DUAL PRICING : THEORY AND APPLICATIONS

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TO MY MOTHER

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CHAPTER I

INTRODUCTION

Even after thirty five years of planned economic development a large fraction of people in India cannot afford to have the basic necessities like food, clothing and shelter. To quote from the Economic Survey [1984-85], "Buoyant agricultural performance thus far has depended mainly upon production gains in wheat and rice. Performance with respect to other critical crops, such as pulses and oilseeds, and dryland farming generally, remains weak. The rate of growth of industry, although somewhat higher than in the ten years preceding the Sixth Plan, remains low... population continues to grow at about 2 percent per year. At this rate, even if we achieve an average GNP growth rate of about 5 percent per year in the Seventh Plan period, we will not be able to make sufficient progress in tackling the pressing problems of poverty and unemployment".

Experience during the last three and a half decades shows that economic growth alone may not lead to an equitable distribution of essential goods. To combine efficiency with equity, the Indian Government has been experimenting with dual pricing schemes for some commodities*. According to this scheme, limited amounts of commodities like wheat, sugar and rice are sold through government operated fair price shops at subsidized prices.

Multiple prices for goods and services are used in many countries. This is fairly common for public utilities like power, transport etc. There are a number of theoretical and empirical studies of such schemes like two-part tariff. They generally make use of the partial equilibrium framework.

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* A good is said to be dual priced if it is sold in the free market at the competitive price and a fixed quota per consumer is sold in the ration shops at a controlled price less than the free market price.

A very general model to study several 'Second best' problems has been formulated by Guesnerie [1979]. He has also characterized the Second Best Pareto Optimal solutions. The purpose of this study is to incorporate dual pricing schemes in a general equilibrium framework and examine their implications for the production and distribution of rice, wheat and edible oils in India.

The plan of the study is as follows:

In Chapter II, we show that for a consumer the utility maximization problem with dual pricing is equivalent to maximizing utility when there is a single price for each good and this price for the good in question is obtained as a weighted average of the free market price and the ration price under dual pricing along with an appropriate income subsidy to compensate for the withdrawal of the public distribution system.

We then examine the validity of many standard propositions in microeconomic theory in the context of dual pricing. These results include Roy's identity, Slutzky equation, adding up property of demand functions etc. We conclude this chapter by obtaining the dual problem for a firm maximising its profits, given that one of the inputs is dual priced. The analysis is similar to that of the consumer.

In Chapter III, we define the concept of a 'dual-price-equilibrium' in an exchange economy without production as a system of prices and allocations together with a ration-quota and a ration price for a single good such that all markets clear, every consumer maximizes utility, and at least one consumer buys more of this good than the prescribed quota with a higher market price than the ration price. Next, we prove that every non-trivial dual-price-equilibrium (which cannot be obtained as a Walras' equilibrium even with a redistribution of resources) is inefficient in the Pareto sense. Lastly, we show that there exists a tax-policy consistent with the existence of a dual-price-equilibrium.

Since a dual-price-equilibrium is inefficient, we study in Chapter IV, locally welfare improving reforms (on the lines similar to Ahmed and Stern [1983 (b)], Dreze [1983] and Guesnerie [1975, 1979, 1980]) in terms of the ration quota and price. This is done by comparing the welfare losses incurred by marginally changing either of the two parameters, namely, the quota and the ration price in order to save a unit of the subsidy provided to the consumers via the public distribution system. For this purpose we use a welfare function of the Bergson-Samuelson type and get possible directions of reform under various assumptions regarding the welfare weights and elasticities of demand.

In Section 3 of this chapter we calculate the welfare weights for a two-class economy consisting of two types of consumers - one type consuming more and the other less than the quota of the rationed good.

In order to see the empirical implications of these results we consider the case of dual pricing in two goods - rice and wheat - using the 1973-74 National Sample Survey (28th round) data on the Indian economy. Since the required data are available in grouped form, care is taken to incorporate this into the model. Considering several levels of inequality aversion, we calculate the welfare losses occurring due to marginal changes in eight instruments, namely, the ration quotas and ration prices of rice and wheat and taxes on four commodity groups. This suggests the possible directions of reform. In particular, we notice that *whatever be the level of inequality aversion*, it is always better to reduce both the ration price and the quota for both rice and wheat consistent with the same government expenditure.

In Chapter V, we see the implications of introducing dual-pricing in edible oils consumption in India. We concentrate on three major edible oils, viz., groundnut oil, mustard oil and vanaspati*. We first estimate the demand and supply functions for these oils and obtain and compare the equilibrium prices with observed prices for the years 1964-65 to 1979-80. Next, we consider certain 'states of nature' depending upon the supply position of groundnut oil since this is the most important among all the oils. After fixing some hypothetical values for the exogenous variables we see the repercussions on prices of a rise or fall in groundnut oil supply and compare various mechanisms to adjust the total oil supply to the initial level in order to stabilize the price of groundnut oil.

Lastly, we consider two schemes of dual pricing-in the first, the government procures fifty percent of the groundnut oil production at a fixed price; in the second, procurement is again fifty percent of the production but at half the prevailing market price. In both cases, only half of the population is covered by the public distribution scheme. It is seen that in both the schemes since the demand in the ration shops goes up, the free market price of groundnut oil increases leading to an overall increase in its supply. Also the changes in this oil's price and quantity are much larger than in the other oils. Since the total expenditure in the economy is fixed and since the percentage gains are related inversely to the levels of expenditure at the micro level, it turns out that there is a redistribution

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* Hydrogenerated oil.

of income from the relatively rich to the relatively poor with an increased per unit profit for the producers. In other words, the buyers in the open market for groundnut oil subsidize those in the ration shops.

Finally, in Chapter VI, we give general equilibrium justification for partial equilibrium analysis of welfare maximization. For this we first consider our model developed in Chapter IV and show that the results obtained by maximizing welfare subject to a given government revenue can also be obtained by maximizing it subject to scarcity constraints.

In the last section of the chapter we generalize the results of Deaton and Stern [1985] in a similar way. Their result is that under a number of different assumptions on household characteristics and preferences etc., optimal transfers are sufficient for the optimality of non-differentiated uniform commodity taxes.

CHAPTER II

CONSUMER AND PRODUCER BEHAVIOUR UNDER DUAL-PRICING

II.1 INTRODUCTION

In India many consumption goods are distributed partly through fair price shops for a more equitable distribution of these goods.* Similar schemes are utilized to distribute the scarce factors of production to those engaged in producing socially more desirable goods. Thus rationing is a regular feature.

Several studies have been undertaken to characterize the consumer behaviour under rationing. Some of them put restrictions on trade or allow prices to be flexible in a given region or have both these constraints. See, e.g., Dreze [1975], Benassy [1975] and Younes [1975]. In some of the recent works on rationing the consumers are forced to consume some given quotas of the rationed goods at some given prices. Their demands are then characterized in terms of those in the absence of any rationing. These works include those by Neary and Roberts [1980] and Guesnerie and Roberts [1984]. None of these researchers, however, work with the more flexible and commonly observed rationing schemes, namely, partial rationing or dual pricing (though a mention is made in Guesnerie and Roberts [1984]).

In this chapter we study the theory of consumer and producer behaviour when there is a dual pricing, i.e., when there are two prices for one or more goods with restrictions on the purchases of these goods at the respective lower prices. We examine the validity of many standard propositions in micro-economic theory, like, the homogeneity of the demand functions and the Slutzky equation and develop the analogues for dual pricing.

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* In 1981, [illegible] million people out of a total population of 685 million were covered by fair price shops. (See George [1985]).

The plan of this chapter is as follows. In the next section we describe the consumer's choice problem under dual pricing and derive the properties of the demand functions. In Section II.3 we study the producer's problem under dual pricing.

II.2. CONSUMER BEHAVIOUR UNDER DUAL-PRICING

II.2.1. The Choice Problem and its Dual

Assume that there are 'ℓ' commodities and that each consumer has a fixed amount of income, denoted by 'm'. Some (rationed) commodities are sold both in the ration shops and in the free market. For ease of exposition, we assume that there is only one commodity which is sold in both these places. Assuming that it is the first commodity, we can let its two prices be $\bar{p}$ per unit in the ration shop and q_1 per unit in the open market. For the rationing scheme to be meaningful we must have $\bar{p} < q_1$. It then becomes necessary to restrict the purchases from the ration shop to, say, D units per consumer. The goods $2,\ldots,\ell$ are sold only in the open market at prices $q_2,\ldots,q_\ell$ per unit respectively. Let $r = (\bar{p}, q_2,\ldots,q_\ell)$ and $q = (q_1, q_2, \ldots, q_\ell)$.

Given a consumption bundle $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_\ell) \in \mathbb{R}^\ell$, the preferences of the consumer can be represented by a utility function $u(\bar{x}_1,\ldots,\bar{x}_\ell)$ which is strictly quasi-concave, bounded, differentiable and increasing in each argument. The consumer's choice problem is to

$$\underset{\bar{x}_1,\ldots,\bar{x}_\ell}{\text{maximise}} \quad u(\bar{x}_1,\ldots,\bar{x}_\ell)$$

subject to

$$\bar{p}\bar{x}_1 + (q_1-\bar{p})\max(\bar{x}_1-D,0) + \sum_{2}^{\ell} q_i\bar{x}_i \leq m \qquad \text{(II.2.1)}$$

The budget constraint is linear in prices and income but is non-linear in quantities. The consumers can now be categorized into three types : (i) those whose consumption of the first good is less than the ration quota, (ii) those who consume exactly D units of this good and (iii) those who buy some positive amount of this good in the open market. These are illustrated in Figure II.1.a - II.1.c.

It is a trivial exercise to show, using standard arguments, that the maximization problem has a unique solution under our assumptions. However, it is slightly difficult to characterize the consumer's response to small changes in the parameters. This depends crucially on whether the optimal consumption bundle lies on the segment AB or BC or at point B of the budget constraint ABC in Figure II.1. In Figure II.1.a, small changes in the open market price of the rationed good have no effect on the demands. This is not true in Figure II.1.c where even small changes in either the control price $\bar{p}$ or the free market prices q_1 and q_2 change the budget set and hence the optimal choice. The situation is more complex in Figure II.1.b where small variations in $\bar{p}$ will affect the demands but similar changes in q_1 may or may not.

The dual-pricing problem can alternatively be viewed as follows. The good with two prices can be treated as two goods with quantity restrictions on the purchase of the good with a lower price. The theory of consumer choice with quantity restrictions can then be used to derive the demand functions. In fact, if we can determine the financial value of the restrictions and add this to the consumer's income as a compensation, then the problem can be studied using the standard tools of consumer theory. The financial value of the restrictions will vary with individuals. However, all the three cases can be studied within the domain of the dual problem which allows for a unified treatment.

The maximization problem in the quantity space has its dual in the price space; the quota restrictions lead to restrictions on prices and income. There are many ways to formulate the dual of a non-linear maximization problem. For our

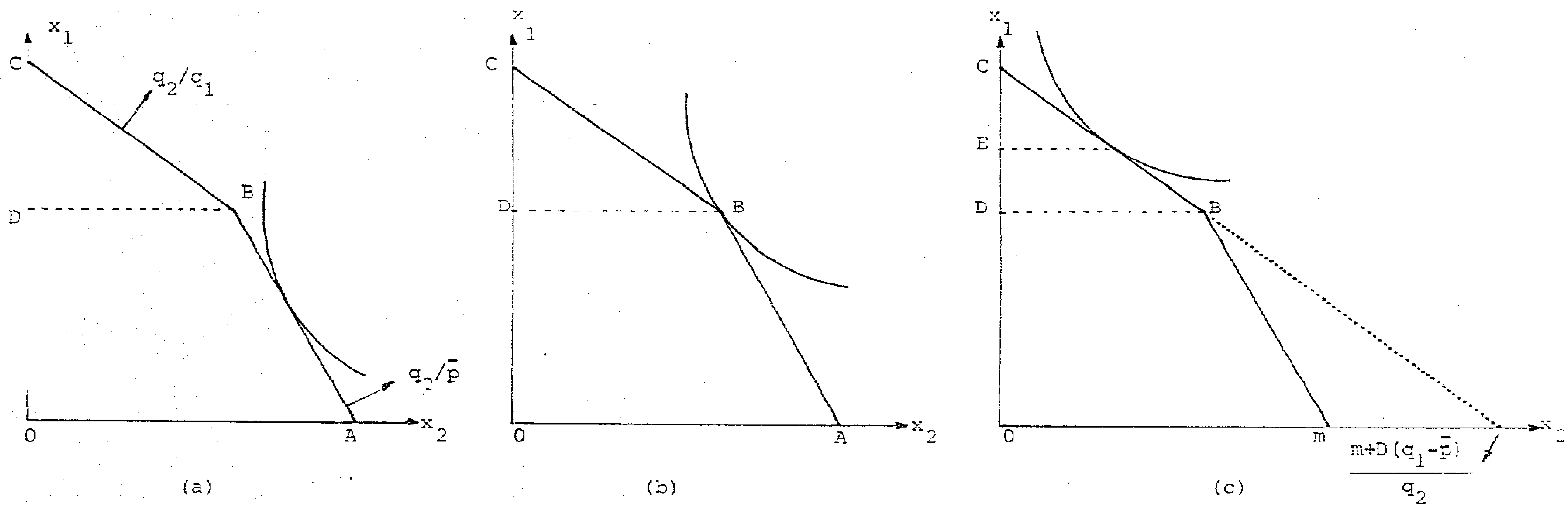

FIGURE II.1

purpose, we shall make use of Fenchel's duality theorem* (see, Luenberger, [1969]).

The consumer's problem under dual-pricing is to

maximize $u(\bar{x})$

subject to $r.\bar{x} + (q_1-\bar{p})\max(\bar{x}_1-D,0) \leq m.$ (II.2.2)

For applying the duality theorem, let

$B = \mathbb{R}^{\ell}_{+}$ and

$C = \{\bar{x} \in \mathbb{R}^{\ell} \mid r.\bar{x} + (q_1-\bar{p})\max(\bar{x}_1-D,0) \leq m\}$

Let the utility function $u(\bar{x})$ be defined over B and a function $g(\bar{x})$ over C with $g(\bar{x}) = 0 \ \forall \ \bar{x} \in C$. Let the objective function $u(\bar{x})-g(\bar{x})$ be defined over the budget set which is the intersection of the two convex sets B and C. Note that $B \cap C$ is exactly the set given by (II.2.2).

The problem is to $\max_{B \cap C} [u(\bar{x})-g(\bar{x})]$. The dual problem is to $\underset{B^* \cap C^*}{\text{minimize}} (g^*-u^*)$ where g^* and u^* are the dual functions and B^* and C^* are the dual regions. Let us first determine the dual regions.

By definition,

$B^* = \{p \in \mathbb{R}^{\ell} \mid \inf_{B} (p.\bar{x}. - u(\bar{x})) > -\infty\}$

Since $u(\bar{x})-p.\bar{x}$ is the consumer surplus, B^* consists of all prices for which the consumer surplus has a supremum in B. Since 'u' is bounded, it is easy to show that

$B^* = \mathbb{R}^{\ell}_{+}$.

The dual function of u is defined for every $p \in B^*$ by

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* See Appendix A for the statement of this theorem.

$$u^*(p) = \inf_{B} (p.\bar{x} - u(\bar{x})).$$

In other words, the dual function is the negative of the supremum of the consumer surplus at price 'p' over the set B.

Determination of the set C* is a little more involved. By definition,

$$C^* = \{p \in \mathbb{R}^\ell \mid \sup_{C} p.\bar{x} < \infty\}.$$

This is the set of prices (including possibly vectors with negative components) for which the set of expenditures on the bundles in C is bounded. It will be easy to determine C* if we can describe the set C in terms of half spaces.

Consider the case of two goods. The set C is shown in Figure II.2. It can be seen from the figure that the set C is nothing but the intersection of the two half spaces whose boundaries are given by the lines AF and BE. These lines are in fact the budget lines corresponding to the prices $(\bar{p},q_2)$ and (q_1,q_2) and incomes m and $m+(q_1-\bar{p})D$ respectively. In the general case, it is easy to show that

$$C = C_1 \bigcap C_2$$

where

$$C_1 = \{\bar{x} \in \mathbb{R}^\ell \mid r.\bar{x} \leq m\}$$

and

$$C_2 = \{\bar{x} \in \mathbb{R}^\ell \mid q.\bar{x} \leq m + (q_1-\bar{p})D\}$$

(See Appendix B for a proof).

To determine the elements of C*, we have to find those $p \in \mathbb{R}^\ell$ for which $\sup_{\bar{x} \in C_1 \bigcap C_2} p.\bar{x} < \infty$. Consider, for example, $p = \lambda(\bar{p},q_2,\ldots,q_\ell)$ where $\lambda \geq 0$. Then

$$\sup_{\bar{x} \in C_1} p.\bar{x} = \sup_{\bar{x} \in C_1} \lambda\, r.\bar{x} = \lambda \sup_{\bar{x} \in C_1} r.\bar{x} = \lambda m.$$

Since $\sup_{\bar{x} \in C_1 \bigcap C_2} p.\bar{x} \leq \sup_{\bar{x} \in C_1} p.\bar{x}$, we have

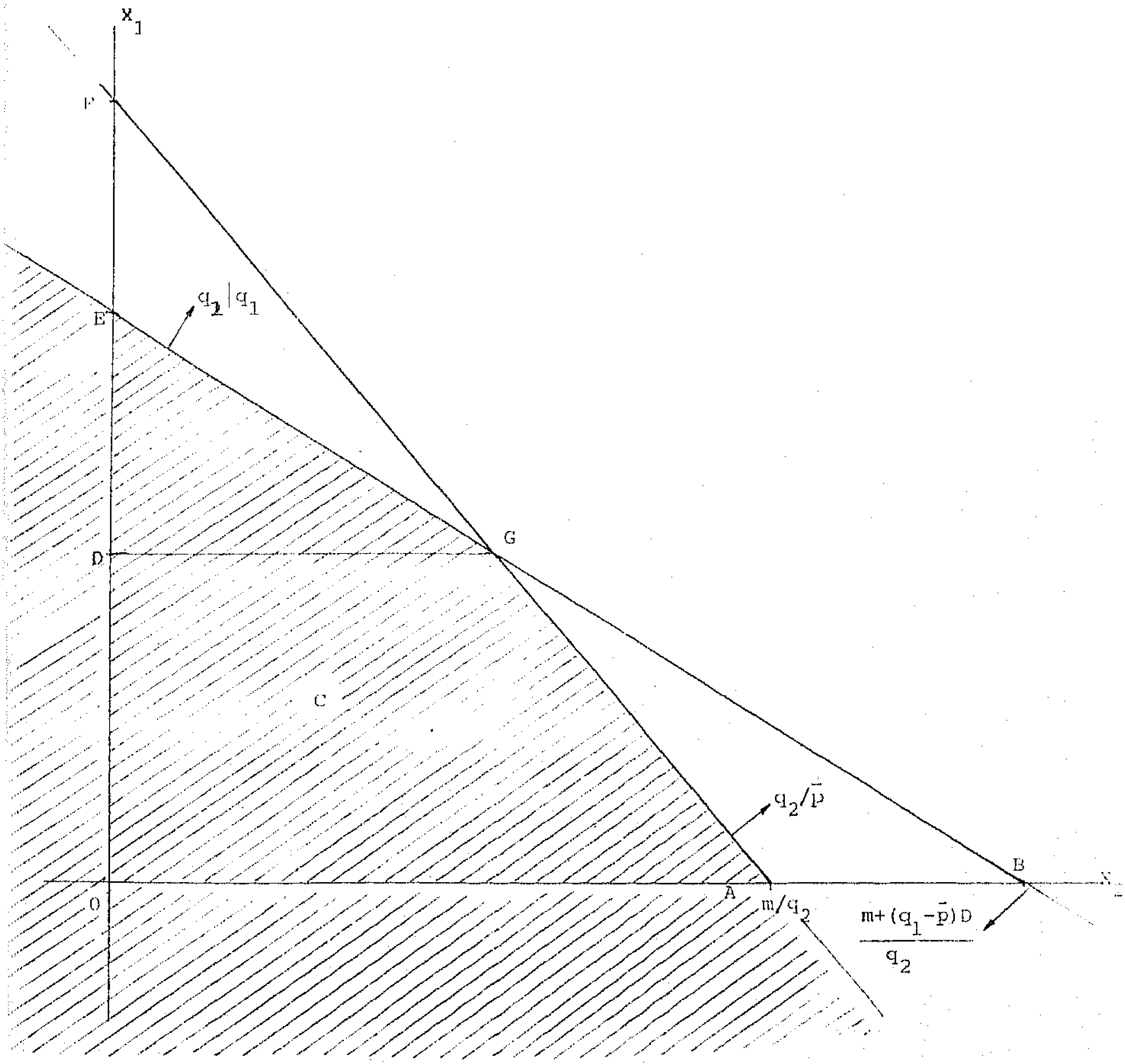

FIGURE II.2

$$\sup_{\bar{x} \in C_1 \cap C_2} \lambda r.\bar{x} \leq \lambda[m+(q_1-\bar{p})D].$$

Thus $\lambda r \in C^*$ for $\lambda \geq 0$.

Similarly $\lambda q \in C^*$ for $\lambda \geq 0$.

Next consider $p = \theta \lambda.q + (1-\theta)\lambda r$, where $0 < \theta < 1$.

Since $p.\bar{x} = \lambda[\theta q.\bar{x} + (1-\theta)r.\bar{x}]$,

$$\sup_{\bar{x} \in C} \lambda[\theta q.\bar{x} + (1-\theta)r.\bar{x}] \leq \sup_{\bar{x} \in C} \lambda\theta q.\bar{x} + \sup_{\bar{x} \in C} \lambda(1-\theta)r.\bar{x}$$

$$\leq \lambda\{\theta[m+(q_1-\bar{p})D] + (1-\theta)m\}.$$

Hence $p \in C^*$.

Next we will show that these are the only prices in C^*. Pick any $p \in C^*$. Then by definition,

$$\sup_{\bar{x} \in C} p.\bar{x} < \infty.$$

It is trivial to check that if the supremum is finite it is in fact achieved, i.e., there exists an $\bar{x}^* \in C$ such that $p.\bar{x} \leq p.\bar{x}^* \;\forall\; \bar{x} \in C$. Then the maximization problem : $\max_{\bar{x}} [p\bar{x} + \lambda\{m-r.\bar{x} - (q_1-\bar{p})\max(\bar{x}_1-D,0)\}]$, where λ is a non-negative Lagrange multiplier, has a solution, $\bar{x}^*$. Suppose $\bar{x}_1^* > D$. Then the necessary conditions for an interior maximum are :

$$p_i - \lambda q_i = 0 \quad \forall\; i = 2,\ldots,\ell.$$

The same conditions are necessary when $\bar{x}_1^* \leq D$. Thus, for a price vector 'p' to belong to C^*, it is necessary that $p_i = \lambda q_i$, $i = 2,\ldots,\ell$, $\lambda \geq 0$. Now we will characterize the first component of 'p'. Since $q_1 > \bar{p}$, there are three possibilities:

(1) $p_1 > \lambda q_1$, (2) $p_1 < \lambda\bar{p}$, (3) $\lambda\bar{p} \leq p_1 \leq \lambda q_1$.

We have already shown above that (3) is sufficient for p to belong to C^*. Now we will show that it is also necessary.

Suppose $p_1 > \lambda q_1$. Let $p_1 = \lambda q_1 + \mu$. Then

$$\sup_C p.\bar{x} = \sup_C \{\lambda q.\bar{x} + \mu\bar{x}_1\}.$$

Now choose a sequence $\bar{x}^n \in C$ such that $\bar{x}_1^n \to \infty$ and

$$q.\bar{x}^n = m + (q_1-\bar{p})D.$$

Then $\lambda q\bar{x}^n + \mu\bar{x}_1^n \to +\infty$. Similarly, it can be shown that $p_1 < \lambda\bar{p}$ ==> p does not belong to C^*. Thus, we have

$$C^* = \{p \in \mathbb{R}^\ell \mid p = \lambda\{\theta q+(1-\theta)r\},\ 0 \leq \theta \leq 1\}.$$

By definition, for $p \in C^*$,

$$\begin{aligned} g^*(p) &= \sup_C p\bar{x} = \sup_C \lambda\{\theta q+(1-\theta)r\}\bar{x} \\ &= \lambda\sup_C[\theta q.\bar{x} + (1-\theta)r.\bar{x}] \\ &= \lambda\sup_C[r\bar{x} + (q_1-\bar{p})\max\{(\bar{x}_1-D),0\} + (q_1-\bar{p})\{\theta\bar{x}_1-\max(\bar{x}_1-D,0)\}] \\ &\leq \lambda\ (m + \theta(q_1-\bar{p})D). \end{aligned}$$

The dual minimization problem is :

$$\min_{p \in B^*\cap C^*} g^*(p)-u^*(p)$$

$$= \min_{\lambda,\theta} [\lambda\{m+\theta(q_1-\bar{p})D\}-\min_B\{\lambda[\theta q+(1-\theta)r]\bar{x}-u(\bar{x})\}],\ \lambda \geq 0, 0 \leq \theta \leq 1$$

$$= \min_{\lambda,\theta}[[\lambda\{m+\theta(q_1-\bar{p})D\} + \max_B\{u(\bar{x})-\lambda[\theta q+(1-\theta)r]\bar{x}\}]$$

$$= \min_{\lambda,\theta}\max_B\{u(\bar{x})+\lambda[m+\theta(q_1-\bar{p})D-\{\theta q+(1-\theta)r\}.\bar{x}]\}$$

For fixed λ and θ , the problem is to maximize utility subject to an income of $m+\theta(q_1-\bar{p})D$ and the price $\theta q+(1-\theta)r$. Thus the financial value of the restriction on trade is $\theta(q_1-\bar{p})D$ at price $\theta q+(1-\theta)r$.

Suppose the maximum is attained by $\bar{x}^*$. Consider the case where $\bar{x}_1^* > D$. Using the Envelope Theorem, minimization of $u(\bar{x}^*)+\lambda[m+\theta(q_1-\bar{p})D-\{\theta q+(1-\theta)r\}\bar{x}^*]$

with respect to θ requires that $\theta = 1$. This means that, for a consumer, who is buying some positive amount of the first good in the free market, the required income compensation is $(q_1-\bar{p})D$ and he will use the price vector q. Similarly, when $\bar{x}_1^* < D$, $\theta = 0$ will minimize the function. This also should be expected. The value of ration quota for a consumer, who is not fully utilising his quota is zero. Also the free market price is irrelevant for his choice. Thus he will use the price vector 'r' and the income 'm'.

If $\bar{x}_1^* = D$, 'θ' drops out from the function and the minimizing value of θ may be anywhere from 0 to 1. When the maximum occurs at the kink as in Figure II.1.b, the supporting price will be between r and q, depending upon the marginal rates of substitution at the kink. Similarly, the income compensation will be between 0 and $(q_1-\bar{p})D$. These results are obvious in the case of two goods from Figure II.1. For our subsequent discussions, whenever there is dual pricing, we will replace this by a single price vector $\theta q+(1-\theta)r$ and the income by $m+\theta(q_1-\bar{p})D$. We will now examine the validity of some of the standard results in consumer theory.

II.2.2. Properties of Demand Functions:

Let $\bar{\phi}(\bar{p},q_1,\ldots,q_\ell,D,m)$ denote the demand correspondence with dual pricing. We will make repeated use of the fact that

$$\bar{\phi}(\bar{p}, q_1,q_2,\ldots,q_\ell,D,m) = \phi(q_\theta,q_2,\ldots, q_\ell, m_\theta) \qquad \text{(II.2.3)}$$

where ϕ is the usual demand correspondence, $q_\theta:=\theta q_1+(1-\theta)\bar{p}$ and $m_\theta = m+\theta(q_1-\bar{p})D$. Since the budget is unchanged when the money income and the prices including the ration price $\bar{p}$ are multiplied by a positive scalar, the demand functions are (positively) linearly homogeneous in prices and income.

