On correlation between two fuzzy sets

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Abstract

In this paper we investigated the correlation between two fuzzy sets defined on the same universal support. It is shown that Spearman's rank correlation coefficient can be applied if the members of the supports are ranked according to the fuzzy membership values of each set. Next a membership-value-based fuzzy correlation measure is proposed and its properties are explained. The proposed measure is compared with an earlier definition [4].

Keywords: Correlation; Random variable; Spearman's rank correlation; Image processing

1. Introduction

Fuzzy set theory is a powerful tool to model imprecise and vague situations where exact analysis is either difficult or impossible. Central to the theory is the concept of fuzzy set, which is the generalisation of the ordinary (crisp) set that captures gradual transition from belongingness to nonbelongingness to the set. It is often theoritically interesting and practically necessary to see how different properties of crisp set can be generalised in the fuzzy situation. Such studies led to enormous interesting results, many of which are discussed in [3,1].

In statistics and in engineering sciences a concept called *correlation* is often used. By correlation analysis the joint relationship of two variables can be examined with the aid of a measure of interdependence of the two variables. Typical subjects for correlation analysis are the interdependence (asssociation) between height of father and height of (adult) son, between score in mathematics and score in statistics, etc. In all these cases the value taken by the variables is objective and the value obtained is the outcome of a purely random event. If $(x_i, y_i;\ i = 1, 2, \ldots, n)$ is the data set, then the correlation between x and y is defined as $r_{xy} = cov(x, y)/s_x s_y$ where $cov(x, y)$ is the covariance between x and y while s_x and s_y are the standard deviations of x and y [2].

But in reality there are many situations where instead of the measured values of the two variables of some objects or entities the ranks of the objects according to the two variables or according to two different qualitative characteristics is available. In this case *rank correlation* is used to correlate the two sets of available data. Such a situation arises when we have a qualitative rather than quantitative knowledge of the samples which can be ranked according to quality. Consider, for example, the quality of handwriting of individuals which has no obvious way to quantify. A typical rank correlation measure is due to Spearman [2].

But an interesting case arises if instead of ranks some subjective grading of the elements is available (the grading may be continuous or discrete). Then a fuzzy set can be formed based on such grading and the relation between two such fuzzy sets can be obtained by comparing the respective membership values in the sets. Thus, we can correlate fuzzy concepts such as big vs. heavy (positively correlated); beautiful vs. ugly (negatively correlated), etc.

It is interesting to see how the notion of correlation can be extended to fuzzy sets. It is possible that elements of the fuzzy sets are ranked in terms of membership and no numerical membership value is assigned to them. In that case an analogue of rank correlation should be found. In this paper fuzzy analogues of both types of correlation measure are proposed. The only related work is that due to Murthy et al. [4] who did not consider rank correlation measure. The relative merits of our proposal with that in [4] are described in Section 4.

2. Proposed measures of fuzzy correlation

The classical correlation measure is inherently statistical in nature. Although the fuzzy membership value lies in $[0, 1]$, the membership value does not have the same significance as that of probability. Moreover, fuzzy membership is often subjective in nature. Thus, the statistical correlation measure is not suitable to define the correlation between two fuzzy sets.

2.1. Fuzzy rank correlation measure

Suppose two fuzzy sets are defined on a universal set X, where we know the rank of any element in each fuzzy set. Let for an element $x_i \in X$ the rank in the first fuzzy set be C_i^1 while that in the second fuzzy set be C_i^2. Let n be equal to the number of elements in X. The difference between the ranks is $d_i = |C_i^1 - C_i^2|$. Now the rank correlation between them can be defined as follows.

Case 1: No ties among ranks.

If within each set the rank of an element is unique then the rank correlation between them is given by $r_R = 1 - 6\sum_i d_i^2/(n(n^2-1))$.

Case 2: Ties within ranks.

If within each set the rank of an element is not unique then the above measure should be modified as follows.

Let there be s tied ranks in the first fuzzy set and t tied ranks in the second fuzzy set. The number of elements which share the same rank is called the length of the tie at that rank.

