Topological Aspects of High-T_c Superconductivity, Fractional Quantum Hall Effect and Berry Phase

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Abstract

We have analysed here the equivalence of RVB states with $\nu = 1/2$ FQH states in terms of the Berry Phase which is associated with the chiral anomaly in 3+1 dimensions. It is observed that the 3-dimensional spinons and holons are chracterised by the non-Abelian Berry phase and these reduce to 1/2 fractional statistics when the motion is confined to the equatorial planes. The topological mechanism of superconductivity is analogous to the topological aspects of fractional quantum Hall effect with $\nu = 1/2$.

1 Introduction

The revival of interest in the resonating valence bond states is due to the discovery of high temperature superconductors. In 1973, Anderson [1] suggested that compared with the classical Neel state, a better ground state could be obtained by allowing nearby spins to couple in singlet pairs. Such singlet bonds can exist in any lattice, in a large variety of spatial configurations. Fazekas and Anderson [2] substantiated this idea with s=1/2 spins on a triangular lattice. The ground state is then a sort of (quantum) liquid of singlet bonds which can resonate among different configurations, whence the name Resonating Valence Bond (RVB).

Kivelson et al [3] analysed the structure of an RVB state on a 2D square lattice and predicted the existence of peculiar topological elementary excitations, having the nature of neutral, spin 1/2 excitations, roughly described as *dangling bonds*, i.e. resulting from the breaking of a singlet pair.

In an earlier paper, Kalmeyer and Laughlin [4] have proposed that the ground state of the frustrated Heisenberg antiferromagnet in two dimensions and the fractional quantum Hall state for bosons might be the *same* in the sense that the two systems could be adiabatically evolved into one another without crossing a phase boundary. This is compatible with the RVB concept of Anderson to be operating in the 2D Heisenbereg antiferromagnet in a triangular lattice where the ground state is understood to be a nondegenerate quantum liquid with an energy gap [1,5,6]. Kalmeyer and Laughlin [4,5] have shown that the fractional Hall state with $\nu = 1/2$ confined to a triangular lattice has the same variational energy as the Anderson RVB state. This strongly suggests that the RVB state and FQH state with $\nu = 1/2$ are the same thing where the ground state is a nondegenerate singlet and the elementary excitations are neutral spin 1/2 fermions. In this note we want to study the topological aspects associated with frustrated Heisenberg antiferromagnet and show the equivalence of RVB state and FQH state with $\nu = 1/2$ in terms of the Berry phase involved in the system.

Indeed some years ago, Libby, Zou and Laughlin [7] have found that adiabatic interchange of three dimensional spinons and holons produces nonzero Berry phase and those reduce to 1/2 fractional statistics when motion is confined to equatorial planes. These results discredict the importance of two dimensionality in this type of problems and raise the possibility of monopole superconductivity in three dimensional Mott insulators. Otherwise, noting that most high temperature superconductors are planar materials, mechanism of superconductivity, namely, pairing by fractional statistics [8] has thus far been investigated only in two spatial dimensions. The discovery of quantum disorder in any translationally invariant spin 1/2 antiferromagnet requires one to consider the existence of neutral spin 1/2 excitations, spinon which may be considered as carrying fractional quantum number. These in turn lead to charged spinless excitations, called holons, which interact by means of a gauge forces potentially capable of using superconductivity. Indeed, Libby, Zou and Laughlin have reported the existence of such forces in three dimensions.

In a series of papers [9-13], we have given a microscopic theory of FQHE in terms of Berry phase which is associated with chiral anomaly in 3+1 dimension. We have considered the 2DEG of N particles on the surface of a three dimensional sphere of an infinitely large radius R in a radial (monopole) magnetic field. As a result, the variational wave function of the homogeneous states of incompressible fluid with finite N are constructed following Haldane [14] and in the spirit of Laughlin [15]. The wavefunction describes a state without gapless excitations and is equivalently characterised as an incompressible fluid at occupations $\nu = 1/m$, m being an odd integer. ν is related to the Berry phase generated by the chiral symmetry breaking caused by the external magnetic field. In the FQH state with even denominator filling factor [11], the Berry phase is removed to the dynamical phase and we can observe the Berry phase when the state is split into pairs of degenerate fermions giving rise to the non-Abelian Berry phase. Indeed, it is found [12] that a $\nu = 1/2$ FQH state can be represented by a pair of spin polarised electrons in a p-wave state which has odd parity. Evidently, this resembles the RVB state.

