On solute dispersion from an elevated line source in an open-channel flow

B.S. MAZUMDER and S. BANDYOPADHYAY
Physics and Applied Mathematics Unit, Indian Statistical Institute, Calcutta 700 035, India

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Abstract. The streamwise dispersion of contaminant molecules due to a turbulent shear flow over a gravel-bed surface is examined when the solute is released from an elevated line source. A finite-difference, implicit method is used to solve the unsteady turbulent convection-diffusion equation, employing a combined scheme of central and four-point upwind differences for the steady-state condition and the Alternating Direction Implicit (ADI) method for the unsteady equation. It is shown how the mixing of contaminant is effected by the mean velocity due to the 'log-wake law' and the corresponding eddy-diffusivity, when the solute is released in terms of a δ-function. The results for the steady-state condition are compared with existing experimental data and some other numerical results. The results obtained by the present method are in much better agreement with the experimental data than those obtained by the previous solution scheme, when the 'log-wake-law' and the corresponding eddy-diffusivity are used. Far from the source, the concentration increases with time (the rate of increase is rapid near the source), asymptotically approaching a steady state after a certain time; concentrations are about two thirds of those near the source. The behaviour of iso-concentration lines in the vertical plane is also studied.

Key words: turbulent flow, dispersion, line source, open channel, finite difference.

1. Introduction

The importance of studies conceming the longitudinal dispersion process is apparent with the current problems of pollution in environments. It has been common practice to discharge waste materials from chemical or industrial plants into streams or the atmosphere. In order to control contamination in a stream or the atmosphere and to predict levels of accidental pollution, an understanding is required of the levels at which a stream is capable of transporting and dispersing pollutants. The present study gives an insight into dispersion phenomena of passive contaminants in solvents, and it has primary importance in industrial and technological fields. The dispersion of a passive tracer from an elevated source is worth studying because of its practical applications in numerous environmental problems.

The longitudinal dispersion of a soluble matter in turbulent flow through a pipe of circular cross-section was first studied by G. I. Taylor [1] in his classic paper. He found that the centre of mass of a cloud has a velocity asymptotically equal to the discharge velocity and that the effect of longitudinal turbulent diffusion is very small. Elder [2], while extending Taylor's model by means of a series of experiments in turbulent open channel flow, showed that the contribution to longitudinal dispersion is approximately 2% of the main flow. Mazumder and Xia [3] presented an analytical solution for the longitudinal dispersion of pollutants in an asymetric flow through a 2-D channel, when the injected solute was uniform over the cross-section of the channel.

Raupach and Legg [4] presented results of an experiment on dispersion of a passive tracer concentration resulting from an elevated continuous line-source within a wind-tunnel turbulent boundary layer over a gravel-bed surface and provided some detailed measurements of the velocity, scalar contaminant concentration values in the relatively sensitive regions at moderate downstream distances. Sullivan and Yip [5] explored the versatile solution scheme developed by Sullivan and Yip [6] to study the contaminant dispersion from an elevated source near an impermeable boundary in turbulent flow considering the effect of longitudinal diffusion. Their scheme was developed for a small-time asymptotic solution for an instantaneous point source and then, through superposition, the small-time solution was used to compile the mean concentration field at any time resulting from non-uniform, instantaneous and continuous sources. The comparison between their results obtained by the solution scheme and the experimental data of Raupach and Legg [4] showed good agreement, except near the source in the downstream distance. Sullivan and Yip [7] also extended their solution scheme, considering a time-dependent eddy diffusivity to study the dispersion of contaminant from an elevated line source and compared the results with the measured data of Raupach and Legg [4] . The comparison shows a better representation of the data only at the nearest downstream measuring station.

