}, \bar{\mu})$ indeed form a PBE.

Now, take any PBE (σ, μ). By the condition for Case 1, $\sigma_M(r|l) = 1$. Then, by Lemma 4.4.1, $\sigma_G(t|\mathcal{W}) = 1$. Then, by the definition of a PBE, $\mu_G(\mathcal{W}|t) \geq \rho^G(\mathcal{W})$. Then condition (i) in the proposition implies $\sigma_M(r|t) = 1$. But then by A4, $\sigma_G(l|\mathcal{S}) = 1$.

[3] This implies that if the masses hold the prior belief about G, then in any regime it is uniquely optimal for them to play r. Since $\pi(\mathcal{S},t) < \pi(\mathcal{W},t)$, this condition is stronger than C2.

Case 2: $\pi(\mathcal{S}, l)V_M - (1 - \pi(\mathcal{S}, l))\Gamma_M = 0$.
In this case, we show that for any PBE (σ, μ) it is true that $\sigma_G(l|\mathcal{S}) = 1$ and $\sigma_G(t|\mathcal{W}) = 1$.

Suppose, there is a PBE (σ, μ) such that $\sigma_G(l|\mathcal{W}) > 0$. Then, by A1 and the condition for Case 2, $\sigma_M(r|l) = 1$. But then, by Lemma 4.4.1, $\sigma_G(t|\mathcal{W}) = 1$ and we get a contradiction. Thus, $\sigma_G(t|\mathcal{W}) = 1$. Then, by the definition of a PBE, $\mu_G(\mathcal{W}|t) \geq \rho^G(\mathcal{W})$. Then condition (i) in the proposition implies $\sigma_M(r|t) = 1$. But then by A4, $\sigma_G(l|\mathcal{S}) = 1$. [4] ■

Next we show that if condition (i) in Proposiiton 4.4.3 does not hold, then even under C1 to C3, there is at least one reasonable PBE, (i.e. a PSE according to our criterion) for which it is not true that $\sigma_G(l|\mathcal{S}) = 1$ and $\sigma_G(t|\mathcal{W}) = 1$.

PROPOSITION 4.4.4 *Suppose C1 to C3 hold but*
(i') $\rho^G(\mathcal{W})[\pi(\mathcal{W}, t)V_M - (1-\pi(\mathcal{W}, t))\Gamma_M] + \rho^G(\mathcal{S})[\pi(\mathcal{S}, t)V_M - (1-\pi(\mathcal{S}, t))\Gamma_M] - C_r \leq 0$.
Then $(\bar{\sigma}, \bar{\mu})$ *as given below is a PSE.*
$\bar{\sigma}_G(t|\mathcal{S}) = 1,\ \bar{\sigma}_G(t|\mathcal{W}) = 1,\ \bar{\sigma}_M(r|l) = 1,\ \bar{\sigma}_M(x|t) = 1;$
$\bar{\mu}_G(.|t) = \rho^G(.),\ \bar{\mu}_G(\mathcal{S}|l) = 1.$

Proof: First, we show that $(\bar{\sigma}, \bar{\mu})$ is a PBE under C1 to C3 and (i'). We have to check that it satisfies the condition of sequential rationality in Definition 4.3.1.

For a type $\theta \in \{\mathcal{S}, \mathcal{W}\}$ consider a strategy $\sigma'_G \in S_G$ such that $0 < \sigma'_G(l|\theta) = p_\theta \leq 1$.
Then, $U_G(\sigma'_G, \sigma_M|\theta)$
$= (1 - p_\theta)(\bar{V} - C_\theta) + p_\theta[(1 - \pi(\theta, l))\bar{V} - \bar{C}]$ whereas,
$U_G(\bar{\sigma}|\theta) = \bar{V} - C_\theta$.
So, $U_G(\bar{\sigma}|\theta) - U_G(\sigma'_G, \sigma_M|\theta)$

[4]However, in this case, we shall not get any unique equilibrium. There are a continuum of equilibria differing in the probability with which M would play r after observing l.

$= p_\theta[\pi(\theta, l)\bar{V} - C_\theta + \bar{C}]$
> 0 (by A3).

By C1 and condition (i') in this proposition, it is clear that for each $m \in A_G$, $U_M(m, \bar{\sigma}_M, \bar{\mu}_G(.|m)) \geq U_M(m, \sigma'_M, \bar{\mu}_G(.|m))$ for every $\sigma'_M \in S_M$.

Now, we show that $(\bar{\sigma}, \bar{\mu})$ is a PSE. We consider two cases.