In this note we shall study the topological aspects associated with frustrated Heisenberg antiferromagnet and consider the equivalence of RVB state and FQH state with $\nu = 1/2$ in terms of the Berry phase involved in the system and consider high-T_c superconductivity from this viewpoint. In section 2 we shall discuss the topological aspects of Heisenberg antiferromagnet on a triangular lattice and RVB states. In section 3 we shall relate RVB states with $\nu = 1/2$ from the viewpoint of Berry phase and finally in section 4 we shall consider the relationship between high- T_c superconductivity and FQH states with $\nu = 1/2$ in terms of non-zero Berry phase.

2 Topological Aspects of Heisenberg Antiferromagnet on a Triangular Lattice and Resonating Valence Bond States

We consider the antiferromagnetic Heisenberg Hamiltonian

$$H = J \sum_{i,j} S_i . S_j \quad , \qquad J > 0 \tag{1}$$

where the sum is over all pairs of nearest neighbour sites of the 2D triangular lattice and

$$\vec{S_i} = \frac{1}{2} \hbar \ \vec{\sigma_i}$$

is the spin operator of the i-th site. The Heisenberg antiferromagnet on a triangular lattice is always characterised by frustration which gives rise to chirality. In our framework this chirality is associated with the Berry phase of our system of interest which is a 2D triangular lattice on the surface of a 3D sphere of large radius R.

Kalmeyer and Laughlin [4] utilising the procedure of Holstein-Primakoff transformation [16] considered the spin problem as a lattice gas by imagining an *atom* to be present on every site with an up spin. The atoms are then bosons with creation operators

$$a_j^\dagger = \hbar^{-1}(S_j^x + iS_j^y)$$

The Hamiltonian (1) then becomes

$$H = T + V \tag{2}$$

where

$$T = \frac{1}{2} \ J \sum_{i,j} \ (\ a_j^\dagger a_i \ + \ a_i^\dagger a_j \ ) \tag{3}$$

and

$$V = J \sum_{i,j} \ (a_j^\dagger a_i a_i a_j^\dagger \ + \ \frac{1}{2} J N_s \ - \ 6J \sum_i \ a_i^\dagger \ a_i \ ) \tag{4}$$

where N_s is the no. of spins or lattice sites. The boson kinetic energy operator T comes from the spin exchange or XY part of the Heisenberg interaction. The near neighbour repulsion of potential energy comes from the Ising part. Now, we see that T given by eqn.(3) does not have the right free particle form because the hopping matrix elements $J_{ij} = J$ are positive. This makes the energy bands disperse down as one moves away from the centre of the Brillouin zone. To avoid this, they considered J_{ij} to be matrix elements of right sign, namely negetive, in the presence of a fictitious vector potential $\vec{A}$, where

$$\vec{A} \ = \ \frac{1}{2} \ B \ (\ x\hat{y} \ - \ y\hat{x} \ ) \tag{5}$$

with a particular value of B. Assigning an arbitary charge e^* to the bosons and coupling them to $\vec{A}$, introduces phases into the matrix element J_{ij} according to

$$J_{ij} \ \rightarrow \tilde{J}_{ij} \ = \ - \ \exp [\frac{2\pi i}{\phi_0} \ \int_i^j \ A.ds] \tag{6}$$

where $\phi_0 = \frac{hc}{e^*}$ is the quantum of flux associated with bosons of charge e^*. If we choose B in such a way that

$$\sqrt{3}\, a_0^2 \;=\; 4\pi\, l_0^2 \tag{7}$$

where a_0 is the lattice constant and $l_0 = (\frac{e^* B}{\hbar c})^{-1/2}$ is the magnetic length then all phase factors are real and corresponds to one fictitious flux quantum per spin. For a closed loop this phase factor corresponds to the Berry phase when the 2D lattice gas is taken to reside on the surface of a 3D sphere in a radial (monopole) magnetic field.

