CONFOUNDED FACTORIAL DESIGNS IN QUASI-LATIN SQUARES

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INTRODUCTION

In factorial experiments the number of treatments become large with increase in the number of factors and also in the number of levels of the factors, so that randomized block designs cannot be used with much efficiency. To combat this all the treatment comparisons are assigned to two or more blocks. By so doing the comparisons arising from the groups of treatments assigned to various blocks cannot be isolated from the fertility differences arising from the blocks. In such a case, the above comparisons are said to be confounded with the blocks. In two or more replications, different sets of comparisons may be confounded in different sets of blocks. But if we choose a Latin square arrangement and arrange the several treatments such that neither should a treatment occur more than once in a row or a column nor all treatments occur in a row or a column, some comparisons will be confounded in the rows and some in the columns. Such arrangements are called Quasi-Latin squares (Yates, 1937).

2. THE GENERAL PROBLEM OF DOUBLE CONFOUNDING.

Let the number of plots in a row or a column be k and kn the total number of treatment combinations. If $k=nr$, then with r replications a latin square arrangement can be found in which some comparisons are confounded with the rows and some with the columns. In order that our experiment may not suffer from the defect of causing apparent effects due to the interaction of fertility and some of the factors it is always better in factorial experiments to group together all the blocks that form a complete replication. This imposes a further restriction on the design that the rows and columns can be broken into r sets each forming a replication with n blocks. For the actual layout we have to randomise the replications, both column and row wise, and then randomise the columns and rows within the replications.

The analysis of such designs does not present any fresh difficulty. We give here the general scheme of analysis in case in which there are m latin square arrangements of size $k\times k$ with r replications in each, confounding some effects with the rows and some with the columns. The rows and columns each have $m(k-1)$ degrees of freedom and variances due to them can be obtained from the row and column totals. The unconfounded interactions and main effects can be calculated from the treatment totals obtained from all the squares. Due to the property of orthogonality of the Latin square it is clear that the effects confounded with the rows (or columns) cannot be estimated from comparisons within the columns (or rows). Another property which gives the clue to the analysis is that any confounded interaction or main effect with the rows (or columns) in a replication can be estimated from other replications constituted by the rows (or columns) in which it is not confounded. This gives a rule which is similar to the rule of finding the sum of squares due to the partially confounded effects; we need choose the treatment totals only from those replications in which the particular effect is not confounded either in the rows or columns. This is at once detected by the two rules, stated above, regarding the estimability of an effect. The loss of information on the partially confounded effects is calculated in the same way as in the randomized incomplete block designs. Loss of information on any effect is one minus the ratio of the number of replications from which the effect is estimable to the total number of replications.

3. CONSTRUCTION OF DESIGNS WITH DOUBLE CONFOUNDING

The next problem is to enunciate the combinational problem and discuss the possibilities of the actual construction of designs. The case of symmetrical factorial confounded designs will be discussed first and some of the results are extended to the asymmetrical case.

Let there be 'a' factors each at s levels. If $k = s^b$ then the relationships $n\,k = s^a$, $nr = s^b$ and $1/r = s^{a-2b}$ hold. Evidently $a \leqslant 2b$. The values of $r = 1, 2, 3$ and 4 only give rise to designs which are of practical value. The useful values of r, s and b are listed below.

TABLE 1

r	s	b
1	prime or prime power	$a/2$
2	2	$(a+1)/2$
3	3	$(a+1)/2$
4	2	$(a+2)/2$
4	4	$(a-1)/2$

In the case of $r = 1$, the construction of designs can be very simply effected by first forming the design confounding certain effects in the rows and adjusting the columns such that an orthogonal set is confounded in them. The adjustment is facilitated by the property that each of the columns confounding an orthogonal set can be built up by chosing one element from each row.