Let the lengths of the s ties in the first fuzzy set be $k_1, k_2, \ldots, k_s$, respectively, and the lengths of the t ties in the second fuzzy set be $k_1', k_2', \ldots, k_t'$, respectively. Then the rank correlation is given by

$$r_R = \frac{(n^2-1)/12 - (T_u + T_v)/2 - \sum d_i^2/2n}{((n^2-1)/12 - T_u)^{1/2}((n^2-1)/12 - T_v)^{1/2}},$$

$$\text{where } T_u = \frac{\sum_{j=1}^{s}(k_j^3 - k_j)}{12n} \quad \text{and} \quad T_v = \frac{\sum_{j=1}^{t}(k_j'^3 - k_j')}{12n}.$$

This rank correlation can still be used if instead of ranks, the membership values of elements of the universe for each fuzzy set are known. This can be done by ranking the elements on the basis of their membership values and then applying Spearman's rank correlation coefficient. The fuzzy rank correlation coefficient defined in this manner satisfies all the properties of Spearman's rank correlation coefficient.

However, in this method only the rank is used while the information about the membership value is neglected in the correlation measure. The membership difference between some consecutive ranks may be very large and the membership difference between some other consecutive ranks may be very small. This is not taken care of in the rank correlation measure. Moreover, the rank correlation measure is defined only if the universal set X is finite. We propose here a correlation measure which is based on membership values and overcomes the above limitations.

2.2. Fuzzy membership correlation measure

Let there be two non-empty fuzzy sets $\tilde{A}$ and $\tilde{B}$ defined on the same universe X.

Let an element $x \in X$ belong to set $\tilde{A}$ with membership value μ and to set $\tilde{B}$ with membership value η where μ and η lie in the interval $[0,1]$ for all $x \in X$.

The proposed correlation measure should be symmetric with respect to μ, η and it must also depend on the membership values of all $x \in X$.

Since correlation essentially compares, it can be taken as a function which depends on $\sum_{x\in X} |\mu - \eta|$.

Since conventional correlation measure lies in $[-1,1]$, it is expected that fuzzy measure should also lie in this range. The measure should also be independent of change of scale of membership values. To achieve both the objectives we use a normalised form

$$\sum_{x\in X} \left| \frac{\mu(x)}{\sum_{x\in X}\mu(x)} - \frac{\eta(x)}{\sum_{x\in X}\eta(x)} \right|. \tag{1}$$

This term brings out the dissimilarity between the two fuzzy sets $\tilde{A}$ and $\tilde{B}$ in terms of the membership but correlation is essentially a similarity measure. Hence, we define the correlation $C_{\tilde{A},\tilde{B}}$ as

$$C_{\tilde{A},\tilde{B}} = 1 - \sum_{x\in X} \left| \frac{\mu(x)}{\sum_{x\in X}\mu(x)} - \frac{\eta(x)}{\sum_{x\in X}\eta(x)} \right|. \tag{2}$$

If the set X is integrable, then the above relation may be modified as

$$C_{\tilde{A},\tilde{B}} = 1 - \int_X \left| \frac{\mu(s)}{\int_X \mu(s)\,\mathrm{d}s} - \frac{\eta(s)}{\int_X \eta(s)\,\mathrm{d}s} \right| \mathrm{d}s. \tag{3}$$

The proposed measure in this form is difficult to compute because of the modulus operator inside the integral. The modulus operator can be avoided if we modify the measure as

$$C_{\tilde{A},\tilde{B}} = 1 - \left[\sum_{x\in X} \left| \frac{\mu(x)}{(\sum_{x\in X}\mu^2(x))^{1/2}} - \frac{\eta(x)}{(\sum_{x\in X}\eta^2(x))^{1/2}} \right|^2 \right]^{1/2}. \tag{4}$$

The above measure may be considered a special case of

$$C_{\tilde{A},\tilde{B}} = 1 - \left[\sum_{x\in X} \left| \frac{\mu(x)}{(\sum_{x\in X}\mu^l(x))^{1/l}} - \frac{\eta(x)}{(\sum_{x\in X}\eta^l(x))^{1/l}} \right|^l \right]^{1/l}, \tag{5}$$

where l is a parameter. For even l the modulus operator is not required but computional work increases as l increases.

3. Mathematical properties of the proposed correlation measure

The proposed measure satisfies the following properties for $0 < l < \infty$ and $\tilde{A}, \tilde{B}, \tilde{C} \neq \emptyset$.

Property 1. $C_{\tilde{A},\tilde{A}} = 1$; $\forall \tilde{A} \subseteq X$.