The purpose of the present study is to explore the dispersion phenomenon of a passive contaminant (soluble matter) injected from a continuous line-source (elevated above the boundary) into a fully-developed turbulent open-channel flow over a gravel-bed surface; and to compare the results with experimental data of Raupach and Legg [4]. For a turbulent open-channel flow over a smooth bed surface, it is well known that the 'log-law' can be applied strictly only to the near wall region having the von-Kármán constant $\kappa = 0{\cdot}412$ and $A = 5{\cdot}29$, whereas the deviation from 'log-law' in the outer region can be expressed well by Coles's wake function (Coles [8]) which depends on the Reynolds number. The knowledge of mean velocity and turbulent characteristics over the rough turbulent boundary layer is limited. In our study, according to Nezu and Nakagawa [9], a modefied 'log-wake-law' and corresponding variable eddy-diffusivity over the rough surface in the turbulent open-channel flow has been assumed when solving the two-dimensional, unsteady, convective diffusion equation. A combined scheme of central differencing and four-point upwind differencing has been used to generate the numerical solution of the convection-diffusion equation for the steady state, whereas a numerical scheme based on the alternating-direction implicit (ADI) method is adopted to solve the general two-dimesional unsteady convection-diffusion equation. This method provides a more general numerical study of the time-dependent problem than the theoretical study of Sullivan and Yip [5] based on a small-time asymptotic solution. A comparison of vertical concentration profiles resulting from the steady-state condition has been made with the experimental data of Raupach and Legg [4] and the results obtained by the solution scheme of Sullivan and Yip [5] at different downstream distances. Results for the unsteady dispersion are presented as concentration contours in the vertical plane.

2. Governing equations

Consider a steady, fully-developed, unidirectional turbulent flow through a channel bounded by a carrier fluid of depth H. We employ a Cartesian co-ordinate system with x^*-axis in the streamwise direction, y^*-axis in the spanwise (cross-stream) direction and z^*-axis (vertical) perpendicular to the flow. Assuming that the mean velocity and the eddy-diffusivity vary with the vertical co-ordinate only and that a slug of a passive contaminant is released into the flow, we find that the concentration $C(x, y, z, t)$ of solute satisfies the dimensionless convection-diffusion equation of the form

$$\frac{\partial C}{\partial t} + u(z)\frac{\partial C}{\partial x} = k_x(z)\frac{\partial^2 C}{\partial x^2} + k_y(z)\frac{\partial^2 C}{\partial y^2} + \frac{\partial}{\partial z}\left(k_z(z)\frac{\partial C}{\partial z}\right) \tag{1}$$

with non-dimensional variables

$$t = \frac{t^* u_*}{H},\ u = \frac{u^*}{u_*},\ x = \frac{x^*}{H},\ y = \frac{y^*}{H},\ z = \frac{z^*}{H},\ k_x = \frac{k_x^*}{u_* H},\ k_y = \frac{k_y^*}{u_* H},\ k_z = \frac{k_z^*}{u_* H}, \tag{2}$$

where u_* is the friction velocity (taken as the reference velocity), H is the depth of the carrier fluid, and k_x, k_y and k_z are the dimensionless eddy diffusivities varying with the vertical co-ordinate. In the field of stationary homogeneous turbulence, if the co-ordinate axes are chosen to coincide with the principal axes of the turbulent fluctuations, the cross-variances become zero. Therefore, in this analysis the off-diagonal terms of the eddy diffusivity tensor k_{ij} have been ignored (Fischer *et al.* [10, Chapter 3]).

The boundary conditions in dimensionless form are

$$C(\pm\infty, y, z, t) = 0, \quad C(x, \pm\infty, z, t) = 0, \quad k_z(z)\frac{\partial C}{\partial z}\bigg|_{z=z_0,1} = 0, \tag{3}$$

where z_0 is the bottom roughness. In the present problem, contaminant is injected at the section $(x = 0,\ y = 0)$ in the form :

$$C(0, 0, z, t) = \delta(z - z_p) \quad \text{at} \quad t = 0, \tag{4}$$

where z_p is the height of the injection point above the bottom *i.e.* zero plane of the surface and $\delta(z - z_p)$ is the Dirac delta function.

The aim of the present analysis is to solve numerically Equation (1) subject to the prescribed boundary conditions (3) and the input condition (4) for the passive scalar contaminant. First, the steady-state form of Equation (1) is solved with the proper boundary and input conditions. The results are compared with the experimental data of Raupach and Legg [4] and also with the numerical results of Sullivan and Yip [5, 7]. Finally, the two-dimensional, unsteady, convection-diffusion equation is solved numerically for the prescribed turbulent velocity field with variable diffusivity.