It may be observed here that the Berry phase is associated with the chiral anomaly which is caused by quantum mechanical symmetry breaking when a chiral current interacts with a gauge field. Indeed, the divergence of the axial vector current in the quantum mechanical case does not vanish and is associated with the topological quantity known as the Pontryagin index through the relation

$$q \;=\; -\frac{1}{16\pi^2}\int Tr^* F_{\mu\nu}\; F_{\mu\nu}\; d^4x \;=\; -\frac{1}{2}\int \partial_\mu\; J_\mu^5\; d^4x \tag{8}$$

where J_μ^5 is the axial vector current and q is the Pontryagin index. The Pontryagin index associated with the integral of the chiral anomaly is related to the Berry phase which arises when a quantum particle described by a parameter dependent Hamiltonian moves in a closed path. Indeed, the Berry phase acquired by such a particle is given by $e^{i\phi_B}$ where

$$\phi_B \;=\; 2\pi\mu \;=\; \pi q \tag{9}$$

with the relation $q = 2\mu$ [17]. Here μ appears as a monopole strength.

In a 3D anisotropic space, we can construct the spherical harmonics $Y_l^{m,\mu}$ with $l = 1/2$, $|m| = |\mu| = 1/2$ when the angular momentum relation is given by

$$\vec{J} \;=\; \vec{r}\times\vec{p} \;-\; \mu\vec{r} \tag{10}$$

where μ can take the values as $0, \pm 1/2, \pm 1, \pm 3/2, \ldots$ This is similar to the angular momentum relation in the case when a charged particle moves in the field of a magnetic monopole. Fierz [18] and Hurst [19] have studied the spherical harmonics $Y_l^{m,\mu}$. Following them we can write

$$Y_l^{m,\mu} \;=\; (1+x)^{-\frac{(m-\mu)}{2}}(1-x)^{\frac{-(m+\mu)}{2}}.\frac{d^{l-m}}{d^{l-m}x}\left[(1+x)^{l-\mu}(1-x)^{l+\mu}\right] e^{im\phi}\; e^{-i\mu\chi} \tag{11}$$

where $x = cos\theta$ and the quantity m and μ just represent the eigenvalues of the operator $i\frac{\partial}{\partial\phi}$ and $i\frac{\partial}{\partial\chi}$ respectively. It is noted that apart from the usual angles θ and ϕ, we have an extra angle χ denoting the rotational orientation around a specified fixed axis attached to a space-time point x_μ giving rise to an anisotropy in the space-time manifold [20]. The fact that in such an anisotropic space the angular momentum can take the value 1/2 is found to be analogous to the result that a monopole charged particle composite representing a dyon satisfying the condition $e\mu = 1/2$ has its angular momentum shifted by 1/2 unit and its statistics shifted accordingly [21].

Now when a 2D triangular lattice is taken to reside on the surface of a 3D sphere of large radius in a radial (monopole) magnetic field, we can associate the chirality with the

Berry phase and the antiferromagnetic Heisenberg Hamiltonian can be represented by an anisotropic Hamiltonian [22].

$$H = J \sum (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) \tag{12}$$

where $J > 0$ and $\Delta \geq 0$, Δ the anisotropic parameter is given by $\Delta = \frac{2\mu+1}{2}$ where μ is related to the Berry phase factor. It is noted that $\Delta = 1$ one has the antiferromagnetic Heisenberg model and for $\Delta \rightarrow \infty$ one has the Ising model. When $\Delta = 0$, we have the XX model. It may be mentioned here that the relationship of the anisotropic parameter Δ with the Berry phase factor μ has been formulated from an analysis of the relationship between the conformal field theory in 1+1 dimension, Chern-Simon theory in 2+1 dimension and chiral anomaly in 3+1 dimension [22]. Indeed the association of conformal field theory with quantum group having the deformation parameter q of the deformed algebra $U_q(SL(2))$ helps us to relate the deformation parameter q with the Berry phase factor μ. In the framework of quantum group, we can consider the following open chain Hamiltonian [23]