Next, we will examine the relation among income elasticities. Differentiating the budget constraint*, we get

$$\frac{\partial}{\partial m}[q_\theta \bar{x}_1 + q_2\bar{x}_2 + \ldots + q_\ell \bar{x}_\ell]$$

$$= q_0 \frac{\partial \bar{x}_1}{\partial m} + \sum_2^\ell q_i \frac{\partial \bar{x}_i}{\partial m} + \bar{x}_1 (q_1 - \bar{p})\frac{\partial \theta}{\partial m}^{**} = 1 + D(q_1-\bar{p})\frac{\partial \theta}{\partial m}$$

$$\Longrightarrow \frac{q_\theta \bar{x}_1}{m} \frac{m}{\bar{x}_1} \frac{\partial \bar{x}_1}{\partial m} + \sum_2^\ell \frac{q_i \bar{x}_i}{m} \frac{m\partial \bar{x}_i}{\bar{x}_i \partial m} = 1 \quad (\because \frac{\partial \theta}{\partial m} = 0 \text{ whenever } \bar{x}_1 \neq D)$$

In practice, θ is not observable. If only q_1 and $\bar{p}$ are observed, one has to evaluate the budget share of $\bar{x}_1$ at the free market price q_1 or the ration price $\bar{p}$. In the former case, the share will be over estimated when $\theta < 1$. Hence the weighted average of the income elasticities will add to more than 1. Similarly, when the ration price is used, the weighted sum of these elasticities will be less than one if $\theta > 0$. Thus one can obtain the lower and upper bounds for the true sum of the income elasticities.

To study some of the properties of the compensated demand functions, let $\tilde{\Psi}(\bar{p}, q_1, q_2, \ldots, q_\ell, D, \bar{u})$ denote the demand vector when the price is q and the utility is $\bar{u}$. Denote the usual compensated demand function by $\Psi(q_1, q_2, \ldots, q_\ell, \bar{u})$.

$$\tilde{\Psi}(\bar{p}, q_1, \ldots, q_\ell, D, \bar{u}) = \Psi(q_\theta, q_2, \ldots, q_\ell, \bar{u}) \qquad \text{(II.2.4)}$$

We also know that $\frac{\partial \Psi_1}{\partial q_i} = \frac{\partial \Psi_i}{\partial q_\theta}$.

Also $\frac{\partial \Psi_i}{\partial q_1} = \frac{\partial \Psi_i}{\partial q_\theta} \cdot \frac{\partial q_\theta}{\partial q_1} = \frac{\partial \Psi_i}{\partial q_\theta} \, q_\theta'$, say, where $q_\theta' = \frac{\partial q_\theta}{\partial q_1}$

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* Throughout this chapter, we will assume that the demand functions are differentiable. Necessary and Sufficient Conditions for this are well known. See, e.g., Debreu [1972].

** Assume that θ is a differentiable function of the parameters $\bar{p}$, $q_1, \ldots, q_\ell, m$.

and $$\frac{\partial \bar{\Psi}_j}{\partial q_i} = \frac{\partial \Psi_j}{\partial q_i}, \quad j = 1,2,\ldots,\ell, \quad i \neq 1.$$

Hence we have

$$\frac{\partial \bar{\Psi}_1}{\partial q_i} = \frac{1}{q_0'} \frac{\partial \bar{\Psi}_i}{\partial q_1} \text{ for } \quad 0 \neq 0, \quad i = 2,\ldots,\ell.$$

By similar argument, we get the following Slutzky equation:

$$\frac{1}{q_\theta'} \frac{\partial \bar{\phi}_i}{\partial q_1} = \frac{1}{q_\theta'} \frac{\partial \bar{\Psi}_i}{\partial q_1} - (\bar{\phi}_1 - D) \frac{\partial \bar{\phi}_i}{\partial m} \qquad \theta > 0.$$

And as usual, we also have

$$\frac{\partial \bar{\phi}_1}{\partial q_i} = \frac{\partial \bar{\Psi}_1}{\partial q_i} - \bar{\phi}_i \frac{\partial \bar{\phi}_1}{\partial m} \quad \forall\, i \neq 1.$$

In standard consumer theory, the sum of the price and income elasticities of any commodity is zero. We will now examine this result. Note that

$$\frac{\partial}{\partial q_i} u(\bar{\Psi}(q,\bar{u})) = 0 \quad \forall\, i = 2,\ldots,\ell$$

$$\Longrightarrow \sum_{j=1}^{\ell} \frac{\partial u}{\partial \bar{\Psi}_j} \cdot \frac{\partial \bar{\Psi}_j}{\partial q_i} = 0$$

$$\Longrightarrow \lambda \left\{ q_0 \frac{\partial \bar{\Psi}_1}{\partial q_i} + \sum_{j=2}^{\ell} q_j \frac{\partial \bar{\Psi}_j}{\partial q_i} \right\} = 0$$

$$\Longrightarrow \left\{ q_\theta \frac{1}{q_\theta'} \frac{\partial \bar{\Psi}_i}{\partial q_1} + \sum_{j=2}^{\ell} q_j \frac{\partial \bar{\Psi}_i}{\partial q_j} \right\} = 0, \quad \theta \neq 0$$

$$\Longrightarrow q_\theta \left(\frac{1}{q_\theta'} \frac{\partial \bar{\phi}_i}{\partial q_1} + (\bar{\phi}_1 - D) \frac{\partial \bar{\phi}_i}{\partial m} \right) + \sum_{j=2}^{\ell} q_j \left(\frac{\partial \bar{\phi}_i}{\partial q_j} + \bar{\phi}_j \frac{\partial \bar{\phi}_i}{\partial m} \right) = 0$$

$$\Longrightarrow q_\theta \cdot \frac{1}{q_\theta' \bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial q_1} + \sum_{j=2}^{\ell} \frac{q_j}{\bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial q_j} + \frac{m}{\bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial m} = 0$$

$$\Longrightarrow \frac{q_0}{\bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial q_\theta} + \sum_{2}^{\ell} \frac{q_j}{\bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial q_j} + \left(1 - \frac{\bar{p}\, D}{m}\right) \frac{m}{\bar{\phi}_i} \frac{\partial \bar{\phi}_i}{\partial m} = 0.^*$$

* Note here that the income elasticity is multiplied by the fraction of expenditure made in the free market; if there is no rationing this equation reduces to the usual sum of elasticities.

In the above equation, the variable that cannot be observed is q_θ. One can substitute it either by q_1 or $\bar{p}$. If q_θ is replaced by q_1, the sum of the elasticities will be positive if the first and i-th goods are gross substitutes, negative if they are not. Use of $\bar{p}$ will result in opposite results. Thus one can only develop bounds for this sum.

We shall now derive Roy's identity when there are dual prices. Let the indirect utility function be $\bar{V}(\bar{p},q_1,q_2,\ldots,q_\ell,D,m)$. Then we know that

$$\bar{V}(\bar{p},q_1,q_2,\ldots,q_\ell,D,m) = V(q_\theta,q_2,\ldots,q_\ell,m_\theta), \qquad \text{(II.2.5)}$$

where V is the usual indirect utility function. The effect of a small change in q_1 on the utility is given by

$$\frac{\partial\bar{V}}{\partial q_1} = \frac{\partial V}{\partial q_\theta} \cdot \frac{\partial q_\theta}{\partial q_1} + \frac{\partial V}{\partial m_\theta} \cdot \frac{\partial m_\theta}{\partial q_1}$$

$$= \frac{\partial V}{\partial q_\theta} \cdot q_\theta' + \frac{\partial V}{\partial m_\theta} \cdot Dq_\theta' .$$

But $$\frac{\partial V}{\partial q_\theta} = \sum_i \frac{\partial u}{\partial x_i} \frac{\partial x_i}{\partial q_\theta} = \lambda(q_\theta \cdot \frac{\partial x_1}{\partial q_\theta} + \sum_2^\ell q_i \frac{\partial x_i}{\partial q_\theta})$$

$$= \lambda \left[\frac{\partial}{\partial q_\theta}[q_\theta x_1 + \sum_2^\ell q_i x_i] - x_1\right]$$

$$= -\lambda \bar{x}_1 \text{ (Note that } m_\theta \text{ is held constant)}.$$

Since $\frac{\partial V}{\partial m_\theta} = \lambda$, we have

$$\frac{\partial\bar{V}}{\partial q_1} = -\lambda q_\theta'(x_1 - D).$$

Again, this is easily understood. Any change in the open market price has no direct effect on the expenditure in the ration shops and it has full effect on purchases in the open market. The net effect should be a mixture of the two, which is precisely what we have. For changes in other prices, the usual Roy's formula applies:

$$\frac{\partial \bar{V}}{\partial q_i} = -\lambda \bar{x}_i \quad i = 2,\ldots,\ell.$$

Next, we study the effect of dual pricing of one commodity on the income and price elasticities of its substitutes and complements. We know from the identity (II.2.3) that

$$\bar{\phi}(\bar{p},q_1,q_2,\ldots,q_\ell,D,m) = \phi(q_\theta,q_2,\ldots,q_\ell,m_\theta).$$

Consider the case where $0 < \theta < 1$. Then $\bar{\phi}_1 = D$. Differentiating the first equation of the system (II.2.3) with respect to m, we get

$$0 = \frac{\partial \bar{\phi}_1}{\partial m} = \frac{\partial \phi_1}{\partial m_\theta} \cdot \frac{\partial m_\theta}{\partial m} + \frac{\partial \phi_1}{\partial q_\theta} \cdot \frac{\partial q_\theta}{\partial \theta} \cdot \frac{\partial \theta}{\partial m}$$

$$= \frac{\partial \phi_1}{\partial m_\theta} [1+(q_1-\bar{p}) D \frac{\partial \theta}{\partial m}] + \frac{\partial \phi_1}{\partial q_\theta} q_1 \frac{\partial \theta}{\partial m}$$

$$\Longrightarrow \frac{\partial \theta}{\partial m} = \frac{\partial \phi_1 / \partial m_\theta}{q_1 (\frac{\partial \phi_1}{\partial q_\theta} + D \frac{\partial \phi_1}{\partial m_\theta}) - \bar{p} D \frac{\partial \phi_1}{\partial m_\theta}}.$$

Since $\frac{\partial \phi_1}{\partial q_\theta} + D \frac{\partial \phi_1}{\partial m_\theta} \le 0$, if the first good is normal, i.e., if $\frac{\partial \phi_1}{\partial m_\theta} > 0$, then $\frac{\partial \theta}{\partial m} > 0$.

Now, consider $\frac{\partial \bar{\phi}_j}{\partial m}$. From (II.2.3) again, we have

$$\frac{\partial \bar{\phi}_j}{\partial m} = \frac{\partial \phi_j}{\partial m_\theta} [1+(q_1-\bar{p}) D \frac{\partial \theta}{\partial m}] + q_1 \frac{\partial \phi_j}{\partial q_\theta} \cdot \frac{\partial \theta}{\partial m}$$

$$= \frac{\partial \phi_j}{\partial m_\theta} + q_1 [\frac{\partial \phi_j}{\partial q_\theta} + D(1-\frac{\bar{p}}{q_1}) \frac{\partial \phi_j}{\partial m_\theta}] \frac{\partial \theta}{\partial m}. \qquad \text{(II.2.6)}$$

If the first and the j-th goods are substitutes, we know that $\frac{\partial \phi_j}{\partial q_\theta} + D \frac{\partial \phi_j}{\partial m_\theta} > 0$. The presence of the term $-D \frac{\bar{p}}{q_1} \frac{\partial \phi_j}{\partial m_\theta}$ makes the sign of the second term on the right hand side of (II.2.6) ambiguous. Suppose $\bar{p}/q_1$ is very small. Then, we have

$$\frac{\partial \bar{\phi}_j}{\partial m} > \frac{\partial \phi_j}{\partial m_\theta}$$

i.e., if the rationed good is normal and is a substitute for this unrationed good 'j', then the income effect on the demand for the unrationed good is more under dual pricing than otherwise. This agrees with and generalizes the earlier findings of Tobin and Houthakker [1951] and Neary and Roberts [1980] (see equation 6.1 of the former and equation 24 of the latter). Note that total rationing of the first good corresponds to the limiting case of $q_1 \to +\infty$. Thus, for large q_1, i.e., for small $\bar{p}/q_1$, we expect our results to be close to those of total rationing.

It may be noted that if $\theta = 1$, i.e., if $\bar{\phi}_1 > D$, or if $\theta=0$, i.e., $\bar{\phi}_1 < D$, $\frac{\partial \theta}{\partial m} = 0$ for normal goods and hence the income effect under dual pricing is the same as under uniform pricing as can be seen from (II.2.6).

We now study the effect of changes in market prices on the compensated demands when there is dual-pricing. Consider an unrationed good $j \neq 1$. If q_j changes by Δq_j an income compensation of approximately $\Delta q_j . \bar{\phi}_j$ is required to maintain the same level of utility. Also, if $0 < \theta < 1$, it can be shown that the change in expenditure on the first good must be zero at the optimum. For any other distribution of this additional income among the commodities, the change in utility is negative. For, let $\varepsilon_i > 0$ be the change in expenditure on good i, $i = 1,2,\dots,\ell$, such that $\sum_{i=1}^{\ell} \varepsilon_i = \bar{\phi}_j \, \Delta q_j$. Then the change in utility is

$$\Delta u \simeq \sum_{i=1}^{\ell} u_i \frac{\varepsilon_i}{q_i} - u_j \bar{\phi}_j \frac{\Delta q_j}{q_j}$$

$$= \frac{u_1}{q_1} \varepsilon_1 + \lambda \sum_{2}^{\ell} \varepsilon_i - \lambda \bar{\phi}_j \Delta q_j$$

$$= \frac{u_1}{q_1} \varepsilon_1 - \lambda \varepsilon_1$$

$$= (\frac{u_1}{q_1} - \lambda) \varepsilon_1 < 0 \quad \because \quad \frac{u_1}{\theta q_1 + (1-\theta)\bar{p}} = \lambda > \frac{u_1}{q_1} .$$

Hence, we get,

$$\left.\frac{\partial \bar{\phi}_j}{\partial q_j}\right|_{u=\text{constant}} = 0 \qquad \forall\ j = 2,\ldots,\ell.$$

Since income is a function of q_j, by differentiating the first equation of (II.2.3) with respect to q_j, we get,

$$0 = \left.\frac{\partial \bar{\phi}_1}{\partial q_j}\right|_{u=\text{constant}} = \frac{\partial \phi_1}{\partial q_j} + \frac{\partial \phi_1}{\partial q_0} \cdot \frac{\partial q_0}{\partial \theta} \cdot \frac{\partial \theta}{\partial q_j} + \frac{\partial \phi_1}{\partial m_0} \cdot \frac{\partial m_0}{\partial \theta} \cdot \frac{\partial \theta}{\partial q_j} + \frac{\partial \phi_1}{\partial m_0} \cdot \frac{\partial m_0}{\partial m} \cdot \frac{\partial m}{\partial q_j}$$

$$= \frac{\partial \phi_1}{\partial q_j} + \phi_j \frac{\partial \phi_1}{\partial m_\theta} + \left[\frac{\partial \phi_1}{\partial m_\theta} q_1 + \frac{\partial \phi_1}{\partial m_\theta} (q_1 - \bar{p}) D\right] \frac{\partial \theta}{\partial q_j}$$

$$\Longrightarrow \frac{\partial \theta}{\partial q_j} = - \frac{\left[\frac{\partial \phi_1}{\partial q_j} + \phi_j \frac{\partial \phi_1}{\partial m_\theta}\right]}{q_1 \left[\frac{\partial \phi_1}{\partial q_\theta} + D\left(1 - \frac{\bar{p}}{q_j}\right) \frac{\partial \phi_1}{\partial m_\theta}\right]}$$

Again, if $\bar{p}/q_1$ is very small, the denominator on the right hand side is negative and hence $\partial\theta/\partial q_j > 0$ if j is a substitute and $\partial\theta/\partial q_j < 0$ if j is a complement for good 1.

From the equations (II.2.3) we also have

$$\left.\frac{\partial \bar{\phi}_j}{\partial q_j}\right|_{u=\text{constant}} = \frac{\partial \phi_j}{\partial q_j} + \frac{\partial \phi_j}{\partial q_0} \cdot \frac{\partial q_0}{\partial \theta} \cdot \frac{\partial \theta}{\partial q_j} + \frac{\partial \phi_j}{\partial m_0} \cdot \frac{\partial m_\theta}{\partial q_j} + \frac{\partial \phi_j}{\partial m_0} \cdot \frac{\partial m_\theta}{\partial \theta} \cdot \frac{\partial \theta}{\partial q_j}$$

$$= \frac{\partial \phi_j}{\partial q_j} + \phi_j \frac{\partial \phi_j}{\partial m_\theta} + q_j \left[\frac{\partial \phi_j}{\partial q_\theta} + D\left(1 - \frac{\bar{p}}{q_1}\right) \frac{\partial \phi_j}{\partial m_\theta}\right] \frac{\partial \theta}{\partial q_j}$$

Thus, the own price effect along the compensated demand curve is smaller in magnitude under dual pricing if $\bar{p}/q_1$ is small, and the unrationed good in question is normal and is a substitute for the ration good. This again agrees with the results in the literature on total rationing (see, for instance, equation 29 of Neary and Roberts [1980]). However, our results for partial rationing (i.e. dual pricing) are ambiguous in general, as it should be expected, since it is a mixture of total rationing and completely free market.

II.3. PRODUCER BEHAVIOUR UNDER DUAL PRICING

In this section, we study the behaviour of a producer*, when its output or an input is subject to dual pricing. For ease of exposition, we will assume that each producer produces only one good and that this is not used as an input in its own production. Assume that there are 'J' producers. The technology of the j-th producer is given by

$$T_j = \{(y,x) \in \mathbb{R}_+^{\ell+1} \mid y \leq f(x)\}$$

where f is strictly concave, increasing in each argument and differentiable. Assume that production is instantaneous. Given a price vector $(q_k, q) \in \mathbb{R}_+^{\ell+1}$, the standard producer's problem is to maximize profit $\pi(x,q) = q_k y - q.x$ subject to $(y,x) \in T_j$, if j-th producer produces the k-th commodity. In this section, we shall use the output of the producer as the numeraire. Then

$$\pi(x,q) = y - q.x.$$

Before we introduce dual pricing, we should make sure that the producer's problem has a solution for all $q >> 0$. In this problem, the profit function is linear but the feasible set T_j is not compact. Hence without further restrictions on the technology set, the problem may have no solution for some prices even when there are decreasing returns to scale.

We will now introduce some restrictions on the technology and prove that the producer's profit maximization problem has a solution for strictly positive price vectors, in the following:

Proposition 1: *Assume that the production function* $f: \mathbb{R}_+^{\ell} \to \mathbb{R}_+$ *is continuous and increasing with* $f(0) = 0$. *Assume further that there exists an* $\alpha \in (0,1)$ *such that*

* The behaviour of a monopolist under these conditions has been studied in Mukherji, Pattanaik and Sundrum [1980]. See also Stern [1985] for effects of a dual pricing scheme in different market structures.

$f(cx) \leq c^{\alpha} f(x)$ $\forall$ $x \in \mathbb{R}_+^{\ell}$ and $c \geq 0$.

Then a profit maximizing solution exists for every $q \in \mathbb{R}_+^{\ell}$ with $q >> 0$.

Proof: The profit function $\pi(x,q) = f(x)-q.x$ is continuous in x. Also the set

$$B(q) = \{x \in \mathbb{R}_+^{\ell} \,|\, \pi(x,q) \geq 0\}$$

is closed and convex. Hence it is enough to show that B(q) is bounded for all q >> 0. Suppose not. Then there exists a 'q' >> 0 and an unbounded infinite sequence $x_n \in B(q)$ for all n. Let

$$\bar{x}_n = \max(x_n^1, x_n^2, \ldots, x_n^{\ell}) \text{ and}$$

$$\underline{q} = \min(q^1, q^2, \ldots, q^{\ell}).$$

We have

$$\pi(x_n, q) = f(x_n) - q.x_n \leq f(x_n) - \underline{q}.x_n \leq f(x_n) - \underline{q}\,\bar{x}_n$$

$$\leq f(\bar{x}_n, \bar{x}_n, \ldots, \bar{x}_n) - \underline{q}\,\bar{x}_n$$

$$\leq \bar{x}_n^{\alpha} f(1) - \underline{q}\,\bar{x}_n$$

$$= \bar{x}_n \{\bar{x}_n^{\alpha-1} f(1) - \underline{q}\}.$$

The expression on the right hand side is unbounded below and hence negative for infinitely many values of n. But $x_n \in B(q) \Longrightarrow \pi(x_n, q) \geq 0$ for all n, a contradiction.

Q.E.D.

Henceforth, we will assume that the conditions of this proposition hold.

We will first study the behaviour of producers under dual pricing of the output. In the case of production, there are at least two ways of introducing a dual pricing scheme. First method is to require each producer to sell a *fixed amount* of the product at a control price to the government.

The second is to require the producers to sell a *fixed fraction* of their output at the control price. Both schemes are practiced in India. For instance, the first scheme applies to the sugar industry while the second to the cement industry. There are a number of inputs for which either formally or informally dual prices exists; consider for example electricity. Commodities, which are traded in the organized as well as unorganized sector, often sell for a higher price in the unorganized market. The cost of credit, for instance, is lower in the organized banking sector, while it is a higher in the rural areas.

If the levy quantity is fixed in absolute terms, it is like a lump sum tax and the marginal conditions relating to the demand for inputs are not affected and hence the supply function of output is not affected. On the other hand, if the levy quota is a fixed fraction of the output, the supply function is likely to be affected. We will consider only the latter.

Suppose a fraction μ of the output produced must be sold at a control price r. If the open market price of this good is q_k, the total revenue from this good is given by

$$r \mu f(x) + q_k (1-\mu) f(x) = f(x) \{\mu r + (1-\mu) q_k\} = f(x) q_\mu$$

where $q_\mu = \mu r + (1-\mu) q_k$. The profit function is given by

$$\pi(x,r,q,\mu) = q_\mu f(x) - q.x.$$

Thus the only difference in the producer's maximization problem due to dual pricing is that a weighted average of the control and open market price is used to calculate the revenue from sales, instead of a single price. This does not create any difficulty. The supply function is obtained in the usual way by using the weighted average of the two prices.

Now, we will study the more interesting aspects of dual pricing in production, viz., dual pricing of inputs. Again, for ease of exposition, we will assume that only one input, say the first, is subject to dual pricing. One can think of charging the control price 's' for a fraction 'ν' of the input used or for a fixed quantity 'F' of the input. The former case is easily handled. Just as in the case of dual pricing for output, the input price should be taken as $\nu s + (1-\nu)q_1$ for the calculation of profits.

Let us consider the case where fixed quotas of the first input are sold at the control price. Assume that the output price is normalized to 1. Then the j-th producer's maximization problem is

$$\max_{(y,x)\varepsilon T_j} \pi(y,x,s,q) = y-sx_1-(q_1-s)\max(x_1-F,0) - \sum_2^{\ell} q_j x_j .$$

As before, it will be convenient if we can eliminate the non-linear budget constraint (in quantities). We will again make use of Fenchel's duality theorem. For this, it will be helpful to write the profit function as

$$\pi(y,x,s,q) = h(y,x,s,q) - g(x)$$

where $$h(y,x,s,q) = \pi(y,x,s,q) \text{ for } (y,x) \varepsilon \mathbb{R}_+^{\ell+1}$$

$$= y-sx_1 - \sum_2^{\ell} q_j x_j - (q_1-s)\max(x_1-F,0)$$

and $$g(x) = 0 \quad \forall \ (y,x) \ \varepsilon \ T_j.$$

Note that

$$\max_{(y,x)\ \varepsilon\ T_j} \pi(y,x,s,q) = \max_{(y,x)\varepsilon T_j \cap \mathbb{R}^{\ell+1}} [h(y,x,s,q)-g(x)]$$

Now the dual region for h is the set of all price vectors $r = (r_0,r_1,\ldots,r_\ell)$ $\varepsilon\ \mathbb{R}^{\ell+1}$ for which

$$\inf_{(y,x)\ \varepsilon\ \mathbb{R}^{\ell+1}} [r(y,x)-h(y,x,s,q)] > -\infty.$$

Clearly

$$r_0 y + \sum_{j=1}^{\ell} r_j x_j - y + sx_1 + (q_1 - s)\max(x_1 - F, 0) + \sum_{2}^{\ell} q_j x_j \qquad \text{(II.3.1)}$$

has a finite infimum only if $r_0 = 1$ and $r_j = -q_j$, for $j = 2,\ldots,\ell$. Hence let $r_0 = 1$ and $r_j = -q_j$ for $j = 2,\ldots,\ell$. We will now determine the possible values for r_1. Consider $r_1 = -s$. Then (II.3.1) reduces to

$$(q_1 - s)\max(x_1 - F, 0) \geq 0 \quad \forall\ (y,x) \in \mathbb{R}^{\ell+1}.$$

Thus the vector $(1, -s, -q_2,\ldots,q_\ell)$ belongs to the dual region. Next consider $r_1 = -q_1$. The expression in (II.3.1) reduces to

$$-(q_1 - s)x_1 + (q_1 - s)\max(x_1 - F, 0) \geq -(q_1 - s)F \text{ for } (y,x) \in \mathbb{R}^{\ell+1}.$$

Thus $(1, -q_1, -q_2,\ldots,q_\ell)$ also belongs to the dual region. It is easy to show that convex combinations of these vectors and no other vector belong to the dual set. Thus a typical element is

$$(1, -\theta q_1 - (1-\theta)s, -q_2,\ldots,-q_\ell) \text{ where } 0 \leq \theta \leq 1.$$

Hence, the dual function $h^*(r)$ is given by

$$h^*(r) = \inf_{x_1 \in \mathbb{R}} [-\theta(q_1 - s)x_1 + (q_1 - s)\max(x_1 - F, 0)] = -\theta(q_1 - s)F$$

Next we will determine the dual function g^*. The domain of g^* is the set of vectors $r \in \mathbb{R}^{\ell+1}$ for which

$$\sup_{(y,x)\in T_j} r(y,x) < \infty.$$

By proposition 1, we know that for all r with $(r_0, -r_1, -r_2,\ldots,-r_\ell) \gg 0$, this supremum is finite. Also

$$g^*(r) = \max_{(y,x)\in T_j} [r_0 y - \sum_{1}^{\ell} r_j x_j]$$

is the maximum profit at the price vector r. Thus the dual problem is

$$\underset{\theta\ \varepsilon[0,1]}{\text{minimize}}\ [g^*(r)-h^*(r)]$$

where

$$r = (1,\ -\theta q_1 - (1-\theta)s,\ -q_2,\ldots,-q_\ell).$$

That is

$$\min_{\theta\varepsilon[0,1]}\ \max_{(y,x)\varepsilon T_j}\ y-[\theta q_1 + (1-\theta)s]x_1 - \sum_2^\ell q_j x_j + \theta(q_1-s)F.$$

Thus the problem is reduced to one of unconstrained maximization over the technology set T_j using the price vector $(1,\ \theta q_1 + (1-\theta)s,\ q_2,\ldots,q_\ell)$ and an income subsidy of $\theta(q_1-s)F$. Again, it is clear from this minimization problem that $x_1 > F \Longrightarrow \theta = 1$ and $x_1 < F \Longrightarrow \theta = 0$. This is analogous to the consumer's maximization problem under dual pricing. Again, extension of dual pricing of inputs to more than one input is straightforward.

CHAPTER III

EXISTENCE AND EFFICIENCY OF DUAL-PRICE EQUILIBRIUM IN AN EXCHANGE ECONOMY WHEN A CONSUMER GOOD IS DUAL-PRICED

III.1 INTRODUCTION

In Chapter II we saw how a consumer's maximization problem under dual pricing can be reduced to one under uniform pricing with an appropriate income subsidy and regulated prices. Given this, one would be interested in defining and studying the properties of a '*Dual-Price-Equilibrium*' when the individuals trade amongst themselves and when a typical consumer maximizes its utility subject to a (kinked) budget constraint of the type studied in Chapter II.

When lumpsum transfers of incomes are permissible, the price mechanism is known to lead to a Pareto efficient allocation of resources. Also, the distributions resulting from very general classes of rationing schemes result in Pareto-inefficient allocations (see, e.g., Nayak [1980]). Yet we find that even in many advanced countries, the prices of some goods and services are regulated according to the composition of consumption, production etc. In this Chapter we thus study the existence and efficiency of an equilibrium with dual pricing of a good.

In section III.2 we describe the notion of a dual price equilibrium in an exchange economy. In the next section, we study the efficiency properties of a Dual Price Equilibrium. In the last section, we prove the existence of such an equilibrium under certain very general conditions.