2.1. Mean-velocity and eddy-diffusivity

Theoretical and experimental studies, by means of some highly sophisticated velocity-measuring devices namely, hot-film anemometry, Laser Doppler anemometry, hydrogen-bubble techniques etc., of the mean flow and the turbulent intensities for uniform open channel flows have been perfomed by many researchers. The logarithmic velocity distribution (log-law) has often been applied to the open channel flow without any detailed verification to fit individual situations by adjusting the values of the von-Kármán constant κ and the integrating constant A. But in many cases, the values due to Nikuradse [11] for turbulent pipe-flow have been adopted as $\kappa = 0{\cdot}4$ and $A = 5{\cdot}5$. Several researchers (Coles [8], Nezu and Rodi [12], Nezu and Nakagawa [9, Chapter 2]) suggested that the deviations of the velocity measurements from the standard log-law *can not be fitted only* by adjusting the von-Kármán constant κ and integrating constant A, but rather by adding a suitable function known as 'wake-function' $w(\xi)$ in the log-law. Nezu and Rodi [12] re-examined the law of the wall and the velocity-defect law in fully-developed open channel flow over smooth beds, measuring with a two-colour LDA and found that the log-law can be applied strictly only to the near-wall region having the universal values $\kappa = 0{\cdot}412$ and $A = 5{\cdot}29$; whereas deviation from the log-law in the outer region can be expressed well by Coles' wake function which depends on the Reynolds number, and is given by

$$u = \frac{1}{\kappa}\log\left(\frac{z^* u_*}{\nu}\right) + A + w(z^*) \qquad \text{for a smooth bed,} \tag{5}$$

where the wake function $w(z^*)$ has the empirical form for wake region as

$$w(z^*) = \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{z^*}{H}\right) \tag{6}$$

with Π is called the wake strength parameter (Coles [8]) which determines the deviations of velocity measurements from the usual log-law. For $\Pi = 0$, Equation (5) turns to the log-profile. The above-mentioned log-wake law is valid only for the turbulence situation over a smooth bed in open-channel flows.

However, according to Nezu and Nakagawa [9], the log-wake law over rough beds in open channel flows can be expressed as

$$u = \frac{1}{\kappa}\log\left(\frac{z^*}{k_s}\right) + A_r + w(z), \tag{7}$$

where A_r is a constant equal to 8·5 for a completely rough wall and k_s is the equivalent sand roughness. In this problem, the velocity distribution for the flow over a gravel bed surface is considered in non-dimensional form as:

$$u = \frac{1}{\kappa}\log\left(\frac{z^*}{{z^*}_0}\right) + w(z), \tag{8}$$

where

$$w(z) = \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}z\right). \tag{9}$$

The constant A_r has been taken into the log-term, ${z^*}_0$ is the equivalent roughness height (0·23 mm.).

The corresponding eddy-diffusivity similar to that of Nezu and Rodi [12] is given by

$$k(z) = \kappa(1-z)\left[\frac{1}{z} + \Pi\pi\sin(\pi z)\right]^{-1}. \tag{10}$$

3. Finite-difference formulations

Analytical solution to (1) subject to boundary conditions with the mean velocity profile $u(z)$, eddy-diffusivity $k(z)$ is not available except for some specified algebraic form of u and k. A small-time asymptotic solution to (1) for an instantaneous point source and near-field contaminant dispersion from an elevated line source was developed by Sullivan and Yip [5, 6].

Therefore, in order to discuss the dispersion pattern from the release of pollutant into the turbulent flow, Equation (1) together with boundary conditions (3) and input condition (4) has been solved by use of a finite-difference technique. For the numerical scheme, boundary conditions not conforming to the coordinate line lead to severe interpolation errors in Cartesian grids and also the application of boundary conditions at large distance causes some difficulties. To resolve this problem, a suitable coordinate transformation along the x and y direction is necessary. So, a transformation of the form

$$x = \frac{1}{2a}\log\left(\frac{1+\xi}{1-\xi}\right), \quad y = \frac{1}{2b}\log\left(\frac{1+\zeta}{1-\zeta}\right), \quad \text{and } z = \eta \tag{11}$$

is used to map the unbounded region (physical plane of (x, y, z)-coordinate) to a bounded one (computational plane of (ξ, ζ, η)-coordinate). The mapping transforms the boundaries $x = \pm\infty$ into $\xi = \pm 1$ and $y = \pm\infty$ into $\zeta = \pm 1$. Here a and b are stretching factors relating the physical domain to the computational domain. A transformation of this form helps to avoid loss of accuracy through discretization in the diffusion and convection terms.