The introduction of the geometrical method of getting at confounded designs, developed by Bose and Kishen (1941) when s is a prime or a prime power may be used at this stage. They have shown that s^{a-b} blocks confounding $(s^{a-b}-1)$ degrees of freedom can be identified with the bundle of flats each of b dimensions passing through a common $(b-1)$ flat at infinity. The nature of confounding depends on the way in which all the $(a-2)$ flats passing through the above $(b-1)$ flat at infinity stand in relation to the 'fundamental simplex' defined by them. From this we have the important deduction that two confounded systems in s^a plots blocks have their effects orthogonal if and only if their corresponding flats at infinity do not have a common $(a-2)$ flat at infinity. In the case $b = a/2$ (a is necessarily even), it is always possible to find $(a/2-1)$ dimensional flats such that they don't have any $(a-2)$ flat in common, but the best designs are only those in which the highest order interactions are confounded. After choosing a suitable pair the arrangement is supplied by the points of interesection of the bundle of flats emanating from the two $(b-1)$ flats at infinity, one set of flats corresponding to the rows and another to the columns.

With more than one latin square, different sets of effects can be confounded in the rows and columns. Sometimes it is possible to get balanced designs with half the number of replications necessary in the case of randomised block designs. We shall discuss only those designs in which $k \leqslant 10$ for only these will be of practical value.

If $s = 2$, $a = 4$, and $b = 2$, there are four factors A, B, C, and D and 3 degrees of freedom are confounded with the rows and 3 with the columns.

(a) Using two latin squares of size 4×4 we can confound the sets

AB ; CD ; ABCD.
AD ; ABC ; BCD.

in the columns and rows of one latin square and the sets

AC ; BD ; ABCD.
BC ; ABD ; ACD.

with those of the other. The relative information on the first and second order interactions is $\frac{1}{2}$ while the third order interaction is completely confounded.

(b) We may use the following sets

AB ; ACD ; BCD.
CD ; ABC ; ABD.

BC ; ABD ; ACD.
AD ; ABC ; BCD.

AC ; BCD ; ABD.
BD ; ACD ; ABC.

in three latin squares and get a balanced arrangement giving $\frac{2}{3}$ of the relative information on the first order interactions and completely confounding the second order interactions.

(c) With five latin squares we can confound the sets

AC ; BD ; ABCD.
AD ; ABC ; BCD.
BC ; ABD ; ACD.

two by two in three latin squares and the sets

A ; B ; AB.
C ; D ; CD.

repeatedly in two latin squares. The relative information on all the effects is $\frac{3}{5}$. If we ignore the gain in efficiency due to arrangement in a latin square, the randomised block design with five replications confounding the above sets is better than the Latin square arrangement in that the relative information on each effect is $\frac{4}{5}$.

In the case $s = 2$, $a = 6$, $b = 3$, there are six factors A, B, C, D, E and F each at two levels. There is no simple balanced design in less than five replications. Unbalanced designs can always be obtained in one or more latin squares by confounding orthogonal sets of effects in the rows and columns.

In the case, $s = 3$, $a = 4$, $b = 2$, it has been shown by Yates (1937) and Nair (1938) that there exists a design in four replications with randomised blocks which gives balance to the partially confounded second order interactions. The fact that the confounded effects in several replications are orthogonal facilitates the balanced arrangement in two Latin squares. The layout is obtained by merging the replications two by two as indicated in para 4. of § 3. The relative information on the partially confounded interactions in $\frac{1}{2}$, where as in the balanced randomised block design is $\frac{3}{4}$. But the Latin square arrangement saves the cost of two more replications. By replicating the two latin squares the design becomes as efficient as the corresponding randomised block design with equal number of replications, even if we ignore the fact that the variance per plot is reduced in the latin square arrangement.

It is not worthwhile discussing the higher types of designs from the practical point of view. But we may give the general result that if in s^a design with s^b plots blocks ($b = a/2$) balance can be obtained in randomised blocks with $2r$ replications and the effects that are confounded in them can be split into r pairs of orthogonal sets, then balance can be achieved in r latin squares. If $2r+1$ replications are necessary in randomised blocks then utmost $2r+1$ latin squares are required. But these will not be as efficient as the randomised block designs.