Case 1: Suppose there exists $p_r \in (0, 1)$ such that the following two inequalities are satisfied:

$$\bar{V} - C_{\mathcal{S}} \leq p_r[(1 - \pi(\mathcal{S}, l))\bar{V} - \bar{C}] + (1 - p_r)\bar{V}.$$

and

$$\bar{V} - C_{\mathcal{W}} \geq p_r[(1 - \pi(\mathcal{W}, l))\bar{V} - \bar{C}] + (1 - p_r)\bar{V}.$$

Let $\tau \in \mathcal{S}_M$ be such that $\tau(r|l) = p_r$. Then $\bar{\mu}_G(.|l)$ is consistent for l. This can be checked by setting $\tau \in BR(l, \bar{\mu}_G(.|l))$, and $h(\mathcal{S}) = 1$, $h(\mathcal{W}) = 0$ where the maps $BR(.,.)$ and $h(.)$ have been defined in the definition of a PSE.

Case 2: Suppose, for every $p_r \in (0, 1)$, whenever $\bar{V} - C_{\mathcal{S}} \leq p_r[(1 - \pi(\mathcal{S}, l))\bar{V} - \bar{C}] + (1 - p_r)\bar{V}$, it is true that $\bar{V} - C_{\mathcal{W}} < p_r[(1 - \pi(\mathcal{W}, l))\bar{V} - \bar{C}] + (1 - p_r)\bar{V}$. Then we claim that there does not exist any belief consistent for l. Suppose $q \in \Delta(T_G)$ is consistent for l. Let $\tau \in BR(l, q)$ be the strategy of M for obtaining this consistent belief q. We show that $\tau(r|l) \neq 0$. Suppose otherwise, i.e., let $\tau(r|l) = 0$. Then, $K^s(l, q, \tau) = \{\mathcal{S}, \mathcal{W}\}$ and therefore, $h(\mathcal{S}) = 1$, $h(\mathcal{W}) = 1$. But then $q(.)$ must be $\rho^G(.)$. By A2, this leads to a contradiction. So, $\tau(r|l) \neq 0$. Moreover, $\tau(r|l) \neq 1$, as then the set K defined in the definition of a consistent belief is empty. Then $0 < \tau(r|l) < 1$. But then, by the condition by which Case 2 has been identified, whenever $\mathcal{S} \in K$, $\mathcal{W}$ is in $K^s(l, q, \tau)$. But this implies that $q(\mathcal{W}) \geq \rho^G(\mathcal{W})$. Then, by C1, $\tau(r|l) = 1$ which leads to a contradiction. Thus, the claim is proved. Therefore, $\bar{\mu}$ is credible with respect to $\bar{\sigma}$. ■

We discuss the significance of the above results in the form of the following remarks.

REMARK 4.4.1 We have noted in Proposition 4.4.1 that condition C1 is necessary for $\mathcal{G}$ to have a PBE in which $\mathcal{S}$ plays l and $\mathcal{W}$ plays t. However, condition C2 is not necessary in this respect. But it is easy to check that if C2 does not hold then there are situations when $\mathcal{G}$ has no PBE in which $\mathcal{S}$ plays l and $\mathcal{W}$ plays t surely. For example, this is true if $\frac{C_{\mathcal{W}}}{\pi(\mathcal{W},l)V+C} > \frac{C_{\mathcal{S}}}{\pi(\mathcal{S},l)V+C}$.

REMARK 4.4.2 Our results show that when the masses are quite prone to revolt (it is at least weakly optimal for them to revolt whenever they face a lenient regime and whenever they get to know surely that the government is weak) then the behavioal pattern that a weak government would repress and a strong government would be lenient is a plausible outcome. The intuition behind this result is that by playing l, a stronger government signals to the masses that it is capable enough to thwart a revolt in case it breaks out and thus saves the expense of taking preventive measures. The masses respond with revolting with a probability low enough. However, a weak government cannot afford to do the same. When the situation is more conducive to revolt (as represented in condition (i) in Proposition 4.4.3) this pattern of behaviour becomes the unique one.

REMARK 4.4.3 There are a number of similarities of our model with that of Acemoglu and Robinson (1999a). As in their model, in our model also the probability of success of a revolt against a relatively stronger government is lower. Repressive measures help in lowering the probability of success of a revolt in our model as well. Again, in our model also, the stronger government can take repressive measure relatively easily. However, the two models differ to a significant extent.

Acemoglu and Robinson assume that the cost of repression is prohibitively high (infinity) for a weak government so that such a type would never be able to repress. So, whenever the masses face repression, they believe that the government must be relatively stronger. Moreover, in their model revolt always fails against a relatively stronger government, so that if the masses know for sure that the government is strong, they have no incentive to revolt. Thirdly, unlike in our model, the acts of concession are more costly in the model of Acemoglu and Robinson. Therefore, a strong government has no incentive to make concessions as it has the costless option of repression. These are some crucial differences between the two models which have generated different sorts of results. So, the present model is complementary to their model as the two models look at two different types of society.

4.5 Conclusion

We have examined one feature of a society facing revolt in a very special and simple model. Our model is evidently skeletal. One immediate direction of further exploration while retaining our simple framework might be to allow the government to have continuous action spaces and examine whether qualitatively similar results hold. Moreover, in our model the players' pay-offs are specified exogenously. Another worthwhile exercise would be to endogenize the pay-offs in a general equilibrium framework similar to that in Grossman (1991).

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