Estimation of Demographic Parameters For India

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This is to certify that the enclosed thesis entitled "Estimation of demographic parameters for India" embodies the research work carried out by Sri Samir Guha Roy under my supervision.

The thesis may kindly be examined and considered for the Ph.D. degree of the institute.

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PREFACE

There has been a persistent emphasis both at the national and international levels on the development and application of techniques of estimation of demographic variables from incomplete or inaccurate data in the less developed regions. Consistent with the theme of research, the present study tries to provide estimates of vital rates and prospective population trends in India and to form a basis for measuring such human needs as education, employment, family welfare, and few others in terms of population size, growth, age structure and nuptiality. The other major objective is to undertake further utilization of data generated by the national population censuses and vital registration systems for estimation and analysis.

The work was an outcome of my applied research in demography and of the lectures for courses in demography and actuarial statistics at the graduate and postgraduate levels in the Indian Statistical Institute and elsewhere. Initial work was undertaken in association with late Professor Ajit Das Gupta, FIA, former UN Consultant, and was continued under the advice of Dr. J. Roy, Research Professor of the Indian Statistical Institute.

Late Professor Ajit Das Gupta was a pioneer in demographic estimation and analysis in India. It is likely, as his research associate, that his views can be found to permeate this work. I am indebted to him for his valuable suggestions in the early stages of this research.

My profound gratitude is due to Professor J. Roy for his very kindly argeeing to supervise the present work and extending all kinds of computer facilities. As a teacher and guide, he has been stimulating, supporting and patient. I am grateful for his thoughtful suggestions, comments and constructive criticisms in refining the work.

Of the several experts with whom I have generally discussed my research, special mention should be made of Professor Asok Mitra whose encouragement, invaluable advice and help, generously given at the time of personal consultations, are gratefully acknowledged.

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As a professional associate of the East-West Population Institute, Hawaii, I had fruitful discussions with its faculty members. Their suggestions and comments are deeply appreciated. Dr. Dorothy Nortman of the Population Council provided me a computer tape for which I am very much thankful.

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1. Introduction

A major concern of all developing countries, in general, and India, in particular, has been the tremendous growth of population during the last three or four decades. The increase in the production of goods and services as a result of planning for socio-economic development during this period has barely kept pace with the growth of population. As a result there has been very little increase in the level of living of the population measured on a per capita basis. This is the negative aspect of population growth, but one should at the same time recognise the enormous possibility inherent in the effective utilisation of the immense man-power of a big country like India. In any case, the Governments concerned have to accept the responsibility for providing food, shelter, education, health and social services and work for the increasing population. Demographic studies relating to current age-sex composition of the population and its future growth have therefore assumed great importance in India and other developing countries.

Any such study must be based on current statistics on the distribution of the population by age and sex, mortality and fertility rates and patterns of in- and out-migration. One would also have to anticipate, either on the basis of past statistical evidence, or realistic logical assumptions, future trends in the rates of mortality and fertility and patterns of migration. Unfortunately, relevant information actually available on these aspects is very meagre in quantity, and whatever is available is also of uncertain quality at best.

2. Inadequacies of empirical data

The main sources of demographic statistics in India are the decennial census, the records of registration of births and deaths and numerous cross-sectional sample surveys conducted by various official and private organisations to assess fertility rates from reproductive histories of couples primarily in connection with the study of knowledge, attitude and practice of family planning methods by married couples.

Considering the scale of operations, the paucity of resources and the extent of illiteracy in the country, the decennial population censuses of India must be recognised to be a great success. Compared to demographic data obtained from other sources, census data for India are definitely much more reliable. But even census data must be very carefully smoothed before these can be used meaningfully. Of the many causes that affect the quality of census data in India the following are very important : (1) incomplete coverage of region because of civil commotion (2) political pressures on certain segments of the population to inflate or deflate the counts (3) inability of a large majority to state their correct ages because of ignorance and illiteracy (4) organisational problems in controlling a very large number of census enumerators most of whom work for a pittance and (5) lack of data processing resources.

Compulsory registration of births and deaths is acknowledged to be a total failure in India : except in important urban centres, the system simply does not exist. A Sample Registration System is in vogue to provide estimates of deaths and births on a sampling-basis. Even these estimates cannot be considered to be very reliable, though these must be taken as perhaps the best available.

Though numerous sample-surveys have been conducted in India to estimate fertility rates, most of them are very limited in scope and coverage. All are cross-sectional and because surveys conducted at different times are by different organisations using different concepts and methodologies, comparison of fertility estimates at different points of time is not valid. Being based on relatively small samples, the estimates obtained from many such surveys are subject to considerable fluctuations of sampling; but most reports do not provide estimates of the margin of sampling error.

3. Broad features of the demographic situation in India

Despite all the deficiencies in the basic demographic data in India, certain features of the growth of population in India are fairly evident. Since attainment of independence in 1947, the population of India has been growing at an alarming rate of about two percent per year. With improved control of infectious diseases and social services, the rate of mortality started declining from as early as 1950, but the rate of decline slowed down in the seventies and most scholars agree that there is little prospect for any further decline in the rate of mortality. The rate of fertility however remained fairly constant upto 1971. As a result of the efforts of the Government to propagate family planning, the rate of fertility started declining during the seventies but very very slowly. The volume of net international migration is insignificant compared to the total population of India. Consequently, with constant fertility and declining mortality, the population of India went on increasing at an increasing rate, until 1971. But with a slight decline in fertility, the 1971-81 decade saw a slight decreases in the rate of growth of the population of India.

4. Objectives of the study

Though the above broad picture is fairly visible even through the untidy mess of Indian demographic statistics, it is only when one wants to measure quantitatively the rates of fertility, mortality and growth that the limitations of the data stand squarely in the way of such estimation. Eminent demographers, working with such fallible demographic data for a number of developing countries, have developed special statistical techniques to make the best use of available data : to smooth the data before using them for purposes of estimation so that the derived estimates do not show unacceptable characteristics. The main objective of this study is to use these methods of smoothing directly, or at times after suitable modification of the method, on some chosen sets of Indian demographic data and to derive simultaneously quantitative estimates of rates of growth and its components : fertility, nuptiality, and mortality for India around the 1970's. This period is of intrinsic interest as the period during which the rate of growth of population in India started decelerating after initial acceleration after independence. But the most important reason of course is that detailed data are available only upto this period. Full results of the 1981 Census are yet to be published.

In this study, we apply a number of different methods on the same or parts of the same data set to estimate demographic parameters. Let us confess at this stage that since the data sets are acknowledged to be defective there is no way for us to demonstrate the applicability of one or the other method of estimation (Statistical tests of significance are not applicable). When however application of a number of different methods of estimation on the same or parts of the same data set, using the same or other related demographic variables, lead to estimates which are visibly close to one another, we tend to have more confidence about the estimated magnitude of the demographic parameter. We do not seek to quantify this degree of confidence in any way as we are of the opinion that formal procedures of statistical inference are not applicable under these circumstances.

The demographic characteristics considered in this study are : current age-sex distribution, age specific mortality and fertility, short-term projection of population and nuptiality. This is done in the next five chapters. The first three chapters are devoted to an examination of the quality of primary statistics relating respectively to (1) the distribution of the population by age and sex (2) the distribution of age at death and (3) the age-specific pattern of fertility. Various methods are adopted to smooth the primary data relating to age-distribution, mortality and fertility. The smoothed data are then utilised to estimate age-specific mortality and fertility rates. An attempt is made to analyse the variation in fertility rates between different States of India in terms of certain socio-economic variables. Short-term forecasts about the population of India is made in the next chapter, using a number of different techniques. In the last chapter an attempt is made to study the pattern of female nuptiality in India.

5. Data-base of the study

The empirical data used in this study have been compiled from a number of sources. The main source of information is of course the decennial population censuses of India : mainly the 1961, 1971 and 1981 censuses. Since full results of the 1981 census were not available at the time, age distributions based on a 5% sample of the census data have been used. To study the pattern of mortality, the main source of information was the Registrar Generals' report for the year 1971 based on the Sample Registration Scheme and the Infant and Child Mortality Survey for the year 1978-79. For studies on fertility, the main sources of information are the Registrar General's fertility survey for 1972, and the early rounds of the National Sample Survey of India. The data base for the study of nuptiality consists of the census reports for 1901 to 1971 and a number of local surveys. It is indeed a weakness of the present study that the empirical data-base is rather outdated, but that is due in general to the delays in availability of statistical information, common for all developing countries.

6: Tools and techniques used

The main demographic tools used in this study are enumerated briefly below. To examine digit-preference-bias in reported ages, the indices used are : Whipple's index, Myers' Index. The " Joint Score" recommended by the United Nations has also been calculated to examine the deviation from smoothness of the reported age-distribution. A "transitional age-structure model" has been used to adjust the reported age distribution at the 1981 census.

To examine the completeness of death-reporting the " completeness index" C introduced by Preston and Hill has been calculated. This has also been recomputed relaxing the assumption of stability of the population using a " modified sectional growth-balance" equation, and the results are more or less similar. To estimate child mortality rates, Brass-Sullivan and Brass-Trussell methods have been used. These have been smoothed by using the logit transformation. The Weibull function has been fitted to the age-specific cumulative survival rates by graphical methods and appears to give a reasonable fit in the range 0-9 years. Adult mortality rates are calculated using widowhood data and techniques due to Brass and Hill and Brass and Trussells.

Age specific fertility rates have been smoothed by using Coale's technique of relating the ratio of reported and natural fertility to an empirical function describing extent of volume of control of natural fertility, through the exponential function. An alternative method of comparing cumulative current fertility with average parity, introduced by Brass has also been tried to estimate age-specific fertility rates. Another method due to Barclay, Coale, Stoto and Trussell has also been tried. A "Relational Gompertz Model" has been fitted to the ratio of the cumulative estimated fertility rates to a corresponding standard. The method of "Path Analysis" has been used to analyze the variation between the age-specific fertility rates in different States of India in term of a number of demographic, socio-economic and family planning variables.

Overall vital rates for 1971, 1972 and 1975 obtained from the Sample Registration System were corrected for incompleteness of birth and death recording by matching the difference between the adjusted birth and death rates with the overall population growth rate, which was independently estimated using a quadratic function in time to describe the rate of growth between the Censuses of 1951, 1961 and 1971. Age specific mortality rates were adjusted with reference to United Nations model set of Life Tables corresponding to the estimated overall death rates. This was done separately for males and females. To estimate the age-sex distribution of the population at a future point of time, one has to start with the age-sex distribution of the population at a base-point (1971). Since the age-sex distribution obtained from Censuses have gross defects which cannot be smoothed mechanically, it was decided to first estimate the size of different age-cohorts by applying overall growth rates on the census data and then applying appropriate age-specific survival rates obtained from relevant UN model Life Tables. This was done for the male population but the age-distribution of the female population was estimated by expressing the ratio of females to males as a quadratic function of age. The moving average procedure tried by the Census Actuary on the 1961 Census data was also tried on the 1971 Cendus data to obtain a second set of adjusted base data. Using such adjusted base data for 1971, estimates were obtained for the population of India in 1981 and 1986. Similarly estimates were made for 1986, 1991, 1996 and 2001 - using adjusted 1981 census as base data. Quadratic growth rate functions were estimated separately for the population in individual States of India using the Census data for 1961, 1971 and 1981 and used to estimate the population in 1991 and 2001 in the different States. For the purpose of ancillary population estimates such as those of students, the populations at individual ages in the range 5-19 were derived from the grouped data using a structural graduation model and Sprague-method.

To graduate the observed proportion of females remaining single at various ages, a linear regression of the logit of the observed proportion on the corresponding logit of a standard proportion was fitted. This was done individually for every decennial census from 1901 to 1971 using two different sets of standards. Using the raw and the smoothed proportions the singulate mean age at marriage for each of these Census years was calculated. An alternative model due to Coale and McNeil was also fitted to the same data. The proportions single at various ages in different cohorts and in different years as well as the corresponding mean age at marriage were estimated for the years 1976 and 1981-86. On assumption of plausible changes in the age at marriage entrance, the tempo at which marriage occurs and the ultimate proportions married, Coale's three-parameter nuptiality model was used for estimating ever married. The marriage behaviour was further studied with reference to Coale's index of nuptiality. Using a computer model, the risk of widowhood was worked out. A new index expressing the inter-sex difference of availability of marriage partners was used.

While appreciating the need to strengthen the demographic data collection system in India, it was found that the use of analytical techniques of estimation appeared rewarding in judging the quality of the basic data for analysis. The choice of methods however could not always be made solely on the basis of scientific objectivity because of the constraint in data availability.

7. Main findings

The main contribution of this study is to try a number of alternative approaches for estimating important demographic parameters for India from empirical data that are widely recognised to be not of a high quality and to show that these different approaches lead to a reasonable estimates, close to one another. Substantive findings are described briefly below.

Compared to previous Censuses, there has been a noticeable improvement in the quality of the age returns in the 1981 Census of India as seen from the very much smaller values of various indices which measure unevenness of the reported age-distribution. As usual however, there is gross underenumeration of children in the age group 0-4. The age distribution for 1981 adjusted through a transitional age-structure model agrees reasonably well with the projected figures using 1971 Census data as base.

The completeness of death-reporting of course varies with the age at death. The present study shows that for the age range 10-59 the completeness of death reporting in 1971 Sample Registration System was around 82.3% for all persons, 89.0% for males and only 75.2% for females. The assumption of over-all 5 per cent underregistration of deaths made by the official agencies is thus not supported by the present study. Various methods of estimation of survivorship ratios for children yield values in the range 0.8720 - 0.8742 at age 2; between 0.8580 and 0.8604 at age 3 and within the range 0.8444 - 0.8470 at age 5. The infant mortality rates were estimated to be around 135 and 155 respectively for males and females during 1961-70. For 1971, the expectation of life for males was estimated as 49.7 years at birth, 41.7 years at age 20, 26.8 years at age 40 and 13.1 years at age 60. The corresponding figures for females were 47.9 years at birth, 39.5 years at age 20, 25.0 years at age 40 and 11.9 years at age 60.

Smoothed values for age-specific marital fertility rates for 1971-72 and 1978 in 5 year age-groups are given below :

age-group	15-19	20-24	25-29	30-34	35-39	40-44	45-49	Total marital ferti-lity
1971-72	0.209	0.328	0.276	0.260	0.185	0.126	0.022	7.0
1978	0.176	0.283	0.217	0.188	0.122	0.072	0.008	5.3

which show a considerable reduction over a short period. A comparison of reported marital fertility with natural fertility revealed some evidence of practice of fertility control in pre-1970 period, but its impact was however weak enough to induce any change in fertility level during the period. 'Path nalysis" carried out on the eight variables measuring fertility, socio-economic condition and family planning efforts for 27 States and Territories of India in 971 indicated that acceptance of family planning programme had only an indirect effect on fertility but social development leading to increase in the age at marriage and level of female education are more likely to bring down the fertility rate.

Based on one period, two periods and cohorts, an analysis of data on age at marriage for females and marital status showed that the mean age at effective marriage of females remained fairly stable at around 16 years till the fifties and started rising rapidly during the sixties and seventies. It may rise to nearly 19 years in 1986 and the average age at effective marriage of females in the birth cohort (1956-61) would be around 18 years. The estimated proportion of evermarried females in-the age group 15-19, estimated by the different methods for the years 1981 to 1986 differ considerably, but are consistent in showing a very rapid decline. The combined proportion of widowed, divorced and separated females in different age groups in 1971 smoothed by the different methods differed considerably between each other and also from the Census data. The number of single males and single females in different age groups estimated for 1971 showed at every age a preponderance of single males over single females. Mortality rates for married persons in different age groups were estimated on the basis of certain assumption separately in males and females for the 5 year periods 1971-76 and 1976-81. A comparison of probability of marriage remaining intact and risk of widowhood between two quinquennia considered reflects slightly improved mortality conditions for married persons. A tentative result derived shows a risk of widowhood of little over 4 per cent at age 40. The combined proportions of widowed-divorced-separated estimated in this study reveal an overall under-estimation of census proportions to the extent of about 22 per cent. The estimated values of an index expressing the inter-sex difference of availability of mates show serious imbalances between the numbers of males and females in marriageable ages. Broadly speaking, consideration of all marital statuses in one comprehensive study permits consistent estimates of proportions single, married and widowed-divorced-separated.

Our projected population agrees broadly with the estimates made by the U.S. Bureau of the Census and World Bank in that the population of India is likely to cross one billion mark in about 2001. The female population in the reproductive age range 15-44 would go up by 124 million in the period 1971-2001. It is however encouraging to note that the proportion of young population of below age 15 is expected to decline by about 9 per cent during 1971-2001 to attain less than 35 per cent at the turn of the century. The working age (15-59) population, on the other hand, would probably increase by about 7 points during the same period. There would be an expected addition of more than 30 million elderly people (60+) to the total population during the three decades preceding 2001. A reversal of the current situation of rejuvenation and ageing processes is thus quite a safe forecast for the 21st century for India.

CHAPTER ONE

AN EVALUATION OF DEMOGRAPHIC DATA FOR INDIA

1.1 Introduction

In the matter of availability of population data, India is relatively well placed among the developing countries. There have been a number of decennial censuses in India, the first one in 1872, and subsequently in 1881, 1891, 1901, 1911, 1921, 1931, 1941, 1951 (first census after independence), 1961, 1971 and 1981 with complete simultaneous coverage of the whole country. The coverage has not however been uniform over all these census years. In 1981 census, no enumeration could be undertaken in the State of Assam due to disturbed conditions there. The original data for the census counts of 1941 are not available, but one can have the tables constructed on the basis of only 2 per cent sample of the census individual slips. Since 1971, fertility data are being collected in the censuses. While 1971 census collected data on live births during the preceding one year, 1981 census elicited information on both current and retrospective fertility.

Apart from the census, birth and death registration had been compulsory in the whole country since 1873. The Sample Registration Scheme (SRS), initiated in 1964-65, provides improved estimates of vital rates for India and the constituent States. During 1970s two All-India surveys were conducted by the Registrar General's Office, one being the 1972 fertility survey and the other the 1979 infant and child mortality survey (India Office of the Registrar General, 1976, 1980 and 1982). While the second survey provides fertility measures directly, the appropriate mortality estimates may also be derived indirectly from the information collected in the survey on the proportion of children dead among those ever born. Intercensal birth rate estimates by reverse survival technique are also regularly provided by the Census Actuaries.

India has been in the research frontier of sample surveys, and the National Sample Surveys (NSS) have in their possession collected information on vital statistics over its different rounds of enquiry. Data on fertility were also collected regionally in the socalled Standard Fertility Surveys sponsored by the Demographic Communication Action Research Advisory Committee, and also by the Population Research Centres (PRC) of the Ministry of Health and Family Welfare. There have been a number of other surveys covering the important demographic variables, but all are local, and thus not representative for the whole country.

1.2 Deficiencies in the demographic data

No estimation of demographic parameters by analytical methods would be needed if the century-old censuses and vital registration system, " the components of the traditional demographic estimation inputs", were perfect. But both these remained far from being complete and reliable, and thus rendered the task of estimation much more difficult and complex.

The Civil Registration System should be the appropriate source of data on fertility, mortality and nuptiality, but gross deficiencies in coverage with respect to both area and subject make it totally unacceptable. Based on dual records system, the SRS vital rates indicate some broad levels and trends. Though no objective verification of the extent of underregistration or sex-age differential in registration has been undertaken, the SRS data are believed to be more deficient (see Chapter Two) than is officially assumed (India Office the Registrar General, 1974).

The general feature of the whole census series is the undercount of infant and young children, and misstatement of age. Differential coverage of the population by sex is also commonly present, the females being more under counted than males. Ages in the older ranges are exaggerated, while those of unmarried and childless females are understated. As regards the overall level of enumeration of the census, the census post-enumeration checks are believed to indicate a lower proportion of the population being omitted than is generally supposed (Visaria, 1971 and Dyson, 1981a, 1981b). In particular, the 1971 census count is suspected to be more deficient in coverage than either the 1961 or 1981 censuses (Dyson and Crook, 1984), thus distorting the intercensal rates of growth.

The true age-sex structure of a population at the current date is a result of the past trend of some demographic events, such as, fertility, mortality and migration. However, any irregular fluctuations in the distribution cannot always be explained by the past variations in these factors. Though the accuracy of sex data is usually ensured (except possibly the differential completeness), the errors in reporting age have been common in all censuses and in all countries. Some well known reasons for age misstatement include under-enumeration of infants, the manipulation of age for reasons of school admissions, voting, marriage, jobs, etc., understatement by unmarried and childless females, over-statement of age by elderly. Besides these errors, which seem to be more or less deliberate, there has been a lack of knowledge of 'duration' among illiterate population, which at most report a round number, such as a multiple of 5.

The non-reporting of age, which is measured by the magnitude of the 'age not stated' category, is another important problem. The frequency of non-response is not age-invariant, rather it increases with age. The common procedure of pro rata distribution of the unknowns is thus not often appropriate (Henry, 1976). The non-reporting of age is also not the same for rural and urban populations, marital statuses, etc. In the Indian census of 1981 (5 percent sample), the non-reporting of age occurred for 3 villagers and for 7 urbanites per ten thousand, not significant anyway. An example of marital status differential is given by Greek data (1951) in which 9 widows, 6 married and 4 single persons per thousand did not report age.

1.3 Appraisal of census data

This study, which is based on 5 percent sample data of 1981 census of India, examines the quality of reporting of age through a few simple and common procedures. Since different scholars evaluated the previous censuses already, the present one is being assessed independent of the preceding enumerations. Distributions of age by single years as well as by 5-year groups are considered, and comparisons are made between males and females, as well as between rural and urban populations. It also considers a transitional age structure model for building up an approximately correct age distribution. Although it is not recommended to smoothe an age distribution prior to applying, say, an indirect technique for demographic estimation, an age distribution free from irregularities forms the basis for population projection (United Nations, 1983). Moreover, the effect of erroneous age data on demographic rates would result in bias.

Like the previous censuses, the single years of age distribution for 1981 census, not presented here for limitation of space, also reveal the usual feature of heaping at ages ending in zero or five. Additional heapings occur at even digits such as 2, 8 etc. Infants are underreported, so that the population in the age range 5-14 is relatively too large, while ages in the older ranges appear to have been exaggerated. Since these ages are subject to different types of errors of reporting other than digital bias, the emphasis is mainly on age range 10-59 in evaluating the 1981 census data on age. Though the patterns of age reporting between males and females, as well as rural and urban areas are broadly similar, the age returns, as in other censuses, are slightly better in urban than in rural areas and for males than for females on the whole. In the young adult age range, the shifting to ages 20 and 25 is significantly pronounced, especially for females. The problem of age misreporting is further compounded when a single respondent gives information on all other members of the household, as is usually the practice in the Indian censuses.

In indicating the degree of digital preference, the Whipple's index can be applied to single years of age data. Thus, on the assumption of rectangularity or linearity in a 10-year age range, heaping on terminal digit 0 or 5 in the age range 10-59 is measured as

$$W_d = \frac{\sum_{i=1}^{5} P_{id}}{1/10 \sum_{i=1}^{5} \sum_{j=0}^{9} P_{ij}}, \quad d = 0 \text{ or } 5,$$

P_{ij} = Population aged 10i + j $\qquad (i = 1, 2, \ldots\ldots\ldots 5)$

and $1 \leq W_d \leq 10$, the lower limit indicating no preference, and the upper one indicating that only the specific digit was reported. The results

(Table 1.1) indicate moderate heaping at these digits, but show more concentration at ages ending in digit 0 than in 5 for both urban and rural areas as well as for both sexes. The index reflects a greater degree of digital preference among rural respondents than among urban respondents. There has generally been less digit preference for males than for females, the difference between them being more marked in the urban areas than in the rural areas, where low education may be a contributing factor to this disparity.

Table 1.1. Whipple's index for heaping on digits zero and five by type of place of residence and sex : 1981

Index	Sex	Total	Rural	Urban
W_0	Male	2.57	2.66	2.29
	Female	2.65	2.72	2.40
W_5	Male	2.14	2.22	1.91
	Female	2.12	2.16	1.99

Though not a strictly monotonically decreasing series, like the life table function l_x, the actual population normally falls with age, so that a simple tabulation of population by terminal digits of age would weigh the smaller digits unduly. Myers' (1951) index (M.I.), which takes care of this factor, calculates a 'blended' population of terminal digits as follows :

$$b_j = (j+1)\, P_{1j} + 10 \sum_{i=2}^{5} P_{ij}, \quad j = 0, 1, \ldots\ldots\ldots, 9$$

and $B = \sum_j b_j$,

P being the reported population at ages 10-59 — the age range selected, as explained above. If there is no digital preference then the expected value is about 10 per cent for the blended population ending in each digit. Then

$$M.I. = \sum_j \left| 100 . \frac{b_j}{B} - 10 \right| ,$$

taking on values between 0 (no preference) and 180 (when only one digit is reported). In the 1981 census, the Myers' blended index is worked out as :

	Myers' blended index		
	Total	Rural	Urban
Male	60.52	64.42	47.50
Female	63.16	66.48	54.02

There is no definite criterion by which the degree of digit preference, indicated by the Myers' index computed, can be judged as high, medium or low. However, the level of reporting bias in the Indian Census may be understood when compared with an index value of mere 3.7 for both sexes in 1980 census of Trinidad and Tabago (Hunte, 1983). On the other hand, examples of higher values are found in Pakistan (1965) for the age range 10-79 (Yusuf, 1967), where the Myers' index was 66 for males and 65 for females.

The results indicate the marked digital preference for rural women (not necessarily the actual respondents in the census) with little or no education. Males have less digital preference than females, but the difference is more pronounced in the urban areas than in the rural areas. This difference is much more pronounced when areas of residence are compared. It is noted that the difference in the index values between the sexes is not as great as the disparity in their literacy rates (57 per cent for males and 29 per cent for females) for ages above 10. The reason for this is probably that being a mere literate does not cause any qualitative change in reporting of age unless certain threshold of education is attained.

The distribution of the blended population for each digit (Table 1.2) shows that the digits 0, 5, 8 and 2 account for three - fifth to three-quarters of the total blended population, and the under-selection occurs most at ages ending in 1 and 9. In general, there has been the greatest avoidance (about 15 to 23 per cent) of odd digits (except 5) while stating age; this is more pronounced for females than for males as well as for the rural areas than for the urban areas (see also Figure 1.1).

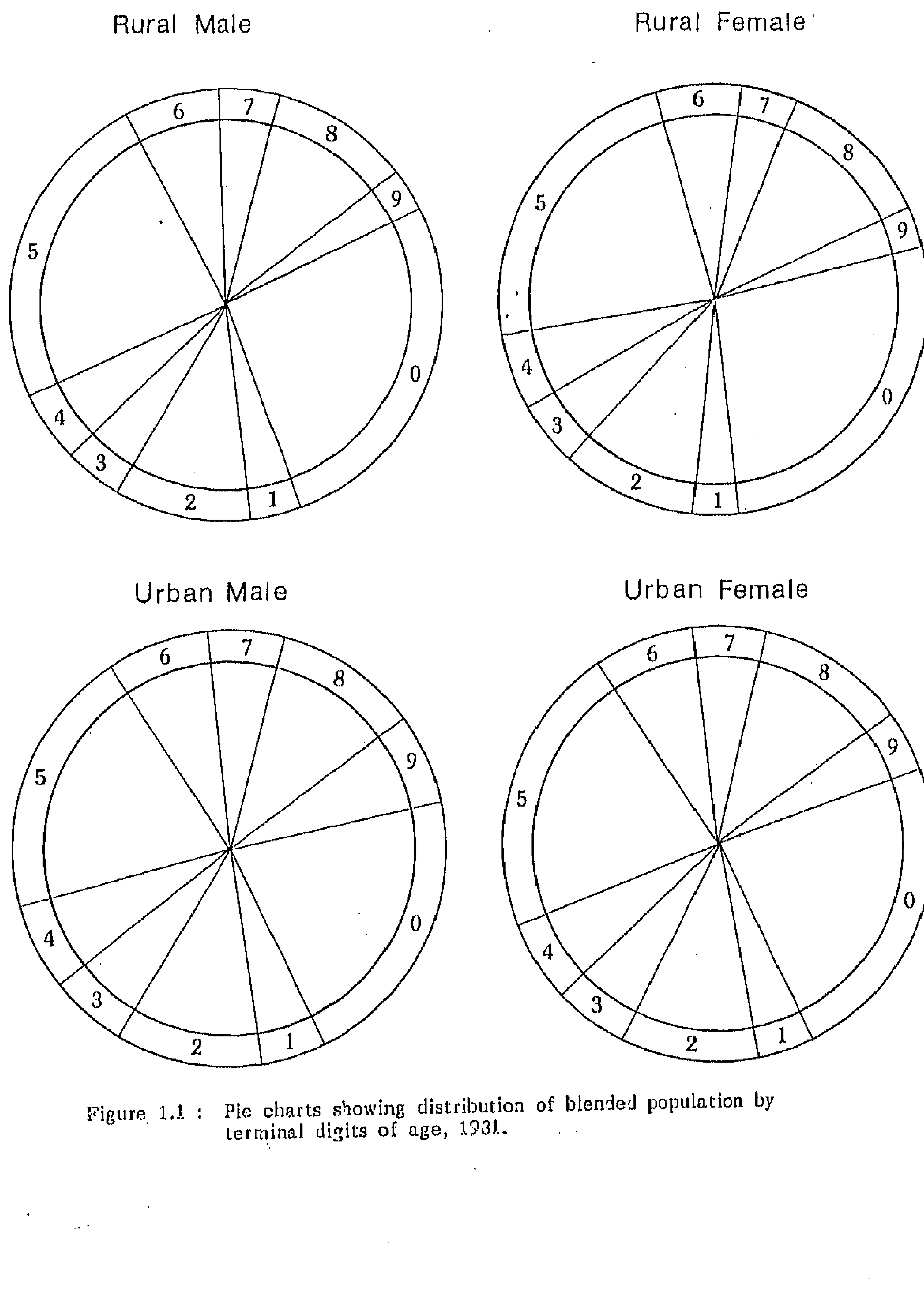


Figure 1.1 : Pie charts showing distribution of blended population by terminal digits of age, 1931.

Table 1.2. Distribution of blended population by terminal digits, sex and areas of residence : 1981

Terminal digit	Males			Females		
	Blended population		Order of prefe-rence	Blended population		Order of prefe-rence
	Figures in '000	Percent		Figures in '000	Percent	
(a) All India						
0	469101	25.60	1	452848	26.57	1
1	71312	3.89	9	57214	3.36	9
2	195913	10.69	4	176876	10.38	4
3	90950	4.97	7	83160	4.88	7
4	101270	5.53	6	94660	5.55	6
5	426128	23.26	2	390096	22.89	2
6	129652	7.08	5	115804	6.79	5
7	87218	4.76	8	76580	4.49	8
8	196184	10.71	3	200068	11.74	3
9	64350	3.51	10	57180	3.35	10
All digits	1832086	100.00		1704486	100.00	
(b) Rural						
0	357645	26.68	1	354579	27.43	1
1	47304	3.53	9	39710	3.07	9
2	142171	10.61	3	134503	10.41	4
3	62398	4.65	7	60264	4.66	7
4	70415	5.25	6	69560	5.38	6
5	324612	24.22	2	301986	23.36	2
6	93180	6.95	5	85192	6.59	5
7	59332	4.43	8	53822	4.17	8
8	142063	10.60	4	153618	11.89	3
9	41270	3.08	10	39320	3.04	10
All digits	1340390	100.00		1292548	100.00	

Contd.../-

Table 1.2 (Continued)

Terminal digit	Males: Blended population, Figures in '000	Males: Blended population, Percent	Males: Order of preference	Females: Blended population, Figures in '000	Females: Blended population, Percent	Females: Order of preference
			(c) Urban			
0	111452	22.26	1	99004	24.04	1
1	24008	4.79	10	16604	4.03	10
2	53715	10.73	4	42375	10.29	4
3	28542	5.70	8	22892	5.56	7
4	30640	6.12	7	25100	6.10	6
5	99906	19.95	2	88110	21.40	2
6	36492	7.29	5	30615	7.44	5
7	27878	5.57	9	22766	5.53	8
8	54131	10.81	3	46440	11.28	3
9	33950	6.78	6	17850	4.33	9
All digits	500714	100.00		411754	100.00	

A method of testing the accuracy of population enumerated at ages 0, 1, 2, 3 and 4 is to use an appropriate life table. Thus, if B_{1981-x} is the number of births x years before 1981 census, then expected population ($\hat{P}$) aged x-1 to x at the time of census is

$$ {}_{1}\hat{P}_{x-1} = B_{1981-x} (L_{x-1}/l_0) $$

The enumeration accuracy of children can be tested in this manner, but the lack of complete and accurate birth registration data makes the application of the method difficult.

Detection of errors in grouped age data is much more difficult than in single year of age data. Various techniques are however available for evaluation of such data; these are intercensal cohort analysis and comparisons with estimates based on birth statistics. But these all require another set of data for comparison. The mathematical graduation of census data and comparison with population models such as stable model are the other methods of evaluation. But the use of the theory of stable population is not appropriate for India, and the graduation methods of wave cutting and running averages have little validity particularly in the reported age ranges where undercount or large under - or over-statement of ages have occurred.

Table 1.3. Reported age distributions of 1961, 1971 and 1981 censuses and model age structure for 1981 per 1,000

Age group	Unadjusted census			Age structure model for 1981*	
	1961	1971	1981	I	II
0 - 4	165	162	126	147	149
5 - 9	132	141	141	132	137
10 - 14	113	119	129	124	122
15 - 19	98	98	96	109	104
20 - 24	87	84	86	93	88
25 - 29	78	74	76	80	77
30 - 34	69	65	64	67	65
35 - 39	59	57	59	58	57
40 - 44	49	49	51	48	48
45 - 49	42	41	44	40	40
50 - 54	34	33	38	31	32
55 - 59	26	26	25	23	25
60+	48	51	65	48	56

* Approximately correct age distributions for 1981 were obtained by projection of adjusted 1971 census (method I) and a directly adjusted 1981 census (method II), adjustments being made by a transitional age structure model discussed later.

The age distributions per 1,000 for the unadjusted censuses of 1961, 1971 and 1981 together with the adjusted 1981 census are shown in Table 1.3. The adjustment was made by a transitional age structure model described later.

In 1981 census, the infants are more under-reported than in 1961 and 1971, so that the population in the 5-14 age range is relatively too large. It may however be noted that due to decline in the birth rate, especially in the last quinquennium, the proportion of infants would have been lower in any case. Further, a slight ageing of population in 1981 compared to 1961 and 1971 is also observed because of the same reason and of improvement in expectancy of life.

An index for measuring accuracy for age distribution developed by the United Nations (1953) requires only the census data collected. It uses both sex ratio for each age group and age ratio for each sex. The sex ratio (S.R.) for the age group (x, x+5) is defined as

$$_5(S.R.)_x = \frac{_5P_x^m}{_5P_x^f} \cdot 100,$$

and the first absolute differences calculated as

$$_5\Delta_x = \quad _5(S.R.)_{x+5} - {}_5(S.R.)_x$$

give, on averaging, what is called the sex ratio score. The sex ratios fluctuate (Table 1.4), and do not decline by age; the fluctuations cannot plausibly be explained by sex differences in mortality and in rates of migration. It may therefore be an indication that the reported age distribution is not accurate or the errors for males are not similar as those for females.

The plausible explanations for low sex ratio in the age range 20-34 in the rural areas are the mis-statement of age of women and under-enumeration of men. Sex ratio scores of about 4-6 do not also indicate any standard of quality of reporting according to the United Nations' criterion, which states that a score of 1.5 implies a highly accurate system of reporting.

The age ratio for age group (x, x+5) is defined as

$$_5I_x = 100 \cdot {}_5P_x \,/\, \left[1/3 \left(_5P_{x-5} + {}_5P_x + {}_5P_{x+5} \right) \right]$$

A summary measure (T) of age ratio scores is

$$T = {}^{1}/_{n} \sum_{x} \left| {}_{5}I_{x} - 100 \right| .$$

Table 1.4. Age- and residence-specific sex ratio and sex ratio scores : 1981

Age group (x, x+5)	Total: Sex ratio	Total: $_5\Delta_x$ *	Rural: Sex ratio	Rural: $_5\Delta_x$ *	Urban: Sex ratio	Urban: $_5\Delta_x$ *
0 - 4	102.55	3.77	102.31	4.16	103.42	2.36
5 - 9	106.32	5.27	106.47	5.53	105.78	4.45
10 - 14	111.59	0.81	112.00	0.42	110.23	4.49
15 - 19	112.40	10.22	111.58	14.39	114.72	1.39
20 - 24	102.18	0.82	97.19	2.23	116.11	2.88
25 - 29	103.00	0.63	99.42	1.28	113.23	7.94
30 - 34	103.63	1.41	98.14	2.18	121.17	0.62
35 - 39	105.04	6.54	100.32	5.27	120.55	12.76
40 - 44	111.58	0.34	105.59	1.62	133.31	7.42
45 - 49	111.24	7.90	107.21	9.08	125.89	4.01
50 - 54	119.14	11.64	116.29	12.69	129.90	6.87
55 - 59	107.50	0.66	103.60	3.14	123.03	15.81
60 - 64	106.84	5.43	106.74	5.13	107.22	6.58
65 - 69	101.41	2.82	101.61	3.88	100.64	1.65
70 - 74	104.23	1.55	105.49	1.80	98.99	0.25
75 - 79	102.68	1.86	103.69	0.23	98.74	7.99
80 - 84	100.82	0.96	103.46	2.96	90.75	4.33
85 - 89	101.78	10.99	106.42	10.61	86.42	14.20
90 - 94	90.79	2.14	95.81	2.40	72.22	0.00
95 - 99	92.93		98.21		72.22	
Sex ratio score		3.99		4.68		5.58

* First absolute differences of sex ratios.

The age ratios (Table 1.5) computed for areas of residence and sexes deviate fairly from 100, especially in the old age range, thereby reflecting at least moderate errors in the reported age distribution. " Some real variations in these age ratios may be expected, however, mainly owing to variations in numbers of births in the past" (United Nations, 1953). An overall degree of accuracy of the age reporting can be judged by the United Nations' standard age ratio scores of around 2.4-2.6 for a population with almost perfect age reporting. As evaluated against the criteria and as against the values of 4.3-5.4 in 1970 census of Philippines (United Nations, 1978), the age ratio

Table 1.5. Age-, sex- and residence-specific age ratios and age ratio scores : 1981

Age group	Total		Rural		Urban	
	Males	Females	Males	Females	Males	Females
5 - 9	106.56	107.02	107.30	107.69	103.94	104.63
10 - 14	106.46	104.69	107.32	105.11	103.60	103.28
15 - 19	94.10	91.38	93.20	89.98	96.70	95.63
20 - 24	98.07	101.85	95.66	99.82	104.21	103.00
25 - 29	101.27	101.09	101.73	100.50	99.98	102.82
30 - 34	96.32	96.48	96.22	97.33	96.58	93.86
35 - 39	100.35	101.66	100.53	101.27	99.87	102.94
40 - 44	101.42	99.03	101.35	99.86	101.61	96.11
45 - 49	97.74	99.79	97.78	99.53	97.63	100.74
50 - 54	109.91	104.34	110.93	104.46	106.58	103.90
55 - 59	80.55	83.97	79.42	84.13	84.55	83.31
60 - 64	124.05	122.95	126.15	123.45	116.06	120.91
65 - 69	78.22	80.81	77.91	80.56	79.49	81.83
70 - 74	118.75	117.01	119.58	117.26	115.25	116.02
75 - 79	69.30	67.65	66.76	67.43	69.58	68.52
80 - 84	123.18	124.40	123.98	124.43	119.96	124.25
85 - 89	54.49	53.39	53.79	52.05	57.52	58.33
90 - 94	103.81	110.62	105.53	112.04	96.46	105.86
Age ratio score	12.72	12.39	13.54	12.53	10.72	11.71

scores of about 11-14 in 1981 census of India indicate a fair degree of inaccuracy in the age distribution. The score is little higher in the rural areas than in the urban areas, but the female score is lower in the rural areas and higher in the urban areas than the corresponding male scores.

The United Nations' index (U.I.), called the joint score, for an over all measure of accuracy of a reported age distribution, which combines sex ratio as well as age ratios for both males and females, is given by

$$U.I. = T_m + T_f + \frac{3}{n} \sum_x {}_5\Delta_x,$$

where T_m and T_f are the age ratio scores for males and females respectively, and the third component is three times the sex ratio score. The reported age distribution is considered

(i) accurate if $U.I. < 20$

(ii) inaccurate if $20 \leq U.I. \leq 40$

and (iii) highly inaccurate if $U.I. > 40$.

The index is only an approximate indicator of the smoothness of the reported data, affected by digital bias, variations in under-enumeration by age, etc. This index, which yields values of about 39 and 40 respectively for the urban and the rural areas, indicates that the age reporting is almost highly inaccurate and that the rural-urban differential is not significant. The data for 1981 census are however much less inaccurate than those for 1961 census (cited in Pollard, Yusuf and Pollard, 1981) as suggested by the following comparison :

Year	Sex ratio score	Age ratio score		Joint score
		males	females	
1981	4.0	12.7	12.4	37.0
1961	7.8	22.8	29.2	75.4

1.4 Adjusting census age distribution

The population distribution for 1981 census was approximately corrected by a transitional age structure model (Das Gupta and Guha Roy, 1976). It assumes the growth-survivor relationship between the successive ages of a closed population formation. Thus, denoting a steady growth rate by r and survival probability by ${}_tp_x$, the population at age x+t is obtained as

$$P_{x+t} = P_x \; e^{-rt} \; {}_tp_x$$

and $$P_x = P_o \; e^{-rx} \; {}_xp_o$$

In the continuous case, the formulations may be stated as follows :
By the definition of the force of mortality

$$\mu_{x+t} = \frac{-d}{dt} \log l_{x+t}$$

$$\therefore \int_0^1 \mu_{x+t} \, dt = -\log \frac{l_{x+1}}{l_x} = -\log {}_1p_x,$$

so that $${}_xp_o = e^{-\int_0^x \mu_x dx}$$

We have

$$P_x = P_o \; e^{-rx} \; e^{-\int_0^x \mu_x \, dx}$$

$$= P_o \; e^{-\int_0^x (r + \mu_x) \, dx}$$

If the growth rates are known at discrete intervals from censuses, and mortality from an appropriate life table, then an approximate formulation of the above relationship may be derived. For example, considering r' and L' for the steady growth rate and life table values in the last decennium, we have

$$_5P_5 = {}_5P_0 \; e^{-2.5r'} \left({}_5L'_5 \,/\, {}_5L'_0 \right)$$

Similarly,

$$_5P_{10} = {}_5P_0 \; e^{-10r' + 2.5r''} \left({}_5L'_{10} \,/\, {}_5L'_5 \right) \left({}_5L''_5 \,/\, {}_5L''_0 \right)$$

where r'' and L'' denote values in the decade previous to the last decade. We can construct an age structure model in this manner. The input data required for the application of the model are then obtained. The average population growth rates (r), derived as

$$r = \exp\left((\log P_2 - \log P_1)/t \right) - 1,$$

P_1 and P_2 being the census populations at the beginning and end of a period, are shown below.

1971-81	1961-71	1951-61	1931-51	1921-31	1911-21	1901-11	Pre-1901
2.28	2.22	1.98	1.30	1.06	-0.04	0.57	0.21

The mortality was assumed to decline at the rate considered in the set of United Nations (UN) Model Life Tables. Thus,

	1976-80	1971-75	1966-70	1961-65	1951-60	1941-50	1931-40	Pre-1931
UN level	65	60	55	50	45	35	25	10
$\overset{o}{e}_o$	52.5	50.0	47.5	45.0	42.5	37.5	32.5	25.0

The age structure model built up was fitted to the 1981 census population, more details of the procedure being given elsewhere (Guha Roy, 1984; see also Chapter Four). Table 1.6 shows the 1981 age distribution of the enumerated population (5 per cent sample data), projected population derived from 1971 adjusted base population (for details, see Guha Roy, 1984; see also Chapter Four) and of the transitional age structure model. It may be noted that the initial age group 0-4 was estimated and the distribution adjusted by fitting the model to the reported population aged 5+, and then extending the fitted distribution below these ages by pro-rating.

Table 1.6. Reported, adjusted (by transitional age structure model) and projected (based on adjusted 1971 census) population for 1981

Age group	Male population (in million) Reported census 1981*	Male Adjusted popl., 1981	Male Projected popl., 1981	Female population (in million) Reported census 1981*	Female Adjusted popl., 1981	Female Projected popl., 1981
0 - 4	42.40	52.90	52.49	41.35	49.31	48.56
5 - 9	48.25	48.99	47.69	45.38	45.43	43.01
10 - 14	45.18	43.74	44.36	40.49	40.42	41.15
15 - 19	33.89	37.01	38.64	30.15	34.20	36.42
20 - 24	28.96	31.07	32.99	28.35	29.12	30.98
25 - 29	25.75	27.30	28.58	25.00	25.20	26.64
30 - 34	21.60	23.13	24.04	20.84	21.34	22.27
35 - 39	19.93	20.25	20.00	18.97	18.64	19.17
40 - 44	18.04	16.98	17.21	16.17	15.72	15.84
45 - 49	15.40	14.36	14.30	13.85	13.22	13.26
50 - 54	13.83	11.14	10.92	11.61	10.48	10.19
55 - 59	8.52	8.68	8.34	7.92	8.30	7.79
60 - 64	9.38	6.94	6.49	8.78	6.80	6.09
65 - 69	4.78	5.32	4.83	4.72	5.40	4.50
70 - 74	4.18	3.58	3.20	4.01	3.82	2.94
75 - 79	1.60	2.04	1.79	1.56	2.27	1.62
80 - 84	1.36	0.91	0.78	1.35	1.08	0.69
85+	0.86	0.53	0.20	0.85	0.07	0.17
Total	343.93	354.40	357.65	321.36	331.78	331.27

Note : Totals may not tally exactly.

* Taken from Census of India 1981, Series - 1, Part-II-Special : Report & Tables based on 5 per cent sample data, Registrar General & Census Commissioner for India, New Delhi, October 10, 1983.

The comparison reveals in the first place an overwhelming under-enumeration of children (0-4), which is in keeping with the earlier observation. It is however possible that the extent of under-enumeration in the 0-4 age group was slightly over-estimated, since the assumption of invariance of fertility in the model was not fully **realised** due to some decline in fertility during 1971-81. The overstatement of ages by the elderly is another reporting error in the data. The pronounced shifting to age group 60-64 from the neighbouring ages for both sexes is yet another pattern of age reporting that offers no plausible explanation. If the transitional model provides reliable estimates of age structure, then the census reporting in the younger age range 5-14 is fairly accurate, but there has been an under-reporting in the range 15-34 and over-reporting in the mid-age range 40-54. On the whole, there was no significant sex-differential in the pattern of reporting age, but the census age distribution lacked a smooth decline of numbers with age, inspite of no appreciable variations in fertility and mortality in the past (see also Figure 1.2).

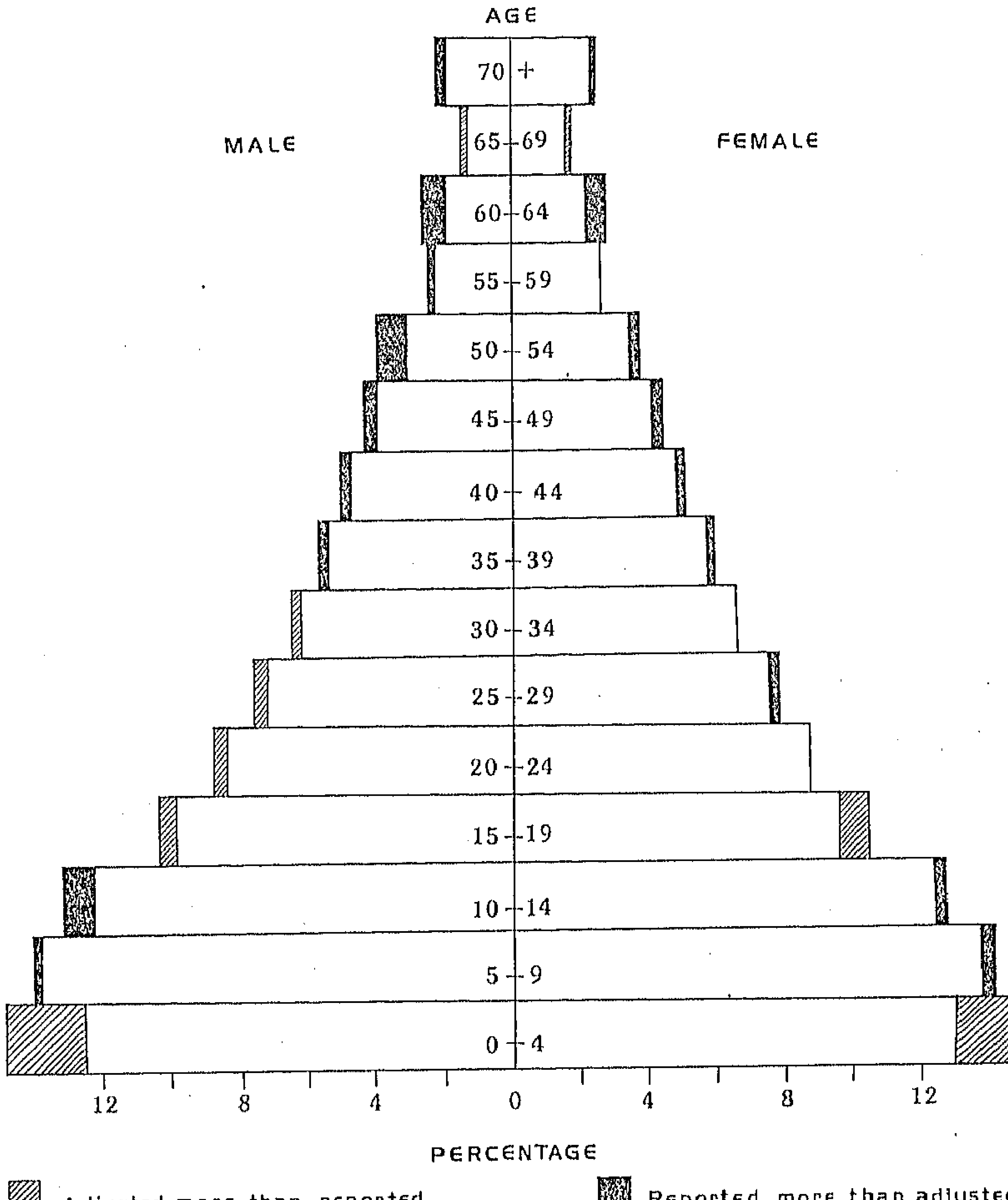


Figure 1.2 : Population pyramid of India, 1981 : reported and adjusted.

CHAPTER TWO

MORTALITY : LEVELS AND PATTERNS

In considering the estimation of mortality for India from the census and vital registration sources, the extent of under-reporting is first examined. The next step uses recent advances in indirect mortality estimation for measuring infant, childhood and adult mortality from survivorship reports of births and spouses. The aspect of modelling age pattern of childhood mortality is re-examined in the light of the Indian experiences.

Estimate of completeness of death reporting uses both stable and non-stable methods, several growth rate inputs, and different initial ages. Using West and South families of model life tables, childhood mortality estimates are based on the recent modifications of the original Brass technique. The Weibull function, defined as the probability of surviving from birth to age x, describes well the age pattern of infant and childhood mortality. Following Brass-Hill procedure, widowhood data provide estimates of adult mortality and of a two-parameter life table.

2.1 Introduction

Interest centres around the current levels of mortality in their sex-age pattern in major segments of population, like infant, child and adult in India. Because of the well-known defects in the vital registration, firm estimates of mortality rates are not available for the country. Civil registration system extends over a full century since 1873 and was made compulsory since 1969, but vital registration remained severely deficient (India Office of the Registrar General, 1970). Even compulsion or legal obligation to register a vital event does not ensure complete or accurate records (UN, 1973). Though it has been demonstrated that properly designed and executed household surveys can render good estimates of vital rates (Das Gupta, 1957; Majumdar, 1962; and Som, 1959), mortality data collected in the various rounds of the National Sample Surveys are considered to be of low quality (Chari, 1977). Attempts to estimate the level of mortality from survival ratios, yielded by longitudinal comparisons of age cohorts between census populations, were unsuccessful because of succession of disturbances, which were superimposed on the gross age reporting errors and biases. Similarly, any estimate of the level of mortality, obtained from an age distribution of deaths would be vitiated by mis-statement of ages and the known deficiency of the regular vital registration system. The improvement of the vital registration system through the use of a set of sample tracts by progressive cluster building (Das Gupta, 1957; Hauser, 1954), appears to be the next logical step. The Sample Registration Scheme (SRS) initiated in 1964-65 in India (India Office of the Registrar General, 1970; Lingner and Wells, 1973) has some element of this approach. Though the scheme provides reasonably reliable estimates of vital rates, the experience suggests some amount of under-estimation (India Office of the Registrar General, 1974).