$$H_N(q,y) = \frac{J}{4}\left(\sum_{i=1}^{N-1} \sigma_i^x \ \sigma_{i+1}^x + \sigma_i^y \ \sigma_{i+1}^y + \frac{q+q^{-1}}{2}\sigma_i^z \ \sigma_{i+1}^z - \frac{q-q^{-1}}{2}(\sigma_1^z - \sigma_N^z)\right) \tag{13}$$

which maintains quantum group symmetry $SU_q(2)$. In the limit $q \rightarrow 1$, one recovers the usual $SU(2)$ algebra. Now if we set $\Delta = \frac{q+q^{-1}}{2}$, we see from eqns.(12) and (13) that the bulk terms of (13) and the one in (12) coincide. The only difference appears in the boundary term of (13) which is essential for the quantum group symmetry. As $\mu = 1/2$ represents a free fermion case corresponding to $q = 1$, we note that in the Hamiltonian (13) (neglecting the boundary term) if we choose $\frac{q+q^{-1}}{2} = \frac{2\mu+1}{2}$, we get the isotropic Heisenberg model.

Now for a 2D triangular lattice on the surface of a 3D sphere in a radial magnetic field, the chirality demands that μ is non-zero and should be given by $|\mu| = 1/2$ in the ground state. However, for $\mu = 1/2$, we get the antiferromagnetic Heisenberg Hamiltonian without frustration. So a frustrated spin system should be given by $\mu = -1/2$ suggesting $\Delta = \frac{2\mu+1}{2} = 0$ in the Hamiltonian (12). But with $\Delta = 0$, this Hamiltonian effectively represents the XX model corresponding to a bosonic system represented by singlets of spin pairs. This eventually leads to the resonating valence bond state giving rise to a nondegenerate quantum liquid.

3 Resonating Valence Bond, Fractional Quantum Hall Effect with $\nu = 1/2$ and Berry Phase

We have shown that when an anisotropic Heisenberg Hamiltonian of an antiferromagnetic spin system is characterised by a particular value of Berry phase factor related to a fixed value of anisotropic parameter, then the system leads to the generation of quantum spin liquid corresponding to a resonating valence bond state. The RVB state is characterised by the anisotropic parameter related to the particular value of Berry phase. We point out here that the spin singlet states forming the quantum liquid are equivalent to FQH liquid with filling factor $\nu = 1/2$. Indeed, in some earlier papers [9-13], we have pointed out that in QHE the external magnetic field causes a chiral symmetry breaking of the fermions (Hall particles) and as a result an anomaly is realised in association with the quantization of Hall conductivity. This helped us to study the behaviour of a quantum Hall fluid from the viewpoint of the Berry phase which is linked with chiral anomaly. We consider a 2DEG of N-particles on the spherical surface of a 3D sphere in a radial (monopole) strong magnetic field. From the description of spherical harmonics $Y_l^{m,\mu}$ given before we can construct spinor

$$\theta = \begin{pmatrix} u \\ v \end{pmatrix}$$

where

$$\begin{aligned} u &= Y_{1/2}^{1/2,1/2} = sin(\theta/2)\ \exp\frac{i(\phi - \chi)}{2} \\ v &= Y_{1/2}^{-1/2,1/2} = cos(\theta/2)\ \exp\frac{-i(\phi + \chi)}{2} \end{aligned} \tag{14}$$

The N-particle wave function is then written as :