This means that the correlation between two identical fuzzy sets is always 1. Property 1 is obvious from Eq. (5).

Property 2. $C_{\tilde{A},\tilde{B}} = C_{\tilde{B},\tilde{A}}$ $\forall \tilde{A}, \tilde{B}$ *defined on* X.

This ensures that the measure is symmetric since the concept of correlation is "between" and not "on". Property 2 is obvious from Eq. (5).

Property 3. $-1 \geqslant C_{\tilde{A},\tilde{B}} \geqslant +1$ $\forall \tilde{A}, \tilde{B}$ *defined on* X.

Proof. Let the membership functions for the sets $\tilde{A}$ and $\tilde{B}$ be μ and η, respectively. Then from Eq. (5) we have

$$C_{\tilde{A},\tilde{B}} = 1 - I_{\tilde{A},\tilde{B}}.$$

But

$$\min(I_{\tilde{A},\tilde{B}}) = 0 \Rightarrow C_{\tilde{A},\tilde{B}} \leqslant 1,$$

also

$$I_{\tilde{A},\tilde{B}} \leqslant \left[\int_X \max \left[\frac{\mu(x)^l}{\int_X \mu^l(x)\,\mathrm{d}s}, \frac{\eta(x)^l}{\int_X \eta^l(x)\,\mathrm{d}s} \right] \mathrm{d}s \right] \leqslant 2.$$

Hence $C_{\tilde{A},\tilde{B}} \geqslant -1$.

This gives the range of values the measure can assume. □

Property 4. $C_{\tilde{A},\tilde{A}}$ *is supremum of all* $C_{\tilde{A},\tilde{B}}$. *That is* $C_{\tilde{A},\tilde{A}} \geqslant C_{\tilde{A},\tilde{B}}$ $\forall \tilde{A}, \tilde{B}$ *defined on* X.

The correlation between a fuzzy set and itself is always greater or equal to the correlation between it and any other fuzzy set defined in the same universe. This property must be satisfied by any correlation measure. Property 4 follows directly from Properties 1 and 3.

Property 5. $C_{\tilde{A},\tilde{B}} = 1$ *if for all* $x \in X$, $\mu(x) = \alpha_1 \eta(x)$ *and* $\alpha_1 \eta(x) \leqslant 1$ *or for all* $x \in X$, $\eta(x) = \alpha_2 \mu(x)$ *and* $\alpha_2 \mu(x) \leqslant 1$, *where* $\alpha_1, \alpha_2 \geqslant 0$ *are constants.*

This property ensures that the measure is independent of change of scale of membership values i.e. if there are two sets of membership values and one set is just a constant multiple of the other then the correlation between them is 1. The Property 5 is obvious from Eq. (5).

Property 6. $C_{\tilde{A},\tilde{B}} = -1$ *if for all* $x \in X$, $\mu(x) > 0$ *and* $\eta(x) = 0$ *or*, $\mu(x) = 0$ *and* $\eta(x) > 0$.

The correlation measure assumes the value −1 i.e. the case of perfect mismatch when an element belongs to one set with some finite membership value and to the other with 0 membership value. The Property 6 is obvious from Eq. (5).

Property 7. *If both* $C_{\tilde{A},\tilde{B}}$ *and* $C_{\tilde{A},\tilde{C}}$ *are high then* $C_{\tilde{B},\tilde{C}}$ *is also high for any* $\tilde{A},\tilde{B},\tilde{C}$ *defined on* X.

Proof. Let the membership functions for the sets $\tilde{A},\tilde{B}$ and $\tilde{C}$ be μ, η and v, respectively.
Let $C_{\tilde{A},\tilde{B}}=1-\delta$ and $C_{\tilde{A},\tilde{C}}=1-\varepsilon$ where δ and ε are small positive numbers.

$$C_{\tilde{B},\tilde{C}}=1-\sum_{x\in X}\left|\frac{\eta(x)}{\sum_{x\in X}\eta(x)}-\frac{v(x)}{\sum_{x\in X}v(x)}\right|.$$