Given then the situation in which the direct information on vital parameters are inadequate, the mortality levels have to be worked out by the application of analytical techniques. Some attempts have already been made to estimate mortality in India by longitudinal comparison of age cohorts at successive censuses, conventional reverse survival method (when fertility is known), central death rate method, quasi-stable method (Coale, 1962; Saxena, 1971) and a few others. Some of these methods were unsuccessful because of lack of uniform completeness of enumeration in different censuses, which were superimposed on the gross age reporting errors and biases. Moreover, any estimate of the level of mortality was vitiated by the severe deficiency of the regular vital registration system. The methodology of demographic estimation has however evolved considerably during the last two decades as compared to the few traditional procedures that constituted the state of the art in earlier years.

Pioneered by William Brass and Ansley Coale (1968), the repertory of indirect mortality estimation techniques has, in recent years, been enriched. With reference to Indian data we experiment with some of the new techniques wherever they are broadly applicable and estimate the relevant mortality indicators. The procedures consist essentially of deriving a life table from data of the type :

Survivorship of children ever born
Orphanhood or parent survival
Widowhood or spouse survival of a first marriage
Survivorship of brothers - sisters or sibling survival

The first information is obviously used to estimate infant and child mortality, and the remaining three for estimating adult mortality. Alternative estimation techniques are tried in order to examine the degree of consistency of the procedures and of the underlying data. An attempt to estimate the extent of underregistration of deaths is also made before deriving national mortality estimates. Our focus is mainly on new techniques of estimation, as far as feasible, so as to shed some light on their working in the situation obtaining in India.

2.2 Extent of under-reporting of deaths

Despite being the best available source of vital rates, the closely supervised dual record system of the Sample Registration Scheme (SRS) still experienced some underregistration. The official estimate (Office of the Registrar General, 1974) put the under-count of deaths roughly at five per cent. There are however reasons to believe that the coverage of vital events in the SRS, though not the same in different years, is deficient by a much more significant margin. Using a few analytical techniques, we try to estimate the level of completeness of deaths in the SRS.

In recent years several methods have been developed for measuring the extent of underreporting in death registration. Some of these methods are : (1) Brass sectional growth balance equation (Brass, 1975); (2) Preston-Hill's method using intercensal growth rate, the recorded death rate, and the age distribution of recorded deaths (Preston and Hill, 1980; Preston, Coale, Trussell and Weinstein, 1980); (3) a modified sectional growth balance equation (Martin, 1980); (4) an intercensal cohort survival method (Brass, 1979; Preston and Hill, 1980; Trussell and Menken, 1979); (5) an extension of the intercensal cohort survival method (Bennett and Horiuchi, 1981); and (6) forward projection technique using data from two censuses.

Whereas all these methods assume age - invariance of completeness in the adult age range, the first two methods are based on the assumption of stability and the remaining four methods use age specific rather than a constant overall growth rate. The latter methods normally use two censuses and provide only estimates of completeness of death reporting relative to the completeness of census enumeration. The applications of these methods to less developed countries were not also successful because of the effects of age misreporting and differential completeness of enumeration between censuses. The interoensal cohort survival method, though not requiring the assumption of stability, uses cohort deaths which are not easily derivable. Moreover, the assumption of an even distribution of deaths within each age interval to apportion deaths between cohorts is not clearly realistic. Though the relaxation of the condition of stability seems more appropriate for India, the non-stable methods impose the constraint of more detailed data requirements than is the case with the stable methods.

We, therefore, consider a procedure for estimating the extent of completeness of death reporting in the Sample Registration Scheme (SRS), which is more straight-forward in application, but based upon assumption of stability. The method proposed by Preston and Hill (1980) uses the age structure of deaths and assumes that the completeness of reporting does not vary substantially with age.

If the annual average growth rate, r, the crude death rate, d, and the age distribution, δ (a), of recorded deaths are known, the completeness, C, of death reporting is obtained from the following relationship (derivation shown in Preston and Hill, 1980) in a stable population in discrete form :

$$C = \frac{D(a+)/P(a+)}{r} \left[\sum_{a}^{w} \delta(a) \exp(ra) - 1 \right],$$

D(a+) and P(a+) being the recorded deaths and the population aged a and above respectively and w the maximum age; the average age, a, at which deaths occur in the 5-year age interval is measured by the distance of mid-point from the lowest age considered for the analysis. For example, if we limit our analysis to mortality beyond childhood, that is, to mortality from age 10 (origin) and the mid-point of the age interval, say, 10-14 is 12.5, the value of a = 12.5 - 10 = 2.5.

In the application of the above technique, the assumption of stability is crucial. If the population does not conform to a stable condition, that is, where fertility and/or mortality have not been constant, the overall growth rate, r, may be a poor approximation of the age specific growth rates, r(a+). For the Indian population, the fertility was high and varied only within narrow limits around the average crude birth rate of about 42 upto 1971. In the absence of any rapid decline of fertility during the period under study, no significant changes in the age structure of the population was contemplated. The observed increasing trend (subject to no differential coverage of censuses) in the average intercensal growth rate is attributed to a substantial decline in mortality. This decline could affect the population structure at older ages. According to Brass (1975) however the violation of underlying stability assumption, especially if only mortality is changing, does not probably affect the estimate of completeness of death reporting to any appreciable extent. Preston, Coale, Trussell, and Weinstein (1980) have also demonstrated that for a destabilized population created by a history of recently declining mortality, the estimates of completeness can be obtained within tolerable margin of error.

Going by these experiences, we use the method to estimate completeness of death reporting of the Sample Registration Scheme (SRS) in 1971. In the first place, we require the annual average growth rate (r), which was calculated here from the enumerated populations of 1961 (March 1) and 1971 (April 1) censuses - the period between them being about 10.083 years. There are some differences of opinions about the differential coverage of the Indian censuses and the extent of underenumeration. Dyson (1981a) believes that the official estimates of around 2 per cent underenumeration, calculated from post-enumeration surveys, are probably much too low. It is believed that the real figure may be as high as 5 per cent. On the other hand, Ashish Bose (1981) lists a series of arguments on why the under-enumeration in the Indian census would be of a small order. The possibility of differential undercount in the recent censuses has been discussed in an article on 'India : Population growth in the 1970s' (Population Council, 1981) and also by Visaria and Visaria (1981). Jain and Adlakha (1982) also cast doubt but could not ascertain whether the post-ponement of census operation by one month resulted in a differential undercount in 1971 census. The U.S. Bureau of the Census (1978) estimated (based on the post-enumeration checks) net underenumerations for total populations in 1961 and 1971 censuses of India as 2.8 and 2.7 per cent respectively. Accepting these estimates, we derived intercensal growth rate in 1961-71.

Using the common procedure of calculating annual average growth rate, namely,

$$r = \frac{1}{10.083} \ln [P(1971)/P(1961)],$$

we get a value of $r = 0.022$, while the growth rate estimated by a quadratic exponential was 0.023. Though not mathematically equivalent with these estimates, the natural increase implied by the excess of birth rate over death rate of the Sample Registration Scheme (SRS) was 0.022. This figure is likely to be little higher because of the known differentials in coverage of births and deaths. On these considerations, we use a rate of 2.3 per cent for our completeness estimates. It may be noted that estimates of completeness depend on the accuracy of r, a too low or too high r may under - or over-estimate completeness.

To start with we illustrate the technique with the initial age 0+ and consider that the average age (a) at which deaths in an age interval occur will approximately be the mid-point of the interval (e.g., 2.5 for 0-4, 7.5 for 5-9, etc.), the exact figure being used does not seem to invalidate the working of the method (Preston and Hill, 1980). For the last open ended age group, which is 70+ in our case, we adopt the procedure prescribed by Preston and Hill and use the model value tabulated by them for calculating average age at death. Bennett and Horiuchi (1981) however considers the procedure as inadequate when the lower bound of the open age interval (involving older persons) lies at a low age. They argue that since the procedure does not consider the variance of the age at death, which is quite high when the lower bound is low, the result may be an underestimation. Their treatment of the open interval is however complex and also based on several assumptions, and since the variance is insignificant for lower bound as high as 70, we did not use any variant of the procedure.

In our calculation, the required elements are $r = 0.023$, as established above, and $e_o^o = 47.5$ years, the estimate (1966-71) being based on the revised report of the Expert Committee on Population (Lal, 1974). The estimate of the average age at death in the age group with no upper bound will be more sensitive error in $\overset{o}{e}_o$ than in r. With these considerations, our estimate for the average age at death for age 70+ is 78.19 when the analysis of mortality starts at age 0+, 68.19 when it starts at 10+ and so on. The other elements required are the crude death rate, d, and the proportionate age distribution of deaths, δ(a), around 1971, which were taken from the Sample Registration Scheme (cited in the Year Book of the Ministry of Health and Family Welfare, 1979). The calculations and results are shown in Tables 2.1a and 2.1b.

Table 2.1a. Calculating the component δ(a).exp (ra) in the estimation of completeness of death reporting by sex : 1971

Age interval	Proportion of recorded deaths, δ(a)			Mid-point of interval (a)	δ(a).exp (ra) (r = 0.023)		
	Males	Females	All		Males	Females	All
0 - 4	0.51981	0.56498	0.54225	2.5	0.55057	0.59842	0.57434
5 - 9	0.04061	0.04243	0.04153	7.5	0.04826	0.05042	0.04935
10 - 14	0.01304	0.01658	0.01458	12.5	0.01738	0.02210	0.01944
15 - 19	0.01428	0.01951	0.01678	17.5	0.02136	0.02918	0.02510
20 - 24	0.01452	0.02439	0.01935	22.5	0.02436	0.04092	0.03247
25 - 29	0.01477	0.01975	0.01764	27.5	0.02780	0.03718	0.03320
30 - 34	0.01501	0.02097	0.01788	32.5	0.03170	0.04428	0.03776
35 - 39	0.01674	0.01683	0.01702	37.5	0.03966	0.03987	0.04032
40 - 44	0.02117	0.01707	0.01935	42.5	0.05627	0.04537	0.05143
45 - 49	0.02461	0.01804	0.02132	47.5	0.07338	0.05379	0.06357
50 - 54	0.03249	0.02439	0.02842	52.5	0.10868	0.08159	0.09507
55 - 59	0.03987	0.02902	0.03442	57.5	0.14962	0.10890	0.12917
60 - 64	0.04997	0.03950	0.04471	62.5	0.21038	0.16630	0.18823
65 - 69	0.05169	0.04072	0.04606	67.5	0.24414	0.19233	0.21755
70+	0.13142	0.10582	0.11869	*	0.79429	0.63795	0.71686
					2.39785	2.14860	2.27386

Source : See text.

* The approximate values of a are 78.22, 78.11 and 78.19 for males, females and all persons respectively. These were estimated from model values (Preston and Hill, 1980, Table 3, p. 354).

Table 2.1b. Estimating completeness of death reporting by sex : 1971

	Annual average growth rate, r*	Crude death rate, d	$\Sigma\ \delta(a)e^{ra}-1$	Estimated completeness, C
Males	0.023	0.0143	1.39785	0.869
Females	0.023	0.0155	1.14860	0.774
All	0.023	0.0149	1.27386	0.825

* Actually, values of $r = \left[\ln \frac{P(1971)}{P(1961)} \right] / 10.085$ are 0.0225, 0.0214 and 0.0220 for males, females and all persons respectively. Considering some differential undercounts between the censuses, all these figures were taken approximately as 0.023. The use of actual values however did not result in any appreciable different estimates.

On the consideration of possible differential underregistration with age, especially in view of the more deficient registration in the infant and childhood age ranges, we recalculate the completeness (Table 2.2) for age 10+, which is given by

$$C = \frac{D(10+)/(P(10+)}{r} \left[\sum_x \delta(x) \exp(r(x-10)) - 1 \right]$$

where x is the mid-point of age interval, the other symbols being defined earlier. This results in an improved estimates of completeness as compared to those starting with infant age. The level of completeness of registration of infant and child deaths thus appears, as expected, to be different from that of young and adult deaths. Later, we have estimated child mortality independently by indirect methods.

Table 2.2. Estimating completeness of death reporting for age 10+ : 1971 (r = 0.023)

	Crude death rate, d(10+)*	$\Sigma\ \delta(x)e^{r(x-10)}-1$	Estimated completeness, C
Males	0.0091	2.25172	0.893
Females	0.0089	2.03531	0.789
All	0.0090	2.14806	0.843

* d(10+) = D(10+)/P(10+).

Using the equation $\frac{b}{r_b} - \frac{d}{r_d} = g$, b and d being the recorded birth and death rates respectively, r_b and r_d the respective completeness of reporting, and g the intercensal growth rate, an independent estimate (Guha Roy, 1984) of completeness of death reporting of 85 per cent was derived for all persons at the national level around 1971. This estimate is of the same order of magnitude as the present one (82.5) based on all ages. The re-estimated result, taking the starting age of 10+, of 84.3 per cent is still closer to the above estimate based on the same rate of population growth.

It may be rewarding to examine completeness estimates based on different initial ages for a given growth rate r. This is illustrated in Table 2.3 for different trial values of r. Several striking features emerge from the results. There have been an upward trends of completeness both with age upto 45 (for each r) and the values of r (for each age). As was found for EL Salvador by John Hobcraft (cited in Preston and Hill, 1980), the completeness estimates are sensitive even to small changes in r; this is more so at the youngest ages. The most disturbing feature however is that no r considered here results in a 'level age-sequence' of completeness estimates. Such a situation may arise from two types of errors : inappropriate choice of r and significant deviation from stability assumption. Since the growth rate seems to be known quite accurately, the lack of age-invariance of completeness (except in a short age range) may be largely due to the second kind of error.

In order to relax the assumption of stability, the Brass growth balance equation b - d = r, in which b, d and r are true birth, death and growth rates respectively, is written as

$$b(a+) - d(a+) = r(a+)$$

where the elements in the equation are the rates in the sectional population aged a and above. If N(a) is the average population aged a, that is,

$$N(a) = ({}_5P_{a-5} + {}_5P_a)/10,$$

P's being the populations in the age groups (a-5, a) and (a, a+5), and N(a+) is the population aged a and above, then the ratio 'N(a)/N(a+) is the " birth" rate

Table 2.3. Age-sequence of completeness of death reporting for plausible growth rates : All persons, 1971

Initial age	Estimated completeness		
	r = 0.0230	r = 0.0235	r = 0.0240
0+	0.825	0.841	0.859
5+	0.826	0.842	0.858
10+	0.843	0.858	0.874
15+	0.860	0.874	0.888
20+	0.884	0.893	0.906
25+	0.898	0.909	0.926
30+	0.920	0.931	0.942
35+	0.942	0.951	0.962
40+	0.965	0.974	0.983
45+	0.972	0.981	0.987
50+	0.963	0.969	0.975
55+	0.949	0.955	0.960
60+	0.944	0.945	0.952

(b(a+)) if persons annually reaching age a are considered to be born into the population of persons aged a and above' (Preston, Coale, Trussell, and Weinstein, 1980). The above sectional growth balance equation can then be expressed as

$$\frac{N(a)}{N(a+)} - d(a+) = r(a+).$$

If the observed death rate, which is often too low, is designated as $\bar{d}$ (a), the above equation is changed to

$$\bar{d}\,(a+) = V \left[\frac{N(a)}{N(a+)} - r(a+) \right],$$

where V is the completeness of death reporting assumed to be constant for all ages. The plot of $\bar{d}$ (a+) against $[N(a)/N(a+) - r(a+)]$ lie on a straight line with a slope equal to V. The estimate of V will however depend on the age range of observations used.

Since mortality has been changing during the period under study, the age differential of growth rates, $r(a+)$, should be considered in the completeness estimate. An accurate estimate of $r(a+)$'s require census enumerations of similar quality. Fortunately, such comparable raw data sets are available (Mukherjee, 1976). As usual, the $r(a+)$'s are being calculated from the reconstructed age distributions of 1961 and 1971 as

$$r(a+) = \frac{1}{10.083} \ln \left[{}^{1971}P(a+)/{}^{1961}P(a+) \right].$$

Similar to an earlier observation (Martin, 1980), we get a j-shaped pattern when $r(a+)$'s are plotted by age with minimum occurring at an age around 20, which was approximately the same as the duration of change in mortality in India.

Table 2.4 presents the sequence of age specific b's, r's and d's and Table 2.5 shows the estimates of completeness of death reporting. Since the tails are more prone to errors, we used the presumably more reliable age ranges 10-49, 10-59, 15-54 and 20-59 for estimating the slope (that is, the completeness) of the modified sectional growth balance equation.

The ranges of completeness estimates (Table 2.5) in the destablized population are 84-91, 68-77 and 76-84 per cent for males, females and all persons respectively. The stable estimates are fairly close to the upper limits (which correspond to the estimates using ages 20-59) of these ranges. Generally speaking, both approaches appear to provide tolerably consistent results, and are further validated by another independent method mentioned above. In the present case, the failure of the assumption of stability due to mortality decline does not lead to an estimate of significantly different order of magnitude from that obtained by using age specific growth rates. On the basis of our analysis, we may possibly conclude that the overall coverage of death reporting in the Sample Registration Scheme is 84-85 per cent when the population above the childhood age range is considered.

Table 2.4. Unsmoothed population age distributions for 1961 and 1971 and the estimated parameters of the sectional growth balance equation for 1971

Age interval (a)	Population (000) 1961	1971	r(a+)	b(a+)	$\bar{d}$(a+)
		(a) Males			
0 - 4	33180	40077			
5 - 9	33064	42717			
10 - 14	26264	36099	0.0229	0.0392	0.0089
15 - 19	18590	25216	0.0210	0.0372	0.0105
20 - 24	18192	21567	0.0195	0.0335	0.0120
25 - 29	18528	20331	0.0200	0.0354	0.0137
30 - 34	15985	18285	0.0223	0.0394	0.0159
35 - 39	13603	17262	0.0245	0.0446	0.0188
40 - 44	12084	15107	0.0248	0.0519	0.0229
45 - 49	9736	12414	0.0257	0.0582	0.0284
50 - 54	9129	11203	0.0262	0.0678	0.0356
55 - 59	5283	6785	0.0291	0.0760	0.0469
60+	12352	16874			
		(b) Females			
0 - 4	32909	39052			
5 - 9	31591	40169			
10 - 14	23022	32189	0.0219	0.0392	0.0087
15 - 19	17278	22241	0.0197	0.0357	0.0101
20 - 24	19129	21525	0.0188	0.0336	0.0112
25 - 29	18047	20477	0.0203	0.0386	0.0125
30 - 34	14853	17900	0.0221	0.0434	0.0145
35 - 39	11860	15625	0.0231	0.0476	0.0170
40 - 44	10772	13267	0.0219	0.0527	0.0205
45 - 49	8322	10378	0.0223	0.0569	0.0254
50 - 54	7979	9500	0.0224	0.0637	0.0315
55 - 59	4549	5664	0.0247	0.0708	0.0407
60+	12353	15826			

Table 2.4 (Continued)

Age interval (a)	Population (000) 1961	1971	r(a+)	b(a+)	$\bar{d}$(a+)
		(c) All persons			
0 - 4	66089	79129			
5 - 9	64655	82886			
10 - 14	49286	68288	0.0224	0.0392	0.0088
15 - 19	35868	47457	0.0204	0.0364	0.0103
20 - 24	37321	43092	0.0191	0.0335	0.0116
25 - 29	36575	40808	0.0201	0.0369	0.0131
30 - 34	30838	36185	0.0222	0.0413	0.0152
35 - 39	25463	32887	0.0238	0.0460	0.0179
40 - 44	22856	28374	0.0234	0.0523	0.0218
45 - 49	18058	22792	0.0241	0.0576	0.0270
50 - 54	17108	20703	0.0244	0.0659	0.0337
55 - 59	9832	12649	0.0270	0.0735	0.0439
60+	24705	32700			

Table 2.5. Estimates of completeness of death reporting for destabilized population compared with stable estimates : 1971

	Non-stable estimates using different age ranges				Stable estimate
	10-49	15-54	20-59	10-59	
Males	0.838	0.860	0.913	0.890	0.893
Females	0.683	0.716	0.766	0.752	0.789
Total	0.760	0.790	0.841	0.823	0.843

2.3 Level of child mortality

As is evident from the above discussion, the availability of information on mortality, including infant and child mortality, by age and sex does not imply a standard of quality. The United Nations and World Health Organization (1981) recommend that emphasis should be given to the analysis of infant and child mortality in every country where the levels of such mortality are considered excessive or problematic. In this context a few recently developed indirect estimation techniques based on Brass basic method (Brass, 1975) seems attractive.

2.3.1 The Brass-Sullivan method

The fertility survey conducted in India (India Office of the Registrar General, 1976), covering the reference period of July, 1971 to June 1972, provides data on the proportions of children dead among those ever born to mothers in standard 5 year age groups. These proportions yield life table q_a-values when multiplied by factors derived by Brass (1975) from a fertility polynomial of age and a scale factor. Sullivan (1972) showed that multiplying factors based on simple linear regression models, fitted to several empirical fertility schedules, yield a much more improved results than the factors developed by Brass. Thus, the probability of dying from birth to age a, q_a, is given by the equation

$$q_a/D_i = A_i + B_i\ (P_2/P_3),$$

where

D_i = proportion of children dead in the ith age group of mother ($i = 1$ for age group 15-19 and so forth)

P_2/P_3 = ratio of the average parity of mothers aged 20-24 to mothers aged 25-29.

A_i, B_i = regression coefficients.

The main assumption underlying the method is the static conditions for age specific fertility and infant and childhood mortality in the recent past. Unlike Brass, who used a single mortality pattern, this method used all the four families of Coale and Demeny (1966) model life tables. Since it is very often not known whether the underlying mortality pattern conforms to West, North, East or South, Sullivan computed the effect of using wrong regression equation. The use of West regression model in the absence of definite knowledge of mortality pattern is recommended. We illustrate the Sullivan estimation with respect to West and South families of life table as these formulations are found to be suitable for India (Srinivasan et al., 1980). The information on surviving children collected from younger mothers are found to be most reliable and the regression analysis is successful with the ratios q_2/D_2, q_3/D_3 and q_5/D_4. The models for estimating q_a are as follows :

West family	South family
$q_2 = D_2\,(1.30 - 0.54\,P_2/P_3)$	$q_2 = D_2\,(1.33 - 0.61\,P_2/P_3)$
$q_3 = D_3\,(1.17 - 0.40\,P_2/P_3)$	$q_3 = D_3\,(1.20 - 0.44\,P_2/P_3)$
$q_5 = D_4\,(1.13 - 0.33\,P_2/P_3)$	$q_5 = D_4\,(1.14 - 0.32\,P_2/P_3)$

Table 2.6. Brass-Sullivan estimates of early childhood mortality for both sexes combined : 1971-72 (P_2/P_3 = 0.606)

Age group of mother	i	a	Average parity, P_i	Proportion of children dead, D_i	q_a	$l_a = 1 - q_a$
(a) West family of mortality pattern						
20 - 24	2	2	1.83	0.1344	0.1307	0.8693
25 - 29	3	3	3.02	0.1500	0.1391	0.8609
30 - 34	4	5	4.19	0.1655	0.1539	0.8461
(b) South family of mortality pattern						
20 - 24	2	2	1.83	0.1344	0.1291	0.8709
25 - 29	3	3	3.02	0.1500	0.1400	0.8600
30 - 34	4	5	4.19	0.1655	0.1566	0.8434

Table 2.6 shows the preliminary estimates of child mortality, but the final choice will be made after considering a further refinement of the conversion mechanism of the observed survival ratios.

2:3.2 The Brass-Trussell method

The regression models used above were fitted only to a few empirical schedules that also lacked early fertility experience of the developing countries. A variant of the procedure was made (Trussell, 1975) by considering additional variables (thus reducing standard error of the regressions) and replacing Sullivan's observed fertility schedules by wide range of Coale - Trussell (1974) model schedules. Using P_1/P_2 (Brass parameter), P_2/P_3 (Sullivan parameter) and D_i, Trussell (1975) developed new multiplying factors, his formulation being

$$q_a = K_i \, D_i^*$$

where

q_a = probability of dying from birth to age a

D_i^* = proportion of children dead in the ith age group of mother

$$K_i = A_i\,(P_1/P_2) + B_i\,(P_2/P_3) + C_i \log_e (P_1/P_2) + D_i \log_e (P_2/P_3) + E_i$$

A_i, B_i, C_i, D_i, E_i = regression coefficients (for the ith age group) tabulated by Trussell.

P_i/P_{i+1} = ratio of the average parities.

The procedure provides a set of alternative estimates of regression coefficients for use exclusively in populations in which fertility starts at an early age. Since entry into consensual unions takes place early in India, the use of these estimates seems most appropriate. Using the West and South mortality patterns as before, the technique is illustrated in Table 2.7.

Table 2.7. Brass-Trussell estimates of early childhood mortality for both sexes combined : 1971-72
(P_1/P_2 = 0.639; P_2/P_3 = 0.606)

Age group of mother	i	a	Regression coefficients A_i	B_i	C_i	D_i	E_i	D_i^*	K_i	q_a	l_a
			(a) West family of mortality pattern								
20 - 24	2	2	-.1340	-.0994	-.0549	-.0234	+.9948	.1344	.8852	.1190	.8810
25 - 29	3	3	-.0778	-.0637	+.0212	-.1592	+.9571	.1500	.9390	.1408	.8592
30 - 34	4	5	-.1430	+.0234	+.0690	-.2378	+.9558	.1655	.9668	.1600	.8400
			(b) South family of mortality pattern								
20 - 24	2	2	-.1215	-.1085	-.0669	-.0314	+.9667	.1344	.8690	.1168	.8832
25 - 29	3	3	-.0750	-.0408	+.0245	-.1938	+.9413	.1500	.9547	.1432	.8568
30 - 34	4	5	-.1512	+.0767	+.0759	-.2733	+.9301	.1655	.9828	.1626	.8374

Table 2.8. Logit smoothing of Brass-Sullivan and Brass-Trussell estimates of survivorship ratios : 1971-72

Age (a)	Standard logit, $Y_{s(a)}$	Estimated $l_{(a)}$	logit $l_{(a)}$, $Y_{(a)}$	$Y_{(a)} - Y_{s(a)}$	$\hat{Y}_{(a)}$ $(6)=(2)+\frac{\Sigma(5)}{3}$	Smoothed estimate, $\hat{l}_{(a)}$
(1)	(2)	(3)	(4)	(5)		(7)
			(a) Brass-Sullivan : West Model			
2	-0.7152	0.8693	-0.9474	-0.2322	-0.9616	0.8725
3	-0.6552	0.8609	-0.9114	-0.2562	-0.9016	0.8585
5	-0.6015	0.8461	-0.8522	-0.2507	-0.8479	0.8450
			(b) Brass-Trussell : West Model			
2	-0.7152	0.8810	-1.0010	-0.2858	-0.9694	0.8742
3	-0.6552	0.8592	-0.9043	-0.2491	-0.9094	0.8604
5	-0.6015	0.8400	-0.8291	-0.2276	-0.8557	0.8470
			(c) Brass-Sullivan : South Model			
2	-0.7152	0.8709	-0.9545	-0.2393	-0.9592	0.8720
3	-0.6552	0.8600	-0.9076	-0.2524	-0.8992	0.8580
5	-0.6015	0.8434	-0.8419	-0.2404	-0.8455	0.8444
			(d) Brass-Trussell : South Model			
2	-0.7152	0.8832	-1.0115	-0.2963	-0.9664	0.8736
3	-0.6552	0.8568	-0.8945	-0.2393	-0.9064	0.8597
5	-0.6015	0.8374	-0.8195	-0.2180	-0.8527	0.8462

Although Trussell's estimates should give better results than those given by Sullivan, the differences in the estimates derived for India are, in fact, negligible compared to the errors in the basic data.

In order to find the best estimate of the survivorship function, l_2 (to be used later in estimating adult mortality), from the Brass-Sullivan and Brass-Trussell estimates, the logit smoothing technique is adopted and illustrated in Table 2.8. Thus the smoothed value of

$$Y(a) = 0.5 \log_e (1 - l_{(a)})/l_{(a)},$$

denoted by $\hat{Y}(a)$, is derived from the estimated functions l_2, l_3 and l_5 as follows

$$\hat{Y}(a) = Y_{s(a)} + \frac{1}{3} \sum_i (Y_{(i)} - Y_{s(i)}) \qquad [a = 2 \text{ or } 3 \text{ or } 5; \; i = 2, 3 \text{ and } 5]$$

where $Y_{s(a)}$ is the logit of the Brass General Standard Life Table smoothed by Hobcraft (cited in Hill and Trussell, 1977). The smoothed $l_{(a)}$ values are then given by

$$\hat{l}_{(a)} = 1/[1 + \exp(2\hat{Y}_{(a)})].$$

The smoothed values of Brass-Sullivan and Brass-Trussell estimates of $l_{(2)}$, are respectively 0.8725 and 0.8742 for West Model and 0.8720 and 0.8736 for South model. The 'best' estimates of $l_{(2)}$, taken as the average of the smoothed estimates, are 0.8734 (West) and 0.8728 (South) - indicating little difference between the mortality patterns considered. As mentioned earlier, it is safer to use West regression model, but in the present case we accept a value of $\hat{l}_{(2)} = 0.8730$ at which the models converge.

Our next task is to get estimates of $l_{(2)}$ for males ($l_{m(2)}$) and females ($l_{f(2)}$) separately. The Census Actuary (India Office of the Registrar General,

1977) gave $l_{m(2)} = 0.8370$ and $l_{f(2)} = 0.8419$ so that, assuming a sex ratio of 1.07 (based on Sample Registration and Census data),

$$l_{(2)} = [1.07\ l_{m(2)} + l_{f(2)}] / 2.07 = 0.8394$$

corresponding to our above estimate of $\hat{l}_{(2)} = 0.8730$. The simple procedure of estimating $l_{m(2)}$ and $l_{f(2)}$ by pro-rating is adopted here. Thus,

$$\hat{l}_{m(2)} = [\hat{l}_{(2)}/l_{(2)}]\ l_{m(2)}$$

$$= (0.8730/0.8394)\ (0.8370) = 0.8705$$

and $$\hat{l}_{f(2)} = [\hat{l}_{(2)}/l_{(2)}]\ l_{f(2)}$$

$$= (0.8730/0.8394)\ (0.8419) = 0.8756.$$

The acceptance of these estimates of childhood mortality as being the 'best' obviously depends on the realisation of the basic assumptions underlying the estimation procedures. Thus, the implicit assumption that child mortality is independent of the order of birth is often questioned. Citing the results of Kenya Fertility Survey of 1977-78, Blacker (1979) shows that the mortality of first order births has often been much higher than that of the second-, third- and fourth- order births. The assumption of static fertility (this is possibly met in the Indian population upto 1971) and mortality and the latter's independence on age of mother are also not satisfied to some extent. These non-realisations of the basic assumptions in full may introduce an element of error into the estimates. The derived results with these possible limitations may however be accepted as an improvement over direct estimates made earlier.

2.4 Modelling age pattern of childhood mortality

The age specific mortality rates in childhood range decrease monotonically with age. An appropriate mathematical formulation for modelling such an age pattern of mortality is the Weibull function (Choe, 1981), which has been used in reliability applications in the biomedical sciences (Gross and Clark, 1975). This function defines the probability of surviving from birth to age x as

$$l_x = \exp[-\lambda x^{\gamma}], \quad \lambda > 0, \quad 0 < \gamma < 1, \quad x \geq 0,$$

where λ, varying with infant mortality, is interpreted as a level parameter, and γ as a shape parameter. A number of observed life tables has been found to fit the model well. A test of the model for India is done graphically after transforming the function into linear form. Thus

$$\ln l_x = -\lambda x^{\gamma}$$

that is, $\ln \frac{1}{l_x} = \lambda x^{\gamma}$

so that $\ln \ln \frac{1}{l_x} = \ln \lambda + \gamma \ln x.$

The test of Weibull function for life tables (1961-70) as reported by the Census Actuary (India Office of the Registrar General, 1977) is made (see Figure 2.1) by plotting ($\ln x$, $\ln \ln \frac{1}{l_x}$) and checking linear trends. Although Preston (cited in Luther, 1983) and Premi (1982) warn that the life table for India may be of poor quality, the model fits tolerably well, but the fit is slightly better for females than for males. Figure 2.1 however shows that for both sexes the points corresponding to very early childhood ages fall below the line indicated by the subsequent point at latter ages, implying underreporting of infant mortality in varying degrees.

The Gompertz function, which is known to describe the pattern of mortality at older ages quite well, may also be applied on a trial basis to fit mortality at young ages. The Gompertz survival function is of the form

$$l_x = e^{-\alpha e^{\beta x}}$$

that is, $\ln \ln \frac{1}{l_x} = \ln \alpha + \beta x.$

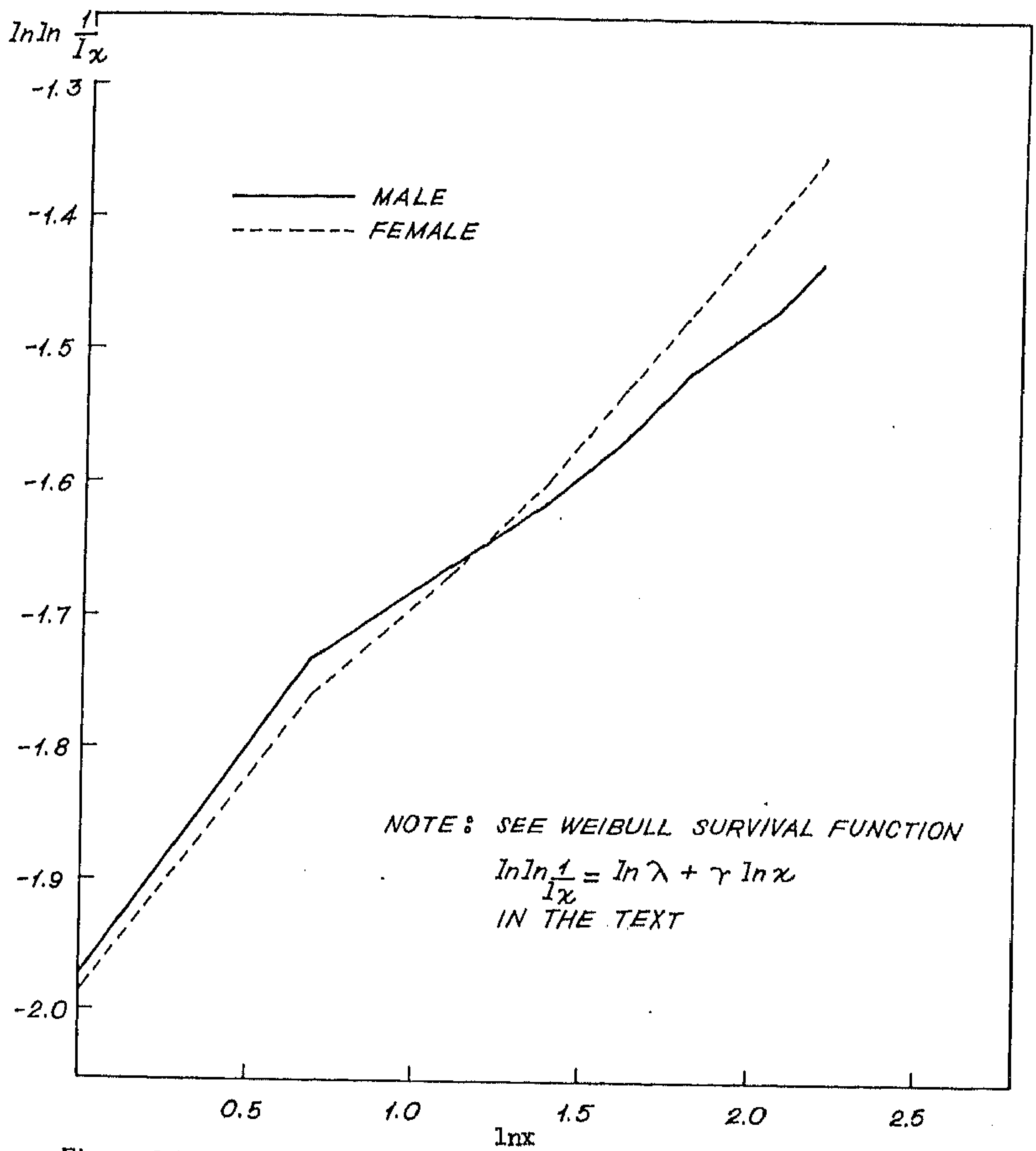


Figure 2.1 : Test of Weibull survival function for the census life tables, 1961-70 (Source : values of l_x were obtained from Registrar General and Census Commissioner, India, 1977)

$\ln \ln \frac{1}{l_x}$ is plotted against age x, rather than against logarithm of age as in the case of Weibull function, to test the fit to the observed Indian life tables (1961-70). As is evident from the data, the trend is not even approximately linear in the very young ages.

Assuming that the infant and childhood mortality pattern follows Weibull distribution, the parameters of the curve may be estimated as follows. We have

$$l_x = \exp [- \lambda x^{\gamma}] \quad \text{and} \quad l_{x+1} = \exp [- \lambda (x+1)^{\gamma}]$$

such that

$$_1q_x = (l_x - l_{x+1})/l_x$$

$$= [\exp (- \lambda x^{\gamma}) - \exp (- \lambda (x+1)^{\gamma})] / \exp (- \lambda x^{\gamma})$$

where $_1q_x$ is the probability of dying between exact ages x and x+1. The fit is made to reported $_1q_x$-values, rather than l_x-values (being more prone to errors than the ratio function), for x excluding very young ages. λ and γ may be estimated from the above equation by using the computer programme package B M D P A R (Dixon and Brown, 1979) for non-linear regression method. Given the estimates of λ and γ, the values of $_1q_x$ for the youngest ages can be extrapolated from the above equation and then used to obtain l_x recursively as $l_{x+1} = l_x (1 - {}_1q_x)$.

Luther (1982) provides a simple linearization procedure as an alternative to non-linear regression method so that a discussion on the approach is in order. We have

$$_1q_x = 1 - \frac{l_{x+1}}{l_x} = 1 - \frac{e^{-\lambda(x+1)^{\gamma}}}{e^{-\lambda x^{\gamma}}}$$

$$= 1 - e^{-\lambda(x+1)^{\gamma} + \lambda x^{\gamma}}$$

$$= 1 - e^{-\lambda[(x+1)^{\gamma} - x^{\gamma}]}$$

$$1 - {}_1q_x = e^{-\lambda[(x+1)^{\gamma} - x^{\gamma}]}$$

so that $-\ln(1 - {}_1q_x) = \lambda[(x+1)^{\gamma} - x^{\gamma}]$ (1)

Now, for the function $f(x) = x^{\gamma}$, $x \geq 0$, which is differentiable on the age interval $[x, x+1]$, there is an intermediate point y such that (by the special case of the Mean Value Theorem for derivatives)

$$f(x+1) - f(x) = (x+1-x)\, f'(y)$$

In other words,

$$(x+1)^{\gamma} - x^{\gamma} = \gamma\,(y)^{\gamma-1} \qquad (2)$$

The equation (1) can then be written as

$$-\ln(1 - {}_1q_x) = \lambda\,\gamma\,(y)^{\gamma-1}$$

that is, $\ln \dfrac{1}{1 - {}_1q_x} = \lambda\,\gamma\,(y)^{\gamma-1}$

$$\therefore \ln\ln \frac{1}{1 - {}_1q_x} = (\ln\lambda + \ln\gamma) + (\gamma-1)\ln y \qquad (3)$$

Thus linearization is attained by the procedure so that the equation (3) can be written in the form

$$Y = \alpha + \beta X$$

where

$$Y = \ln \ln \frac{1}{1 - {}_1q_x}, \quad X = \ln y, \text{ and } \alpha = \ln \lambda + \ln \gamma \text{ and } \beta = \gamma - 1$$

being respectively the intercept and slope of the line. The goodness of fit of the Weibull curve may be determined if the plot of the points (X, Y) is linear for ages for which ${}_1q_x$-values are reliable.

The equation (2) gives the value of y as follows :

$$y = \left[((x+1)^{\gamma} - x^{\gamma})/\gamma \right]^{\frac{1}{\gamma - 1}}$$

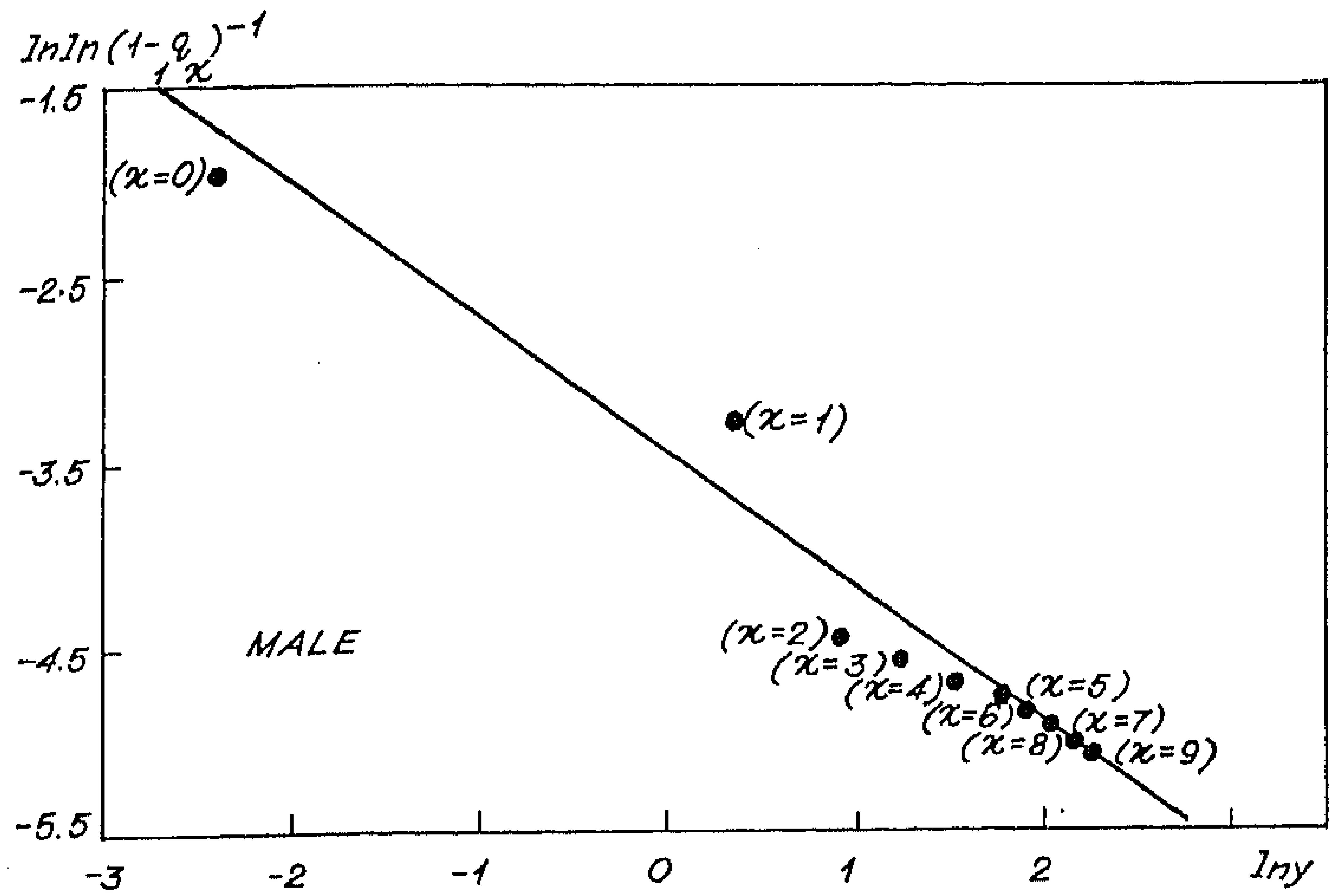
and hence $\ln y = \frac{1}{\gamma-1} \ln \left[((x+1)^{\gamma} - x^{\gamma})/\gamma \right]$.

Luther (1982) tabulates the values of ln y for $\gamma = 0.05$ to $\gamma = 0.30$ and shows that ln y is not sensitive to changes in γ for age $x \geq 3$.

An ordinary least square fit is not considered appropriate as the ln y values for the fitted points are not equally spaced. Instead, a graphical approach, rather than a laborious weighted least squares, is suggested for obtaining the estimates.

We use the procedure to estimate early childhood mortality from the reported ${}_1q_x$ -values (Table 2.9) for India (1961-70). The points $\left[\ln \ln (1 - {}_1q_x)^{-1}, \ln y \right]$ are plotted for ages 0 to 9 in Figure 2.3, which shows that for ages 5 to 9 the points fall almost on a line, implying that the fit is most satisfactory in this age range. The fitted lines represented in equation (3) were made to pass through the points at ages 5 and 9 (and fits the intermediate points as well) for females and 7 and 9 for males and were extended to extrapolate the values at the youngest ages.

The graphically fitted lines (Figure 2.2) are above the point at age 0, implying that the infant mortality (${}_1q_0$) level was under-estimated. The higher level of reported ${}_1q_x$ values at age 1 suggest a spurious shifting of deaths to this age from the neighbouring ages. On the other hand, some deceased children appear to have been omitted (at a decreasing rate by age) altogether from reporting in the age range 2 to 4.



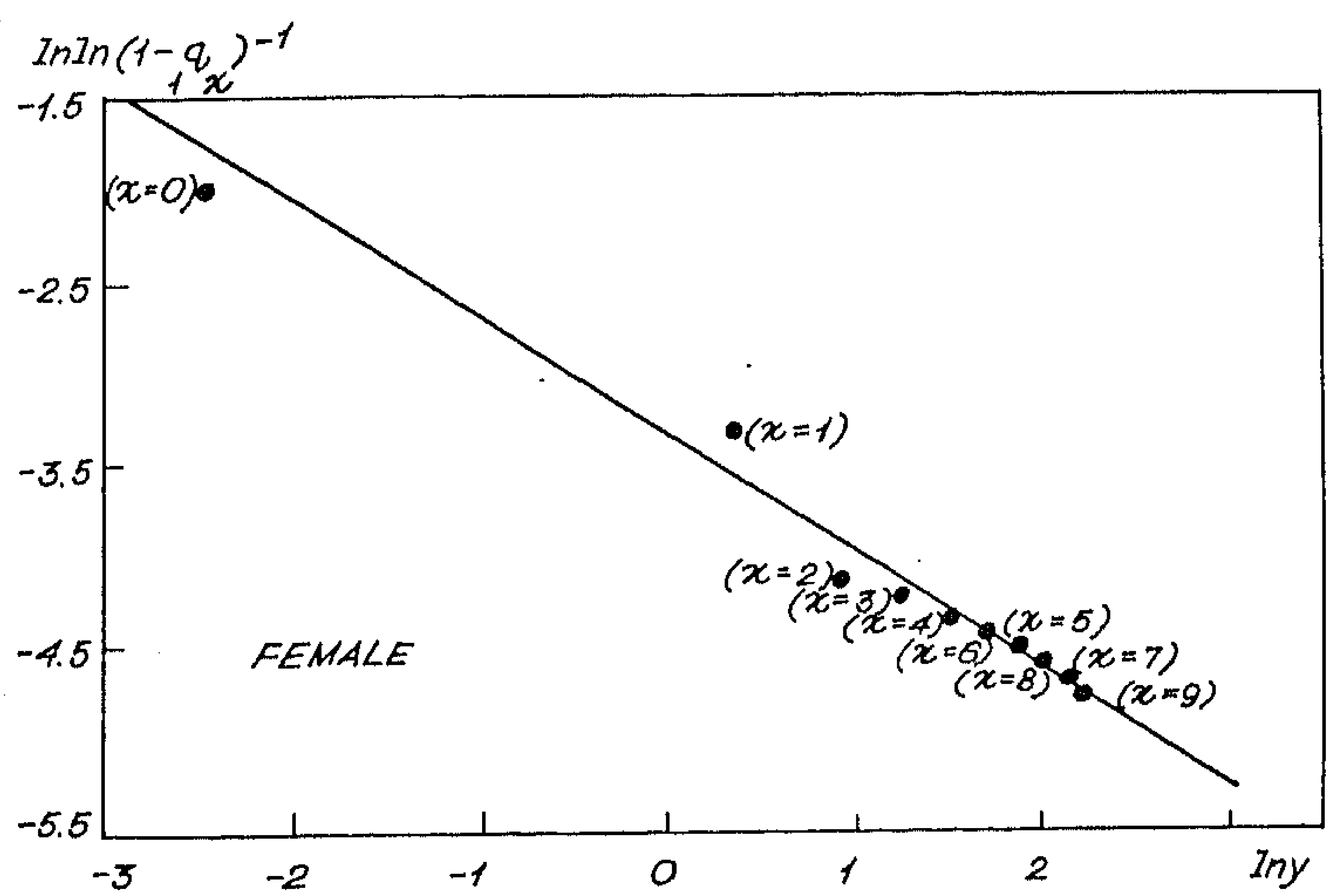


Figure 2.2 : Fitting a straight line to the points [ln ln(1-$_1q_x$)$^{-1}$, ln y], taken from Table 2.9, to find the Weibull Curve parameters

Table 2.9. Census and Weibull estimates of l_x and ${}_1q_x$ values ($0 \leq x \leq 9$): 1961-70

Age (x)	$X = \ln y$ *	$Y = \ln \ln (1-{}_1q_x)^{-1}$ **	${}_1q_x$ Census	${}_1q_x$ Weibull	l_x Census	l_x Weibull
			(a) Male			
0	-2.43668	-1.97033	0.13013	0.13546	100,000	100,000
1	0.37000	-3.25813	0.03773	0.02129	86,987	86,454
2	0.90392	-4.44573	0.01166	0.01485	83,705	84,613
3	1.24650	-4.55399	0.01047	0.01177	82,729	83,356
4	1.50030	-4.66020	0.00942	0.00991	81,863	82,375
5	1.70223	-4.76462	0.00849	0.00864	81,092	81,559
6	1.87000	-4.86399	0.00769	0.00772	80,404	80,854
7	2.01355	-4.95834	0.00700	0.00700	79,786	80,230
8	2.13901	-5.04356	0.00643	0.00643	79,227	79,668
9	2.25045	-5.11970	0.00596	0.00596	78,718	79,156
			(b) Female			
0	-2.43668	-1.98493	0.12837	0.15488	100,000	100,000
1	0.37000	-3.36236	0.03406	0.02787	87,163	84,512
2	0.90392	-4.12649	0.01601	0.01993	84,194	82,157
3	1.24650	-4.22910	0.01446	0.01607	82,846	80,520
4	1.50030	-4.32626	0.01313	0.01369	81,648	79,226
5	1.70223	-4.41264	0.01205	0.01205	80,576	78,141
6	1.87000	-4.48442	0.01122	0.01084	79,605	77,199
7	2.01355	-4.54443	0.01057	0.00990	78,712	76,362
8	2.13901	-4.66769	0.00935	0.00915	77,880	75,606
9	2.25045	-4.76107	0.00852	0.00852	77,152	74,914

[The estimated Weibull parameters are given in the text]

* ln y - values (mid-range) are from Luther (1982).

** Reported ${}_1q_x$ - values are from the Life Tables (1961-70) for India (paper 1 of 1977, Census of India, 1971).

The fitted lines passing through the two points at ages 7 and 9 for males, (2.01355, −4.54443) and (2.25045, −4.76107), and at ages 5 and 9 for females, (1.70223, −4.76462) and (2.25045, −5.11970), are as follows :

$$Y_m = -3.58685 - 0.68113\ X$$
$$Y_f = -3.33076 - 0.63556\ X$$

(m and f denote males and females respectively)

where $Y = \ln [\ln (1 - {}_1q_x)^{-1}]$ and $X = \ln y$, y being defined earlier.

For males,

$$\beta = \gamma - 1 = -0.68113, \text{ so } \gamma = 0.31887, \text{ and}$$

$$\alpha = \ln y + \ln \lambda = -3.58685, \text{ so } \lambda = 0.08682.$$

Similarly for females,

$$\gamma = 0.36444 \text{ and } \lambda = 0.09814.$$

The fitted Weibull curves can now be written as

$$l_x^m = \exp [-0.08682\ x^{0.31887}]$$
$$l_x^f = \exp [-0.09814\ x^{0.36444}],$$

$0 \leq x \leq 10.$

The estimates of l_x can thus be obtained from these equations. ${}_1q_x$ may also be derived as

$${}_1q_x = \frac{\exp [\exp (Y)] - 1}{\exp [\exp (Y)]}$$

The resulting estimates are shown in Table 2.9. As was mentioned earlier, an ordinary least squares (OLS) fit may not be appropriate in the present case. But for the purpose of comparison with the graphical estimates, the OLS estimates are also derived. The OLS produces the values of ${}_1q_0$ as 0.13541 for males and 0.15805 for females, compared to our estimated values of 0.13546 and 0.15488 respectively. The estimates for males are almost the same, and those for females, the agreement is not so good, but still of the same order of magnitudes. For other ${}_1q_x$ values, the results of comparison lead to similar conclusions, but the advantage lies with the graphical approach, being simpler to apply.