$$\psi_N^{(m)} = \prod_{i<j} (u_i v_j - u_j v_i)^m \tag{15}$$

where m is the inverse of the filling factor ν. Evidently, for odd (even) m, we will have the fermionic (bosonic) states. Following Haldane [14] we identified $m = J_{ij} = J_i + J_j$ for an N-particle system. It is noted that the $m = 1$ state which describes the complete filling of the lowest Landau level corresponds to the ground state for the contribution of the factor $\vec{r} \times \vec{p} = 0$ with $\mu = 1/2$. Indeed, following the Dirac quantization condition $e\mu = 1/2$ we note that the quasiparticle for $m = (\frac{1}{\nu}) = 1$ will exhibit the IQHE with fermion number 1. However, if we consider the state with $\vec{r} \times \vec{p} = 1$, the respective angular momentum is changed to $J = 3/2$ for $\mu = 1/2$. This can be viewed as a system with $\mu_{eff} = 3/2$ having $\vec{r} \times \vec{p} = 0$. This will give rise to $m = \frac{1}{\nu} = 3$ when the fermion number of the quasiparticle is found to be 1/3 which is evident from the relation $e\mu_{eff} = 1/2$. In this way, we can study all FQH states with $\nu = 1/m$, m being an odd integer given by $m = 2\mu_{eff}$. For states with $\nu = \frac{n}{2\mu_{eff}}$, we consider the higher Landau level with the Dirac quantization condition $e\mu = \frac{1}{2}n$ where n can be viewed as a vortex of strength 2l+1. The states with $\nu = \frac{n'}{2\mu_{eff}}$, n' being an even integer, can be generated through conjugate states [11,13].

This analysis suggests that for the FQH fluid with even denominator filling factor we face a peculiar situation. For example, for the FQH state with $\nu = 1/2$ the Dirac quantization condition $e\,\mu = 1/2$ suggests that $\mu = 1$. Then in the angular momentum relation

$$\vec{J} = \vec{r} \times \vec{p} - \mu\vec{r}$$

for $\mu = 1$ (or an integer) we can use a transformation which effectively suggests that we can have a dynamical relation of the form

$$\vec{J} = \vec{r} \times \vec{p} - \mu\vec{r} = \vec{r'} \times \vec{p'} \tag{16}$$

This equation indicates that the Berry phase which is associated with μ may be unitarily removed to the dynamical phase. This implies that the average magnetic field may be taken to be vanishing in these states. However, to observe the effect of the Berry phase, we can split the state into a pair of electrons, each with the constraint of representing the state $\mu = \pm 1/2$. Now as $|\mu| = 1$ is achieved for a pair of electrons each having either $\mu = +1/2$ or $-1/2$. We can construct the two-component spinor

$$\theta = \begin{pmatrix} u \\ v \end{pmatrix}$$

from $Y_l^{m,\mu}$ with $|\mu| = |m| = 1/2$ where

$$u = Y_{1/2}^{1/2,1/2} \quad , \quad v = Y_{1/2}^{-1/2,1/2} \tag{17}$$

The charge conjugate states are

$$\tilde{u} = Y_{1/2}^{-1/2,1/2} \quad , \quad \tilde{v} = Y_{1/2}^{1/2,-1/2} \tag{18}$$

This indicates that electrons in the pair have a definite chirality given by $\mu = 1/2(-1/2)$. So each electron in the pair is spin polarised. Thus for $|\mu| = 1$ we can depict the pair as a p-wave state of spin polarised electrons [24].

From this analysis, it appears that these pairs will give rise to the SU(2) symmetry, as we can consider the state of these two electrons as a SU(2) doublet. We know that the usual Laughlin state for a quantum Hall fluid can be represented by the Abelian U(1) symmetry having an Abelian Berry phase. Due to pairing, in the even denominator quantum Hall states we will have $SU(2) \times U(1)$ symmetry indicating that there will be a non-Abelian Berry phase. Thus these states will represent non-Abelian Hall fluid.

It may be observed that for an odd parity p-wave paired FQH state for spin singlets of polarised electrons, the total wave function is given by the product of the Pfaffian wave function ϕ_{pf} and the Laughlin wave function ϕ_m as

$$\phi_p = \phi_{pf}\phi_m \tag{19}$$

$$\phi_{pf} = \mathcal{A}\left(\frac{1}{(z_1 - z_2)\ (z_3 - z_4)\ \ldots\ldots}\right) \tag{20}$$

$$\phi_m = \left(\prod_{i<j}(z_i - z_j)^m\ exp(-1/4\sum |z_i|^2)\right) \tag{21}$$

where $z_i = x_i + iy_i$ and $\mathcal{A}$ is the antisymmetrization operator and $m = \frac{1}{\nu} = 2$. It has been shown that the non-Abelian part of the bulk wave function ϕ_{pf} can be written as a correlation of the primary fields ψ in the Ising model where ψ represents the free fermion field in a free Majorana fermion theory. In the Berry phase formalism each fermion is characterised by $|\mu| = 1/2$ forming the pair. The Abelian part ϕ_m with $m = 2$ represents the singlet formed by a spin pair and corresponds to a bosonic case.