We shall investigate the possibilities of constructing designs in the case $r>1$ and derive the necessary and sufficient conditions in particular cases. Whenever a design with conditions stated in para. 1 of Section 2 exists then it is always possible by rearrangement to adjust such that the first, second etc., sets of rows and columns form complete replications. By adjustment the treatments in the quasi-latin square form into r^2 groups of n^2 treatments each, the groups coming in the same row or column not having any element in common. These groups need not necessarily be identical in sets of r, but this holds automatically in the case $r=2$. If we take any n columns forming a complete replication together with the division into r groups as formed above we get an arrangement in blocks of size k/r confounding $(nr-1)$ degrees of freedom. In these the $(n-1)$ degrees of freedom confounded in the n columns are necessarily involved. Dealing with columns only we find r such replications in blocks of size k/r confounding $(nr-1)$ degrees of freedom each. In order that the effects confounded in any replication of the rows may be estimated completely from the replications in which it is not confounded, it is necessary that any row of the latin square should be capable of being built up by choosing one element from each of nr blocks of size k/r of any replication of the columns. This shows that the effects to be confounded in the rows should be orthogonal to the totality of the confounded effects in the various replications obtained from the columns in blocks of size k/r. A similar result holds for the effects confounded in the columns and the two results are complementary.

In the case $r=2$, there are four groups and the treatment combinations in the diagonal groups are the same. If we consider any replication of the columns, confounding $(n-1)$ degrees of freedom, then the effect with one degree of freedom, confounded in the two groups into which it is split in the manner indicated above, together with $(n-1)$ degrees of freedom determines the totality of the $(2n-1)$ d.f. confounded in a replication with blocks of size $k/2$. Similarly the same effect confounded with the groups in the other replication of the columns determines the totality of the $(2n-1)$ degrees of freedom confounded in blocks of size $k/2$. The effects that are confounded with the rows must be orthogonal to these different effects confounded above. So the necessary and sufficient condition for the existence of a quasi-latin square design with two replications is that there exists a common effect which together with effects confounded in the columns (or rows) and their interactions with it are orthogonal to the effects to be confounded in the rows (or columns). The number of different solutions will depend on the number of such available effects. The actual construction of some designs are given below.

If $s=2$, $a=5$, $b=3$, there are 5 factors A, B, C, D and E each at 2 levels. The higher level may be indicated by the small letter and lower level by the absence of a letter. It is desired to confound the effects.

ABC ; ADE ; BCDE.
ABD ; BCE ; ACDE. .. (3.3)

with the rows and the effects

ACE ; BCD ; ABDE.
ACD ; BDE ; ABCE. .. (3.31)

with the columns. It is found that there are two effects ABCD and ABCDE which satisfy the conditions given above. The solution of the above problem given by Yates (1937) is that corresponding to ABCD. The alternative solution is given below. ABCDE in conjunction with (3.3) gives

ABC ; ADE ; BCDE ; ABCDE ; A ; BC ; DE.

ABD ; BCE ; ACDE ; ABCDE ; B ; AD ; CE.

Two replications are formed in blocks of 4 plots and arranged as indicated below.

TABLE 2. QUASI-LTAIN SQUARE FOR 2^5 DESIGN.

abcde	*ade*	*cd*	*bd*	*ce*	*a*	*be*	*abc*	R_1
ad	*abcd*	*bde*	*cde*	*ae*	*c*	*abce*	*b*	R_2
ab	*ac*	*e*	*bce*	*d*	*acde*	*bcd*	*abde*	R_3
ace	*abe*	*bc*	1	*acd*	*de*	*abd*	*bcde*	R_4
de	*b*	*abd*	*ae*	*abcde*	*bce*	*ac*	*cd*	R_5
be	*d*	*a*	*abde*	*bc*	*abcd*	*cde*	*ace*	R_6
c	*bcde*	*abce*	*acd*	*ab*	*bd*	*ade*	*e*	R_7
bcd	*ce*	*acde*	*abe*	*bde*	*abc*	1	*ad*	R_8
C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	

The comparisons (3.3) are confounded in the rows R_1 to R_4 and R_5 to R_8 and ABCDE is confounded in the groups differentiated above. Now there remains only shifting of the treatments in each group without disturbing the rows, so that the comparisons (3.31) are confounded in the columns C_1 to C_4 and C_5 to C_8. The design given above is arranged in this manner and gives a suitable layout only after proper randomisation discussed in introduction. The relative information on the partially confounded effects is $\frac{1}{2}$. We may arrive at a balanced arrangement in 5 latin squares by confounding in each four sets of the five sets of comparisons obtained by adding :

$$ABE ; \quad CDE ; \quad ABCD. \qquad .. \quad (3.4)$$

to (3.3) and (3.31). The relative information on each of the confounded interactions is only $\frac{2}{5}$. Designs confounding any four of the sets.