The values of the parameter γ derived above, which indicate the speed with which mortality declines, are quite high, and fall outside the usual range 0.05 to 0.30. However, the re-estimated values of l_x and ${}_1q_x$, using ln y corresponding to such high values of γ, differ from the earlier estimates only slightly.

The estimates of l_x, obtained here as an adjustment of reported values, are not strictly comparable with our earlier Brass-Sullivan and Brass-Trussell estimates (l_2, l_3 and l_5) because of the differences in reference periods. In spite of this, the general compatability of the estimates are encouraging.

Further interesting results may be derived from the Weibull survival distribution, once λ and γ are estimated. The instantaneous force of mortality or the hazard rate is defined as

$$\mu_x = -\frac{1}{l_x}\frac{d}{dx}l_x,$$

where l_x is given by the Weibull function. We have then

$$\mu_x = \lambda\gamma x^{\gamma-1}.$$

Based on the above values of λ and γ, the Weibull hazard functions are graphed in Figure 2.3. We find that μ_x is a monotonically decreasing function in the young age range, confirming the appropriateness of the use of Weibull model for graduating infant and childhood mortality. In an application to Australian data, Krishnamoorthy (1982) also obtained a better fit for the Weibull model and also for the logarithmic model (Hartman, 1980)

$$l(x) = [1 + e^{2a} x^{2b}]^{-1}$$

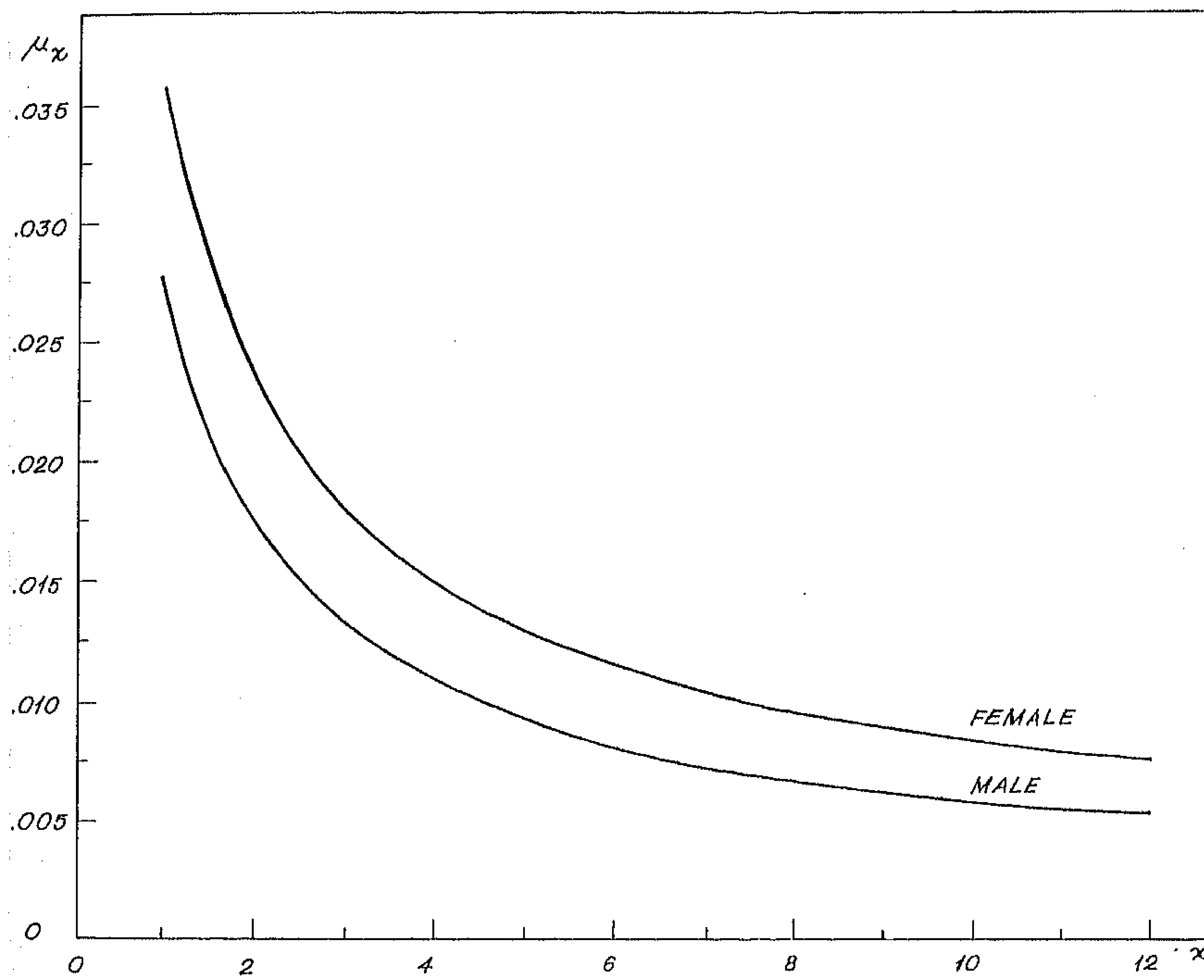


Figure 2.3 : Weibull hazard functions, $\mu_x = \lambda\gamma x^{\gamma-1}$, based on $\lambda = 0.087$ and $\gamma = 0.319$ for males and $\lambda = 0.098$ and $\gamma = 0.364$ for females (values derived from life tables, 1961-70, for India)

when compared to a hyperbolic function (Keyfitz, 1966)

$$l(x) = (\alpha x + \beta) / (x + \beta),$$

and recommended the Weibull curve for its elegance for analytic treatment. This does not however imply that it will be applicable to all sets of data; for example, the extremely poor fit of the curve to the mortality experience of Taiwan male children for 1970-71 (Luther, 1982) may be cited.

2.5 Level of adult mortality

The United Nations and World Health Organization working group (Bangkok, 20-23 October, 1981) on mortality studies noted the several inadequacy of reliable mortality data for adults in developing countries. In view of a very serious economic and social consequences of adult death, it recommended that serious efforts be made for analysis of data on adult mortality, using a wide range of approaches and procedures. Several techniques are available for estimating adult mortality based on spouse survival, parent survival and sibling survival. But the only input available for India, that too not very accurately, are the widowhood data collected in the censuses. The indirect estimation of adult mortality can thus be tried by the application of only the spouse survival method.

Besides the data constraint, the technique may be more suitable than the other methods for several reasons. It is not subject to the difficulties of multiple counting and adoption effect commonly associated with the orphanhood data used in the parent survival method (Brass and Hill, 1973). Assuming that all respondents would marry at the mean age at first marriage, thus fixing the exposure to risk, the spouse survival method avoids the use of dated data such as the distributions of age at first marriage. It was shown (Hill, 1977) that this drastic simplification would have a significant effect only for young ages.

The incidence of widowhood being mainly determined by mortality and age pattern of marriage, the method derives the proportions widowed of first spouse from model first marriage functions, which are simple polynomials rather than sophisticated Coale's model (Coale, 1971), and model life tables.

The proportions not widowed in two adjacent five-year age groups are then related to appropriate life table probability of surviving from a fixed starting age to the central point of the adjacent pair of age groups to obtain weighting factors (as functions of ages at first marriage) for converting the reported proportions not widowed into estimates of levels of adult mortality. Hill's (1977) spouse survival method considers a stable age distribution and assumes the same rate of mortality for the widows (widowers as well) as for the currently married persons.

It may be noted that the mortality derived by the method is under-estimated when both the spouses die. If the prevalent remarriage rates of widows (and widowers) of first marriage are not taken into consideration, the method, on the other hand, over-estimates the expectancy of life. The widowhood information collected only from first spouse may however avoid this problem. The spouse survival method actually estimates mortality experience of ever-married population, which approximates the total population (in the adult age range) when marriage is nearly universal as in the case of India.

2.5.1 Male adult mortality from widowhood data

The Brass-Hill equations for estimating male mortality from widowhood of female respondents are given as follows :

(i) when the mean age at first marriage of the females is below 20 :

$$\frac{l_{(N+5)}}{l_{(22.5)}} = W(N)\ {}_5P_{N-5} + (1 - W(N))\ {}_5P_N$$

(ii) when the mean age at first marriage of the females is above 20 :

$$\frac{l_{(N+5)}}{l_{(27.5)}} = W(N)\ {}_5P_{N-5} + (1 - W(N))\ {}_5P_N$$

where $l_{(22.5)}$, $l_{(N+5)}$ and $l_{(27.5)}$ are the life table survivors at ages 22.5, N+5 and 27.5 respectively; W (N) are the appropriate weighting factors derived from Brass - Hill tabulation on the basis of Hajnal's (1953) singulate mean age at first marriage for females and weighted (by age distribution) mean age at first marriage for males; ${}_5P_{N-5}$ and ${}_5P_N$ are the proportions of females not widowed in the adjacent five-year age groups of N-5 to N and N to N+5 respectively; N is the central age of two adjacent five-year age groups. For example, N will be 30 when the age groups are 25-29 and 30-34, and survival probability will be $l_{(35)}/l_{22.5)}$ or $l_{(35)}/l_{(27.5)}$, implying survivorship of 12.5 years from an age 22.5 or 7.5 years from an age 27.5.

This indirect estimation of adult mortality may not be successful if the widowhood data lack reliability. The Indian Censuses provide data on age specific proportions widowed, but the data are deficient as discussed elsewhere (Krishnamoorthy, 1977; Guha Roy, 1981b, Malaker, 1981). In situations of substantial remarriages of widows, the information on ever-widowed is clearly distorted. But in India, only some broad observations on widow remarriages are available. Though it is believed that there has been a liberalised outlook to widow remarriage in the culture (India Office of the Registrar General, 1974), no significant rise in widow remarriage is guessed (Krishnamoorthy, 1977). During 1901-71, there has however been a steady decline of proportions widowed, largely due to improvement in mortality. The UN survey (United Nations, 1961) in Mysore reported a maximum of 4.5 per cent remarriage among ever-married females. A survey in Calcutta City (Poti, et al., 1960), conducted for a different purpose, estimated remarriage rate to be of the order of 2 per cent. Both these studies were however local and possibly outdated now, and thus not representative of the national situation.

We look to another culture in the Indian subcontinent for the relevant data because we feel that it would be more realistic even to take a token allowance for widow remarriages rather than ignore them. The Bangladesh Retrospective Survey of Fertility and Mortality conducted in 1974 provide the necessary information for that country. Whereas 83 per cent of the population in India are Hindus, and 11 per cent Muslims, Bangladesh has predominantly Muslim population. The religion differential in patterns of marriage is also well pronounced. We, therefore, assume that the age pattern of remarriages among Hindu females in Bangladesh (United Nations, 1981, Table 96), would broadly apply to India. These remarriage rates were smoothed and adjusted by us to make them broadly consistent with the general age pattern (known from the censuses and Sample Registration Scheme of the Registrar General) of nuptiality. The remarriage pattern shown by the Mysore Study was used as an additional guideline during the adjustment process. The adjusted age (x) specific remarriage proportions (p_x) are presented in Table 2.10.

Following Brass, a correction technique may be applied to the reported proportions widowed, distorted by remarriages. If

W_{Ex} = proportions of ever widowed of the first husband,

W_x = reported proportions widowed of the first husband,

p_x = proportions remarried and

x = five-year age group, then

$$W_{Ex} \quad \frac{W_x + p_x}{1 + p_x} .$$

The adjusted proportions never widowed (P_x) required for the Brass-Hill formulations are obtained (see Table 2.10) as complement of W_{Ex}.

The singulate mean age at first marriage of females is 17 years, so that the first Brass-Hill equation estimating survival ratio from age 22.5 to age N+5 is used in our case. The age distribution weighted mean age at first marriage of males is worked out to be 21.7 years. These mean ages at marriage for males and females have been used as the points of entry into the standard table (Hill, 1977) of weights, W(N), for converting proportions not widowed into life table survival probability.

At this stage we estimate the survival ratio $l_{(N+5)}/l_{(22.5)}$ from the Brass-Hill equation (see Table 2.10). $l_{(N+5)}$ is estimated from Brass two-parameter logit model

$$\text{logit} \left[l_{(x)} \right] = \alpha + \beta \ \text{logit} \left[l_{s(x)} \right]$$

where $l_{(x)}$ and $l_{s(x)}$ are observed and Brass Standard survivorship function respectively. An independent estimate of male child mortality, $l_{(2)}$ = 0.871,

Table 2.10. Estimates of male adult survivorship by the Hill widowhood method (Widowhood data : 1971)

Age group of female respondents (x)	Observed proportions widowed (W_x)	Estimated proportions of widow remarriages (p_x)	Estimated proportions widowed of 1st husband ($W_{Ex} = \frac{W_x + p_x}{1 + p_x}$)	Proportions not widiwed of 1st husband	Central age (N)	Weight, W(N)	$\frac{l_{(N+5)}}{l_{(22.5)}}$	$l_{(N+5)}$	Implied β
15 – 19	.0034	.026	.0287	.9713	20	.741	.966	.761	1.131
20 – 24	.0090	.039	.0462	.9538	25	.570	.947	.746	1.040
25 – 29	.0190	.043	.0594	.9406	30	.599	.930	.733	0.966
30 – 34	.0410	.049	.0858	.9142	35	.623	.903	.712	0.941
35 – 39	.0699	.052	.1159	.8841	40	.640	.857	.675	0.970
40 – 44	.1440	.057	.1902	.8098	45	.644	.791	.623	1.014
45 – 49	.2060	.050	.2438	.7562	50	.624	.700	.552	1.343
50 – 54	.3655	.044	.3922	.6078	55	.604	.593	.467	1.104
55 – 59	.4108	.033	.4296	.5704					

Sources : W_x taken from the Census of India (1971), and W(N) from Hill (1977).

$l_{(2)}$ = 0.871, Hajnal's singulate mean age at first marriage of females = 17.0, period male mean age at first marriage = 21.7, α = −0.19, β (average) = 1.06 and $l_{(22.5)}$ = 0.788.

was derived by us by the Brass-Sullivan and Brass-Trussell techniques. If another value, $l_{(N+5)}$, is known, then we have two simultaneous equations

$$\text{logit}\left[l_{(2)}\right] = \alpha + \beta\ \text{logit}\left[l_{s(2)}\right]$$

$$\text{logit}\left[l_{(N+5)}\right] = \alpha + \beta\ \text{logit}\left[l_{s(N+5)}\right]$$

to obtain

$$\beta = \frac{\text{logit}\left[l_{(N+5)}\right] - \text{logit}\left[l_{(2)}\right]}{\text{logit}\left[l_{s(N+5)} - \text{logit}\left[l_{s(2)}\right]\right.}$$

Taking N+5 = 22.5 and assuming β = 1 initially, we estimate $l_{(22.5)}$ = 0.788 and hence $l_{(N+5)}$. Corresponding to each $l_{(N+5)}$, we get a value of β, and an average of these values obtained as $\bar{\beta}$ = 1.06 is used to make a new estimate of $l_{(22.5)}$. The process is continued until $l_{(22.5)}$ remains fairly constant. In our case, a second estimate of $l_{(22.5)}$ = 0.782 (when $\bar{\beta}$ = 1.06) is very close to the first estimate, which ($l_{(22.5)}$ = 0.788) is thus finally accepted.

Corresponding to $\bar{\beta}$ = 1.06, α is found as −0.19 so that the prediction equation is

$$\text{logit}\left[l_{(x)}\right] = -0.19 + 1.06\ \text{logit}\left[l_{s(x)}\right],$$

in which α stands roughly for changes in the level of mortality and β alters the link between child and adult mortality (Hill and Trussell, 1977). A life table (Table 2.11) is next generated by the above equation, which yields a male expectation of life at birth ($\mathring{e}_o^m$) of 49.7 years. A direct estimate of mortality by the U.S. Bureau of the Census (1978) is $\mathring{e}_o^m$ = 48.7 years for India (the official estimate for 1961-70 was 46.4 years). Relative to this, a little overestimate of $\mathring{e}_o^m$, obtained by us, indicates that the estimates of widowhood proportions should possibly be adjusted slightly more upward for remarriages than had been done here.

Table 2.11. Brass two-parameter abridged life table (males, 1971) generated by logit $[l_{(x)}]$ = $-.19 + 1.06$ logit $[l_{s(x)}]$

Age (x)	logit $l_{s(x)}$	logit $l_{(x)}$	l_x	L_x	T_x	$\mathring{e}_x$
0	-	-	100000	92639[a]	4972589	49.7
1	- .8670	-1.1090	90185	347665[b]	4879950	54.1
5	- .6015	- .8276	83959	415965	4532285	54.0
10	- .5498	- .7728	82427	409248	4116320	49.9
15	- .5131	- .7339	81272	401498	3707072	45.6
20	- .4551	- .6724	79327	390082	3305574	41.7
25	- .3829	- .5959	76706	376852	2915492	38.0
30	- .3150	- .5239	74035	363295	2538640	34.3
35	- .2496	- .4546	71283	348815	2175345	30.5
40	- .1816	- .3825	68243	332442	1826530	26.8
45	- .1073	- .3037	64734	313002	1494088	23.1
50	- .0212	- .2125	60467	288995	1181086	19.5
55	+ .0821	- .1030	55131	258752	892091	16.2
60	+ .2100	+ .0326	48370	220725	633339	13.1
65	+ .3721	+ .2044	39920	174480	412614	10.3
70	+ .5818	+ .4267	29872	122495	238134	8.0
75	+ .8593	+ .7209	19126	71790	115639	6.0
80	+1.2375	+1.1218	9590	32230	43849	4.6
85+	+1.7722	+1.6885	3302	11619[c]	11619	3.5

a. $L_0 = .25\, l_0 + .75\, l_1$

b. $L_1 = 1.9\, l_1 + 2.1\, l_5$

c. $L_{85+} = l_{85} \times \log_{10} l_{85}$.

2.5.2 Female adult mortality from widowhood data

The procedure for estimating female adult mortality is the same as that of male mortality except that the exposure to risk and the level of the risk are determined by male and female first marriage distributions respectively (just the reverse of the foregoing estimation).

The Brass-Hill estimating equation takes the forms :

(i) when the male mean age at marriage is below 25 years :

$$\frac{l_{(N-5)}}{l_{(17.5)}} = W(N)\ {}_5P_{N-5} + (1 - W(N))\ {}_5P_N$$

(ii) when the male mean age at marriage is above 25 years :

$$\frac{l_{(N-5)}}{l_{(22.5)}} = W(N)\ {}_5P_{N-5} + (1 - W(N))\ {}_5P_N$$

where the symbols have the usual meanings. In these equations, the ages of exposure are considered 5 years less than those of the males because of the same average age difference between the spouses. The weights, W(N), which take account of deviations from this average, are located by the distribution of male mean age at marriage (22.2 years) and the period mean age at marriage of females (17.1 years). The mean age at marriage of the cohort of males being less than 25, the first estimating equation is used in the present case. The adjusted widowhood data were built up from the reported proportions of widowers and estimated proportions of widower-remarriages in the same way as in the foregoing estimation. An independent estimate of female childhood mortality, $l_{(2)} = 0.876$, was derived earlier by us by the Brass-Sullivan and Brass-Trussell techniques. The widowhood technique is exemplified in Table 2.12, and the prediction equation derived is

$$\text{logit}\,[\,l_{(x)}\,] = -0.13 + 1.18\ \text{logit}\,[\,l_{s(x)}\,]$$

The life table generated (Table 2.13) gives rise to a value of $\overset{o}{e}{}_o^f = 47.9$ compared to an estimated value of 47.4 years by the U.S. Bureau of the Census (1978). These independent estimates of female expectation are much closer than

Table 2.12. Estimates of female adult survivorship by the Hill widowerhood method (Widowerhood data : 1971)

Age group of male respondents (x)	Observed proportions of widowers (W_x)	Estimated proportions of widower remarriages (p_x)	Estimated proportions widowered of 1st wife $(W_{ex} = \frac{W_x+p_x}{1+p_x})$	Proportions not widowered of 1st wife (P_x)	Central age (N)	Weight, $W(N)$	$\frac{l_{(N-5)}}{l_{(17.5)}}$	$l_{(N-5)}$	Implied β
20 – 24	.0063	.030	.0352	.9648	25	.659	.955	.781	1.31
25 – 29	.0123	.056	.0647	.9353	30	.430	.919	.752	1.27
30 – 34	.0210	.080	.0935	.9065	35	.434	.889	.727	1.22
35 – 39	.0277	.111	.1248	.8752	40	.469	.856	.700	1.19
40 – 44	.0457	.137	.1607	.8393	45	.503	.824	.674	1.15
45 – 49	.0593	.163	.1911	.8089	50	.536	.793	.648	1.11
50 – 54	.0971	.167	.2263	.7737	55	.557	.763	.624	1.04
55 – 59	.1189	.176	.2508	.7492					

Sources : W_x taken from the Census of India (1971), and W(N) from Hill (1977).

$l_{(2)}$ = 0.876, Hajnal's singulate mean age at first marriage of males = 22.2, period female mean age at first marriage = 17.1, α = –0.13, β(average) = 1.18 and $l_{(17.5)}$ = 0.818.

Table 2.13. Brass two-parameter abridged life table (females, 1971) generated by logit $[l_{(x)}] = -.13 + 1.18$ logit $[l_{s(x)}]$

Age (x)	logit $l_{s(x)}$	logit $l_{(x)}$	l_x	L_x	T_x	$\mathring{e}_x$
0	-	-	100000	93204[a]	4794618	47.9
1	- .8670	-1.1531	90938	349781[b]	4701414	51.7
5	- .6015	- .8398	84285	417212	4351633	51.6
10	- .5498	- .7788	82600	409800	3934421	47.6
15	- .5131	- .7355	81320	401175	3524621	43.3
20	- .4551	- .6670	79150	388370	3123446	39.5
25	- .3829	- .5818	76198	373425	2735076	35.9
30	- .3150	- .5017	73172	358018	2361651	32.3
35	- .2496	- .4245	70035	341500	2003633	28.6
40	- .1816	- .3443	66565	322800	1662133	25.0
45	- .1073	- .2566	62555	300608	1339333	21.4
50	- .0212	- .1550	57688	273355	1038725	18.0
55	+ .0821	- .0331	51654	239478	765370	14.8
60	+ .2100	+ .1178	44137	197890	525892	11.9
65	+ .3721	+ .3091	35019	149375	328002	9.4
70	+ .5818	+ .5565	24731	98275	178627	7.2
75	+ .8593	+ .8840	14579	52782	80352	5.5
80	+1.2375	+1.3303	6534	21188	27570	4.2
85+	+1.7722	+1.9612	1941	6382[c]	6382	3.3

a. $L_0 = 0.25\, l_0 + 0.75\, l_1$

b. $L_1 = 1.9\, l_1 + 2.1\, l_5$

c. $L_{85+} = l_{85} \times \log_{10} l_{85}$

in the case of male value. Though both males and females now live longer in India, the life expectancy is still not high. As found earlier, the rate of children dying before their first birth day is even now high, resulting in a lower life expectancy at birth.

2.5.3 Reference period for adult survivorship probabilities

It is of interest to relate adult mortality estimates prepared from spouse survival to a specific period. The widowhood method estimates mortality about 3 years and more before the census; as the age of the respondents increases, the time location of the estimate is pushed further back in the past. Brass and Bamgboye (1981) suggested a method of estimation of time period to which the adult survivorship probabilities refer. Employing the procedure, described in detail below, the estimates of time reference periods for the estimated adult survivorship probabilities are shown in Table 2.14.

Table 2.14. Time reference periods for male and female survivorship probabilities derived from spouse survival for 1971

Central age N	Male		Female	
	Survival probability $l_{(N+5)}/l_{(22.5)}$	Approx. reference date	Survival probability $l_{(N-5)}/l_{(17.5)}$	Approx. reference date
25	.947	October 1968		
30	.930	June 1966	.919	July 1966
35	.903	April 1964	.889	March 1964
40	.857	February 1962	.856	October 1961
45	.791	May 1960	.824	August 1959
50	.700	August 1958	.793	October 1957
55	.593	July 1957	.763	May 1956

Note : $l_{(N+5)}/l_{(22.5)}$ and $l_{(N-5)}/l_{(17.5)}$ are taken from Tables 2.10 and 2.12.

On the assumption that mortality level has been changing linearly over time (on the logit scale) during the past 1.5 - 2.0 decades, the adult survivorship estimates obtained from widowhood data are considered equal to those prevalent at specific time, t(N), N being the central age of a 5-year age group. While the theoretical treatment is given in Brass and Bamgboye (1981), the broad computational procedure for t(N), the number of years before the census, is being described.

2.5.3a Time reference to which male survivorship estimates refer

Denoting by SMAM the singulate mean age at marriage and by MAM the population weighted mean age at marriage, the length of exposure indicator is given by

$$X_m(N) = MAM_m + N - 2.5 - SMAM_f$$

$Z(x_m(N))$ = values of the standard function for calculation of the time reference (UN, 1983, Table 88)

The correction function is obtained as

$$U_m(N) = 0.3333\, l_n(1 - P_f(N - 5)) + Z(x_m(N))$$
$$+ 0.0037\,(27 - MAM_m),$$

P(N-5) being the female proportion widowed. The estimate of time is then computed from the following equation

$$t_m(N) = 0.5\,(N - 2.5 - SMAM_f)\,(1.0 - U_m(N))$$

.5.3b Time reference to which female survivorship estimates refer

$$X_f(N) = MAM_f + N + 2.5 - SMAM_m$$

$Z(x_f(N))$ = values of the standard function

$$U_f(N) = 0.3333 \ln(1 - P_m(N)) + Z(x_f(N))$$
$$+ 0.0037\,(27 - MAM_f),$$

$P_m(N)$ being the male proportion with first spouse alive.

$$t_f(N) = 0.5\,(N + 2.5 - SMAM_m)\,(1.0 - U_f(N))$$

The subscripts m and f refer to males and females respectively.

With the input given in Tables 2.15 and 2.16, we illustrate the above procedure for male survivorship probability in the case of $N = 25$. Thus,

$$X_m(25) = 21.7 + 25 - 2.5 - 17$$
$$= 27.2$$
$$Z(x_m(25)) = Z(27.2)$$
$$= 0.09$$

The value of this standard function is obtained by linear interpolation from the UN (1983) table.

$$U(25) = 0.3333 \ln(0.9538) + 0.09 + 0.0037(27 - 21.7)$$
$$= 0.0938$$

The number of years (before the census) to which mortality estimate refers is given by

$$t_m(25) = 0.5(25 - 2.5 - 17)(1 - 0.0938)$$
$$= 2.5$$

The census of 1971 was taken on 1 April, which is equivalent to 1971.2. The decimal equivalent has been obtained (UN, 1983) as

(Number of days from 1 January to 1 April)/365

= 91/365 = 0.249

The male survivorship estimate $l_{(30)}/l_{(22.5)}$ therefore refers approximately to 2.5 years before 1971.2, that is, to 1968.7 or September 1968.

In the estimation of time reference period, we used both singulate mean age at marriage (SMAM) and period mean age at marriage (MAM) rather than using only SMAM for both sexes. The procedure for estimating period mean age at marriage (Hill, 1977) is illustrated in Tables 2.17 and 2.18. It may be noted that using both approaches, the estimates of reference dates were found to be

Table 2.15. Calculations of time location for male survivorship probabilities estimated from widowhood data for 1971

Central age N	Proportion not widowed $1-P_f(N-5)$	Adult survivorship probability $l_{(N+5)}/l_{(22.5)}$	Length of exposure $x_m(N)$	Standard function $Z(x_m(N))^a$	Correction factor $u_m(N)$	No. of years preceding the census $t_m(N)$	Approx. reference date
25	.9538	.947	27.7	.090	.0988	2.5	October 1968
30	.9406	.930	32.7	.090	.0927	4.8	June 1966
35	.9142	.903	37.7	.094	.0921	7.0	April 1964
40	.8841	.857	42.7	.120	.1085	9.1	February 1962
45	.8098	.791	47.7	.168	.1451	10.9	May 1960
50	.7562	.700	52.7	.227	.1753	12.6	August 1958
55	.6078	.593	57.7	.300	.2256	13.7	July 1957

[a] The values of the standard function $Z(x)$ were taken from UN (1983, Manual X).

Note : The population weighted mean age at marriage for males (MAM_m) and the singulate mean age for females ($SMAM_f$) are respectively 21.7 and 17.0 years.

Table 2.16. Calculations of time location for female survivorship probabilities estimated from widowerhood data for 1971

Central age N	Proportion not widowed $1-P_m(N)$	Adult survivorship probability $l_{(N-5)}/l_{(17.5)}$	Length of exposure $x_f(N)$	Standard function $Z(x_f(N))$[a]	Correction factor $u_f(N)$	No. of years preceding the census $t_f(N)$	Approx. reference date
30	.9065	.919	27.3	.090	.0943	4.7	July 1966
35	.8752	.889	32.3	.090	.0826	7.0	March 1964
40	.8393	.856	37.3	.094	.0722	9.4	October 1961
45	.8089	.824	42.3	.117	.0834	11.6	August 1959
50	.7737	.793	47.3	.163	.1148	13.4	October 1957
55	.7492	.763	52.3	.222	.1626	14.8	May 1956

[a] The values of the standard function Z(x) were taken from UN (1983, Manual X).

Notes : (1) The singulate mean age at marriage for males ($SMAM_m$) and the population weighted mean age at marriage for females (MAM_f) are respectively 22.2 and 17.1 years.

(2) Data for N = 25 have been excluded because the tabulated values of the standard function Z(x) do not cover the value of x(25) = 22.3.

Table 2.17. Calculations of population weighted mean age at first marriage of males (1971)

Age group x, x+5	No. of males ('000)	Male proportion single $_5S_x$	Approx. rate for first marriages	New age group	Corresponding no. of males ('000)	Synthetic no. of marriages ('000)	Central age
10-14	36091	.956	.044	10.0 - 12.5	19255	847	11.25
15-19	25211	.823	.133	12.5 - 17.5	32812	4364	15
20-24	21563	.504	.319	17.5 - 22.5	23314	7437	20
25-29	20327	.189	.315	22.5 - 27.5	20516	6463	25
30-34	18275	.073	.116	27.5 - 32.5	20320	2357	30
35-39	17265	.040	.033	32.5 - 37.5	17315	571	35
40-44	15102	.035	.005	37.5 - 42.5	16810	84	40
45-49	12413	.030	.005	42.5 - 47.5	13919	70	45
						22193	

Population weighted mean age at first marriage of males (1971) = Σ (7) X (8)/ Σ (7) = 21.7

Notes : Col. (2) : Obtained from Table A-1, U.S. Bureau of Census, 1978.

Col. (3) : obtained from the Census.

Col. (4) : Derived as $_5S_x - {_5S_{x+5}}$; for x < 10, $_5S_x$ = 1.

Col. (5) : New age groups formed with the mid-points of two successive age groups.

Col. (6) : These estimates were obtained from Col. (2) by using age splitting coefficients derived by Carrier and Hobcraft (1971). The coefficients used are as follows :

10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
.5335	1.3015	1.0812	1.0093	1.1119	1.0029	1.1131	1.1213

Col. (7) : These numbers were obtained by multiplication of Col. (4) and Col. (6).

Table 2.18. Calculations of population weighted mean age at first marriage of females (1971)

Age group x, x+5	No. of females ('000)	Female proportion single $_5S_x$	Approx. rate for first marriages	New age group	Corresponding no. of females ('000)	Synthetic no. of marriages ('000)	Central age
10–14	32186	.881	.119	10.0 – 12.5	17171	2043	11.25
15–19	22236	.429	.452	12.5 – 17.5	28940	13081	15
20–24	21520	.091	.338	17.5 – 22.5	23267	7864	20
25–29	20472	.020	.071	22.5 – 27.5	20662	1467	25
30–34	17889	.009	.011	27.5 – 32.5	19891	219	30
35–39	15629	.005	.004	32.5 – 37.5	15674	63	35
40–44	13261	.004	.001	37.5 – 42.5	14761	15	40
45–49	10379	.004	.000	42.5 – 47.5	11638	0	45
						24752	

Population weighted mean age at first marriage of females (1971) = $\frac{\Sigma (7) \times (8)}{\Sigma (7)}$ = 17.1

Note : Explanations for columns are given in Table 2.17.

very close, the differences in period and singulate mean ages at marriage being small anyway.

2.5.4 An alternative estimate of adult mortality

The methodology of indirect mortality estimation from data on survival of first spouse, as discussed above, has further been simplified (Hill and Trussell, 1977). The modified method uses regression analysis to relate survival probabilities to the proportions of first spouses surviving to respondents. Though the method is much simpler for direct application, the data needed are not so readily available as the information on (complements of) proportions widowed used in the original method. It uses singulate mean age at marriage for both males and females rather than using singulate mean for one sex and period mean for the other sex. This approach however shares some of the underlying assumptions of the earlier method. These are : (i) all respondents marry at the mean age of marriage ; (ii) single and ever-married persons experience the same mortality; and (iii) the age distribution conforms to stable form. Also, various model life tables, fertility and nuptiality schedules form the basis of this method for estimating mortality.

A series of proportions of first spouses surviving to respondents is generated from a derived theoretical expression by choosing appropriate values of the parameters of mortality and nuptiality schedules and the population growth rate. The regression equation for predicting the survivors at age N, that is, $l_{(N)}$ is then derived from the given values of (i) the probability of surviving to age 2, $l_{(2)}$, (ii) the proportion of first spouse surviving, ${}_5S_N$, to respondents aged N to N+5, and (iii) the singulate mean ages at marriage of females, S_f, and males, S_m. The equation is of the form

$$l_{(N)} = a + b\,S_f + c\,S_m + d\,{}_5S_N + e\,l_{(2)}$$

where the estimated coefficients (a, b, c, d and e) were tabulated by Hill and Trussell (1977) on the basis of 900 observations.

We illustrate the technique with reference to estimation of male mortality, data limitations not withstanding. Although the questions on spouse survival are asked in the Indian Censuses, the proportions of first husbands surviving to wives have never been tabulated. We estimated earlier the female proportions not widowed (designated here as ${}_5P_N$ rather than P_x), which along with female $({}_5F_N)$ and male $({}_5M_N)$ age distributions of 1971 population are used to estimate the proportions of husbands surviving to females : ${}_5S_N = ({}_5P_N \cdot {}_5F_N)/{}_5M_N$. Given the values of S_f, S_m, ${}_5S_N$ and $l_{(2)}$, the estimates of $l_{(N)}$ are shown in Table 2.19.

The estimated survivorship function, $l_{(N)}$, can be used in the Brass scheme

$$\frac{1}{2}\log_e \frac{1 - l_{(N)}}{l_{(N)}} = \alpha + \beta \log_e \frac{1 - l_{s(N)}}{l_{S(N)}}$$

Table 2.19. Hill-Trussell estimate of adult male survivorship : 1971

Age of females (N, N+5)	Proportions of husbands surviving to females, $_5S_N$	$l_{(N)}$	β
25 - 29	0.8794	0.728	1.392
30 - 34	0.8488	0.713	1.249
35 - 39	0.8139	0.696	1.161
40 - 44	0.7378	0.651	1.205
45 - 49	0.6809	0.624	1.154
50 - 54	0.5399	0.543	1.252
55 - 59	0.4989	0.526	1.132

$l_{(2)}$ (male) = 0.871, S_f = 17.0, S_m = 22.2.

β derived from Brass logit relation :

$$\text{logit}\,[l_{(x)}] = \alpha + \beta\ \text{logit}\,[l_{s(x)}]$$

where $l_{s(x)}$ is the Brass standard survivorship function.

to generate life tables for various values of α and β. The value of β for each $l_{(N)}$ is obtained (see Table 2.15) and average β is calculated as 1.22, the corresponding value of α being -0.08. The life table generated by these estimates of α and β yield $\overset{o}{e}{}_o^m$ = 46.4 years for 1971, which is exactly the same as the estimate (1961-70) made by the Census Actuary (India Office of the Registrar General, 1977). This suggests no improvement in mortality (even if the acturial estimate is exact), which is unrealistic and thus not acceptable. The inadequacy of the basic data, as mentioned earlier, is possibly responsible for the underestimation of mortality and the failure of the Hill-Trussell first spouse survival method with the currently available Indian data.

CHAPTER THREE

FERTILITY : LEVELS AND PATTERNS

The estimates of fertility for India obtained directly from the vital registration, censuses and demographic surveys appear mutually inconsistent so that analytical methods may be employed to improve the estimates.

We first examine the age pattern of marital fertility and seek to find evidence of family limitation practices in the population. Following Coale, the age curves of marital fertility are then modelled by two parameters describing the extent of voluntary control of fertility and the level of fertility relative to natural fertility. By a successful use of Brass P/F ratio method on the Indian fertility data, certain patterns of errors of reporting fertility are investigated. Relational Gompertz fertility model, based on a standard schedule of fertility transformed mathematically, is used to graduate and parametrize fertility.

A recursive path analytic model is used in analysing how social and economic factors condition the relationships between fertility and acceptance of birth control measures. It uses States and Union Territories of India (rural) as units of observation. A few tests are being made to ensure that the model is appropriate.

A slightly shorter version of section 3.5 was presented at the ninth annual conference (December 1983) of the Indian Association for the Study of Population. The contents of section 3.6 presented at the eleventh Summer Seminar in Population (June-July 1980) of the East-West Population Institute, Hawaii was later published in Rural Demography.

3.1 Introduction

The study of fertility as the main component of population growth has assumed considerable importance ever since the acceptance of India's national population policy for arresting the growth of population. Vital registration system is usually the basic source of fertility data, but several local or regional studies were conducted in the past, the most notable being the Mysore Population Study (United Nations, 1961) of 1951-52. Earlier censuses tried to collect retrospective data on fertility and a few sample censuses during 1960s collected data on current fertility. The National Sample Survey (NSS) started to collect information on fertility since 1951 in single-round surveys. However, the estimates of fertility based on sample surveys are not only subject to sampling error (about 2 per cent in the NSS surveys : Government of India, 1961), but also subject to several times higher response error (Som, 1973).

An important breakthrough in the collection of information on fertility in India was made through the introduction of Sample Registration System (SRS) in 1964-65, which started to generate valuable data on fertility trends and differentials. This source has further been substantiated by the collection of information on live births from currently married women in the censuses since 1971. There have been also two competent surveys at the national level conducted by the Office of the Registrar General, namely, a 1972 fertility survey and a 1979 infant and child mortality survey (India Office of the Registrar General, 1976 and 1982), providing estimates of fertility for 1971-72 and 1978 respectively.

The decline in fertility indicated by these surveys is twice that indicated by the SRS, which is known to underestimate the level of fertility (Jain and Adlakha, 1982). Again, the extent of underestimation in the 1978 estimates may be higher than that in the 1972 estimates because the former was not adjusted for the births missed by the survey. Thus the available information on fertility appear mutually inconsistent, and alternative estimates by using different methods may be useful.

3.2 Evidence of family limitation practice in the population

We first focus on age patterns of marital fertility in India and compare them with the age pattern of natural fertility. Empirical evidence confirms that the characteristic feature of natural fertility in populations not practising deliberate control of family size is significantly different from that found in the populations with controlled fertility. We consider two reported marital fertility schedules (presented in Table 3.1) for India, one taken from 1972 fertility survey and the other derived from 1958-59 National Sample Survey 14th Round (Government of India, 1973) and compare them with a standard natural fertility schedule presented later in this section. The Panel A of Figure 3.1, in which these age speicifc marital rates (for ages 20-24 and above) are plotted, shows significant differences in the levels of observed and natural fertility. The fertility curves turn out to be convex for both the populations — one with natural and the other with observed fertility, the convexity tending to become more flattened for the latter. Although the scientific family limitation practices in India had not been much widespread, the folk methods and social - religious taboos, commonly practised, are believed to have fertility regulating effect on the populations that thus differed from the natural fertility population. Inspite of the fact that the shapes of the fertility curves are, more or less similar, the fertility declined more rapidly in the observed population at the earlier ages than in the 'natural' population in which sharp drops occurred after ages 35-39 because of decline in fecundity. Though separated by a time interval of 13 years, the similarity of the two observed schedules of fertility, both in level and shape, implies that the fertility level has remained almost steady (upto early 1970s) even under a prolonged programme of fertility regulation. This may be due to differential bias in the two surveys (SRS and NSS); but our analysis tends to confirm that there has not been any substantial decline in fertility during the period.

Following Knodel (1977), the comparison between natural and observed fertility situations is also depicted in Panel B of Figure 3.1 in which age specific marital fertility rates (r) for age groups 25-29 and above have been expressed as ratios to the level at age group 20-24. In symbols, the index values (I) of age specific marital fertility rates are given by

$$_5I_{20+5i} = \frac{_5r_{20+5i}}{_5r_{20}}$$

where i = 1, ..., 5.

The populations considered under regimes of observed and natural fertility in Panel B of the Figure 3.1 are the same as before, and they reveal the same kind of contrast in levels and similarity in shapes as in Panel A. The shape of the observed fertility curves show moderate divergence from the standard convex natural fertility age pattern, implying that some degree of fertility control (possibly confounded with some other factors that affect this departure) was present.

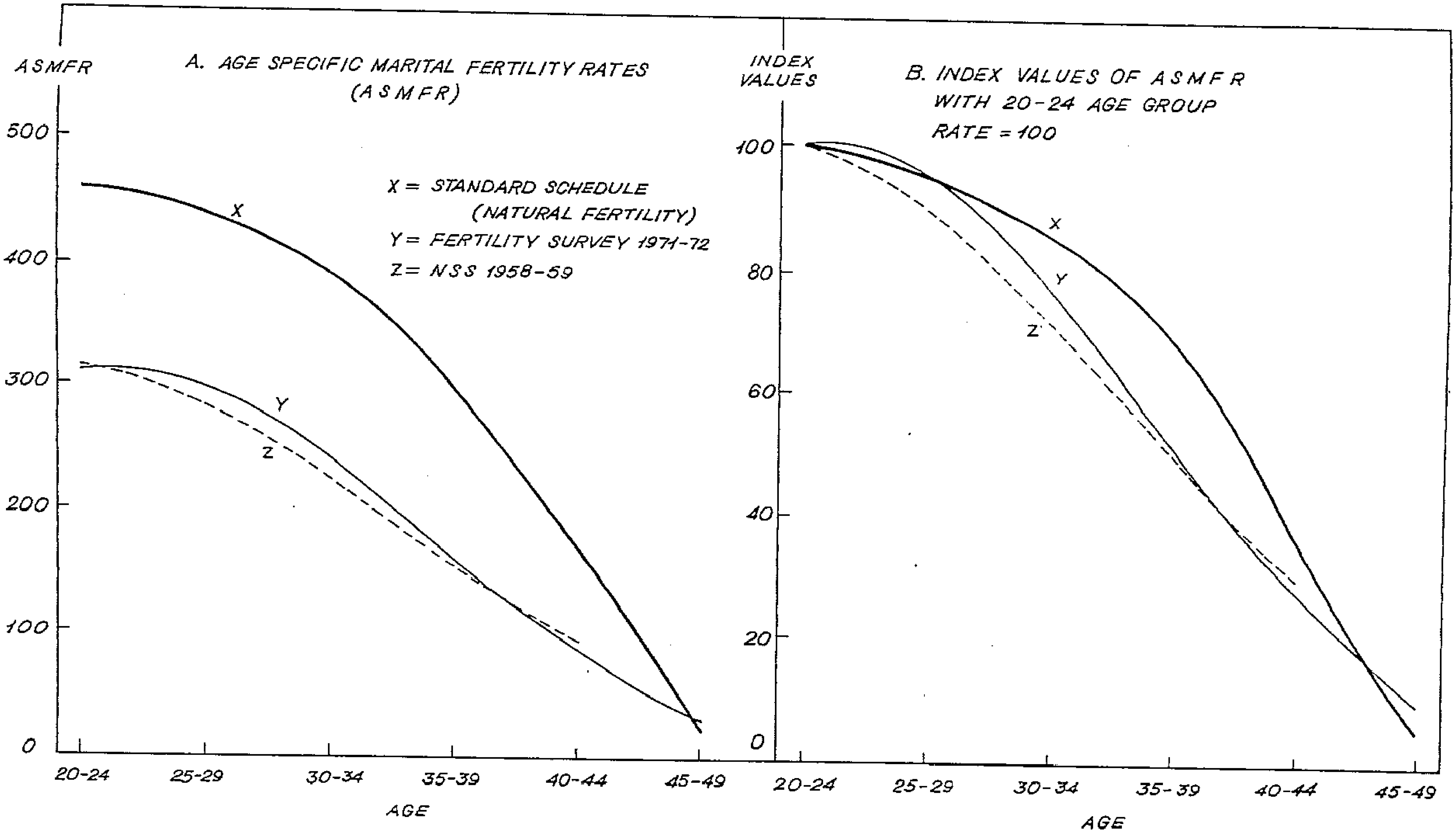


Figure 3.1 : Comparisons of two reported fertility schedules (1958-59 & 1971-72) with a standard schedule of natural fertility

3.3 Coale's model of marital fertility

Based on a comparison of natural and controlled fertility within marriage, a method of estimating levels of fertility from data that are deficient was developed by Coale (1971). A single standard natural fertility schedule was derived from ten of the thirteen schedules of Henry's (1961) age patterns of natural fertility, and the observed age curves of marital fertility were modelled by a two-parameter transformation of that standard. The formulation fitted to the more reliable parts of the data (that is, ages above 20) is given by

$$r(a)/n(a) = M \exp (m \cdot v(a))$$

where a stands for age, r(a) and n(a) are the reported and the standard natural fertility schedules respectively, v(a) is an empirically derived function expressing the universal pattern of voluntary control of fertility, and the two parameters M and m are interpreted respectively as the level at which natural fertility is experienced and the extent of voluntary control of marital fertility. The values of n(a) and v(a), stated to be invariant over time and population groups, are taken from Coale and Trussell (1978, Table 1, p. 205) in this analysis :

	20–24	25–29	30–34	35–39	40–44
n(a)	0.460	0.431	0.395	0.322	0.167
v(a)	0.000	–0.279	–0.667	–1.042	–1.414

The above model can be written in a more convenient form as

$$\ln [r(a)/n(a)] = \ln M + m\, v(a)$$

so that the parameters m and M that specify a model schedule of marital fertility can be estimated by ordinary least square (OLS) regression as

$$\Sigma \ln [r(a)/n(a)] = n (\ln M) + m \Sigma v(a)$$

$$\Sigma v(a) \ln [r(a)/n(a)] = (\ln M) \Sigma v(a) + m \Sigma [v(a)]^2$$

where n is the number of data points used.

The OLS has been found adequate for the estimation of m and M as the final estimates do not differ significantly by choice of technique (Coale and Trussell, 1978). The Coale's model fertility estimates [$\hat{r}(a)$] for several Indian schedules of marital fertility [r(a)] are shown in Table 3.1. The estimates for r(a), obtained by Brass method later, are also compared with the Coale's estimates for All India, 1971-72.

Table 3.1. Reported and estimated age specific marital fertility rates : 1958-59 and 1971-72

Age group (a)	All India, 1971-72			Rural India, 1971-72		Urban India, 1971-72		All India, 1958-59	
	r(a)	$\hat{r}(a)$ Brass	$\hat{r}(a)$ Coale	r(a)	$\hat{r}(a)$	r(a)	$\hat{r}(a)$	r(a)	$\hat{r}(a)$
15 - 19	0.213	(0.171)	(0.289)	0.212	(0.288)	0.221	(0.299)	0.228	(0.275)
20 - 24	0.313	0.330	0.323	0.313	0.322	0.313	0.334	0.315	0.308
25 - 29	0.299	0.317	0.283	0.303	0.287	0.284	0.266	0.284	0.275
30 - 34	0.240	0.249	0.236	0.249	0.244	0.201	0.194	0.225	0.235
35 - 39	0.161	0.171	0.176	0.170	0.186	0.124	0.127	0.161	0.179
40 - 44	0.087	0.097	0.083	0.094	0.090	0.052	0.053	0.095	0.087
percent error		8.1	9.5		9.4		10.7		7.6
Total marital fertility rate	6.56	6.68	6.95	6.70	7.08	5.98	6.36	6.54	6.80

Source : See text.

Note : $\hat{r}(a)$s are Coale-estimates except otherwise stated.

The obvious advantage of using several techniques on the same set of data (in this case the 1972 Fertility Survey data) is that it is possible to test the uniqueness or otherwise of the estimate. In the age range 20-44 for which the reporting is more accurate than for 15-19, both Brass-and Coale-procedures yield similar and improved estimates, and at least the results appear internally consistent. In the other fertility schedules considered here, the fit of the Coale's model appears quite satisfactory. The errors of estimation associated with the use of the model may be evaluated by examining the percent errors obtained as

$$\text{percent error} = 100 \cdot \Sigma \left| r(a) - \hat{r}(a) \right| / \Sigma r(a),$$

where $r(a)$ and $\hat{r}(a)$ are the reported and estimated values of marital fertility over all observed values including ages 15-19. The errors of less than 10 percent in most cases (See Table 3.1) are indicative of moderately good fit, which is further substantiated by the low values (0.002 - 0.004) of mean square error (m.s.e.) of regression obtained as

$$\text{m.s.e.} = \Sigma \left[\ln r(a) - \ln \hat{r}(a) \right]^2 / n.$$

In fitting the model described above, the parameter m estimated over age range 20-24 serves as a convenient summary measure of the degree of family limitation implicit in the age structure of the reported marital fertility schedule. The value of m can thus be an appropriate measure to test the extent of deviation of the observed schedule from the natural pattern. As shown in Table 3.2, a very moderate increase in m (0.18 - 0.24) is indicated over a period of 13 years from 1958-59 to 1971-72, and may be taken as evidence of some weak levels of fertility control in the population. The values of m however reveal substantial rural-urban differential, and are compatible with relatively strong and weak levels of fertility control respectively in the urban and rural areas.

The index m for individual age groups may be obtained from the basic equation of Coale's model as

$$m = \ln \left[r(a)/M \cdot n(a) \right] / v(a),$$

where $M = r(20\text{-}24)/n(20\text{-}24)$ is used as a scale factor in the calculation of m to make it independent of the level of fertility. The values of m for age group 25-29 and above are presented in Table 3.2.

" If the equation used to calculate m were fully valid and the typical age pattern of controlled fertility appropriately estimated, the separately determined values of m for each age group would all be identical" (Knodel, 1977). The different values for m (Table 3.2) across the age groups are all nearly equal, and the magnitudes of these values are also consistent with the trend and pattern of family limitation practices in India. The accuracy of

Table 3.2. Indices of fertility level (M) and fertility control (m) : 1958-59 and 1971-72

Population groups	Index of fertility level, M	Mean index of fertility control, $\bar{m}$	m by age groups			
			25-29	30-34	35-39	40-44
All India (1971-72)	0.702	0.242	0.240	0.242	0.240	0.244
Rural India (1971-72)	0.700	0.186	0.190	0.187	0.184	0.185
Urban India (1971-72)	0.726	0.585	0.582	0.586	0.586	0.585
All India (1958-59)	0.670	0.178	0.175	0.178	0.179	0.178

the estimates of age pattern of marital fertility by the method could thus be attained. Since the Coale's model does not take into account the differences in composition of population groups by marriage duration (Page, 1977), this may however affect the above fertility estimates.

The level of fertility for each population group relative to the natural fertility schedule is indexed by M (Table 3.2). The M-values are all below unity, apparently indicating substantial family limitation practices. But it seems reasonable to assume that the moderately low level of M is not necessarily dependent solely on voluntary control of fertility in the situation of poor nutritional levels affecting fecundity (Frish, 1975).

3.4 Brass method of fertility estimation

In a fertility survey the direct question on births during the preceding year very often results in answers biased by timing errors. Reported fertility might be lower or higher than the true level according as the responses cover a shorter or longer period than 12 months. The number of children ever born by a woman is also subject to reporting bias. The older women understate their parities by wider margin than the younger ones possibly because of progressive (with age) foregetting of children ever born and of tendency to omit selectively those older children not present in the household. Thus, both types of fertility information, namely the age specific fertility rates (current measure) and the average parity (retrospective measure) may be clearly deficient, but the nature of the errors is different. Brass assumes that the imprecision in the reference period in current fertility is not likely to be influenced by the ages of women and that the number of children ever born is reported with acceptable accuracy at the younger ages of mother, such as 20-24 and 25-29. So he combines the advantages of both measures by accepting the age specific fertility pattern from the current births and the level of fertility from the reported average number of children ever born to younger women, and comparing them.