III.2. DUAL-PRICE EQUILIBRIUM IN AN EXCHANGE ECONOMY

Let us consider an exchange economy with 'ℓ' commodities, traded and consumed by 'n' consumers of the type discussed earlier. That is, if

$A = \{1,\ldots,n\}$ denotes the set of finitely many consumers, then the consumption set, the preference preordering and the initial endowment of the i-th consumer are represented by (X^i, u^i, e^i) where $X^i \subset \mathbb{R}^{\ell}_{+}$, $u^i : X^i \to \mathbb{R}_{+}$ and $e^i \in X^i$. We make the usual assumptions on the consumption set and the intial endowment. That is, we assume X^i to be closed and convex with a lower bound and no satiation. We make the same assumptions on the utility function as given in Chapter II, such as it is differentiable and the preferred set is strictly convex etc. Assuming that good ℓ is the numeraire, the price set is given by $Q = \mathbb{R}^{\ell}_{+} \times \{1\}$. Assume again that only the first commodity is subject to dual pricing with prices $\bar{p}$ and q_1 in the ration shop and the free market respectively and with a ration quota D. With the fixed constants $\bar{p}$ and D, for any given $q \in Q$, the i-th consumer's budget correspondence is defined as

$$\beta^i(\bar{p},q,D) := \{x \in X^i \mid \bar{p}(x_1 - e^i_1) + (q_1 - \bar{p})[\max(x_1 - D, 0) - \max(e^i_1 - D, 0)] + \sum_{j=2}^{\ell} q_j(x_j - e^i_j) \leq 0\}.$$

In this scheme e^i lies on the boundary of the budget set. Note that the evaluation procedure for all the transactions in good one is the same, namely, that the consumption as well as the initial endowments are evaluated upto D units at the price $\bar{p}$ and above that at the price $q_1 > \bar{p}$.

The demand correspondence for consumer i is then given by

$$\bar{\phi}^i(q) = \{x \in \beta^i(\bar{p},q,D) \mid x \succsim_i y \quad \forall\, y \in \beta^i(\bar{p},q,D)\}.$$

For $\bar{x}^i \in \bar{\phi}^i(q)$, excess demand is denoted by $\bar{Z}(q)$. An exchange economy is denoted by

$\mathcal{E} := (X^i, u^i, e^i)$.

<u>Definition III.1.</u>

An exchange economy is said to have a Dual Price Equilibrium (DPE) if there exists a pair $(\bar{p},D)$ of the ration price and the quota for the first commodity, a price vector $q^* \in Q$ and a set of allocations $\{\bar{x}^{i*}, i \in A\}$ satisfying the following conditions.

(i) $\sum_{1}^{n} (\bar{x}^{i*} - e^{i}) = 0$

(ii) $\bar{x}^{i*} \in \bar{\phi}^{i}(q^*)$

(iii) $q_1^* > \bar{p}$ *and* $\bar{x}_1^{i*} > D$ *for at least one* $i \in A$.

Note that if either of the inequalities in the last condition does not hold the DPE reduces to a competitive equilibrium.

Definition III.2.

A DPE is said to be trivial if the associated allocations can be obtained as a Walras equilibrium without any redistribution of initial endowments, i.e., if there is a $\bar{q} \in Q$ *such that* $\bar{x}^*$ *is a Walras equilibrium at prices* $\bar{q}$ *and the initial endowments* e^i, $i \in A$.

Again, it is interesting to see that the violation of either inequality in condition (iii) of definition III.1 means that it is a trivial DPE.

III.3 INEFFICIENCY OF A DPE

Given that a DPE exists, our concern then is to see if it has some desirable properties at least with respect to the distribution of the good with two prices. For instance, we can make sure that every consumer demands at least a prespecified minimum, say D units, of the first commodity (assuming that nD is physically feasible) irrespective of the initial distribution of resources. This will hold good if, in particular, $\bar{p}$ is set equal to zero and the good has no satiation*.

Given the well known trade-off between equity and efficiency and given that a DPE allocation can be more equitable than, say, a competitive allocation, we shall now examine whether DPE allocations are Pareto-efficient. For this

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* For a detailed analysis of this see section 4.A of Guesnerie and Roberts [1984].

purpose, we shall use the following result which was obtained in Chapter II as an immediate consequence of Fenchel's duality theorem.

Let $\bar{x}^{i*} \varepsilon \bar{\phi}^{i}(q)$, $i \varepsilon A$. Then, there exists a $\theta^{i} \varepsilon [0,1]$ such that $\bar{x}^{i*}$ maximizes u^{i} subject to the following budget constraint

$$\bar{x}_1^i[\theta^i q_1 + (1-\theta^i)\bar{p}] + \sum_{j=2}^{\ell} \bar{x}_j^i q_j \leq \bar{p}\, e_1^i + (q_1-\bar{p})\max(e_1^i-D,0) + \sum_{j=2}^{\ell} q_j e_j^i + \theta^i(q_1-\bar{p})D$$

This means that if the i-th consumer uses the price $\theta^i q_1+(1-\theta^i)\bar{p}$ to evaluate the demand for the first good with an income subsidy on the initial income, it will maximize its utility by the *same vector* $\bar{x}^{i*}$ which gives maximum utility under dual pricing. We also know that

$$\bar{x}_1^{i*} > D \implies \theta^i = 1,$$

$$\bar{x}_1^{i*} < D \implies \theta^i = 0$$

and

$$\bar{x}_1^{i*} = D \implies \theta^i \varepsilon [0,1].$$

We can now prove the following

Theorem 1: *Every non-trivial Dual Price Equilibrium is Pareto inefficient.*

Proof: Suppose not. Then there exists a price vector $\bar{q}$ and an allocation $\bar{x}^i$, $i = 1,\ldots,n$ which is Pareto efficient and is also a non-trivial DPE. We know that, under our assumptions, any Pareto efficient allocation can be obtained as a Walras equilibrium with a suitable redistribution of income. Let q^* be a price vector associated with such a Walras equilibrium. Necessary conditions for Walras equilibrium imply that*, for any pair of consumers (i,k)

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* u^{ij} stands for the partial derivative of u_i with respect to the j-th good. The context will make clear the point at which the derivative is taken.

$$\frac{u^{i1}}{u^{ij}} = \frac{q_1^*}{q_j^*} = \frac{u^{k1}}{u^{kj}} \quad \forall\ j = 2,\ldots,\ell \qquad \text{(III.3.1)}$$

We have proved earlier that there exists a $\theta^i \in [0,1]$ such that the consumer demand is unaltered if the dual price $(\bar{p},\bar{q}_1)$ is replaced by $\theta^i\bar{q}_1+(1-\theta^i)\bar{p}$ and the current expenditure by $\bar{p}e_1^i+(\bar{q}_1-\bar{p})\max(e_1^i-D,0)+\sum_{j=2}^{\ell} e_j^i\bar{q}_j + \theta^i D(\bar{q}_1-\bar{p})$. Thus the necessary conditions for the consumer's maximization problem imply that

$$\frac{u^{i1}}{u^{ij}} = \frac{\theta^i\bar{q}_1+(1-\theta^i)\bar{p}}{\bar{q}_j}, \quad i = 1,2,\ldots,n, \quad j = 2,\ldots,\ell \qquad \text{(III.3.2)}$$

By definition, $\bar{x}_1^{i*} > D$ for at least one consumer, say the k-th. That is, $\theta^k = 1$. Hence we have, from (III.3.1) and (III.3.2).

$$\frac{q_1^*}{q_j^*} = \frac{u^{k1}}{u^{kj}} = \frac{\bar{q}_1}{\bar{q}_j} \implies q_j^* = c\ \bar{q}_j \text{ for some positive scalar 'c'.}$$

In view of the homoegeneity of the demand functions for DPE (see page 16 of Chapter II) we could choose 'c' such that $q_j^* = \bar{q}_j\ \forall j$. Now consider any consumer $i \neq k$. We know that

$$\frac{\bar{q}_1}{\bar{q}_j} = \frac{q_1^*}{q_j^*} = \frac{u^{i1}}{u^{ij}} = \frac{\theta^i\ \bar{q}_1 + (1-\theta^i)\bar{p}}{\bar{q}_j} \implies \theta^i = 1.$$

But $\theta^i = 1$ only if $\bar{x}_1^i \geq D$. In view of this, the budget constraint of any consumer can be written as

$$\bar{p}\ \bar{x}_1^i + (q_1^*-\bar{p})(\bar{x}_1^i-D) + \sum_{j=2}^{\ell} q_j^*\bar{x}_j^i = \bar{p}\ e_1^i + (q_1^*-\bar{p})\max(e_1^i-D,0) + \sum_{j=2}^{\ell} q_j^*\ e_j^i$$

$$\implies \bar{p}(\bar{x}_1^i-e_1^i)+(q_1^*-\bar{p})\{\bar{x}_1^i-D-\max(e_1^i-D,0)\} + \sum_{j=2}^{\ell} q_j^*(\bar{x}_j^i-e_j^i) = 0.$$

Summing over i and using the fact that all markets clear, we get

$$\sum_{i=1}^{n}(\bar{x}_1^i - D) - \sum_{i=1}^{n}\max(e_1^i-D,\ 0) = 0.$$

Notice that the above equation cannot be satisfied if $e_1^i - D < 0$ for any i. Hence $e_1^i - D \geq 0 \ \forall \ i \in A$. The budget constraint then becomes

$$q_1^* \bar{x}_1^i + \sum_{j=2}^{\ell} q_j^* \bar{x}_j^i = q_1^* e_1^i + \sum_{j=2}^{\ell} q_j^* e_j^i,$$

which shows that $\bar{x}^i$ can be purchased at price $q^* = \bar{q}$ without any redistribution of endowments. Hence $\bar{x}^i$, $i=1,\ldots,n$, is a trivial DPE, contrary to our assumption.

Q.E.D.

We hasten to add that the Pareto inefficiency of an allocation is not so serious that we should discard DPE allocations, especially in view of its desirable properties of distributing goods. Hence it will be interesting to investigate the conditions under which dual price equilibrium will exist.

III.4. EXISTENCE OF A DPE

As we have hinted before, the need for dual pricing will arise only when the initial distribution of the first good is not even and the demand for this good (in a Walrasian set up) is high when its price is low. For example, there is no need for dual pricing of this good, if every one initially has at least D units of this good. Hence it is *reasonable* to assume that* (henceforth we shall use $\bar{\phi}$ and x interchangeably)

$$\sum_{i=1}^{n} [\max(D-e_1^i,0) - \max(D-\bar{\phi}_1^i,0)] > 0 \text{ for all } q \text{ with } q_1 < \bar{p} \quad \text{(III.A.1)}$$

For example, this assumption holds if $e_1^i < D$ for at least one i, and $\bar{\phi}_1^i > D$ for all i, which will be the case when $\bar{p}$ and D are small. We can now prove the following

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* For our purpose, this assumption could be weakened to $\sum_1^n [\max(D-e_1^i,0) - \max(D-\bar{\phi}_1^i,0)] \neq 0$.

Theorem 2: *If assumption (III.A.1) holds, there does not exist any dual price equilibrium.*

Proof. Suppose not. Then there exists a price vector $(\bar{p}, q_1, q_2, \ldots, q_\ell)$ such that

$$\sum_{i=1}^{n} \bar{\phi}_j^i(\bar{p}, q, D) - e_j^i = 0 \quad \forall j \qquad \text{(III.4.1)}$$

From the definition of the budget set, we have,

$$\bar{p}(\bar{\phi}_1^i - e_1^i) + (q_1 - \bar{p})\{\max[\bar{\phi}_1^i - D, 0] - \max[e_1^i - D, 0]\} + \sum_{j=2}^{\ell} q_j(\bar{\phi}_j^i - e_j^i) = 0.$$

Summing over all i, we have

$$\bar{p} \sum_{i=1}^{n} (\bar{\phi}_1^i - e_1^i) + (q_1 - \bar{p}) \sum_i [\max(\bar{\phi}_1^i - D, 0) - \max(e_1^i - D, 0)] + \sum_i \sum_j q_j(\bar{\phi}_j^i - e_j^i) = 0.$$

Using (III.4.1) we have, since $q_1 > \bar{p}$,

$$\sum_{i=1}^{n} [\max(\bar{\phi}_1^i - D, 0) - \max(e_1^i - D, 0)] = 0 \qquad \text{(III.4.2)}$$

But

$$\max(\bar{\phi}_1^i - D, 0) = (\bar{\phi}_1^i - D) + \max(D - \bar{\phi}_1^i, 0)$$

and

$$\max(e_1^i - D, 0) = (e_1^i - D) + \max(D - e_1^i, 0)$$

Hence

$$\sum_{i=1}^{n} [\max(\bar{\phi}_1^i - D, 0) - \max(e_1^i - D, 0)] = \sum_{i=1}^{n} \{(\bar{\phi}_1^i - D) - (e_1^i - D) - [\max(D - e_1^i, 0) - \max(D - \bar{\phi}_1^i, 0)]\}$$

$$= -\sum_{i=1}^{n} [\max(D - e_1^i, 0) - \max(D - \bar{\phi}_1^i, 0)] < 0 \quad \text{by (III.A.1)}$$

Q.E.D.

But this contradicts (III.4.2).

Let us now find out the reason for the non-existence of an equilibrium. This is more easily understood in the case of two consumers A and B and two goods 1 and 2. In Figure III.1, the good on the horizontal axis is taken as the 'numeraire' and the other good is subject to dual pricing. The two prices of the second good are shown by the budget lines KLM and K'L'M'. The ration quota D is given by the length OD = O'F. Suppose the endowment distribution is 'e', below the line DG. Then

$$\max(D-e^A,0) + \max(D-e^B,0) = D-e^A > 0 \qquad \text{(III.4.3)}$$

What is the total value, i.e. the total income of the two consumers? With two prices for one good, there are several ways of defining the income. But we shall stick to our procedure. That is, for buying or selling, the second good is evaluated at price '$\bar{p}$' upto D units and then at price $q_1 > \bar{p}$. The total amount of the second good is OB = O'A. Its value at price 'q_1' is AQ. But we have to evaluate some units at price '$\bar{p}$'. For the consumer B (with origin O'), this involves a deduction of $(q_1-\bar{p})D = \varepsilon$, say, since its initial endowment of this good is greater than D. For consumer A, who owns less than D units of this good, we have to subtract $(q_1-\bar{p})\ e_2^A = \eta$ (say) $< \varepsilon$. Thus the total income is found to be the point shown in the figure as Y_A+Y_B.

Suppose the point at which both consumers maximise their utilities is 'P' (shown in the figure). First note that

$$\max(D-\bar{\phi}_2^A,0) + \max(D-\bar{\phi}_2^B,0) = 0. \qquad \text{(III.4.4)}$$

From (III.4.3) and (III.4.4), we see that our assumption (III.A.1) is satisfied. In fact (III.A.1) is satisfied whenever the initial endowment point is outside the rectangle DCFG and the equilibrium point 'P' is inside this rectangle.

What is the expenditure at 'P' ? It is easy to check that this is obtained by subtracting $2\varepsilon = 2D(q_1-\bar{p})$ from AQ. This is shown as E_A+E_B in the figure. Thus $E_A+E_B < Y_A+Y_B$, contradicting Walras law.

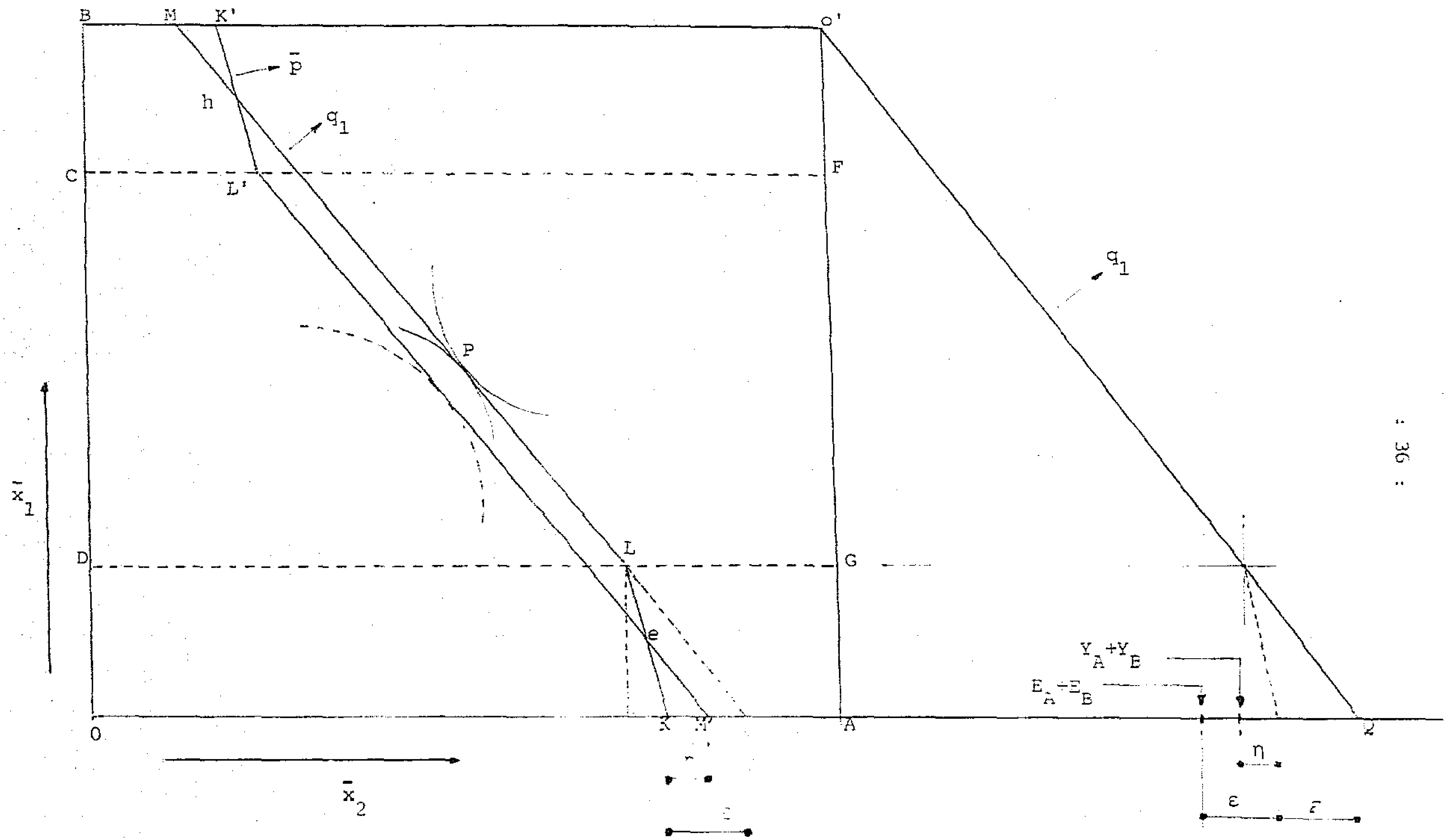

FIGURE III.1

Alternatively, we can verify the non-existence of an equilibrium as follows. Only those allocations which are the points of tangency of the indifference curves of both the consumers to a common budget line can be equilibrium allocations. For any price $q_1 > \bar{p}$, the only allocations common to both budget lines are the point 'h' and the initial endowment point 'e'. But, since the budget lines are different, these cannot be equilibrium allocations.

The source of the difficulty is the following : The trader who has more than D units of the good subject to dual pricing values it at price 'q_1' at the margin, while the buyer, who has less than D, values and pays '$\bar{p}$' < q_1. The problem, therefore, is due to different evaluations by buyers and sellers at the margin. During this process, we may say that some 'spurious' income is generated for some sellers of this good, making the total expenditure less than total income (as we also saw in Figure III.1). One way to resolve this problem is to create some 'spurious' expenditure of the same magnitude by another transaction.

The evaluation of purchases or consumption of good 1 at two prices is quite practicable. To sight an example, such dual pricing schemes are operated in various states of India for essential commodities such as sugar, rice, wheat, other cereals and pulses etc. But when it comes to evaluation of supplies, there is no reason to believe that a consumer will evaluate the first D units of the endowment of this good at a price smaller than the market price. Another major problem here is to mark some units of this endowment as the "first D units". This involves the problem of "truth-telling" as also of resale.

If we allow for resale and if each consumer can always sell any amount (possibly more than the endowment) of good 1 and buys at most D at $\bar{p}$ and more at q_1 then each agent will always buy D at $\bar{p}$ and make a profit of $(q_1-\bar{p})D$. However we shall assume that resale is not allowed since the dual price system does not work with this.

Suppose, therefore, that the government (which is the (n+1)-th agent) operates the public distribution system by buying the first good in the free market and selling it at a controlled price to the consumers. For this purpose it purchases nD units of this good and provides a quota of D units per head. But since some consumers may not utilise the full quota, it effectively supplies an amount $G = \sum_{i=1}^{n} \min(\bar{\phi}_1^i, D)$ through the ration shops. For simplicity assume that (nD-G) is kept as public stock.

Assume that the government finances its purchase of nD units of the rationed good by taxing free market consumption. That is, the owners of endowment receive prices $p' = (p_1,\ldots,p_\ell)$ for their sales whereas the consumers pay $(q_1,\ldots,q_\ell)$ for their purchases in the free market and $\bar{p}$ for the purchase of the ration quota. The tax vector $t' = (t_1,\ldots,t_{\ell-1},0)$ is such that $q_j = p_j(1+t_j)\ \forall j = 1,\ldots,\ell$.

A typical consumer's budget constraint is then given by*

$$\bar{p}\bar{x}_1^i + (q_1-\bar{p})\max(\bar{x}_1^i - D, 0) + \sum_{j=2}^{\ell} q_j \bar{x}_j^i = \sum_{j=1}^{\ell} p_j e_j^i .$$

The government's budget restriction is

$$p_1 t_1 \sum_{i=1}^{n} \max(\bar{x}_1^i - D, 0) + \sum_{j=2}^{\ell} \sum_{i=1}^{n} p_j t_j \bar{x}_j^i + \bar{p} \sum_{i=1}^{n} \min(\bar{x}_1^i, D) = p_1 nD.$$

Its demand for (stock of) good 1 is

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* The i-th consumer's budget and demand correspondences are modified accordingly.

$$\bar{x}_1^{n+1} = nD - \sum_1^n \min(\bar{x}_1^i, D).$$

Summing the budget constraints of all the consumers and the government we have

$$p_1 \sum_{i=1}^{n+1} \bar{x}_1^i + \sum_{j=2}^{\ell} \sum_{i=1}^{n} p_j \bar{x}_j^i = \sum_1^{\ell} \sum_1^{n} p_j e_j^i$$

or

$$\sum_1^{\ell} \sum_1^{n+1} p_j (\bar{x}_j^i - e_j^i) = 0 \quad (\text{for } \bar{x}_j^{n+1} = 0 \quad \forall j \neq 1)$$

Given a vector $(\bar{p}, p_1, \ldots, p_\ell)$ of nominal sellers' prices, let us denote the vector of relative prices by $(\bar{P}, P_1, \ldots, P_\ell) \in \Delta^{\ell+1}$, the $(\ell+1)$ dimensional unit simplex. Note that with our specifications if demand functions are homogenous of degree zero in $(\bar{p}, p_1, \ldots, p_\ell)$ then they are homogenous of degree zero in $(\bar{p}, q_1, \ldots, q_\ell)$ also where $q_j = p_j(1+t_j)$ $\forall j = 1, \ldots, \ell$. Hence, given the budget and demand correspondences in terms of $(\bar{p}, p_1, \ldots, p_\ell)$ they can be easily reduced to functions of $p' = (\bar{P}, P_1, \ldots, P_\ell) \in \Delta^{\ell+1}$ given that good 'ℓ' is the numeraire. Walras law can therefore be stated in terms of P as

$$\sum_1^{\ell} \sum_1^{n+1} P_j (\bar{x}_j^i - e_j^i) = 0 \quad (\text{if } p_i > 0 \text{ for at least one } i)$$

or, $\quad p'\bar{z} = 0$

where $\bar{z}' = (\bar{z}_1, \ldots, \bar{z}_\ell)$ is the vector of excess demands with $\bar{z}_j = \sum_{i=1}^{n+1} \bar{z}_j^i$ $\forall j = 1, \ldots, \ell$.

Definition III.3.

An exchange economy is said to be in a Dual Price Equilibrium with taxes (DPET) if, for given $t' = (t_1, \ldots, t_\ell) > 0$ *with* $t_\ell = 0$, $\exists$ *a vector of prices* $P' \in \Delta^{\ell+1}$ *and a quota D per head of the first good together with a set of allocations* $\{\bar{x}^{i*}, i \in A\}$ *satisfying*

(i) $\bar{z}_j^* = 0 \quad \forall j$

(ii) $\bar{x}^{i*} \in \bar{\phi}^{i*}(p)$

(iii) $q_1 > \bar{p}$ *and* $\bar{x}_1^{i*} > D$ *for at least one* i.*

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* If $\bar{x}_1^{i*} \leq D$ $\forall i$ then the economy is said to be in an equilibrium with rationing.

To prove the existence of an equilibrium we make the following assumptions.

$\forall i,\ X^i \subset \mathbb{R}^{\ell}_{+}$ *is closed, convex and* $0 \in X^i$. X^i has local non-satiation property, i.e., for every $x \in X^i$ and for every neighbourhood V of x in X^i, $\exists\ y \in V$ such that $y \succsim_i x$. Let X^i be such that $x + \mathbb{R}^{\ell}_{+} \subset X^i$. (III.A.2)

$\succsim_i$ is continuous and strictly convex, i.e.,

$x \sim y \Longrightarrow \lambda x + (1-\lambda)y \succ x$ *for* $\lambda \in (0,1)$ (III.A.3)

$e^i \in X^i\ \forall i$ and $e^i > 0$ (We do not consider goods 'j' for which $\sum_i e^i_j = 0$). Note that $0 \in X^i$ and $e^i > 0$ imply $\inf_{x \in X^i} E_p(x) < \sum_{j=1}^{\ell} p_j e^i_j$ for $p \gg 0$, where $E_p(x) = \bar{p}x_1 + (q_1-\bar{p})\max(x_1-D,0) + \sum_2^{\ell} q_j x_j$. Note also that demand functions are homogenous of degree zero in $p' = (\bar{p}, p_1, \dots, p_{\ell})$. (III.A.4)

For any $P'_m = (\bar{P}_m, P_{1m}, \dots, P_{\ell m}) \in \Delta^{\ell+1}$ with $P_{hm} \to 0$ for some $h = 1, \dots, \ell$, $\lim_{m \to \infty} \inf \|z(P_m)\| = \infty$. (III.A.5)

<u>Lemma 1:</u> *If* $\beta^i(p)$ *is compact then the set of maximal elements* $\bar{\phi}^i(p)$ *in* $\beta^i(p)$ *is non-empty. If* β^i *is continuous at p then* $\bar{\phi}^i$ *is upper hemi continuous* at p.*

See theorem A.III.3 in Hildenbrand and Kirman [1976] for a proof of Lemma 1.

<u>Lemma 2</u>. *The correspondence* $\beta^i(p)$ *is lower hemi continuous** at any* $p \in \Delta^{\ell+1}$ *if* $\inf_{x \in X^i} E_p(x) < \sum_{j=1}^{\ell} p_j e^i_j$. *If* $p \gg 0$ *then* β^i *is continuous at p.*

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* Let S and T be subsets of metric spaces $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively. Let Ψ be a correspondence (a point-to-set mapping) from S into T. Then the correspondence Ψ is said to be <u>upper hemi-continuous (uhc)</u> at $s \in S$ if, for every neighbourhood M of $\Psi(s)$, $\exists$ a neighbourhood O_s of s such that $x \in O_s \Longrightarrow \Psi(x) \subset M$. Ψ is said to be uhc if it is uhc at every $s \in S$.

** A correspondence Ψ: $S \to T$ is said to be <u>lower hemi-continous (lhc)</u> at $s \in S$ if, for every open set u with $\Psi(s) \cap u \neq \phi$, $\exists$ a neighbourhood v of s such that $\Psi(s') \cap u \neq \phi\ \forall s' \in v$. Ψ is said to be lhc if it is lhc at every $s \in S$.

See Appendix - C for a proof of this lemma.

Theorem 3. *If assumptions (III.A.2) to (III.A.5) hold there exists on equilibrium with rationing.*

Proof: We shall first show the existence of an equilibrium price vector and then that of an equilibrium ration quota. It is easy to prove (see lemmas 1 and 2) that $\bar{\phi}^i(p)$ is non-empty and continuous at any $p' = (\bar{p}, p_1, \ldots, p_\ell) >> 0$ given (III.A.2) to (III.A.4).

For obtaining ration price we define the following rule. The ration price $\bar{P}$ of the controlled commodity is simply assumed to be a fraction $0 < \delta < 1$ of the corresponding free market price p_1 for the sellers.

