Orthogonal Even Nonlinear Coherent States

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We introduce orthogonal even nonlinear coherent states and study their statistical properties.

1. INTRODUCTION

The coherent states introduced by Glauber [1] are important in the study of quantum optics. These states are defined as the right-hand eigenstates $|\alpha\rangle$ of the non-Hermitian boson annihilation operator a. Recently nonlinear coherent states have been studied by various authors [2]. These states are generated as right-hand eigenstates of the product of the boson annihilation operator and a nonlinear function f of the number operator. The even and odd nonlinear coherent states were constructed in ref. 3 and their statistical properties were investigated.

In this paper we introduce orthogonal even nonlinear coherent states [4] as another superposition of nonlinear coherent states and study their statistical properties.

2. ORTHOGONAL EVEN NONLINEAR COHERENT STATES

As in ref. 3, we consider the annihilation operator A and the creation operator $A^\dagger$ as the distortions of the usual annihilation operator a and creation operator $a^\dagger$. They are given by

$$A = af(\hat{n}) = f(\hat{n}+1)a$$
$$A^\dagger = f(\hat{n})a^\dagger = a^\dagger f(\hat{n}+1) \tag{1}$$

where

$$\hat{n} = a^\dagger a, \qquad [A, \hat{n}] = A, \qquad [A^\dagger, \hat{n}] = -A^\dagger \tag{2}$$

f is an operator-valued function of the Hermitian number operator.

Using the relations

$$A = \sum_{n=0}^{\infty} \sqrt{n} f(n) |n-1\rangle\langle n|$$
$$A^\dagger = \sum_{n=0}^{\infty} \sqrt{n} f(n) |n\rangle\langle n-1| \tag{3}$$

we obtain the commutation relation between A and $A^\dagger$ as

$$[A, A^\dagger] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}) \tag{4}$$

where f is chosen to be real, nonnegative and $f^\dagger(\hat{n}) = f(\hat{n})$.

The nonlinear coherent state satisfies the equation

$$A|\alpha, f\rangle = \alpha|\alpha, f\rangle \tag{5}$$

Using the completeness relation of the number state

$$1 = \sum_{n=0}^{\infty} |n\rangle\langle n| \tag{6}$$

and the decomposition of $|\alpha, f\rangle$ in the number state basis $\{|n\rangle\}$, we observe that

$$|\alpha, f\rangle = c_0 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}\, f(n-1)!} |n\rangle \tag{7}$$

where

$$c_0 = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{\sqrt{n!}\, f(n-1)!} \right)^{-1/2} \tag{8}$$

with the convention $f(n)! = f(0)f(1)\ldots f(n)$.

The even nonlinear coherent state is defined as

$$|\alpha, f\rangle_+ = c_+(|\alpha, f\rangle + |-\alpha, f\rangle) \tag{9}$$

where

$$c_+ = \left(2 + 2c_0^2 \sum_{n=0}^{\infty} \frac{(-|\alpha|^2)^n}{n!\,[f(n-1)!]^2}\right)^{-1/2} \tag{10}$$

The odd nonlinear coherent state is defined as

$$|\alpha, f\rangle_- = c_-(|\alpha, f\rangle - |-\alpha, f\rangle) \tag{11}$$

where

$$c_- = \left(2 - 2c_0^2 \sum_{n=0}^{\infty} \frac{(-|\alpha|^2)^n}{n!\,[f(n-1)!]^2}\right)^{-1/2} \tag{12}$$

Now, we define the orthogonal even nonlinear coherent state as

$$|\alpha, f\rangle_+^0 = c_+^0(|\alpha, f\rangle_+ + |i\alpha, f\rangle_+) \tag{13}$$

where

$$c_+^0 = (c_+ A)^{-1} \tag{14}$$

with

$$A^2 = 4 + 4c_0^2 \sum_{n=0}^{\infty} \frac{(-|\alpha|^2)^n}{n!\,[f(n-1)!]^2} + 4c_0^2 \sum_{n=0}^{\infty} \frac{(i|\alpha|^2)^n}{n!\,[f(n-1)!]^2} + 4c_0^2 \sum_{n=0}^{\infty} \frac{(-i|\alpha|^2)^n}{n!\,[f(n-1)!]^2} \tag{15}$$

3. STATISTICAL PROPERTIES

3.1. Number Distribution

Expanding orthogonal even nonlinear coherent states in terms of the number state basis, we have

$$\begin{aligned}
|\alpha, f\rangle_+^0 &= c_+^0(|\alpha, f\rangle_+ + |i\alpha, f\rangle_+) \\
&= c_+^0[c_+(|\alpha, f\rangle + |-\alpha, f\rangle) + c_+(|i\alpha, f\rangle + |-i\alpha, f\rangle)] \\
&= c_+^0 c_+[|\alpha, f\rangle + |-\alpha, f\rangle + |i\alpha, f\rangle + |-i\alpha, f\rangle] \\
&= c_+^0 c_+\Bigg[c_0 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}\,f(n-1)!}|n\rangle + c_0 \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}\,f(n-1)!}|n\rangle \\
&\quad + c_0 \sum_{n=0}^{\infty} \frac{(i\alpha)^n}{\sqrt{n!}\,f(n-1)!}|n\rangle + c_0 \sum_{n=0}^{\infty} \frac{(-i\alpha)^n}{\sqrt{n!}\,f(n-1)!}|n\rangle
\end{aligned}$$

$$= c_+^0 c_+ c_0 \sum_{n=0}^{\infty} \frac{\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n}{\sqrt{n!}\, f(n-1)!} |n\rangle \tag{16}$$

The number distribution in the state $|\alpha, f\rangle_+^0$ is given by

$$W_n[|\alpha, f\rangle_+^0] = |\langle n|\alpha, f\rangle_+^0|^2$$

$$= (c_+^0 c_+ c_0)^2 \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2} \tag{17}$$

which shows that the fluctuation is regulated by the nonlinear function f.