The cumulated fertility (F') from the earliest age of childbearing to an exact age should match the mean parity (P) to that age, if it is assumed that the women dying have the same fertility as those surviving and that there are no errors in the reference period. If P and F' do not agree, the reported age specific fertility is multiplied by the ratio P/F' for younger women to adjust the entire series of reported current fertility rates so as to be consistent with the mean parity of younger women. But there is no exact correspondence between P and F' with respect to location of age, and they are thus not directly comparable. When the conventional quinquennial age groups (e.g., 15-19, 20-24, etc.) are considered, F' indicates cumulated fertility upto ages 19.5, 24.5, etc., whereas P relates roughly to the midpoints 17.5, 22.5, etc. It is possible to adjust F' to these ages by linear interpolation, but the assumption of rectangular distribution of fertility within each age interval is possibly not realistic. A more sophisticated procedure (Brass, 1968) is, therefore, used to derive adjustment factors (k). These factors are derived with the help of an age specific fertility model function

$$f(x) = \begin{cases} C(x - s)(s + 33 - x)^2 & \text{for } s \leq x \leq s+33 \\ 0 & \text{otherwise} \end{cases}$$

where the length of the childbearing period is taken as 33 years, and s is the starting age of childbearing and C is a constant linked to the level of fertility, and are related respectively to the mean age of childbearing ($\bar{m}$) and total fertility rate (F) by

$$\bar{m} = s + 13.2 \quad \text{and } C = F \div 98826.75$$

These relations can easily be established by putting $x - s = a$ in the Brass fertility polynomial and integrating.

Now, the integral of f(x) from s to a particular age t gives the cumulative fertility to age t :

$$F(t) = \int_s^t f(x)\,dx$$

$$= C \cdot \left[\frac{1}{4} (s + 33 - x)^4 - 11 (s + 33 - x)^3 \right]_s^t$$

The age specific fertility rate for the age group i to i+5 is then given by

$${}_5f_i = \frac{1}{5} \left[F(i+5) - F(i) \right]$$

Similarly, the integral of F(x) within a 5-year age interval (i to i+5, say) divided by the age interval gives the mean number of children ever born (${}_5MC_i$) per woman in the age group.

The adjustment factor, k, for each age group (x, x+5) is then obtained as

$${}_5k_x = \left[{}_5MC_x - F(x) \right] / {}_5f_x$$

Now, the unadjusted cumulative fertility F'_{i+5} upto age i+5 is obtained from the reported age specific fertility rates ${}_5f_j$ as

$$F'_{i+5} = 5 \sum_{j=0}^{i-1} {}_5f_j$$

and finally the adjusted average cumulative fertility is given by

$${}_5F_x = F'_x + {}_5k_x \cdot {}_5f_x$$

The k - factors can be directly obtained from the Brass - tabulation (Brass and Coale, 1968, Table 3.1, p. 94) by linear interpolation guided by the observed ratio ${}_5f_{15}/{}_5f_{20}$ ('the steepness of the take off of the fertility curve') for the first three age groups and by the mean age of childbearing ($\bar{m}$) of the reported fertility schedule for the other age groups.

There are now two options for correcting the observed age specific fertility. On the assumption, justified by experience, that the forgetting of children ever born (P) at ages 20-24 and 25-29 of women being less likely, either of the ratios ${}_5P_{20}/{}_5F_{20}$ or ${}_5P_{25}/{}_5F_{25}$ (or their average) may be used as multipliers to adjust upwards (or downwards) the reported fertility schedule.

The 1972 fertility survey, as mentioned earlier, provide information on children ever born (P) for only ever-married women. In order to compute P-values corresponding to age specific fertility rates (f) for all women, the given P-values are being adjusted by the proportion of all women who were ever-married in the 1971 census.

The results of comparison of cumulated current fertility and average parity are shown in Table 3.3. Since the P/F ratios are greater than unity (upto age 40) and since they exhibit a downward trend with age, it seems that the possible omission of births by older women had coupled with either error of shorter reference - period size or an occurrence of fertility decline. But in view of the fact that fertility has been, more or less, steady in the recent past, the pattern shown by P/F ratios indicate most probably the reference size error and the usual under-reporting by older women. Moreover, the Parity/Fertility ratios are not likely to be affected by any possible temporal misallocation of births unless it varied with the reported ages of women. Going by the P_2/F_2 ratio (subscript 2 stands for age group 20-24), a moderate understatement of current fertility of the order of 7 percent is indicated. The existence of errors in the basic data on fertility is thus implied. As mentioned above, these errors seem to be due to reporting of current births (period data) in a shorter reference period than a year and omission of children ever born to cohorts of women in the older age range. The high value of P/F = 1.24 for the age group 15-19 may however be due to the tendency of Brass multiplying factors to lower the estimate of fertility at younger ages.

Table 3.3. Estimation of age specific fertility rates by Brass method : India, 1971-72

Age group	i	Reported f_i	$F'_i = 5f_{i-1}+F'_{i-1}$	k_i	$F_i = F'_i+k_if_i$	P_i	P_i/F_i	Adjusted f_i
15 - 19	1	0.0909		1.9910	0.181	0.225	1.24	0.097
20 - 24	2	0.2631	0.4545	2.8458	1.203	1.286	1.07	0.282
25 - 29	3	0.2762	1.7700	3.0129	2.602	2.737	1.05	0.296
30 - 34	4	0.2169	3.1510	3.1170	3.827	3.944	1.03	0.232
35 - 39	5	0.1428	4.2355	3.2414	4.698	4.746	1.01	0.153
40 - 44	6	0.0755	4.9495	3.5010	5.214	5.050	0.97	0.081
Total fertility (F)		5.33						5.71
Crude birth rate		37.8						40.5

Notes : 1. See text for explanation of symbols.

2. f_is correspond to age groups one-half year younger than shown.

3. f_i and P_i (after preliminary adjustment) were taken from 1972 fertility survey (India Office of the Registrar General, 1976, Table 4, p. 6 and Table 40, p. 24).

4. k_is were determined on the basis of $f_1/f_2 = 0.345$ for the first three age groups and of $\bar{m} = 28.82$ years for the remaining age groups.

In selecting appropriate adjustment factor, we go by the recommended procedure (Brass and Coale, 1968) of using P/F ratio for age group 20-24. The recall lapse at this age group being supposedly minimum, the level of fertility is best judged by the average parity reported by the mothers of these ages. Moreover, in the present case, the P/F ratio corresponding to this age group is quite consistent with the trend of the ratios of latter ages. Since teen-age sub-fecundity affects irregularly the teen-age fertility, fertility rates below age 20 are not considered for the purpose. Also, P/F corresponding to 15-19 is sensitive to sampling error. We thus accept 1.07 (Table 3.3), the value of P/F for age group 20-24, for an upward adjustment of reported births under the assumption that the reference size error is same for all ages and that the average number of children is reported correctly by younger mothers.

In order to test the accuracy of estimates of average cumulative fertility by the k - factor procedure, we consider an alternative method of estimation (Barclay, Coale, Stoto and Trussell, 1976). The procedure consists in fitting a quadratic polynomial to the cumulative fertility rates and integrating the function over a 5-year period and dividing the result by 5 to find average cumulative fertility (F). The estimating equations are

$$F_1 = F_0' + 3.392 f_1 - 0.392 f_2$$

and $$F_i = F_{i-1}' + 0.392 f_{i-1} + 2.608 f_i,$$

where the symbols have the same meanings as explained earlier and the values of i for the age groups 10-14, 15-19, 20-24, ... are 0, 1, 2, ... The estimates of F_i by the two methods are presented below :

Method	15-19	20-24	25-29	30-34	35-39	40-44
Brass	0.181	1.203	2.602	3.827	4.698	5.214
Barclay, Coale, et al.	0.205	1.176	2.594	3.825	4.693	5.202

As is evident, the two approaches make only a trivial difference in the results, particularly at ages 25+.

In estimating the birth rate from the Brass fertility method, we note that the reported number of births last year being tabulated by age of women at the time of survey, the births really occurred to mothers one-half year younger on the average. The age specific fertility rates, which thus correspond to age groups 14.5 - 19.5, 19.5 - 24.5, ... of mothers, were converted (Trussell and Hill, 1980) into rates for standard age groups 15-19, 20-24, ... before calculating the birth rate. The present estimate of the birth rate is 40.5 compared with the rate of 37.8 derived from the reported fertility schedule for the period 1971-72. Our estimates of total fertility rate (5.71) and crude birth rate (40.5) compare quite well with Jain - Adlakha's (1982). Their corresponding estimates were 5.75 and 40.6 for 1972.

The age specific marital fertility rates are being obtained by dividing the adjusted age specific fertility rates (Table 3.3) by the proportions married (1971 Census) in each age interval. The results, already presented in Table 3.1 along with the Coale-estimates, make a remarkable similarity in pattern with the observed rates, though the estimate for age group 15-19 seems unstable.

Further illumination on the patterns of errors of reporting fertility could be made by comparing current and retrospective fertility separately for each birth order and sex of offspring. Unfortunately, the fertility data by such classifications are not presently available. We could however study the differentials by rural and urban regions of India. The P/F ratios (Table 3.4) for rural areas are extremely consistent across age groups and somewhat lower than the ratios for the country as a whole. The urban ratios are consistent only at ages 20+, but much higher than the national ratios. Contrary to expectation, the underreporting of births in the year before the survey is remarkably higher for urban sector as compared to the rural sector, the level of fertility implied by P/F ratio for age group 20-24 being 28 per cent higher than that indicated by current rates. It is however possible that because of the relatively small number of births, the sampling error of the measures for the urban part (covering only one-fifth or so of the total population) is high. In spite of this reporting differentials, the additional comparisons of P and F confirm our estimate of overall adjustment factor of around 7 per cent.

Table 3.4. P/F ratios by area of residence : India, 1971-72

Area	Age group of mothers at the time of survey					
	15-19	20-24	25-29	30-34	35-39	40-44
Rural	1.25	1.05	1.02	1.00	0.98	0.93
Urban	1.72	1.28	1.22	1.21	1.20	1.18
All	1.24	1.07	1.05	1.03	1.01	0.97

In the above estimation of age specific fertility rates for India, the information on current fertility were taken from the same survey as the data on retrospective fertility so that age misstatements are the same for the two sets of measurements. Although the procedure is generally robust to errors in age, the use of the same source of data reduces the effect of any extensive age misreporting on the estimates. Further improvements in the results may be achieved by replacing the direct question on births during the preceding year by the one on most recent births (UNECA, 1971; Blacker, 1971) in the demographic surveys.

3.5 Parametrizing Indian fertility experience by Gompertz function

Originally developed as a description of mortality curve, the Gompertz function (Gompertz, 1825) conveniently restated as

$$F(x) = FA^{B^x},$$

F(x) being the cumulative fertility rates to age x, F the total fertility, A and B the positive constants less than unity, was later found to be a good representation of fertility distributions (Brass, 1974, 1977, 1980, 1981; Martin, 1967; Murphy and Nagnur, 1972; Romaniuk and Tawny, 1969; Wunsch, 1966). Besides F, the other two parameters A and B of the curve have explicit demographic interpretations. Thus, A is the proportion of total fertility (F) completed by age x_o (origin) and B is related to the extent of peakedness of the fertility distribution. An important property of the Gompertz curve is that the age specific fertility rates obtained as

$$\frac{d}{dx} F(x) = F \log_e A \log_e B(A^{B^{x-x_o}} \cdot B^{x-x_o})$$

(x_o being a suitable origin on the age axis) increase monotonically to a maximum and then decrease monotonically, and by the age at which the maximum fertility is attained, exactly $^1/_e$ or about 37 per cent of F is completed irrespective of the values of F, A, B, and x_o (Titus, 1972). In order to absorb some irregularities in the data, the model uses cumulative fertility rather than age specific fertility.

The problems of estimation of the parameters have been investigated in several studies. Wunsch (1966) used the method of partial totals and Martin (1967) applied selected points method to fit the Gompertz function

$$F(x) = FA^{B^{(x-x_o)}}$$

Thus, taking d as the interval between two consecutive selected age points (x_o, x_1, and x_2) and the observed cumulative fertility rates F(0), F(1) and F(2) at the corresponding selected points, the parameters were estimated as follows :

$$\hat{B} = [(\log_e F(2) - \log_e F(1))/(\log_e F(1) - \log_e F(0))]^{1/d}$$

$$\hat{A} = \mathrm{Exp}\,[(\log_e F(1) - \log_e F(0))/(B^d - 1)]$$

$$\hat{F} = \mathrm{Exp}\,[\log_e F(0) - \log_e A]$$

The method is however considered unsatisfactory as different sets of selected points give rise to widely varying estimates (Miller, 1946). The problem is reduced, but not completely eliminated by fitting a cumulative curve. Murphy and Nagnur (1972) used least squares fits by taking recourse to an iterative procedure. As many as four iterations were required to reach the convergence. Although the method provides a much better fit, it requires computer facilities to be available.

In order to simplify the comparison of the Gompertz model with observations required for exploratory analysis, Brass made double logarithm $[-\log_e(-\log_e)]$ linearity transformation of the function

$$F(x)/F = A^{B^x}$$

Thus,

$$\log_e [F(x)/F] = B^x \log_e A$$

$$\log_e [-\log_e F(x)/F] = \log_e (-B^x \log_e A)$$

$$-\log_e [-\log_e F(x)/F] = -\log_e (-\log_e A) - x \log_e B$$

$$\text{Colog}\,[\,\text{Colog}\,F(x)/F\,] = \text{Colog}\,(\text{Colog}\,A) - x\,\text{Colog}\,B$$

that is, $Y(x) = a + bx$

where $Y(x) = \text{Colog}\,[\text{Colog}\,F(x)/F\,]$

$a = \text{Colog}\,(\text{Colog}\,A)$ and

$b = \text{Colog}\,B, \quad -\infty < a < \infty$ and $0 < b < \infty$

The equation has been found to be a good representation of the fertility distribution in the central age range, but the fit to observations at the early and late ages is not good enough. As in the Brass logit model life table system in which the assumption of a linear relationship of a given life table, $l(t)$, to a common standard, $l_s(t)$, has proved successful, the fit of the Gompertz model at the tails of the fertility distribution can be much improved by relating similarly $Y(x)$ to a suitable standard, $Y_s(x)$, rather than to x. Thus, the modified Gompertz function is written as

$$Y(x) = \alpha + \beta\, Y_s(x),$$

where α and β measure location and scale of the transformed variable and

$$Y_s(x) = \text{Colog}\,[\,\text{Colog}\,F_s(x)/F_s\,].$$

The standard schedule, $Y_s(x)$, can be derived from a set of fertility rates to represent an average pattern. Such a standard has been developed (Booth, 1979) from a series of reported fertility distributions and Coale-Trussell fertility models (Coale and Trussell, 1974). If the modified Gompertz model were exact, any one reported fertility schedule could be taken as $Y_s(x)$. In reality however the model does not hold exactly, and an appropriate standard for a specific country can be developed as an average of the available series of fertility schedules so as to minimize the deviations of individual schedule from linearity.

A. 'Relational' Gompertz Fertility Model (this is a relational system as it relates an observed $Y(x)$ - schedule to the standard $Y_s(x)$ by a simple function) can be fitted to the mean parities, P_i (P_1 for age group 15-19, P_2 for 20-24, etc.), as well so that

$$Y(i) = \text{Colog}\ (\text{Colog}\ P_i/F) = \alpha + \beta\ Y_s(i).$$

In the application of the relational model, the total fertility, F, must either be reliably known or estimated. If P_{45-49}, that is, the number of children ever born as reported by mothers of age 45-49, is accurate, it is then roughly taken as F. But experience suggests that the total fertility reported by older women is low owing to omission of births and probably selection bias. In such a case, we have a three-parameter system in which F, α and β are to be estimated.

Zaba (1981) has developed a simple and elegant method of fitting the model by separating the fertility pattern measured by α and β from the estimation of fertility level (F). Instead of using $F(x)/F$ and P_i/F, which require reliable information on total fertility, the ratios $F(x)/F(x+5)$ and P_i/P_{i+1} are used and assumed to follow a Gompertz form. Thus defining

$$Z(i) = \text{Colog}\ (\text{Colog}\ P_i/P_{i+1})$$

$$= \alpha - \log_e [\exp(-\beta\ Y_s(i)) - \exp(-\beta\ Y_s(i+1))]$$

and expanding it by a Taylor series in the vicinity of the standard value of $\beta=1$, one gets

$$Z(i) - e(i) = \alpha + 0.48\ (\beta - 1)^2 + \beta\ g(i),$$

where $g(i)$, put for the result of differentiating Z(i) with respect to β, and $e(i) = Z_s(i) - g(i)$ are calculated from the standard distribution.

The Relational Gompertz Model (RGM) is illustrated (Guha Roy, 1983; India Office of the Registrar General, 1983) with the application to the Indian fertility data collected in the 1972 fertility survey and 1979 infant and child mortality survey of the Registrar General. The model however is more appropriate for the cohort data rather than cross sectional data on fertility considered here. We thus implicitly assume that fertility remained constant in the past, which it really did upto 1970, at least approximately. Moreover, the decline of fertility thereafter has been slow, and may not possibly affect significantly the estimation of total fertility from the average parities in the age group 15-35. Besides the difficulty of application of the model to cross sectional data, the major limitation of the collected information on fertility has been the error of response. There has been a tendency to understate the number of children ever born due to memory lapse and to omit the children born alive but now dead. We have found that the ratios of cumulated current fertility and mean parity (P/F) show a gradual decline with age, reflecting omission of births by older women. Further, the discrepancy between the value of P/F and the expected value of 1 at young ages indicates most probably a period reference error in the current fertility reports. The misstatement of ages further complicates the matter.

The relational system discussed above is believed to be well suited to deal with such situations. We use the synthetic measure of the cumulative current marital fertility rather than using the direct information on children ever born to ever married women. This is because a fit of the model to the latter data has been found to be much poorer than that obtained in the case of parity of the synthetic cohort. This synthetic measure ($\hat{P}_i$) is derived from the cumulated fertility, F_i (upto ages 19.5, 24.5, etc.), as follows :

$$P_i = F_i + k_i f_i$$

$$= 5 \sum_{j=0}^{i-1} f_j + k_i f_i,$$

where f_i is the observed age (i) specific marital fertility rate and k_i is a set of adjustment factors derived by Brass (Brass and Coale, 1968, Table 3.1, p. 94) to obtain values of adjusted cumulated current fertility corresponding to mid-points of 5-year age groups, that is, to 17.5, 22.5, etc.

The fitted Gompertz function is obtained for different population groups as follows :

1971-72

Rural : $Y(i) = 0.019 + 0.873\, Y_s(i)$

Urban : $Y(i) = 0.169 + 0.949\, Y_s(i)$

All India : $Y(i) = 0.053 + 0.894\, Y_s(i)$

1978

All India : $Y(i) = 0.179 + 0.984\, Y_s(i)$,

where $Y(i)$ is as defined above, and $Y_s(i)$ is taken from Booth (1979). The development of an independent standard schedule, $Y_s(i)$, from the historical series of fertility measures for India does not seem to be appropriate as they lack uniformity in their quality. Moreover, some of the earlier series were derived from the civil registration system, and thus totally unacceptable.

Table 3.5 and Figure 3.2 present the results of fitting the modified Gompertz function to period data for the Indian currently married women. The detail calculations for these fittings are shown in the Appendix. The present estimate of total marital fertility (TMF) of 7.00 for 1971-72 is of the same order of magnitude as the estimates of 6.68 and 6.95 derived earlier by us using Brass P/F correction and Coale's model of marital fertility respectively.

The efficiency of the fit of the Gompertz curve may be tested by estimating standardized errors. Choosing the age pattern of Indian females in 1971 Census, adjusted by a transitional age structure model (Guha Roy, 1980), as the standard population (S_i), we calculate the errors of fit as

$$\text{net error} = \sum_i (f_i - \hat{f}_i)\ S_i \,/\, \Sigma f_i S_i$$

$$\text{gross error} = \sum_i \left| f_i - \hat{f}_i \right| \ S_i \,/\, \Sigma f_i S_i$$

Table 3.5. Modified Gompertz fertility estimates (model fitted to mean parities of married women 15-35) : India, 1971-72 and 1978

	Age group of married women							TMF
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	
(a) Rural India : 1971-72								
Mean parity :								
synthetic	0.59	1.98	3.55	4.92	5.94	6.57	6.85	
model	0.59	2.01	3.55	4.94	6.11	6.95	7.26	
Marital fertility :								
reported	0.212	0.313	0.303	0.249	0.170	0.094	0.032	6.9
model	0.213	0.321	0.287	0.267	0.206	0.136	0.025	7.3
(b) Urban India : 1971-72								
Mean parity :								
synthetic	0.62	2.03	3.54	4.72	5.51	5.91	5.87	
model	0.61	2.06	3.50	4.70	5.61	6.19	6.38	
Marital fertility :								
reported	0.221	0.313	0.284	0.201	0.124	0.052	0.016	5.9
model	0.216	0.333	0.246	0.230	0.147	0.089	0.016	6.4
(c) All India : 1971-72								
Mean parity :								
synthetic	0.59	1.99	3.55	4.88	5.85	6.44	6.70	
model	0.58	2.01	3.53	4.88	5.97	6.74	7.02	
Marital fertility :								
reported	0.213	0.313	0.300	0.240	0.161	0.089	0.029	6.7
model	0.209	0.328	0.276	0.260	0.185	0.126	0.022	7.0
(d) All India : 1978								
Mean parity :								
synthetic	0.48	1.69	2.97	3.97	4.67	5.07	5.26	
model	0.48	1.71	2.96	3.97	4.72	5.19	5.33	
Marital fertility :								
reported	0.178	0.272	0.236	0.174	0.114	0.056	0.024	5.3
model	0.176	0.283	0.217	0.188	0.122	0.072	0.008	5.3

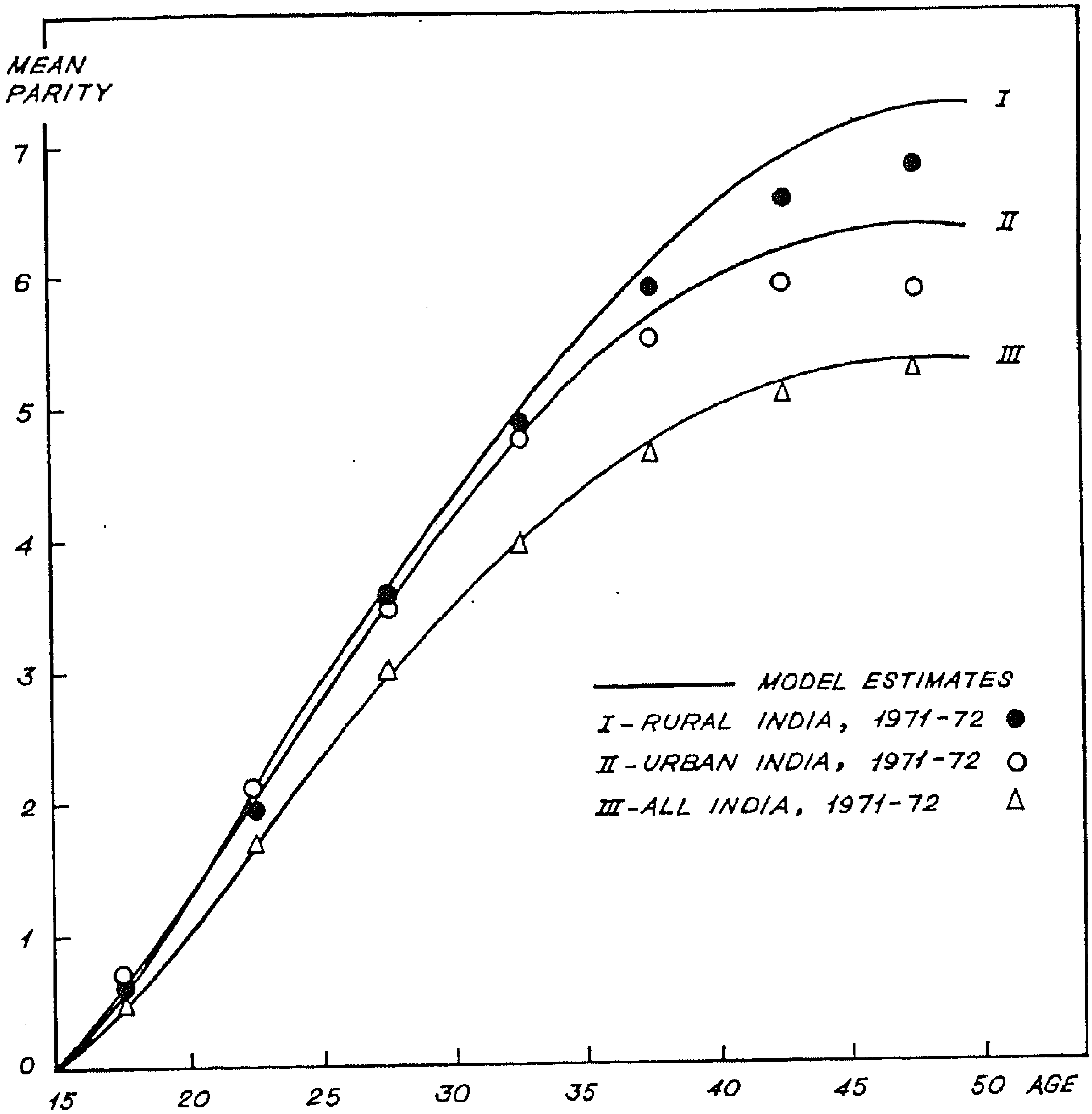


Figure 3.2 : Relational Gompertz parity estimates compared with observed values : rural, urban and all India, 1971-72

where f_i is the reported, and $\hat{f}_i$ the estimated marital fertility rates by age i. Thus, the errors (%) for current period rates are

	Net error	Gross error
Rural India, 1971-72	4.08	6.91
Urban India, 1971-72	3.14	10.38
All India, 1971-72	2.91	7.82
All India, 1978	0.94	6.38

Given the criterion that the net error is most appropriate measure of fit for period fertility distribution (Murphy and Nagnur, 1972), the errors of less than one per cent to four per cent are not overwhelming in relation to the convenience of estimating age specific fertility by only three parameters of the Gompertz function. Though the estimate of the fertility rate for age group 15-19 is unstable when derived by the conventional P/F method, the Gompertz fit is good enough even at this age range.

The per cent change in the estimated marital fertility between 1971-72 and 1978 is as follows :

15-19	20-24	25-29	30-34	35-39	40-44
-15.8	-13.7	-21.4	-27.7	-34.1	-42.8

Consistent with the age pattern of practice of fertility control methods in India, the fertility declined most among married women of over 30 years of age, and least in the peak fertility age group 20-24. The life time fertility rate (TMFR) declined by 24 per cent from 7.0 to 5.3 during 1972-78. The estimate of TMFR agrees well with that obtained earlier by Coale's procedure for All India, 1971-72, but the agreement is not so good with that derived by the parity/fertility ratio method of Brass.

Perhaps it would be useful to fit a relational Gompertz model to mean parities of all women rather than married women only*. The results of fitting are shown in Table 3.6. Consideration of married women, as done above, may

* This was suggested by Prof. K.S. Srikantan, Gokhale Institute of Politics & Economics, Pune in a personal communication.

Table 3.6. Modified Gompertz fertility estimates (model fitted to mean parities of all women aged 15-35) : India, 1971-72 and 1978

	Age group of women							
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	TFR
				(a) 1971-72				
Mean parity :								
synthetic	0.17	1.18	2.58	3.81	4.68	5.19	5.43	
model	0.17	1.19	2.58	3.83	4.75	5.28	5.43	
Fertility rates :								
reported	0.087	0.262	0.276	0.217	0.143	0.076	0.029	5.4
model	0.087	0.265	0.272	0.227	0.152	0.075	0.009	5.4
				(b) 1978				
Mean parity :								
synthetic	0.12	0.98	2.16	3.13	3.79	4.15	4.31	
model	0.12	0.98	2.15	3.12	3.78	4.12	4.20	
Fertility rates :								
reported	0.068	0.228	0.228	0.166	0.106	0.049	0.019	4.3
model	0.065	0.232	0.221	0.170	0.105	0.044	0.003	4.2

create problems about interpreting cumulated age specific marital fertility rates since it does not properly correspond to any cohort. The total fertility rate for Taiwan in 1981, for example, was only 2.46, whereas the cumulated age specific marital fertility rates stood at 7.23. The reason put forward for this large difference is the multiple counting of first and low order births in cumulated age speicifc marital fertility rates. Indeed for India, the fit of the model to fertility of all women is slightly better (except perhaps the age group 45-49) than that to marital fertility, but the disparity is nowhere near Taiwan's. Thus, the estimation of total fertility rates, being the main focus of interest, is broadly consistent between married women and all women. The level of this estimate is however somewhat lower than that derived earlier by us by Brass procedure and also by Bongaart's model of proximate determinants of fertility (Jain and Adlakha, 1982), the implementation of the latter model in the case of India requiring several assumptions.

3.6 Path analysis of areal fertility in rural India

In recent years there has been widespread recognition that the transition from high to low fertility and the acceptance of population control programme are interacting processes, and that their interaction does not take place all of their own, but in the setting of a certain level of socio-economic development. One important conceptual development that has emerged is the dispelling of the simplistic notion that family planning programme alone would bring about a fertility decline. As noted in some studies (Davis, 1967), the programmes have been most successful in rapidly developing societies where people would adopt small family norm whether there was a fertility regulation programme or not. In less developed societies efforts should be made to alter the structural factors (e.g., economic) obtaining in the society that motivate couples to high fertility performance. As long as the desire for children remains high, mere institution of the programme will be of little consequence. It has thus become necessary to unravel the mechanism of how social and economic factors condition the relationships between fertility performance and programme acceptance. The improved understanding of the interrelationship between demographic, economic and social factors would provide a more complete conceptual framework for the formulation of population policy (Asian Population Studies, 1978). It is in this context that the present section attempts to assess Indian population control programme impact on fertility when a certain socio-economic threshold has been reached (Guha Roy, 1981a). The ultimate object of the analysis is to estimate the relative impact of the programme on the fertility measure, after eliminating or adjusting for the influence of non-programme factors.

The analysis adopts a recursive path - analytic model, which excludes feedback between variables. It uses geographic area as unit of observation since the interest focuses on the total aggregate of individuals associated with it.

The selection of the variables depends upon the model used and upon the availability of data. Broadly, the data required for each area are current fertility measures, some programme factors and measures of non-programme variables having a direct or indirect effect on fertility.

Though the States and Union Territories provide lesser number of cases for reliable estimation of parameters (Hermalin, 1975), this larger level of aggregation is chosen due to lack of adequate information on non-programme factors and the vital registration data at the district level. It cannot however be said at this stage that the relationship studied at this level will not differ from those based on the lower level. Of the 28 rural areas, one unit, that of Chandigarh, is excluded because of unusually high fertility rate reported.

The choice of the variables utilized in this study depends primarily upon the availability of data. In addition, the choice of specific data is also guided by the particular interrelations between variables that have been assumed (United Nations, 1978). The variables measured on an interval scale are given (see Table 3.7a) below :

A.	Dependent variables :	1. Age (30-34) specific fertility in 1972;
		2. Age (30-34) specific marital fertility in 1972;
		3. Age (30-34) specific proportion of women married in 1972;
B.	Demographic variables :	1. Infant mortality in 1970;
		2. Singulate mean age at marriage for women in 1971;
C.	Socio-economic variables :	1. Percent of females with at least primary level education in 1971;
		2. Activity rate : Total workers per 100 population in 1971;
D.	Programme factor :	Cumulative per cent of couples protected by sterilization and IUD (Intra-uterine contraceptive device) through first quarter of 1972.

Table 3.7a. Variables for multivariate areal analysis : Rural India, 1971-72

States/Union Territories	Variable No.							
	1	2	3	4	5	a	b	c
Andhra Pradesh	160.6	.901	178.3	7.7	16.5	7.6	43.9	122.2
Assam	243.0	.940	258.4	3.8	18.5	10.8	28.1	142.4
Bihar	228.6	.938	243.8	5.3	16.2	3.1	27.2	106.1
Gujarat	254.4	.944	269.4	10.6	18.1	5.7	33.0	159.7
Haryana	318.1	.972	327.2	12.0	16.7	3.4	26.5	82.1
Himachal Pradesh	270.6	.908	297.9	5.0	17.7	9.1	37.2	151.3
Jammu & Kashmir	196.1	.907	216.2	5.6	17.4	2.9	30.5	93.0
Karnataka	170.8	.929	183.9	9.1	17.6	11.9	14.5	101.0
Kerala	200.0	.870	230.3	13.1	20.8	35.4	29.5	55.9
Madhya Pradesh	278.3	.975	285.3	10.3	15.9	2.8	38.4	151.7
Maharashtra	188.8	.926	203.9	18.1	17.0	9.7	38.6	102.5
Manipur	202.6	.932	217.4	1.7	21.7	7.4	35.8	106.6
Meghalaya	204.1	.801	254.7	0.4	19.7	10.7	46.2	138.4
Orissa	169.6	.918	184.8	12.0	17.4	10.4	31.3	139.8
Punjab	283.3	.973	291.1	15.8	19.8	12.7	29.1	106.6
Rajasthan	273.4	.963	283.9	4.4	16.1	1.6	32.4	148.5
Tamil Nadu	177.8	.923	192.6	11.4	19.3	10.0	38.2	133.9
Tripura	316.9	.942	336.4	4.1	18.1	17.2	28.2	96.3
Uttar Pradesh	302.9	.969	312.5	6.0	16.2	4.5	31.5	165.4
West Bengal	206.6*	.906	228.0*	9.1	17.5	9.0	27.2	113.4
A. & N. Islands	301.9	.950	317.8	2.7	18.2	9.6	38.0	98.2
Arunachal Pradesh	320.3	.959	333.9	0.7	19.5	2.8	35.8	91.0
Dadra & N.H.	259.3	.896	289.3	2.7	18.3	4.3	47.2	111.0
Delhi	273.5	.997	274.2	5.5	17.5	10.8	26.6	95.7
Goa, Daman & Diu	167.0	.873	191.2	8.3	20.9	15.8	31.4	74.6
Lakshadweep	196.1	.883	222.2	2.8	17.9	12.6	26.1	146.0
Pondicherry	227.7	.933	244.1	12.3	18.8	12.9	33.0	52.0

* Estimated from local surveys.

Sources : See text.

Notes : Variable 1 : Age (30-34) specific fertility in 1972;

2 : Age (30-34) specific proportion married in 1972;

3 : Age (30-34) specific marital fertility in 1972;

4 : Cumulative percent of couples protected by sterilization and IUD through first quarter of 1972;

5 : Singulate mean age at marriage for women in 1971;

a : Percent of females with at least primary level education in 1971;

b : Activity rate in 1971;

c : Infant mortality in 1970.

These variables were lagged one to two years to allow the possible time in their impact on current fertility.

This analysis draws heavily on Hermalin (Hermalin, 1975). The variables employed in this study are more or less similar to those utilized by him in his analysis of the Taiwan programme. There are however some differences in the definition of the socio-economic variables and of the programme factor taken as indicators of " modernization" .

In India, the major sources of demographic data are the population censuses, the civil registration system, national sample surveys and the sample registration scheme (SRS). Though the censuses and vital registration extend over a full century, they remained severely defective. Adjustments of raw data were, therefore, necessary in a number of cases. The main sources from which the above data have been compiled are as follows :

1. " Age specific fertility rate" and " age specific marital fertility rate" : Registrar General, India, Fertility Differentials in India, 1972, Tables 14 and 16, pp. 46-50, 1976.

2. " Proportion married" : Obtained mostly by dividing ASFR by ASMFR. Also obtained directly from Census 1971.

3. " Cumulative per cent of couples protected" : Estimated from : Family Welfare Programme in India : Year Books 1972-73 & 1974-75, Ministry of Health and Family Planning, and Pocket Book of Population Statistics, Registrar General, Census Centenary 1972.

4. " Singulate mean age at marriage" : Estimated by Hajnal's method from proportion single reported in Census 1971.

5. " Female education" : Census 1971, Series 1, Part II-C(ii) : Social and Cultural Tables.

6. " Activity rate" : Census 1971, Economic Tables.

7. " Infant mortality Rate" : Registar General, India, Sample Registration Bulletin, Vol. VII, No. 1, January-March, 1973, and Vol. IX, No. 4, October, 1975.

The dependent variables are made to refer to 1972 since it is known that completeness of coverage by SRS improved markedly in 1972. The age specific fertility used as a dependent variable is composite. Thus, it is the product of age specific proportion married and age specific marital fertility. It is, therefore, possible to examine the relative contributions of the components of age specific fertility and to study how the effects of various factors on the composite variable is transmitted through each component. By taking logarithm of the composite variable and its components, their multiplicative relationship is made additive.

Following the Health Ministry's procedure, the number of couples currently protected by sterilization was obtained from the total sterilization performed since inception by allowing attrition due to mortality and aging at the rate of 2.5 per cent in the initial year and 5 per cent each thereafter. For estimating the number of couples currently protected by IUD, attrition due to pregnancy, removals and expulsion was allowed, in addition to aging and mortality. As a working approximation attrition rate was considered at the rate of 1.25 per cent in the initial month and 2.5 per cent thereafter. Since no firm information on their magnitude of use and regularity of use are available, the conventional control measures have not been included for computing the acceptance rate.

The socio-economic factors and infant mortality are considered independent or exogenous variables, which are not determined within the model. The remaining variables included are all endogenous. Each variable is in standardized form, (shown in Table 3.7b) that is,

$$Y = (\tilde{Y} - Y)/\sigma,$$

where

Y is the standardized variable;

$\tilde{Y}$ is the unstandardized variable; and

σ is the standard deviation of $\tilde{Y}$.

Each dependent variable is treated as completely determined by a set of other variables in the model, including as necessary the residual effects of unmeasured variables (R). It is however obvious that all the determinants of fertility, if known, have not been taken into consideration. Besides, the

Table 3.7b. Variables in standard form for multivariate areal analysis : Rural India, 1970-72
(Suffixes indicate variable numbers as in Table 3.7a)

States/Union Territories	Y_1	Y_2	Y_3	Y_4	Y_5	X_a	X_b	X_c
Andhra Pradesh	-1.587	-0.616	-1.736	0.060	-1.072	-0.279	1.756	0.293
Assam	0.218	0.342	0.180	-0.790	0.250	0.208	-0.938	0.973
Bihar	-0.048	0.297	-0.124	-0.463	-1.271	-0.965	-1.092	-0.249
Gujarat	0.418	0.457	0.392	0.692	-0.015	-0.569	-0.103	1.555
Haryana	1.391	1.119	1.391	0.997	-0.940	-0.920	-1.211	-1.057
Himachal Pradesh	0.689	-0.434	0.912	-0.529	-0.279	-0.051	0.613	1.272
Jammu & Kashmir	-0.715	-0.457	-0.742	-0.398	-0.477	-0.996	-0.529	-0.690
Karnataka	-1.321	0.068	-1.577	0.365	-0.345	0.376	0.477	-0.421
Kerala	-0.623	-1.416	-0.417	1.236	1.770	3.959	-0.700	-1.938
Madhya Pradesh	0.811	1.187	0.690	0.626	-1.469	-1.011	0.818	1.286
Maharashtra	-0.881	0	-1.041	2.326	-0.742	0.041	0.852	-0.370
Manipur	-0.575	0.160	-0.716	-1.248	2.365	-0.310	0.374	-0.232
Meghalaya	-0.541	-3.288	0.103	-1.531	1.043	0.193	2.148	0.138
Orissa	-1.351	-0.205	-1.551	0.997	-0.477	0.147	-0.393	0.885
Punjab	0.889	1.142	0.793	1.825	1.109	0.498	-0.768	-0.232
Rajasthan	0.732	0.890	0.665	-0.659	-1.337	-1.194	-0.205	1.178
Tamil Nadu	-1.142	-0.068	-1.334	0.866	0.779	0.086	0.784	0.687
Tripura	1.378	0.388	1.535	-0.725	-0.015	1.184	-0.921	-0.579
Uttar Pradesh	1.177	1.050	1.154	-0.311	-1.271	-0.752	-0.359	1.747
West Bengal	-0.488	-0.502	-0.469	0.365	-0.411	-0.066	-1.092	-0.003
A. & N. Islands	1.164	0.594	1.242	-1.030	0.052	0.025	0.750	-0.515
Arunachal Pradesh	1.421	0.799	1.499	-1.466	0.911	-1.011	0.374	-0.757
Dadra & N.H.	0.501	-0.753	0.757	-1.030	0.118	-0.783	2.318	-0.084
Delhi	0.732	1.689	0.484	-0.420	-0.411	0.208	-1.194	-0.599
Goa, Daman & Diu	-1.417	-1.347	-1.376	0.190	1.836	0.971	-0.376	-1.309
Lakshadweep	-0.715	-1.073	-0.603	-1.008	-0.147	0.483	-1.280	1.094
Pondicherry	-0.065	0.183	-0.118	1.062	0.448	0.529	-0.103	-2.070

social phenomena are complex enough to identify all required factors and, many variables have been excluded because of the non-availability of relevant data. In addition to this " specification error", there is the problem of errors of measurement. Further, the data which are available often violate the assumptions of the statistical model (Hermalin, 1975). Under the circumstances, the results of analysis must be interpreted with caution.

After constructing the zero-order correlation matrix (Table 3.8) with all the variables, a forward stepwise inclusion procedure was employed to arrange the variables in order of importance (Taucher and Bocaz, 1978). The arrangement of the variables are shown in Figure 3.3. According to convention of path diagrams, " each variable occurs earlier in time than those appearing to the right of it and later than those to the left, and can be affected by all the variables that precede it". The one-way straight arrows are used to indicate direct effect of one variable to another and the sign and magnitude of these are path coefficients, p_{ij}. The unanalyzed zero-order correlations, r, between the exogenous variables are shown by curved bidirectional arrows.

Table 3.8. Correlation matrix for areal fertility analysis : Rural India, 1970-72

	Y_2	Y_3	Y_4	Y_5	X_a	X_b	X_c	Mean	S.D.
Y_1	0.579	.942	-.236	-.201	-.274	-.175	.092	5.44	.292
Y_2		.453	.178	-.405	-.397	-.390	.135	-0.08	.038
Y_3			-.320	-.146	-.235	-.119	.079	5.52	.194
Y_4				-.090	.284	-.174	-.184	7.43	4.583
Y_5					.545	.078	-.445	18.12	1.536
X_a						-.199	-.461	9.43	6.564
X_b							.127	33.60	5.888
X_c								113.50	29.714

The algebraic representation of the system shown in Figure 3.3 can be written by the following set of equations :

$$Y_1 = p_{12} Y_2 + p_{13} Y_3 \qquad (1)$$

$$Y_2 = p_{25} Y_5 + p_{2a} X_a + p_{2b} X_b + p_{2u} R_u \qquad (2)$$

$$Y_3 = p_{34} Y_4 + p_{35} Y_5 + p_{3b} X_b + p_{3c} X_c + p_{3w} R_w \qquad (3)$$

$$Y_4 = p_{45} Y_5 + p_{4a} X_a + p_{4b} X_b + p_{4v} R_v \qquad (4)$$

$$Y_5 = p_{5a} X_a + p_{5b} X_b + p_{5z} R_z \qquad (5)$$

where,

Y_1 = logarithm of age specific fertility,

Y_2 = logarithm of age specific proportion of women married,

Y_3 = logarithm of age specific marital fertility,

Y_4 = cumulative per cent of couples protected by sterilization and IUD,

Y_5 = singulate mean age at marriage of women,

X_a = per cent of women with at least primary level education,

X_b = activity rate,

X_c = infant mortality rate.

Since the system is assumed to be recursive, the ordinary least squares can be employed in the above system of equations. The procedure assumes that (i) the association between the dependent and the explanatory variables is linear; (ii) the error term has an independent distribution with mean zero and a constant variance; (iii) the explanatory variables are independent of the error term. The unknown path coefficients are however solved in terms of the observed correlations. For example,

$$r_{4a} = \Sigma\, Y_4 X_a / N \quad [\text{the variables being in standard form}]$$

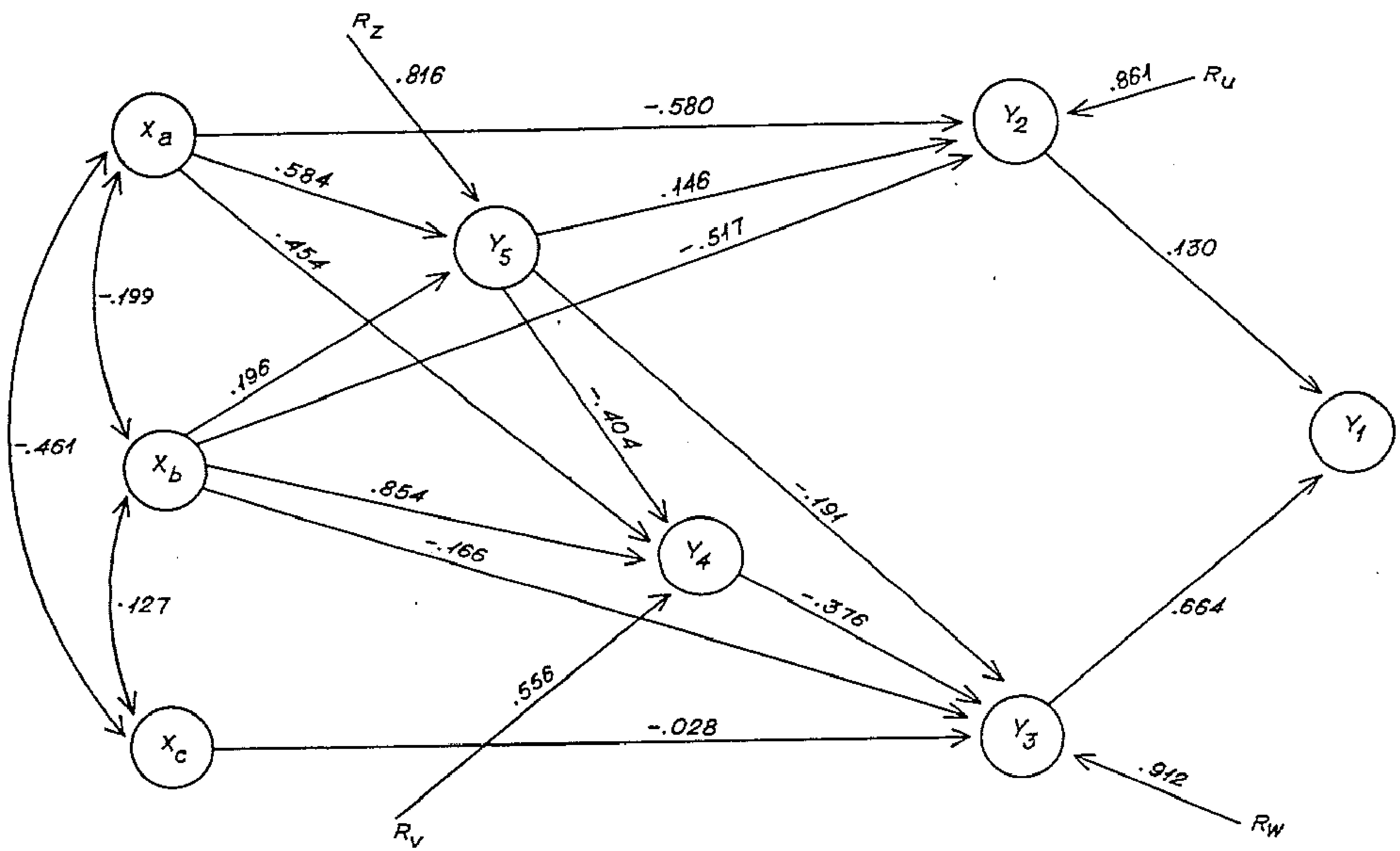


Figure 3.3. : Path diagram showing effects of female education (X_a), activity rate (X_b) and infant mortality (X_c) on logarithm of age specific fertility rate (Y_1) through the components of logarithm of age specific proportion married (Y_2) and logarithm of age specific marital fertility rate (Y_3)

$$= \frac{1}{N} \Sigma X_a (p_{45} Y_5 + p_{4a} X_a + p_{4b} X_b + p_{4v} R_v)$$

[substituting from (4) the expression for Y_4]

$$= p_{45} r_{5a} + p_{4b} r_{ab} \qquad (6)$$

since $r_{aa} = 1$ and $r_{av} = 0$.

Similarly, $r_{4b} = \Sigma Y_4 X_b/N$

$$= p_{45} r_{5b} + p_{4a} r_{ab} + p_{4b} \qquad (7)$$

and $r_{45} = \Sigma Y_4 Y_5/N$

$$= p_{45} + p_{4a} r_{5a} + p_{4b} r_{5b} \qquad (8)$$

It may be noted that the variable, which appears later in the causal sequence is expanded for convenience (Duncan, 1966). Equations (6), (7) and (8) can be solved simultaneously for the path coefficients p_{45}, p_{4a} and p_{4b}, after substituting the observed values of the coefficient of correlation. The other coefficients can be similarly obtained, or directly calculated from the basic theorem of path analysis. Because of algebraic identity, the path coefficients in equation (1) can be obtained as

$$p_{12} = \frac{\sigma_2}{\sigma_1} \text{ and } p_{13} = \frac{\sigma_3}{\sigma_1} .$$

The values of the path coefficients are shown in Table 3.9.

The calculation of the so-called residual paths is being made following Duncan (Duncan, 1966). The general form of the basic theorem of path analysis is

$$r_{ij} = \sum_q p_{iq} r_{jq}$$

Table 3.9. Path coefficients for areal fertility analysis : Rural India, 1970-72

Dependent variable	Independent variable						
	X_a	X_b	X_c	Y_5	Y_4	Y_3	Y_2
Y_5	.584	.196					
Y_4	.454	.854		-.404			
Y_3		-.166	-.028	-.191	-.376		
Y_2	-.580	-.517		.146			
Y_1						.664	.130

where i and j are two variables and q runs over all the variables which have paths leading directly to variable i. By setting i = j,

$$r_{ii} = 1 = \sum_q p_{iq} r_{iq},$$

which, on expanding, gives

$$1 = \sum_q p_{iq}^2 + 2 \sum_{q,q'} p_{iq} r_{qq'} p_{iq'},$$

where q' is greater than q and they include all measured and unmeasured variables. The residual paths p_{2u}, p_{3w}, p_{4v} and p_{5z} are obtained from this expression as follows :

$$p_{2u}^2 = 1 - (p_{25}^2 + p_{2a}^2 + p_{2b}^2) - 2(p_{25} r_{5a} p_{2a} + p_{25} r_{5b} p_{2b} + p_{2a} r_{ab} p_{2b})$$

$$p_{3w}^2 = 1 - (p_{34}^2 + p_{35}^2 + p_{3b}^2 + p_{3c}^2) - 2(p_{34} r_{45} p_{35} + p_{34} r_{4b} p_{3b}$$

$$+ p_{34} r_{40} p_{3c} + p_{35} r_{5b} p_{3b} + p_{35} r_{5c} p_{3c} + p_{3b} r_{bc} p_{3c})$$

$$p_{4v}^2 = 1 - (p_{45}^2 + p_{4a}^2 + p_{4b}^2) - 2(p_{45} r_{5a} p_{4a} + p_{45} r_{5b} p_{4b} + p_{4a} r_{ab} p_{4b})$$

$$p_{5z}^2 = 1 - (p_{5a}^2 + p_{5b}^2) - 2\, p_{5a}\, r_{ab}\, p_{5b}.$$

The values of the residual paths thus obtained are as under.

$p_{2u} = .861$	$p_{3w} = .912$
$p_{4v} = .556$	$p_{5z} = .816$

After computing the path coefficients for required combinations of variables, the set of equations (1) through (5) can now be written as

$$Y_1 = .130\, Y_2 + .664\, Y_3$$

$$Y_2 = .146\, Y_5 - .580\, X_a - .517\, X_b + .861\, R_u$$

$$Y_3 = -.376\, Y_4 - .191\, Y_5 - .166\, X_b - .028\, X_c + .912\, R_w$$

$$Y_4 = -.404\, Y_5 + .454\, X_a + .854\, X_b + .556\, R_v$$

$$Y_5 = .584\, X_a + .196\, X_b + .816\, R_z$$

In the model described above, the influences of all antecedent variables on a subsequent one are not considered. For example, no direct effect of X_c on Y_5 or on Y_4 is assumed. This assumption can be tested by comparing the correlations, r_{5c} and r_{4c}, obtained from the model with those observed from the data. Thus,

		" model" correlation	observed correlation
$r_{5c} = p_{5a}\, r_{ac} + p_{5b}\, r_{bc}$	=	−.244	−.445
$r_{4c} = p_{45}\, r_{5c} + p_{4a}\, r_{ac} + p_{4b}\, r_{bc}$	=	.079	−.184

The significant discrepancy between the correlations implied by the model and those observed gives indication that the model could be improved by considering direct effects to Y_4 and Y_5 from X_c.