Define,

$$S_m = \{P \in \mathbb{R}^{\ell+1} | \bar{P} + \sum_1^\ell P_i = 1,\ \bar{P} = \delta P_1 \text{ and } P_h \geq \frac{1}{(1+\delta)m}\ \forall h,\ 0 < \delta < 1\}.$$

For $m > \ell$ define the map

$$P_m : S_m \to S_m \text{ by}$$

$$P_{hm} \to \frac{1}{m} + (1 - \frac{\ell}{m}) \{\frac{P_{hm} + \max(\bar{z}_h, 0)}{1 + \sum_1^\ell \max(\bar{z}_j, 0)}\} \quad \forall h = 2, \ldots, \ell.$$

$$P_{1m} \to \frac{1}{(1+\delta)m} + (1 - \frac{\ell}{m}) \{\frac{P_{1m} + \max(\bar{z}_1, 0)/(1+\delta)}{1 + \sum_1^\ell \max(\bar{z}_j, 0)}\}$$

and

$$\bar{P}_m = \delta P_{1m}.$$

Note that S_m is non-empty, compact, convex and this map from S_m into itself is continuous. Hence, by Brouwer's fixed point theorem, we have a fixed point, say, P_m^*. Since P_m^* is a bounded sequence, let $P_m^* \to P^*$.

We claim that $P^* >> 0$.

Suppose not. Let $P_{hm}^* \to 0$. Using (III.A.5) we then have $||\bar{z}(P_m^*)|| = +\infty$, i.e., for some $k = 1, \ldots, \ell$ $\bar{z}_k(P_m^*) \to +\infty$. Now, P_{km}^* cannot go to zero. If $k = 1$,

$$P^*_{1m} \rightarrow \frac{1}{(1+\delta)m} + (1-\frac{\ell}{m})\{\frac{P^*_{1m} + \bar{z}_1(P^*_m)/(1+\delta)}{1+\bar{z}_1(P^*_m) + \sum_2^{\ell} \max(\bar{z}_j,0)}\}$$

$$= \frac{1}{(1+\delta)m} + \frac{(1-\frac{\ell}{m})}{(1+\delta)\ \{1 + \frac{1+\sum_2^{\ell}\max(\bar{z}_j,0)}{\bar{z}_1(P^*_m)}\}} + \frac{(1-\frac{\ell}{m})\ P^*_{1m}}{1+\bar{z}_1(P^*_m) + \sum_2^{\ell}\max(\bar{z}_j,0)}$$

If $k = 2,\ldots\ldots,\ell$.

$$P^*_{km} \rightarrow \frac{1}{m} + \frac{(1-\frac{\ell}{m})}{\{1 + \frac{1+\sum_2^{\ell}\max(\bar{z}_j,0)}{\bar{z}_1(P^*_m)}\}} + \frac{(1-\frac{\ell}{m})\ P^*_{km}}{1+\bar{z}_1(P^*_m)+\sum_2^{\ell}\max(\bar{z}_j,0)}$$

If, for some k, $P^*_{km} = 0$ then we can easily check that P^*_{km} is not mapped to itself, i.e., P^*_m is not a fixed point. Hence, we must have $P^*_{km}\ \bar{z}_k(P^*_m) > 0$ for large m. And $P^*_{km} > 0$, $\bar{z}_k(P^*_m) \rightarrow +\infty$ imply $P^*_{km}\ \bar{z}_k(P^*_m) \rightarrow +\infty$. This, together with Walras' law implies $P^*_{rm}\ \bar{z}_r(P^*_m) < 0$ for some $r = 1,\ldots\ldots,\ell$. Since $P^*_m \in S_m$ we must then have $P^*_{rm} > 0$ and $\bar{z}_r(P^*_m) < 0$ for large m. Then Walras's law will hold only if $\bar{z}_r(P^*_m) \rightarrow -\infty$. This is a contradiction since X^i is bounded below $\forall i$.

Thus, P^*_{hm} does not converge to zero for any $h = 1,\ldots,\ell$, i.e., $P^* >> 0$. Since $P^* >> 0$ we know that $\bar{z}(P^*)$ is continuous. We will now show that $\bar{z}_h(P^*) = 0 \quad \forall h = 1,\ldots,\ell$. Suppose not. Let $\bar{z}_h(P^*) > 0$ for some $h = 1,\ldots,\ell$. By Walras' law $\bar{z}_k(P^*) < 0$ for some k, i.e.,

$$P^*_{km} \rightarrow \frac{1}{m} + (1-\frac{\ell}{m})\ \frac{P^*_k}{1+\bar{z}_h(P^*)} \neq P^*_k \text{ if } k = 2,\ldots,\ell$$

$$P^*_{1m} \rightarrow \frac{1}{(1+\delta)m} + (1-\frac{\ell}{m})\ \frac{P^*_1}{1+\bar{z}_h(P^*)} \neq P^*_1 \quad \text{if} \quad k = 1$$

i.e., P^* is not a fixed point. Hence, we have $\bar{z}_h(P^*) = 0\ \forall h$.

We shall now construct a map for calculating the equilibrium quota of the rationed good.

Note that the government's budget constraint can be written as

$$p_1 t_1 \sum_{i=1}^{n} \max(\bar{x}_1^i - D, 0) + \sum_{2}^{\ell} \sum_{1}^{n} p_j t_j \bar{x}_j^i - p_1 \bar{x}_1^{n+1} = (p_1 - \bar{p}) \sum_{1}^{n} \min(\bar{x}_1^i, D)$$

Define $TR = p_1 t_1 \sum_{1}^{n} \max(\bar{x}_1^i - D, 0) + \sum_{2}^{\ell} \sum_{1}^{n} p_j t_j \bar{x}_j^i - p_1 \bar{x}_1^{n+1}$. Define the "average real subsidy" for the dual pricing scheme by the function

$$g(D) = \frac{TR}{n(p_1 - \bar{p})}$$

Consider the map $\Psi : [0, \frac{e_1 + \varepsilon}{n}] \to [0, \frac{e_1 + \varepsilon}{n}]$ given by

$$D \xrightarrow{\Psi} \min[\max\{0, g(D)\}, \frac{e_1 + \varepsilon/2}{n}] \text{ where } e_1 = \sum_{i=1}^{n} e_1^i \text{ and } 0 < \varepsilon < 1.$$

Since Ψ is a continuous map from a compact, convex set into itself by Brouwer's fixed point theorem there exists a D^* such that $\Psi(D^*) = D^*$.

Since p^* is also a fixed point we make the following claims. First $(e_1+\varepsilon/2)/n$ cannot be a fixed point* for if $g(D) \geq (e_1+\varepsilon/2)/n$ then $D \to (e_1+\varepsilon/2)/n$ and demand in the first market becomes

$$\sum_{i=1}^{n+1} \bar{x}_1^i = nD + \sum_{i=1}^{n} \{\bar{x}_1^i - \min(\bar{x}_1^i, D)\} = e_1 + \varepsilon/2 + \sum_{i=1}^{n} \max(\bar{x}_1^i - D, 0) > e_1,$$

the supply on the first market. This contradicts $\bar{z}_h(P^*) = 0 \ \forall h$, i.e . P^* cannot be a fixed point.

Secondly, zero cannot be a fixed point for the map Ψ since it will violate Walras law by violating government's budget constraint. For, if $g(D) \leq 0$ then $D = 0$ and the government's expenditure is zero whereas income is positive since $p_1^* \bar{x}_1^{n+1} = 0$ and $p_1^* t_1 \sum_{1}^{n} \max(\bar{x}_1^i - D, 0) + \sum_{2}^{\ell} \sum_{1}^{n} p_j^* t_j \bar{x}_j^i > 0$ because $t > 0$, $p^* \gg 0$ and $\bar{x}_j^i > 0 \ \forall j$ for at least one i (since $\bar{z}_h = 0 \ \forall h$ and $e_j > 0 \ \forall j$). Hence we must have $0 < D \leq e_1/n$.

Q.E.D.

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* In fact $(e_1 + \varepsilon/_2 \, r)/n$ cannot be a fixed point for all $r = 1, 2, \ldots,$

From the above theorem, it is clear that we cannot rule out D* being equal to e_1/n i.e., the ration quota may exhaust all the supply and the open market may not function. Hence it will useful to find some conditions under which a DPET exists. In fact, it is interesting to note that there is a vector of taxes such that a DPET exists.

Since $\sum_i e_j^i > 0 \ \forall j$, we can show that at every convex exchange economy, $\mathcal{E} := ((X^i, \succsim_i, e^i), t)$ with taxes the equilibrium price correspondence is uhc (see 2.2 Proposition 4 and B Theorem 1 in Hildenbrand [1974]). Also, since D is a continuous function of x, using lemmas 1 and 2 and the corollary to B Proposition 1 in Hildenbrand [1974] we know that D is uhc at every $\mathcal{E}$. Hence, if we take a sequence $t_m \to 0$ then we know that $D(t_m) \to 0$ with $D(0) = 0$. Thus, starting with a tax vector $t >> 0$, if, in equilibrium, $D^* = e_1/n$ then by reducing t slightly we can reduce the equilibrium quota such that $D^* < e_1/n$ at the new equilibrium.

CHAPTER IV

WELFARE IMPROVEMENTS IN A DUAL-PRICE EQUILIBRIUM

IV.1. INTRODUCTION

In the present and the following chapters we make use of the theoretical framework for some empirical analysis of the public distribution schemes for wheat, rice and edible oils.

We begin this chapter with a DPE and ask the question: Is there a feasible welfare improvement? That is, can one change the quota and the controlled price marginally so that the subsidy provided does not increase but the social welfare does? In order to answer this question we make use of the theory of reforms (see e.g., Ahmed and Stern [1983], J.P.Dreze [1984], Guesnerie [1980]).

The plan of this chapter is as follows:

In section 2, we assume that there is a dual-price equilibrium with some ration quota and ration price for the first good. Then we calculate the loss in welfare due to (i) an increase in the ration price and (ii) a decrease in the quota in order to save one unit of subsidy. Using a social welfare function we compare the two losses and suggest possible directions of reform.

In section 3, instead of specifying welfare weights, we actually calculate them for the simple case of a two-class economy - with one class consisting of consumers who consume more than the quota of the rationed good and the other consuming less.

In the concluding section, we determine the welfare losses due to reduction in quotas and increases in ration prices of wheat and rice and due to increases in tax rates for four commodity groups and compare them for the year 1973-74 using the National Sample Survey data for the Indian economy.

IV.2. WELFARE IMPROVING DIRECTIONS

Consider the economy described in Chapter III. Let the collection of vectors $((q_1,\ldots,q_\ell),(\bar{x}^1,\ldots,\bar{x}^n))$ be a dual-price equilibrium with some price-quota pair $(\bar{p},D)$. Rearrange the consumers so that for $i = 1,\ldots,n_1$, $\bar{x}_1^i > D$, for $i = n_1+1,\ldots,n_2$, $\bar{x}_1^i = D$ and for $i = n_2+1,\ldots,n$, $\bar{x}_1^i < D$. Then, we know that

$$\frac{u^{ij}}{q_j} = \frac{u^{ik}}{q_k} = \text{(say) } \lambda^i \quad \forall i = 1,\ldots,n_1, \ \forall (j,k) \in (1,\ldots,\ell) \qquad \text{(IV.2.1(i))}$$

$$\frac{u^{i1}}{\theta^i q_1 + (1-\theta^i)\bar{p}} = \frac{u^{ij}}{q_j} = \lambda^i \quad \forall i = n_1+1,\ldots,n_2, \ \forall j = 2,\ldots,\ell \quad \text{for some } \theta^i,\ 0 \le \theta^i \le 1 \qquad \text{(IV.2.1(ii))}$$

and $$\frac{u^{i1}}{\bar{p}} = \frac{u^{ij}}{q_j} = \lambda^i \quad \forall i = n_2+1,\ldots,n, \ \forall j = 2,\ldots,\ell \qquad \text{(IV.2.1.(iii))}$$

We are now interested in finding possible locally welfare-improving directions from the present position, namely, $(q,(\bar{x}^i),\bar{p},D)$. For this we assume that the welfare function is of the Bergson-Samuelson type, namely,

$$W = W(u^1,\ldots,u^n).$$

Since the given position is not an optimum one can alter $\bar{p}$ and D marginally in such a way that, with a fixed or reduced government subsidy, the welfare increases. Consider first a change in $\bar{p}$ (We shall assume that all demand functions are differentiable and use the notation $x^i_{j\alpha}$ to denote $\frac{\partial \bar{x}^i_j}{\partial \alpha}$ where 'α' is some parameter). We have

$$\frac{\partial W}{\partial \bar{p}} = \sum_{i=1}^{n} \sum_{j=1}^{\ell} \frac{\partial W}{\partial u^i} u^{ij} \bar{x}^i_{j\bar{p}} = \sum_{i=1}^{n_1} \frac{\partial W}{\partial u^i} \lambda^i \sum_{j=1}^{\ell} q_j \bar{x}^i_{j\bar{p}}$$

$$+ \sum_{i=n_1+1}^{n_2} \frac{\partial W}{\partial u^i} \lambda^i [(\theta^i q_1 + (1-\theta^i)\bar{p})\bar{x}^i_{1\bar{p}} + \sum_{j=2}^{\ell} q_j \bar{x}^i_{j\bar{p}}]$$

$$+ \sum_{i=n_2+1}^{n} \frac{\partial W}{\partial u^i} \lambda^i (\bar{p}\, \bar{x}^i_{1\bar{p}} + \sum_{j=2}^{\ell} q_j \bar{x}^i_{j\bar{p}}) \quad \text{(using IV.2.1)} \qquad \text{(IV.2.2)}$$

Now, for $i = 1,\ldots,n_1$, the following identity holds

$$\sum_{j=1}^{\ell} q_j e_j^i \equiv \bar{p}D + q_1(\bar{x}_1^i - D) + \sum_{j=2}^{\ell} q_j \bar{x}_j^i \qquad (IV.2.3)$$

Therefore, we can differentiate both sides with respect to $\bar{p}$ to obtain

$$0 = D + \sum_{j=1}^{\ell} q_j \bar{x}_{j\bar{p}}^i$$

or, $$\sum_{j=1}^{\ell} q_j \bar{x}_{j\bar{p}}^i = -D \quad \forall i = 1,\ldots,n_1 \qquad (IV.2.4)$$

Similarly, for $i = n_1+1,\ldots,n_2$, we can differentiate

$$\sum_{j=1}^{\ell} q_j e_j^i \equiv \bar{p}\,\bar{x}_1^i + \sum_{j=2}^{\ell} q_j \bar{x}_j^i$$

or, $$\sum_{j=1}^{\ell} q_j e_j^i + \theta^i(q_1-\bar{p})\bar{x}_1^i \equiv (\theta^i q_1+(1-\theta^i)\bar{p})\bar{x}_1^i + \sum_{j=2}^{\ell} q_j \bar{x}_j^i \qquad (IV.2.5)$$

partially with respect to $\bar{p}$ to get

$$-\theta^i \bar{x}_1^i + \theta^i(q_1-\bar{p})\bar{x}_{1\bar{p}}^i = (1-\theta^i)\bar{x}_1^i + (\theta^i q_1+(1-\theta^i)\bar{p})\bar{x}_{1\bar{p}}^i + \sum_{j=2}^{\ell} q_j \bar{x}_{j\bar{p}}^i$$

or, $$(\theta^i q_1+(1-\theta^i)\bar{p})\bar{x}_{1\bar{p}}^i + \sum_{j=2}^{\ell} q_j \bar{x}_{j\bar{p}}^i = -D \quad \forall i = n_1+1,\ldots,n_2 \qquad (IV.2.6)$$

(since $\bar{x}_1^i = D$).

Henceforth we shall use the notation $q_{\theta i}$ to denote $(\theta^i q_1+(1-\theta^i)\bar{p})$.

Also, for $i = n_2+1,\ldots,n$ the budget constraint in equilibrium is

$$\sum_{j=1}^{\ell} q_j e_j^i \equiv \bar{p}\bar{x}_1^i + \sum_{j=2}^{\ell} q_j \bar{x}_j^i. \qquad (IV.2.7)$$

Differentiating this with respect to $\bar{p}$ yields

$$\bar{p}\bar{x}_{1\bar{p}}^i + \sum_{j=2}^{\ell} q_j \bar{x}_{j\bar{p}}^i = -\bar{x}_1^i \quad \forall i = n_2+1,\ldots,n \qquad (IV.2.8)$$

Substituting (IV.2.4), (IV.2.6) and (IV.2.8) into (IV.2.2) gives us

$$\frac{\partial W}{\partial \bar{p}} = - \sum_{i=1}^{n} \beta^i \min(\bar{x}_1^i, D)$$

where $\beta^i = \frac{\partial W}{\partial u^i} \lambda^i$ is the social marginal utility of income to consumer i.

Let us now look at the changes in welfare when the ration quota D is changed slightly

$$\frac{\partial W}{\partial D} = \sum_{i=1}^{n} \sum_{j=1}^{\ell} \frac{\partial W}{\partial u^i} u^{ij} \bar{x}_{jD}^i .$$

Differentiating the budget constraints (IV.2.3), (IV.2.5) and (IV.2.7) with respect to D, we have

$$\sum_{j=1}^{\ell} q_j \bar{x}_{jD}^i = q_1 - \bar{p} \qquad \forall i = 1,\ldots,n_1$$

$$q_{\theta^i} \bar{x}_{1D}^i + \sum_{j=2}^{\ell} q_j \bar{x}_{jD}^i = \theta^i (q_1 - \bar{p}) \quad \forall i = n_1+1,\ldots,n_2$$

and $\bar{p}\, \bar{\bar{x}}_{1D}^i + \sum_{j=2}^{\ell} q_j \bar{x}_{jD}^i = 0 \quad \forall i = n_2+1,\ldots,n.$

Hence, we can write

$$\frac{\partial W}{\partial D} = \sum_{i=1}^{n_1} \beta^i (q_1 - \bar{p}) + \sum_{i=n_1+1}^{n_2} \beta^i \theta^i (q_1 - \bar{p})$$

$$= (q_1 - \bar{p}) \sum_{i=1}^{n_2} \beta^i \theta^i \quad \text{with } \theta^i = 1 \quad \forall\, i = 1,\ldots,n_1 .$$

Since we want to find simultaneous changes in $\bar{p}$ and D which increase welfare with the same or less subsidy, we have to see their effects on subsidy also.

In equilibrium the total subsidy* availed by consumers is

$$S = (q_1 - \bar{p}) \; (n_2 D + \sum_{n_2+1}^{n} \bar{\bar{x}}_1^i)$$

Then a marginal change in $\bar{p}$ induces S to change by

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* For ease of exposition we work here with the gross subsidy on the scheme. However, for empirical analysis we shall modify our formulae suitably by considering the subsidy net of tax revenue.

$$\frac{\partial S}{\partial \bar{p}} = -(n_2 D + \sum_{n_2+1}^{n} \bar{x}_1^i) + (q_1 - \bar{p}) \sum_{n_1+1}^{n} \bar{x}_{1\bar{p}}^i$$

$$= - \sum_{1}^{n} \min(\bar{x}_1^i, D) + (q_1 - \bar{p}) \sum_{n_1+1}^{n} \bar{x}_{1\bar{p}}^i .$$

If we want to raise an extra rupee of revenue by changing $\bar{p}$, i.e. if $\Delta S = \frac{\partial S}{\partial \bar{p}} \Delta \bar{p} = -1$, the controlled price must change by $\Delta \bar{p} = -(\partial S/\partial \bar{p})^{-1}$. Thus, the fall in social welfare from an increase in ration price to make a rupee of revenue is

$$\lambda_{\bar{p}} = - \frac{\partial W}{\partial \bar{p}} \Delta \bar{p}$$

$$= \frac{\sum_{1}^{n} \beta^i \min(\bar{x}_1^i, D)}{\sum_{1}^{n} \min(\bar{x}_1^i, D) - (q_1 - \bar{p}) \sum_{n_1+1}^{n} \bar{x}_{1\bar{p}}^i}$$

Similarly, we have to calculate the fall in welfare due to a fall in the ration quota. For this purpose, first note that

$$\frac{\partial S}{\partial D} = (q_1 - \bar{p}) \; (n_2 + \sum_{n_2+1}^{n} \bar{x}_{1D}^i)$$

The government can save one unit of subsidy if the per head quota changes by

$$\Delta D = - (\partial S/\partial D)^{-1}.$$

The loss in welfare due to a fall in quota to gain one rupee of revenue is then given by

$$\lambda_D = - \frac{\partial W}{\partial D} \Delta D$$

$$= \frac{\sum_{1}^{n_2} \beta^i \theta^i}{n_2 + \sum_{n_2+1}^{n} \bar{x}_{1D}^i}$$

To simplify matters, we shall make use of the identity (II.2.3) of Chapter II,

$$\bar{x}^i(\bar{p}, D, q_1, \ldots, q_\ell; \sum_{j=1}^{\ell} q_j e_j^i) \equiv x^i(q_{\theta^i}, q_2, \ldots, q_\ell; \sum_{j=1}^{\ell} q_j e_j^i + \theta^i(q_1 - \bar{p})D),$$
$$0 \leq \theta^i \leq 1$$

where $\bar{x}^i$ and x^i are demands under dual pricing and uniform pricing respectively. Then, clearly, for $i = n_2+1, \ldots, n$, i.e., for $\theta^i = 0$, we have

$$\bar{x}_D^i = x_D^i = 0.$$

Hence, we may write

$$\lambda_D = \frac{\sum_1^{n_2} \beta^i \theta^i}{n_2}$$

If $\lambda_{\bar{p}} > \lambda_D$, we can increase welfare by decreasing D sufficiently to raise one rupee of revenue and then spend this rupee again as subsidy to reduce $\bar{p}$ appropriately.* And vice-versa. This happens because if $\lambda_{\bar{p}} > \lambda_D$, then a decrease in D affects only the "well-to-do" ($\bar{x}_1^i \geq D$) but a decrease in $\bar{p}$ benefits all so that the loss in welfare due to a reduction in D is more than compensated when $\bar{p}$ is subsidized accordingly.

Clearly, this is Pareto improving in the case of a single consumer as can be seen diagrammatically.

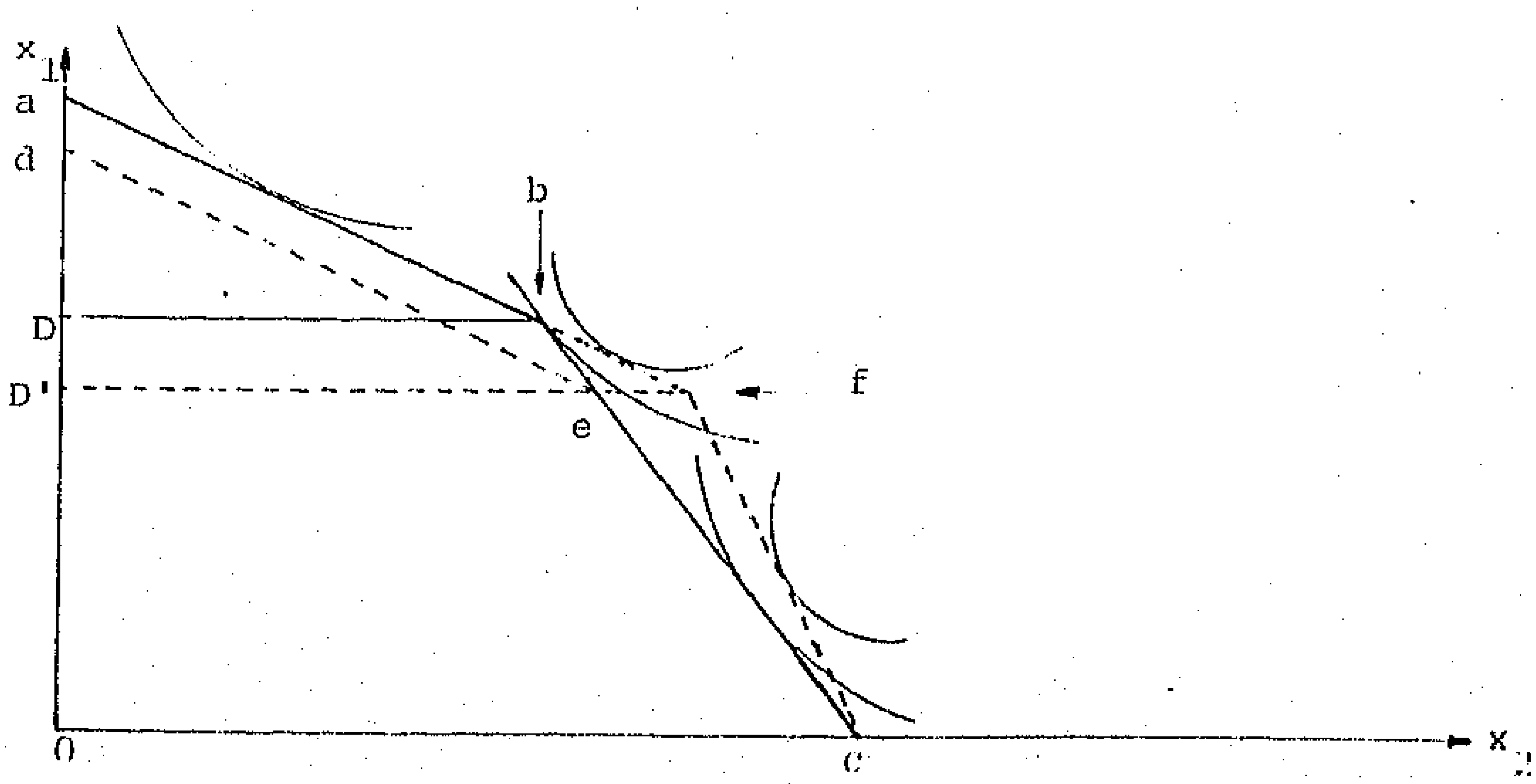

FIGURE IV.1

Given the budget contraint abc, the utility maximizing consumer will be either

* We shall show in Chapter VI that this analysis has a general equilibrium justification.

on the segment ab or the segment bc or the point b. When the ration quota is reduced from D to D' to gain a rupee of revenue (∿ ad), the budget constraint becomes dec affecting the consumer adversely only if its consumption bundle lies on the segment ab. But, with the quota fixed at D', when this one rupee is spent to subsidise the ration price*, the final budget constraint comes afc. Clearly, a move from abc to afc is a Pareto improvement.

Thus, we see that whenever $\lambda_{\bar{p}} \neq \lambda_D$, there are possible directions of welfare improving reforms. Since we want to improve welfare without loss of revenue, we can find a convex cone of directions as the intersection of the two half spaces, $dW > 0$ and $dS \leq 0$, where

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* Another way of verifying that the government's net gain from this excercise is zero is to consider the following figure

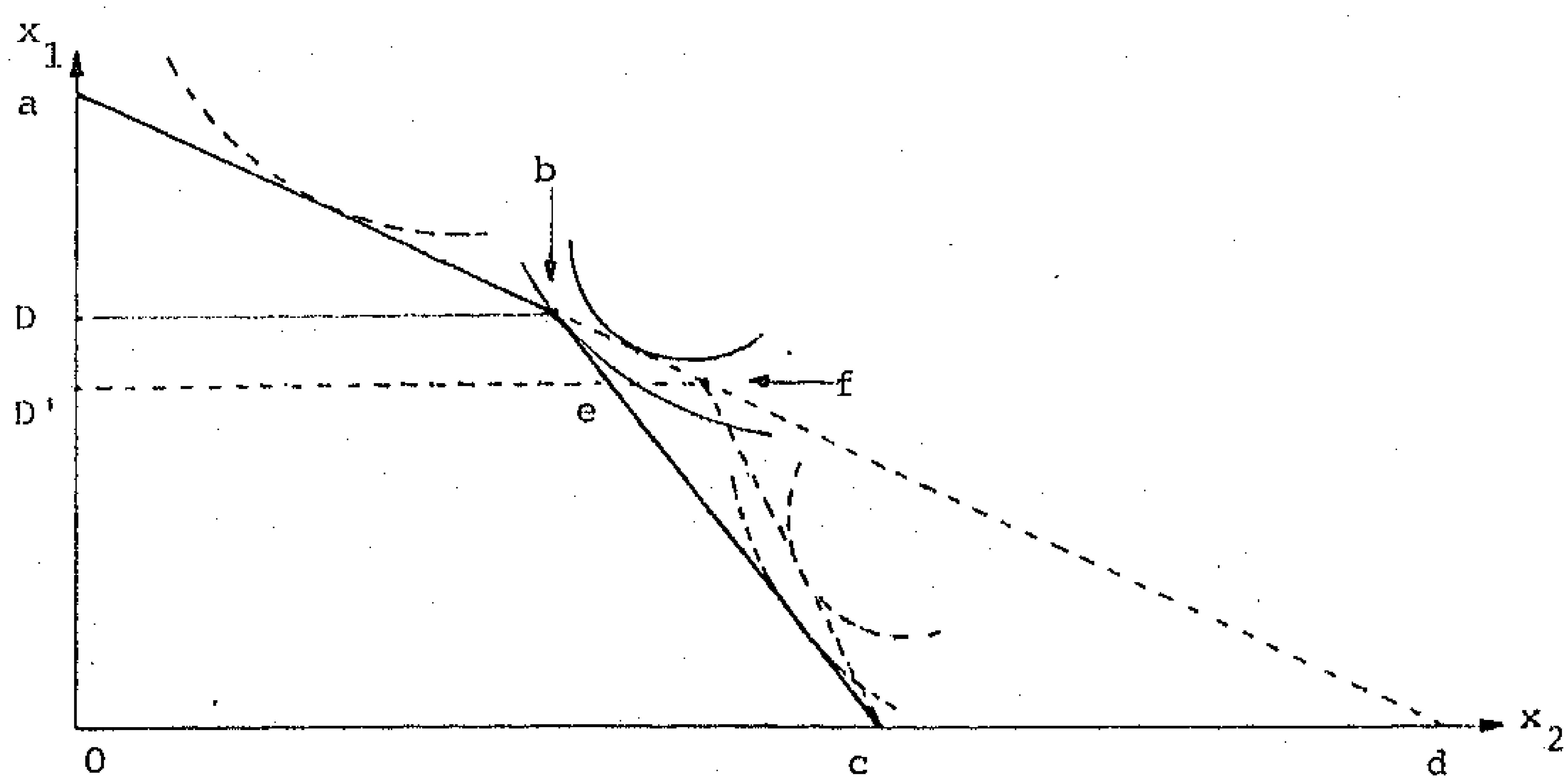

FIGURE IV.2

With the budget constraint abc, the subsidy provided is given by the quota D times the difference between the prices given by bd and bc. This is shown by cd in the figure. After the government's exercise, the new budget line becomes afc and the subsidy is now calculated using the prices given by fd and fc and the new quota D'. This again amounts to cd in Figure IV.2.

$$dW = \frac{\partial W}{\partial \bar{p}} \cdot d\bar{p} + \frac{\partial W}{\partial D} \cdot dD$$

and

$$dS = \frac{\partial S}{\partial \bar{p}} \cdot d\bar{p} + \frac{\partial S}{\partial D} .dD.$$

Having seen the implications of the inequality between $\lambda_{\bar{p}}$ and λ_D, a question that immediately arises is : Can one compare the two welfare losses under various value judgements?