Using the transformation (11), Equation (1) and the boundary and initial conditions, (3) and (4), in the computational domain become

$$\begin{aligned}\frac{\partial C}{\partial t} + u(\eta)a(1-\xi^2)\frac{\partial C}{\partial \xi} \;=\; & a^2 k_\xi(\eta)(1-\xi^2)\left[(1-\xi^2)\frac{\partial^2 C}{\partial \xi^2} - 2\xi\frac{\partial C}{\partial \xi}\right] + \\ & + b^2 k_\zeta(\eta)(1-\zeta^2)\left[(1-\zeta^2)\frac{\partial^2 C}{\partial \zeta^2} - 2\zeta\frac{\partial C}{\partial \zeta}\right] + \\ & + \frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right)\end{aligned} \tag{12}$$

and

$$\left.\begin{aligned} & C(\pm 1, \zeta, \eta, t) \;=\; 0, \;\; C(\xi, \pm 1, \eta, t) \;=\; 0, \;\; k_\eta(\eta)\left.\frac{\partial C}{\partial \eta}\right|_{\eta=\eta_0, 1} = 0, \\ & C(0, 0, \eta, t) \;=\; \delta(\eta - z_p) \quad \text{at } t = 0 \end{aligned}\right\}. \tag{13}$$

Now, since the flow is fully-developed turbulent flow, convection along the horizontal direction will be much larger than the longitudinal diffusion of the solute. If the solute is injected at $\xi = 0$, it cannot reach the negative side of the $\xi = 0$ line because of the dominance of convection. Hence the concentration of the solute for $-1 \le \xi < 0$ is assumed to be zero.

3.1. Steady-state concentration equation

The steady, two-dimensional form of (12) is

$$u(\eta)a(1-\xi^2)\frac{\partial C}{\partial \xi} - a^2 k_\xi(\eta)(1-\xi^2)\left[(1-\xi^2)\frac{\partial^2 C}{\partial \xi^2} - 2\xi\frac{\partial C}{\partial \xi}\right] - \frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right) = 0 \tag{14}$$

and the boundary and input conditions from (13) are

$$C(1, \eta) \;=\; 0, \;\; k_\eta(\eta)\left.\frac{\partial C}{\partial \eta}\right|_{\eta=\eta_0, 1} = 0, \quad C(0, \eta) = \delta(\eta - z_p). \tag{15}$$

A combined scheme of central differencing and four-point upwind differencing is used (Fletcher [13, pp. 295–304]) to generate a numerical solution to (14) subject to the boundary and initial conditions (15). The diffusion terms have been discretised by use of central differencing technique, while for the convective term a four-point upwind scheme is adopted. The oscillatory behaviour of the three-point central-differencing representation and the dissipative nature of the two-point upwind scheme for the convective term suggest that a four-point upwind representation for the convective term $u\partial C/\partial\xi$ is necessary to obtain satisfactory results.

The following discretizations are used for the convective term: for $u > 0$ it will be represented by

$$\left.\frac{\partial C}{\partial \xi}\right|_{jk} = \frac{C(j+1,k) - C(j-1,k)}{2\Delta\xi} + \\ + q\left[\frac{C(j-2,k) - 3C(j-1,k) + 3C(j,k) - C(j+1,k)}{3\Delta\xi}\right] + O(\Delta\xi^2) \quad (16)$$

and that is for $u < 0$

$$\left.\frac{\partial C}{\partial \xi}\right|_{jk} = \frac{C(j+1,k) - C(j-1,k)}{2\Delta\xi} + \\ + q\left[\frac{C(j-1,k) - 3C(j,k) + 3C(j+1,k) - C(j+2,k)}{3\Delta\xi}\right] + O(\Delta\xi^2), \quad (17)$$

where q is the control parameter, which is effective in reducing the dispersion error (*i.e.*, controls the size of modification). The case $q = 0{\cdot}5$ provides a more accurate but slightly oscillatory solution. Increasing q produces a smoother but more dispersed solution, similar to that obtained by a two-point upwind scheme. $C(j,k)$ is the value of C at the mesh point $(j,k) : \xi = (j-1)\Delta\xi$ and $\eta = (k-1)\Delta\eta$, $\Delta\xi$ and $\Delta\eta$ are the grid spacing along the x-axis and z-axis, respectively. The mesh point where the solute is injected is described as $(1, k_p)$, where $j = 1$ corresponds to $\xi = 0$, $j = N+1$ to $\xi = 1$ and $k = k_p$ corresponds to $\eta = z_p$.