The values of the path coefficients for the components " marital fertility" (.664) and " proportion married" (.130) indicate their relative contribution in influencing " age specific fertility". The way in which the effect of a factor on " age specific fertility" (Y_1) is transmitted through " proportion married " (Y_2) and " marital fertility" (Y_3) is illustrated for " age at marriage" (Y_5) as follows :

$$r_{15} = p_{12}\, r_{52} + p_{13}\, r_{53} \tag{9}$$

Substituting the values of correlation and path coefficients from Tables 3.8 and 3.9,

$$-.201 \simeq (.130)\,(-.405) + (.664)\,(-.146)$$
$$\simeq -.053 - .097$$

It is seen that " age at marriage" influences " age specific fertility" to a greater extent through its effect on " marital fertility" and to a lesser extent through " proportion married" at age 30-34 (Hermalin, 1975). The equation (9) can be further analyzed as follows :

$$r_{35} = p_{34}\, r_{54} + p_{35} + p_{3b}\, r_{5b} + p_{3c}\, r_{5c}$$
$$-.146 \simeq (-.376)(.090) - .191 + (-.166)(.078) + (-.028)(-.445)$$
$$\simeq .034 - .191 - .013 + .012$$

Thus, earlier " age at marriage" mainly increases " marital fertility" directly rather than indirectly through its effect on " activity rate", " infant mortality rate" and " cumulative per cent of couples protected" . The correlation between " age specific fertility" (Y_1) and " cumulative per cent of couples protected" (Y_4) can be decomposed as follows :

$$r_{14} = p_{12}\, r_{24} + p_{13}\, r_{34}$$

and expanding further,

$$r_{14} = p_{12}\,(p_{25}\, r_{45} + p_{2a}\, r_{4a} + p_{2b}\, r_{4b}) + p_{13}\,(p_{34} + p_{35}\, r_{45} + p_{3b}\, r_{4b} + p_{3c}\, r_{4c})$$

$$r_{14} = p_{13}\, p_{34} + p_{12}\, p_{2a}\, r_{4a} + p_{13}\, p_{3c}\, r_{4c} + r_{45}\,(p_{12}\, p_{25} + p_{13}\, p_{35}) + r_{4b}\,(p_{12}\, p_{2b} + p_{13}\, p_{3b})$$

It is noted that there is no direct effect of acceptors on " age specific fertility". The former's impact is only through indirect and joint effects. Thus, the first term on right hand side in the above expression is the indirect effect of the number of acceptors on " age specific fertility" through its effect on " marital fertility". The second and third terms are joint effects with " educational level" and " infant mortality", which in turn act on " age specific fertility" through its components. The remaining terms are the effects on " age specific fertility" which the number of acceptors shares with " age at marriage" and " activity rate". The relative intensity of effect of each term in explicating the impact of acceptance on fertility can be understood from the following magnitudes :

$$-.236 \simeq -.250 - .021 + .003 - .002 + .011 + .012 + .019$$

The result illustrates that acceptance (Y_4) decreases " age specific fertility" primarily through " marital fertility" (Y_3). This is substantiated further by the following decomposition of correlation between Y_3 and Y_4 :

$$r_{34} = p_{34} + p_{35}\, r_{45} + p_{3b}\, r_{4b} + p_{3c}\, r_{4c}$$

or

$$-.320 \simeq -.376 + .017 + .029 + .005$$

The present study breaks down the composite variable (age specific fertility) into its components (proportion married and marital fertility) before conducting a search for its variability. The results of analysis indicate that the acceptance of programme has only an indirect effect on age specific fertility which may mean that the national antinatalist policy is unlikely to contribute much to acceleration of fertility decline. With socio-economic development and institution of social measures such as raising the minimum age at marriage, motivations to accept effective control measures at a low parity level will take place and fertility will expectedly move downward.

APPENDIX TO CHAPTER THREE

Table A3.1. Standard values for mean parities

Age	i	$Y_s(i)$	e(i)	g(i)
15 - 19	1	-1.0787	1.2897	-1.7438
20 - 24	2	-0.3119	1.4252	-1.0157
25 - 29	3	0.3538	1.3725	-0.3353
30 - 34	4	1.0569	1.1421	0.4391
35 - 39	5	1.9534	0.7061	1.5117
40 - 44	6	3.4130	0.2763	3.2105
45 - 49	7	6.0557	-	-

Source : Brass (1981).

Table A3.2. Fitting a relational Gompertz model to mean parities (P_i) of younger married women : <u>All India Rural, 1971-72</u>

Age	i	P_i	$Z(i) = -\ln(-\ln \frac{P_i}{P_{i+1}})$	Z(i) - e(i) observed	Z(i) - e(i) fitted*	$Y(i) = \hat{\alpha} + \hat{\beta} Y_s(i)$	$P_i/\bar{F}$	$\hat{F}$ = (3) ÷ (8)	$\hat{P}_i$ = (8) X $\bar{F}$ **	$\hat{f}_i$ ***
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15 - 19	1	0.59	-0.1912	-1.4809	1.4956	-0.9227	0.0808	7.30	0.59	0.213
20 - 24	2	1.98	0.5381	-0.8871	-0.8600	-0.2533	0.2757	7.18	2.01	0.321
25 - 29	3	3.55	1.1198	-0.2527	-0.2660	0.3279	0.4865	7.30	3.55	0.287
30 - 34	4	4.92	1.6692	0.5271	0.4101	0.9417	0.6771	7.27	4.94	0.267
35 - 39	5	5.94	2.2946	1.5885	1.3465	1.7243	0.8367	7.10	6.11	0.206
40 - 44	6	6.57	3.1764	2.9001	2.8295	2.9985	0.9514	6.91	6.95	0.136
45 - 49	7	6.85	-	-	-	5.3056	0.9950	6.88	7.26	0.025

* The fitted values of Z(i) - e(i) are obtained from $Z(i) - e(i) = \alpha + 0.48(\beta - 1)^2 + \beta g(i)$.
β and α are estimated by fitting a line to Z(1), Z(2) and Z(3). Thus,

$-1.4809 = \alpha + 0.48\ (\beta - 1)^2 - 1.7438\ \beta$ .... (1) $\quad \hat{\beta}_1 = 0.816$

$-0.8871 = \alpha + 0.48\ (\beta - 1)^2 - 1.0157\ \beta$ .... (2) $\quad \hat{\beta}_2 = 0.872 \quad \hat{\beta} = \frac{1}{3}(\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3) = \underline{0.873}$

$-0.2527 = \alpha + 0.48\ (\beta - 1)^2 - 0.3353\ \beta$ .... (3) $\quad \hat{\beta}_3 = 0.932$

Substituting $\hat{\beta} = 0.873$ in equations (1), (2) and (3), $\hat{\alpha}_1 = 0.034$, $\hat{\alpha}_2 = -0.008$, $\hat{\alpha}_3 = 0.032$ and

$\hat{\alpha} = \frac{1}{3}(\hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3) = \underline{0.019}$

** $\bar{F} = \frac{1}{4}(7.30 + 7.18 + 7.30 + 7.27) = 7.3$

*** $\hat{P}_i$ in Col. (10) is converted to $\hat{f}_i$ (age specific marital fertility rate) as follows :

We obtained $P_i = 5 \sum_{j=0}^{i-1} f_j + k_i f_i$ so that $\hat{f}_i = \left[\hat{P}_i - 5\ (\hat{f}_1 + \ldots\ldots\ldots\ldots\ldots + \hat{f}_{i-1}) \right] / k_i$,

where k_i factors are taken from Brass (Brass and Coale, 1968, Table 3.1, p. 94).

Table A3.3. Fitting a relational Gompertz model to mean parities (P_i) of younger married women : <u>All India Urban, 1971-72</u>

Age	i	P_i	$Z(i) = -\ln(-\ln \frac{P_i}{P_{i+1}})$	Z(i) - e(i) observed	Z(i) - e(i) fitted	$Y(i) = \hat{\alpha} + \hat{\beta} Y_s(i)$	P_i/F	$\hat{F} =$ (3) ÷ (8)	$\hat{P}_i =$ (8) X $\bar{F}$	$\hat{f}_i$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15 - 19	1	0.62	-0.1706	-1.4603	-1.4854	-0.8548	0.0953	6.51	0.61	0.216
20 - 24	2	2.03	0.5868	-0.8384	-0.7937	-0.1263	0.3215	6.31	2.06	0.333
25 - 29	3	3.54	1.2459	-0.1266	-0.1473	0.5061	0.5472	6.47	3.50	0.246
30 - 34	4	4.72	1.8659	0.7238	0.5884	1.1741	0.7341	6.43	4.70	0.230
35 - 39	5	5.51	2.6581	1.9520	1.6074	2.0257	0.8764	6.29	5.61	0.147
40 - 44	6	5.91	-	-	3.2212	3.4124	0.9676	6.11	6.19	0.089
45 - 49	7	5.87	-	-	-	5.9229	0.9973	5.89	6.38	0.016

Table A3.4. Fitting a relational Gompertz model to mean parities (P_i) of younger married women : All India, 1971-72

Age	i	P_i	$Z(i) = -\ln(-\ln \frac{P_i}{P_{i+1}})$	$Z(i) - e(i)$ observed	$Z(i) - e(i)$ fitted	$Y(i) = \hat{\alpha} + \hat{\beta} Y_s(i)$	P_i/F	$\hat{F}$ = (3) ÷ (8)	$\hat{P}_i$ = (8) X $\bar{F}$	$\hat{f}_i$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15 - 19	1	0.59	-0.1954	-1.4851	-1.5005	-0.9113	0.0831	7.09	0.58	0.209
20 - 24	2	1.99	0.5468	-0.8784	-0.8496	-0.2258	0.2855	6.97	2.01	0.328
25 - 29	3	3.55	1.1450	-0.2275	-0.2414	0.3693	0.5009	7.09	3.53	0.276
30 - 34	4	4.88	1.7076	0.5655	0.4509	0.9979	0.6917	7.05	4.88	0.260
35 - 39	5	5.85	2.3425	1.6364	1.4099	1.7993	0.8475	6.90	5.97	0.185
40 - 44	6	6.44	3.2294	2.9531	2.9286	3.1042	0.9561	6.73	6.74	0.126
45 - 49	7	6.70	-	-	-	5.4668	0.9958	6.73	7.02	0.022

Table A3.5. Fitting a relational Gompertz model to mean parities (P_i) of younger married women : <u>All India, 1978</u>

Age	i	P_i	$Z(i) = -\ln(-\ln \frac{P_i}{P_{i+1}})$	$Z(i) - e(i)$ observed	$Z(i) - e(i)$ fitted	$Y(i) = \hat{\alpha} + \hat{\beta}\, Y_s(i)$	P_i/F	$\hat{F} =$ (3) ÷ (8)	$\hat{P}_i =$ (8) X $\bar{F}$	$\hat{f}_i$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15 - 19	1	0.48	-0.2301	-1.5198	[illegible]	[illegible].8824	0.0892	5.38	0.48	0.176
20 - 24	2	1.69	0.5730	-0.8522	-0.8203	-0.1279	0.3209	5.27	1.71	0.283
25 - 29	3	2.97	1.2372	-0.1353	-0.1509	0.5271	0.5541	5.36	2.96	0.217
30 - 34	4	3.97	1.8177	0.6756	0.6111	1.2155	0.7433	5.34	3.97	0.188
35 - 39	5	4.67	2.4988	1.7927	1.6665	2.1011	0.8849	5.28	4.72	0.122
40 - 44	6	5.07	3.3025	3.0262	3.3381	3.5374	0.9713	5.22	5.19	0.072
45 - 49	7	5.26	-	-	-	6.1378	0.9978	5.27	5.33	0.008

CHAPTER FOUR

POPULATION PROJECTIONS FOR INDIA, 1971-2001

Two sets of short-term population projections are prepared for India for 1971-1986 and 1981-2001. The projections are based on the historical trends and the current estimates of vital rates. Accepting the vital rate estimates provided by the Sample Registration System as the first approximation, consistency criteria are used to improve the estimates. A mathematical model is developed to generate an approximately corrected age structure consistent with the probable mortality levels and the observed growth rates. The projection of the populations of the States is based on mathematical curves, but made consistent with the projected national population. Using a structural graduation model and the Sprague's method, single year populations in the age range 5-19 are derived to estimate the size of student population under full enrolment.

The chapter is an extension of my published paper on " Population estimates for India", Demography India, Vol. 13, Nos. 1 & 2, 1984, which was reviewed in M. Chaudhry's paper (1985) on " Comparative study of India's population : Projections based on the 1981 Census", Royal Military College of Canada and Queen's University, Kingston, Ontario.

4.1 Introduction

India, with a current population of over 700 million, ranks second in the world in population size. Even with moderate annual growth rate of around two per cent, this large population base has immense population potential.

The importance of the knowledge of population trends for educational, manpower and overall national planning and development is well known. Several attempts have been made (Ambannavar, 1975; Cassen and Dyson, 1976; Frejka, 1973) to project the population of India with a view to help formulating and implementing population policies and programmes. The results from the census also indicate that major changes in earlier estimates may be required. These estimates varied widely, and refining operations are continuing. The present study originated in this context.

In India, the major sources of demographic data are the population censuses, the civil registration system, national sample surveys and the sample registration scheme. Though the censuses and vital registration in the country extend over a full century, these remained severely defective (Som, 1973).

Based on historical trends and current estimates, we made a short-run demographic projection for India. In the first place, the working hypotheses for future trends are less likely to be fallible if projections are made only for a short period from the base date. A population estimate, longer-run in its perspectives, is merely academic, because a country's future demographic profile is dependent upon unpredictable factors and innovations affecting prospective trends in the components of growth. Secondly, a short-run projection syncronizing with a short-term development planning, such as five-year plans adopted by India, is more illuminating than projection drawn into a distant future. Moreover, a short-run projection of population provides a general framework for further projection at a later date when more information demographically relevant are available.

We prepare two sets of short-term population projections for India and its major states for 1971-86 and 1981-2001. For our present purpose, the administrative divisions as existed at Census 1971 are adopted. The take-off points of the projections are 1st March, 1971 and 1981. The state level estimates of population are also made to get disaggregated estimates as well as the sectoral growth pattern of the country. The usual graduation methods of adjustment of the base populations being not satisfactory, we evolved a mathematical model to generate an approximately corrected age structure for 1971 and 1981. The estimates could be improved by iterations, and the algebra underlying the techniques developed and made more precise. But this was not considered to be worthwhile as the first approximations we arrived at appear to be good enough for the purpose in hand.

4.2 Components of population growth

The population of India may be taken as virtually closed, but this is not true of the constituent states. For an understanding of the population dynamics of the country as a whole we would look only to fertility and mortality components. India has a long history of civil registration system, but this contimued to be severely defective, so, that no firm estimates of vital rates from this source could be made. Several household surveys on national scale have been undertaken by the National Sample Survey (NSS) Organisation to bridge the information gap on the components of growth. Because of gross underestimation due to large ascertainment errors, little substantive use could be made of the NSS vital statistics. The average decadal vital rates obtained from the age distribution of the population censuses are not altogether acceptable because of the serious nature of the distortions.

The Sample Registration Scheme (SRS) initiated by Registrar General's Office (1964-65) is believed to be the best available estimates of vital rates to-date. The process combines the advantages of longitudinal registration and periodic retrospective sample surveys. Continuous recording of events is maintained by resident enumerators to be compared with the results of independent surveys conducted at regular intervals with short reference period. An application of a simplified Chandrasekhar-Deming model (Chandrasekhar and Deming, 1949) for estimation is made on the assumption that events missed and recording of spurious reports by both the systems are insignificant. While the estimates obtained from this scheme are as good as in any other closely supervised survey, the technique used is biased. The final estimates depend on the characteristic items selected as criteria for matching, and residual uncertainty is associated with the matching procedure (Das Gupta and Guha Roy, 1976). These difficulties in utilizing the method are apparent in the under-estimation of the SRS birth and death rates. The Registrar General and Census Commissioner, India, (1974) suggested that the vital rates computed from the SRS data would have to be inflated by about 5 per cent.

We however decide to accept the vital rate estimates of the Sample Registration Scheme as the first approximation, for subsequent improvement. The birth and death rates estimated from the SRS are 36.9 and 14.9 respectively in 1971. Gross underenumeration is indicated, and consistency criteria are used to improve the estimates.

It is known that, during this century, rates of population growth, though varying from year to year, remained relatively steady at a low level, and then started accelerating. For an independent estimate of growth of population of

India, a quadratic exponential is fitted to the reported census populations of 1951, 1961 and 1971. Base (1971) population P(0) is subjected exponentially to a linear compound of a long-run constant growth u and acceleration parameter f to get P(t), the population in the t-th year. In other words,

$$P(t) = P(0) \exp (ut + ft^2)$$

The fit can be made exact by using two equations in two unknowns. In fact, only two equations arise, namely,

$$P(1951) = P(1971) \exp (u (-20.083) + f (403.3269))$$

and

$$P(1961) = P(1971) \exp (u (-10.083) + f (101.6669))$$

taking into account th the reference dates for the 1951 and 1961 censuses are the same (March 1), wh e that of the 1971 census was April 1. Therefore, there is only one solution f the values of u and f and, hence, the fitted values would be identical wi the observed ones. The current rate of population increase is given by

$$g = \frac{d}{dt} \quad (ut + ft^2),$$

t being measured in years from 1971 census. Annual net migration being assumed insignificant, the above derivative need not be adjusted on this account to get the natural population growth rate. The theoretical curve was intended to describe the trends around 1971; generally it will not be valid in the long-run.

With P(1951) = 361, P(1961) = 439 and P(1971) = 548, the resulting curve is

$$P(t) = P(1971) \exp (0.023211t + 0.00012095t^2)$$

It yields plausible growth rates of 2.3 and 2.5 per cent in 1971 and 1981 respectively.

It may be of interest also to use 1981 population as a base for curve fitting. Thus, with P(1961), P(1971), P(1981) = 685 and t measured from 1981 census, the resulting curve is

$$P(t) = P(1981) \exp(.022752t + .0000253t^2)$$

It yields growth rates of 2.2 and 2.3 per cent in 1971 and 1981 respectively. The latter estimates are slightly lower compared to those based on 1971.

Going by the SRS records on birth (b) and death rates (d), we have

$$\frac{b}{r_b} - \frac{d}{r_d} = g$$

or

$$\frac{36.9}{r_b} - \frac{14.9}{r_d} = 23.2,$$

where r_b and r_d denote completeness of birth and death registrations respectively. To get consistent solutions of reported vital rates from the above single equation, we used the birth rate of 41.0 per 1,000 around 1971. This estimate was obtained by reverse survival technique from the base 0-4 population (1971), adjusted by a transitional age structure model developed later. We used the United Nations Model Life Table at level 55 corresponding to prevalent value of about $\mathring{e}_o$ = 47.5 years. Our estimate of birth rate agrees with that given in the Actuarial Reports, Census of India, which used the same technique of estimation after adjustment for under-enumeration of young children. We would not repeat the review of the previous estimates but refer to Robert Cassen and Tim Dyson (1976) who examined in detail the baseline assumptions on fertility in previous projections for India (Ambannavar, 1975; Frejka, 1973; Government of India, 1974; Operations Research Group, undated; Raghavachari, 1974; Zachariah and Cuca, 1972) and concluded that the initial level assumed in the projections (crude birth rate ranges from 38.6 - 40.4 during 1965-71) was low. Pooling various indications, they assumed that the crude birth rate for the period 1966-71 was in the region of 42. Our estimate of 41 lies in between those of previous projections and Cassen - Dyson.

The completeness of birth reporting thus turns out to be 90 per cent, which when substituted in the above equation yields a value of 85 per cent for completeness of death reporting. An alternative estimate of completeness by Preston and Hill's (1980) method is also of the same order of magnitude. This gives a value of 17.8 per 1,000 for the death rate in 1971. One interesting point is that like some other cultures, deaths appear to be less completely reported than births in the Sample Registration System (SRS). There are irregular variations in the level of reporting in the SRS. It is believed that the SRS improved markedly in 1972, probably because a fertility survey was carried out simultaneously with the SRS operations and a more careful supervision was instituted. We may, therefore, stipulate an improved completeness of reporting of 95 per cent (Registrar General, 1974) during 1972-75. However, due to budget cuts in later years, the high quality achieved during this period was probably not maintained in recent years.

Adding back the estimated amount of under-enumeration, the estimated birth rates for the years 1971, 1972 and 1975 are :

	SRS rates	r_b	rates adjusted by r_b
1971	36.9	0.90	41.0
1972	36.6	0.95	38.5
1975	35.2	0.95	37.1

For comparison we present below the following birth rates for India estimated by different international agencies for the period around 1975 (Kirk, 1979) :

UN	US Census Bureau	AID	World Bank	Population Council	Population Reference Bureau	Environmental Fund	World Watch
36	36-37	35	36	36	34	40	36

Our estimate may be taken to be close to the value of 36 reported by majority of the agencies. There are clear indications that the birth rate, assumed to be invariant so far, had begun declining after 1971. The experience of the SRS confirmed such a trend for the period 1971-75. An exponential curve was fitted to the time series of adjusted birth rates from the SRS. Thus we have

$$b(t) = b(0) \exp(-\beta t)$$

or $$b(t) = 41.0 \exp(-0.012213t)$$

Continuing this trend, birth rates during the quinquennia 1971-75, 1976-80 and 1981-85 were found respectively as 40.1, 37.7 and 35.5 as compared to the corresponding figures of 39.4, 37.5 and 36.0 estimated in F_1M_1 projection of Cassen and Dyson (1976). As there was a short-fall of young adult females during the period, given the lower proportions observed in the age range 15-24 at the Census 1971, the birth rate would have been lower in any case.

Our next task is to split the births by sex. The sex ratio of the Indian population is biased towards males. The adverse sex ratio seems to be the result of a higher female mortality due to neglect of females after birth (Registrar General and Census Commissioner, India, 1974). In the absence of any significant migration either way, sex differential in mortality is the only major factor in the imbalance in sex ratio. The SRS provided nearly a comprehensive and representative record of births and showed a high sex ratio at birth of 107-108 in 1960-69. Considering greater omission in counting of females compared with males in the social setting now obtaining in India, we however accepted a value of 107 for our purpose. This high sex ratio at birth was consistent with the adjusted population base of 1971 adopted for the forward estimates. Since the sex ratio at birth is a 'biological constant' which only varies within a narrow margin of tolerance over time, we have assumed a constant value of 107 for the purpose of our short-term estimates.

In obtaining consistent solutions of vital rates, we have seen that the death rates estimated in the SRS were lower to the extent of 15 per cent. On the other hand, the technique of estimating contemporary level of mortality from survival ratios yielded by longitudinal comparisons of age cohorts across census populations at 1961 and 1971, did not work owing to gross age reporting errors and biases. Estimating the level of mortality through the technique of age distribution of deaths would not similarly apply to the general population due to over-statement of age in the old age ranges, and the severe deficiency of the regular vital registration system.

We used the United Nations model set of life tables at various levels (United Nations, 1956), for these tables were as good as any others for our purpose. Both the United Nations and Coale-Demeny (1966) model sets of tables have their advantages and short-comings (Adlakha, 1972), and it should be obvious that no specific mortality experience can be expected to cling to the very heterogenous averages of the model system. We did not however adhere formally to any particular model level at all age groups but switched across the levels to give effect to the recent specific age pattern of mortality for India.

The pooled age specific death rates in the SRS for 1970-72 period, centered on 1971, were adjusted to the consistent death rate of 17.8. In terms of life expectancy (for both sexes combined), this corresponds to 47.5 years (and UN level of 55) in our mortality base-assumption. This estimate is consistent with those of Cassen and Dyson (1976) who took male life expectation of 48 years, and that of females 46 years. Though it is not expected that the death rate would go back to its earlier higher levels, the deceleration in mortality decline is nevertheless evident (Guha Roy, 1984). Marvels of modern medicine led to a sharp decline of the death rate during last decades, but lack of improvement in nutritional level had arrested further decline in mortality, which tended to reach a plateau. We therefore assumed that the 1971 mortality as obtained above would prevail during the period 1971-75 and decline under normal conditions at half the rate assumed in the set of UN model tables, that is, by about two and a half levels during each quinquennium of 1976-80 and 1981-85. As the public health programme in the country is still at a low ebb, particularly in the predominant rural sector, this assumption about mortality trend seems plausible. The estimated age specific mortality as of 1971 and the pertinent survival factors corresponding to the projected mortality periods, given by UN model levels, are shown in Table 4.1.

4.3 Age structure and estimates for the future

During the last decades, India did not experience any serious natural and man-made calamities that could disturb the demographic development of her

Table 4.1. Age specific death rates for 1971 (SRS average 1970-72) and estimated UN Model Levels (UNL) along with the projected survival ratios

(a) Male

Age (x)	Mortality experience for 1971		Survival rates for the projected quinquennia			m_x X 10^3 for 1981
	m_x X 10^3	UNL	1971-75	1976-80	1981-85	
0	133.0	69	0.8674	0.8755	0.8842	118.3
0 - 4	52.6	55	0.9369	0.9407	0.9445	48.3
5 - 9	4.8	53	0.9801	0.9813	0.9825	4.3
10 - 14	2.0	71	0.9870	0.9879	0.9887	1.8
15 - 19	2.3	83	0.9862	0.9873	0.9884	2.0
20 - 24	3.0	85	0.9847	0.9860	0.9873	2.6
25 - 29	3.3	84	0.9834	0.9848	0.9861	2.8
30 - 34	4.1	79	0.9786	0.9801	0.9816	3.6
35 - 39	5.3	74	0.9703	0.9722	0.9739	4.8
40 - 44	8.2	66	0.9533	0.9559	0.9583	7.4
45 - 49	11.7	63	0.9333	0.9365	0.9395	10.7
50 - 54	18.8	54	0.8947	0.8989	0.9029	17.6
55 - 59	26.2	53	0.8547	0.8596	0.8642	24.7
60 - 64	41.2	46	0.7823	0.7886	0.7947	33.8
65 - 69	56.9	49	0.7087	0.7155	0.7220	54.1
70 - 74	83.4	50*	0.6054	0.6124	0.6194	80.0
75 - 79	122.9	50*	0.4757	0.4830	0.4903	118.2
80 - 84	182.5	50*	0.2768	0.2823	0.2878	176.1
85+	284.1	50*				278.0

* Estimated in order to obtain a consistent series.

Table 4.1 (Continued)

(b) Female

Age (x)	Mortality experience for 1971		Survival rates for the projected quinquennia			m_x X 10^3 for 1981
	m_x X 10^3	UNL	1971-75	1976-80	1981-85	
0.	137.0	60	0.8594	0.8666	0.8739	124.7
0 - 4	59.2	45	0.9209	0.9254	0.9300	54.0
5 - 9	5.3	50	0.9773	0.9787	0.9801	4.7
10 - 14	2.4	67	0.9852	0.9863	0.9873	2.1
15 - 19	3.4	69	0.9803	0.9818	0.9832	3.0
20 - 24	4.2	71	0.9778	0.9796	0.9813	3.8
25 - 29	4.8	70	0.9756	0.9774	0.9792	4.2
30 - 34	5.6	67	0.9716	0.9735	0.9754	4.9
35 - 39	5.9	67	0.9685	0.9706	0.9726	5.3
40 - 44	6.8	67	0.9624	0.9646	0.9666	6.1
45 - 49	9.4	63	0.9471	0.9498	0.9523	8.6
50 - 54	15.2	53	0.9146	0.9185	0.9222	14.1
55 - 59	20.2	54	0.8839	0.8883	0.8929	18.8
60 - 64	36.0	43	0.8049	0.8117	0.8180	33.4
65 - 69	49.3	47	0.7357	0.7429	0.7496	46.2
70 - 74	76.4	47*	0.6261	0.6342	0.6419	n72.0
75 - 79	116.8	47*	0.4936	0.5023	0.5130	111.0
80 - 84	173.8	47*	0.2902	0.2966	0.3027	166.0
85+	276.8	47*				269.5

* Estimated in order to obtain a consistent series.

large population. As in other countries of the subcontinent, census enumeration was however deficient; there were undercounts of infants and females and mis-statements of age. Our purpose here is not to discuss the well known errors and biases in reporting ages, which are embedded in the culture, but we proceed to adjust the base population of 1971 census for irregularities in age distribution. Figure 4.1 gives an illustrative representation of the reported and the model distributions for males at Census

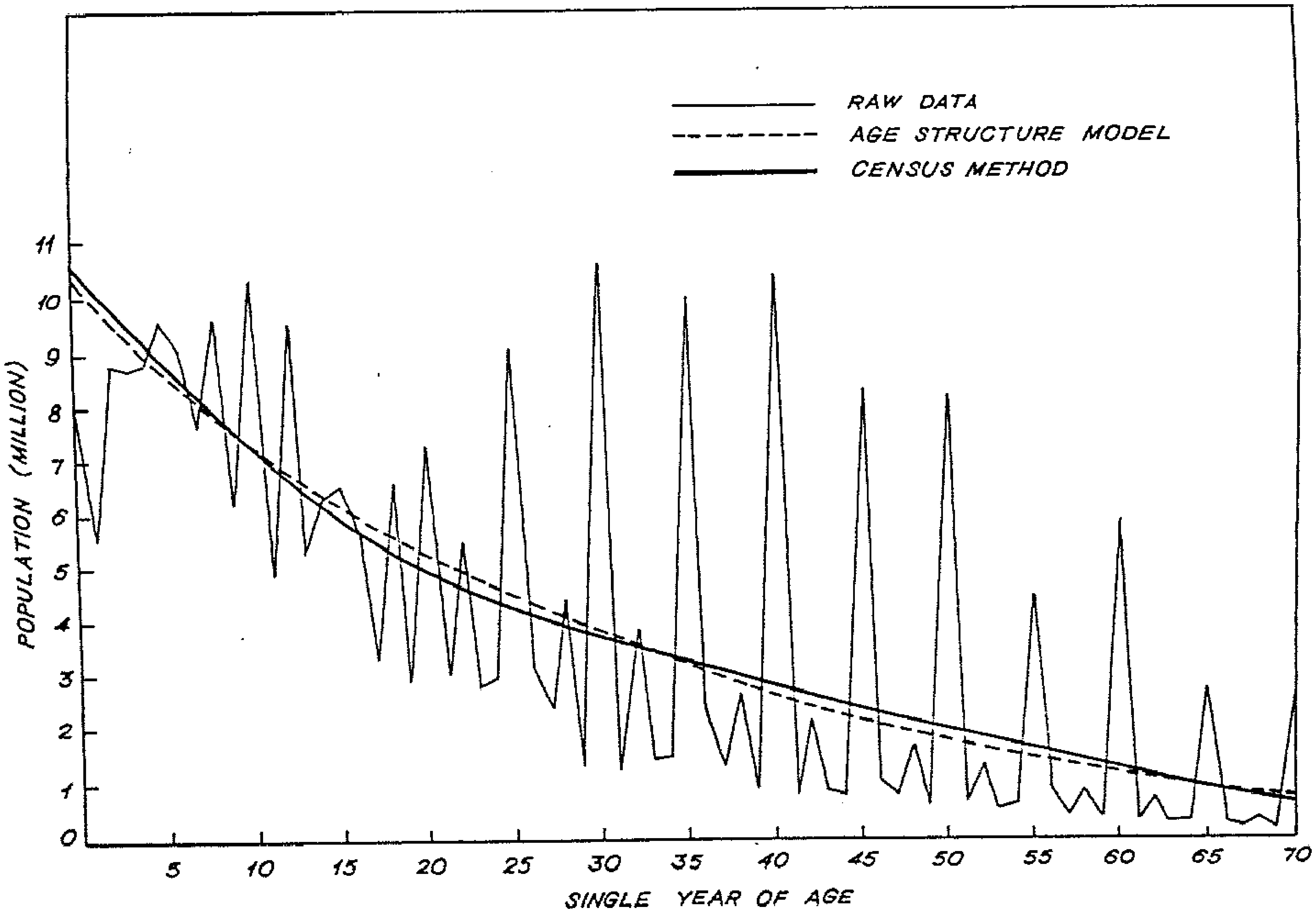


Figure 4.1 : Raw and adjusted population age distributions of India : 1971 census (Males)

1971 base. The distributions for females are almost the same as males and are therefore not shown graphically.

The usual general purpose graduation methods of wave cutting and running averages do smoothe the reported age distribution, but have little validity particularly in reported age ranges where undercounts or large under-or over-statement of ages have occurred. " Mechanical smoothing processes work well only when deficits and excesses in persons reporting each age are balanced over fairly short intervals. When, as in the present instance, there are large displacements of reported age, disproportionate omissions or improper inclusions of persons in broad age groups, mechanical smoothing will yield an age distribution still strongly affected by the original errors" (Coale and Hoover, 1959). To construct an approximate age structure, the theory of stable populations is often applied. But the use of stable models is not appropriate to India under transition of declining mortality (Das Gupta, et al., 1965).

In such a situation prevailing in India with declining fertility, an approximate population model was evolved to generate corrected age structure consistent with the probable mortality levels and the observed growth rates at successive censuses. The UN model mortality sets before modification to conform to local pattern were used for convenience, in view of the broad approximation involved anyway.

The model supposes the growth survivor relationship between the successive ages of a closed population formation; fertility was assumed broadly invariant till 1971. The essence of the model is that suitably chosen stationary population (${}_tL_x$) is given the stimulus of actual growth (g) and survivorship (${}_tp_x$) to evolve an age structure to be fitted to the reported census population.

4.4 The mathematical model in discrete form

We consider a population which (i) has been closed to migration and (ii) has been growing at a constant decadal rate $\bar{g}$; (iii) its mortality, as described by a life table, is subject to change over a quinquennium (e.g., 1966-70, 1961-65) or a decennium (e.g., 1951-60, 1941-50 etc.). The fertility was assumed to have been broadly constant upto 1971.

Evidently the population under 5 at the end of 1970 will be survivors of the birth cohort of 1966-70, the 5-9 years olds will be the survivors of the births in 1961-65; and so on. Thus to find the size of the population in each age group we require (i) the calculation of the birth cohorts preceding 1971 and (ii) the calculation of the survivors at the end of 1970.

The input data used to implement this model are the average population growth rates ($\bar{g}$), known at discrete intervals, and were taken from censuses. The values of $\bar{g}$ were computed as :

$$\bar{g} = \exp((\ln P_2 - \ln P_1)/t) - 1$$

where P_2 is the population enumerated at the end of the period, P_1 that at the beginning and t is the length of the intercensal period (e.g., t = 10.083 for 1961-71). The annual average growth (g_i) rates are shown as follows :

1961-71	1951-61	1931-51	1921-31	1911-21	1901-11	pre-1901
2.22	1.98	1.30	1.06	-0.04	0.57	0.21
g_6	g_5	g_4	g_3	g_2	g_1	g_0

A single value for the growth rate during 1931-51 was taken as it changed only slightly in this period. The decade 1911-21 experienced severe epidemics, resulting in a negative growth.

The mortality patterns for the different periods were described by United Nations Model Life Tables, and the levels (UNL) broadly considered (Ministry of Health and Family Planning, 1972-73) were as given below :

	1966-70	1961-65	1951-60	1941-50	1931-40	pre-1931
UNL	55	50	45	35	25	10
e_o	47.5	45.0	42.5	37.5	32.5	25.0

Since in 1968 (mid-year of 1966-70) there was ${}^{55}_{5}L_0$ population on the average (as per the life table assumed) and since this number was growing at the average annual rate of g_6 (according to census decadal growth rate), we have

(1) Size of the cohort (1966-70) at the end of 1970

$$= {}^{55}_{5}L_0 \;\Big/\; (1 + g_6)^{2.5} \simeq {}^{55}_{5}L_0 \; e^{-2.5 g_6}$$

Similarly, the average size of the birth cohort in 1963 (mid-year of 1961-65) was ${}^{50}_{5}L_0$ (as per the life table of the previous quinquennium) and this was growing again at the rate of g_6 (the cohort originating within the same decade 1961-71), so that (without mortality assumption for the time being)

(2) Size of the cohort (1961-65) at the end of 1970

$$= {}^{50}_{5}L_0 \;\Big/\; (1 + g_6)^{7.5} \simeq {}^{50}_{5}L_0 \; e^{-7.5 g_6}$$

Again, the average size of the birth cohort (1956-60) in 1958 was ${}^{45}_{5}L_0$ (according to the life table for the period 1951-60) and this cohort passed through two decades, 1951-61 and 1961-71, with growth rates of g_5 and g_6 respectively. Now, the time difference between 1958 and 1970 is 12.5 years, which should be apportioned between the two decades. After 1958 there were 2.5 years that fell in 1951-61 decade, and 10 years in 1961-71 decade. Thus we get

(3) Size of the cohort (1956-60) at the end of 1970

$$= {}^{45}_{5}L_0 \;\Big/\; (1 + g_6)^{10} \; (1 + g_5)^{2.5}$$

$$\simeq {}^{45}_{5}L_0 \; e^{-10 g_6} \; e^{-2.5 g_5}$$

and so on. The population alive at the end of 1970 can now be easily calculated, using appropriate survival factors, ${}_{5}S_x$. Thus,

$$\text{Persons at age 0-4} = {}^{55}_{5}L_0 \times e^{-2.5g_6}$$

$$\text{Persons at age 5-9} = {}^{50}_{5}L_0 \times e^{-7.5g_6} \times {}^{55}_{5}S_0$$

$$(\text{where } {}^{55}_{5}S_0 = {}^{55}_{5}L_5 \,/\, {}^{55}_{5}L_0)$$

$$\text{Persons at age 10-14} = {}^{45}_{5}L_0 \times e^{-10g_6} \times e^{-2.5g_5} \times {}^{50}_{5}S_0 \times {}^{55}_{5}S_5$$

The approximate age structure model could thus be built up in this manner. The model assumes that an estimate of true age structure of a population may be obtained by inflating the ${}_5L_x$ values derived from a series of model life tables at different levels according to a few broadly consistent changes in growth and mortality. The complete presentation for the quinquennial age groups are given below :

0- 4 : ${}^{55}_{5}L_0\,(1+g_6)^{-2.5}$

5- 9 : ${}^{50}_{5}L_0\,(1+g_6)^{-7.5}$

10-14 : ${}^{45}_{5}L_0\,(1+g_6)^{-10}\,(1+g_5)^{-2.5}$

15-19 : ${}^{45}_{5}L_5\,(1+g_6)^{-10}\,(1+g_5)^{-7.5}$

20-24 : ${}^{35}_{5}L_0\,(1+g_6)^{-10}\,(1+g_5)^{-10}\,(1+g_4)^{-2.5}$

25-29 : ${}^{35}_{5}L_5\,(1+g_6)^{-10}\,(1+g_5)^{-10}\,(1+g_4)^{-7.5}$

30-34 : ${}^{25}_{5}L_0\,(1+g_6)^{-10}\,(1+g_5)^{-10}\,(1+g_4)^{-12.5}$

35-39 : ${}^{25}_{5}L_5\,(1+g_6)^{-10}\,(1+g_5)^{-10}\,(1+g_4)^{-17.5}$

40-44 : ${}^{10}_{5}L_0\,(1+g_6)^{-10}\,(1+g_5)^{-10}\,(1+g_4)^{-20}\,(1+g_3)^{-2.5}$

45-49 : ${}^{10}_{5}L_5\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-7.5}$

50-54 : ${}^{10}_{5}L_{10}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-2.5}$

55-59 : ${}^{10}_{5}L_{15}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-7.5}$

60-64 : ${}^{10}_{5}L_{20}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}\ (1+g_1)^{-2.5}$

65-69 : ${}^{10}_{5}L_{25}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}\ (1+g_1)^{-7.5}$

70-74 : ${}^{10}_{5}L_{30}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}$
$(1+g_1)^{-10}\ (1+g_0)^{-2.5}$

75-79 : ${}^{10}_{5}L_{35}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}$
$(1+g_1)^{-10}\ (1+g_0)^{-7.5}$

80-84 : ${}^{10}_{5}L_{40}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}$
$(1+g_1)^{-10}\ (1+g_0)^{-12.5}$

85+ : ${}^{10}_{5}L_{45}\ (1+g_6)^{-10}\ (1+g_5)^{-10}\ (1+g_4)^{-20}\ (1+g_3)^{-10}\ (1+g_2)^{-10}$
$(1+g_1)^{-10}\ (1+g_0)^{-17.5}$

The population alive at the end of 1971 can be obtained by subjecting the above expressions to the following survival factors (${}_5S_x$) in corresponding ages :

5- 9 : ${}^{55}_{5}S_0$

10-14 : ${}^{50}_{5}S_0 \times {}^{55}_{5}S_5$

15-19 : ${}^{50}_{5}S_5 \times {}^{55}_{5}S_{10}$

20-24 : ${}^{45}_{5}S_{0} \times {}^{45}_{5}S_{5} \times {}^{50}_{5}S_{10} \times {}^{55}_{5}S_{15}$

25-29 : ${}^{45}_{5}S_{5} \times {}^{45}_{5}S_{10} \times {}^{50}_{5}S_{15} \times {}^{55}_{5}S_{20}$

30-34 : ${}^{35}_{5}S_{0} \times {}^{35}_{5}S_{5} \times {}^{45}_{5}S_{10} \times {}^{45}_{5}S_{15} \times {}^{50}_{5}S_{20} \times {}^{55}_{5}S_{25}$

35-39 : ${}^{35}_{5}S_{5} \times {}^{35}_{5}S_{10} \times {}^{45}_{5}S_{15} \times {}^{45}_{5}S_{20} \times {}^{50}_{5}S_{25} \times {}^{55}_{5}S_{30}$

40-44 : ${}^{25}_{5}S_{0} \times {}^{25}_{5}S_{5} \times {}^{35}_{5}S_{10} \times {}^{35}_{5}S_{15} \times {}^{45}_{5}S_{20} \times {}^{45}_{5}S_{25} \times {}^{50}_{5}S_{30} \times {}^{55}_{5}S_{35}$

45-49 : ${}^{25}_{5}S_{5} \times {}^{25}_{5}S_{10} \times {}^{35}_{5}S_{15} \times {}^{35}_{5}S_{20} \times {}^{45}_{5}S_{25} \times {}^{45}_{5}S_{30} \times {}^{50}_{5}S_{35} \times {}^{55}_{5}S_{40}$

50-54 : ${}^{25}_{5}S_{10} \times {}^{25}_{5}S_{15} \times {}^{35}_{5}S_{20} \times {}^{35}_{5}S_{25} \times {}^{45}_{5}S_{30} \times {}^{45}_{5}S_{35} \times {}^{50}_{5}S_{40} \times {}^{55}_{5}S_{45}$

55-59 : ${}^{25}_{5}S_{15} \times {}^{25}_{5}S_{20} \times {}^{35}_{5}S_{25} \times {}^{35}_{5}S_{30} \times {}^{45}_{5}S_{35} \times {}^{45}_{5}S_{40} \times {}^{50}_{5}S_{45} \times {}^{55}_{5}S_{50}$

60-64 : ${}^{25}_{5}S_{20} \times {}^{25}_{5}S_{25} \times {}^{35}_{5}S_{30} \times {}^{35}_{5}S_{35} \times {}^{45}_{5}S_{40} \times {}^{45}_{5}S_{45} \times {}^{50}_{5}S_{50} \times {}^{55}_{5}S_{55}$

65-69 : ${}^{25}_{5}S_{25} \times {}^{25}_{5}S_{30} \times {}^{35}_{5}S_{35} \times {}^{35}_{5}S_{40} \times {}^{45}_{5}S_{45} \times {}^{45}_{5}S_{50} \times {}^{50}_{5}S_{55} \times {}^{55}_{5}S_{60}$

70-74 : ${}^{25}_{5}S_{30} \times {}^{25}_{5}S_{35} \times {}^{35}_{5}S_{40} \times {}^{35}_{5}S_{45} \times {}^{45}_{5}S_{50} \times {}^{45}_{5}S_{55} \times {}^{50}_{5}S_{60} \times {}^{55}_{5}S_{65}$

75-79 : ${}^{25}_{5}S_{35} \times {}^{25}_{5}S_{40} \times {}^{35}_{5}S_{45} \times {}^{35}_{5}S_{50} \times {}^{45}_{5}S_{55} \times {}^{45}_{5}S_{60} \times {}^{50}_{5}S_{65} \times {}^{55}_{5}S_{70}$

80-84 : ${}^{25}_{5}S_{40} \times {}^{25}_{5}S_{45} \times {}^{35}_{5}S_{50} \times {}^{35}_{5}S_{55} \times {}^{45}_{5}S_{60} \times {}^{45}_{5}S_{65} \times {}^{50}_{5}S_{70} \times {}^{55}_{5}S_{75}$

85+ : $\dfrac{{}^{25}T_{55}}{{}^{25}T_{45}} \times \dfrac{{}^{35}T_{65}}{{}^{35}T_{55}} \times \dfrac{{}^{45}T_{75}}{{}^{45}T_{65}} \times \dfrac{{}^{50}T_{80}}{{}^{50}T_{75}} \times \dfrac{{}^{55}T_{85}}{{}^{55}T_{80}}$

where the life table function T_x is obtained as usual by cumulation of L_x - function starting with L_{85+}.

The age structure model thus developed was fitted to the reported 1971 Census male population. The transitional model may still differ from reality because possible cyclical variations in population movements were left out of account. A different procedure had to be used to adjust under-enumeration in the initial age group 0-4. The age distribution was adjusted by fitting the model to the reported population aged five and over and then the fitted distribution was extended below those ages by pro-rating. When the male population reported at Census 1971 had been distributed according to the age structure model, the female population suffering from even worse irregularities of reporting, was estimated with reference to the finally adjusted male distribution. For this purpose, smoothed regression functions were derived from the reported male and female populations by age group. A quadratic regression was found appropriate (Das Gupta and Guha Roy, 1976); the ratio of the female population F_x to the male population M_x worked out for census 1971 was as follows :

$$F_x / M_x = 0.94688204 + 0.00025716x - 0.00002835x^2$$

The proportionate age distributions derived by the above method are presented in Table 4.2 and the finally adjusted male and female base populations at Census 1971 are shown in Table 4.3.

The aggregate population at Census 1971 was not adjusted for shifting the reference date from April to the usual census month of March, but to take at least a token account of under-enumeration, revealed in 1971 post-enumeration check, the published census total was made to refer to March.

A second procedure adopted for smoothing the raw Census (1971) age distribution for comparative purposes was the 11-point moving-average method used by the census actuary (Census of India, 1961). In this procedure, the individual age (x) population between the ages 8 and 67 was obtained by averaging the 11 years x-5 to x+5. The five-year age groups were then formed with multiples of five as mid-points, namely, 8-12, 13-17, etc. These group totals were again smoothed by a three-point moving-weighted-averaged method. Thus, an adjusted population (P′) of age group (a) was obtained as

$$P'_a = 0.25\,P_{a-1} + 0.50\,P_a + 0.25\,P_{a+1}$$

These adjusted totals were redistributed to individual ages by using Sprague — multipliers and regrouped into conventional five-year age groups, such as 10-14,

Table 4.2. Reported and adjusted population age distributions per 1,000 of each sex : 1971

Age	Unadjusted census, 1971*		Adjusted census, 1971: U.S. Bureau of the Census**		Census method*		Age structure model*	
	Male	Female	Male	Female	Male	Female	Male	Female
0 - 4	141	148	158	161	174	172	170	173
5 - 9	150	152	135	134	144	144	141	144
10 - 14	127	122	120	119	118	116	119	121
15 - 19	89	84	104	102	95	95	104	105
20 - 24	76	82	87	86	79	83	87	88
25 - 29	72	78	74	73	70	75	76	76
30 - 34	64	68	65	63	63	66	64	64
35 - 39	61	59	58	56	56	56	54	54
40 - 44	53	50	50	49	49	47	43	42
45 - 49	44	39	42	42	41	38	35	34
50 - 54	39	36	34	35	33	31	30	28
55 - 59	24	22	27	27	26	25	25	24
60 - 64	26	26	19	21	20	19	20	19
65 - 69	13	12	13	14	13	13	15	13
70 - 74	11	11	8	9	8	8	9	8
75 - 79	4	4	3	4	5	5	5	4
80+	6	7	3	5	6	7	3	3

* Figures exclude data for Sikkim.

** Figures refer to all areas of India.

15-19, and so on. The smoothed distributions for older ages (67+) were obtained by assuming constant second-order differences above that age (Census of India, 1961; Srinivasan, 1978). The populations for ages 0-7 were estimated using the equation

$$y_x = A + Hx + BC^x,$$

y_x being the population at age x, which was an adaptation of Makeham's formula. The statement explaining the use of the above equation is quoted below from Census of India, Paper Number 2 of 1963 :

Table 4.3. Adjusted base population of India at Census 1971 and estimated populations (000) in 1981 and 1986

Age group	Adjusted base population 1971			Current estimates 1981			Current estimates 1986		
	Males	Females	Total	Males	Females	Total	Males	Females	Total
0 - 4	48253	45658	93911	52492	48559	101051	57601	53206	110807
5 - 9	39907	37785	77692	47690	43008	90698	49579	45160	94739
10 - 14	33856	32026	65882	44363	41151	85514	46855	42152	89007
0 - 14	122016	115469	237485	144545	132718	277263	154035	140518	294553
15 - 19	29392	27738	57130	38640	36421	75061	43861	40628	84489
20 - 24	24788	23301	48089	32992	30978	63970	38191	35810	74001
25 - 29	21584	20180	41764	28581	26637	55218	32573	30398	62971
30 - 34	18085	16791	34876	24038	22269	46307	28184	26083	54267
35 - 39	15421	14196	29617	20803	19166	39969	23595	21721	45316
40 - 44	12233	11146	23379	17206	15835	33041	20260	18640	38900
15 - 44		113352			151306			173280	
45 - 49	9941	8951	18892	14303	13262	27565	16489	15306	31795
50 - 54	8435	7492	15927	10921	10188	21109	13438	12629	26067
55 - 59	7170	6271	13441	8340	7786	16126	9861	9396	19257
15 - 59	147049	136066	283115	195824	182542	378366	226452	210611	437063
60 - 64	5712	4910	10622	6487	6087	12574	7207	6952	14159
65 - 69	4121	3474	7595	4832	4499	9331	5155	4979	10134
70 - 74	2664	2199	4863	3197	2936	6133	3489	3373	6862
75 - 79	1468	1183	2651	1788	1621	3409	1981	1885	3866
80 - 84	862	677	1539	779	692	1471	877	832	1709
85+	45	35	80	197	173	370	224	209	433
60+	14872	12478	27350	17280	16008	33288	18933	18230	37163
Total	283937	264013	547950	357649	331268	688917	399420	369359	768779
Birth rate			41.0			36.4			34.2
Death rate			17.8			14.8			13.9
Growth rate			23.2			21.6			20.3

From the nature of the problem H must be negative so that $A + Hx$ would represent a population decreasing linearly with age. The role of the component corresponding to BC^x is to regulate largely the element of extra mortality in childhood. The multiplication of the component by C (a positive fraction around half), for every unit increase in age, makes for a rapid fall in y_x in childhood ages. The above equation, thus, defines a population decreasing with age in a manner conforming to the general pattern of variation in mortality risks in childhood and later ages.

The above paper however did not give a very robust procedure for estimating the parameters A, H, B and C in the above model. The method used by us was an iterative least squares method, described in details as follows :

We have

$$y_x = A + Hx + BC^x,$$

Writing

$$W_x = \log \Delta^2 y_x$$

We get

$$W_x = \beta + \gamma x$$

where

$$\beta = \log B (C - 1)^2$$

$$\gamma = \log C$$

We apply least squares method to estimate the values of β and γ in

$$W_x = \beta + \gamma x$$

These estimates give

$$C = e^{\gamma}$$

and $$B = e^{\beta}/(C - 1)^2$$

To estimate the parameters A and H, we compute

$$yy_x = y_x - BC^x$$

and note that

$$yy_x = A + Hx.$$

We apply least square method once again to estimate A and H in

$$yy_x = A + Hx$$

The first approximations thus obtained are designated as A_0, H_0, B_0 and C_0.

The least square estimates for the four parameters, A, H, B and C are computed by minimizing

$$L = \sum_x (y_x - A - Hx - BC^x)^2$$

with respect to A, H, B and C.

We start with A_0, H_0, B_0 and C_0 as the initial approximations for the required values of A, H, B and C respectively. Evaluating

$$L_0 = \sum_x (y_x - A_0 - H_0x - B_0C_0^{\ x})^2$$

we find the value of L_0.

Let us write

$$A = A_0 + \delta_1$$
$$H = H_0 + \delta_2$$
$$B = B_0 + \delta_3$$
$$C = C_0 + \delta_4$$

and $f(A, H, B, C, x) = A + Hx + BC^x$

Then

$$f(A, H, B, C, x) = f(A_0 + \delta_1, H_0 + \delta_2, B + \delta_3, C_0 + \delta_4, x)$$

$$= f(A_0, H_0, B_0, C_0, x) + \delta_1 \left(\frac{\delta f}{\delta A}\right)_0$$

$$+ \delta_2 \left(\frac{\delta f}{\delta H}\right)_0 + \delta_3 \left(\frac{\delta f}{\delta B}\right)_0 + \delta_4 \left(\frac{\delta f}{\delta C}\right)_0,$$

neglecting higher order terms.