To answer this question, we start with an extreme case. Suppose that $\beta^i = 0 \;\; \forall\, i = 1,\ldots,n_2$, and $\beta^i = 1 \;\forall i = n_2+1,\ldots,n$. That is, we define the 'rich' to be those who are able to buy enough of the essential (rationed) good ($\bar{x}_1^i \geq D$) and give them a weight of 'zero' each, whereas we give a constant weight of 'one' each to the 'poor'. Also, assuming that the demand curve for good 1 is downward sloping for $i = n_1+1,\ldots,n$, with at least one of them consuming some positive amount of this good, we get

$$\lambda_D = 0 < \frac{\sum_{i=n_2+1}^{n} \bar{x}_1^i}{\sum_{i=n_2+1}^{n} \bar{x}_1^i + n_2 D - (q_1-\bar{p}) \sum_{n_1+1}^{n} \bar{x}_{1\bar{p}}^i} = \lambda_{\bar{p}} < 1$$

Thus, welfare would go up if we reduce both the quota and the controlled price in the manner described earlier without affecting the total subsidy involved. This is only to be expected since the poor are net gainers and the rich are not assumed to contribute to the social welfare. So, let us consider an alternative welfare function given by $\beta^i = \beta \;\forall i$. Then we have

$$\lambda_D \leq \beta$$

and

$$\lambda_{\bar{p}} = \beta \frac{\sum_{1}^{n} \min(\bar{x}_1^i, D)}{[\sum_{1}^{n} \min(\bar{x}_1^i, D) - (q_1-\bar{p}) \sum_{n_1+1}^{n} \bar{x}_{1\bar{p}}^i]} < \beta$$

assuming again that $\bar{x}^i_{1\bar{p}} \leq 0 \; \forall i = n_1+1,\ldots,n$, with strict inequality holding for at least one i. In this case we are not able to compare the two losses unless we assume that $\theta^i = 1 \; \forall i = n_1+1,\ldots,n_2$, in which $\lambda_D = \beta > \lambda_{\bar{p}}$. That is, welfare can be improved with the same expenditure by increasing both the ration price and the quota appropriately. Notice that this means that the poor are net losers since they are not responsive to small changes in the quota and they also subsidize the rich in order to provide for a unit of revenue to the government. This can be explained if we assume that the marginal utility of income falls as income rises, then an equal weight for all means that a one unit increase in the utility of the rich contributes more to welfare than that of the poor*.

From the fact that it is possible to find welfare improving reforms *from any DPE one may argue* that it is better to reach an optimum by eliminating the dual pricing schemes and using lumpsum transfers**. However, this is not feasible due to limited possibilities for lumpsum transfers on a large scale. It is precisely for this reason that in many countries less efficient schemes like subsidizing input prices or distribution of essential commodities at subsidized prices are resorted to.

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* Recall that $\beta^i = \frac{\partial W}{\partial u^i} \cdot \frac{\partial u^i}{\partial m}$ where $m = \sum_{j=1}^{\ell} q_j e^i_j$.

** We have seen that given a DPE with a ration price-quota pair, it is preferable to reduce both in an appropriate manner given a "proper" welfare function. In the limit, it would mean that the best way of operating dual pricing is to supply some quota $\bar{\bar{D}}$ per head with the ration price $\bar{p}$ being zero as can be seen in the figure below. (This is to be expected because lumpsum transfers are known to lead to Pareto optimality). In this case, the government's net subsidy would equal its expenditure on purchasing the good for rationing which could as well have been distributed as money transfers. So, the choice is between distributing cash vis-a-vis distributing goods.

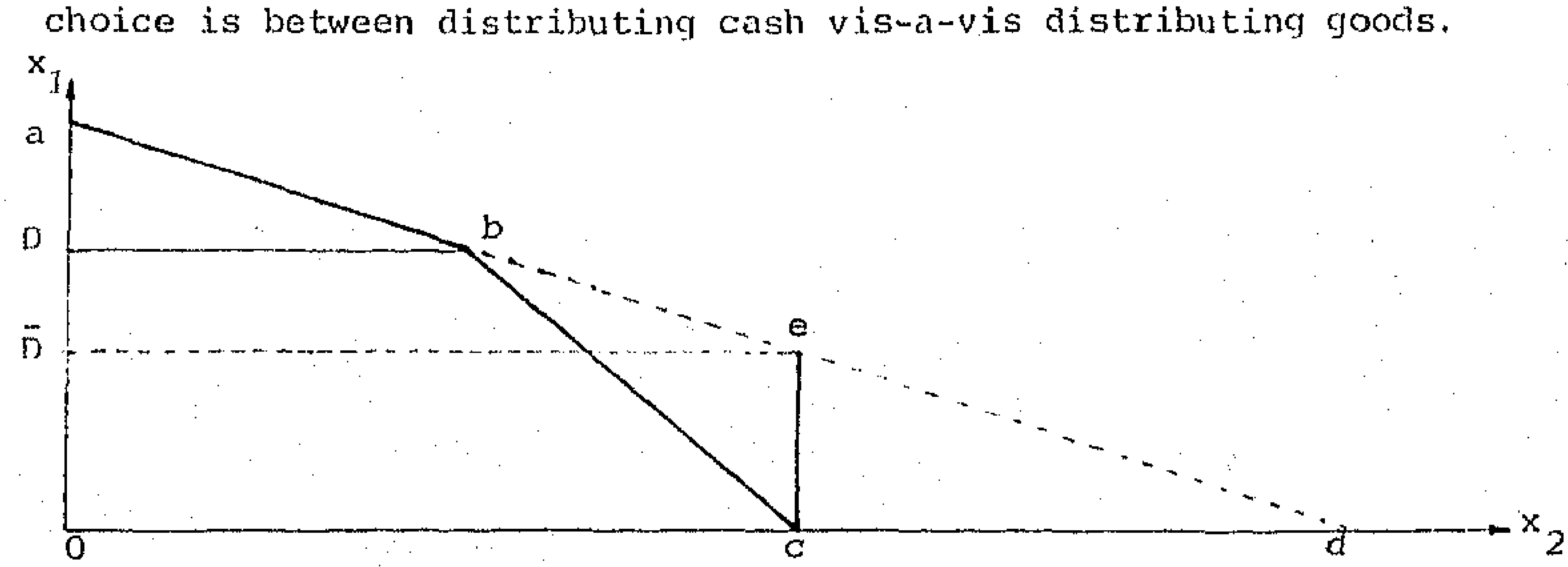

Figure IV.3

IV.3. CALCULATING THE WELFARE WEIGHTS IN A 'TWO-CLASS ECONOMY'

Instead of specifying the welfare weights, one alternative would be to obtain them in order to benefit the poor. For this we consider a "two-class economy"* with two types of people, namely, with $\bar{x}_1^i = \bar{x}_1^1 > D$ and a welfare weight of β_1 $\forall i = 1,\ldots,n_1$ and $\bar{x}_1^i = \bar{x}_1^2 < D$ with a weight of β_2 $\forall i = n_1+1,\ldots,n$. Also, let $n_2 = n-n_1$. Then we may rewrite

$$\frac{\partial W}{\partial D} = (q_1-\bar{p})n_1\beta_1$$

$$\frac{\partial W}{\partial \bar{p}} = -\sum_1^n \beta^i \min(\bar{x}_1^i, D) = -(n_1\beta_1 D + n_2\beta_2\bar{x}_1^2)$$

$$S = (q_1-\bar{p})(n_1 D + n_2\bar{x}_1^2)$$

$$\frac{\partial S}{\partial D} = (q_1-\bar{p})(n_1+n_2\bar{x}_{1D}^2) = (q_1-\bar{p})n_1$$

$$\frac{\partial S}{\partial \bar{p}} = (q_1-\bar{p})n_2\bar{x}_{1\bar{p}}^2 - (n_1 D+n_2\bar{x}_1^2)$$

Consequently,

$$\lambda_{\bar{p}} = \frac{n_1\beta_1 D + n_2\beta_2\bar{x}_1^2}{n_1 D+n_2\bar{x}_1^2-(q_1-\bar{p})n_2\bar{x}_{1\bar{p}}^2}$$

$$\lambda_D = \frac{n_1\beta_1}{n_1} = \beta_1.$$

Let us denote by 'A' the expression

$$n_1 D + n_2\bar{x}_1^2 - (q_1-\bar{p})n_2\bar{x}_{1\bar{p}}^2 > 0$$

if $\bar{x}_{1\bar{p}}^2 \leq 0$. Then, we can write

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* This was suggested by Professor Debraj Ray during one of the discussions.

$$\lambda_{\bar{p}} - \lambda_D = \frac{n_2\bar{x}_1^2(\beta_2-\beta_1)+(q_1-\bar{p})n_2\beta_1\bar{x}_{1\bar{p}}^2}{A}$$

$$= \frac{n_2}{A}\,[\bar{x}_1^2(\beta_2-\beta_1)+(q_1-\bar{p})\beta_1\bar{x}_{1\bar{p}}^2].$$

Hence, if either the controlled price is close to the free market price or if $\bar{x}_{1\bar{p}}^2$ is small, then $\lambda_{\bar{p}} > \lambda_D \Longrightarrow \beta_2 > \beta_1$.

IV.4. RESULTS

We present in this section some results making use of the 28th round National Sample Survey data for the Indian economy for 1973-74. Assuming that consumers are utility maximizers and given that they consume positive amounts of the goods concerned, it is easy to see that the expressions we have obtained for welfare losses in section 2 still hold good.

Since rice and wheat are two major items of rationing in India, we consider their ration quotas and prices besides various commodity taxes as the instruments to work with and use our previous results in a form modified to take care of two rationed goods together with subsidy net of tax revenue. To obtain the ration quota per capita per month we use the data on public distribution of foodgrains and the population covered by fair prices shops/ration shops as given in Bulletin on Food Statistics, (BFS) 1975. (By good 1 we shall mean rice and by good 2 wheat). The issue prices ($\bar{p}_1$ and $\bar{p}_2$) and free market prices (q_1 and q_2) were also obtained from BFS, 1975.

For urban India we consider four expenditure groups and for rural areas, eight expenditure groups. These groups are formed according to expenditure per month. The NSS data gives the consumption of each item in value and quantity terms on a per capita per monthly basis. For each group, the NSS also provides for the size (number of persons) of rice eating population and wheat-eating population. Aggregate consumption of rice and wheat for each group is then obtained by multiplying the respective per capita consumption and the size of the population consuming it. Aggregate quota for each group for rice and wheat is also obtained in the same way. Naturally, the quotas for different groups differ. Here we have assumed that the quota per person and issue and market prices are the same in both the urban as well as the rural areas. These figures are presented in Tables IV.1 to IV.3.

Table IV.1

	Ration quota per capita (D) per month (Kg)	Issue Price ($\bar{p}$) (Rs./Kg.)	Market price (q) (Rs./Kg)
Rice	0.618 (D_1)	1.438 ($\bar{p}_1$)	1.84 (q_1)
Wheat	1.321 (D_2)	0.870 ($\bar{p}_2$)	1.03 (q_2)

Table IV.2
All-India (Urban)

Expenditure groupss (mean expenditure)		0-265 (132.5)	265-517 (391)	517-724 (620.5)	724-969 (846.5)
Per capita consumption (Kg.)	Rice	6.32	6.99	6.28	2.47
	Wheat	5.78	8.30	10.00	6.72
Size (no. of persons)	Rice	29020.00	336.00	6.00	10.00
	Wheat	5543.00	304.00	6.00	18.00
Aggregate consumption (Kg.)	Rice	183406.40	2348.60	38.28	44.46
	Wheat	32038.50	2523.20	60.00	120.96
Aggregate quota (Kg.)	Rice	17934.36	207.65	3.71	11.12
	Wheat	7322.30	401.58	7.93	23.78

Table IV.3
All-India (Rural)

Expdr.group (mean Expdr.)		0–8 (4)	8–11 (9.5)	11–13 (12)	13–21 (17)	21–34 (27.5)	34–75 (54.5)	75–100 (112.5)	150–250 (200)
Per capita consumption (Kg.)	R	1.14	2.65	3.17	4.91	6.83	9.24	10.14	10.60
	W	1.47	1.58	1.25	2.35	4.40	6.79	9.85	12.60
Size	R	22.00	61.00	89.00	2110.00	12475.00	40286.00	10131.00	995.00
	W	15.00	19.00	40.00	848.00	6617.00	25403.00	8141.00	961.00
Aggregate consumption (Kg.)	R	25.08	161.65	282.13	10360.10	85204.25	372242.64	102728.34	10547.00
	W	22.05	30.02	50.00	1992.80	29114.80	17286.37	80188.85	12108.60
Aggregate quota (Kg.)	R	13.60	37.70	55.00	1303.98	7709.55	24896.75	6260.96	614.91
	W	19.82	25.10	52.84	1120.21	8741.06	33557.36	10754.26	1269.48

Before proceeding further, let us see how our results get altered when we introduce dual pricing in goods, say, 1 and 2. First note that for none of the groups $\bar{X}_1^i = D_1^i$ or $\bar{X}_2^i = D_2^i$ where $\bar{X}^i$ and D^i denote the group consumption and quota respectively. Recall that in section IV.2 we had defined individuals as rich or poor depending on whether they consumed more or less of the rationed good than the quota. However, using available data, it is not possible to say whether a person consumed more or less than the specified quota of a good because only *average consumption for each group* is available. Hence, we shall say that a consumer is rich compared to another if it falls in a group with a higher mean expenditure than the latter. The expenditure groups arranged in an ascending order of expenditure are presented in Table IV.4.

Table IV.4

Expenditure group	1	2	3	4	5	6	7	8	9	10	11	12
Mean expenditure (in Rs.per month)	4.0	9.5	12.0	17.0	27.5	54.5	112.5	132.5	200.0	391.0	620.5	846.5

Note that for all the groups i, $\bar{x}_1^i > D_1^i$ where $\bar{X}^i$ and D^i are consumption and quota for group i=1,...,12. Also, except for group 3, (11-13) $\bar{x}_2^i > D_2^i$, i.e. i = 1,....,12, i ≠ 3.

Now, getting back to re-doing our exercise, let us group the consumers in to four groups, say, A_1, A_2, A_3 and A_4 with the property that on an average $\bar{x}_1^i > D_1$, $\bar{x}_2^i > D_2$ $\forall i \in A_1$, $\bar{x}_1^i > D_1$, $\bar{x}_2^i < D_2$ $\forall i \in A_2$; $\bar{x}_1^i < D_1$, $\bar{x}_2^i > D_2$ $\forall i \in A_3$ and $\bar{x}_1^i < D_1$, $\bar{x}_2^i < D_2$ $\forall i \in A_4$. Then the first order conditions for utility maximization imply that

$$\frac{u^{ij}}{q_j} = \frac{u^{ik}}{q_k} \ \forall i \in A_1,\ \forall j,k = 1,\dots,\ell,$$

$$\frac{u^{i2}}{\bar{p}_2} = \frac{u^{ik}}{q_k} \ \forall i \in A_2 \ \forall\ k \neq 2. \tag{IV.4.1}$$

$$\frac{u^{i1}}{\bar{p}_1} = \frac{u^{ik}}{q_k} \ \forall i \in A_3 \ \forall\ k \neq 1$$

$$\frac{u^{i1}}{\bar{p}_1} = \frac{u^{i2}}{\bar{p}_2} = \frac{u^{ik}}{q_k} \ \forall i \in A_4 \ \forall\ k \neq 1,2.$$

At the maximum utility, the budget constraints are the following identities

$$\bar{p}_1D_1+\bar{p}_2D_2+q_1(\bar{x}_1^i-D_1)+q_2(\bar{x}_2^i-D_2)+\sum_{j=3}^{\ell} q_j\bar{x}_j^i = \sum_{j=1}^{\ell} p_je_j^i \ \forall i \in A_1$$

$$\bar{p}_1D_1+q_1(\bar{x}_1^i-D_1)+\bar{p}_2\bar{x}_2^i+\sum_{j=3}^{\ell} q_j\bar{x}_j^i = \sum_{j=1}^{\ell} p_je_j^i \ \forall i \in A_2 \tag{IV.4.2}$$

$$\bar{p}_1\bar{x}_1^i+\bar{p}_2D_2+q_2(\bar{x}_2^i-D_2) + \sum_{3}^{\ell} q_j\bar{x}_j^i = \sum_{1}^{\ell} p_je_j^i \ \forall i \in A_3$$

$$\bar{p}_1\bar{x}_1^i+\bar{p}_2\bar{x}_2^i + \sum_{3}^{\ell} q_j\bar{x}_j^i = \sum_{1}^{\ell} p_je_j^i \ \forall i \in A_4.$$

Differentiating the set of equations (IV.4.2) with respect to $\bar{p}_1$, $\bar{p}_2$, D_1, D_2, t_1, t_2, (t_k, k = 3,...,ℓ) we get the following sets of equalities

$$\sum_{1}^{\ell} q_j \bar{x}^i_{j\bar{p}_1} = -D_1 \quad \forall i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2\bar{p}_1} + \sum_{j\neq 2} q_j \bar{x}^i_{j\bar{p}_1} = -D_1 \quad \forall i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1\bar{p}_1} + \sum_{j\neq 1} q_j \bar{x}^i_{j\bar{p}_1} = -\bar{x}^i_1 \quad \forall i \in A_3 \qquad \text{(IV.4.3)}$$

$$\bar{p}_1 \bar{x}^i_{1\bar{p}_1} + \bar{p}_2 \bar{x}^i_{2\bar{p}_1} + \sum_{j\neq 1,2} q_j \bar{x}^i_{j\bar{p}_1} = -\bar{x}^i_1 \quad \forall i \in A_4$$

$$\sum_{1}^{\ell} q_j \bar{x}^i_{j\bar{p}_2} = -D_2 \quad \forall i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2\bar{p}_2} + \sum_{j\neq 2} q_j \bar{x}^i_{j\bar{p}_2} = -\bar{x}^i_2 \quad \forall i \in A_2 \qquad \text{(IV.4.4)}$$

$$\bar{p}_1 \bar{x}^i_{1\bar{p}_2} + \sum_{j\neq 1} q_j \bar{x}^i_{j\bar{p}_2} = -D_2 \quad \forall i \in A_3$$

$$\bar{p}_1 \bar{x}^i_{1\bar{p}_2} + \bar{p}_2 \bar{x}^i_{2\bar{p}_2} + \sum_{j\neq 1,2} q_j \bar{x}^i_{j\bar{p}_2} = -\bar{x}^i_2 \quad \forall i \in A_4$$

$$\sum_{j=1}^{\ell} q_j \bar{x}^i_{jD_1} = q_1 - \bar{p}_1 \quad \forall i \in A_1$$

$$\text{(IV.4.5)}$$

$$\bar{p}_2 \bar{x}^i_{2D_1} + \sum_{j\neq 2} q_j \bar{x}^i_{jD_1} = q_1 - \bar{p}_1 \quad \forall i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1D_1} + \sum_{j\neq 1} q_j \bar{x}^i_{jD_1} = 0 \quad \forall i \in A_3$$

$$\bar{p}_1 \bar{x}^i_{1D_1} + \bar{p}_2 \bar{x}^i_{2D_1} + \sum_{j\neq 1,2} q_j \bar{x}^i_{jD_1} = 0 \quad \forall i \in A_4$$

$$\sum_{1}^{\ell} q_j \bar{x}^i_{jD_2} = q_2 - \bar{p}_2 \quad \forall i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2D_2} + \sum_{j\neq 2} q_j \bar{x}^i_{jD_2} = 0 \quad \forall i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1D_2} + \sum_{j\neq 1} q_j \bar{x}^i_{jD_2} = q_2 - \bar{p}_2 \quad \forall i \in A_3 \qquad \text{(IV.4.6)}$$

$$\bar{p}_1 \bar{x}^i_{1D_2} + \bar{p}_2 \bar{x}^i_{2D_2} + \sum_{j\neq 1,2} q_j \bar{x}^i_{jD_2} = 0 \quad \forall i \in A_4$$

$$\sum_{1}^{\ell} q_j \bar{x}^i_{jt_1} = D_1 - \bar{x}^i_1 \quad \forall\, i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2t_1} + \sum_{j\neq 2} q_j \bar{x}^i_{jt_1} = D - \bar{x}^i_1 \quad \forall\, i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1t_1} + \sum_{j\neq 1} q_j \bar{x}^i_{jt_1} = 0 \quad \forall\, i \in A_3 \qquad \text{(IV.4.7)}$$

$$\bar{p}_1 \bar{x}^i_{1t_1} + \bar{p}_2 \bar{x}^i_{2t_1} + \sum_{3}^{\ell} q_j \bar{x}^i_{jt_1} = 0 \quad \forall\, i \in A_4$$

$$\sum_{1}^{\ell} q_j \bar{x}^i_{jt_2} = D_2 - \bar{x}^i_2 \quad \forall\, i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2t_2} + \sum_{j\neq 2} q_j \bar{x}^i_{jt_2} = 0 \quad \forall\, i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1t_2} + \sum_{j\neq 1} q_j \bar{x}^i_{jt_2} = D_2 - \bar{x}^i_2 \quad \forall\, i \in A_3 \qquad \text{(IV.4.8)}$$

$$\bar{p}_1 \bar{x}^i_{1t_2} + \bar{p}_2 \bar{x}^i_{2t_2} + \sum_{3}^{\ell} q_j \bar{x}^i_{jt_2} = 0 \quad \forall\, i \in A_4$$

and $\forall\, k = 3,\ldots,\ell$,

$$\sum_{1}^{\ell} q_j \bar{x}^i_{jt_k} = -\bar{x}^i_k \quad \forall\, i \in A_1$$

$$\bar{p}_2 \bar{x}^i_{2t_k} + \sum_{j\neq 2} q_j \bar{x}^i_{jt_k} = -\bar{x}^i_k \quad \forall\, i \in A_2$$

$$\bar{p}_1 \bar{x}^i_{1t_k} + \sum_{j\neq 1} q_j \bar{x}^i_{jt_k} = -\bar{x}^i_k \quad \forall\, i \in A_3 \qquad \text{(IV.4.9)}$$

$$\bar{p}_1 \bar{x}^i_{1t_k} + \bar{p}_2 \bar{x}^i_{2t_k} + \sum_{3}^{\ell} q_j \bar{x}^i_{jt_k} = -\bar{x}^i_k \quad \forall\, i \in A_4$$

Again assuming a welfare function of the type $W = W(u^1,\ldots,u^n)$, one can write the partial derivatives of $W(.)$ with respect to a parameter α as follows.

$$\frac{\partial W}{\partial \alpha} = \sum_{i=1} \sum_{j=1}^{\ell} \frac{\partial W}{\partial u^i} u^{ij} \bar{x}^i_{j\alpha} = \sum_{i\varepsilon A_1} \beta^i \sum_{j=1}^{\ell} q_j \bar{x}^i_{j\alpha} + \sum_{i\varepsilon A_2} \beta^i (\bar{p}_2 \bar{x}^i_{2\alpha} + \sum_{j\neq 2} q_j \bar{x}^i_{j\alpha})$$

$$+ \sum_{i\varepsilon A_3} \beta^i (\bar{p}_1 \bar{x}^i_{1\alpha} + \sum_{j\neq 1} q_j \bar{x}^i_{j\alpha}) + \sum_{i\varepsilon A_4} \beta^i (\bar{p}_1 \bar{x}^i_{1\alpha} + \bar{p}_2 \bar{x}^i_{2\alpha} + \sum_{j\neq 1,2} q_j \bar{x}^i_{j\alpha}).$$

using equations (IV.4.1) and defining β^i as before. Then, making use of equations (IV.4.3) to (IV.4.9) and putting $\alpha = \bar{p}_1, \bar{p}_2, D_1, D_2, t_1, t_2, (t_k; k = 3,\ldots,\ell)$ in succession, one obtains

$$\frac{\partial W}{\partial \bar{p}_1} = -[\sum_{i\varepsilon A_1} \beta^i D_1 + \sum_{i\varepsilon A_2} \beta^i D_1 + \sum_{i\varepsilon A_3} \beta^i \bar{x}^i_1 + \sum_{i\varepsilon A_4} \beta^i \bar{x}^i_1] = -\sum_{i=1}^{n} \beta^i \min(D_1, \bar{x}^i_1)$$

since $D_1 < \bar{x}^i_1 \ \forall\ i \ \varepsilon\ A_1 \cup A_2$ and $D_1 > \bar{x}^i_1 \ \forall\ i\ \varepsilon\ A_3 \cup A_4$

$$\frac{\partial W}{\partial \bar{p}_2} = -\sum_{i=1}^{n} \beta^i \min(D_2, \bar{x}^i_2)$$

$$\frac{\partial W}{\partial D_1} = \sum_{i\varepsilon A_1} \beta^i (q_1 - \bar{p}_1) + \sum_{i\varepsilon A_2} \beta^i (q_1 - \bar{p}_1) = (q_1 - \bar{p}_1) \sum_{i\varepsilon A_1 \cup A_2} \beta^i$$

$$\frac{\partial W}{\partial D_2} = (q_2 - \bar{p}_2) \sum_{i\varepsilon A_1 \cup A_3} \beta^i$$

$$\frac{\partial W}{\partial t_1} = -\sum_{i\varepsilon A_1} \beta^i (\bar{x}^i_1 - D_1) - \sum_{i\varepsilon A_2} \beta^i (\bar{x}^i_1 - D_1) = -\sum_i \beta^i \max(\bar{x}^i_1 - D_1, 0)$$

$$\frac{\partial W}{\partial t_2} = -\sum_i \beta^i \max(\bar{x}^i_2 - D_2, 0)$$

$$\frac{\partial W}{\partial t_k} = -\sum_i \beta^i \bar{x}^i_k \ \forall\ k = 3,\ldots,\ell.$$

Since we have only the expenditure groupwise data, we shall allot the same β^i to all the persons in a particular expenditure group i.e., we shall have to specify 12 β^i's for the 12 groups. As we have seen earlier, in our case the sets A_3 and A_4 are empty, for there is no group with $\bar{x}^i_1 < D^i_1$. Also A_1 includes all the groups except for the expenditure group (11-13), i.e., group 3; and A_2 is same as group 3. Therefore, we shall use the following formulae:

$$\frac{\partial W}{\partial \bar{p}_1} = - \sum_{i=1}^{12} \beta^i D_1^i$$

$$\frac{\partial W}{\partial \bar{p}_2} = - \left(\sum_{i \neq 3} \beta^i D_2^i + \beta^3 \bar{x}_2^3 \right)$$

$$\frac{\partial W}{\partial D_1} = (q_1 - \bar{p}_1) \sum_{i=1}^{12} \beta^i$$

$$\frac{\partial W}{\partial D_2} = (q_2 - \bar{p}_2) \sum_{i \neq 3} \beta^i .$$

$$\frac{\partial W}{\partial t_1} = - \sum_{i=1}^{12} \beta^i (\bar{x}_1^i - D_1^i)$$

$$\frac{\partial W}{\partial t_2} = - \sum_{i \neq 3} \beta^i (\bar{x}_2^i - D_2^i)$$

$$\frac{\partial W}{\partial t_k} = - \sum_{i=1}^{12} \beta^i \, \bar{x}_k^i \quad \forall \; k = 3, \ldots, \ell .$$

where D_1 and D_2 represent per capita per month quotas of rice and wheat respectively.