The boundary conditions are then reduced to

$$\left.\begin{array}{l} C(N+1,k) = 0 \quad \text{for} \quad 0 \le k \le M+1 \\ C(1,k_p) = 0, \;\; C(1,k) = 0 \quad \text{for} \quad 0 \le k \le M+1 \;\; \text{except} \quad k = k_p \\ C(j,0) = C(j,2), \;\; C(j,M+2) = C(j,M) \;\; \text{for} \quad 1 \le j \le N+1 \end{array}\right\}, \quad (18)$$

where $k = 1$ and $k = M+1$ represent the values of k at the boundary $\eta = 0$ and $\eta = 1$, respectively. This gives us a system of algebraic equations which are then solved by means of the successive overrelaxation (SOR) method. The appropriate relaxation parameter has been found by numerical inspection. An inverse transformation has been applied to get back to the physical plane from the computational plane.

For a stable and accurate solution the following conditions have been incorporated

$$\Delta\xi = \left.\frac{k_\xi(\eta)}{u(\eta)}\right|_{\min} \quad \forall\, j,k$$

and

$$q = \frac{1}{2}\left[1 + \frac{a\xi k_\eta(\eta)}{u(\eta)}\right].$$

3.2. Unsteady concentration equation

For the unsteady, two-dimensional form of (12) neglecting cross-stream diffusion k_ζ, we have

$$\begin{aligned}&\frac{\partial C}{\partial t} + u(\eta)a(1-\xi^2)\frac{\partial C}{\partial \xi} - a^2 k_\xi(\eta)(1-\xi^2)\left[(1-\xi^2)\frac{\partial^2 \xi}{\partial \xi^2} - 2\xi\frac{\partial C}{\partial \xi}\right] - \\ &-\frac{\partial}{\partial \eta}\left(k_\eta(\eta)\frac{\partial C}{\partial \eta}\right) = 0\end{aligned} \tag{19}$$

with boundary conditions given by (13).

An alternating-direction implicit (ADI) method is used to obtain a numerical solution of (19). The ADI-scheme in two dimensions has the desirable attributes of being unconditionally stable, second-order accurate and economical to formulate. To solve (19), central differencing is used on the diffusion terms, while backward differencing is used for the convective term. This scheme prevents numerical oscillations in the solutions, although the local spatial resolution is only first order. Equation (19) is written in two half time steps with one spatial variable implicit in one half step and the other spatial variable implicit in the other half step. Thus, each half step involves the direct solution of a tri-diagonal system of equations .

During the first half step, the value of the concentration C is known at time level n and unknown at the $(n + 1/2)$, denoted by $*$. These unknown values, $C(j, k, *)$, are associated with the x-direction (*i.e.*, for ξ-implicit and η-explicit). The system of equations can be written as

$$a_j\, C(j-1,k,*) \;+\; b_j\, C(j,k,*) \;+\; c_j\, C(j+1,k,*) \;=\; d_j \tag{20}$$

with

$$\begin{aligned}a_j &= \frac{1}{2}\left[a^2 k_\eta(\eta)(1-\xi^2)^2\frac{\Delta t}{(\Delta\xi)^2} + a^2 k_\eta(\eta)(1-\xi^2)\xi\frac{\Delta t}{\Delta\xi} + au(1-\xi^2)\frac{\Delta t}{\Delta\xi}\right],\\ b_j &= -\left[1 + a^2 k_\eta(\eta)(1-\xi^2)^2\frac{\Delta t}{(\Delta\xi)^2} + \frac{1}{2}au(1-\xi^2)\frac{\Delta t}{\Delta\xi}\right],\\ c_j &= \frac{1}{2}a^2 k_\eta(\eta)(1-\xi^2)\left[(1-\xi^2)\frac{\Delta t}{(\Delta\xi)^2} - \xi\frac{\Delta t}{\Delta\xi}\right]\end{aligned} \tag{21}$$

and

$$\begin{aligned}d_j = &\left[k_\eta(\eta)\frac{\Delta t}{(\Delta\eta)^2} - 1\right]C(j,k,n) + \left[\frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} - \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2}\right]C(j,k-1,n) - \\ &-\left[\frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} + \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2}\right]C(j,k+1,n).\end{aligned} \tag{22}$$

Equation (20) is a tri-diagonal system of algebraic equations which can be solved by means of the Thomas algorithm [14, pp. 125–129]. Sequentially, the system of equations is solved for $C(j, k, *)$, $\quad j = 2, \ldots, N$, for each row $k = 1, \ldots, M + 1$.