AB ; CD ; ABCD.
AC ; DE ; ACDE.
AD ; BE ; ABDE.
AE ; BC ; ABCE.
BD ; CE ; BCDE.

are also possible but may not be of use for the first order interactions are partially affected.

If $s = 2, a = 3, b = 2$, there are three factors A, B and C each at two levels. Various types of confounding and the common effects with whose help the designs can be formed are given in Table 3.

In the general case $r > 2$ we may state the necessray and sufficient condition as follows :

(a) If we denote the effects to be confounded in the Ist, 2nd .... rth replication of the columns by $C_1, C_2, \ldots, C_r$ respectively, then there exists a set of effects $S_1, S_2, \ldots, S_r$ each with $(r-1)$ degrees of freedom which together with the effects $C_1, C_2, \ldots, C_r$ and corresponding interactions which we denote by C_iS_j, i, j=1,2 ....r are all orthogonal to the effects confounded in the rows.

(b) The r^2 groups obtained from r complete replications confounding the effects $S_1, S_2, \ldots, S_r$ are such that r complete replications can be obtained by choosing one group from each of the above r replications.

Table 3. Quasi-latin square designs for 2×2×2

number of latin squares	effects confounded with: rows	groups	columns	relative information
1	AB ; BC	B	ABC ; ABC	AB = AC = ½, ABC = 0
3	AB ; BC BC ; CA CA ; AB	B C A	ABC ; ABC ABC ; ABC ABC ; ABC	AB, BC, CA = ⅓, ABC = 0
1	BC ; CA	C	AB ; ABC	AB = BC = CA = ABC = ½
3	AB ; BC BC ; CA CA ; AB	B C A	AC ; ABC AB ; ABC BC ; ABC	AB, BC, CA = ⅓, ABC = ½
3	A ; AB B ; BC C ; AC	A or B B or C C or A	ABC ; ABC ABC ; ABC ABC ; ABC	A = AB, B = BC, C = CA = ½, ABC = 0

In the case $s = 3$, $a = 3$, $b = 2$, with a latin square of size 9×9 only some degrees of freedom belonging to the triple factor interaction need be confounded. The 9 degrees of freedom belonging to the triple factor interaction can be split into 4 orthogonal sets, T_1, T_2, T_3 and T_4 of 2 degrees of freedom each. They are given below :

Table 4. Orthogonal set for the 2nd order interaction 3×3×3

combination of 1st. and 2nd factors		T_1			T_2			T_3			T_4		
		levels of the third factor											
1	00	0	2	1	0	1	2	0	2	1	0	1	2
2	10	1	0	2	2	0	1	1	0	2	2	0	1
3	20	2	1	0	1	2	0	2	1	0	1	2	0
4	01	2	1	0	1	2	0	1	0	2	2	0	1
5	11	0	2	1	0	1	2	2	1	0	1	2	0
6	21	1	0	2	2	0	1	0	2	1	0	1	2
7	02	1	0	2	2	0	1	2	1	0	1	2	0
8	12	2	1	0	1	2	0	0	2	1	0	1	2
9	22	0	2	1	0	1	2	1	0	2	2	0	1

(a) Two sets T_i and T_j may be chosen for confounding in all the replications of the rows and columns respectively so that no information on T_i and T_j can be had from one square. The design is constructed with the help of choosing any third set T_k, $k \neq i$ or j or any suitable second order interaction for the effects S_1, S_2 and S_3 defined in para. (b) above. With two squares confounding T_1 and T_2 in one and T_3 and T_4 in another we have a balanced arrangement which gives ½ the relative information on all the four.

(b) A second system of arrangement is to confound the sets T_1, T_2 and T_3 in the replications of the rows and the sets T_2, T_3 and T_4 in those of the columns. The relative information on T_2 and T_4 is $\frac{1}{3}$ and that on T_1 and T_3 is $\frac{2}{3}$. The construction is effected by choosing a second order interaction for S_1, S_2 and S_3. The solution is given below.