Next, consider the net subsidy involved in the operation.

$$S = (p_1 - \bar{p}_1) \left(\sum_{i \in A_1 \cup A_2} D_1 + \sum_{i \in A_3 \cup A_4} \bar{x}_1^i \right) + (p_2 - \bar{p}_2) \left(\sum_{A_1 \cup A_3} D_2 + \sum_{A_2 \cup A_4} \bar{x}_2^i \right)$$

$$- t_1 \sum_{A_1 \cup A_2} (\bar{x}_1^i - D_1) - t_2 \sum_{A_1 \cup A_3} (\bar{x}_2^i - D_2) - \sum_{j=3}^{\ell} \sum_i t_j \, \bar{x}_j^i$$

Differentiating S partially with respect to the parameters one obtains

$$\frac{\partial S}{\partial \bar{p}_1} = - \left[\sum_{A_1 \cup A_2} D_1 + \sum_{A_3 \cup A_4} \bar{x}_1^i \right] + (p_1 - \bar{p}_1) \sum_{A_3 \cup A_4} \bar{x}_{1\bar{p}_1}^i + (p_2 - \bar{p}_2) \sum_{A_2 \cup A_4} \bar{x}_{2\bar{p}_1}^i$$

$$- t_1 \sum_{A_1 \cup A_2} \bar{x}_{1\bar{p}_1}^i - t_2 \sum_{A_1 \cup A_3} \bar{x}_{2\bar{p}_1}^i - \sum_{3}^{\ell} \sum_{1}^{n} t_j \, \bar{x}_{j\bar{p}_1}^i .$$

$$\frac{\partial S}{\partial \bar{p}_2} = -\left[\sum_{A_1 \cup A_3} D_2 + \sum_{A_2 \cup A_4} \bar{x}_2^i\right] + (p_1-\bar{p}_1)\sum_{A_3 \cup A_4} \bar{x}^i_{1\bar{p}_2} + (p_2-\bar{p}_2)\sum_{A_2 \cup A_4} \bar{x}^i_{2\bar{p}_2}$$

$$- t_1 \sum_{A_1 \cup A_2} \bar{x}^i_{1\bar{p}_2} - t_2 \sum_{A_1 \cup A_3} \bar{x}^i_{2\bar{p}_2} - \sum_3^{\ell} \sum_1^{n} t_j \bar{x}^i_{j\bar{p}_2}$$

$$\frac{\partial S}{\partial D_1} = (p_1-\bar{p}_1)\left[\#(A_1 \cup A_2) + \sum_{A_3 \cup A_4} \bar{x}^i_{1D_1}\right] + (p_2-\bar{p}_2)\sum_{A_2 \cup A_4} \bar{x}^i_{2D_1}$$

$$+ t_1\left[\#(A_1 \cup A_2) - \sum_{A_1 \cup A_2} \bar{x}^i_{1D_1}\right] - t_2 \sum_{A_1 \cup A_3} \bar{x}^i_{2D_1} - \sum_3^{\ell}\sum_1^{n} t_j \bar{x}^i_{jD_1}$$

$$\frac{\partial S}{\partial D_2} = (p_2-\bar{p}_2)\left[\#(A_1 \cup A_3) + \sum_{A_2 \cup A_4} \bar{x}^i_{2D_2}\right] + (p_1-\bar{p}_1)\sum_{A_3 \cup A_4} \bar{x}^i_{1D_2}$$

$$- t_1 \sum_{A_1 \cup A_2} \bar{x}^i_{1D_2} + t_2\left[\#(A_1 \cup A_3) - \sum_{A_1 \cup A_3} \bar{x}^i_{2D_2}\right] - \sum_3^{\ell}\sum_1 t_j \bar{x}^i_{jD_2}$$

$$\frac{\partial S}{\partial t_1} = (p_1-\bar{p}_1)\sum_{A_3 \cup A_4} \bar{x}^i_{1t_1} + (p_2-\bar{p}_2)\sum_{A_2 \cup A_4} \bar{x}^i_{2t_1} - \sum_{A_1 \cup A_2} (\bar{x}^i_1 - D_1)$$

$$- t_1 \sum_{A_1 \cup A_2} \bar{x}^i_{1t_1} - t_2 \sum_{A_1 \cup A_3} \bar{x}^i_{2t_1} - \sum_3^{\ell}\sum_1^{n} t_j \bar{x}^i_{jt_1}$$

$$\frac{\partial S}{\partial t_2} = (p_1-\bar{p}_1)\sum_{A_3 \cup A_4} \bar{x}^i_{1t_2} + (p_2-\bar{p}_2)\sum_{A_2 \cup A_4} \bar{x}^i_{2t_2} - \sum_{A_1 \cup A_3} (\bar{x}^i_2 - D_2)$$

$$- t_1 \sum_{A_1 \cup A_2} \bar{x}^i_{1t_2} - t_2 \sum_{A_1 \cup A_3} \bar{x}^i_{2t_2} - \sum_3^{\ell}\sum_1^{n} t_j \bar{x}^i_{jt_2}$$

$$\frac{\partial S}{\partial t_k} = (p_1-\bar{p}_1)\sum_{A_3 \cup A_4} \bar{x}^i_{1t_k} + (p_2-\bar{p}_2)\sum_{A_2 \cup A_4} \bar{x}^i_{2t_k} - t_1 \sum_{A_1 \cup A_2} \bar{x}^i_{1t_k} - t_2 \sum_{A_1 \cup A_3} \bar{x}^i_{2t_k}$$

$$- \sum_3^{\ell}\sum_1^{n} t_j \bar{x}^i_{jt_k} \quad \forall\ k = 3,\ldots,\ell.$$

In our data the number of rice-eaters is different from that of wheat-eaters in each expenditure group. Thus, we cannot use the set $\{A_i\}$ as we have defined because it is not possible in the given situation to specify, e.g., how many individuals consume more of rice and less of wheat than the quota. Therefore we shall take the set A_1UA_2 to represent those income groups for which $\bar{X}_1^i > D_1^i$ and A_3UA_4 consists of groups with $\bar{X}_1^i < D_1^i$ which, we have noticed before, is a null set. Similarly, A_1UA_3 represents the groups with $\bar{X}_2^i > D_2^i$ and A_2UA_4 represents those with $\bar{X}_2^i < D_2^i$. Then, clearly, A_1UA_2 covers all the twelve groups, A_3UA_4 is empty, A_1UA_3 consists of all the groups except for the third and A_2UA_4 is the same as group 3. Given this, we can rewrite the above equations as:

$$\frac{\partial S}{\partial \bar{p}_1} = -\sum_1^{12} D_1^i + (q_2-\bar{p}_2)\bar{X}^3_{2\bar{p}_1} - \sum_1^{12}\sum_1^{\ell} t_j\,\bar{X}^i_{j\bar{p}_1}$$

$$\frac{\partial S}{\partial \bar{p}_2} = -\left(\sum_{i\neq 3} D_2^i+\bar{X}_2^3\right) + (q_2-\bar{p}_2)\bar{X}^3_{2\bar{p}_2} - \sum_1^{12}\sum_1^{\ell} t_j\bar{X}^i_{j\bar{p}_2}$$

$$\frac{\partial S}{\partial D_1} = (q_1-\bar{p}_1)\ (\text{total number of rice-eaters}) + (q_2-\bar{p}_2)\bar{X}^3_{2D_1} - \sum_1^{12}\sum_1^{\ell} t_j\bar{X}^i_{jD_1}$$

$$\frac{\partial S}{\partial D_2} = (q_2-\bar{p}_2)[(\text{total number of wheat-eaters less those in group 3}) + \bar{X}^3_{2D_2}] - \sum_1^{12}\sum_1^{\ell} t_j\bar{X}^i_{jD_2}$$

$$\frac{\partial S}{\partial t_1} = (q_2-\bar{p}_2)\bar{X}^3_{2t_1} - \sum_1^{12}(\bar{X}_1^i-D_1^i) - \sum_1^{12}\sum_1^{\ell} t_j\bar{X}^i_{jt_1}$$

$$\frac{\partial S}{\partial t_2} = (q_2-\bar{p}_2)\bar{X}^3_{2t_2} - \sum_{i\neq 3}(\bar{X}_2^i - D_2^i) - \sum_1^{12}\sum_1^{\ell} t_j\bar{X}^i_{jt_2}$$

$$\frac{\partial S}{\partial t_k} = (q_2-\bar{p}_2)\bar{X}^3_{2t_k} - \sum_1^{12}\bar{X}_k^i - \sum_1^{12}\sum_1^{\ell} t_j\,\bar{X}^i_{jt_k},\quad \forall\, k = 3,\ldots,\ell.$$

Since $\bar{X}_2^3 < D_2^3$, we know that on an average any consumer in this group is consuming less than its quota of wheat. From a previous result we then know that this consumer will consume the same bundle if dual pricing is replaced by uniform pricing using the price vector $(\bar{p}_1, q_1, \ldots, q_\ell)$ without any compensations in income i.e. $\bar{X}^3_{j\bar{p}_2}$ can be obtained from the price elasticity of demand as

$$\bar{x}^3_{j\bar{p}_2} = e^3_{j\bar{p}_2} \frac{\bar{x}^3_j}{\bar{p}_2}$$

where $e_{2\bar{p}_2}$ is the own price elasticity of demand for wheat. However, since for all other groups, $\bar{x}^i_2 > D^i_2$, we can write $\bar{x}^i_{j\bar{p}_2}$ in terms of expenditure elasticity as $-D^i_2\, e^i_{jm}\, \bar{x}^i_j/m^i$ where m^i represents the value of expenditure under dual pricing In a similar fashion we may express $\bar{x}^i_{j\bar{p}_1}$ for all i and all j because $\bar{x}^i_1 > D^i_1$ for every group. This follows from the identity (II.2.3) of Chapter II. The same identity also implies that $\bar{x}^3_{jD_2} = 0\ \forall j$ and $\bar{x}^i_{jD_2} = (q_2-\bar{p}_2)e^i_{jm}\bar{x}^i_j/m^i$ for all other groups. Similarly, $\bar{x}^i_{jD_1} = (q_1-\bar{p}_1)e^i_{jm}\bar{x}^i_j/m^i$. We can also write for all j, $\bar{x}^i_{jt_k} = e^i_{jk}\bar{x}^i_j/q_k$ for all $k \neq 2$. However, $\bar{x}^3_{jt_2} = \bar{x}^3_{jq_2} = 0$, whereas, for all other i, $\bar{x}^i_{jt_2}$ can again be written as $e^i_{j2}\ \bar{x}^i_j/q_2$.

The expressions for the welfare losses generated due to alterations in the instruments to yield a unit of revenue each to the government are given by

$$\lambda_{\bar{p}_1} = \frac{\sum_1^{12} \beta^i\, \bar{p}_1\, D^i_1}{\sum_1^{12} \bar{p}_1 D^i_1 - (q_2-\bar{p}_2)\, \bar{p}_1 \bar{x}^3_{2\bar{p}_1} + \sum_1^{12}\sum_1^{\ell} t_j\, \bar{p}_1 \bar{x}^i_{j\bar{p}_1}}$$

$$\lambda_{\bar{p}_2} = \frac{\sum_{i\neq 3} \beta^i\, \bar{p}_2\, D^i_2 + \beta^3\, \bar{p}_2\, \bar{x}^3_2}{\sum_{i\neq 3} \bar{p}_2\, D^i_2 + \bar{p}_2\bar{x}^3_2 - (q_2-\bar{p}_2)\, \bar{p}_2\, \bar{x}^3_{2\bar{p}_2} + \sum_1^{12}\sum_1^{\ell} t_j\, \bar{p}_2\, \bar{x}^i_{j\bar{p}_2}}$$

$$\lambda_{D_1} = \frac{(q_1-\bar{p}_1)\sum_1^{12}\beta^i}{(q_1-\bar{p}_1)\ (\#\text{ rice eaters}) + (q_2-\bar{p}_2)\ \bar{x}^3_{2D_1} - \sum_1^{12}\sum_1^{\ell} t_j \bar{x}^i_{jD_1}}$$

$$\lambda_{D_2} = \frac{(q_2-\bar{p}_2)\sum_{i\neq 3}\beta^i}{(q_2-\bar{p}_2)\,[(\#\text{ wheat eaters}) - (\#\text{ wheat eaters in group 3}) + \bar{x}^3_{2D_2}] - \sum_1^{\ell}\sum_1^{12} t_j \bar{x}^i_{jD_2}}$$

$$\lambda_{t_1} = \frac{-\sum_1^{12} \beta^i q_1(\bar{X}_1^i - D_1^i)}{(q_2-\bar{p}_2)q_1\bar{X}^3_{2t_1} - \sum_1^{12} q_1(\bar{X}_1^i - D_1^i) - \sum_1^{12}\sum_1^{\ell} t_j q_1 \bar{X}^i_{jt_1}}$$

$$\lambda_{t_2} = \frac{\sum_{i\neq 3} \beta^i q_2(\bar{X}_2^i - D_2^i)}{\sum_{i\neq 3} q_2(\bar{X}_2^i - D_2^i) - (q_2-\bar{p}_2)q_2 \bar{X}^3_{2t_2} + \sum_1^{12}\sum_1^{\ell} t_j q_2 \bar{X}^i_{jt_2}}$$

$$\lambda_{t_k} = \frac{\sum_1^{12} \beta^i q_k \bar{X}_k^i}{\sum_1^{12} q_k\bar{X}_k^i - (q_2-\bar{p}_2)q_k\bar{X}^3_{2t_k} + \sum_1^{\ell}\sum_1^{12} t_j q_k \bar{X}^i_{jt_k}}, \quad \forall\, k = 3,\ldots,\ell$$

Note that all these expressions can be written in value terms. For instance, we may write

$$\sum_{i,j} t_j\, q_k\, \bar{X}^i_{jt_k} = \sum_{i,j} t_j\, \bar{X}^i_j\, e^i_{jk}$$

$$= \sum_i \sum_{j=3}^{\ell} \frac{t_j}{q_j}(q_j\bar{X}^i_j)e^i_{jk} + \sum_i \sum_{j=1,2} \frac{t_j}{q_j}\{exp^i_j + (q_j-\bar{p}_j)\min(\bar{x}^i_j, D^i_j)\}e^i_{jk}$$

where t_j/q_j is the tax rate on good j, $q_j\bar{X}^i_j$ is the expenditure on good j=3,...,ℓ by group i, exp^i_j is the expenditure on good j = 1,2 by group i and $(q_j-\bar{p}_j)$ min $(\bar{x}^i_j, D^i_j)$ is the subsidy accruing to the i-th group. Other expressions in the equations can be expanded in a similar way.

To work out the results empirically, we have divided the commodities into four parts, namely, rice, wheat, inferior cereals and other commodities. We obtained the various elasticities from Binswanger and Swamy [1985] (henceforth BS). Since the consumer price elasticity matrix does not satisfy the adding up property namely,

$$\sum_j \alpha_j\, e_{jk} = -\,\alpha_k \quad (k=1,\ldots,4) \quad \text{where } \alpha_j = p_j\bar{x}_j$$

we adjust it to satisfy this requirement. For this we follow the procedure given in Dreze [1983] (see Appendix D for the details). These matrices are presented in Appendix E.

To obtain the groupwise expenditure elasticities from the common elasticities* given in BS, we have devised the following procedure. For j = rice, wheat and inferior cereals, we write

$$e^{i}_{jm} = e_{jm} \frac{\bar{X}_j}{m} \cdot \frac{m^i}{\bar{x}^i_j}$$

where e_{jm} is the expenditure elasticity for good j from BS, m is the per capita expenditure and $\bar{X}_j$ is obtained as $(S_j \times m)/p_j$ where S_j is the share of good j in expenditure and p_j is the corresponding price index. However, for the group of 'other commodities' it is not very meaningful to use $\bar{x}^i_j$. Therefore, we define the expenditure elasticity as

$$e^{i}_{4m} = e_{4m} \frac{\bar{X}_j}{m} \cdot \frac{m^i}{\alpha^i_4} \; p_4$$

where α^i_j is the share of expenditure of good j for consumer group i.

For obtaining t_j/q_j, we used the effective tax rates* given in Ahmed and Stern [1983(a)]; t_4 was obtained as a simple average of effective tax rates for 'other commodities'. The welfare weights for various groups were obtained using a concave function of expenditure for each group i

$$\Psi^i(m^i) = \begin{cases} \dfrac{k(m^i)^{1-e}}{1-e} & \text{if } e \neq 1, \quad e \geq 0 \\[2ex] k \log m^i & \text{if } e = 1 \end{cases}$$

where 'e' is a measure of inequality aversion. Then β^i is defined as $\Psi^{i\,'}(m^i)$ with m^1 being the expenditure for the poorest group. k is chosen such that $\beta^1 = 1$. This implies that $\beta^i = (m^1/m^i)^e \; \forall i$. Since $\{m^i\}$ is an increasing sequence, is a decreasing one with e >0. This means that a rupee added to a poor group's expenditure is more valuable than that added to the richer groups. However, $\beta^i = 1 \; \forall i$ if e = 0 implying thereby that the value of a rupee of expenditure is the same for the poor as well as the rich groups.

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* We assume that these were applicable in 1973-74 also.

For a few values of 'e we have derived the welfare weights for all the groups and the corresponding welfare losses arising from changes in the parameters. These are presented in Table IV.5 below.

Table IV.5 : Welfare weights and losses for all groups

Expenditure group	Wlfare weight	Inequality Aversion (e) 0.0	0.5	1.0	3.0
1. 4.0	β^1	1.0	1.0	1.0	1.0
2. 9.5	β^2	1.0	0.6489	0.4210	0.0746
3. 12.0	β^3	1.0	0.5773	0.3333	0.0370
4. 17.0	β^4	1.0	0.4851	0.2352	0.0130
5. 27.5	β^5	1.0	0.3814	0.1455	0.0031
6. 54.5	β^6	1.0	0.2709	0.0734	0.0004
7. 112. 5	β^7	1.0	0.1686	0.0350	0.0000
8. 132.5	β^8	1.0	0.1737	0.0302	0.0000
9. 200.0	β^9	1.0	0.1414	0.0200	0.0000
10. 391.0	β^{10}	1.0	0.1011	0.0102	0.0000
11. 620.5	β^{11}	1.0	0.0803	0.0064	0.0000
12. 896.5	β^{12}	1.0	0.0687	0.0047	0.0000
Welfare losses due to various instruments	$\lambda_{\bar{p}_1}$	16.9392	4.0817	1.1198	0.0199
	$\lambda_{\bar{p}_2}$	1.9752	.4272	.0995	.0011
	λ_{D_1}	.0001	.0000	.0000	.0000
	λ_{D_2}	.0002	.0001	.0000	.0000
	λ_{t_1}	1.0000	.2420	.0651	.0007
	λ_{t_2}	1.0006	.2744	.1198	.0045
	λ_{t_3}	1.0186	.3126	.1320	.0328
	λ_{t_4}	1.2189	.1543	.0223	.0001

Note that as e increases all the entries in the second column, except for β^1, decrease. As is clear from this trend, not only will β^i's, $i \neq 1$, go to zero as e becomes large but the losses in welfare arising from marginal changes in our instrumental values will also go to zero, thereby, making the given situation an optimum. However, all depends on the value judgements that the policy makers may make. Also, at every stage, there is a discrepancy between the welfare losses, λ's. One thing worth noting here is that *whatever be the level of inequality aversion*, the best way to increase welfare with the same government expenditure is to reduce the ration quota for rice and increase its controlled price appropriately. This happens because the loss in welfare is the least when D_1 is reduced to raise one unit of revenue and this same additional revenue, if spent to subsidize $\bar{p}_1$, will yield a much larger increase in welfare.

Though in theory it is always possible to find a welfare improving allocation from any DPE, in practice it may not be possible. It is quite likely that at an observed equilibrium our calculations may lead to directions of reform very different from those predicted by theory. It is interesting to note that in our case the data support the theoretical propositions.

CHAPTER V

PRICE AND DISTRIBUTION POLICIES FOR EDIBLE OILS : An Application of Dual-Price Equilibrium

V.1 INTRODUCTION

In the last chapter we analysed the efficiency properties of dual pricing schemes for wheat and rice. In this chapter, we study the effects of introducing dual pricing schemes for edible oils.

Edible oils constitute the major source of supply of fats in India. For the year 1984-85 the provisional estimate for per capita availability of edible oils including vanaspati* is 6.7 Kg. as against the per capita consumption of 26 Kg. in devélcped countries and a world average of 11 Kg. The domestic production of these oils is not only very low but also highly uncertain. Consequently, their prices fluctuate a lot. Since oil seeds, oil cakes, edible oils and oil-based products constitute about 13 per cent of the wolesale price index, such fluctuations would seriously affect the masses. The uncertain supply of oilseeds also affects the employment of about half a million persons engaged in milling and processing of oilseeds and fats.

There is no question that the production of edible oils must be increased. But this can be done only in the long run. In the short run, some policy measure for an equitable distribution should be introduced. It is not difficult if a commodity is storable and has no close substitutes. However, in a technical sense, the five major edible oils, viz., groundnut, mustard, coconut, sesamum and vanaspati are perfect substitutes as cooking media. But, whether the consumers consider them as substitutes or not is an empirical question.

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* Vanaspati is a brand name for hydrogenated oil.

The purpose of this chapter is to study the degree of substitutability and the sensitivity of demand and supply of three major oils - groundnut, mustard and vanaspati - to prices and income. This chapter is palnned as follows.

In sections 2 and 3, we present the methodology and the estimates of a system of demand and supply functions for the major oils.

In section 4, we compare the observed and equilibrium prices under the assumption of exogenous supplies. Given that groundnut oil is the most important edible oil, we obtain various combinations of adjustment mechanisms to stabilize its price when there are external disturbances affecting its supply.

Next, we consider two types of dual-pricing schemes - one with an exogenous and the other with an endogenous ration price - and obtain steady state equilibria corresponding to various "states of nature". The resulting outputs and prices are then compared in terms, inter-alia, of gains to various income groups.

V.2. DEMAND FOR OILS

The demand for any commodity depends, in principle, on the prices of all commodities, assets and income. But for practical purposes, it is convenient to make some restrictive assumptions regarding consumer behaviour. For example we can assume that utility functions are additive. At the same time we need the functional forms to be flexible enough to accommodate different degrees of substitutability among commodities. It should also be easy to estimate the parameters. In this context, the almost ideal demand system (AIDS) proposed by Deaton and Muellbauer [1980] is quite useful.

In this system, the value shares are related linearly to the logarithms of prices and the logarithm of the total expenditure in real terms. Thus

$$W_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(E/P) \qquad (V.2.1)$$

where

E = total expenditure

W_i = share of ith good in the expenditure

p_j = price of the jth good

and P = is the price index defined by

$$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j$$

"The most interesting feature of (V.2.1) from an econometric view point is that it is very close to being linear. Apart from the expression P, which involves the parameters (V.2.1) can be estimated equation by equation using ordinary least squares". Also, as noted by Deaton and Muellbauer, the price index P can often be calculated directly before estimation so that (V.2.1) becomes straightforward to estimate. In fact, in our study, we found that the use of nominal expenditures yielded more meaningful estimates in terms of price and expenditure elasticities than the real expenditure.

The share equations were estimated for groundnut, and rapeseed/mustard oils using data for the period 1963-64 to 1979-80. (See Appendix F for sources of data). For vanaspati, the share equation did not yield meaningful results. A simple log-linear demand function was found to be better than the linear or the share equation. The equations estimated using ordinary least squares are as follows:

$$D_G = \frac{E}{P_G}\,[\,\underset{(2.45)^*}{10.08} - \underset{\substack{(-0.4)\\ [-1.12]}}{0.33\log P_G} + \underset{\substack{(1.78)\\ [0.91]}}{0.83}\{\log P_M + \log P_c + \log P_V\} - \underset{\substack{(-2.17)^*\\ [\ 0.40]}}{1.67\log E}\,] \qquad (V.2.2)$$

$$\bar{R}^2 = 0.24, \quad DW = 1.9, \quad DF = 13$$

$$D_M = \frac{E}{P_M}\,[\,\underset{(2.57)^*}{8.02} + \underset{\substack{(1.08)\\ [-0.54]}}{0.72\log P_M} + \underset{\substack{(1.46)\\ [0.79]}}{0.41}\{\log P_G + \log P_c + \log P_V\} - \underset{\substack{(-2.62)^*\\ [\ 0.05]}}{1.48\log E}\,] \qquad (V.2.3)$$

$$\bar{R}^2 = 0.33, \quad DW = 2.04, \quad DF = 13$$

$$\log D_V = \underset{(-1.10)}{-1.368} + \underset{(0.17)}{0.026\log P_G} + \underset{(0.17)}{0.012\log P_M} + \underset{(0.17)}{0.003\log P_S} \qquad (V.2.4)$$

$$+ \underset{(0.17)}{0.002\log P_c} - \underset{(-3.46)}{1.359\log P_V} + \underset{(4.81)^{**}}{1.309\log E}$$

$$\bar{R}^2 = 0.68, \quad DW = 1.45, \quad DF = 10$$

where p_i = wholesale prices index of oil i

D_i = demand for ith oil for direct consumption purposes where

i = G(groundnut oil), M(mustard oil), S(sesamum oil), C(coconut oil) and V(vanaspati).

E = private final consumption expenditure at current prices.

Figures within circular brackets indicate the t values and those within square brackets indicate the elasticities.

The own, cross-price and income elasticities of demand for all the oils have the expected signs and appear to be reasonable. While interpreting the regression coefficients, it should be kept in mind that, in the case of share equations, the own-price elasticity is obtained by dividing the regression coefficient by the mean share and then subtracting one. Similar adjustments are made for other elasticities.

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* significant at 5% level of significance

** significant at 1 per cent level of significance.

V.3 SUPPLY OF OILS

The supply of all oils except vanaspati depends mainly on the supply of the oilseeds. The supply of any oilseed should depend upon the area allotted for the particular crop , and the rainfall. The area and the application of other inputs, in turn, will depend upon the relevant prices.

First, the logarithm of the prices of oilseeds were assumed to be linear functions of the logarithm of the prices of the corresponding oils. Using the data for the period 1963-64 to 1978-79, we have the following equations:

$$\log P_{GS} = \underset{(1.33)}{0.31} + \underset{(18.8)^{**}}{.93} \log P_G \qquad (V.3.1)$$

$$\bar{R}^2 = .96, \quad DW = 2.7, \quad DF = 14$$

$$\log P_{MS} = \underset{(0.17)}{0.02} + \underset{(31.8)^{**}}{1.01} \log P_M \qquad (V.3.2)$$

$$\bar{R}^2 = .99, \quad DW = 2.2, \quad DF = 14$$

The fits are excellent and the coefficients are meaningful. These equations could be interpreted to yield the expected price of the particular oil seed in the next season.

Next, we face the area allocation problem.

The major costs of cultivation of a crop are wages and the imputed cost of land. It is reasonable to assume that they do not vary significantly among substitutable crops. Then the area allocation must depend primarily on the expected revenues and their variability. This is a problem involving the allocation of a fixed resource among several risky prospects. In this

** significant at 1 per cent level of significance.

study, we do not attempt to solve this problem. Hence, we assume that the supply of an oilseed depends upon the expected price of the same oilseed, expected price of competing crops, the area allocated and the rainfall index. Ideally we would have liked to replace the area allocated by a function of prices.

The supply equations for the two oilseeds have been estimated using the data for the years 1951-52 to 1979-80. They are

$$S_{GS} = \underset{(-2.26)^*}{-1894.60} + \underset{(2.5)^*}{17.74} P_{GS_{-1}} + \underset{(5.68)^*}{0.58}\, A_{GS} - \underset{(-1.4)}{9.25} P_{J_{-1}} + \underset{(5.1)^{**}}{25.8}\, RI \qquad (V.3.3)$$

$$\bar{R}^2 = 0.81, \quad DW = 2.7, \quad DF = 22$$

$$S_{MS} = \underset{(-0.6)}{-234.00} + \underset{(1.4)}{4.50} P_{MS_{-1}} + \underset{(3.1)^{**}}{0.44}\, A_{MS} - \underset{(0.25)}{0.75} P_{W_{-1}} + \underset{(6.6)^{**}}{0.17}\, RI \qquad (V.3.4)$$

$$\bar{R}^2 = 0.76, \quad DW = 2.6, \quad DF = 22.$$

where the subscript (-1) for a price means that the price is lagged by one year.