Let us consider

$$Y Z_x = y_x - A_0 - H_0 x - B_0 C_0^{\,x}$$

(Note that $L_0 = \Sigma Y Z_x^2$)

Then the model is

$$Y Z_x = \delta_1 + x \delta_2 + C_0^x \ \delta_3 + (x B_0 C_0^{\,x-1}) \delta_4$$

and the parameters δ_1, δ_2, δ_3 and δ_4 have to be fitted by least squares.

Let us write

$$x_{1i} = 1$$

$$x_{2i} = x_i$$

$$x_{3i} = C_0^{\,x_i}$$

$$x_{4i} = x_i B_0 C_0^{\,x_i - 1}$$

Then the model is

$$Y Z_i = x_{1i} \delta_1 + x_{2i} \delta_2 + x_{3i} \delta_3 + x_{4i} \delta_4$$

The least squares normal equations are

$$\varphi_1 = C_{11} \delta_1 + C_{12} \delta_2 + C_{13} \delta_3 + C_{14} \delta_4$$

$$\varphi_2 = C_{21} \delta_1 + C_{22} \delta_2 + C_{23} \delta_3 + C_{24} \delta_4$$

$$\varphi_3 = C_{31}\delta_1 + C_{32}\delta_2 + C_{33}\delta_3 + C_{34}\delta_4$$

$$\varphi_4 = C_{41}\delta_1 + C_{42}\delta_2 + C_{43}\delta_3 + C_{44}\delta_4$$

where

$$\varphi_j = \sum_{i=1}^{N} Y Z_i x_{ji}$$

$$C_{jk} = \sum_{i=1}^{N} x_{ji} x_{ki}$$

Solving the above equations, we get δ_i's and hence A_1, H_1, B_1 and C_1 to fit $A_1 + H_1x + B_1C_1^x$. Now we compute

$$L_1 = \Sigma \left[y_x - (A_1 + H_1x + B_1C_1^x) \right]^2$$

We check if L_1 is very much less than L_0. We stop if the reduction $L_0 - L_1$ is small. That is,

$$L_0 - L_1 < \varepsilon,$$

otherwise we take $A_0 = A_1$, $H_0 = H_1$, $B_0 = B_1$, $C_0 = C_1$ and repeat the procedure.

To estimate the parameters A, H, B and C in the above model, computer programme in FORTRAN IV language was used, which is shown in the Appendix. The age distributions per 1,000 obtained by the above method for the base year 1971, are also shown in Table 4.2, together with unadjusted census figures; the population distribution of the 1971 age structure model, as developed earlier and the adjusted 1971 figures made by the U.S. Bureau of the Census (1978) are also presented in this table for comparison. Figure 4.1 given earlier, shows the raw and smoothed population distributions.

The reported age distributions show the nature of age distortions; some features of this irregularity in the population distributions are well known for the Indian sub-continent. Infants are underreported, so that the population in the 5-14 age range is relatively too large. The other inaccuracies in the Census distributions have already been discussed by different scholars (Ambannavar, 1975; Frejka, 1973; Poti, 1983). The different methods which are independently applied here to smoothe reported age distribution seem to take account of these deficiencies in the data. On the whole, the adjusted base population distribution obtained by the age structure model is, more or less, similar to that of the Indian Census method than to that by the U.S. Bureau of the Census.

At this stage, we tested the mutual consistency of the adjusted base population and the estimated components of growth (Das Gupta, et al., 1965). Following the usual procedure for the sex-age component projection, the adjusted population thus obtained was survived by five-year intervals to the census anniversary months in 1976, 1981 and 1986, by application of the five-yearly sex-age specific improving survival factors shown in Table 4.1. As the period of projection was short and the sex-age structure of the population was changing very slowly, the quinquennial birth rates estimated earlier were applied to the average quinquennial female population aged 15-44 to estimate the emerging births. The average was worked out in the manner as described by Ajit Das Gupta and others (1969). The aggregate births were then split into males and females in accordance with the steady sex ratio at birth established above, and survived in a similar manner.

Adjusted base population of India at Census 1971 and estimated populations in 1981 and 1986 are shown in Table 4.3. The corresponding estimates of birth, death and growth rates, consistent with the assumptions underlying the projections, are also shown at the base of the same table. The Planning Commission's Expert Committee on Population Projection gave a revised estimate of 657.3 million population in 1981 (Ministry of Health and Family Planning, 1972-73). The other projections made by the Office of the Registrar General (1974) estimated (high fertility assumption) 677.5 and 753.6 million populations in 1981 and 1986, compared to our estimates of 668.9 (Census 1981 reports 685.2 million) and 768.8 million respectively.

4.5 Application of the model to census 1981

A few results of 1981 census being available, it may be of interest to fit the transitional age structure model to the latest reported population as well. The model was constructed for 1981 under similar assumptions about past paths of growths and mortality rates as for 1971, and the population growth rate for 1971-81 was similarly calculated as 22.8 per cent (designated as g_7); the mortality patterns during 1971-75 and 1976-80 were assumed to be described respectively by UN model life tables of levels 60 and 65. Thus the population ${}_5P_x$ from age x to x+5 at the end of 1980 is given by

$${}_5P_0 = {}^{65}_{5}L_0 \quad e^{-2.5g_7}$$

$${}_5P_5 = {}^{60}_{5}L_0 \quad e^{-7.5g_7} \quad ({}^{65}_{5}L_5 \;/\; {}^{65}_{5}L_0)$$

$${}_5P_{10} = {}^{55}_{5}L_0 \quad e^{-(10g_7 + 2.5g_6)} \quad ({}^{60}_{5}L_5 \;/\; {}^{60}_{5}L_0)\,({}^{65}_{5}L_{10} \;/\; {}^{65}_{5}L_5)$$

and so on, the symbols having the same significance as explained earlier.

The age structure model thus being built up was fitted to the 1981 census population in the same manner as in 1971. Table 4.4 shows the per cent age distributions of the enumerated population (5 per cent sample) of 1981, the projected 1981 population derived earlier from 1971 adjusted population and population distribution of the transitional age structure model of 1981. While the model distributions broadly agree, the census distribution shows the usual under-emmeration in the age group 0-4.

For deriving a new set of projections, 1981-2001, from the adjusted 1981 base, we assumed that whereas the expectation of life ($\overset{o}{e}_0$) for males would gain 2.5 years each quinquennium, that for females the rate of gain would be the same upto 1986-90, but it would be 3 years in the next two quinquennia. Consistent with the Government policy for greater emphasis on health care for women (and children) in the coming decades, a faster rate of improvement in life expectancy for females is assumed. Though the overall level of mortality in terms of the UN model set for South Asia (United Nations, 1982) has been used as a first

Table 4.4. Reported, projected (based on adjusted 1971 Census) and transitional model age structures (%) for 1981 : Both sexes combined

Age group	Reported Census 1981*	Transitional model for 1981** 1971 base	1981 base
0 - 4	12.6	14.7	14.9
5 - 9	14.1	13.2	13.7
10 - 14	12.9	12.4	12.2
15 - 19	9.7	10.9	10.4
20 - 24	8.6	9.3	8.8
25 - 29	7.6	8.0	7.7
30 - 34	6.4	6.7	6.5
35 - 39	5.8	5.8	5.7
40 - 44	5.1	4.8	4.8
45 - 49	4.4	4.0	4.0
50 - 54	3.8	3.1	3.2
55 - 59	2.5	2.3	2.5
60+	6.5	4.8	5.6

* Based on unsmoothed age data (5%) for 1981 census (excluding Assam and data for unknown age).

** Includes Assam.

approximation, national age patterns of mortality were expected to deviate from the model. The reported mortality pattern of the SRS, though lower overall, was assumed to represent the age pattern of mortality for the country. A smoothed set of values of age specific mortality was chosen from the UN tables to correspond approximately to the national pattern. The pertinent survival factors given by the UN model tables are shown in Table 4.5a, the overall mortality levels in terms of $\mathring{e}_o$ being presented below.

$\mathring{e}_o$	1981-85	1986-90	1991-95	1996-2000
Males	54.5	57.0	59.5	62.0
Females	53.5	56.0	59.0	62.0

Table 4.5a. Five-year life table survival ratios for India : 1981-2000

(a) Male

Age group	Survival ratio 1981-85	1986-90	1991-95	1996-2000
0 - 4	.9498	.9579	.9654	.9722
5 - 9	.9886	.9905	.9921	.9936
10 - 14	.9926	.9937	.9947	.9956
15 - 19	.9910	.9924	.9936	.9946
20 - 24	.9893	.9908	.9922	.9935
25 - 29	.9868	.9886	.9904	.9919
30 - 34	.9828	.9851	.9872	.9892
35 - 39	.9755	.9785	.9814	.9840
40 - 44	.9640	.9679	.9717	.9753
45 - 49	.9452	.9503	.9553	.9602
50 - 54	.9179	.9243	.9307	.9371
55 - 59	.8774	.8853	.8934	.9015
60 - 64	.8198	.8296	.8398	.8502
65 - 69	.7448	.7567	.7690	.7820
70 - 74	.6546	.6677	.6816	.6964
75 - 79	.5612	.5738	.5874	.6021
80+	.4100	.4181	.4268	.4363

(b) Female

0 - 4	.9405	.9492	.9587	.9676
5 - 9	.9866	.9888	.9911	.9931
10 - 14	.9906	.9921	.9937	.9951
15 - 19	.9874	.9895	.9917	.9936
20 - 24	.9856	.9880	.9904	.9925
25 - 29	.9836	.9861	.9888	.9911
30 - 34	.9804	.9831	.9861	.9888
35 - 39	.9760	.9789	.9822	.9853
40 - 44	.9690	.9722	.9760	.9795
45 - 49	.9550	.9592	.9640	.9687
50 - 54	.9300	.9358	.9426	.9492
55 - 59	.8910	.8987	.9081	.9176
60 - 64	.8356	.8456	.8579	.8703
65 - 69	.7600	.7721	.7871	.8028
70 - 74	.6571	.6714	.6896	.7090
75 - 79	.5426	.5572	.5759	.5962
80+	.3713	.3805	.3924	.4054

The quinquennial birth rates derived on the basis of the trend implied by a negative exponential curve fitted earlier were as follows :

	1981-85	1986-90	1991-95	1996-2000
Birth rate	35.5	33.3	31.0	29.0

We used here the following procedure to estimate the 0-4 age group population.

Let $\bar{b}_{t,\ t+5}$ = average birth rate for the quinquennium (t, t+5)

^{t}P = aggregate population in year t

$^{t+5}P_{5+}$ = population aged 5 and over in year t+5

L_{0-4}/l_0 = survival ratio from birth to ages 0-4

S.R. = sex ratio at birth

x_m = population aged 0-4 for males

x_f = population aged 0-4 for females

x = $x_m + x_f$

m, f = suffixes for males and females.

Then

$$(1)\quad 5\,\bar{b}_{t,\ t+5}\left(\frac{^{t}P + {}^{t+5}P_{5+} + x}{2}\right)\left(\frac{L_{0-4}}{l_0}\right)_m\left(\frac{S.R.}{1 + S.R.}\right) = x_m$$

$$(2)\quad 5\,\bar{b}_{t,\ t+5}\left(\frac{^{t}P + {}^{t+5}P_{5+} + x}{2}\right)\left(\frac{L_{0-4}}{l_0}\right)_f\left(\frac{1}{1 + S.R.}\right) = x_f$$

Writing B = $5\,\bar{b}_{t,\ t+5}$ and denoting the products of last two bracketed quantities as R and T respectively in equations (1) and (2), we get

$$(3)\quad \frac{1}{2}\,B\,R\,x + \frac{1}{2}\,BR\,(\,^{t}P + {}^{t+5}P_{5+}\,) = x_m$$

(4) $\frac{1}{2}$ B T x + $\frac{1}{2}$ BT $({}^{t}P + {}^{t+5}P_{5+}) = x_f$

Equations (3) and (4) may be written as

(5) $A + Dx = x_m$

(6) $C + Ex = x_f$

where

$$A = D({}^{t}P + {}^{t+5}P_{5+})$$

$$D = \frac{1}{2}BR$$

$$C = E({}^{t}P + {}^{t+5}P_{5+})$$

$$E = \frac{1}{2}BT$$

Adding equations (5) and (6), and noting that

$x = x_m + x_f$, we get

$$(A + C) + (D + E)x = x$$

so that $x = \frac{A + C}{1 - D - E}$.

The 0-4 age group population was estimated in this manner. We assumed a constant sex ratio at birth of 107 males per 100 females for the short-run projection (Tables 4.5b and 4.5c). It appears that the two separate projections with 1971 and 1981 base populations did not give rise to important differences in projected population sizes as well as in population age structures.

A review of the recent population projections for India has been made elsewhere (Chaudhry, 1985). We present in Table 4.5d some of these estimates for comparative purpose. Because of the different methodologies of projections and various assumptions as to the most plausible future demographic path, the estimates vary to different degrees among the agencies and years. According to our projections and those by the U.S. Bureau of the Census (1984) and World

Table 4.5b. Projected populations (000) for India upto 2001 (based on the 1981 census) : Males

Age group	Adjusted population, 1981	Projected population			
		1986	1991	1996	2001
0 - 4	52902	57205	61106	64427	67906
5 - 9	48994	50246	54797	58992	62636
10 - 14	43745	48435	49769	54364	58614
0 - 14	145641	155886	165672	177783	189156
15 - 19	37010	43421	48130	49505	54125
20 - 24	31070	36677	43091	47822	49238
25 - 29	27303	30738	36340	42755	47511
30 - 34	23126	26943	30388	35991	42409
35 - 39	20250	22728	26542	29999	35602
40 - 44	16978	19754	22239	26048	29519
45 - 49	14356	16367	19120	21610	25405
50 - 54	11144	13569	15554	18265	20750
55 - 59	8683	10229	12542	14476	17116
15 - 59	189920	220426	253946	286471	321675
60 - 64	6935	7618	9056	11205	13050
65 - 69	5324	5685	6320	7605	9526
70 - 74	3584	3965	4302	4860	5947
75 - 79	2036	2346	2647	2932	3385
80 - 84	907	1143	1346	1555	1765
85+	53	394	643	849	1049
60+	18839	21151	24314	29006	34722
Total	354400	397463	443932	493260	545553

Table 4.5c. Projected populations (000) for India upto 2001 (based on the 1981 census) : Females

Age group	Adjusted population, 1981	Projected population			
		1986	1991	1996	2001
0 - 4	49306	52892	56406	59586	62907
5 - 9	45428	46372	50205	54076	57655
10 - 14	40417	44819	45853	49758	53703
0 - 14	135151	144083	152464	163420	174265
15 - 19	34199	40037	44465	45564	49514
20 - 24	29120	33768	39617	44096	45272
25 - 29	25198	28701	33363	39237	43765
30 - 34	21337	24785	28302	32989	38888
35 - 39	18643	20919	24366	27909	32620
40 - 44	15715	18196	20478	23932	27499
45 - 49	13216	15228	17690	19987	23441
50 - 54	10477	12621	14607	17053	19361
55 - 59	8295	9744	11811	13769	16187
15 - 59	176200	203999	234699	264536	296547
60 - 64	6802	7391	8757	10726	12634
65 - 69	5395	5684	6250	7513	9335
70 - 74	3820	4100	4389	4919	6031
75 - 79	2267	2510	2753	3027	3488
80 - 84	1079	1230	1399	1585	1805
85+	66	425	630	796	965
60+	19429	21340	24178	28566	34258
Total	330780	369422	411341	456522	505070

Table 4.5d. Different population projections compared : India, 1981-2001
(Medium variant projections mainly based on 1981 census)

Agency/Author	Population (millions)				
	1981	1986	1991	1996	2001
1. Registrar General, India (1984)	685	758	836	915	991
2. UN Population Division (1984)	703*	-	845*	-	972*
3. UN ESCAP (Natrajan, 1982)	684	759	834	905	978
4. U.S. Census Bureau (1984)	700	779	865	955	1046
5. World Bank (Vu and Zachariah, 1983)	691*	-	853*	-	1017*
6. Guha Roy (Present estimates)	685	767	855	950	1050

* Estimates are linear interpolation of populations in 1980, 1990 and 2000.

Bank (Vu and Zacharish, 1983), the population of India is likely to cross one billion mark in about 2001. It can be generally said that meeting the Indian Planning Commissions target of reaching the replacement level fertility by the end of the present century is bleak indeed.

4.6 Implications of the projected population

According to our projections, the proportion of the aggregate population below age 15 is expected to decline from 43.3 per cent in 1971 (41.0 per cent in 1981) to 34.6 per cent in 2001. It will be appreciated that under the assumed future course, the moderate fall in fertility makes its effect felt in the pre-adult age range. On the other hand, the proportion of working age (15-59) population would increase by about 7 points during the period 1971-2001. Though the change in the proportion of old age (60+) population continues to be slow, increasing from a little less than 6 per cent in 1971 to a possible level of about 7 per cent in 2001, the absolute increase during 1971-2001 would be much higher than 30 million (Guha Roy, 1985). With the changes in the age structure as indicated by the projections, the reproductive stock of the population would also increase. This is the female population aged 15-44 whose proportion in the total population would increase from 20.7 per cent in 1971 to 22.6 per cent in 2001, adding 124 million females in the reproductive age range over the period.

Another important consideration of the implication of the projection is the child dependency ratio (P_{0-14} / P_{15-59}). This ratio, which was about 84 per cent in 1971 and 77 per cent in 1981, would probably drop to about 59 per cent in 2001. While in 1971 one 'worker' had to shoulder the burden of about one dependant (both young and old), in 2001 every three 'workers' would have two dependants to support.

4.7 Mathematical projection of state populations

The transitional age structure model, successfully applied to the national population, could not be used for the populations of the constituent States of India; the demographic disturbances were apparently too large in some of these States (e.g., imbalance caused by heavy immigration in Assam) to be cured by this technique. The State populations are not closed like the country as a whole and the magnitude of internal migration was not precisely known. In such a situation, the quadratic exponential growth curves,

$$P(t) = P(0) \exp (ut + ft^2),$$

used earlier for independent estimates of population, were fitted to the reported populations of the States and Union Territories at censuses 1961, 1971 and 1981 (for Assam the official estimate for 1981 was used) and the trend was extrapolated to 1991 and 2001. While the disaggregated State population extrapolations are not acceptable as their population estimates, the extrapolated values were used as relative weights to prorate them to the aggregated estimates just made above.

With t measured in years from 1971, the resulting curves for different units were derived as follows :

Male

States/Union Territories	Estimating equations $P(t) = P(0)\exp(ut + ft^2)$
1. Andhra Pradesh	$22009\exp(.019907t + .000086t^2)$
2. Assam	$7714\exp(.029572t + .000124t^2)$
3. Bihar	$28847\exp(.021582t + .000037t^2)$
4. Gujarat	$13803\exp(.024841t - .000099t^2)$
5. Haryana	$5377\exp(.026056t - .000170t^2)$
6. Himachal Pradesh	$1767\exp(.020130t + .000059t^2)$
7. Jammu & Kashmir	$2459\exp(.023952t - .000182t^2)$
8. Karnataka	$14972\exp(.022484t + .000086t^2)$
9. Kerala	$10588\exp(.020012t - .000339t^2)$
10. Madhya Pradesh	$21455\exp(.024105t - .000147t^2)$
11. Maharashtra	$26116\exp(.022931t - .000141t^2)$
12. Orissa	$11041\exp(.020609t - .000220t^2)$
13. Punjab	$7266\exp(.019323t + .000044t^2)$
14. Rajasthan	$13485\exp(.025981t + .000176t^2)$
15. Tamil Nadu	$20828\exp(.018331t - .000232t^2)$
16. Uttar Pradesh	$47016\exp(.021008t + .000152t^2)$
17. West Bengal	$23436\exp(.021341t - .000159t^2)$
18. Other States	$2140\exp(.030400t - .000056t^2)$
19. Union Territories	$3623\exp(.038709t - .000045t^2)$

Female

States/Union Territories	Estimating equations $P(t) = P(0) \exp(ut + ft^2)$
1. Andhra Pradesh	$21494 \exp(.019601t + .000100t^2)$
2. Assam	$6911 \exp(.031334t + .000004t^2)$
3. Bihar	$27506 \exp(.019183t + .000209t^2)$
4. Gujarat	$12895 \exp(.024970t - .000028t^2)$
5. Haryana	$4660 \exp(.026584t - .000104t^2)$
6. Himachal Pradesh	$1693 \exp(.021924t + .000028t^2)$
7. Jammu & Kashmir	$2158 \exp(.028107t + .000237t^2)$
8. Karnataka	$14327 \exp(.022692t + .000129t^2)$
9. Kerala	$10759 \exp(.020620t - .000223t^2)$
10. Madhya Pradesh	$20199 \exp(.023484t - .000089t^2)$
11. Maharashtra	$24296 \exp(.023105t - .000064t^2)$
12. Orissa	$10904 \exp(.019665t - .000182t^2)$
13. Punjab	$6285 \exp(.021174t + .000101t^2)$
14. Rajasthan	$12281 \exp(.026704t + .000217t^2)$
15. Tamil Nadu	$20371 \exp(.017631t - .000162t^2)$
16. Uttar Pradesh	$41325 \exp(.019762t + .000359t^2)$
17. West Bengal	$20876 \exp(.023192t - .000117t^2)$
18. Other States*	$2017 \exp(.029920t + .000015t^2)$
19. Union Territories	$3051 \exp(.038370t + .000047t^2)$

* "Other States" comprise smaller units like Manipur, Meghalaya, Nagaland, Tripura and Sikkim.

Tables 4.6 and 4.7 show relative population weights and estimated populations of the States in 1991 and 2001. The shifts in the geographic distribution of the national population result from the effects of the variations in population growth rates. These variations across regions or States occur due to differences in the components of growth, namely, births, deaths and migrations. It however takes a longer time period than has been considered here to find any changing distribution of population among the States. Thus Uttar Pradesh, Bihar, Maharashtra, West Bengal, Andhra Pradesh, Madhya Pradesh and Tamil Nadu remain the most populous States of India and maintain a fairly constant share of country's population of about 66-67 per cent during 1971-2001. However, there has been an indication that some population redistribution among the States may take place before the turn of the century. On the basis of the present estimation, Andhra Pradesh and Madhya Pradesh, which have now smaller aggregate population than West Bengal, are likely to equal or exceed the West Bengal's population in around 2001 due to their higher growth rates potential. Similarly, Rajasthan will be more populous State than Karnataka in the next decade if the present trend continues.

The change in the share (per cent) of zonal populations (excluding Union Territories) to total can be seen as follows :

Zone*	1971	1981	1991	2001
South	24.7	24.0	23.0	22.3
West	14.1	14.1	13.9	13.6
Central	23.7	23.8	24.6	26.3
East	25.9	25.8	25.6	25.6
North	10.5	10.8	11.3	12.2

* South : Andhra Pradesh, Karnataka, Kerala, Tamil Nadu.

West : Gujarat, Maharashtra.

Central : Madhya Pradesh, Uttar Pradesh.

East : Assam, Bihar, Orissa, West Bengal, Manipur, Meghalaya, Nagaland, Tripura, Sikkim.

North : Haryana, Punjab, Himachal Pradesh, Rajasthan, Jammu & Kashmir.

Table 4.6. Relative population weights and estimated population of the States and Union Territories (U.T.) : 1991 and 2001

Male

	States/Union Territories	Population weights (%)		Census 1981 population (million)	Estimated Population (million)	
		1991	2001		1991	2001
1.	Andhra Pradesh	7.7	7.9	27.11	34.18	43.10
2.	Assam	3.3	3.8	10.47*	14.65	20.73
3.	Bihar	10.3	10.4	35.93	45.72	56.75
4.	Gujarat	5.0	4.9	17.55	22.20	26.73
5.	Haryana	1.9	1.8	6.91	8.43	9.82
6.	Himachal Pradesh	0.6	0.6	2.17	2.66	3.27
7.	Jammu & Kashmir	0.8	0.8	3.17	3.55	4.36
8.	Karnataka	5.5	5.8	18.92	24.42	31.64
9.	Kerala	3.1	2.6	12.53	13.76	14.18
10.	Madhya Pradesh	7.5	7.1	26.89	33.29	38.73
11.	Maharashtra	8.9	8.4	32.41	39.51	45.83
12.	Orissa	3.5	3.1	13.31	15.54	16.91
13.	Punjab	2.5	2.5	8.94	11.10	13.64
14.	Rajasthan	5.5	6.3	17.85	24.42	34.37
15.	Tamil Nadu	6.2	5.4	24.49	27.52	29.46
16.	Uttar Pradesh	17.3	18.6	58.82	76.81	101.48
17.	West Bengal	7.7	7.1	28.56	34.18	38.73
18.	Other States	0.9	0.9	3.04	4.00	4.91
19.	Union Territories	1.8	2.0	5.32	7.99	10.91
	India			354.40	443.93	545.[illegible]

* Official estimate (No census could be taken in Assam in 1981).

Table 4.7. Relative population weights and estimated population of the States and Union Territories (U.T.) : 1991 and 2001

Female

	States/Union Territories	Population weights (%)		Census 1981 population (million)	Estimated Population (million)	
		1991	2001		1991	2001
1.	Andhra Pradesh	7.9	7.8	26.44	32.50	39.40
2.	Assam	3.1	3.3	9.43*	12.75	16.67
3.	Bihar	10.4	10.8	33.98	42.78	54.54
4.	Gujarat	5.0	4.9	16.54	20.57	24.75
5.	Haryana	1.8	1.7	6.01	7.40	8.59
6.	Himachal Pradesh	0.6	0.6	2.11	2.47	3.03
7.	Jammu & Kashmir	1.0	1.1	2.82	4.11	5.56
8.	Karnataka	5.6	5.8	18.22	23.04	29.29
9.	Kerala	3.5	3.0	12.92	14.40	15.15
10.	Madhya Pradesh	7.4	6.9	25.29	30.44	34.85
11.	Maharashtra	8.9	8.5	30.37	36.61	42.93
12.	Orissa	3.6	3.1	13.06	14.81	15.66
13.	Punjab	2.4	2.4	7.85	9.87	12.12
14.	Rajasthan	5.4	6.1	16.41	22.21	30.81
15.	Tamil Nadu	6.5	5.5	23.92	26.74	27.78
16.	Uttar Pradesh	16.9	18.9	52.04	69.51	95.45
17.	West Bengal	7.5	6.9	26.02	30.85	34.85
18.	Other States	0.9	0.9	2.86	3.70	4.55
19.	Union Territories	1.6	1.8	4.49	6.58	9.09
	India			330.78	411.34	505.07

* Official estimate (No census could be taken in Assam in 1981).

Regional redistribution of population being a long-run process, the time pattern of shifts of population from one zone to another would not be fully reflected over a short period (1971-2001) considered. It is however indicated that the South zone will have a steady decline in its share of national population, while the North and Central zones will experience a steady increase in their population shares. The East and West zones, after a somewhat static positions initially, may show a slow decline in the proportions of the country's population. These changes are mostly the results of changing rates of natural increase and net migration due mainly to regional differential in economic opportunities.

4.9 Estimated population in individual ages and years

In development planning, the educational variable plays an important role, and is used for a number of different concepts such as " an index of social development", " a measure of receptiveness to different sources of information", " a representative of the influence of modernization and urbanization", and so on. It is a general experience that investment in education often pays off better in providing skilled manpower required for social and economic development of a country. The national plans, therefore, aim at providing universal education, in stages, in the age ranges from which primary and secondary students are drawn. The need for estimation of student population (approximately 5-19 years) in individual ages is obvious enough for such educational planning.

The estimated populations given in 5-year age-intervals can be broken up into single ages within the range 5-19 by application of a structural graduation model (Das Gupta et al., 1965 and 1971) :

$$\log {}^{T}P_{x} = a(T) + b(T) \cdot x + c(T) \cdot x^2$$

where T = years of estimation (e.g. 1971, 1981, 2001),

x = individual ages, and

a, b and c = parameters.

The model assumes steady growth and survivorship conditions for the distribution of population in the late childhood and younger adult age. The basic relationship between populations at ages 0 and x, with average growth rate g and survival ratio ${}_xS_0$, given by

$$P_x \cdot e^{gx} = P_0 \cdot {}_xS_0$$

or

$$\log_e P_x = \log_e P_0 - gx + \log_e {}_xS_0$$

can be expanded by Euler-Maclaurin Theorem in which higher order derivatives can be neglected in good approximation. This expansion gives the above quadratic expression of age x for the logarithm of population at age x.

The values of the parameters a(T), b(T) and c(T) for any year T may be obtained from the populations at the individual ages 7, 12 and 17 (approximate mid-points of age groups 5-9, 10-14 and 15-19) for the year. The populations at the 5-year age groups are taken as five times the populations at these individual ages. The errors introduced in this calculation are substantively adjusted by prorating the estimated individual age populations by the given 5-year grouped age data.

The favourable characteristics of the structural model are that it is simple to apply with fewer number of basic data; it is relatively less sensitive to the errors in the data; and it is flexibile to use for intervals of any length. The Sprague's six-term fifth-difference osculatory interpolation formula, on the other hand, allows a continuity between consecutive sets of five single-year estimates. This continuity can be achieved by the structural model only at the expense of a certain decrease in efficiency of the estimates (Das Gupta, et al., 1971).

The values of the parameters a(T), b(T) and c(T) for the years 1971, 1981 and 2001 are shown in Table 4.8. It appears that the parametric values for males and females are very often sufficiently close to permit the use of one set of values for both sexes. We however use here both sets for the purpose of disaggregation of population.

Table 4.8. Values of individual age parameters

Parameter	Sex	Year (T)		
		1971	1981	2001
a(T)	male	3.90206	3.97946	4.09785
	female	3.87835	3.93460	4.06187
b(T)	male	-.01528	-.00342	-0.00519
	female	-.01531	-.00046	-0.00572
c(T)	male	.00020	-.00057	-0.00012
	female	.00019	-.00068	-0.00009

The single age populations in the 5-19 range, for the census years 1971 and 1981 and projection year 2001, are derived by both structural graduation model and Sprague's formula. These are shown in Table 4.9, separately for sexes. On the basis of comparison between the two methods, it appears that their efficiency is approximately of the same order.

4.9 Size of student population under full enrolment

On the basis of our estimates, the student age (approximately 5-19) population would rise from 200.7 million in 1971 to 251.3 million in 1981 and 336.2 million in 2001. The share of the school age stratum in the total population of India seems to be more or less static at around 36 per cent in the decade 1971-81, but is likely to decline to 32 per cent by 2001 (Table 4.10). The arrested growth rate of population (decadal growth rates for 1961-71 and 1971-81 being almost same : Census 1981) caused by slightly higher decline in fertility than in mortality in recent times is likely to reduce the high proportion of the youthful population. The average annual growth rate (about 2.5 per cent) of the total population in the decade 1971-81, as given by the Census, is about the same as the school-age population.

Between 1971 and 1981, the populations in age ranges 6-13 (roughly corresponding to elementary stage : I-VIII) and 14-17 (roughly corresponding

Table 4.9. Individual age population (000), Aged 5-19 : 1971, 1981 and 2001

Age	1971		1981		2001	
	Structural model	Sprague formula	Structural model	Sprague formula	Structural model	Sprague formula
(a) Male						
5	8561	8599	9663	9613	12822	12824
6	8253	8263	9626	9606	12679	12680
7	7964	7955	9562	9577	12532	12531
8	7693	7674	9475	9502	12380	12379
9	7436	7416	9364	9392	12223	12222
10	7198	7180	9226	9251	12061	12060
11	6971	6961	9069	9083	11896	11895
12	6759	6759	8892	8892	11727	11727
13	6558	6568	8695	8681	11554	11554
14	6370	6388	8481	8456	11376	11378
15	6193	6221	8244	8217	11195	11203
16	6027	6067	7999	7966	11014	11037
17	5869	5900	7739	7719	10828	10851
18	5721	5706	7469	7483	10640	10633
19	5582	5498	7189	7255	10448	10401
5 - 19	103155		130693		175375	
(b) Female						
5	8106	8143	8657	8834	11831	11829
6	7815	7824	8638	8682	11684	11684
7	7541	7532	8602	8572	11534	11534
8	7283	7265	8580	8491	11381	11382
9	7040	7021	8531	8429	11225	11226
10	6813	6796	8480	8390	11065	11068
11	6597	6587	8379	8377	10905	10906
12	6393	6394	8253	8309	10743	10742
13	6202	6211	8105	8148	10578	10577
14	6021	6038	7934	7927	10412	10410
15	5851	5877	7737	7716	10244	10246
16	5691	5729	7527	7506	10074	10090
17	5539	5569	7299	7287	9904	9920
18	5396	5382	7057	7067	9732	9729
19	5261	5181	6801	6845	9560	9529
5 - 19	97549		120580		160872	

Table 4.10. School-age population : 1971, 1981 and 2001

A. Share (per cent) of school-age (5-19) stratum in the total population

Year	Male	Female	Both sexes
1971	36.3	36.9	36.6
1981	36.5	36.4	36.5
2001	32.1	31.9	32.0

B. School-age population (000) by educational level

(i) Elementary stage (Classes I-VIII) : 6-13 years

Year	Male	Female	Both sexes
1971	58832 (57.0)	55684 (57.1)	114516 (57.1)
1981	73909 (56.6)	67568 (56.0)	141477 (56.3)
2001	97052 (55.3)	89115 (55.4)	186167 (55.4)

(ii) Secondary stage (classes IX - XII) 14-17 years

Year	Male	Female	Both sexes
1971	24459 (23.7)	23102 (23.7)	47561 (23.7)
1981	32463 (24.8)	30436 (25.2)	62899 (25.0)
2001	44413 (25.3)	40634 (25.3)	85047 (25.3)

[Figures in the brackets indicate the per cent of respective age range populations in the total school-age population (5-19 years). The efficiency of structural model and Sprague-method being almost equal disaggregated populations were taken here from the former.]

to secondary stage : IX-XII) seem to have increased annually on the average by 27 and 15 millions respectively (Table 4.10). These rates of increase in the period 1981-2001 are however likely to be only 2.2 and 1.1 millions respectively. On the other hand their respective proportions in the total school-age population (5-19 years) would likely to decrease for the age range 6-13 but increase for the age range 14-17 during the period 1971-2001.

The implications of extending free and compulsory elementary education upto age 14, as contemplated under the Sixth and Seventh Five Year Plans, merit attention. Going by the enrolment rates of 82.1 per cent for boys and 51.2 per cent for girls in 1976-77 (Ministry of Health and Family Welfare, Year Book, 1978-79, Table C.5, p. 65) at elementary level (age 6-13), the enrolment figures (including repeaters) for boys and girls in 1981 would be 61 and 35 millions respectively. According to our projection, the number of enrolled boys and girls in elementary level (Classes I-VIII) is expected to be 80 and 46 millions respectively in 2001, if the same rates of enrolment prevail. Under full enrolment upto age 14, the additional volume of students to be covered in 2001 would be more than 17 million boys and 43 million girls. Almost twice as much arrangements as at present will possibly have to be made in elementary schools for ensuring female education similar to males. The school age population sex ratio (males per 100 females) seems to increase steadily from 106 at census 1971 base to 108 in 1981 and to 109 in 2001. The enrolment statistics however may not be meaningful as the dropout rate in primary education is estimated to be more than 60 per cent. The recent spurt in overall enrolment has thus been negated by the high dropout rates. Without a significant improvement in economic conditions, the dropout rate will probably remain high, for children (girls get married early or work at home) will have to work either to earn a living for themselves or to supplement their parents' meagre income.

APPENDIX TO CHAPTER FOUR

A FORTRAN IV PROGRAMME TO ESTIMATE THE PARAMETERS
A, H, B, AND C IN THE MODEL
Y = A + H * X + B * C ** X

```
      DOUBLE PRECISION X(10), Y(10), WX(10), ZX(10), A(4, 4) ,
      B(4), Z(4), YY(10), T(4, 10), YZ(10)
      DOUBLE PRECISION AO, BO, CO, HO, A1, B1, C1, H1, LO, L1, EPS
      DOUBLE PRECISION CWX(10), CYY(10), CY(10)
      COMMON A, B/NORM/T/SOL/Z
      EQUIVALENCE (WX, YY, YZ)
      INTEGER CR
      CR = 5
      LP = 6
      EPS = .1D - 3

999   READ (CR, 1, END = 400)N, (X(I), Y(I), I = 1, N)
      WRITE (LP, 16)
      WRITE (LP, 2)
      WRITE (LP, 3) (X(I), Y(I), I = 1, N)
      NN = N - 2
      DO 20     I = 1, NN
      ZX(I) = Y(I + 2) - 2 * Y (I * 1) + Y(I)
      IF (ZX(I) . LT. 0) ZX (I) = -ZX(I)

20    WX(I) = DLOG (ZX(I))
      WRITE (LP, 17)
      WRITE (LP, 10)
      WRITE (LP, 5) (ZX(I), WX(I), I = 1, NN)
      DO 30   J = 1, NN
      T (1, J) = 1
      T (2, J) = X(J)
      T (3, J) = 0

30    T(4, J) = 0
      CALL NORMAL (WX, NN, 2)
```

```
      WRITE (LP, 18)
      DO 35 I = 1, 4
35    WRITE (LP, 11) I, (A(I, J), J = 1, 4), I, B(I)
      CALL SOLVE (2, M)
      IF (M.EQ.0) GO TO 100
      WRITE (LP, 19)
      WRITE (LP, 15) Z(1), Z(2)
      CO = DEXP (Z(2))
      BO = DEXP (Z(1))/(CO - 1)** 2
      WRITE (LP, 6) CO, BO
      WRITE (LP, 37)
      DO 38 I = 1, NN
      CWX(I) = Z(1) + Z(2)* X(I)
38    WRITE (LP, 39) WX(I), CWX(I)
      DO 40 I = 1, N
40    YY(I) = Y(I) - BO*CO**X(I)
      WRITE (LP, 4)
      WRITE (LP, 41)
      WRITE (LP, 42) (YY(I), I = 1, N)
      DO 45 J = 1, N
      T(1, J) = 1
      T(2, J) = X(J)
      T(3, J) = 0
45    T(4, J) = 0
      CALL NORMAL (YY, N, 2)
      WRITE (LP, 65)
      DO 48 I = 1, 4
48    WRITE (LP, 11) I, (A(I, J), J = 1, 4), I, B(I)
      CALL SOLVE (2, M)
      IF (M.EQ.0) GO TO 100
```

```
      AO =  Z(1)
      HO =  Z(2)
      WRITE (LP, 19)
      WRITE (LP, 13)  AO, HO
      WRITE (LP, 51)
      LO =  0
      DO  50  I =  1, N
      CYY(I) =  AO + HO*X(I)
      CY(I) =  AO + HO*X(I) + BO*CO**X(I)
      WRITE (LP, 79) YY(I), CYY(I), Y(I), CY(I)
      YZ(I) =  Y(I) - CY(I)

50    LO =  LO + YZ(I)**2
      WRITE (LP, 12) LO

99    DO  55 J =  1, N
      T(1, J) =  1
      T(2, J) =  X(J)
      T(3, J) =  CO**X(J)

55    T(4, J) =  X(J)*BO*CO**(X(J) - 1)
      CALL NORMAL (YZ, N, 4)
      WRITE (LP, 67)
      DO  58  I =  1, 4

58    WRITE (LP, 11) I, (A(I, J), J =  1, 4), I, B(I)
      CALL SOLVE (4, M)
      IF (M.EQ.0)  GO  TO  100
      WRITE (LP, 19)
      DO  59  I =  1, 4

59    WRITE (LP, 61) I, Z(I)
      A1 =  AO + Z(1)
      H1 =  HO + Z(2)
      B1 =  BO + Z(3)
      C1 =  CO + Z(4)
```

```
      WRITE (LP, 14) A1, H1, B1, C1
      WRITE (LP, 64)
      L1 = 0
      DO 60 I = 1, N
      CY(I) = A1 + H1*X(I) + B1 * C1 ** X(I)
      WRITE (LP, 63) Y(I), CY(I)
      YZ(I) = Y(I) - CY(I)

 60   L1 = L1 + YZ(I)**2
      WRITE (LP, 62) L1
      IF (L1.GT.LO) GO TO 200
      IF (LO - L1.LT.EPS) GO TO 300
      AO = A1
      LO = L1
      HO = H1
      BO = B1
      CO = C1
      GO TO 99

100   WRITE (LP, 7)
      GO TO 999

200   WRITE (LP, 8)
      WRITE (LP, 69) AO, HO, BO, CO, LO
      GO TO 999

300   WRITE (LP, 9) L1, A1, H1, B1, C1
      GO TO 999
    1 FORMAT (I5/(D9.2, D15.7))
    2 FORMAT (///, 24X, 'X', 25X, 'Y')
    3 FORMAT (20X, D10.2, 10X, D20.7)
    4 FORMAT (1H1)
    5 FORMAT (20X, D20.8, 10X, D20.8)
    6 FORMAT (/// 10X, 'HENCE  CO = ', D20.12, 5X, 'BO = ', D20.12)
    7 FORMAT (/ 20X, 'NORMAL EQUATIONS CANNOT BE SOLVED')
    8 FORMAT (// 20X, 'L1 > LO')
```

```
 9  FORMAT (/// 20X, 'L = ', D20.12, 5X, 'A = ', D20.12, 5X,
   1'H = ', D20.12, *// 20X, 'B = ', D20.12, 'C = ', D20.12)
10  FORMAT (30X, 'ZX(I)', 26X, 'WX(I)')
11  FORMAT (// 20X, 'A(', I1', J)', 5X, 4D14.5, 5X,
   1'B (', I1', ) = ', D15.8)
12  FORMAT (//// 25X, 'LO = ', D20.12)
13  FORMAT (//// 20X, 'AO = ', D20.12, 10X, 'HO = ', D20.12)
14  FORMAT (// 10X, 'HENCE   A1 = ', D20.12, 15X,
   1'H1 = ', D20.12  // 20X, 'B1 =  ', D20.12, 15X,
   2'C1 = ', D20.12)
15  FORMAT (/// 20X, 'Z(I) =  ', D20,12, 5X, 'Z(2) =  ', D20.12)
16  FORMAT (1H1, 10X, 'THE GIVEN DATA IS'/10X, 17 (' — '))
17  FORMAT (1H1, 10X, 'ZX DENOTE THE SECOND
   1 DIFFERENCES OF Y AND WX(I) =  DLOG (ZX(I))')
18  FORMAT (// 10X, 'THE SYSTEM OF NORMAL EQUATIONS
   1 OF WX(I) =  DLOG (B*(C - 1) ** 2) + X(I) *
   2 DLOG(C)  IS GIVEN BY : ')
19  FORMAT (/// 10X, 'SOLUTION OF THE ABOVE SYSTEM IS')
37  FORMAT (1H1, 30X, 'WX(I)', 30X, 'CWX(I)')
39  FORMAT (20x, D20.12, 10X, D20.12)
41  FORMAT (20X, 'YY(I) =  Y(I) - BO * CO ** X(I)')
42  FORMAT (20X, D20.12)
51  FORMAT (1H1, 20X, 'YY(I)', 25X, 'CYY(I)', 25X, 'Y(I)',
   1 25X, 'CY(I)')
61  FORMAT (// 20X, 'Z(', I1, ') = ', D20.12)
62  FORMAT (//// 25X, 'L1 = ', D20.12)
63  FORMAT (20X, D20.12, 20X, D20.12)
64  FORMAT (1H1, 30X, 'Y(I)', 30X, 'CY(I)')
65  FORMAT (// 10X, 'THE SYSTEM OF NORMAL EQUATIONS
   1 OF YY(I) =  A + H * X(I)  IS GIVEN BY :')
67  FORMAT (//// 10X, 'THE SYSTEM OF NORMAL EQUATIONS
   1 OF  YZ(I) =  DEL1 + DEL 2 * X(I) + DEL 3 * CO ** X(I) +
   2 DEL 4 * X(I) * BO * CO ** (X(I) - 1)  IS GIVEN BY :')
69  FORMAT (// 10X, 'HENCE  A = ', D20.12, 15X, 'H = ',
   1 D20.12 // 20X, 'B = ', D20.12, 15X, 'C = ', D20.12 //
   2 10X, 'AND', 7X, 'L =  ', D20.12)
```

```
79   FORMAT (15X, D20.12, 10X, D20.12, 10X, D20.12,
     1  10X, D20.12)

400  STOP
     END

     SUBROUTINE NORMAL (YZ, L, M)
     COMMON A, B
     COMMON/NORM/T
     DOUBLE PRECISION A(4, 4), B(4), T(4, 10), YZ(10)
     DO 10 I = 1, 4
     B(I) = 0
     DO 10 J = 1, 4

10   A (I, J) = 0
     DO 15 I = 1, M
     DO 15 J = 1, L

15   B(I) = B(I) + YZ(J) * T(I, J)
     DO 25 I = 1, M
     DO 25 J = 1, M
     DO 25 K = 1, L

25   A(I, J) = A(I, J) + T(I, K)* T(J, K)
     RETURN
     END

     SUBROUTINE SOLVE (N, M)
     COMMON A, B
     COMMON/SOL/X
     DOUBLE PRECISION A(4, 4), B(4), X(4)
     DO 1 I = 1, N
     IF (A(I, I).EQ.0) CALL CHANGE (N, I, M)
     IF (M.EQ.0) RETURN

1    CONTINUE
     N1 = N - 1
```

```
      DO 20 K = 1,' N1
      K1 = K + 1
      DO 20 I = K1,' N
      IF (A(I,' K)) 10, 20, 10
10    B(I) = B(I) - A(I,' K) * B(K)/A(K,' K)
      DO 20 J = K1,' N
      A(I,' J) = A(I,' J) - A(I,' K)/A(K,' K) * A(K,' J)
20    CONTINUE
      DO 25 I = 1,' N
      IF (A(I,' I).NE.0) GO TO 25
      M = 0
      RETURN
25    CONTINUE
      M = 1
      X(N) = B(N)/A(N,' N)
      DO 40 I = 1,' N1
      K = N - I
      L = K + 1
      DO 30 J = L,' N
30    B(K) = B(K) - X(J) * A(K,' J)
40    X(K) = B(K)/A(K,' K)
      RETURN
      END
      SUBROUTINE CHANGE (N,' K, M)
      COMMON A,' B
      DOUBLE PRECISION A(4,' 4), B(4),' BIG
      BIG = DABS (A(1,' K))
      JJ = 1
      DO 5 J = 1,' N
      IF (J.EQ.K) GO TO 5
```

```
      IF (A(J, K) . EQ . 0) GO TO 5
      IF (A(K, J) . EQ . 0) GO TO 5
      IF (DABS (A(J, K)) . LE . BIG) GO TO 5
      BIG = DABS (A(J, K))
      JJ = J
 5    CONTINUE
      J = JJ
      IF (BIG.NE.0) GO TO 10
      M = 0
      RETURN
10    DO 11 I = 1, N
      TEMP = A(K, I)
      A(K, I) = A(J, I)
11    A(J, I) = TEMP
      TEMP = B(K)
      B(K) = B(J)
      B(J) = TEMP
      RETURN
      END
```

CHATER FIVE

FEMALE NUPTIALITY IN INDIA : A RECONSTRUCTION

Inadequacy of census data on marital status, whether through unavailability or unknown reliability is the crux of the problem. The fact base may be improved through demographic surveys. A reasonable accuracy in the available data may be achieved by using a logit-linear model.

A structural graduation formula appears to provide a fairly good approximation of the schedules of female proportions single and a set of short-run projections (1981-86). The proportions ever-married estimated by Coale's three-parameter model are being matched with the proportions indirectly obtained from the estimated proportions single. From the combined estimates of widowed-divorced-separated, a consistent set of estimates of female proportions married is derived. Using a computer model, the risk of widowhood is also worked out.

A new index termed " Sexual Dimorphism Percentage" is used to indicate availability of marriage partners.

5.1 Introduction

The study describes the national marriage pattern changes which have occurred. An attempt is made to provide a perspective on the future of Indian nuptiality, including some forward estimates. The subject is treated demographically rather than linking it with the matrix of social development. It is however true that the shifting patterns of marriage is in interaction with the changing socio-cultural environment of the population. Despite this, it is incumbent upon us in this study to confine ourselves to the arithmetic of demographic measurement.

The general framework for marriage in this country is characterised as follows : (a) marriage mostly pre-arranged by parents; (b) a dowry paid to the bridegroom's family (in some lower racial-cultural communities, brides are bought in cash or kind); (c) one wife (however, the prevalence of polygamy is found in some minor sections of the population); (d) permanent union; (e) early and universal marriage with very low celibacy rates; and (f) incidence of remarriage insignificant among female population. It is argued that the rise in the female age at marriage and the increase in proportions single in teen ages during the recent times seem to be the result of urbanization, the marginal rise in female education and the greater female participation in labour force outside agriculture.

5.2 Proportions single

Proportion single is an important indicator of the extent of first marriages in a population. These proportions at the younger ages indicate the tempo of recent nuptiality, and those still single at older ages show the prevalence of lifelong non-marriage (Smith, 1978a). The Indian censuses provide marital status data cross-classified by sex and age. But the data are often defective and may be quite seriously distorted. In some cases the ratios of proportions single in successive age groups in two consecutive censuses exceed unity because of faulty data (or selective mortality or migration). This happens, for example, between age groups 25-29 in 1941 and 35-39 in 1951, and similarly between 30-34 and 40-44, 35-39 and 45-49, and 40-44 and 50-54. Bias in age reporting may affect the proportions single at different ages. The observed distributions of proportions single are subject to fluctuations that may conceal the true nature of the phenomenon to a large extent. In the present study, we try to smooth out the inconsistent data as much as the proposed logit linear model would permit.

5.2.1 Developing model schedules of female proportions single

The logit of the proportion p $(0 < p < 1)$ is defined as logit $p = 0.5 \log_e p/(1 - p)$. (In demographic applications the factor 0.5 is customary, but it is usually not used by Statisticians). Our interest focuses on the proportions single, $S(a)$, at age a so that p is replaced by $S(a)$ in this analysis. The graph of logit $S(a)$ against a, though more linear than the graph of $S(a)$ against a, is still not linear enough. So we use the following " relational" form equation instead. When several schedules of proportions single can be related linearly to the same function $S_s(a)$, this common function may be regarded as a standard schedule. Proportion single schedules have been recorded in many countries for a number of censuses, making it possible to search for underlying common pattern. We can thus find $S_s(a)$ such that

$$\text{logit } S(a) = \alpha + \beta \ \text{logit } S_s(a) + e$$

or

$$P(a) = \alpha + \beta \ P_s(a) + e$$

where $P(a) = $ logit $S(a)$, $P_s(a) = $ logit $S_s(a)$ and e is the error term, the amount by which any individual $P(a)$ may fall off the line implied by the model. Now the linear fit is better. Then α and β give a two-parameter summary of the shape of the $S(a)$ curve. Subject to an appropriate choice of $S_s(a)$, all of the temporal variation may be described by the trajectory of the parameters α and β. By analogy with Brass relational mortality model we see that α varies directly with singulate mean age at marriage (SMAM); β varies inversely with the dispersion around the SMAM.

The principle underlying the logit system originates from bio-assay, and has been used and described extensively for mortality and postpartum variables (Brass, 1971; 1975; Hill and Trussell, 1977; and Lesthaeghe and Page, 1980). Only a brief description of the analogous system is therefore in order. At a given age a, each woman may be single or not. The proportion of women remaining in the original state was designated as $S(a)$ for each age a. The shape of $S(a)$ curve may often be described by a function similar to a logistic (Hyrenius et al., 1967). The derivation of the logit transformation from the underlying logistic distribution has been made elsewhere (Hill and Trussell, 1977). This transformation has the useful property of preserving (when transformed back) the end points 0 and 1, and is therefore applicable to distributions with these end points (Brass, 1971; 1975; and Hill and Trussell, 1977). It should however be noted that the logit fit may not be very good if nuptiality is changing rapidly over time. For example, if age at marriage oscillates, as it did during World War 2 in many countries, the cross-sectional proportions single by age could show some strange patterns.

We considered least squares procedures adequate for our purpose of estimating the parameters in the above model. Since the tails of the distribution are sometimes prone to reporting errors, a more robust procedure of estimation (Brass, 1971; 1975; and Lesthaeghe and Page, 1980), considering only the central portion of the P(a) schedule in the age range 20 to 44, was also used in a few cases on a trial basis. This involves splitting suitably the observations into two portions, A and B. Designating the mean values for each portion as $\overline{P}_A$ and $\overline{P}_B$ for the observed schedule and $\overline{P}_{S,A}$ and $\overline{P}_{S,B}$ for the standard schedule, the estimates for α and β are as follows :

$$\beta = (\overline{P}_B - \overline{P}_A)/(\overline{P}_{S,B} - \overline{P}_{S,A})$$

$$\alpha = \overline{P}_A - \beta \overline{P}_{S,A}$$

Since the estimates by the two methods did not differ much ultimately, we persisted with the least square fitting procedure applied to observed data for age range excluding 0-4, 5-9 and 50+. Yet another alternative procedure is to use median regressions (Barclay, et al., 1976). But the number of data points, as given by the age groups for which P(a) is reported, being small, this method is difficult to apply.