TABLE 5. THE QUASI-LATIN SQUARE DESIGN FOR $3 \times 3 \times 3$

10	01	82	50	71	32	00	21	42	
60	81	12	70	31	52	20	41	92	T_1
80	11	62	30	51	72	40	91	22	
31	72	50	22	90	41	81	62	10	
51	32	70	42	20	91	11	82	60	T_2
71	52	30	02	40	21	61	12	80	
22	90	41	81	62	10	72	50	31	
42	20	91	11	82	60	32	70	51	T_3
92	40	21	61	12	80	52	30	71	
T_4			T_2					T_6	

The confounded effect in the group of any replication row or column wise is the comparison defined by the following table.

TABLE 6. THE COMMON CONFOUNDED EFFECTS IN GROUPS OF TABLE 5.

levels of the third factor	comparison
0 + 1 + 2	1 3 2
0 + 1 + 2	6 5 4
0 + 1 + 2	8 7 9

The design of Table 5 is useful for layout only after proper randomization. We may choose another square confounding T_2, T_1, T_3 and T_3, T_2, T_4 together with the above for balancing. The relative information is $\frac{1}{4}$ on all.

TABLE 7. 8×8 QUASI-LATIN SQUARE FOR 2^4 DESIGN.

ABCD		ABCD		ABCD		ABCD		
44	42	23	31	33	21	14	12	ABC
41	43	22	34	32	24	11	13	
22	34	44	12	41	13	23	31	BCD
33	24	11	21	14	43	32	42	
14	13	32	43	22	31	44	21	ABD
11	12	41	24	23	42	33	34	
23	31	14	42	11	34	22	43	ACD
32	21	33	13	44	12	41	24	

In the case $r = 4, s = 2, a = 4, b = 3$, there are four factors A, B, C and D. The effects ABC, BCD and CDA can be confounded with the rows and the effect ABCD with the columns. The unrandomized design is given in Table 7 where the first and second figures stand for the following. $1 = (1)$, $2 = a$, $3 = b$, $4 = ab$ and $1 = (1)$, $2 = c$, $3 = d$, $4 = cd$. Thus 44 stands for *abcd*.

4. Extension to the general factorial design.

We can, now, extend some of the principles enunciated above to the asymmetrical factorial designs of the form $p \times q^2$ in pq plots blocks. When q is a prime or a prime power it is possible to divide the $(q-1)^2$ degrees of freedom belonging to BC into $(q+1)$ orthogonal sets of $(q-1)$ degrees of freedom each arising from the comparisons.

$$Q^i_1, Q^i_2, \ldots . Q^i_q;\ i = 1, 2, \ldots . (q-1) \qquad .. \quad (4)$$

It has been shown by Nair and Rao (1941) that a design for pq^2 in pq plots blocks can be found in $(q-1)$ replications if $p \leqslant q$ obtaining balance on any of the comparisons (4) and its interaction with A. For complete balance $(q-1)^2$ replications are necessary. Since $k^2/r = pq^2$, $k = pq$, $r = p$; $(q-1)$ replications can be arranged in the columns and another set of $(q-1)$ replications giving balance over an orthogonal set can be arranged in the rows if $p = (q-1)$. It is easily seen that A is the only possible effect that can be confounded among the groups. Thus only one type of square is possible for any sets of replications. A completely balanced arrangement can be found in $(q-1)/2$ (if q is odd) or $(q-1)$ (if q is even) latin squares.

For the asymmetrical designs $p \times q$ in blocks of p plots the combinatorial restrictions give,

$$k^2/r = pq,\ k = p,\ p = rq \qquad .. \quad (4.1)$$

so that $pq = rq^2$ and the design with two factors A and B can be reduced to a design with three factors by the introduction of pseudovariates A_1 and A_2 at levels r and q, building up the levels of A. The construction of designs may be followed as in para. 1 of Section 3. The analysis may also be carried with the use of pseudovariates. Thus 6×3 design is possible in a quasi-latin square 6×6 confounding some effects of the interaction AB with the rows and some with the columns. The main effects are completely preserved. The design and analysis will be identical with that of 2×3^2. The designs 9×3, 8×4, etc. can always be arranged in 9×9, 8×8 etc. quasi-latin squares confounding the first order interaction only, with the use of pseudovariates.

The use of quasi-latin squares in confounded arrangements for factorial designs is explained in this article. The necessary randomization, the proper analysis together with the method of construction of the possible designs are discussed. The possibilities of arranging varietal trials in quasi-latin squares and the various types of designs have been already found (Rao : 1943) and a description of the methods is given below.