By substituting for P_{GS} and P_{MS} from equations (V.3.1) and (V.3.2) respectively into equations (V.3.3) and (V.3.4) and then by applying the corresponding conversion factors (see Appendix F) for the two oilseeds and for direct consumption, we obtained the following.

Edible Oil Supply Equations : (1964-65 to 1978-79).

$$S_G = -316.10 + 3.0\, P_{G_{-1}}^{0.93} + 0.1\, A_{GS} - 1.56\, P_{J_{-1}} + 4.36\, RI \qquad (V.3.5)$$
$$[0.93] \quad [0.73] \quad [-0.19] \quad [0.4]$$

$$S_M = -71.12 + 1.4\, P_{M_{-1}}^{1.01} + 0.14\, A_{MS} - 0.23\, P_{W_{-1}} + 0.05\, RI \qquad (V.3.6)$$
$$[1.01] \quad [0.93] \quad [-0.05] \quad [0.01]$$

The following are the notations used:

iS = ith oilseed with 'i' defined as earlier

A_{iS} = gross cropped area under ith oilseed

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* significant at 5 per cent level of significance

** significant at 1 per cent level of significance

J = jowar

W = wheat

The other notations used are same as earlier.

For the production of vanaspati, more than 95% of the cost consists of oils used. Hence, under normal conditions, the supply should depend upon the prices of input oils and the price of vanaspati. In fact, it should depend upon the relative prices of inputs. But, for the past two and a half decades, the input mix has been regulated by the government. Under these circumstances, the supply will increase, for example, when the price of vanaspati increases. But, if the price of a regulated input decreases, the supply may not increase, even though the relative price of that input has decreased. In view of this, we have introduced the input and output prices separately in the supply equation. Also, since the prices of cottonseed oil and soyabean oil were not available, we have used their respective seed prices. The estimated equation using the data for the period 1971-72 to 1978-79 is:

$$S_V = \underset{(8.3)^{**}}{692.03} + \underset{\substack{(4.18)^{**} \\ [0.53]}}{1.62}\, W\, p_{soy+cot} - \underset{\substack{(-4.57)^{**} \\ [-1.41]}}{5.14}\, P_S + \underset{\substack{(2.02) \\ [0.58]}}{2.31}\, P_V \qquad (V.3.7)$$

$$\bar{R}^2 = 0.78, \quad DW = 2.75, \quad DF = 4$$

where

$W\ P_{spy+cot}$ = weighted price index of soyabean and cottonseed, the weights being the respective fractions constituted by these oils in total oils used in vanaspati production.

It may be noted that the coefficient of the weighted price of soyabean and cottonseed is opposite to what we usually expect. As noted earlier, due to restrictions in their use, their signs cannot be predicted. It really depends upon whether the quota restrictions were binding or not.

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* significant at 1 per cent level of significance.

V.4. ECONOMIC POLICIES

We will now make use of the estimated system of demand and supply functions to analyse the effects of some price and distribution policies. Before we study the effects of specific economic policies, we will examine the reliability of the system of demand functions by computing the equilibrium prices of the three oils corresponding to the estimated values of their supplies and then comparing them with the observed prices. For this purpose we fix the values of all the exogenous variables like the consumption expenditure, lagged prices etc. at the observed levels. The computed equilibrium prices are reported in Table V.1.

Table V.1

Observed and Equilibrium Prices (OP and EP)

Year	Groundnut oil		Mustard oil		Vanaspati	
	OP	EP	OP	EP	OP	EP
1964-65	50.6	52.0	66.9	62.3	60.0	66.2
1965-66	71.4	51.2	69.9	64.8	68.2	65.9
1966-67	91.9	57.7	83.9	74.1	93.5	65.3
1967-68	64.6	80.3	82.6	90.6	81.5	91.7
1968-69	76.5	82.3	76.6	78.1	74.3	88.5
1969-70	97.3	100.4	86.8	117.1	90.2	83.0
1970-71	95.5	93.1	98.2	103.4	100.0	88.1
1971-72	83.3	79.0	95.3	86.7	92.0	81.8
1972-73	115.9	112.6	109.3	105.8	100.1	95.4
1973-74	161.7	153.0	163.5	157.6	131.8	136.2
1974-75	171.6	196.8	163.1	153.7	171.5	192.6
1975-76	1 15.8	147.4	104.1	139.8	160.0	168.9
1976-77	157.0	205.4	163.2	235.6	148.0	141.8
1977-78	161.3	178.1	202.2	156.7	167.5	170.4
1978-79	146.2	178.2	181.7	148.5	161.3	165.2
1979-80	207.7	227.4	232.6	193.0	188.0	178.1

Note that the equilibrium prices are reasonably close to the observed prices for many years.

Now, we shall do some simulation exercise. Assume that all the exogenous variable (including all the supplies) are fixed at their average levels of the last five years of the given data -

$P_S = 170$, $P_C = 160$, E = Rs. 62360 crs.

$S_G = 900,000$ tonnes, $S_M = 615,000$ tonnes, and $S_V = 585,000$ tonnes

Now, suppose that the supply of groundnut oil changes by 10%, i.e. by 90,000 tonnes. An interesting question is : what should be the change in the supply of mustard oil or vanaspati to stabilize the price of groundnut oil? In particular, will a change of 90,000 tonnes of mustard oil or vanaspati be adequate for this purpose? In Table V.2, we present the equilibrium prices before and after the change in the supply of groundnut oil, with the last four rows showing some adjustment mechanisms to stabilize groundnut oil price.

Table V.2

Uniform Pricing

	Edible oil/ vanaspati	Supply (in '000 tonnes)	Equilibrium price	Percentage change in equilibrium price compared to (1)(b)
(1)	G	900	186.4	-
	M	615	173.1	-
	V	585	164.6	-
(2)	G	810	207.3	11.2
	M	615	180.8	4.4
	V	585	165.0	0.2
(3)	G	810	186.7	0.2
	M	730	126.5	-26.9
	V	585	164.2	-0.2
(4)	G	810	186.4	0.0
	M	615	156.5	-9.5
	V	790	131.9	-19.9
(5)	G	810	187.2	0.4
	M	780	134.0	-22.5
	V	625	156.5	-4.9
(6)	G	810	187.1	8.4
	M	685	137.5	-20.5
	V	650	152.1	-7.6

To compensate for a reduction of 90,000 tonnes of groundnut oil, we find that we need to augment the supply of mustard oil by 115000 tonnes. This amount is arrived at by trial and error. Notice with this compensatory change, the price of groundnut oil is 186.7 which is close to the initial equilibrium price of 186.4. The price of mustard oil decreases by 26.9% while the total supply of oil increases by 1.2%.

We can achieve the same result by increasing the supply of vanaspati by 205000 tonnes. Thus the marginal rate of substitution between groundnut oil and vanaspati appears to be around 1:2, while an unit of groundnut oil is substituted by 1.2 units of mustard oil. Thus, depending upon the price rates of these oils we can work out the type of oil to be imported.

We will now study the effect of dual pricing of groundnut oil. To start with, assume that the oil markets are in equilibrium without dual pricing, i.e. oils are sold at uniform prices. We will consider the following four possible states of nature:

1. Normal state
2. Good state, 10% increase in the supply of all oils*
3. Bad state, 10% decrease in the supply of all oils
4. 40% decrease in groundnut oil, 28% decrease in mustard oil and 22% decrease in vanaspati**

Suppose dual pricing is introduced for groundnut oil. What will be the open market price of groundnut oil and other oils? To answer this question, we need to know the demand functions for all oils under dual pricing. Such demand functions cannot be determined empirically since dual pricing has

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* When we say that there is 10% increase in the supply, we mean that the supply function is multiplied by 1.1. In other words supply curve shifts.

** Maximum annual decrease in supply during 1964-65 to 1979-80.

never been tried so far*. The free market demand functions under dual pricing must be deduced theoretically from the estimated demand functions under uniform pricing. We know that dual pricing is equivalent to an income subsidy and an increase in the open market price. Thus, with the knowledge of income and price elasticities, we can deduce the demand functions under dual pricing. Let '$\bar{p}$' be the control price, 'D' the ration quota and 'q' be the open market price under dual pricing. Then there exists a $\theta \in [0,1]$ such that the demand for groundnut oil with a uniform price $\theta q+(1-\theta)\bar{p}$ and an income subsidy , $\theta D(q-\bar{p})$, is exactly the same as that under dual pricing.

Here we will have a slight diversion for aggregation of demand functions.

V.4.1. Exact Aggregation of Demand with Kinked Budget-Sets

We know that the θ varies among individuals. This will pose problems for aggregation since both the prices as well as the exogenous incomes vary across households. In this context one could consider aggregation with respect to a stochastic specification of the demand functions. For instance, see Hausman [1985] in which all individuals face the same price vectors but consume at different marginal prices depending on their demand. However, such stochastic aggregation requires, e.g., restrictions on the joint distribution of preferences and the exogenous variables.

In a seminal contribution, Gorman [1953] derives the conditions for exact aggregation assuming that the individual attributes are fixed and the consumers maximize utility. The restrictiveness of the necessary conditions for exact aggregation have been explored by Deaton-Muellbauer [1981] and Lau [1982] among others. Muellbauer [1981] demonstrates exact linear aggregation when labour income is endogenous.

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* Only very recently the supply of edible oils through the public distribution system has been started.

In our case, since θ varies across individuals, generalized* demand functions are of the form

$$w_i^h = a_i + b_{i1}\log q_{\theta^h} + \sum_{j\neq 1} b_{ij}\log q_j + c_i\log(m_{\theta^h}/k^h) \qquad \text{(V.4.1)}$$

where w_i^h is the share of expenditure on good i, for household h, $q_{\theta^h} = \theta^h q_1+(1-\theta^h)\bar{p}$ and $m_{\theta^h} = m^h + \theta^h D(q_1-\bar{p})$ where m^h is the initial income and k^h is a "sophisticated measure of household size".

For obtaining an exact aggregation, assume that there are $N = N_1+N_2+N_3$ consumers where N_1 is the number of consumers with $\bar{x}_1^h > D$, N_2 with $\bar{x}_1^h = D$ and N_3 with $\bar{x}_1^h < D$. We know that $\theta = 1$ for the first category and $\theta = 0$ for the last.

The share of aggregate expenditure on good i in the aggregate budget of all households is given by

$$\bar{w}_i = \sum_h q_i \bar{x}_i^h/\sum_h m_{\theta^h} \equiv \sum_h m_{\theta^h} w_i^h/\sum_h m_{\theta^h} \qquad \forall\ i \neq 1$$

and $$\bar{w}_1 = \sum_h q_{\theta^h}\bar{x}_1^h/\sum_h m_{\theta^h} \equiv \sum_h m_{\theta^h} w_1^h/\sum_h m_{\theta^h}$$

which using (V.4.1) can be written as

$$\bar{w}_i = a_i+b_{i1}\sum_h \frac{m_{\theta^h}\log q_{\theta^h}}{\sum_h m_{\theta^h}} + \sum_{j\neq 1} b_{ij}\log q_j + c_i\sum_h \frac{m_{\theta^h}\log(m_{\theta^h}/k^h)}{\sum_h m_{\theta^h}}$$

$$= a_i+b_{i1}[\log q_1(\sum_{N_1} m_{\theta^h}) + \sum_{N_2} m_{\theta^h}\log q_{\theta^h} + \log\bar{p}(\sum_{N_3} m_{\theta^h})]/\sum_{h\in N} m_{\theta^h}$$

$$+ \sum_{j\neq 1} b_{ij}\log q_j + c_i\sum_h m_{\theta^h}\log(m_{\theta^h}/k^h)/\sum_h m_{\theta^h}.$$

We can write

$$q_{\theta^h} = \theta^h q_1+(1-\theta^h)\bar{p} = q_1[\theta^h + (1-\theta^h)\bar{p}/q_1].$$

When $\bar{p}/q_1$ is close to 1, i.e., $q_{\theta^h} \simeq q_1$, we get

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* In the following our approach is same as that of Deaton and Muellbauer [1980].

$$\bar{w}_i \simeq a_i + b_{i1}[\log q_1(\sum_{N_1+N_2} m_{\theta h}) + \log \bar{p}(\sum_{N_3} m_{\theta h})]/\sum_h m_{\theta h}$$

$$+ \sum_{j\neq 1} b_{ij} \log q_j + c_i \sum_h m_{\theta h} \log(m_{\theta h}/k^h) / \sum_h m_{\theta h}$$

$$= a_i + b_{i1} \log q_1 + \frac{\sum_{N_3} m_{\theta h}}{\sum_h m_{\theta h}} \log(\bar{p}/q_1) + \sum_{j\neq 1} b_{ij} \log q_j$$

$$+ c_i \sum_h m_{\theta h} \log(m_{\theta h}/k^h) / \sum_h m_{\theta h}$$

Define the index k by

$$\log(\bar{m}/k) = \sum_h m_{\theta h} \log(m_{\theta h}/k^h) / \sum_h m_{\theta h}$$

where $\bar{m}$ is the mean expenditure of households. Then, the aggregate demand function becomes

$$\bar{w}_i = a_i + b_{i1} \log q_1 + \sum_{j\neq 1} b_{ij} \log q_j + c_i \log(\bar{m}/k).$$

But since $q_{\theta h} \simeq q_1$ ($\because \bar{p}/q_1 \simeq 1$), we can approximate this function by

$$\bar{w}_i = a_i + b_{i1} \log \bar{q}_\theta + \sum_{j\neq 1} b_{ij} \log q_j + c_i \log(\bar{m}/k) \qquad \text{(V.4.2)}$$

where $\bar{q}_\theta$ is the average of individualized prices $q_{\theta h}$, which is of the same form as (V.4.1).

If all consumers have the same tastes, i.e., $k^h = 1\ \forall h$, the index k in the "representative budget level" $\bar{m}/k$ shows the equality of the distribution of household budgets. When k^h varies across households, k also reflects the demographic structure.

Note, however, that such aggregation is not permitted if the ratio of controlled price to free market price of the rationed good is significantly different from one. In fact, if we look at some of the rationed goods in India we find that this proportion is very close to one. For example, for rice it was

.82 in West Bengal and .76 in Uttar Pradesh for the year 1981. These are two major rice growing states in India. Similarly, for major wheat growing states, Punjab and Uttar Pradesh, these ratios were as high as .93 and .89 respectively for the same year.

For empirical estimation of equation (V.4.2), we need observations on $\bar{m}$ where

$$\bar{m} = \frac{1}{N}\sum_h m_{\theta^h} = \frac{1}{N}\sum_h m^h + D\ (q_1-\bar{p})\ \frac{1}{N}\sum_h \theta^h.$$

Instead of assuming a probability distribution for θ^h we can obtain the sensitivity of the estimates with respect to a single parameter, namely, its mean $\bar{\theta} = \frac{1}{N}\sum_h \theta^h$. And $\bar{q}_\theta$ can be replaced by q_1 given that $\bar{p}/q_1$ is approximately equal to one. However, we do not attempt this exercise here.

Instead we follow the procedure given below.

V.4.2. Some Policy Simulations

The individual demand functions for the rationed good are of the form

$x = m_\theta/q_\theta(a+b \log q_\theta + c \log m_\theta)$.

(For simplicity we have dropped the other price terms). We know that

$$\frac{1}{q_\theta} = \frac{1}{q_1[1-(1-\theta)(1-\bar{p}/q_1)]} \simeq \frac{1}{q_1}[1+(1-\theta)(1-\frac{\bar{p}}{q_1})].$$

Assume that there are $N = N_1+N_2$ consumers, where N_1 is the number of (the first category) consumers buying more than D units and N_2 consumers (the second category) buy less than D units*. We know that $\theta = 1$ for the first category and $\theta = 0$ for the second. Assume that the income compensation is small relative to the income. Hence instead of m_θ, we will use m. Then the demand for the first category consumer is

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* The same analysis will go through when N_1 is the number of consumers consuming more than or equal to D units of the rationed good.

$$x = \frac{m}{q}\,(a+b \log q + c \log m) \qquad \text{(i)}$$

and for the consumer in the second category, it is

$$x = \frac{m}{q}\,(a+b \log p + c \log m) + \frac{m}{q}\,(1-\frac{\bar{p}}{q})\,(a+b \log \bar{p} + c \log m) \qquad \text{(ii)}$$

Aggregating over both categories, we get

$$\sum_1^N x = [\sum_1^N m/q]\{a+b([\sum_1^{N_1} m / \sum_1^N m]\log q + [\sum_1^{N_2} m/\sum_1^N m]\log \bar{p}) + c\ [\sum_1^N m \log m/\sum_1^N m]\}$$

$$+ \quad [\sum_1^{N_2} m/q]\,(1-\frac{\bar{p}}{q})\{a+b \log \bar{p} + c\ [\sum_1^{N_2} m \log m/ \sum_1^{N_2} m]\}.$$

When p/q is close to one, we can ignore the second term, and write the aggregate demand function as

$$\sum_1^N x = [\sum_1^N m/q]\ \{a+b\ [\sum_1^{N_1} m/ \sum_1^N m]\ \log q + b[\sum_1^{N_2} m/\sum_1^N m]\ \log \bar{p} + c \log \bar{m}\}$$

where, for practical purposes, we have used $\sum_1^N m/N$ (for m) as the income of a representative consumer, i.e., the simple average of the incomes. Since N_1 and N_2 are not observed, we use various proportions to represent $\sum_1^{N_i} m/\sum_1^N m$, i = 1,2. However, here we present results only for the case $\sum_1^{N_i} m/\sum_1^N m = 0.5$ for i = 1,2.

To study the impact of dual pricing, it is important to keep in mind that the demand for oils depends upon current prices, among other variables, while supply depends upon lagged prices. Now suppose we introduce dual pricing in period 1. Then the market prices of oils will change from the previous prices under uniform pricing. This will affect the supplies in the next period and hence the prices in the next period. Suppose we go through many periods and the prices converge. Then the supplies will also converge. We will refer to this as the Steady State.

We have used the following procedure to calculate the steady states under dual-pricing. Let $(p,q) \in R^6_+$, where p is the vector of current prices and q that of lagged prices. Let $Z(p,q) \in R^3$ denote the vector of excess demands. Consider the following map from $R^6_+ \to R^6_+$:

$$p^i \to p^i + Z^i \qquad i = 1,2,3$$
$$q^i \to p^i \qquad i = 1,2,3.$$

Suppose there is a fixed point for this map, say $(\bar{p},\bar{q})$. Then $Z(\bar{p},\bar{q}) = 0$ is the excess demand vector in the steady state.

In order to analyse the effects of dual pricing we used, inter-alia, the NSS (National Sample Survey), 28th round, data for 1973-74 on per capita consumption in India. The values of the exogenous variables used are

Price Indices	
Coconut oil	: 159.9
Sesamum oil	: 166.0
Jowar (lagged)	: 126.0
Wheat (lagged	: 106.5
Cottonseed and soyabean (weighted)	: 111.5
Rainfall Index	: 101.5
Area under groundnut (lagged)	: 6990.0 ('000 hectares)
Yield of groundnut	: 845.0 (Kg./hectare)
Aggregate consumption expenditure	: Rs. 42933.0 crores
Population	: 57.57 crores.

We consider two types of dual price schemes* - DP-1 and DP-2. In both the schemes, the government procures 50% of the groundnut oil production at the market price.

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* For a similar analysis of cereals see Chetty and Srinivasan [1984].

However, the two schemes differ in the calculation of issue price which is exogenous (fixed at 50% of the free market price under uniform pricing) in DP-1 and endogenous (obtained as 50% of the prevailing free market price under dual pricing) in DP-2*. The equilibrium prices for various states of nature are given in Table V.3

Table V.3 : Equilibrium Price Indices

State of Nature	Groundnut oil			Mustard oil			Vanaspati			Ration price	
	UP**	DP-1	DP-2	UP	DP-1	DP-2	UP	DP-1	DP-2	DP-1	DP-2
1.	147.7	195.6	183.5	136.7	131.0	133.2	125.4	125.3	125.4	73.9	91.9
2	133.6	177.6	166.2	121.2	115.4	117.5	119.1	118.9	118.9	66.8	83.1
3.	164.3	216.6	203.4	154.5	148.9	151.0	132.8	132.6	132.7	82.2	101.7
4.	227.7	296.1	279.5	197.2	191.5	193.5	143.7	143.6	143.7	113.9	139.8

** UP stands for uniform pricing.

It is worth noting that compared to the uniform pricing scheme, the open market price of groundnut oil under DP-1 increases by about 30% and under DP-2 by about 24% only in all states of nature. Increase in groundnut oil production is therefore much more under DP-1 than under DP-2 as shown in table V.4. The introduction of dual-pricing leads also to a decrease in both the equilibrium price and the quantity of mustard oil. The reduction in price ranges from 3% to 5% for DP-1 and is slightly lower for DP-2. These schemes have practically no effect on vanaspati. Thus, as far as the prices are concerned, the dual pricing schemes appear to be beneficial from the point of view of consumers. The increased demand for groundnut oil can be explained by the expected high price elasticity of demand for the poorer sections.

* We are not reporting here the results obtained with other combinations of ration supply, price etc.

Table : V.4 : Equilibrium Quantities

State of Nature	Groundnut oil			Mustard oil			Vanaspati		
	UP	DP-1	DP-2	UP	DP-1	DP-2	UP	DP-1	DP-2
1	848.8	942.2	918.7	591.3	579.3	583.9	514.8	514.4	514.6
2	903.0	997.9	973.5	619.2	600.7	605.7	550.1	549.8	549.5
3	793.2	884.4	861.6	566.2	555.5	559.5	478.5	478.3	478.4
4	602.9	680.0	661.4	518.1	509.4	512.5	434.5	434.3	434.4

We can make use of our analysis to estimate the effect of dual pricing on the income distribution. We know the prices before and after the introduction of dual-pricing. From this, we can calculate the monetary value of the ration card, i.e., the income subsidy due to rationing. The utility of a consumer at price 'q' and money income 'm' is given by the indirect utility function, V(q,m). If the open market price under dual pricing is q', with the control price $\bar{p}$ and ration quota D, the utility of the consumer is given by $V\{q', m + D(q'-\bar{p})\}$. The change in utility is approximately

$$(q'-q)\frac{\partial V}{\partial q} + D(q'-\bar{p})\frac{\partial V}{\partial m} = -\mu x(q'-q) + \mu D(q'-\bar{p})$$

where μ is the marginal utility of income and x is the initial consumption. Thus, if we know the initial consumption of groundnut oil by expenditure groups we can calculate the gains or losses due to dual pricing in terms of rupees.

These welfare gains are presented in Table V.5 and the corresponding values of the dual pricing variables are given in the next table. The following expenditure groups are used in the calculations.

	Expenditure groups	Average expenditure per capita per month (in Rs.)
All-India Urban	1	132.5
	2	391.0
	3	620.5
	4	846.5
All-India Rural	5	4.0
	6	9.5
	7	12.0

Expenditure groups	Average expenditure per capita per month(in Rs,)
8	17.0
9	27.5
10	54.5
11	112.5
12	200.0

Table V.5: Welfare Gains from Dual-Pricing under "normal" conditions

Expenditure group	DP-1		DP-2	
	(in Rs.)	(gain in percent)	(in Rs.)	(gain in percent)
1	1.45	1.092	1.18	0.89
2	2.18	0.558	1.73	0.44
3	1.99	0.321	1.59	0.26
4	0.59	0.069	0.59	0.06
5	0.71	17.800	0.63	15.77
6	0.59	6.170	0.54	5.65
7	0.67	5.580	0.60	4.99
8	0.75	4.440	0.66	3.89
9	0.85	3.120	0.74	2.69
10	1.09	2.000	0.91	1.68
11	1.45	1.290	1.18	1.05
12	1.85	0.920	1.48	0.74

Table V.6: Dual-Pricing Variables in a normal state of nature

	DP-1	DP-2
Ration Price (Rs./quintal)	325.02	403.64
Market Price (Rs./quintal)	860.00	807.00
Ration Supply ('000 tonnes)	471.11	459.36
Subsidy provided (Rs. crores)	202.42	185.29
Subsidy taken (Rs. crores)	101.21	92.64
Quota per head (Kg./month)	0.14	0.13
Value of quota (Rs./month)	0.59	0.54

It can be seen that the percentage gains are proportional to the expenditures. Or, in other words, the gain as a percentage of expenditure is higher, the lower the expenditure is. Also, the gains are much higher under DP-1 than under DP-2. One explanation for this is the lower ration price under DP-1 as compared to DP-2, although the total subsidy provided is more. But, the quota per head and also the value of quota are higher in the first scheme than in the second.

CHAPTER VI

WELFARE MAXIMIZATION : GENERAL EQUILIBRIUM JUSTIFICATION FOR A PARTIAL EQUILIBRIUM ANALYSIS

VI. 1 *INTRODUCTION*

We carried out some partial equilibrium calculations in Chapter IV by trying to increase welfare "at the margin" with the same public subsidy involved in the dual pricing scheme. In other words, we found conditions to maximize social welfare with a given (shadow) government revenue. In Section 2 of this Chapter we show that the same necessary conditions hold when welfare is maximized in a general equilibrium framework with *appropriate shadow prices for public production.*

We have examined earlier the possibility of replacing dual pricing scheme by lumpsum transfers. For instance, if all the consumers consume more than the quota of a dual-priced good, then we can provide a uniform subsidy of quota times the difference between free market and ration price of this good. We may also give individualized quotas based on tastes, preferences, earnings and other household characteristics such as composition in terms of age, sex, etc. Here again we can think of dual-pricing involving individualized transfers as functions of household characteristics. Deaton and Stern [1985] (henceforth called DS) show that optimal transfers imply (i.e., are sufficient for) the optimality of uniform non-differentiated commodity taxes.

In section 3 we study this problem in the context of a general equilibrium framework.

VI.2 GENERALIZATION OF THE PLANNER'S MODEL OF CHAPTER IV

We now describe our model including production.

Private agents consist of consumers and producers. Producers are denoted by $g = 1,\dots,G$. For a producer, say the gth, a production plan (a specification of the quantities of all inputs and outputs) is represented by a point y^g of $\mathbb{R}^\ell$, the commodity space. The set of all production plans or the production set is denoted by Y^g. We make all the standard assumptions on the production set (see, e.g., Debreu [1959]). The gth producer, facing a price vector p and a vector of rations $\bar{y}^g = (\bar{y}^g_-, \bar{y}^g_+)$ indicating lower and upper bounds on g's net production of each commodity chooses a production plan $y^g(p, \bar{y}^g)$ solving

$$\max_{y^g} \pi^g \equiv p \cdot y^g \text{ s.t.}$$
$$y^g \in Y^g = \{y^g | F^g(y^g) \leq 0\}$$
$$\bar{y}^g_- \leq y^g \leq \bar{y}^g_+$$

where F^g is a strictly concave production function. Producer inputs are not taxed so that the prices p paid by the producers go to owners of endowments. The aggregate net supply vector of producers is denoted by y=

Consumers are denoted by h=1,...,H. For the h-th consumer a consumption plan is represented by $x^h \in \mathbb{R}^\ell_+$. The set X^h of all the possible consumption plans is called the consumption set. The utility function u^h is an increasing function from X^h to $\mathbb{R}$. We make the necessary assumptions on the consumption set and the utility function for the existence of an equilibrium consumption for the h-th consumer (see, e.g., page 29 and Debreu [1959]).

The consumers buy goods at prices q = p+t where 't' is a vector of indirect taxes (excluding non-linear taxes and taxes on intermediate goods). The income of the h-th consumer consists of a profit income and a lumpsum transfer r^h from the government ,

$$m^h = \sum_h \theta^{gh} \pi^g + r^h$$

where θ^{gh} represents the share of the h-th consumer in g's profits. We assume that the first commodity is dual priced for consumers in the same way as defined earlier. Consumer h $(h = 1,\ldots,H)$ then chooses a consumption plan $x^h(q,\bar{p},D,m^h)$ to solve

$$\max_{x^h} u^h(x^h) \text{ subject}$$

$$\bar{p}\, x_1^h + (q_1-\bar{p}) \max\,(x_1^h-D,0) + \sum_2^{\ell} q_j\, x_j^h = m^h.$$

(Assume that the solution to this problem is in the interior of x^h).

The indirect utility function is $V^h(q,\, \bar{p},\, D,\, m^h)$. The aggregate consumption vector is denoted by $x \equiv \sum_h x^h$.