But,

$$\sum_{x\in X}\left|\frac{\eta(x)}{\sum_{x\in X}\eta(x)}-\frac{v(x)}{\sum_{x\in X}v(x)}\right|$$

$$\leqslant\sum_{x\in X}\left|\frac{\eta(x)}{\sum_{x\in X}\eta(x)}-\frac{\mu(x)}{\sum_{x\in X}\mu(x)}\right|+\sum_{x\in X}\left|\frac{\mu(x)}{\sum_{x\in X}\mu(x)}-\frac{v(x)}{\sum_{x\in X}v(x)}\right|\leqslant\delta+\varepsilon.$$

Hence, $C_{\tilde{B},\tilde{C}}\geqslant 1-(\delta+\varepsilon)$.
Thus, if δ and ε are small then $C_{\tilde{B},\tilde{C}}$ is high. □

3.1. Geometrical interpretation

The proposed measure of correlation has a simple geometrical interpretation if X is a simple bounded one-dimensional Euclidean space. If the area under the membership functions μ and η are standardised to unity, then the sum of the areas of nonoverlap when subtracted from unity gives the correlation between the sets $\tilde{A},\tilde{B}$.

Example. Let $X=[0,1]$ $\tilde{B}$: $\eta(x)=x$ $\tilde{A}$: $\mu(x)=1-x$, $x\in[0,1]$, see Fig. 1.
Here, area under $\mu(x)=\frac{1}{2}$ and area under $\eta(x)=\frac{1}{2}$.
Therefore, the standardisation factor for both sets $\tilde{A},\tilde{B}=\frac{1}{2}$.
Hence, the standardised area of nonoverlap $=(A_1+A_3)/\frac{1}{2}=1\Rightarrow C_{\tilde{A},\tilde{B}}=0$.
Using Eq. (3) we also get the same result:

$$C_{\tilde{A},\tilde{B}}=1-\int_0^1\left|\frac{1}{1/2}-\frac{2x}{1/2}\right|\,\mathrm{d}x=0.$$

4. Comparison with an earlier measure

Murthy et al. also proposed a measure for fuzzy correlation [4]. They defined fuzzy correlation as

$$C_{f_1,f_2}=1-\frac{4\int_\Omega(f_1-f_2)^2\,\mathrm{d}x}{X_1+X_2},\tag{6}$$

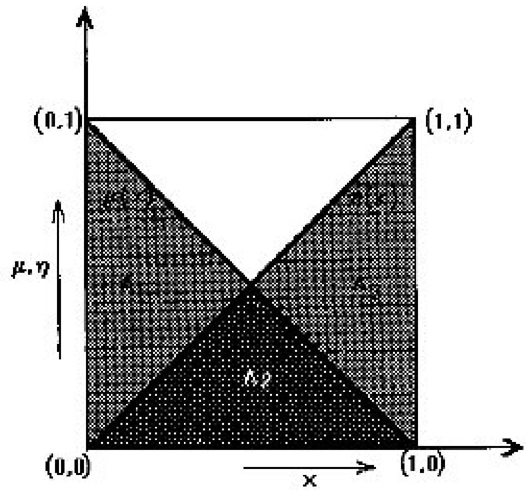

Fig. 1.

where

$$X_1 = \int_{\Omega} (2f_1 - 1)^2 \, \mathrm{d}x, \tag{7}$$

$$X_2 = \int_{\Omega} (2f_2 - 1)^2 \, \mathrm{d}x \tag{8}$$

and the domain Ω is a closed interval in R and the membership functions f_1 and f_2 are such that

(a) $f_1 : \Omega \to [0,1]$ and $f_2 : \Omega \to [0,1]$ are continuous.

(b) $f_1(\Omega) = f_2(\Omega) = [0,1]$, and

(c) $\forall x \in \Omega^c,\ f_i(x) = 0$ or 1 or undefined $\forall i = 1,2$.

If Ω is finite then the $\int$ can be replaced by $\sum$ over Ω and the measure is modified as

$$C_{f_1, f_2} = \begin{cases} 1 - \dfrac{4\sum_{x \in X} [f_1(x) - f_2(x)]^2}{X_1 + X_2}, \\ 1 \quad \text{if } X_1 + X_2 = 0, \end{cases} \tag{9}$$

where

$$X_1 = \sum [2f_1(x) - 1]^2 \quad \text{and} \quad X_2 = \sum [2f_2(x) - 1]^2. \tag{10}$$

Condition (b) is very restrictive for general application of the measure because a practical fuzzy set may not have all values in $[0,1]$.