To advance to the next half step with z-implicit and x-explicit, we solve the following set of equations for the unknown values of concentration $C(j, k - 1, n + 1)$ using the known intermediate values $C(j, k, *)$ from

$$a_k\, C(j,k-1,n+1) \;+\; b_k\, C(j,k,n+1) \;+\; c_k\, C(j,k+1,n+1) \;=\; d_k \tag{23}$$

with

$$\left.\begin{aligned} a_k &= \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2} - \frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta},\ b_k = -\left[1 + k_\eta(\eta)\frac{\Delta t}{(\Delta\eta)^2}\right] \\ c_k &= \frac{k'_\eta(\eta)}{4}\frac{\Delta t}{\Delta\eta} + \frac{k_\eta(\eta)}{2}\frac{\Delta t}{(\Delta\eta)^2} \end{aligned}\right\}, \tag{24}$$

and

$$\begin{aligned} d_k = &\left[a^2 k_\eta(\eta)(1-\xi^2)^2\frac{\Delta t}{(\Delta\xi)^2} + \frac{1}{2}au(1-\xi^2)\frac{\Delta t}{\Delta\xi} - 1\right] C(j,k,*) - \\ &- \frac{1}{2}\left[a^2 k_\eta(\eta)(1-\xi^2)^2\frac{\Delta t}{(\Delta\xi)^2} + a^2 k_\eta(\eta)(1-\xi^2)\xi\frac{\Delta t}{\Delta\xi} + \right. \\ &\left. + au(1-\xi^2)\frac{\Delta t}{\Delta\xi}\right] C(j-1,k,*) - \\ &- \frac{1}{2}a^2 k_\eta(\eta)(1-\xi^2)\left[(1-\xi^2)\frac{\Delta t}{(\Delta\xi)^2} - \xi\frac{\Delta t}{\Delta\xi}\right] C(j+1,k,*). \end{aligned} \tag{25}$$

This system of equations are then solved for $C(j,k,n+1)$, $k = 1,\ldots,M+1$, for each row $j = 2,\ldots,N$.

4. Discussion of results

In the present study the velocity distribution and corresponding eddy-diffusivity k_η (Equations (8) and (10)) due to Nezu and Nakagawa [9] for flow over a rough bed are used. These relations are different from the standard log-law $u(z) = \kappa^{-1}\log(z/z_0)$ and eddy-diffusivity $k(z) = \kappa z$. As knowledge of the mean velocity and turbulence characteristics over a rough bed is limited, modified smooth-bed relations are applied to a rough bed surface. Bed roughness plays an important role in the inner region very close to the wall. Keeping the values of the universal constant $\kappa = 0{\cdot}4$ and the integrating constant $A_r = 8{\cdot}5$ for a completely rough bed (Nezu and Nakagawa [9]) fixed, we use the velocity profile with adjustable wake strength parameter $\Pi = 0{\cdot}09$ in the numerical model to get closer agreement with the experimental results of Raupach and Legg [4] for the steady-state concentration distribution. The value of wake strength parameter ($\Pi = 0{\cdot}09$) is comparable with the Π-value reported in the literature by Song, Graf and Lemmin [15].