These odd parity singlets may be taken to be equivalent to spin singlets of RVB states generating a quantum liquid. Indeed, we may observe that a many body system comprising the pairs of spin singlet states will give rise to an antiferromagnetic chain. In other words, an anisotropic antiferromagnetic Heisenberg model with a particular value of anisotropic parameter $\Delta = 0$, (which corresponds to a particular value of Berry phase factor) will effectively give rise to the RVB states generating the quantum spin liquid. Equivalently the anisotropic Heisenberg Hamiltonian (12) which reduces the bosonic XX model with anisotropic parameter $\Delta = 0$, represents the FQH bosonic state with $\nu = \frac{1}{m} = 1/2$. The correlation of FQH states with $\nu = 1/2$ and RVB spin singlet states can be represented in our formalism as

$$\psi_{\nu=1/2} = \phi_{pf} \; \psi_N^{m=2} \equiv \psi_{RVB} \tag{22}$$

where ϕ_{pf} and $\psi_N^{m=2}$ is given by (20) and (15) respectively.

4 High T_c Superconductivity, Fractional Quantum Hall Effect with $\nu = 1/2$ and Berry phase

The equivalence of RVB states with $\nu = 1/2$ FQH states are characterised by neutral spin 1/2 excitations called spinons and charged spinless excitations called holons. To study these excitations in the framework of our present analysis, we note that a single spin down electron at a site j is surrounded by an otherwise featureless spin liquid representing a RVB state. The 3D formulation of such a system will be taken to be such that 2D frustrated spin system lies on the surface of a 3D sphere in a radial (monopole) magnetic field which gives rise to the chirality associated with the magnetic monopole. This in turn gives rise to the Berry phase factor $\mu = -1/2$. Now the single spin down state characterised by $\mu = -1/2$ when coupled with this *monopole* represented by $\mu = -1/2$ will give rise to a state having $\mu = -1$. As described in previous section, the Berry phase factor $|\mu| = 1$ effectively gives rise to a FQH state with $\nu = 1/2$. Indeed, the state characterised by $|\mu| = 1$ is formed by the single spin state ($\mu = 1/2$) in the spin liquid and the *orbital spin* caused by the *monopole* represented by the $\mu = -1/2$ characteristic of a triangular lattice in three dimensions. In this framework, the neutral spin 1/2 excitation, the spinon is such that, the elementary spin 1 excitation characterised by $|\mu| = 1$ may split into two parts, with one spin 1/2 excitation in the bulk and the other part is due to the *orbital spin* which is in the background characterised by the chirality of a triangular lattice. This is analogous to the idea of Laughlin [5,6] that spinons obey spin 1/2 statistics.

Now when a hole is introduced into the system by doping, this may combine with this spinon giving rise to a spinless charged excitation known as holons. In these frame work holons may also be represented by FQH liquid $\nu = 1/2$ corresponding to a singlet characterised by a flux $\phi_0 = \frac{hc}{2e}$. Obviously these resemble Cooper pairs. As these states correspond to FQH liquid with $\nu = 1/2$, the ground state will represent a quantum liquid with an energy gap. This corroborates with the idea of Laughlin [25] that a gas of such particles might actually be a superconductor with a charge 2 order parameter. As stated by Laughlin [25], the analog of the fractionally charged quasiparticle is the *spinon*, the analog of the compressional sound wave is an *antiferromagnetic spin wave*, the analog of Wigner crystalisation is *antiferromagnetic ordering* and the analog of the magnetoroton gap is a *magnetic fluctuation gap*. The gap of the spin wave spectrum of the magnet gives the measure of how *liquid* the spin liquid is.