The measure proposed by Murthy et al. [4] does not satisfy Property 5 that the correlation measure should be independent of change of scale. Also, their measure does not satisfy Property 6. However they have $C_{f_1, f_2} = -1$, if $f_1 = 1 - f_2$ for all elements in the universe Ω where f_1 and f_2 are the membership functions of the first and second fuzzy sets defined on the same universe Ω. But this property comes in conflict with Property 1 as $f_1 \to 0.5$ and $f_2 \to 0.5$ for all elements in the universe Ω. The restriction imposed by condition (b) i.e. $f_1(\Omega) = f_2(\Omega) = [0,1]$ is not strong enough to prevent this contradiction. This case has been considered by Murthy et al. [4] and they modified their measure as

$$C_{f_1,f_2} = \begin{cases} \dfrac{\lambda_1(D)}{\lambda_1(\Omega)} + \dfrac{\lambda_1(\Omega - D)}{\lambda_1(\Omega)} \left[1 - \dfrac{4}{X_1 + X_2} \displaystyle\int_{\Omega - D} (f_1 - f_2)^2 \, \mathrm{d}x\right], \\ 1 \quad \text{if } X_1 + X_2 = 0 \end{cases} \tag{11}$$

where $D = \{x\colon f_1(x) = f_2(x),\ x \in X\}$ and λ_1 represents the Lebesgue measure on R [length of a set]. If Ω is finite then Lebesgue measure is replaced by cardinal number of the set. Still the measure does not give intuitively correct results and also has a stability problem as $X_1 \to 0$ or $X_2 \to 0$. This is illustrated in the following example.

Let $\Omega = [0,1]$ and

$$f_1(x) = \begin{cases} \dfrac{x}{2k} & \text{if } 0 \leqslant x \leqslant k, \\ \frac{1}{2} & \text{if } k < x \leqslant 1 - k, \\ \dfrac{x - 1 + 2k}{2k} & \text{if } 1 - k < x \leqslant 1, \end{cases}$$

$$X_1 = \int_0^1 (2f_1 - 1)^2 \, \mathrm{d}x,$$

clearly,

$$\lim_{k \to 0} X_1 = 0.$$

But the condition $f_1(\Omega) = [0,1]$ is not violated no matter how small k is.

If the membership function $f_2(x)$ defined on [0,1] is such that $f_1 \neq f_2$ for any $x \in [0,1]$ or $f_1 = f_2$ only at some points such that $\lambda_1(D) = 0$ then

$$\begin{aligned} C_{f_1,f_2} &= \lim_{k \to 0} \left[1 - \frac{4}{X_1 + X_2} \int_\Omega (f_2 - f_1)^2 \, \mathrm{d}x\right] \quad \text{since } \int_D (f_1 - f_2)^2 \, \mathrm{d}x = 0 \\ &= \lim_{k \to 0} \left[1 - \frac{4}{X_2} \left[\int_0^k (f_1 - f_2)^2 \, \mathrm{d}x + \int_k^{1-k} (f_2 - 0.5)^2 \, \mathrm{d}x + \int_{1-k}^1 (f_2 - f_1)^2 \, \mathrm{d}x\right]\right]. \end{aligned}$$

As f_1 and f_2 are bounded in the interval [0,1], $(f_1 - f_2)^2$ is also bounded in this region. Hence the first and third integral tend to 0 as $k \to 0$, i.e.

$$\lim_{k \to 0} \int_0^k (f_1 - f_2)^2 \, \mathrm{d}x = 0 \quad \text{and} \quad \lim_{k \to 0} \int_{1-k}^1 (f_1 - f_2)^2 = 0.$$

Therefore,

$$C_{f_1,f_2} = \lim_{k \to 0} \left[1 - \frac{\int_k^{1-k} (2f_2 - 1)^2 \, \mathrm{d}x}{\int_0^1 (2f_2 - 1)^2 \, \mathrm{d}x}\right] = 1 - 1 = 0.$$

Thus, we get $C_{f_1,f_2} = 0$ for any f_2 which satisfes the stated conditions. This is not intuitively correct as we can see from Fig. 2.

All the membership functions f_2, f_3 and f_4 in Fig. 2 are very positively correlated with f_1, but correlation by the method due to Murthy et al. [4] are all 0.