An approach similar to the one adopted for postpartum variables by Lesthaeghe and Page (1980) in the development of a standard schedule may be adopted. The procedure is as follows : An observed schedule of proportions single for a population is first selected. The data are then smoothed by three-point moving averages, and the resulting schedule is converted to logits. Linear relations emerge when logits of other schedules are plotted against this preliminary standard, which is further smoothed graphically to get a standard without irregularities. Since the relational form equation used is not, in practice, exact, the estimates are likely to be affected by the choice of standard. Recently, Thapa and Retherford (1982) had shown divergence of indirect estimates (by Brass method) of infant mortality based on different standard life tables. In the case of nuptiality, it may also be useful to try at least two standards instead of just one to see if the results came out about the same.

5.2.2 Application to Indian nuptiality

The general framework for marriage in India being characterised by early and almost universal marriage with very low celibacy rates, S(a) attain unity near age 10, and 0 around age 50. If we include those Indian "marriages" that are formal but unconsummated child marriages then the assumption of S(a) = 1 around a = 10 is possibly not realistic. In our formulation we therefore consider the transition from single state to only effective (that is, women attaining reproductive age) marriage state.

In the present analysis, we considered eight observed schedules for the period 1901-71. More or less uniform definitions have been adopted in the classification of population by marital status in different censuses of India (Shrivastava, 1971). This ensures the comparability of data, and thus provides an analytic basis for study of nuptiality trend. However, only smoothed data for 1931 census were available, and Agarwala (1962) noted a bias in this census caused by smoothing. We had not explicitly used this schedule for building up a standard, but the result derived from the logit relational procedure for 1931 should be interpreted with caution.

We had chosen two standard schedules reflecting Indian nuptiality patterns, rather than relying on any "universal" standard. One (designated as Standard I) is based on the Census 1961, as it is thought to be better organized and nuptiality pattern, as broadly indicated by mean age at marriage, did not change very significantly (as in 1971) at this point. The other (designated as Standard II) is taken as a sort of average of 1901-71 reported schedules to eliminate the peculiarities of the individual years (Stoto, 1981). The standard schedules (Table 5.1), derived from the logit system, describe the essence of observed ones (Table 5.2).

Preliminary examination reveals the existence of fairly strong linear relations between the observed and the standard. Even for earlier periods in which data irregularities are more likely, these relations hold quite fairly. Figure 5.1 illustrates the relations between the logits of a pair of observed schedules at an early (1901) and a recent (1971) census dates and that of a standard schedule, and compared them with the lines represented by the model derived later. This figure is complemented with the Table 5.3 that gives differences between observed and fitted proportions single.

Table 5.1. Standard schedules of proportions single

Age (a)	$s_s(a)$	
	Standard I	Standard II
< 10	1.000	1.000
10 - 14	0.695	0.633
15 - 19	0.395	0.315
20 - 24	0.110	0.096
25 - 29	0.013	0.028
30 - 34	0.010	0.016
35 - 39	0.008	0.012
40 - 44	0.006	0.010
45 - 49	0.005	0.009

Table 5.2. Observed schedule of proportions single

Age (a)	1901	1911	1921	1931	1941	1951	1961	1971
10 - 14	0.543	0.555	0.601	0.493	0.755	0.827	0.805	0.881
15 - 19	0.156	0.163	0.188	0.166	0.252	0.280	0.292	0.429
20 - 24	0.042	0.044	0.052	0.042	0.041	0.065	0.060	0.091
25 - 29	0.025	0.022	0.025	0.018	0.014	0.042	0.019	0.020
30 - 34	0.020	0.017	0.019	0.013	0.009	0.018	0.010	0.009
35 - 39	0.018	0.014	0.015	0.011	0.009	0.017	0.007	0.005
40 - 44	0.013	0.013	0.014	0.009	0.009	0.015	0.006	0.005
45 - 49	0.011	0.011	0.013	0.008	0.009	0.011	0.005	0.004

Source : Official census reports.

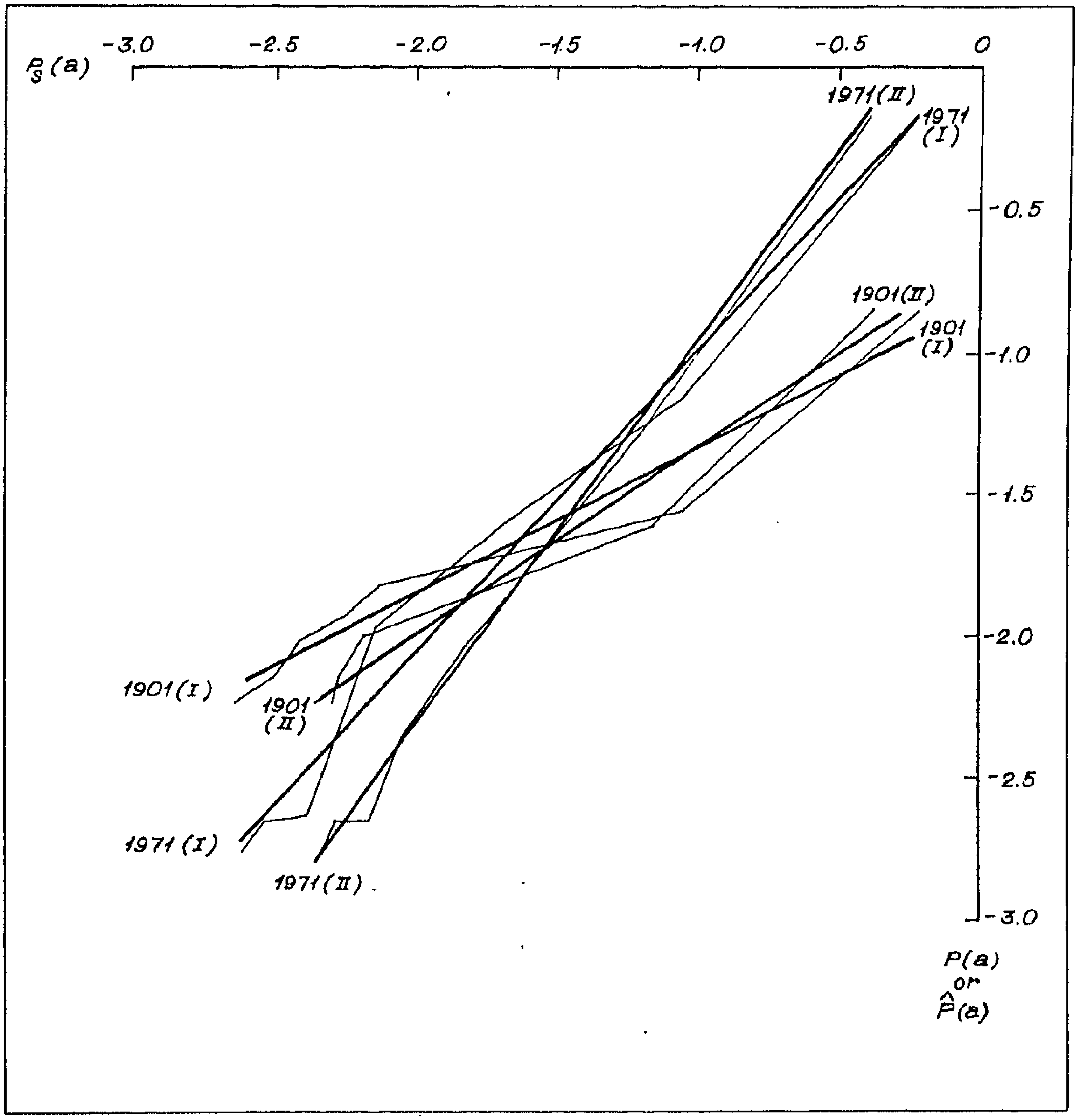


Figure 5.1 : Proportions single schedules after conversion to logits : relationships between (i) Observed (1901 & 1971) and Standards I and II (thin lines) and (ii) Model and Standards I and II (thick lines)

Table 5.3. Age specific differences between observed and fitted proportions single

Age (a)	Differences between percentages single, S(a) - $\hat{S}$ (a)							
	1901	1911	1921	1931	1941	1951	1961	1971
				Standard I				
15 - 19	2.3	2.5	-0.5	2.2	-6.9	3.6	2.4	1.2
20 - 24	-1.9	-1.7	-1.8	-1.4	-5.4	-3.2	-1.9	-1.8
25 - 29	0.4	0.3	0.3	0.3	0.0	1.8	0.7	0.8
30 - 34	0.2	0.1	0.1	0.0	-0.2	-0.2	0.0	0.0
35 - 39	0.2	-0.1	-0.1	0.0	0.0	-0.1	-0.1	-0.2
40 - 44	-0.1	0.0	0.0	0.0	0.2	0.0	0.0	0.0
45 - 49	-0.2	0.0	0.0	0.0	0.3	-0.2	0.0	0.0
M.S.E*	0.011	0.006	0.006	0.005	0.044	0.020	0.012	0.016
				Standard II				
15 - 19	2.1	2.3	2.4	2.0	5.8	2.7	1.7	0.1
20 - 24	-1.5	-1.2	-1.3	-1.0	-2.7	-2.5	-1.1	-0.5
25 - 29	-0.1	-0.2	-0.2	-0.1	-1.0	1.0	0.1	0.2
30 - 34	0.2	0.1	0.1	0.0	-0.6	-0.2	0.0	0.1
35 - 39	0.3	0.0	0.0	0.1	-0.4	0.1	0.0	-0.1
40 - 44	0.0	0.1	0.1	0.0	-0.2	0.2	0.0	0.1
45 - 49	-0.2	0.0	0.0	0.0	-0.1	-0.2	0.0	0.0
M.S.E*	0.007	0.005	0.003	0.003	0.020	0.010	0.002	0.002

* Mean Square Error = $\sum_{a=1}^{n}$ (logit S(a) - logit $\hat{S}$ (a))^2/n,

where age group index (a) 1 stands for 15-19, 2 for 20-24, etc.

An examination of the deviations of observed from fitted provides a means of testing for the presence of errors in the original data or in the fit. In absolute terms, the deviations are not large, and show some systematic patterns. Larger deviations occur almost uniformly in the younger age range (around 20) than above it. Substantial misreporting in the population age distributions have been a common feature in India (United Nations, 1967), and is likely to affect the distribution by age and marital status. The common tendency to understate age by single, but marriageable females, probably account for a large proportion of the deviations in the younger age range.

The mean square error of the regression is computed (Table 5.3) as a measure of goodness of fit of the model to the reported values. Since the statistic does not attain a value of zero, the fit is obviously not perfect. In the absence of an objective criterion, it is difficult to infer on the nature of the fit, but following Coale and Trussell (1978) we consider a value of 0.005 as a " mediocre " fit. On this basis, the Standard II schedule, as expected, shows a far closer fit than is the case with Standard I. Even for the former, 1941 and, to a lessor extent, 1951 schedules exhibit a somewhat poor fit.

To understand this anomaly we should consider the disruptive events during these periods. The Second World War, 1939-45, interfered most with the 1941 census operation. The age composition of certain States in India at this census was, in addition, seriously distorted because of the major communities making a determined move to inflate their respective numbers at the enumerations. Moreover, the war had profound effects on the economy and on the way of life of the population (Gupta, 1969). Coupled with the possible secondary repercussion of war, the partition in 1947, resulting in communal riots and the movements of millions of refugees, had influenced demographic events subsequently. Because of the war, 1941 may thus have been an atypical year for nuptiality (and for that matter for other demographic phenomena). Also, the logit model cannot possibly remove the major deliberate errors in the data completely. This is perhaps a part of the reason why the logit fit does not work too well for 1941.

While the fit of the logit model seems to be good to most data on historical series of proportions single in India with almost unchanged marriage custom, it is pertinent to apply the model to a strongly different population. The case of Japan with late marriage pattern is illustrated for the purpose. The standard was chosen, following similar procedure described earlier, from the census series available, and two reported schedules for the years 1920 and 1960 were tested. Since separate points could not be represented in Figure 5.1, the fit being very close indeed inspite of some changes in nuptiality pattern during 1920-60, the Japanese data are shown only in Table 5.4. It may be worth noting that the age pattern of deviations of observed from fitted values are not similar between the populations of India and Japan, and the deviations are probably more due to errors in the original data than in the fit.

Table 5.4. Standard, observed and fitted schedules of proportions single : Japan, Selected years

Age (a)	$S_s(a)$	1920		1960	
		$S(a)$	$\hat{S}(a)$	$S(a)$	$\hat{S}(a)$
15 - 19	0.966	0.823	0.820	0.986	0.985
20 - 24	0.553	0.314	0.310	0.683	0.708
25 - 29	0.152	0.092	0.096	0.216	0.239
30 - 34	0.057	0.041	0.046	0.094	0.092
35 - 39	0.030	0.027	0.028	0.055	0.047
40 - 44	0.020	0.021	0.021	0.032	0.031
45 - 49	0.015	0.019	0.017	0.021	0.023

Source : Official census reports (cited in Smith, 1978a).

From the above considerations we assumed deviations from the standard as measurement errors, and adjusted the data to generate model schedules as follows. Corresponding to each observed schedule we estimated the best fitting straight line :

$$\hat{P}(a) = \hat{\alpha} + \hat{\beta} P_s(a)$$

where $\hat{P}$ (a) is the estimator of P(a) = logit S(a). The model schedules of proportions single $\hat{P}$ (a), covering a wide range of observed schedules, may be derived from standard ones by varying only two parameters. The estimates of α and β are shown in Table 5.5, for decennial years between 1901 and 1971.

Table 5.5. Estimates of α and β across census years for each of the two standard schedules

Year	Standard I		Standard II	
	α	β	α	β
1901	-0.83053	0.50834	-0.67750	0.64482
1911	-0.80213	0.54126	-0.64229	0.68479
1921	-0.70445	0.55730	-0.53854	0.70584
1931	-0.76047	0.61640	-0.57840	0.77988
1941	-0.18488	0.89867	-0.39630	0.81025
1951	-0.42715	0.65565	-0.21420	0.84063
1961	-0.32102	0.86443	-0.05710	1.09863
1971	0.05503	1.05290	0.37615	1.33797

In interpreting the trends in these parameters, it is useful to have an understanding of the nuptiality pattern in the culture. During 1901-71 there has been an increase in proportions single at younger ages and a decrease at older ages, implying rising age at marriage but declining celibacy rates. The upward trends in the parameters α and β appear to correspond to these trends in nuptiality. αs have values less than unity and barring 1971, they are all negative, implying shift of the peaks of the distributions towards younger ages — lowering the average ages at marriage compared to the standard schedules (around 17.6 years). The values of β are all less than unity in the earlier censuses, and tend to decrease the concentrations around the central values. The estimates of these parameters for 1971 notably stand out of the line for the earlier census series. The impact of this departure in raising the age at marriage is likely to be much more telling in the long run.

Table 5.6 shows the trend in model proportions single. While studying the trends and differentials in nuptiality of the country and their patterns

Table 5.6. Model schedules of proportions single derived from the standard schedules

Age	1901	1911	1921	1931	1941	1951	1961	1971
(a) Standard I								
15 - 19	0.133	0.138	0.193	0.144	0.321	0.244	0.268	0.417
20 - 24	0.061	0.061	0.070	0.056	0.095	0.097	0.079	0.109
25 - 29	0.021	0.019	0.022	0.015	0.014	0.024	0.012	0.012
30 - 34	0.018	0.016	0.018	0.013	0.011	0.020	0.010	0.009
35 - 39	0.016	0.015	0.016	0.011	0.009	0.018	0.008	0.007
40 - 44	0.014	0.013	0.014	0.009	0.007	0.015	0.006	0.005
45 - 49	0.013	0.011	0.013	0.008	0.006	0.013	0.005	0.004
(b) Standard II								
15 - 19	0.135	0.140	0.164	0.146	0.194	0.253	0.275	0.428
20 - 24	0.057	0.056	0.065	0.052	0.068	0.090	0.071	0.096
25 - 29	0.026	0.024	0.027	0.019	0.024	0.032	0.018	0.018
30 - 34	0.018	0.016	0.018	0.013	0.015	0.020	0.010	0.008
35 - 39	0.015	0.014	0.015	0.010	0.013	0.016	0.007	0.006
40 - 44	0.013	0.012	0.013	0.009	0.011	0.013	0.006	0.004
45 - 49	0.013	0.011	0.013	0.008	0.010	0.013	0.005	0.004

across time, it is important to note that till 1931 no minimum age for marriage was prescribed by law. The Child Marriage Restraint Act or more commonly known as Sarda Act, enacted in April 1930, provided penalties for marriages taken place under 14 years of age of females and under 18 of males. 'The slight lowering of the mean age at marriage to 12.7 in 1931 from 13.7 in 1921 brings out the fact that a large number of child marriages seem to have taken place before the act came into force' (Registrar General of India, 1974). As a matter of fact, there had not been much time lag between the enactment as well as strict enforcement of the act and the subsequent census, and the people aged below the prescribed minimum apparently went for marriages with more than normal tempo in fear of punishment later. This partly explains that this act, instead of raising proportions single at the younger ages, lowered them initially. The disruptive events that interrupted demographic phenomena around 1941 have already been mentioned earlier. Also, the 1921 estimates are slightly counter to the trend, particularly at ages 15-19. The great influenza epedemic of 1918, believed to have taken a toll of 12 million lives in the country, and many famines prior to 1921 (Registrar General of India, 1974) appear to have been reflected in this fluctuation. It is likely that marriages were temporarily postponed for some years during that period, thus causing a bulge in proportions never married.

It is of interest to examine what the proportions single imply in terms of their first differences. The graphs of these differences for the fitted (derived from Standard II) compared with observed (1901 and 1971) are shown in Figure 5.2. We define the proportions declining between the age groups a-5 and a as

$$D\,(a-5,\ a) = S\,(a-5) - S(a)$$

Since everyone does not marry by age 50 (the single condition is assumed definitive by age 50), D(a-5, a) is only approximately equal to the proportion of marriages that occur in each age group. If D is divided by S(50), then this interpretation assumes that the proportions S(a) characterize a synthetic cohort as it moves through age and time. Thus the model works best if nuptiality patterns do not change from one census to the next. If nuptiality changes rapidly, then, as mentioned earlier, there are problems in giving a cohort interpretation to what are essentially cross-sectional data.

In Figure 5.2 the observed schedules expressed as proportions declining closely follow the same form of distribution as the standard schedules. The distribution has a longer tail on the right implying that the proportions drop off more slowly on the right of the curve than on the left with steep initial rise. In the population with nearly universal marriage, only a few women remain single at older ages. All the schedules give rise to unimodal (and

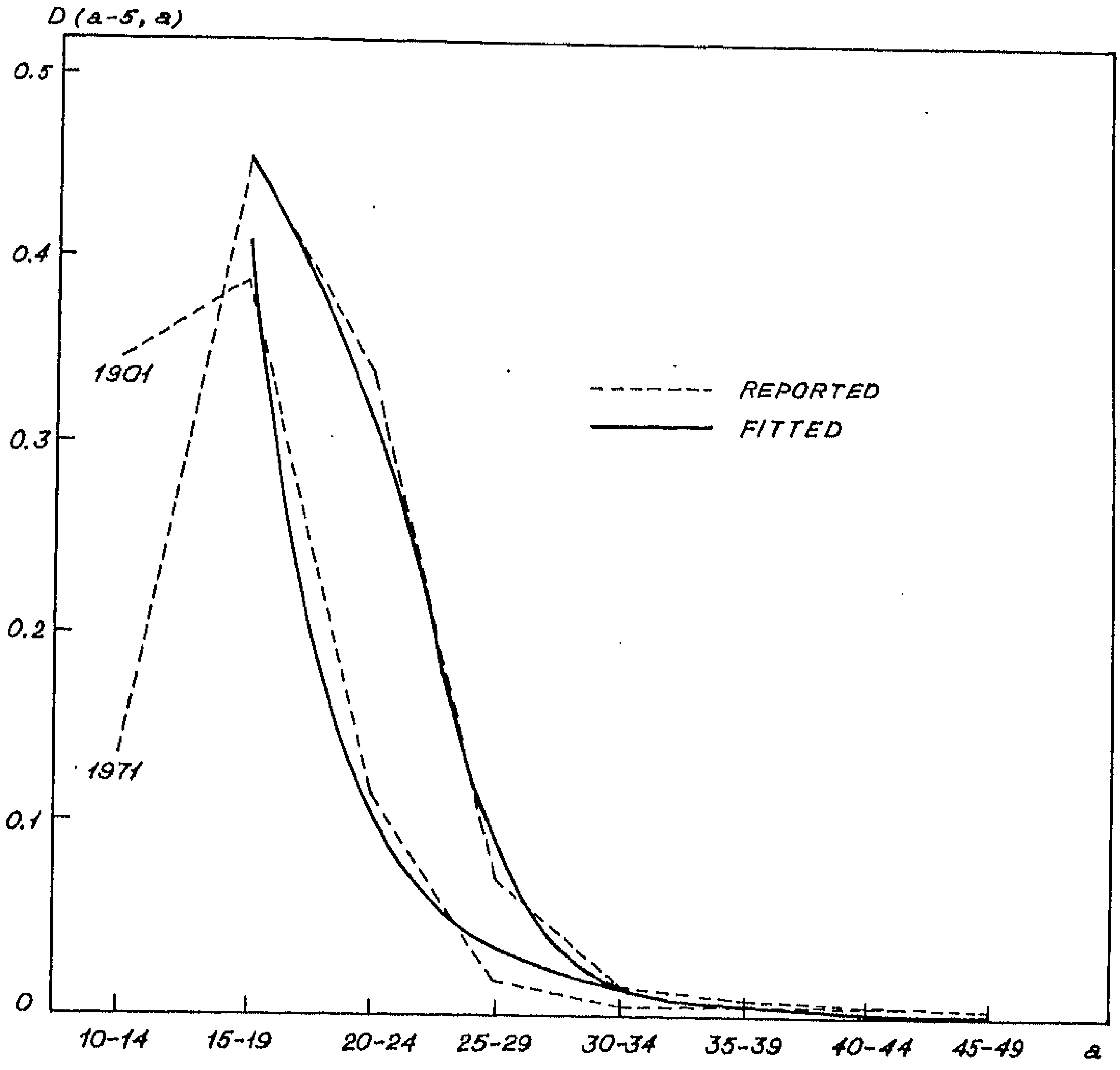


Figure 5.2 : Single proportions declining (D (a-5, a)) by age (a) for selected observed (1901 & 1971) and fitted schedules (derived from Standard II)

leptokurtic) distributions characterized by rapid transition from single state to married state at the younger ages with a peak at age 15-19. Beyond that point it declines at first rapidly and then slowly.

It would be useful to compare the singulate mean ages at marriage (SMAM) computed from the raw data and logit model for each census year. In India, exposure to the risk of conception and child-bearing is confined to period of marriage, and fertility performance below age 15 is assumed insignificant (Guha Roy, 1972). We thus begin at age 15 assuming that no effective marriages occur earlier than age 15. This is however not to suggest that formal marriages below age 15 do not occur at all in India. The SMAMs or more strictly SMAEMs (Singulate Mean Age at Effective Marriage) for one period (Hajnal, 1953) are found as follows :

	1901	1911	1921	1931	1941	1951	1961	1971
Raw SMAM	16.1	16.0	16.2	16.1	16.4	16.9	16.8	17.7
Model SMAM	15.9	16.0	16.3	16.0	16.4	16.7	16.8	17.7

Given the limitation of the reported data, the agreement between observed and model SMAMs can be considered as quite remarkable. It is however more appropriate to derive mean age at marriage by calculating proportions remaining single from one decade to the next in order not to assume the census cross section as a cohort. This has been done elsewhere (Agarwala, 1962), and as stated above, the present purpose is merely to compare census and model estimates.

5.2.3 Comparison of the logit model with the Coale-McNeil model

Coale and McNeil (1972) have developed a closed expression of the first marriage frequency function,

$$g(a) = (0.19465/K) \exp \left\{ (-0.174/K)(a - a_0 - 6.06\,K) - \exp \left[(-0.2881/K)(a - a_0 - 6.06\,K) \right] \right\},$$

where a_0 is the age at which first marriages begin, and K, the tempo at which first marriages take place. C G(a), the approximate proportion ever married (C being the proportion who will ever marry), can be calculated numerically from g(a). Since a demographic estimate is generally non-unique, a consistent value of S(a) obtained from the relation S(a) + C G(a) = 1 may provide an alternative estimate of proportions single. This offers a possible comparison of the logit model with the Coale - McNeil procedure. Considering 1971 data for illustration, the results of comparison are shown in Table 5.7. Following Coale's recommended procedure (Coale, 1971), the parametric values are being estimated as a_0 = 10.16 and K = 0.65 (C = 0.995), which compare well with the values, a_0 = 10 and K = 0.624 (C = 0.996), estimated in another study (Smith, 1978a).

Upto age 25, the proportions single are corrected in the same direction by both methods. If the data are systematically in error, then may be the Coale-McNeil model actually provides a better fit. That is, the Coale-McNeil model may fit the true proportions single better but the reported proportions single worse than the logit model.

Table 5.7. Reported and estimated (by two procedures) female proportions single in 1971

Age	Census	Logit model	Coale-McNeil Model
15 - 19	0.429	0.417	0.4144
20 - 24	0.091	0.109	0.1166
25 - 29	0.020	0.012	0.0308
30 - 34	0.009	0.009	0.0082
35 - 39	0.005	0.007	0.0021
40 - 44	0.005	0.005	0.0005
45 - 49	0.004	0.004	0.0001

In fitting the Coale-McNeil behavioral model to the empirical data for India one finds difficulty in identifying the stages preceding marriage as conceived in the model. The stages such as entry into the marriage market, meeting a suitable partner, and getting engaged, interpreted in Western cultures, cannot be exactly equated with the stages leading to marriages in the Indian society where marriages are mostly arranged by the parents. However, a very rough correspondence between the two cultures in respect of marriage practices may be assumed. Thus, the sequence of steps antecedent to marriage in India can be stated as the starting of negotiations between the boy's parents and the girl's parents (analogous to entry into the 'state of mubility'), settlement of dowries and other details (similar but not at all identical to the stage between ultimate spouse and engagement in the Western culture), and solemmizing the marriage (corresponding to the stage between engagement and marriage). These stages are not all non-overlapping, especially the first two. It is not surprising that because of the lack of exact behavioral correspondence on the one hand and the irregularities in the basic data on the other, the fit of the Coale-McNeil model to the Indian data of 1971 (found to be more so for the earlier censuses) is not good enough. Coale (1977) himself warned, even after being successful with French data, that reality is more complex than this model, and the good fit in one instance may not be replicated in the other. Though not wholly incontestable, the logit model allows us to tune the estimates of proportion single to a specific culture subject to a choice of an appropriate standard schedule.

5.2.4 Projecting proportions single

Our interest focuses on the period 1981-86, and we attempt to make a set of estimates of the female proportions single from the time series (1901-71) of the model schedules. We examined several methods, but choice of a mathematical curve is more or less subjective as there is no way of knowing whether one curve is superior to the other in absolute terms. We however believe that even approximate methods could make modest estimates that would serve our purpose at hand. We argue that the force which have prompted marriage delay in the recent past resulting in the on-going shift in nuptiality should gather momentum in the coming years. At present there seems to be no tendency for proportions single to stabilise by the period for which estimates are made. Our projections might thus be viewed as reflecting merely a contimued trend. We however used consistency criteria later to check these estimates.

A single comprehensive formula to give the individual age group and year values from the time series of proportions single was tried. A little trial-and-error calculation showed that a structural graduation formula provides a fairly good approximation of the proportion single, ${}^{t}S_{m}(a)$:

$$\log_{10} {}^{t}S_{m}(a) = \alpha^{*}(a) + \beta^{*}(a) \cdot t + \gamma^{*}(a) \cdot t^{2},$$

where a = age groups for estimation (e.g. 15-19, 20-24, etc.),

t = measured in years from 1936 (approximate mid-point of the period considered), and

$\alpha^{*}, \beta^{*}, \gamma^{*}$ = parameters defining the polynomial.

The least square estimates of these parameters for any age group were obtained by fitting the above curve to the model proportions single corresponding to census-years 1901-71. The estimates of the parameters for each age group are shown as follows :

Age group (a)	$\alpha^{*}(a)$	$\beta^{*}(a)$	$\gamma^{*}(a)$
15 - 19	-0.74379	0.006369	0.0000908
20 - 24	-1.15423	0.003433	0.0000477
25 - 29	-1.71171	-0.002929	-0.0000825
30 - 34	-1.78936	-0.003453	-0.0000946
35 - 39	-1.84396	-0.004309	-0.0001126
40 - 44	-1.91571	-0.005469	-0.0001384
45 - 49	-1.97512	-0.006188	-0.0001456

It appeared that linear functions of age were not enough to provide good approximations to the parameters. On the other hand, a quadratic exponential seems to give satisfactory fit; the parameters of this curve varied fairly smoothly with age to permit efficient use of the graduation formula. An objection may however be raised for the unasymptotic estimates of proportions single resulting from the use of this model*. Since there is no

* This argument was made by Prof. S. Mitra, Emory University, Atlanta, in a personal discussion with the author.

Table 5.8. Estimated proportions single by cohorts and quinquennial years, and mean ages at marriage, 1901-71

Age group	1901	1906	1911	1916	1921	1926	1931	1936	1941	1946	1951	1956	1961	1966	1971	
15 - 19	.133	.137	.138	.146	.193	.159	.144	.180	.194	.213	.244	.263	.268	.338	.417	
20 - 24	.061	.061	.061	.063	.070	.065	.056	.070	.076	.077	.097	.086	.079	.098	.109	
25 - 29	.021	.020	.019	.021	.022	.020	.015	.019	.020	.022	.024	.016	.012	.012	.012	
30 - 34	.018	.017	.016	.017	.018	.017	.013	.014	.016	.019	.020	.013	.010	.010	.009	
35 - 39	.016	.015	.015	.015	.016	.015	.011	.012	.014	.016	.018	.011	.008	.008	.007	
40 - 44	.014	.013	.013	.014	.014	.013	.009	.010	.012	.014	.015	.008	.006	.006	.005	
45 - 49	.013	.012	.011	.012	.013	.012	.008	.009	.010	.011	.013	.007	.005	.005	.004	
Cohort SMAEM*								16.1	16.0	16.0	16.0	16.2	16.2	16.3	16.4	16.5
(Birth cohort)								(1881-1886)	(1886-1891)	(1891-1896)	(1896-1901)	(1901-1906)	(1906-1911)	(1911-1916)	(1916-1921)	(1921-1926)
Period SMAEM*																
One period (Hajnal)	15.9	16.0	16.0	16.0	16.3	16.1	16.0	16.3	16.4	16.5	16.7	16.8	16.8	17.2	17.7	
Two periods (Agarwala)		16.0	16.0	16.0	16.3	16.1	16.1	16.2	16.2	16.5	16.7	16.8	16.8	17.4	17.9	

* SMAEM = Singulate mean age at effective marriage (assumed to start at age 15).

Table 5.9. Forward estimates of proportions single, and mean ages at marriage : 1976-86

Age group	1976	1981	1982	1983	1984	1985	1986	
15 - 19	0.453	0.533	0.551	0.570	0.590	0.611	0.633	
20 - 24	0.115	0.125	0.127	0.130	0.132	0.134	0.137	
25 - 29	0.011	0.010	0.010	0.009	0.009	0.009	0.009	
30 - 34	0.008	0.007	0.007	0.007	0.007	0.007	0.006	
35 - 39	0.006	0.005	0.005	0.005	0.005	0.005	0.005	
40 - 44	0.004	0.004	0.003	0.003	0.003	0.003	0.003	
45 - 49	0.004	0.003	0.003	0.003	0.002	0.002	0.002	
Cohort SMAEM		16.6	16.7	16.8	16.9	17.3	17.7	18.0
(Birth cohort)		(1926-1931)	(1931-1936)	(1936-1941)	(1941-1946)	(1946-1951)	(1951-1956)	(1956-1961)
Period SMAEM :								
One period (Hajnal)	17.9	18.3	18.4	18.5	18.7	18.8	18.9	
Two periods (Agarwala)	18.0	18.5				19.1		

reason to expect that the current nuptiality trend would change significantly in the near future, the above curve of an underlying set of model schedules could be used to project proportions single for the short period considered.

The proportions single in the 15-49 age range for the years 1981-86 were first obtained to extend the census cross sections available decennially from 1901 to 1971. The proportions single for an intermediate quinquennium (such as 1966, for example) were subsequently estimated to conform model schedules of the two bordering years (such as 1961 and 1971). The schedules for the period 1901-71 now found quinquennially are presented in Table 5.8. In Table 5.9, the estimated values for the years upto 1986 are presented. For the age group 15-19, there is almost an uniform tendency for the proportions single to rise across time, whereas the smooth upward trends at ages 20-24 started after 1961. In the middle and older age ranges, the proportions single are, more or less, stabilized by 1970s at lower levels than in the first half of the century.

5.2.5 Singulate mean age at effective marriage

The synthetic indicator of " age at marriage" takes into account the fact that the mean age at marriage is the mean duration of single life. We considered women in the age range 15-50, assuming that no effective marriage occurred earlier than age 15. The assumption made for calculation of singulate mean age at marriage (Hajnal, 1953) that there is no marital status differential for migration is possibly partly realistic, but the same cannot be said with certainty for mortality differential. Data on this later aspect however are scarce in the Indian context.

For any census cross-section, the singulate mean age at effective marriage (SMAEM) is

$$\text{SMAEM} = \left[15 + 5 \sum_{a=15-20}^{45-50} S(a) - 50\,(1 - S) \right] / S$$

where S(a) is the proportion single in age group a and

$$S = 1 - S\,(45\text{-}50)$$

is the 'propensity to marry' or the proportion ever-married. We have taken here the proportion single at age 50 simply as S (45-50) rather than the usually used average of S (45-50) and S (50-55). The term 50 (1 - S) has been subtracted from the numerator to remove the never marrying from consideration. Proportions single under age 15 are presumed equal to unity.

The marriage patterns as manifested from the trends of the mean age at marriage are shown in Tables 5.8 and 5.9. At about the middle of the current century the age at effective marriage of the Indian females started rising from its earlier approximately stable levels. The assumption of unchanged nuptiality in the above calculations of singulate mean age at marriage is thus not met in practice for the whole period. In the situation of steadily rising marriage ages, the measure indicates the average experience of an indeterminant period preceding the census (Smith, 1978a). Agarwala's (1962) mean ages at marriage calculated from two consecutive censuses are considered a solution to this problem. The method is as follows :

Let ${}^{t}S(a)$ = proportion single in age group a (e.g., 15–19, 20–24, etc.) at time t

$M(a)$ = proportion marrying between ages a–5 and a in the t–5 to t quinquennium.

Now, the ratio of proportions single in successive age groups in two periods gives the proportion who married between the two ages. Thus, for the t–5 to t quinquennium,

$$M(a) = {}^{t}S(a) \;/\; {}^{t-5}S(a-5)$$

To find the proportions left single at each age, these ratios are applied sequentially as follows, assuming proportions single under age 15 as unity.

$$M\,(15\text{-}19) = \pi\,(15\text{-}19)$$

$$M\,(20\text{-}24) \times \pi\,(15\text{-}19) = \pi\,(20\text{-}24)$$

$$M\,(25\text{-}29) \times \pi\,(20\text{-}24) = \pi\,(25\text{-}29)$$

and so on. Here πs are the "quinquennia synthetic" proportions single, and are presented in Table 5.10. For a hypothetical cohort, the calculation of mean ages at marriage from the synthetic proportions single was done in the usual manner, that is, by Hajnal's method.

The diagonals in Tables 5.8 and 5.9 give the proportions single for genuine birth cohorts. The calculation of mean age is done with Hajnal's method, which is more appropriate for cohort data than cross-section data. The SMAEMs for one period (Hajnal), two periods (Agarwala) and cohorts are also shown in these tables. For the first six birth cohorts, 1881-86 to 1906-11, the SMAEMs remained steady at around 16 years, and then started accelerating. The period SMAEMs estimated by two methods present interesting comparison. When there is a clear out upward trend in nuptiality, the period means obtained by Agarwala's method are higher than (as from the quinquennium 1946-51 through 1981-1986) or equal to the neighbouring one-period (Hajnal) means. Because of a lack of regular upward trend, earlier quinquennia do not show such systematic relationship between the two period means.

Table 5.10. Quinquennia synthetic proportions single : 1901-1971

Age	1901-06	1906-11	1911-16	1916-21	1921-26	1926-31	1931-36
			(a) 1901-36				
15 - 19	0.137	0.138	0.146	0.193	0.159	0.144	0.180
20 - 24	0.063	0.061	0.067	0.092	0.054	0.051	0.087
25 - 29	0.021	0.019	0.023	0.032	0.015	0.012	0.030
30 - 34	0.017	0.015	0.021	0.028	0.012	0.008	0.028
35 - 39	0.014	0.013	0.019	0.026	0.010	0.005	0.026
40 - 44	0.011	0.012	0.018	0.024	0.008	0.003	0.023
45 - 49	0.010	0.010	0.017	0.023	0.007	0.002	0.023

(b) 1936-71

Age	1936-41	1941-46	1946-51	1951-56	1956-61	1961-66	1966-71
15 - 19	0.194	0.213	0.244	0.263	0.268	0.338	0.417
20 - 24	0.082	0.085	0.111	0.093	0.080	0.124	0.134
25 - 29	0.023	0.024	0.035	0.015	0.011	0.019	0.016
30 - 34	0.020	0.023	0.031	0.008	0.007	0.016	0.012
35 - 39	0.020	0.023	0.030	0.005	0.004	0.013	0.009
40 - 44	0.020	0.023	0.028	0.002	0.002	0.009	0.005
45 - 49	0.020	0.021	0.026	0.001	0.001	0.008	0.004

Since we considered only effective marriage age, our estimated means were higher and not strictly comparable with other estimates. However, corresponding to our estimates of 17.7 (1971) and 17.9 (1966-71), an all-India survey (Operations Research Group, Baroda, 1970-71) gives the estimated mean age at marriage as 18.3 years for the period of marriage 1966-70. The fertility survey conducted by the Registrar General in 1972 also included age at effective marriage. For the period of marriage 1968-72, the mean age was 17.4 years for rural areas, and 18.5 years for urban areas (Registrar General, India, 1976). On the whole, the indication from available sources, including our analysis, is that the later-marriage norm is being accepted generally, thus raising the age at marriage.

5.3 Proportions married and ever-married

5.3.1 Building-up data

The proportions of married females in each quinquennial age group were only available in the censuses of 1961 and 1971 at the time of analysis. Such data for earlier censuses are not readily available for building up trends. In the absence of an alternative, we built up the data for 1951 as follows : From the given percentage distribution of 1951 population by age and sex (Mukherjee, 1976) and 1951 census aggregate for females, the number of females by five-year age groups was obtained. The proportions married could only be obtained by ten-year age groups as residuals from the given proportions single and widowed or divorced (Census of India, 1951; Mitra, 1978). The absolute number of married females for the age group 10-19 to 45-54 could thus be found. To reduce the data available by ten-year age groups to the required five-year groups, we used quadratic interpolation, also known as Newton's method of halving an age group (United Nations, 1956), as follows :

Let ${}_5M_a$ = estimated number of married females in the first half (a, a+5) of the given ten-year age group (a, a+10);

${}_{10}M_a$ = given number of married females in the entire ten-year group (a, a+10);

${}_{10}M_{a-10}$ = given number of married females in the preceding ten-year group (a-10, a); and

${}_{10}M_{a+10}$ = given number of married females in the following ten-year group (a+10, a+20).

Then

$${}_5M_a = \frac{1}{2}\left({}_{10}M_a + \frac{1}{8}\left({}_{10}M_{a-10} - {}_{10}M_{a+10}\right)\right)$$

The estimates for the number of married females by five-year age groups in the reproductive age range 15-44 were thus derived. And the proportions married reported in the censuses of 1961 and 1971, and estimated for 1951 are shown in Table 5.11.

Table 5.11. Observed and standard schedules of proportions married : 1951, 1961 and 1971

Age	Proportions married at age a, C(a)				logit C(a)			
	1951	1961	1971	standard	1951	1961	1971	standard
15 - 19	0.711	0.696	0.561	0.650	0.45	0.41	0.12	0.31
20 - 24	0.891	0.918	0.894	0.840	1.05	1.21	1.07	0.83
25 - 29	0.881	0.942	0.956	0.915	1.00	1.39	1.54	1.19
30 - 34	0.902	0.914	0.950	0.910	1.11	1.18	1.47	1.16
35 - 39	0.856	0.870	0.917	0.855	0.89	0.95	1.20	0.89
40 - 44	0.709	0.777	0.842	0.795	0.45	0.62	0.84	0.68

At this point, and before adjusting the schedule of proportions, we verified that the quinquennia proportions married as reported in the censuses of 1961 and 1971, could approximately be reproduced by the above method. The estimates made for 1951 are taken as first approximation to be improved later.

5.3.2 Model schedules for female proportions married

The estimated schedule for 1951 shows bimodality, one at ages 20-24 and the other at ages 30-34. But it seems unlikely that female proportions of first marriages would attain a peak during early 20s' ages, and again during early 30s. Since the other two available schedules (that is, 1961 and 1971) do not exhibit this feature, the 1951-schedule seems to reflect patterns different from reality. Moreover, the proportions married are probably subject to same kind of fluctuations and errors as in proportions single that confound the phenomenon.

We attempt to adjust the data on proportions married by the same procedure as has been used for single schedules. Since the 1961 schedule is intermediate between the other two, we have simply smoothed the reported proportions from this schedule to produce a standard. Figure 5.3 shows the reported and standard schedules of proportions married. The values are given in Table 5.11. As before, these schedules are converted to logits to see the extent to which the reported schedules linearise on the standard. While the linearisation is almost perfect for 1971-schedule, it is not as good for other schedules as it was for proportions single (Figure 5.4). However, the linearisation is fairly apparent, and accept linear relationship with the standard as defining the distribution underlying the data.

Taking the standard as an effective tool, we consider the logit-linear model as appropriate for fitting to the reported schedules of proportions married. Then the fitted values may be considered as corrected schedules. These are given below for three census years of 1951, 1961 and 1971.

1951 : $\text{logit } C(a) = 0.17394 + 0.77200 \text{ logit } C_s(a)$

1961 : $\text{logit } C(a) = 0.06949 + 1.05594 \text{ logit } C_s(a)$

1971 : $\text{logit } C(a) = -0.35513 + 1.63269 \text{ logit } C_s(a)$

where a ranges from age group 15-19 to 40-44. The model schedules are shown in Table 5.12. A comparison of reported and adjusted schedules for the available data sets is shown in Figure 5.5. The deviations from the models indicate most probably the measurement errors.

The proportions married in the more frequent marriage ages (e.g. 15-19) have declined during the recent decades. Considering the effects upon nuptiality of on-going social and economic changes, it is believed that the decline in the teen ages is likely to continue. The few observation points available do not however provide a sufficient basis for assessing the trend. From the trend of proportions single based on long period of observations and its strong association with that of proportions married, it may, nevertheless, be expected that the trend for the period 1951-71 may apply to, at least, near future.

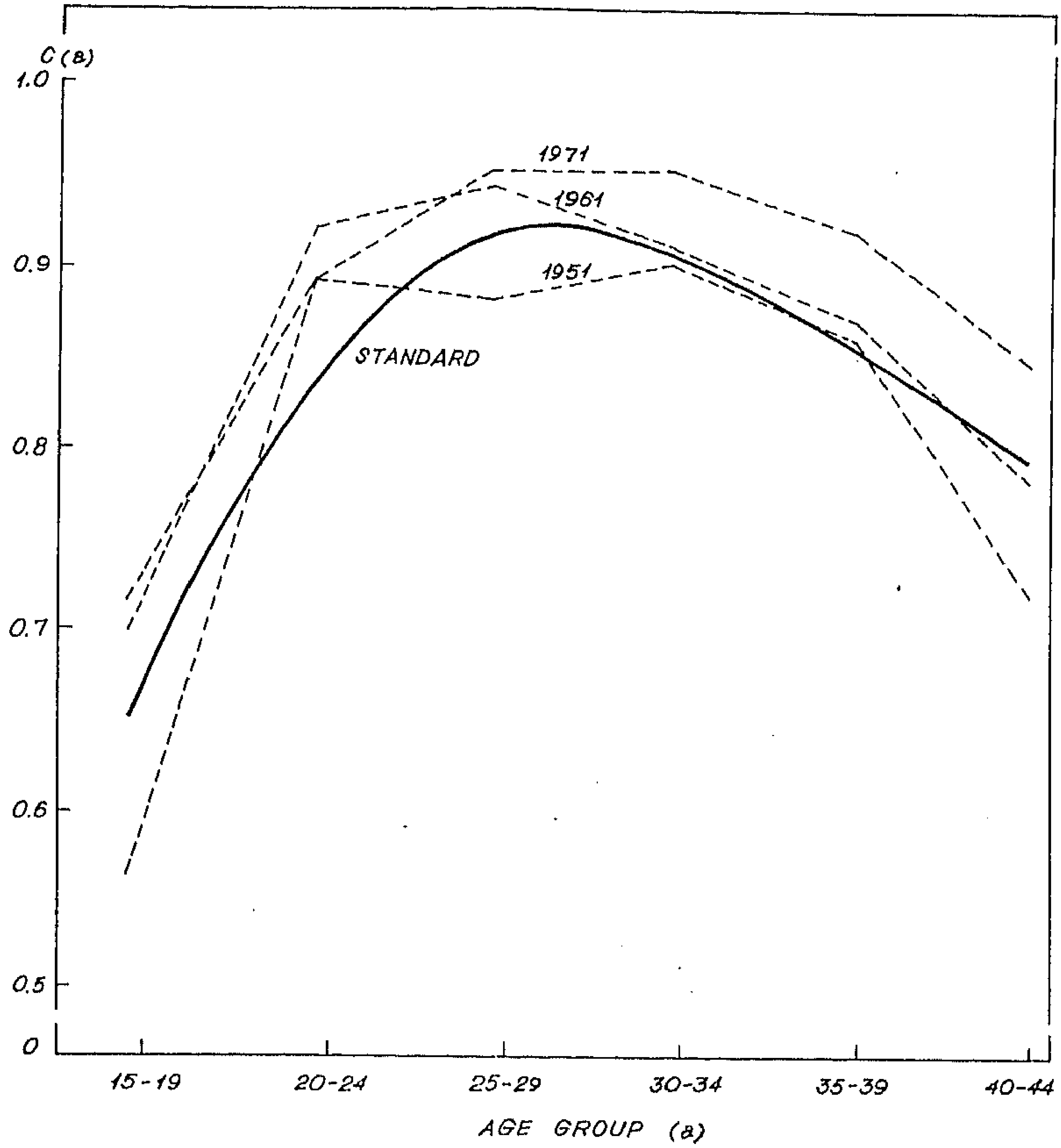


Figure 5.3 : Reported (broken lines) and standard (solid line) schedules of proportions married, C(a) : 1951-71

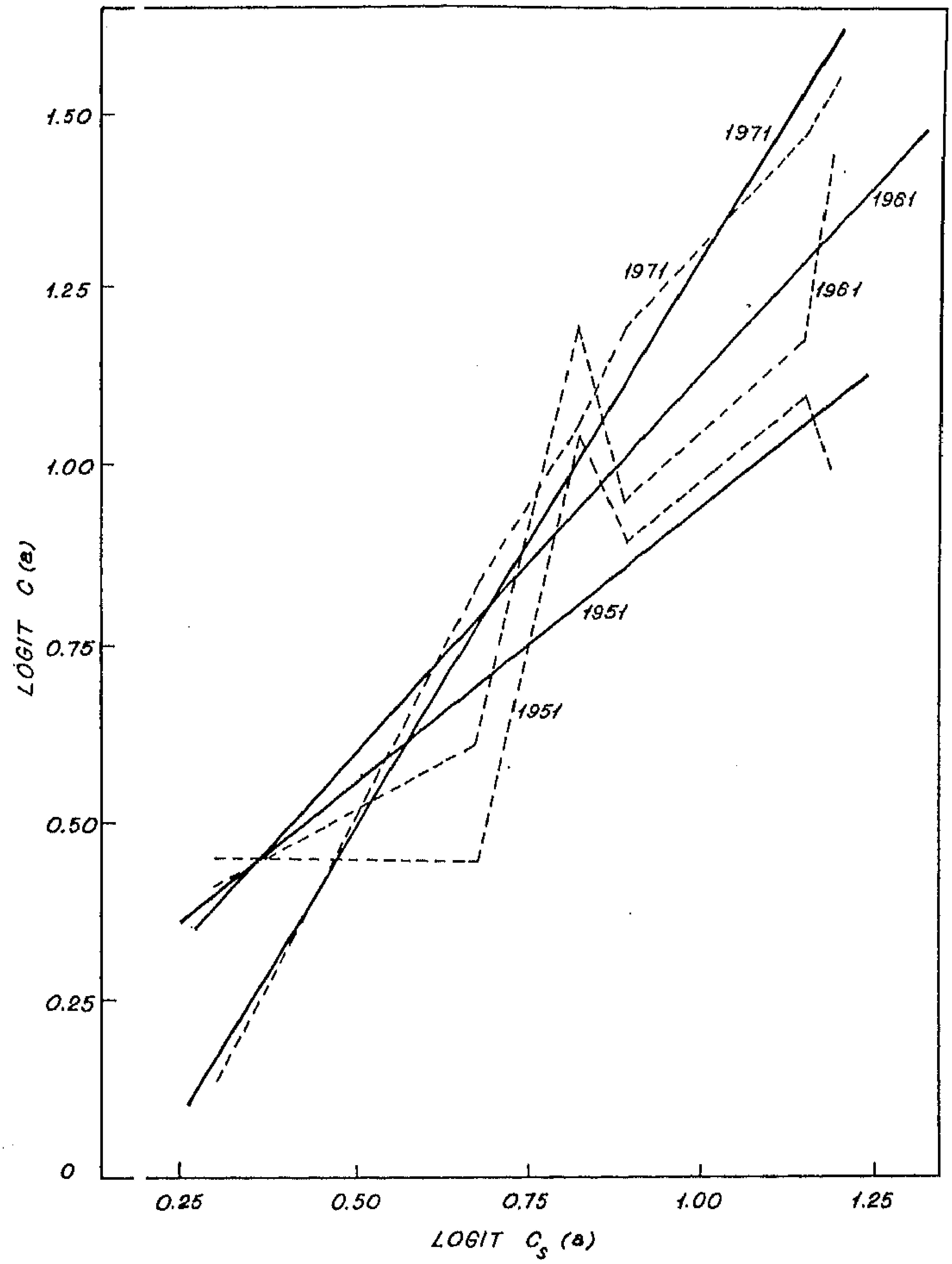


Figure 5.4 : Relations between the logits of three reported (broken lines) schedules of proportions married and those of a single standard schedule (Solid lines indicate best fitting straight lines for each observed schedule)

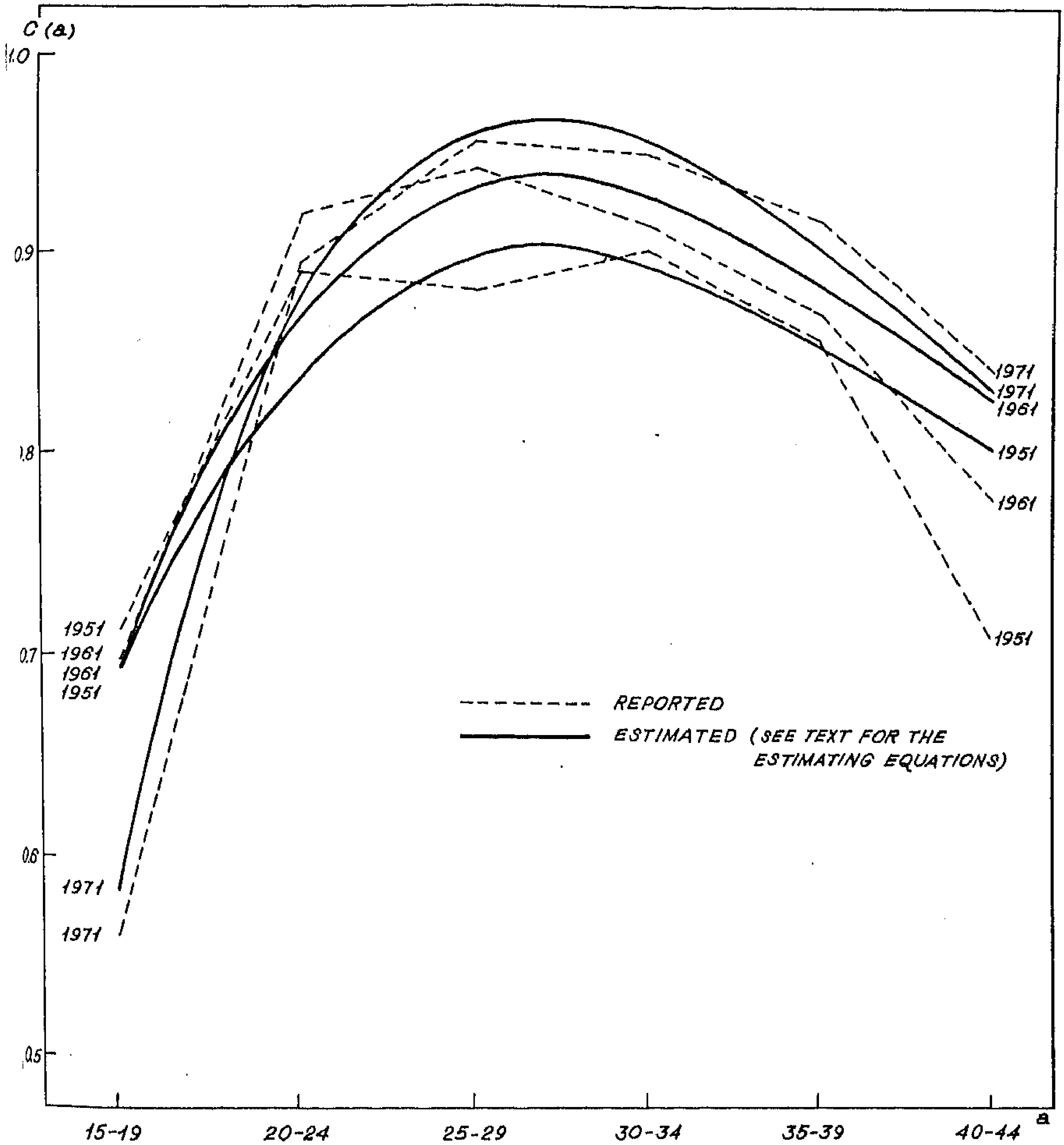


Figure 5.5 : Reported schedules of proportions mar.ie: (C(a)) and their standard-based estimated distributions : 1951-71

Table 5.12. Model schedules of proportions married ($C_m(a)$) derived from the standard schedule : 1951, 1961 and 1971

Age	Model expressed as logit $C_m(a)$			Model expressed as proportions married at age a, $C_m(a)$		
	1951	1961	1971	1951	1961	1971
15 - 19	0.41	0.40	0.17	0.696	0.690	0.582
20 - 24	0.81	0.95	1.00	0.836	0.869	0.881
25 - 29	1.09	1.33	1.59	0.899	0.934	0.960
30 - 34	1.07	1.29	1.54	0.895	0.930	0.956
35 - 39	0.86	1.01	1.10	0.848	0.883	0.900
40 - 44	0.70	0.79	0.80	0.802	0.828	0.831

5.3.3 Projecting proportions ever-married

The conventional projection procedure is to start with the distribution of males by marital status and nuptiality, and then derive the distribution of females by nuptiality and marital status (Widen, 1969). The future nuptiality is estimated on the basis of observed marriage rates for males, projected graphically. These rates are then used to control the rates for females so that the total number of newly married males during the year is the same as the number of newly married females. The procedure of this kind could not however be adopted in the case of India due to lack of information on the annual number of marriages in the absence of compulsory marriage registration.