The expection of the number distribution is given by

$$\langle \hat{n} \rangle_+^0 = (c_+^0 c_+ c_0)^2 \sum_{n=0}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2} n \tag{18}$$

and the variance is

$$V(\hat{n})_+^0 = (c_+^0 c_+ c_0)^2 \left[\sum_{n=0}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2} n^2 - (c_+^0 c_+ c_0)^2 \left(\sum_{n=0}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2} n \right)^2 \right] \tag{19}$$

The second-order correlation function is obtained from the relation

$$g_+^{0(2)}(0) = \frac{V(\hat{n})_+^0 - \langle \hat{n} \rangle_+^0}{\langle \hat{n} \rangle_+^{0(2)}} + 1$$

$$= \left((c_+^0 c_+ c_0)^2 \sum_{n=0}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2} n \right)^{-2} \times \sum_{n=2}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{(n-2)!\,[f(n-1)!]^2} \tag{20}$$

Depending on the particular form of f, the right-hand side of equation (20) can be less than or greater than one, producing either antibunching or bunching.

3.2. Quadrature Variance for Orthogonal Even Nonlinear Coherent States

Two quadrature components are defined as

$$x = \frac{A + A^\dagger}{2}, \qquad p = \frac{A - A^\dagger}{2i} \tag{21}$$

We know that

$$(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 \tag{22}$$

To calculate (22) we observe that

$$_{+}\langle \alpha, f \mid \alpha, f\rangle_{-} = 0 \tag{23}$$

Again,

$$A|\alpha, f\rangle_{+} = \alpha \frac{c_+}{c_-} |\alpha, f\rangle_{-} \tag{24}$$

Similarly,

$$A|\alpha, f\rangle_{-} = \alpha \frac{c_-}{c_+} |\alpha, f\rangle_{+} \tag{25}$$

Thus, arbitrary nonlinear even and nonlinear odd coherent vectors are orthogonal and can be exchanged by the operator A.

Also we observe that

$$
\begin{aligned}
{}_{+}\langle \alpha, f|i\alpha, f\rangle_{-} &= 0\\
{}_{+}\langle i\alpha, f|\alpha, f\rangle_{-} &= 0\\
{}_{-}\langle \alpha, f|i\alpha, f\rangle_{-} &= 2c_-^2 c_0^2 \sum_{n=0}^{\infty} \frac{(i|\alpha|^2)^n - (-i|\alpha|^2)^n}{n!\,[f(n-1)!]^2}\\
{}_{+}\langle \alpha, f|i\alpha, f\rangle_{+} &= 2c_+^2 c_0^2 \sum_{n=0}^{\infty} \frac{(i|\alpha|^2)^n + (-i|\alpha|^2)^n}{n!\,[f(n-1)!]^2}\\
{}_{+}^{0}\langle \alpha, f|A|\alpha, f\rangle_{+}^{0} &= 0\\
{}_{+}^{0}\langle \alpha, f|A^2|\alpha, f\rangle_{+}^{0} &= 0\\
{}_{+}^{0}\langle \alpha, f|A^\dagger A|\alpha, f\rangle_{+}^{0} &= 2|\alpha|^2\left(\frac{c_+^0 c_+}{c_-}\right)^2 \left[1 + 2c_-^2 c_0^2 \,\mathrm{Re} \sum_{n=0}^{\infty} \frac{(i|\alpha|^2)^n - (-i|\alpha|^2)^n}{n!\, f(n-1)!]^2}\right]
\end{aligned} \tag{26}
$$

Hence we have

$$
\begin{aligned}
(\Delta x)^2 = \frac{1}{4}\left[2|\alpha|^2\left(\frac{c_+^0 c_+}{c_-}\right)^2\left[1 + 2c_-^2 c_0^2 \,\mathrm{Re} \sum_{n=0}^{\infty} \frac{(i|\alpha|^2)^n - (-i|\alpha|^2)^n}{n!\,[f(n-1)!]^2}\right]\right]\\
+ \frac{1}{4}(c_+^0 c_+ c_0)^2 \sum_{n=0}^{\infty} \frac{|\alpha^n + (-\alpha)^n + (i\alpha)^n + (-i\alpha)^n|^2}{n!\,[f(n-1)!]^2}\\
\times [(n+1)f^2(n+1) - nf^n]
\end{aligned} \tag{27}
$$

where $\alpha = re^{i\theta}$.

Depending on the particular form of f, the right-hand side of (27) can be ≤ 1 if $r^2 \ll 1$ and so we see that orthogonal even coherent states can exhibit squeezing.

4. CONCLUSION

We have introduced nonlinear orthogonal even coherent states which show nonclassical features. These nonclassical effects depend on the introduced nonlinearity rather than on the symmetry of the states. In view of their singular properties, these states may be of use in optical and microwave fields. They may be also interesting from the point of view of quantum groups.

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