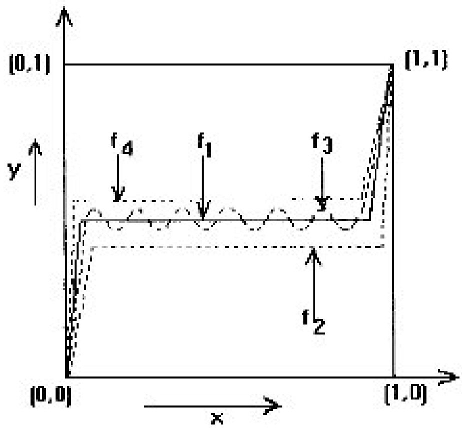

Fig. 2. The membership values are plotted along the y-axis and the Domain Ω is represented along x-axis.

If the functions f_2, f_3 and f_4 now approach f_1 maintaining the properties stated above, the value of the correlation will remain zero no matter how closely it approximates f_1. Therefore,

$$\lim_{f_i \to f_1} C_{f_1, f_i} = 0, \quad \forall i \in 2, 3, 4.$$

But

$$C_{f_1, f_1} = 1.$$

Thus the measure makes a drastic jump from 0 to 1 and hence it is not stable as $X_1 \to 0$ or $X_2 \to 0$.

5. Discussions

A modified fuzzy correlation measure is proposed in this paper which overcomes some of the limitations of an earlier proposed measure. Moreover, an equivalent of rank correlation measure is also described.

The proposed measure can be used where the concept of correlation is necessary but the presence of uncertainity and ambiguity prevents classical analysis.

The fuzzy correlation measure can be used in pattern matching if suitable fuzzy sets can be constructed from the image and pattern templates. This applicability of the proposed measure in pattern matching was tested experimentally by taking a binary image set of handwritten arabic numerals 0–9, as shown in Fig. 3a.

The fuzzy set corresponding to the pattern templates was constructed by assigning a membership value to each pixel. The pixels belonging to the background of the patterns was assigned a membership value 0 and the pixels which belonged to the pattern was assigned a membership value proportional to its gray level. Since the pattern templates were binary the corresponding fuzzy sets were easily constructed.

The various image templates corresponding to each pattern templates was generated by blurring the pattern template and by adding varying amounts of noise to it. The templates were blurred by spatial averaging. The selected window size was 3×3 and all the weights were equal to $\frac{1}{9}$. The noise added was uniform random. This was done to simulate practical situations where some noise and some uncertainity regarding the belongingness of a pixel to a pattern is bound to be present. The resulting images are shown in Fig. 3.

The correlation between the image fuzzy set and the pattern fuzzy set was computed for every pattern position in the template and the best match was taken as the result. This is necessary since the patterns can occur anywhere in the image templates.

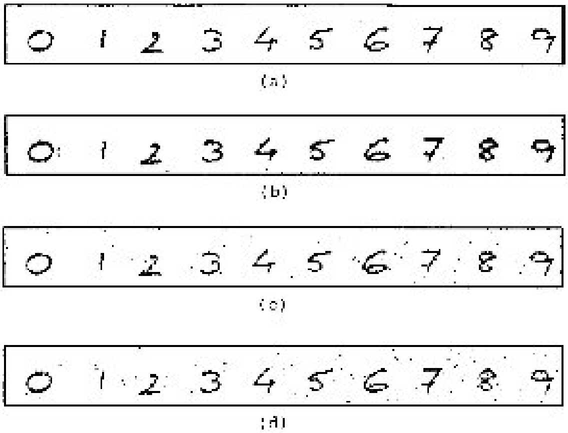

Fig. 3. A set of typical hand-written numerals: (a) original set; (b) blurred; (c) with 5% noise; (d) with 10% noise.