Besides the private agents there is a government whose share in g's profits is given by $\zeta^g \equiv 1- \sum_h \theta^{gh}$. Denoting a public production vector by $z \in Z$ (the public production set) $\subset \mathbb{R}^{\ell}$ the government revenue at prices p is given by

$$R_p = p.z + \sum_g \zeta^g \pi^g - \sum_h r^h + t_1 \sum_h \max(x_1^h-D,0)$$

$$+ \sum_{j=2}^{\ell} t_j x_j - (p_1-\bar{p}) \sum_h \min(x_1^h,D)$$

$$= p.z + t.x + \sum_g \zeta^g \pi^g - \sum_h r^h - (q_1-\bar{p}) \sum_h \min(x_1^h,D)$$

The government can interfere by imposing quantity or price restrictions or controlling some other variables. We assume that its signals consist of control variables (denoted by $s = (p_i),\ (t_i),\ (r^h),\ ((\bar{y}_i^g)),\ ((\theta^{gh})))$, and predetermined or exogenous variables (denoted by $w = (\bar{p},D)$).

Private excess demand is then

$$E(s;\, w) = \sum_h x^h(p+t,\, \bar{p},\, D,\, m^h) - \sum_g y^g(p,\bar{y}^g).$$

The government's problem is to

$$\text{(P)} \quad \max_s W(\ldots,\, V^h(p+t,\, \bar{p},\, D,\, r^h + \sum_g \theta^{gh}\pi^g(.)),\ldots)$$

$$\text{subject to} \quad \sum_h x^h(.) - \sum_g y^g(.) - z = 0 \qquad (\nu)$$

and $\quad s \in S$

where S is the "opportunity set"* of the government and W(.) is a social welfare function of Bergson-Samuelson type*.

Let the solution to the above problem be $V^*(z)$. And let z^* solve

$$\max_{z} V^*(z) \text{ subject to } z \in Z.$$

If Z is convex, z^* has maximum shadow profits in Z, i.e., $\nu^* z^* = \max_{z \in Z} \nu^* z$ where the shadow price vector $\nu^* = \frac{\partial V^*}{\partial z}$ at z^*. (For further details on ν^* see Dreze [1983]).

Let us consider two cases.

Case 1: Let $\nu = \lambda p^{**}$ (for simplicity, we drop the superscript '*' from ν^*). We shall now show that the solution to the problem (P) is the same as that to the problem

$$\max_{s \in S} W(.) \text{ subject to } R_p = 0 \qquad (\lambda)$$

The first order conditions to problem (P) can be written as

$$\frac{\partial W}{\partial s} - \nu \frac{\partial x}{\partial s} = \frac{\partial W}{\partial s} - \lambda p \frac{\partial x}{\partial s} = \frac{\partial W}{\partial s} - \lambda [q \frac{\partial x}{\partial s} - t \frac{\partial x}{\partial s}] = 0$$

Again suppose that for $h = 1,\ldots,n_1$, $x_1^h > D$, for $h = n_1+1,\ldots,n_2$, $x_1^h = D$ and finally $x_1^h < D$ for $h = n_2+1,\ldots,n$. Then we may write their respective budget constraints as

(i) $\bar{p}D + q_1(x_1^h - D) + \sum_2^{\ell} q_j x_j^h = m^h$, $h = 1,\ldots,n_1$

(ii) $\bar{p} x_1^h + \sum_2^{\ell} q_j x_j^h = m^h$, $h = n_1+1,\ldots,n$.

Differentiating the above equations with respect to $s = t_1,\ldots,t_\ell,\bar{p},D$ and substituting for $q.\frac{\partial x}{\partial s}$, we can write the first order conditions as

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* For more details see Dreze and Stern [1985].

** Conditions under which this holds are given in Diamond and Mirrlees [1971] Dreze and Stern [1985] etc.

$$\frac{\partial W}{\partial t_1} - \nu \frac{\partial x}{\partial t_1} = \frac{\partial W}{\partial t_1} + \lambda[\sum_h \max(x_1^h - D, 0) - (q_1 - \bar{p}) \sum_{n_1+1}^{n} \frac{\partial x_1^h}{\partial t_1} + t.\frac{\partial x}{\partial t_1}] = \frac{\partial W}{\partial t_1} + \lambda \frac{\partial R_p}{\partial t_1} = 0$$

$$i \neq 1, \frac{\partial W}{\partial t_i} - \nu \frac{\partial x}{\partial t_i} = \frac{\partial W}{\partial t_i} + \lambda[x_i - (q_1 - \bar{p}) \sum_{n_1+1}^{n} \frac{\partial x_1^h}{\partial t_i} + t.\frac{\partial x}{\partial t_i}] = \frac{\partial W}{\partial t_i} + \lambda \frac{\partial R_p}{\partial t_i} = 0$$

$$\frac{\partial W}{\partial \bar{p}} - \nu \frac{\partial x}{\partial \bar{p}} = \frac{\partial W}{\partial \bar{p}} + \lambda[\sum_h \min(x_1^h, D) - (q_1 - \bar{p}) \sum_{n_1+1}^{n} \frac{\partial x_1^h}{\partial \bar{p}} + t.\frac{\partial x}{\partial \bar{p}}] = \frac{\partial W}{\partial \bar{p}} + \lambda \frac{\partial R_p}{\partial \bar{p}} = 0$$

and

$$\frac{\partial W}{\partial D} - \nu \frac{\partial x}{\partial D} = \frac{\partial W}{\partial D} + \lambda[t.\frac{\partial x}{\partial D} - (q_1 - \bar{p})n_2] = \frac{\partial W}{\partial D} + \lambda \frac{\partial R_p}{\partial D} = 0$$

Hence, we have, for $\nu = \lambda p$,

$$\max_{s \in \bar{S}} W(.) \text{ subject to } x(.) - y(.) - z = 0 \qquad (\nu)$$

$$\Longleftrightarrow \max_{s \in S} W(.) \text{ subject to } R_p = 0 \qquad (\lambda)$$

Case 2: Next consider the case $\nu \neq p$. Then the shadow revenue for the planner is

$$R_\nu = \nu.z + \tau^c.x + \sum_g \zeta^g \pi^g - \sum r^h - (\nu_1 + \tau_1^c - \bar{p}) \sum_h \min(x_1^h, D) + (\nu - p).y$$

where τ^c is the vector of shadow taxes for consumers, $\tau^p = \nu - p$ is that for the producers and $q = \nu + \tau^c$. Note here that for a given ν, if q changes then τ^c changes.

Again, differentiating the identities (i) and (ii) with respect to t_1 and substituting into the first order conditions of problem (P), we have

$$\frac{\partial W}{\partial t_1} - \frac{\partial x}{\partial t_1} = \frac{\partial W}{\partial t_1} + [\frac{\partial}{\partial t_1}(x.\tau^c) - (q_1 - \bar{p}) \sum_{n_1+1}^{n} \frac{\partial x_1^h}{\partial t_1} - \sum_1^n \min(x_1^h, D)] = \frac{\partial W}{\partial t_1} + \frac{\partial R_\nu}{\partial t_1} = 0$$

In a similar fashion we can verify that

$$\frac{\partial W}{\partial s} - \nu \frac{\partial x}{\partial s} = \frac{\partial W}{\partial s} + \frac{\partial R_\nu}{\partial s} = 0$$

for $s = t_2, \ldots, t_\ell, \bar{p}$, and D.

Hence, even when the shadow prices are different from the producers prices, we can say that the solution to problem (P) is same as that to

$$\max_{s \in S} W(.) + R_{\nu} .$$

We will next generalize the results of Deaton-Stern [1985].

VI.3. OPTIMALITY OF UNIFORM COMMODITY TAXES

In this section, we give general equilibrium justification for the results obtained in DS.

One of their assumptions is regarding the separability of goods from leisure with linear Engel curves for goods in terms of total commodity expenditure. However, the intercepts of these curves are functions of household characteristics thus allowing for taste variations. The government pays a "demogrant" (possibly negative) to each household depending on its characteristics.

When the Government maximizes social welfare subject to its budget constraint and individual utility constraints, we find that "an optimal structure of the benefit or demogrant scheme implies the optimality of uniform commodity taxation" if the planner is not *interested* in the unobservable characteristics of households.

In this section we show that we obtain the same results by solving problem (P) of the last section by appropriately modifying it to take care of the assumptions of DS.

Suppose there are (ℓ+1) goods, good '0' being leasure separable from other goods, i.e., h-th individual's utility function can be written as

$$v^h = u^h(x_o^h,\ x^h)$$

where x^h is an ℓ-vector of goods and x_o^h is the consumption of leisure. The consume

$$\text{Max } u^h(x_o^h, x^h) \quad \text{subject to}$$

$$q_o^h x_o^h + q.x^h = r^h + \sum_{g=1}^{G} \theta^{gh}\ \pi^g + d^h + q_o^h\ T^h$$

where q_o^h is the wage rate for household h and T^h its time endowment. The demogrant $d^h = \alpha_o + \sum_{j=1}^{J} \alpha_j\ z^{jh}$ is a function of observable household characteristics $(z^{jh},\ j = 1,\ldots,J)$, for household h.

If u_1^h is the maximum utility from goods and $q.\ x^h = m^h$, then goods' Engel curves will be linear if u_1^h is of the form

$$u_1^h = f^h\ \{\frac{m^h - a^h(q)}{b(q)}\}$$

where f^h is monotone increasing and $a^h(q)$ and $b(q)$ are positively linear homogeneous in goods prices q.

Adopting the specification of DS, we have

$$a^h(q) = a(q) + \sum_{j=1}^{J} \alpha^j(q)\ z^{jh} + \varepsilon^h(q)$$

The functions $a(.)$, $\alpha^j(.)$ and $\varepsilon^h(.)$ are homogeneous of degree 1. The demand functions for goods are then

$$x_i^h = \beta_i(q)m^h + \{a_i(q) - \beta_i(q)a(q)\} + \sum_{j=1}^{J} \{\ \alpha_i^j(q) - \beta_i(q)\alpha^j(q)\}z^{jh}$$
$$+ \{\varepsilon_i^h(q) - \beta_i(q)\ \ \varepsilon^h(q)\} \qquad \text{(VI.3.1)}$$

where $\beta_i(q) = b_i(q)/b(q)$ is the marginal propensity to consume good i and the functions with subscript i are the ith partial derivatives of the respective functions without the subscript.

The production conditions are the same as in the last section.

The welfare maximization problem is ggain

(P') Max $W(\ldots, V^h(.),\ldots)$ subject to

$$x(.) - y(.) - z = 0 \qquad (\nu)$$

The first order conditions to this problem imply the following.

$$(\alpha_o) \quad \sum_h \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial m^h} \frac{\partial m^h}{\partial \alpha_o} - \nu \cdot \sum_h \frac{\partial x^h}{\partial m^h} \frac{\partial m^h}{\partial \alpha_o} = 0$$

or $$\sum_h \beta^h - \nu \sum_h \frac{\partial x^h}{\partial m^h} = 0$$

or, $$\sum_h b^h = 0 \qquad \text{(VI.3.2)}$$

where β^h is the social marginal utility of money to consumer h. Equation (VI.3.2) says that the aggregate 'net' social marginal utility of money is zero.

$$(\alpha_j) \quad \sum_h \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial m^h} \frac{\partial m^h}{\partial \alpha_j} - \nu \sum_h \frac{\partial x^h}{\partial m^h} \frac{\partial m^h}{\partial \alpha_j} = 0$$

or, $$\sum_h b^h z^{jh} = 0 \qquad \text{(VI.3.3)}$$

"so that demogrants are set so as to exploit any correlation between observable characteristics and the social value of money given to h".

Finally,

$$(t_j) \quad \tau^c \frac{\partial \tilde{x}}{\partial q_j} = \sum_h b^h x_j^h \text{ (where } \tilde{x} \text{ is compensated demand)}$$

or, $$\sum_{k=1}^{\ell} \tau_k^c \frac{\partial \tilde{x}_j}{\partial q_k} = \sum_h b^h x_j^h \qquad \text{(VI.3.4)}$$

where the shadow taxes, $\tau_k^c = q_k - \nu_k$. Now we shall show that optimal demogrants are sufficient for optimality of proportional taxes.

If we substitute $\delta\nu_k$ for τ_k^c (see Deaton [1979] for the conditions under which this holds), the left hand side of equation (VI.3.4) becomes

$$\delta \sum_k \nu_k \frac{\partial \tilde{x}_j}{\partial q_k}$$

$$= \frac{\delta}{1+\delta} \sum_k q_k \frac{\partial \tilde{x}_j}{\partial q_k} \quad (\tau_k^c = q_k - \nu_k = \delta \nu_k)$$

$$= \frac{\delta}{1+\delta} \left(- \sum_k q_o^h \frac{\partial \tilde{x}_j^h}{\partial q_o^h}\right)$$

$$= - \frac{\delta}{1+\delta} \sum_h q_o^h \frac{\partial \tilde{x}_j^h}{\partial m^h} \frac{\partial m^h}{\partial q_o^h}$$

$$= - \beta_j(q) \frac{\delta}{1+\delta} \sum_h q_o^h (T^h - x_o^h)$$

which is proportional to $\beta_j(q)$.

Thus, if we show that the right hand side of (VI.3.4) is also proportional to $\beta_j(q)$, we get the required result. We can write

$$\sum_h b^h x_j^h = \beta_i(q) \sum_h b^h m^h + \sum_h [\alpha_i(q) - \beta_i(q) a(q)] b^h$$

$$+ \sum_{j=1}^{J} [\alpha_i^j(q) - \beta_i(q) \alpha^j(q)] \sum_h b^h z^{jh} + \sum_h [\varepsilon_i^h(q) - \beta_i(q) \; \varepsilon^h(q)] b^h$$

The second and third terms of this expression are zero using equations (VI.3.2) and (VI.3.3). Hence, "if the pattern of idiosyncratic commodity demands over individuals is independent of social marginal utilities of money, and given the other assumptions, optimal commodity taxes should be uniform".

APPENDIX - A

Fenchel Duality Theorem:

Assume that f and g are, respectively, convex and concave functionals on the convex sets B and C in a normed space X. Assume that $B \cap C$ contains points in the relative interior of B and C and that either [f, B] or [g,C] has non empty interior. Suppose further that

$$\mu = \inf_{x \in B \cap C} \{f(x)-g(x)\}$$

is finite. Then

$$\mu = \inf_{x \in B \cap C} \{f(x)-g(x)\} = \max_{x^* \in B^* \cap C^*} \{g^*(x^*)-f^*(x^*)\}$$

where the maximum on the right is achieved by some $x_o^* \in B^* \cap C^*$.

If the infimum on the left is achieved by some $x_o \in B \cap C$, then

$$\max_{x \in B} [\langle x, x_o^* \rangle - f(x)] = \langle x_o, x_o^* \rangle - f(x_o)$$

and

$$\min_{x \in C} [\langle x, x_o^* \rangle - g(x)] = \langle x_o, x_o^* \rangle - g(x_o).$$

APPENDIX - B

Define

$$C = \{x\varepsilon R^{\ell} | \bar{p}x_1 + \sum_2^{\ell} q_i x_i + (q_1-\bar{p})\max(x_1-D,0) \leq m\}$$

$$C_1 = \{x\varepsilon R^{\ell} | \bar{p}x_1 + \sum_2^{\ell} q_i x_i \leq m\}$$

$$C_2 = \{x\varepsilon R^{\ell} | \sum_1^{\ell} q_i x_i \leq m + (q_1-\bar{p})D\}$$

Claim : $C = C_1 \cap C_2$

Pick any $x \varepsilon C$. Suppose $x_1 \leq D$.

Then, $x \varepsilon C \Longrightarrow \bar{p}x_1 + \sum_2^{\ell} q_i x_i \leq m \Longrightarrow x \varepsilon C_1$

Consider $\sum_1^{\ell} q_i x_i = \bar{p}x_1 + \sum_2^{\ell} q_i x_i + (q_1-\bar{p})x_1$

$$\leq m + (q_1-\bar{p})x_1$$

$$\leq m + (q_1-\bar{p})D.$$

$\Longrightarrow x \varepsilon C_2$

$\therefore \quad x \varepsilon C_1 \cap C_2$

Suppose $x_1 > D$.

Let $x \varepsilon C$. Then $\bar{p}x_1 + \sum_2^{\ell} q_i x_i + (q_1-\bar{p})(x_1-D) \leq m$

$\Longrightarrow \quad \bar{p}x_1 + \sum_2^{\ell} q_i x_i \leq m - (q_1-\bar{p})(x_1-D) < m$

$\Longrightarrow \quad x \varepsilon C_1$

Also, $\bar{p}x_1 + \sum_2^{\ell'} q_i x_i + (q_1-\bar{p})(x_1-D) \leq m$

$\Longrightarrow \quad \bar{p}x_1 + \sum_2^{\ell} q_i x_i + (q_1-\bar{p})(x_1-D) + (q_1-\bar{p})D \leq m + (q_1-\bar{p})D$

$\Longrightarrow \quad \sum_1^{\ell} q_i x_i \leq m + (q_1-\bar{p})D$

$\Longrightarrow x \varepsilon C_2$

$\therefore \quad x \varepsilon C_1 \cap C_2.$

Next, consider $x \in C_1 \cap C_2$.

Let $x_1 \leq D$. Then $x \in C_1 \Rightarrow \bar{p}x_1 + \sum_2^{\ell} q_i x_i \leq m \Rightarrow x \in C$.

Let $x_1 > D$. Since $x \in C_2 \Rightarrow q_1 x_1 + \sum_2^{\ell} q_i x_i \leq m + (q_1 - \bar{p})D$

$$\Rightarrow \bar{p}x_1 + (q_1 - \bar{p})x_1 + \sum_2^{\ell} q_i x_i \leq m + (q_1 - \bar{p})D$$

$$\Rightarrow \bar{p}x_1 + (q_1 - \bar{p})(x_1 - D) + \sum_2^{\ell} q_i x_i \leq m$$

$$\Rightarrow x \in C.$$

Hence $C = C_1 \cap C_2$.

Q.E.D.

APPENDIX - C

Lemma 2: The correspondence $\beta(p)$ is lhc at any $P \in \Delta^{\ell+1}$ with $p = (1/P_\ell)P$ if $\inf_{x \in X} E_p(x) < \sum_1^\ell p_j e_j$, where $E_p(x) = \bar{p}x_1 + (q_1 - \bar{p})\max(x_1 - D, 0) + \sum_2^\ell q_j x_j$. If $p >> 0$ then β is continuous at p.

Proof: Consider any sequence $p_n \to p$ and $y \in \beta(p)$. We have to prove that $\exists\, y_n \in \beta(p_n)$ with $y_n \to y$. (See Theorem A.III.2 in Hildenbrand and Kirman [1976]). Note that $E_p(y) \leq \sum_1^\ell p_j e_j$.

Case(i): $E_p(y) < \sum_1^\ell p_j e_j$.

Since $p_n \to p$ and $E_p(y) - \sum_1^\ell p_j e_j$ is a continuous function of p, for large n, say $n \geq N$, $E_{p_n}(y) - \sum_1^\ell p_{jn} e_j < 0$. Choose the sequence y_n as

$$y_n = \begin{cases} y & \forall\, n \geq N \\ \text{any point of } \beta(p_n) & \text{for } n < N \end{cases}$$

Case (ii): $E_p(y) = \sum_1^\ell p_j e_j$.

Since $\inf_{x \in X} E_p(x) < \sum_1^\ell p_j e_j$ $\exists$ $z \in X$ such that $E_p(Z) < \sum_1^\ell p_j e_j$. Choose $\lambda_n \in [0,1]$ such that $\lambda_n \downarrow 0$ and $E_{p_n}[\lambda_n Z + (1-\lambda_n)y] < \sum_1^\ell p_{jn} e_j$ for large n. Hence construct the sequence y_n as follows

$$y_n = \begin{cases} \lambda_n Z + (1-\lambda_n)y & \forall\, n \geq N \\ \text{any point of } \beta(p_n) & \forall\, n < N . \end{cases}$$

Note that N is chosen such that

$$E_{p_n}(Z) < \sum_1^\ell p_{jn} e_j \quad \forall\, n \geq N .$$

We now have to prove that β is lhc at $p >> 0$. Note first that $\beta(p)$ is bounded. Also $\beta(p)$ is closed and convex.

Consider the sequence $p_n \longrightarrow p$ and $y_n \in \beta(p_n)$. Since β is compact valued it is enough to prove that there is a subsequence $y_{n_q} \to y \in \beta(p)$. (See theorem A.III.1 in Hildenbrand and Kirman [1976]). Also, since β is closed, it is enough to prove that the sequence $\{y_n\}$ is bounded. Suppose not. Since β is lhc and $y \in \beta(p)$. $\exists$ $z_n \in \beta(p_n)$ with $z_n \to y$. Since $\beta(p)$ is compact, consider an ε-sphere around $\beta(p)$. Since $z_n \to y$, for large n, $z_n \in \beta(p)$. Define

$$v_n = \mu_n z_n + (1-\mu_n) y_n$$

by choosing μ_n such that the distance $d(v_n, \beta(p)) = \varepsilon \; \forall n$. Since β is convex valued, $v_n \in \beta(p_n)$. But β is closed and therefore there is a subsequence $v_{n_q} \to v \in \beta(p)$. That is, $\exists$ N such that $\forall\, n_q > N$, $v_{n_q} \in \beta(p)$, i.e., $d(v_{n_q}, \beta(p)) = 0$ – a contradiction. Hence $\{y_n\}$ is bounded.

Q.E.D.

APPENDIX - D

To minimize the Euclidean distance between the adjusted and unadjusted matrices, let e^o denote the original elasticity matrix and e^1 the adjusted matrix. Let Δ denote the matrix $((e^1_{jk} - e^o_{jk}))$ with $\Delta_{jk} = e^1_{jk} - e^o_{jk}$. We have to

$$\underset{e^1}{\text{Min}} \sqrt{\sum_{j,k} (e^1_{jk} - e^o_{jk})^2} \text{ subject to } \sum_j \alpha_j e^1_{jk} = - \alpha_k \quad (\chi_k) \quad (k=1,\ldots,4).$$

The first order conditions imply

$$(e^1_{jk}) \quad \frac{\Delta_{jk}}{||\Delta||} = \chi_k \alpha_j \quad (j,k = 1,\ldots,4) \tag{1}$$

$$\text{or,} \quad \Delta_{jk} = \chi_k \alpha_j ||\Delta|| \tag{2}$$

Also, since

$$\sum_j \alpha_j e^1_{jk} = -\alpha_k, \tag{3}$$

we have, using (2)

$$\sum_j \alpha_j (e^o_{jk} + \chi_k \alpha_j ||\Delta||) = -\alpha_k \quad (k=1,\ldots,4) \tag{4}$$

$$\text{or} \quad \chi_k = \frac{\hat{\alpha}_k - \alpha_k}{||\alpha||^2 \, ||\Delta||} \tag{5}$$

where $\hat{\alpha}_k = - \sum_j \alpha_j e^o_{jk}$.

Therefore,

$$\Delta^*_{jk} = \frac{\alpha_j (\hat{\alpha}_k - \alpha_k)}{||\alpha||^2} .$$

APPENDIX - E

Unadjusted Price Elasticity Matrix

	Rice	Wheat	Inferior cereals	Other commodities
Rice	-0.6987	0.0616	0.0748	-0.3103
Wheat	0.0594	-0.3200	-0.0477	-0.9365
Inferior cereals	0.1896	0.0184	-0.6619	0.0510
Other commodities	-0.0845	0.0861	-0.0470	-0.8474

Adjusted Price Elasticity Matrices

Group	All India Rural	Rice	Wheat	Inferior cereals	Other commodities
1	Rice	-0.8018	-0.1048	-0.0172	-0.0831
	Wheat	-0.1084	-0.5909	-0.1974	-0.5667
	Inferior cereals	-0.0235	-0.3255	-0.8520	0.5204
	Other commodities	-0.1160	0.0352	-0.0751	-0.7779
2	Rice	-0.9585	-0.1392	-0.1079	0.0608
	Wheat	-0.0628	-0.4144	-0.1336	-0.7619
	Inferior cereals	-0.0393	-0.1585	-0.8229	0.3779
	Other commodities	-0.0604	0.1046	-0.0301	-0.8816
3	Rice	-0.8314	0.0238	-0.0603	-0.2077
	Wheat	0.0217	-0.3307	-0.0860	-0.9074
	Inferior cereals	-0.0806	-0.0585	-0.9371	0.2598
	Other commodities	0.0255	0.1174	0.0651	-0.9324
4	Rice	-0.8609	-0.0157	-0.0825	-0.1622
	Wheat	-0.0014	-0.3489	-0.1066	-0.8809
	Inferior cereals	-0.0916	-0.1156	-0.9346	0.3076
	Other commodities	-0.0362	0.1090	-0.0002	-0.8914
5	Rice	-0.8160	-0.0411	-0.0447	-0.1963
	Wheat	-0.0015	-0.3732	-0.1097	-0.8773
	Inferior cereals	-0.0170	-0.1623	-0.8723	0.2516
	Other commodities	-0.1841	-0.0011	-0.1485	-0.7506
6	Rice	-0.7740	-0.0384	0.0015	-0.2275
	Wheat	0.0171	-0.3760	-0.0887	-0.8901
	Inferior cereals	0.0825	-0.1235	-0.7659	0.1684
	Other commodities	-0.2108	-0.0814	-0.1697	-0.7087
7	Rice	-0.7076	0.0229	0.0626	-0.3032
	Wheat	0.0533	-0.3460	-0.0558	-0.9317
	Inferior cereals	0.1772	-0.0350	-0.6787	0.0608
	Other commodities	-0.1254	-0.0906	-0.1026	-0.8149

Group	All India Rural	Rice	Wheat	Inferior cereals	Other commodities
8	Rice	-0.6978	0.0605	0.0752	-0.3118
	Wheat	0.0600	-0.3207	-0.0473	-0.9377
	Inferior cereals	0.1907	0.0169	-0.6612	0.0488
	Other commodities	-0.0067	-0.0067	-0.0055	-0.9892
Group	All India Urban				
9	Rice	-0.6958	0.0557	0.0761	-0.3157
	Wheat	0.0613	-0.3240	-0.0467	-0.9402
	Inferior cereals	0.1927	0.0120	-0.6604	0.0451
	Other commodities	-0.0285	-0.0274	-0.0212	-0.9530
10	Rice	-0.6962	0.0580	0.0762	-0.3143
	Wheat	0.0612	-0.3227	-0.0465	-0.9395
	Inferior cereals	0.1900	0.0176	-0.6615	0.0501
	Other commodities	-0.0131	-0.0195	-0.0037	-0.9655
11	Rice	-0.6972	0.0598	0.0756	-0.3126
	Wheat	0.0607	-0.3217	-0.0468	-0.9387
	Inferior cereals	0.1896	0.0184	-0.6619	0.0510
	Other commodities	-0.0065	-0.0130	-0.0005	-0.9778
12	Rice	-0.6982	0.0610	0.0750	-0.3110
	Wheat	0.0603	-0.3210	-0.0471	-0.9380
	Inferior cereals	0.1896	0.0184	-0.6619	0.0510
	Other commodities	-0.0023	-0.0079	0.0001	-0.9876

APPENDIX - F

DATA : The sample period chosen is from 1950-51 to 1979-80.

1. Consumption: For the two edible oils, the estimated production during the period under consideration was taken from the Commercial Crop Statistics published by the Directorate of Economics and Statistics. These estimates relate to the agricultural year, i.e. July-June. While estimating the vegetable oil production adjustments have been made for exports and imports and the oil recovery ratios[1] applied to the quantities of oilseeds available for crushing. From the estimated oil production direct consumption figures are obtained by applying the following conversion factors[2].

Edible oil	Direct Consumption (as Percentage of total production)
Groundnut	73.6
Rapeseed/Mustard	100.0

For vanaspati, the production figures were obtained from Directorate of Vanaspati, Oils and Fats. These relate to the financial years.

2. Prices: The wholesale price indices, with 1970-71 = 100, were taken from Chandhok [1978]. For the edible oils, since the consumption figures were available for the crop years, the wholesale price indices were also computed for the same years. For vanaspati, corresponding to financial year production data, financial year wholesale price indices were used with base year 1970-71.

3. Area and Production: Production of oilseeds and area under oilseeds and substitute crops are from various issues of "Area, Production and Yield in India", published by the Directorate of Economics and Statistics.

4. Private Final Consumption Expenditure: These figures at current prices are from the National Accounts Statistics, Central Statistical Organization.

5. Rainfall Index: These indices for oilseeds are from Ray [1983].

1.

Seed	Proportion of oil extracted (as percentage of seeds crushed)
Groundnut	39.3
Rapeseed/mustard	33.0

2. Obtained as averages from Pavaskar [1979].

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