It may added that Weigmann [26,27] has studied topological mechanism of superconductivity where it is argued that strongly correlated electronic systems represent physical systems where topological fluid may appear. Noting that there are two operators which characterize the ground state of an antiferromagnet, namely density of energy

$$\epsilon_{ij} \;=\; (1/4 \;+\; \vec{S}_i.\vec{S}_j) \tag{23}$$

and chirality or measure of topological order

$$W(C) \;=\; Tr \prod_{i \in C} (1/2 \;+\; \vec{\sigma}.\vec{S}_i) \tag{24}$$

where σ are Pauli matrices and C is the lattice contour. Weigmann has related these operators with the amplitude and phase Δ_{ij} of Anderson's RVB through

$$\epsilon_{ij} \;=\; |\Delta_{ij}|^2 \tag{25}$$

$$W(C) \;=\; \prod_{C} \Delta_{ij} \tag{26}$$

This suggests that Δ_{ij} is a gauge field. It can be locally transformed by a U(1) transformation

$$\Delta_{ij} \;\longrightarrow\; \Delta_{ij}\; e^{i(\alpha_i \;-\; \alpha_j)} \tag{27}$$

The topological order parameter W(C) aquires the form of a lattice Wilson loop

$$W(C) \;=\; e^{i\phi(C)} \tag{28}$$

which is associated with the flux of the RVB field

$$e^{i\phi(C)} \;=\; \prod_{C} e^{iA_{ij}} \tag{29}$$

where A_{ij} is a phase of Δ_{ij} representing a magnetic flux which penetrates through a surface enclosed by the contour C. This phase is essentially the Berry phase related to chiral anomaly when we describe the system in three dimensions through the relation

$$W(C) = \; e^{i2\pi\mu} \tag{30}$$

when the 2D surface is considered to be the surface of a 3D sphere of large radius in a radial magnetic field. Infact, we have formulated here topological superconductivity and showed that it is associated with the topological aspects of FQH fluid with $\nu \;=\; 1/2$.

5 Discussion

Kalmeyer and Laughlin [4] have pointed out the equivalence of the ground states of the frustrated Heisenberg antiferromagnet in 2D and FQH states with $\nu \;=\; 1/2$ in the sense that the two system could be adiabatically evolved into one another without crossing a phase boundary. We have studied here the topological aspects of RVB states in terms of the Berry phase and have shown its relationship with the topological aspects of FQH states with $\nu = 1/2$. Indeed, we have shown here that three dimensional spinons and holons produce nonzero Berry phase and these reduce to 1/2 fractional statistics when motion is confined to equatorial phases. This is in confirmity with the idea of Laughlin. In view of this we have interpreted here high T_c superconductivity as topological superconductivity and is not just confined to 2D planes. In fact, there is a possibility that 3D Mott insulators may also produce superconductivity with proper doping. Weigmann [27] have argued that topological superconductivity - a result of developing of topological order - takes place in some models of highly correlated electron systems in three dimensions. We shall study the Meissner effect in this framework in our further report.

It may be pointed out here that the equivalence of RVB states with FQH states with $\nu = 1/2$ also raises the possibilities that high T_c superconductivity may be studied in the framework of conformal field theory as the FQH states are being studied from this viewpoint also. In fact we have noted that RVB states are formed when the anisotropic parameter $\Delta = \frac{2\mu + 1}{2}$ in the Hamiltonian (12) is zero indicating that $\mu = -1/2$. Now from an analysis of the relationship between the conformal field theory in 1+1 dimension, Chern-Simons theory in 2+1 dimension and chiral anomaly in 3+1 dimension a relationship between the central charge c in conformal field theory and the Berry phase factor μ associated with the chiral anomaly may be established and it is found to be given by $c = 1 - \frac{6}{m(m+1)}$ with $m = 2\mu + 2$ [22]. This suggests that $\mu = 1/2$ effectively corresponds to $c = -2$. Indeed, the non-Abelian part of the wave function for some FQH states with even denominator filling factor have been studied in the framework of conformal field theory with $c = -2$. It may be added here that there exists a class of conformal field theories with a chiral algebra which may be associated with Wess-Zemino-Witten (WZW) theories. In view of this, we may have another version of topological superconductivity in terms of WZW theories [28].

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