Coale's three-parameter model (Coale, 1971) may be used for projection by assuming plausible changes in the age at marriage entrance (a_0), the tempo at which marriage occurs (K), and the final proportion married (C). This approach, following Smith (1978b), will give independent estimates of ever-married proportions of females, and should be comparable to the proportions indirectly estimated from the proportions single obtained earlier. The proportions of currently married can then be derived from these estimates and the estimates of proportions widowed, divorced or separated.

We first fit the standard schedules of first marriage frequencies to the marriage data of 1941-1971. In determining the parameters a_0, K and C the ratios R_1, R_2 and R_3 calculated as the proportion ever-married in one five-year age group to that in the next five-year group are first obtained as follows on the consideration that a_0, the earliest age at which most of the marriages begin to take place, lies between 10 and 15 years of age. Thus, R_1 is proportion ever-married (10-15)/proportion ever-married (15-20), R_2 is proportion ever-married (15-20)/proportion ever-married (20-25) and R_3 is proportion ever-married (20-25)/proportion ever-married (25-30).

Ratio	1941	1951	1961	1971
R_1	0.297	0.235	0.230	0.162
R_2	0.872	0.837	0.795	0.654
R_3	0.943	0.925	0.932	0.902

In an earlier study (Malaker, 1978), the fitting of standard schedule was made to female marriage data of 1961 for four groups of states of India classified according to mean age at marriage. The ranges of the three ratios were 0.047 - 0.392 (R_1), 0.393 - 0.866 (R_2) and 0.840 - 0.985 (R_3). Our estimates for 1961, obtained after adjusting the raw census data, fall within these ranges.

Following Coale's recommended procedure, the (R_1, R_2) combination of ratios were used to estimate the parameters K and a_0 for the years 1941-1971. the estimates of the three nuptiality parameters are :

Parameter	1941	1951	1961	1971
a_0	10.07	10.49	10.15	10.16
K	0.39	0.40	0.47	0.65
C	0.938	0.955	0.959	0.995

While the estimates for 1961 agree with the results of the study referred above, our 1971 estimates compare well with the values 10.0 (a_0), 0.624 (K), and 0.966 (C) estimated in another study (Smith, 1978a). The nuptiality estimates by single ages for the years 1941-1971 are shown in Tables 5.13a-5.13d.

The nuptiality assumptions employed for the projections are made in terms of Coale's three parameters (Coale, 1971) rather than in terms of the proportions single directly (Smith, 1978b). To estimate nuptiality for the 1981-86 period, we first considered the experience of neighbouring Sri Lanka, which is an important exception in South Asia for its high SMAM and broad time span of marriage entrance (Smith, 1978a). Corresponding to its values of a_0 = 13.5, K = 0.93 and C = 0.94 in 1971, we assumed, as one alternative, the values of a_0 = 13.0, K = 0.90 and C = 0.95 for India in 1986. The resulting estimates of proportions ever-married were too low, and did not seem to be attainable by this period, as evidenced from the preliminary results of 1981 census. Since the above " low" projection reflecting accelerated marriage delay does not work well in the current Indian context, we resort to a "medium" projection, assuming that the two timing parameters (a_0 and K) would continue to shift linearly at 1941-1971 average annual rate over the projection period, while C held its 1971 level. The assumption implies a slower tempo of marriage-entrance, a little change in the age at which marriage first begins, and no change in the universality of marriage in the quinquennium in question.

As a variant of the " medium" projection, a " high" series was also obtained on the assumption that all the three parametric values at 1971 levels would remain constant throughout the projection period. However, only the plausible medium projections of female proportions ever-married along with the independent estimates, obtained as complement of the estimated proportions single, are shown in Table 5.14. Barring a few cases, the comparison reveals small differences between the estimates.

Table 5.13a. Estimating proportion ever-married, first marriage frequency, and risk of first marriage from Coale's model : 1941

Age a	$(a-a_0)/K = x_s$	$C\,G_s(x_s)$	$g_s\,(x_s)$	$r(a)$
11	2.38	0.0204	0.0204	0.05
12	4.95	0.1156	0.0952	0.17
13	7.51	0.2858	0.1702	0.29
14	10.08	0.4699	0.1841	0.37
15	12.64	0.6198	0.1499	0.41
16	15.21	0.7270	0.1072	0.43
17	17.77	0.7999	0.0729	0.44
18	20.33	0.8486	0.0487	0.44
19	22.90	0.8807	0.0321	0.44
20	25.46	0.9007	0.0200	0.45
21	28.03	0.9140	0.0133	0.45
22	30.59	0.9237	0.0097	0.45
23	33.15	0.9302	0.0065	0.45
24	35.72	0.9343	0.0041	0.45
25	38.28	0.9368	0.0025	0.45
26	40.85	0.9380	0.0012	0.45

Notes : a_0 – age after which first marriages generally begin

K – tempo at which marriages take place

C – ultimate proportion ever-married

(a_0, K and C were estimated using Coale's three-parameter model)

$C\,G_s\,(x_s)$ – proportion ever-married (interpolated from Coale's standard schedule)

$g_s\,(x_s)$ – first marriage frequency [differences in $C\,G_s\,(x_s)$]

$r(a)$ – risk of first marriage at age a, derived from a double exponential :

$$r(a) = (0.174/k)\, e^{-4.411\, e^{-(0.309/k)\,(a - a_0)}}$$

Table 5.13b. Estimating proportion ever-married, first marriage frequency, and risk of first marriage from Coale's model : 1951

Age a	$(a-a_0)/K = x_s$	$C\ G_s\ (x_s)$	$g_s\ (x_s)$	r(a)
11	1.28	0.0057	0.0057	0.02
12	3.78	0.0617	0.0560	0.11
13	6.28	0.2020	0.1403	0.23
14	8.78	0.3856	0.1836	0.32
15	11.28	0.5558	0.1702	0.38
16	13.78	0.6844	0.1286	0.41
17	16.28	0.7749	0.0905	0.42
18	18.78	0.8364	0.0615	0.43
19	21.28	0.8777	0.0413	0.43
20	23.78	0.9048	0.0271	0.43
21	26.28	0.9218	0.0170	0.43
22	28.78	0.9338	0.0120	0.43
23	31.28	0.9425	0.0087	0.43
24	33.78	0.9483	0.0058	0.43
25	36.28	0.9519	0.0036	0.43
26	38.78	0.9542	0.0023	0.43
27	41.28	0.9550	0.0008	0.43

Notes : Same as Table 5.13a.

Table 5.13c. Estimating proportion ever-married, first marriage frequency, and risk of first marriage from Coale's model : 1961

Age a	$(a-a_0)/K = x_s$	$C\ G_s\ (x_s)$	$g_s\ (x_s)$	r(a)
11	1.81	0.0115	0.0115	0.03
12	3.94	0.0685	0.0570	0.10
13	6.06	0.1877	0.1192	0.19
14	8.19	0.3431	0.1554	0.26
15	10.32	0.4967	0.1536	0.31
16	12.45	0.6239	0.1272	0.34
17	14.57	0.7197	0.0958	0.35
18	16.70	0.7903	0.0706	0.36
19	18.83	0.8409	0.0506	0.37
20	20.96	0.8770	0.0361	0.37
21	23.09	0.9023	0.0253	0.37
22	25.21	0.9192	0.0169	0.37
23	27.34	0.9312	0.0120	0.37
24	29.47	0.9404	0.0092	0.37
25	31.60	0.9474	0.0070	0.37
26	33.72	0.9522	0.0048	0.37
27	35.85	0.9554	0.0032	0.37
28	37.98	0.9576	0.0022	0.37
29	40.11	0.9590	0.0014	0.37

Notes : Same as Table 5.13a.

Table 5.13d. Estimating proportion ever-married, first marriage frequency, and risk of first marriage from Coale's model : 1971

Age a	$(a-a_0)/K = x_s$	$C\ G_s\ (x_s)$	$g_s(x_s)$	r(a)
11	1.29	0.0060	0.0060	0.01
12	2.83	0.0321	0.0261	0.04
13	4.37	0.0912	0.0591	0.09
14	5.91	0.1842	0.0930	0.13
15	7.45	0.2985	0.1143	0.17
16	8.98	0.4171	0.1186	0.20
17	10.52	0.5291	0.1120	0.23
18	12.06	0.6257	0.0966	0.24
19	13.60	0.7049	0.0792	0.25
20	15.14	0.7690	0.0641	0.26
21	16.68	0.8194	0.0504	0.26
22	18.22	0.8592	0.0398	0.26
23	19.75	0.8903	0.0311	0.27
24	21.29	0.9147	0.0244	0.27
25	22.83	0.9334	0.0187	0.27
26	24.37	0.9476	0.0142	0.27
27	25.91	0.9582	0.0106	0.27
28	27.45	0.9667	0.0085	0.27
29	28.98	0.9737	0.0070	0.27
30	30.52	0.9796	0.0059	0.27
31	32.06	0.9842	0.0046	0.27
32	33.60	0.9877	0.0035	0.27
33	35.14	0.9903	0.0026	0.27
34	36.68	0.9922	0.0019	0.27
35	38.22	0.9937	0.0015	0.27
36	39.75	0.9947	0.0010	0.27
37	41.29	0.9950	0.0003	0.27

Notes : Same as Table 5.13a.

Table 5.14. Estimated female proportions ever-married obtained by two methods : 1981-86

Age group	Proportion ever-married					
	1981	1982	1983	1984	1985	1986
(a) Coale's model						
15 - 19	0.485	0.466	0.449	0.432	0.424	0.408
20 - 24	0.806	0.792	0.779	0.766	0.759	0.746
25 - 29	0.930	0.924	0.917	0.910	0.906	0.899
30 - 34	0.973	0.970	0.967	0.963	0.962	0.958
35 - 39	0.990	0.989	0.988	0.986	0.985	0.983
40 - 44	0.995	0.995	0.995	0.994	0.994	0.993
(b) Estimates from proportion single						
15 - 19	0.467	0.449	0.430	0.410	0.389	0.367
20 - 24	0.875	0.873	0.870	0.868	0.866	0.863
25 - 29	0.990	0.990	0.991	0.991	0.991	0.991
30 - 34	0.993	0.993	0.993	0.993	0.993	0.994
35 - 39	0.995	0.995	0.995	0.995	0.995	0.995
40 - 44	0.996	0.997	0.997	0.997	0.997	0.997

5.3.4 Estimating proportions of marital disruption and deriving proportions married

Our next task is to obtain estimates for female proportions widowed and divorced. We note that the declining trend in the proportions widowed, recorded upto 1971, was largely due to falling mortality and to a lesser extent due to broader outlook to widow remarriage (Registrar General of India, 1974). But the deceleration in mortality decline in recent years is likely to cause no further significant reduction in the proportions widowed in near future. The other less important factor (in the Indian context) responsible for marital disruption is divorce or separation of spouses. Although divorce is not culturally acceptable, the liberalization of divorce laws, reflected in the Special Marriage Act, 1954, Hindu Marriage Act 1955 and the "mutual consent" clause added in 1976 to the Hindu Marriages Act and inclusion in early 1981 of "irretrievable breakdown" as a ground for divorce, may marginally raise the low proportions divorced or separated in India. This is partially evident from an unpublished study conducted in Hooghly District, West Bengal in 1981 by the Indian Statistical Institute. Although it is nowhere near as frequent as in the West, some indication of this rise is also evident from the fact that in Delhi, 25 divorce petitions are filed every day, whereas 10 years ago, the number was less than five (Statesman, October 9, 1981). We thus examined recent marital history and behaviour, pooled the information from various available sources, and made plausible assumptions about the short-run future trend. As one alternative, we assume that the combined proportions widowed and divorced-separated, in which we are presently interested, will remain at 1971 levels at least upto 1986. In the second alternative, we assume that the combined proportions will decline, more or less, at the same annual average rate as observed during 1961-71.

Using life table techniques, a recent study (Malaker, 1981) showed as high as 40 per cent under enumeration in the reported proportions widowed in one census. The mortality levels chosen in the study from the model life tables, derived mainly from Western sources, is however unlikely to reflect the age patterns of mortality as presently obtaining in India. We have also found that fixing a (model) life table for all ages on the basis of a single measure of $\overset{o}{e}_o$ does not always work well for developing countries. In view of this and other available evidence, we believe that the extent of under-enumeration derived in the above study may have been over estimated. Even if such estimates are accepted, the reported female proportions single and married have to be drastically revised to conform to consistent sets of estimated values of the components of marital structure.

Going however by the consideration that the reported proportions widowed (and also divorced) are clearly deficient, we attempt to estimate them indirectly as follows. As is known, the fertility survey conducted by the office of the Registrar General, covering the reference period of July, 1971 to June, 1972

(Registrar General of India, 1976) provides firm data on fertility at the national level. The age specific fertility rates (ASFR) and age specific marital fertility rates (ASMFR) are taken by us from this study, but made consistent with the adjusted population age structure (Guha Roy, 1979) to yield a crude birth rate of about 41 per thousand around 1971. Since in India, illegitimacy is not a significant factor and the marital fertility rates are calculated from the same total births as the general fertility rates, an independent estimate of proportions currently married (PM), in addition to those obtained by model schedule, may be obtained from the algebraically exact relation :

$$PM = ASFR \div ASMFR$$

We earlier estimated proportions ever-married $[C\ G_s\ (x_s)]$ by Coale's model (method 1), and also as complement of model schedule of proportions single, $S_m(a)$ (method 2). The combined proportions widowed-divorced-separated, designated here as R(a), can then be obtained easily from these estimates. Thus, we have

$$\text{Method 1} \ : \ R(a) = [C\ G_s(x_s)]' - PM(a)$$

$$\text{Method 2} \ : \ R(a) = [1 - S_m(a)] - PM(a)$$

As is obvious, $C\ G_s\ (x_s)$ and PM(a) are not directly comparable. The former refers to single ages, whereas the latter relates roughly to the midpoint of 5-year age groups (a), that is, to 17.5, 22.5, ..., etc. By taking $x_s = \frac{a - a_0}{K} = 17.5,\ 22.5$ etc., $C\ G_s(x_s)$ are adjusted to these ages to obtain age-equivalent values designated as $[C\ G_s(x_s)]'$.

The estimated female proportions of widowed-divorced-separated obtained by the two methods are presented in Table 5.15, and compared with the census proportions, which were obtained from the age-sex and marital status distribution, ignoring unspecified marital status. The overall census proportions turn out to be under-enumerated by 6.5 per cent in relation to method 1, and by 21.6 per cent in relation to method 2, whereas the two methods agree very closely in the age range 30-44 because most marriages occur below age 30 of females. Although the census proportions are clearly deficient in level, the method 1 estimates are closer to age pattern of reported proportions than those of method 2.

Table 5.15. Estimated combined proportions of widowed, divorced and separated females according to two methods : 1971

Age group	Adjusted		Estimated	Proportions widowed, divorced or separated		
	ASFR	ASMFR	PM	Method 1	Method 2	Census[1/]
15 - 19	0.0950	0.1675	0.5672	0.0102	0.0158	0.0072
20 - 24	0.2824	0.3302	0.8552	0.0196	0.0358	0.0153
25 - 29	0.2968	0.3194	0.9351	0.0273	0.0529	0.0248
30 - 34	0.2325	0.2498	0.9307	0.0583	0.0603	0.0462
35 - 39	0.1597	0.1774	0.9002	0.0948	0.0928	0.0770
40 - 44	0.0972	0.1170	0.8308	0.1642	0.1642	0.1485

Note : See text for explanation of different columns.

[1/] Census of India, 1971, Series 1, Part IIc(ii), Social and Cultural Tables, pp. 5-9.

Compared to currently married, its complement of proportions widowed, divorced and separated is small in India. Aggregation of these components, as is done above, is believed to result in more stable estimate of " marital disruption". Later, we also estimate the annual risk of widowhood, that is, death of either spouses.

We assume, as one alternative, that the change in proportions ever-married is due largely to change in the proportions currently married, whereas the components comprising " marriage disruption" hold, more or less, the 1971-levels as revised above (Table 5.15). Following this assumption, the proportions currently married for 1981-86 are estimated from the corresponding proportions ever-married (Table 5.14) obtained by Coale's model (method 1), and also as complement of model schedule of proportions single (method 2). The estimates are presented in Table 5.17A.

On the consideration that the reporting biases of the proportions widowed, divorced or separated in the censuses of 1961 and 1971 are similar, we find their observed rate of decline during 1961-71 as follows :

Item	Proportions widowed-divorced-separated							
	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Census 1961	0.002	0.011	0.022	0.039	0.075	0.123	0.217	0.298
Census 1971	0.001	0.007	0.015	0.025	0.046	0.077	0.148	0.211
Percent annual decline (1961-71)								
(i) observed	5.0	3.6	3.2	3.6	3.9	3.7	3.2	2.9
(ii) 3-point moving average		3.9	3.5	3.6	3.7	3.6	3.3	

We thus observe that the rate of annual decline (1961-71) over the age groups varies within a narrow margin. Though the magnitudes of the observed proportions are known to be deficient, the age pattern of their decline can plausibly be accepted as approximately correct. The data are however smoothed by three-point moving averages before employing them for deriving proportions married during 1981-86 under the second alternative assumption already mentioned. The combined proportions widowed, divorced or separated, estimated for 1981-86 according to rate of decline (1961-71) derived above, are presented in Table 5.16. The estimates of proportions married, derived from Table 5.14 and Table 5.16, are presented in Table 5.17B.

At this point we verified that the proportions single, S(a), the proportions married, C(a), and the proportions widowed-divorced-separated, R(a), estimated for 1971 and 1981-86 do not give rise to inconsistent results. In other words, the estimates satisfy for each age group (a) and time (t),

$$^{t}S(a) + {}^{t}C(a) + {}^{t}R(a) = 1,$$

the variables included in this consistency check being however not all independently estimated.

Table 5.16. Estimated female proportions widowed-divorced-separated according to two methods : 1981-86

Age group	1981	1982	1983	1984	1985	1986
			Method 1			
15 - 19	0.0062	0.0060	0.0058	0.0055	0.0053	0.0051
20 - 24	0.0127	0.0123	0.0119	0.0114	0.0110	0.0107
25 - 29	0.0175	0.0168	0.0162	0.0156	0.0151	0.0145
30 - 34	0.0367	0.0354	0.0341	0.0328	0.0316	0.0304
35 - 39	0.0607	0.0585	0.0564	0.0544	0.0524	0.0505
40 - 44	0.1100	0.1064	0.1029	0.0995	0.0962	0.0930
			Method 2			
15 - 19	0.0096	0.0093	0.0089	0.0086	0.0082	0.0079
20 - 24	0.0233	0.0225	0.0217	0.0209	0.0202	0.0195
25 - 29	0.0339	0.0326	0.0315	0.0303	0.0292	0.0282
30 - 34	0.0380	0.0366	0.0352	0.0339	0.0327	0.0315
35 - 39	0.0594	0.0573	0.0552	0.0532	0.0513	0.0494
40 - 44	0.1100	0.1064	0.1029	0.0995	0.0962	0.0930

Table 5.17. Four sets of estimates of female proportions married : 1981-86

A. Assuming constant proportions widowed-divorced-separated at 1971 levels

Age group	Proportions married by years					
	1981	1982	1983	1984	1985	1986
Method 1						
15 - 19	0.4748	0.4558	0.4388	0.4218	0.4138	0.3978
20 - 24	0.7864	0.7724	0.7594	0.7464	0.7394	0.7264
25 - 29	0.9027	0.8967	0.8897	0.8827	0.8787	0.8717
30 - 34	0.9147	0.9117	0.9087	0.9047	0.9037	0.8997
35 - 39	0.8952	0.8942	0.8932	0.8912	0.8902	0.8882
40 - 44	0.8308	0.8308	0.8308	0.8298	0.8298	0.8288
Method 2						
15 - 19	0.4512	0.4332	0.4142	0.3942	0.3732	0.3512
20 - 24	0.8392	0.8372	0.8342	0.8322	0.8302	0.8272
25 - 29	0.9371	0.9371	0.9381	0.9381	0.9381	0.9381
30 - 34	0.9327	0.9327	0.9327	0.9327	0.9327	0.9337
35 - 39	0.9022	0.9022	0.9022	0.9022	0.9022	0.9022
40 - 44	0.8318	0.8328	0.8328	0.8328	0.8328	0.8328

B. Assuming average rate of decline of the period 1961–71 in the proportions widowed-divorced-separated

Age group	Proportions married by years					
	1981	1982	1983	1984	1985	1986
Method 1						
15 – 19	0.4788	0.4600	0.4432	0.4265	0.4187	0.4029
20 – 24	0.7933	0.7797	0.7671	0.7546	0.7480	0.7353
25 – 29	0.9125	0.9072	0.9008	0.8944	0.8909	0.8845
30 – 34	0.9363	0.9346	0.9329	0.9302	0.9304	0.9276
35 – 39	0.9293	0.9305	0.9316	0.9316	0.9326	0.9325
40 – 44	0.8850	0.8886	0.8921	0.8945	0.8978	0.9000
Method 2						
15 – 19	0.4574	0.4397	0.4211	0.4014	0.3808	0.3591
20 – 24	0.8517	0.8505	0.8483	0.8471	0.8458	0.8435
25 – 29	0.9561	0.9574	0.9595	0.9607	0.9618	0.9628
30 – 34	0.9550	0.9564	0.9578	0.9591	0.9603	0.9625
35 – 39	0.9356	0.9377	0.9398	0.9418	0.9437	0.9456
40 – 44	0.8860	0.8906	0.8941	0.8975	0.9008	0.9040

5.4 Coale's index of nuptiality

On the basis of evidence given by our estimates, the earlier tempo of female nuptiality in the peak ages 15-19 and 20-24 would considerably slow down. This was also evident from the rising singulate mean age at marriage worked out earlier (Tables 5.8 and 5.9). The emergence of near-modern marriage behaviour would be further substantiated with reference to an index which summarizes the nuptiality pattern of the population (Coale, 1969a and 1969b).

If m_i = number of currently married women in age i,

W_i = number of all women in the same age; and

F_i = natural marital fertility rates in the corresponding age,

Coale's index of marriage pattern, I_m, is given as

$$I_m = \sum_{i=15}^{44} m_i F_i \Big/ \sum_{i=15}^{44} W_i F_i$$

where $0 \leq I_m \leq 1$ theoretically.

An age-standardized measure of Coale's index is given by

$$I_m^* = \sum_{i=15}^{44} (m_i / W_i) F_i \Big/ \sum_{i=15}^{44} F_i$$

Analogous to index for currently married (I_m), indexes for wodowed (I_w), divorced (I_d), separated (I_p), single (I_s) and ever-married (I_{em}) can be worked out (Hull, 1978; Smith, 1978a, 1978b). We however find here a combined index of marital dissolution (I_r),

$$I_r = I_w + I_d + I_p,$$

and use the index, I^*, for convenience of using proportions. The natural marital fertility rates used by us are as given by Henry (1961), which are lower than those of Hutterites (Coale, 1971) in all ages. Henry's data represent the average age specific rates for thirteen non-Malthusian populations in which family limitation was not practised.

	Natural marital fertility rates						
	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Hutterites (Coale)	(0.819)	0.550	0.502	0.447	0.406	0.222	0.061
Non-Malthusion Populations (Henry)	(0.648)	0.435	0.407	0.371	0.298	0.152	0.022

Since the natural marital fertility for age group 15-19 is not given, we take it as 1.49 times the rate at ages 20-24 (Coale, 1971).

The indexes I^*_m, I^*_r, I^*_{em}, I^*_s and I^*_u ($= I^*_r + I^*_s$) are shown in Table 5.18. The I^*_m-values distinguish between the overall fertility obtained from a pattern of universal marriage starting at age 15 and that estimated from the observed marriage pattern. They measure the extent to which marriage contributes to the achievement of maximum fertility potential of the population studied. Over the period of study, the population experiences rising female ages at marriage (SMAEM), which are associated with the declining values of I^*_m.

Table 5.18. Estimated component indexes of female marriage pattern : 1961, 1971, 1981-86

Year	Index (I): Single (I^*_s)	Currently married (I^*_m)	Div. + Sep. + Widowed ($I^*_d + I^*_p + I^*_w = I^*_r$)	Ever-married (I^*_{em})	Not currently married ($I^*_s + I^*_r = I^*_u$)
1961	.095			.905	
1971	.142	.805	.053	.858	.195
1981	.177	.770 - .789	.053 - .034	.823	.230 - .211
1982	.182	.765 - .785	.053 - .033	.818	.235 - .215
1983	.188	.759 - .780	.053 - .032	.812	.241 - .220
1984	.194	.753 - .775	.053 - .031	.806	.247 - .225
1985	.200	.747 - .771	.053 - .029	.800	.253 - .229
1986	.207	.740 - .765	.053 - .028	.793	.260 - .235

Notes : I^*_m, I^*_r and I^*_{em} computed by Method 2 (see text). The first and second terminal values of the ranges (1981-86) for I^*_m, I^*_r and I^*_u correspond to assumptions A and B (see Table 5.17) respectively.

Sources : Tables 5.8, 5.9, 5.15, 5.16 and 5.17, and Henry's (1961) data on natural marital fertility.

5.5 The availability of mates

The availability of marriage partners, as measured by Dixon (1971), is the ratio of males to females in different groups of marriageable ages differing by a constant figure of five years, grooms being older than their brides. The Indian census reports show that the mean ages at marriage for men and women differed by about seven years before 1931, but the difference came down to around five years during 1931-71. Recent studies indicate that the age difference is again tending towards the pre-1931 average of seven years. The reason for the shift may not merely be the short supply of females coupled with the trend in sex ratio, which continues to be unfavourable to females in spite of marginal improvement in the Census of 1981 (Office of the Registrar General, 1981). In the socio-cultural setting obtaining in India, 'there is no bar to older men marrying very young women and marrying in widowhood and recruiting wives from a much wider age group of women than in other countries' (Mitra, 1978). In view of this, we try an exception to the simplified assumption of general five-year difference, and consider all possible interacting ages of the marriageable number of females (F) and males (M) to express the inter-sex difference of availability by an arithmetic index termed here as Sexual Dimorphism Percentage (S.D.P) :

$$P_j = 100 \left(\frac{M_i}{F_j} - 1\right)$$

where the females of jth category would choose mates from males of ith category. This index, similar to that used by Entomologists, measures the numerical superiority of one sex over the other.

To illustrate the calculation of S.D.P.s, we start from the premise that the joint distribution of single males and females desiring marriages in different age groups would approximate that of married spouses. We further assume that this joint distribution remains unchanged and that the marriage of females is universal. Since no bisexual age distribution is available in the census reports, we pool the pertinent data from the National Sample Surveys, second and fourth rounds (rural), 1951-52 (Das gupta, Som, Majumdar and Mitra, 1955), and from the data collected by the Indian Statistical Institute in the following studies : (a) Growth of family size, Calcutta, 1968; (b) Employees' population, Indian Statistical Institute, 1968; (c) Matrimonial advertisers, Calcutta, 1970–71 and (d) Rural couples, South 24-Parganas, West Bengal, 1972. Some of these sources are local, but the average experience could be applied to India as a whole, subject to inter-locality variations in the age patterns of marriage. India is still predominantly rural and our samples biased towards rural population is not unrepresentative in this regard. We may however try to adjust the average of five joint age distributions of males and females to the census distribution of 1971 (the results of 1981 census being not available at the time of the analysis) as follows :

Let p_{rs} = unadjusted (reported) proportions of (potential) spouses in the r-th age category of males and s-th age category of females

$$\sum_s p_{rs} = p_{r0}, \quad \sum_r p_{rs} = p_{0s}, \quad \sum_s \sum_r p_{rs} = 1$$

and

π_{r0} and π_{0s} = marginal proportions derived from census.

Then we have :

male \ female	1	2 . . . . s	total: reported	total: census
1	p_{11}	p_{12} . . . . p_{1s}	p_{10}	π_{10}
2	p_{21}	p_{22} . . . . p_{2s}	p_{20}	π_{20}
.	.	. . . .	.	.
.	.	. . . .	.	.
.	.	. . . .	.	.
.	.	. . . .	.	.
r	p_{r1}	p_{r2} . . . . p_{rs}	p_{r0}	π_{r0}
total (reported)	p_{01}	p_{02} . . . . p_{0s}	$p_{00} = 1$	
total (census)	π_{01}	π_{02} . . . . π_{0s}		$\pi_{00} = 1$

We compute

$$D = \frac{(\pi_{11} - p_{11})^2}{p_{11}} + \frac{(\pi_{12} - p_{12})^2}{p_{12}} + \cdots\cdots + \frac{(\pi_{rs} - p_{rs})^2}{p_{rs}}$$

and minimise D with respect to $\pi_{11}, \pi_{12}, \ldots\ldots, \pi_{rs}$, subject to the conditions

$$\pi_{11} + \pi_{12} + \cdots\cdots + \pi_{1s} = \pi_{10}$$

$$\cdots\cdots\cdots\cdots$$

$$\pi_{r1} + \pi_{r2} + \cdots\cdots + \pi_{rs} = \pi_{r0}$$

and

$$\pi_{11} + \pi_{21} + \cdots\cdots + \pi_{r1} = \pi_{01}$$

$$\cdots\cdots\cdots\cdots$$

$$\pi_{1s} + \pi_{2s} + \cdots\cdots + \pi_{rs} = \pi_{0s}$$

In this manner we could have obtained least squares solutions for the above problem. But we prefer to make a subjective adjustment of the reported proportions for illustration of the calculation of numbers available for first marriage. The derived joint distribution of potential spouses, reflecting age preferences for marriage, is shown in Table 5.19.

We propose to illustrate the calculation of S.D.P.s with reference to 1971, and our next step is to obtain estimates for single populations for this year. For sake of uniformity, we use census proportions single for both sexes to get estimates of single populations from the sex-age distribution of 1971-population (Table 5.20). These estimates are subjected to the distribution presented in Table 5.19 to obtain the totals of opposite sexes, paired in specified age combinations so as to lead to possible marital unions (Table 5.21).

The S.D.P. values in various possible age combinations of potential bride and groom are shown in Table 5.22. They show the relationships between

Table 5.19. Proportionate age distribution[1] (adjusted average of five distributions) of potential brides and grooms

Age of potential groom	Age of potential bride						
	15-19	20-24	25-29	30-34	35-39	40-44	total
15 - 19	0.321						(2)
20 - 24	0.844	0.156					
25 - 29	0.403	0.529	0.068				
30 - 34	0.118	0.431	0.412	0.039			
35 - 39	0.014	0.103	0.535	0.320	0.028		
40 - 44		0.054	0.255	0.453	0.227	0.011	
45 - 49		0.018	0.100	0.326	0.357	0.199	
50 - 54		0.003	0.087	0.099	0.258	0.553	
55 - 59						0.194	(2)

(1) Based on a combined sample of 21,290 persons.

(2) Since marriages of females below age 15 and above age 44 are not considered here, the proportions do not add up to unity for the terminal age groups of males.

Table 5.20. Estimated populations of single females and males by age : 1971

Age group	All females (000)	Female proportions single	Single females (000)	All males (000)	Male proportions single	Single males (000)
15 - 19	27738	0.429	11900	29392	0.823	24190
20 - 24	23301	0.091	2120	24788	0.504	12493
25 - 29	20180	0.020	404	21584	0.189	4079
30 - 34	16791	0.009	151	18085	0.073	1320
35 - 39	14196	0.005	71	15421	0.040	617
40 - 44	11146	0.005	56	12233	0.035	428
45 - 49				9941	0.029	288
50 - 54				8435	0.028	236
55 - 59				7170	0.026	186

Table 5.21. Estimates of the number of single persons available for marriage : 1971

Age of potential bride	No. of single females (000)	Age of potential groom	No. of single males (000)
15 - 19	11900	15 - 19	7765*
		20 - 24	10544
		25 - 29	1644
		30 - 34	156
		35 - 39	9
20 - 24	2120	20 - 24	1949
		25 - 29	2158
		30 - 34	569
		35 - 39	64
		40 - 44	23
		45 - 49	5
		50 - 54	0
25 - 29	404	25 - 29	277
		30 - 34	544
		35 - 39	330
		40 - 44	109
		45 - 49	29
		50 - 54	21
30 - 34	151	30 - 34	51
		35 - 39	197
		40 - 44	194
		45 - 49	94
		50 - 54	23
35 - 39	71	35 - 39	17
		40 - 44	97
		45 - 49	103
		50 - 54	61
40 - 44	56	40 - 44	5
		45 - 49	57
		50 - 54	131
		55 - 59	36

* 7765 = 24190 (Table 5.20) X 0.321 (Table 5.19).

Table 5.22. Sexual dimorphism percentage (S.D.P.) : 1971

Age of potential bride	Number of single females (000)	Age range of potential groom	Number of single males (000)*	S.D.P.
15 - 19	11900	15 - 39	20118	69.1
20 - 24	2120	20 - 54	4768	125.0
25 - 29	404	25 - 54	1310	224.3
30 - 34	151	30 - 54	559	270.2
35 - 39	71	35 - 54	278	291.5
40 - 44	56	40 - 59	229	308.9

* Sum of the number of single males corresponding to single females of particular age group in Table 5.21.

the number of females and males in various age groups that is likely to contact marriages with each other. The S.D.P.s being high and positive, there are serious imbalances between the number of males and of females in marriageable ages. The numerical superiority of males over females in each age bracket explains the greater proportion of never-married males compared to never-married females in the population. Under the situation, a good number of male marriages should be delayed, and the males of similar ages would be competing in the search for desirable mates.

5.6 Finding the risk of widowhood

As is known, the data on widowhood in the Indian Censuses are deficient, and no systematic analysis in this regard was thus possible. However, even an approximately corrected data on widowhood by age may be useful in projection of population by marital status. The adjustment of census proportions widowed (and divorced-separated) has been discussed earlier. In this section, we attempt to derive mortality for married persons for estimation of risk of widowhood.

A computer model, MORT, converts male and female survival rates into an annual age specific risk of death of either spouse (Nortman, et al., 1978). The data required are the $_5L_x$ life table values for both sexes, thereby determining the joint probability of survival of the couple. The simplified assumptions are : independence of risks of deaths (μ) of the spouses, age difference (D) of five years or its multiple between the sexes, and constant risks of marital dissolution over the projection period (T).

Denoting by x the lower age limit of female spouse at the beginning of the projection period, the probability, P, of the marriage remaining intact over the period is

$$P = \frac{_5L^h_{x+D+T}}{_5L^h_{x+D}} \times \frac{_5L^w_{x+T}}{_5L^w_x}$$

the superscripts h and w denoting husband and wife respectively. Now

$$P = \exp (- \mu T)$$

That is, $\mu = \frac{1}{T}$ (Colog P).

The question of widowhood is naturally the study of mortality of spouses. Empirical evidences from different studies show mortality differentials by marital status. It is noted that married persons experience lower mortality in all ages. Pressat (1972) argues as follows : this excess mortality (for single persons) is not due only to the condition of bachelorhood, but equally to a selection process in marriage that rejects certain among the physically and mentally deficient bachelors.

Though no national data on mortality by marital status are available, it is nevertheless expected that this global pattern should be applicable to India as well. The adjusted sex-age mortality experience (1971-81) for the general population (Table 5.23) was derived earlier (Guha Roy, 1979) from the Sample Registration System and United Nations set of Model Life Tables, without using any particular level ($\mathring{e}_0$) for all age groups, but switched across levels depending on the observed age specific pattern of mortality for India. This general level should be adjusted suitably to get the mortality rates for currently married persons.

Table 5.23. **Estimated mortality rates for general population ($_5M_x$) by sex : India, 1971-81**
(Rates per thousand)

Age group (x, x+5)	$_5M_x$			
	1971-76		1976-81	
	Males	Females	Males	Females
15 - 19		3.4		3.1
20 - 24	3.0	4.2	2.8	4.0
25 - 29	3.3	4.8	3.0	4.4
30 - 34	4.1	5.6	3.7	5.2
35 - 39	5.3	5.9	5.0	5.6
40 - 44	8.2	7.6	7.7	7.1
45 - 49	11.7	9.4	11.1	8.9
50 - 54	18.8			

Source : Guha Roy (1979).

The actual mortality experiences of married persons in developed countries are obviously not applicable to India, but the relation between the married ($_5m_x$) and general population ($_5M_x$) mortality rates obtaining in these countries may be assumed to hold approximately for India. We therefore look to average Swedish experience 1921-60 (Larsson, 1965), for these data are as good as any others for our illustration, and express the relative mortality differences as an index (Table 5.24), $_5I_x = 100\ _5m_x\ /\ _5M_x$. This index increases with age for both sexes, but males have smaller values compared to females, which is due mainly to " extra" mortality risks associated with pregnancy and childbearing. The range of this index for males varies much less widely than that for females over the age groups.

On the consideration that mortality differentials between married and general population in India will be less pronounced as compared to contemporary developed regions, the upper values of $_5I_x$ (Table 5.24) are taken and assumed to apply for the period 1971-81. The $_5M_x$ values (Table 5.23) are next subjected to these indexes to get estimates of mortality ($_5m_x$) for married

Table 5.24. Index ($_5I_x$) for mortality rates for married persons with overall mortality by sex and age = 100

Age group (x, x+5)	$_5I_x$	
	Males	Females
20 - 24	50 - 64	71 - 88
25 - 29	63 - 68	71 - 93
30 - 34	69 - 73	84 - 95
35 - 39	76 - 82	87 - 96
40 - 44	83 - 86	90 - 97[a]
45 - 49	87 - 89	90 - 97[a]
50 - 54	90 - 93	

Source : Derived from Swedish Life Tables, 1921-60 (Larsson, 1965).

[a]Adjusted slightly to have a smooth series.

persons. These, along with levels of the UN Model Life Tables (UML) and corresponding $\mathring{e}_0$ values, are shown in Table 5.25. The $_5L_x$ function at these levels are found, and used to estimate risk of widowhood (μ) for the quinquennia 1971-76 and 1976-81 (Table 5.26). In this calculation, we take spousal age difference, D = 5 years and projection period, T = 5 years. It is to be noted that the same rate of mortality decline has been assumed for married persons as for general population.

The effect of improving mortality conditions is reflected in more stable marital unions and reduced risks of widowhood, both varying with age but in opposite directions. These estimates, which may be improved further, should ideally be compared with corresponding parameters in other developing countries. Since no comparable data are known to be available, we estimate life expectancy at age x ($\mathring{e}_x$) for married females from our data for comparison with those available for Philippines (Table 5.27). In respect of $\mathring{e}_x$, the disparity between the countries is very large in all ages, and the married Filipino women are expected to live about 7-10 years longer than their Indian

Table 5.25. Mortality rates and interpolated levels (UNL and $\overset{o}{e}_o$) for married persons by age and sex : India, 1971-81 (Rates per thousand)

Age group (x, x+5)	Males			Females		
	$_5m_x$	UNL[a]	$\overset{o}{e}_o$	$_5m_x$	UNL[a]	$\overset{o}{e}_o$
1971-76						
15 - 19				2.9	74	56.8
20 - 24	1.9	96	68.7	3.7	76	57.6
25 - 29	2.2	94	67.6	4.5	72	56.1
30 - 34	3.0	88	64.8	5.3	69	54.3
35 - 39	4.3	82	61.3	5.7	69	54.3
40 - 44	7.1	72	56.0	7.4	64	51.9
45 - 49	10.4	68	54.1	9.1	64	52.2
50 - 54	17.5	58	49.1			
1976-81						
15 - 19				2.8	75	57.6
20 - 24	1.8	97	69.0	3.5	77	58.8
25 - 29	2.0	96	68.5	4.1	75	57.6
30 - 34	2.7	91	66.2	4.9	71	55.7
35 - 39	4.1	83	62.2	5.4	71	55.3
40 - 44	6.6	75	57.6	6.9	66	53.2
45 - 49	9.9	70	55.2	8.6	67	53.4
50 - 54	16.8	60	50.2			

[a]Approximate, to the nearest unit.

Table 5.26. Estimating probability of marriage remaining intact (P) over the period and annual risk of wodowhood (μ) : India, 1971-81

Age x	Person-years lived by married females in 5-year interval $_5L_x$	$_5L_{x+5}$	Person-years lived by married males in 5-year interval $_5L_{x+5}$	$_5L_{x+10}$	P	μ
			1971-76			
15	423765	420372	465635	455581	.9706	.0060
20	420372	409192	455581	434711	.9288	.1048
25	409192	384439	434711	406577	.8787	.0259
30	384439	374006	406577	362788	.8681	.0283
35	374006	346408	362788	334912	.8654	.0289
40	346408	334583	334912	278220	.8024	.0440
			1976-81			
15	427280	426230	466997	460168	.9830	.0034
20	426230	412238	460168	442650	.9304	.0144
25	412238	390638	442650	411960	.8819	.0251
30	390638	375317	411960	373330	.8707	.0277
35	375317	355331	373330	342443	.8684	.0282
40	355331	343215	342443	286148	.8071	.0429

Sources : Derived from Table 5.25 and the formulae given in the text.

counterparts. This is borne out by the fact that the general mortality conditions in the Philippines (CDR = 10.5, $\mathring{e}_0$ = 60 in 1970-75) are much improved (so should be for married persons) than for some developing countries, such as India (CDR = 17.5, $\mathring{e}_0$ = 50 or so in 1971-75). The risks of widowhood are thus likely to be much lower in Philippines than in India.

Table 5.27. Comparison of estimated life expectancies ($\overset{o}{e}_x$) for married females : India (1971) and Philippines (1970)

Age x	India, 1971 T_x	India, 1971 l_x	India, 1971 $\overset{o}{e}_x$	Philippines, 1970 $\overset{o}{e}_x$	Difference ($\overset{o}{e}_x$) in years
15	3839835	85370	44.98	54.73	9.75
20	3416070	85127	40.13	50.22	10.09
25	2995698	81736	36.65	45.82	9.17
30	2586506	77907	33.20	41.46	8.26
35	2202067	75868	29.02	37.13	8.11
40	1828061	70560	25.91	32.85	6.94
45	1481653	68556	21.61	28.61	7.00
50	1093017	54083	20.21	26.96	6.75

Sources : Data on Philippines are obtained from Adelamer N. Alcantara (1975) and those on India are estimated from UN Model Life Tables.

5.7 Summary and discussion

The Indian censuses provide marital status data cross-classified by sex and age. A direct question on age at marriage has also been included in the recent censuses. The main purpose for collecting marriage history is to study family formation as it affects fertility to a large extent. There are however several limitations in the general state of information and in the quality of data on marital status. Firstly, no data on the number of annual marriages are available for computing marriage rates. There exists no complete national registration system, and the National Sample Surveys (NSS) do not also help much in this respect. Another important item that has been ignored in the statistical system is the collection of data on mortality by marital status. Secondly, the reported data too may be subject to substantial ascertainment errors, as has been indicated by various scholars. The nature of such errors, among others, are likely to be misclassification of marital status, misreporting of age, evasive answer bias, and possible lack of uniformity in adhering to a common de facto definition of marital status. One indication of errors in the data is given by the ratios of proportions single in successive age groups in two consecutive censuses, and they have been found, in some cases, to exceed unity in the absence of any significant migration. Any nuptiality analysis based on raw data of unknown reliability might thus be misleading.

The inadequacy of information on nuptiality, whether through unavailability or unknown reliability, is the crux of the problem. The imperative need for improvement of the fact base through demographic surveys and studies has been voiced time and again by eminent Indian Demographers. While little has so far been done in this direction, no systematic analysis has also been undertaken to evaluate the accuracy of data on marital status. In spite of these limitations, interesting analyses were made by S.N. Agarwala and others. In the present study, we focus mainly on adjustment of data to the extent possible and make plausible future estimates of the components of marital status for females. We believe that consideration of proportions single, married and widowed-divorced-separated in one comprehensive study makes it possible to arrive at consistent estimates.

In order to achieve reasonable accuracy in the data on marital status, we use a logit-linear model as a checking device. The model, successfully used for mortality and postpartum variables, seems to have been useful in adjusting reported schedules single and married, and in estimating population characteristics from the adjusted distributions.

The proportions single can be studied by reference to the eight observed schedules for the period 1901-1971. A comparison of the model and observed schedules reveals the nature of age distortions. Specifically, the model seems to reduce the inflated (due to understatement of age)proportions single reported in age group 15-19 by transferring the estimated excess to higher ages, and thus adjust the apparent shortfall in age range 20-29.

The data sets for proportions married by age are available only in the censuses of 1961 and 1971, and estimated for 1951 by pooling relevant data. While the agreement between observed and model schedules is very close for 1971 — revealing better reporting in this census, the marked differences in these schedules in the earlier censuses of 1951 and 1961 indicate higher misreporting errors.

We attempt to make a set of short-run projections of the female proportions single from the time series of the model schedules. In this context we argue that the forces which have prompted marriage delay in the recent decades resulting in the on-going shift in nuptiality should gather momentum in the coming years. At present there seems to be no tendency for proportions single to stabilise by the period for which estimates are made. Several mathematical curves are tried, and a quadratic exponential seems to give satisfactory fit. The parameters of this curve vary fairly smoothly with age to permit efficient use of the graduation method. This yields first approximations for the proportions single, improved later by using consistency criteria.

The proportions single in the 15-49 age range for each of the years 1981-86 are first obtained to extend the census cross-sections available decennially. The proportions single for an intermediate quinquennium are subsequently estimated to conform to model schedules of two bordering years. For the age group 15-19, there is almost an uniform tendency all through for the proportions to rise across time, whereas the smooth upward trends at ages 20-24 start after 1961. In the middle and older age ranges, the proportions single are, more or less, stabilised by 1970s at lower levels than in the first half of the century.

The conventional projection procedure for proportions married being difficult to apply for lack of annual number of marriages, Coale's three-parameter model is first used to project proportion ever-married. This is done on the assumptions of three (low, medium and high) plausible changes in the age at marriage entrance, the tempo at which marriage occurs, and the final proportions married. The accepted medium projection is based on the assumptions that the two timing parameters will continue to shift linearly at 1941-71 average annual rate over the study period (upto 1986), while the near universal marriage pattern will, more or less, persist. These independent estimates of ever-married proportions of females are compared with the corresponding proportions indirectly estimated from the model schedules of proportions single. Barring a few cases, the comparison reveals small differences between the estimates.

We next try to obtain combined estimates for widowed and divorced females. The short-run future trend is derived on two assumptions. As one alternative, we assume that the changes in the proportions ever-married is due largely to change in the proportions currently married, whereas the components comprising " marriage disruption" remain, more or less, at the adjusted 1971 levels at least upto 1986. In the second alternative, we assume that these proportions will decline approximately at the same annual rate as observed during 1961-71.

The estimates of the female proportions married are derived for the period 1981-86 from the above estimates of proportions ever-married and widowed-divorced-separated. Four sets of estimates thus obtained are compared. At this stage, we verify that the proportions single and married, and combined proportions of widowed-divorced-separated do not give rise to inconsistent results.

The marriage patterns as manifested from the trends in the singulate mean ages at effective marriage (SMAEM) are studied with reference to Hajnal's one-period, Agarwala's two-period and genuine birth cohorts. In India, exposure to the risk of conception and childbearing is confined to intervals of marriage, and no " effective" marriage is assumed in this study to occur earlier than age 15. At about the middle of the current century, the age at effective marriage of the Indian females started rising from its earlier approximately stable levels. For the first six birth cohorts, 1881-86 to 1906-11, the SMAEMs remained steady at around sixteen years, and then started accelerating. The period SMAEMs estimated by two methods present interesting comparison. When there has been a clear cut upward trend in nuptiality, the period means obtained by Agarwala's method are higher than (as from the quinquennium 1946-51 through 1981-86) or equal to the neighbouring one-period means. Because of a lack of regular upward trend, earlier quinquennia do not show such systematic relationship between the two period means. Corresponding to our estimates of one-period and two-period SMAEMs of 17.7 (1971) and 17.9 (1966-71) years respectively, the all-India fertility survey (1972) gives the ages at effective marriage as 17.4 years for rural areas, and 18.5 years for urban areas. On the whole, it may be concluded that the later-marriage norm is being accepted generally, thus raising the age at marriage.

The emergence of the above trend in marriage behaviour is substantiated with reference to Coale's index (I). Analogous to this index for currently married, indexes for single, ever-married, not currently married and marital dissolution are also derived for 1961 and 1971, and for each of the years for the period 1981-86. The values of the component indexes change over the period in a manner consistent with the rising female ages at marriage.

As is already mentioned, not much is known about the Indian mortality patterns of currently married persons in general and age patterns of widowhood in particular. The input data are either not available or available in defective state. Without foregoing the issue, we arrive at first approximations of these parameters by trying to build up required data, sometimes looking to other cultures for relevant information. The main conclusion from the analysis made with the help of a computer model, MORT, is that the risk of widowhood shows a steady rise with older cohorts, a feature to be expected on the basis of accumulated risks. This risk also seems to decline over time because of improving (slowed down recently) mortality trend.

The Indian Census reports show that the mean ages at marriage for boys and girls differed by about seven years before 1931, but the difference came down to around five years during 1931-71. Recent indication is that the age difference may rise again. In view of this, we try an exception to the simplified assumption of general five-year difference, and consider all interacting ages (derived from the distributions of married spouses) of the marriageable number of boys and girls to express the inter-sex difference of availability of mates by an arithmetic index termed as " Sexual Dimorphism Percentage". The index shows the relationship between the numbers of males and females in various age groups that are likely to contact marriages with each other. Our estimated values show serious imbalances between the numbers of the sexes in marriageable ages. The numerical superiority of males over females in each age bracket (e.g., 15-19 for females and 15-39 for males) explains why there are greater proportions of never-married males compared to never-married females in the population.

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