5. Qasi-latin squares in varietal trials.

The methods developed in previous Sections can be extended to the case of experiments involving a number of varieties. These designs not only enhance the precision of comparisons but the number of replications is sometimes halved. The fundamental problem of arrangements in quasi-latin squares connected with varietal trial may be stated as follows.

There are v treatments which are denoted by $T_1, T_2, \ldots . T_v$, to be tested in n lattice squares of k^2 cells each such that the i-th variety is replicated r_i times on the total and the treatments T_i and T_j appear together λ_{ij} times in the rows and columns combined in all the squares and μ_{ij} times in all the squares.

$$\lambda_{ii}=0,\ \lambda_{ij}=\lambda_{ji},\ \mu_{ij}=\mu_{ji} \quad .. \quad (5.1)$$

$$r_1+r_2+\ldots+r_v=n\,k^2. \quad .. \quad (5.2)$$

$$2(k-1)\,r_i=\lambda_{i1}+\ldots+\lambda_{iv}$$

$$(k^2-1)r_i=\mu_{i1}+\ldots+\mu_{iv} \quad .. \quad (5.3)$$

The analysis of the most general type of design can be carried out as indicated below. The normal equations giving varietal comparisons (the varietal effects are denoted by $t_1,\ldots t_v$ for comparative purposes) are

$$Q_i=\frac{r_i\,(k-1)^2}{k^2}\,t_i-\frac{1}{k}\sum_j\lambda_{ij}\,t_j+\frac{1}{k^2}\sum_j\mu_{ij}\,t_j \quad .. \quad (5.4)$$

where Q_i = the sum of observations on the i-th treatment
— the sum of row and column means in which the i-th treatment occurs
+ the weighted total of square means, the weight being the number of times the i-th treatment occurs in a square.

If $t_1, t_2, \ldots t_v$ constitute a solution of the system of normal equations (5.4) then the analysis of variance table leading to tests of significance can be given as follows.

TABLE. 8. ANALYSIS OF VARIANCE TABLE

variation due to	d.f.	sum of squares
squares	$n-1$	U
columns	$n(k-1)$	U
rows	$n(k-1)$	U
treatments	$v-1$	$\Sigma t_i Q_i$
error	—	—
total	nk^2-1	U

The expressions marked by U are to be calculated in the usual manner. The sum of squares due to columns and rows are to be calculated from 'within squares'. The expressions marked by — are obtained by subtraction.

The difference of i-th and j-th treatment effects is estimated by (t_i-t_j) and if this is equivalent to $l_1Q_1+\ldots+l_vQ_v$ then the variance of (t_i-t_j) is given by $(l_i-l_j)\,\sigma^2$, where σ^2 is the error variance so that using the estimate of σ^2 given by the error line in the above analysis of variance table we can test for treatment comparisons.

The simplest case of the quasi-latin square arrangements for varietal trials is when all the varieties occur in each square and $\lambda_{ij}=1$ for all i and j. These designs are derivable from the solution of the balanced incomplete block designs for the system of parameters

$$v=m^2,\ b=m^2+m,\ k=m,\ r=m+1 \quad .. \quad (5.5)$$

where $m = p^s$ (p being a prime and s an integer). This is a resolvable series so that the design can be reduced to $(m+1)$ groups of m blocks each containing m plots. If $(m+1)$ is even we can combine the groups two by two to form $n = (m+1)/2$ quasi-latin squares. This is possible due to the orthogonal property of these groups, the rows and columns of a quasi-latin square representing two separate replications in the original design. Thus the number of replications can be halved.

In this special case $\mu_{ij} = 1$ for all i and j so that the normal equations become

$$Q_i = \left(\frac{n(k-1)^2}{k^2} + \frac{1}{k}\right)t_i - \frac{1}{k}\Sigma t_i + \frac{1}{k^2}\Sigma t_i \qquad \text{.. (5·6)}$$

so that

$$t_i - t_j = (Q_i - Q_j)\frac{k^2}{n(k-1)^2 + k}$$

and

$$V(t_i - t_j) = \frac{2k^2}{n(k-1)^2 + k}\sigma^2 \qquad \text{.. (5·7)}$$

Various other types of designs can be derived by allowing λ's and μ's to assume assigned values.

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