Table 1
Fuzzy correlation for blurred images

Templates	Image									
	0	1	2	3	4	5	6	7	8	9
Pattern										
0	0.515	−0.501	−0.170	0.166	−0.304	−0.084	−0.083	−0.117	−0.204	−0.128
1	−0.732	0.468	−0.573	−0.639	−0.631	−0.675	−0.762	−0.610	−0.645	−0.630
2	−0.354	−0.275	0.531	−0.138	0.052	−0.238	−0.317	−0.060	−0.179	−0.502
3	−0.146	−0.385	−0.137	0.456	−0.376	−0.351	−0.377	−0.200	−0.252	−0.107
4	−0.488	−0.425	−0.053	−0.432	0.485	−0.327	−0.261	−0.325	−0.351	−0.119
5	−0.222	−0.395	−0.200	−0.275	−0.168	0.515	−0.272	−0.110	−0.101	−0.249
6	−0.013	−0.517	−0.094	−0.133	−0.002	−0.068	0.502	−0.279	−0.151	−0.163
7	−0.305	−0.367	−0.026	−0.219	−0.245	−0.194	−0.448	0.493	−0.188	−0.245
8	−0.384	−0.350	−0.019	−0.129	−0.181	−0.090	−0.342	−0.067	0.503	−0.222
9	−0.323	−0.373	−0.418	−0.064	0.010	−0.291	−0.365	−0.191	−0.307	0.460

Table 2
Fuzzy correlation under 5% noise

Templates	Image									
	0	1	2	3	4	5	6	7	8	9
Pattern										
0	0.915	−0.680	−0.584	−0.140	−0.526	−0.086	−0.029	−0.580	−0.240	−0.220
1	−0.731	0.883	−0.927	−0.707	−0.582	−0.640	−0.769	−0.701	−0.653	−0.590
2	−0.290	−0.892	0.795	−0.398	−0.541	−0.491	−0.255	−0.483	−0.566	−0.397
3	−0.021	−0.627	−0.636	0.882	−0.426	−0.287	−0.326	−0.632	−0.240	0.036
4	−0.558	−0.559	−0.685	−0.448	0.895	−0.317	−0.238	−0.327	−0.435	−0.446
5	−0.137	−0.833	−0.579	−0.381	−0.310	0.925	−0.273	−0.476	−0.153	−0.122
6	0.074	−0.815	−0.476	−0.315	−0.167	−0.172	0.916	−0.463	−0.259	−0.556
7	−0.635	−0.605	−0.515	−0.255	−0.358	−0.156	−0.470	0.894	−0.240	−0.328
8	−0.270	−0.628	−0.565	−0.192	−0.326	−0.019	−0.290	−0.186	0.868	−0.209
9	−0.251	−0.544	−0.790	−0.230	0.038	−0.172	−0.574	−0.527	−0.262	0.904

Table 3
Fuzzy correlation under 10% noise

Templates	Image									
	0	1	2	3	4	5	6	7	8	9
Pattern										
0	0.870	−0.667	−0.594	0.035	−0.541	−0.191	−0.389	−0.840	−0.258	−0.200
1	−0.770	0.722	−0.814	−0.661	−0.664	−0.647	−0.794	−0.796	−0.647	−0.642
2	−0.277	−0.541	0.720	−0.225	−0.534	−0.604	−0.278	−0.546	−0.269	−0.621
3	−0.029	−0.600	−0.531	0.816	−0.442	−0.316	−0.372	−0.796	−0.249	−0.307
4	−0.546	−0.500	−0.675	−0.499	0.728	−0.356	−0.278	−0.524	−0.388	−0.104
5	−0.275	−0.619	−0.647	−0.380	−0.405	0.853	−0.280	−0.569	−0.167	−0.372
6	0.071	−0.722	−0.481	−0.295	−0.222	−0.182	0.850	−0.500	−0.294	−0.333
7	−0.749	−0.579	−0.535	−0.442	−0.466	−0.377	−0.536	0.724	−0.181	−0.374
8	−0.466	−0.605	−0.564	−0.186	−0.372	−0.073	−0.332	−0.206	0.871	−0.240
9	−0.385	−0.541	−0.752	0.041	−0.089	−0.206	−0.710	−0.592	−0.279	0.771

The results obtained are shown in Tables 1–3. From the results it is evident that in each case the proposed correlation measure can be used for pattern matching. It is also noted that blurring reduces the correct match correlation values from +1.0 to near about +0.5 for all the templates. Addition of noise reduces the correct match correlation values in proportion to the percentage of noise added. For nonmatched templates the correlation values are negative in most cases because the area of nonoverlap region is more than the area of overlap region. In general, it is noted that for nonmatched templates the correlation values are higher when the image templates are blurred. This is due to the increased area of overlap due to blurring.

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