Results for steady-state dispersion are compared with the experimental data of Raupach and Legg [4] and the numerical results of Sullivan and Yip [5, 7]. In their wind-tunnel experiments, Raupach and Legg [4] used a flow depth H of 540 mm, a rough surface of 7 mm gravel glued to a wooden base board, and a heat source at a height of $h = 60$ mm above the zero plane of the surface. Their measured data yielded an approximate logarithmic velocity profile $u^* = u_*/\kappa\log(z^*/z_0)$, with the roughness height $z_0 = 0{\cdot}12$ mm and $\kappa = 0{\cdot}38$. The vertical and downstream distances were normalised by the source height h for the comparison with the measured concentration values. The values of concentration C has been normalised with the temparature scale

$$\theta_* = \frac{F}{hu(h)},$$

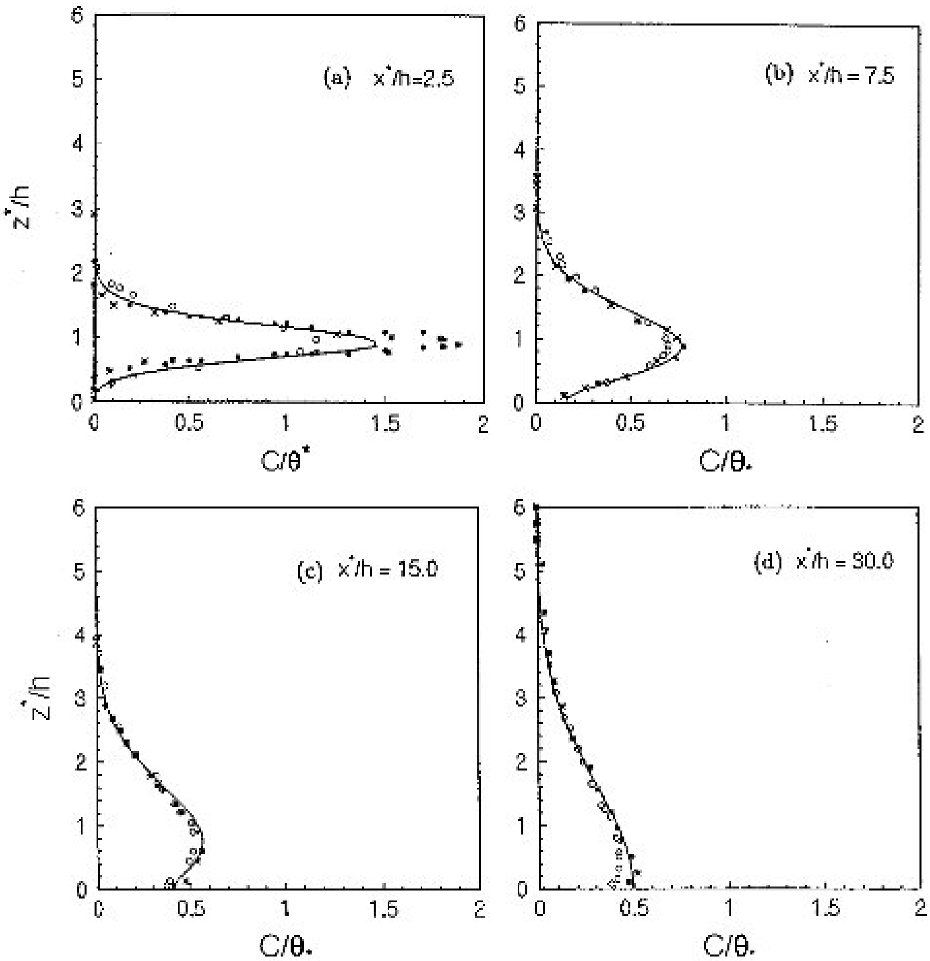

Figure 1. Comparison of dimensionless, steady, concentration profiles at various downstream distances among the experiments of Raupach and Legg [4] (∗ ∗ ∗), the solution-scheme of Sullivan and Yip [5] (∘ ∘ ∘), time-dependent eddy-diffusivity of Sullivan and Yip [7] (• • •) and the present solution (—).

where F is the constant flux of contaminant through a plane normal to the flow (Sullivan and Yip [5]).

Comparison of the present results with the experimental data of Raupach and Legg [4] and the numerical results of Sullivan and Yip [5] for the steady-state concentration distribution at four downstream distances ($x^*/h = 2{\cdot}5, 7{\cdot}5, 15{\cdot}0, 30{\cdot}0$) are shown in Figure 1(a–d). A remarkably good comparison is achieved, except the closest downstream station (Figure 1a) $x^*/h = 2{\cdot}5$, if the log-wake-law with the value ($\Pi = 0{\cdot}09$) is used, although the off-diagonal diffusion terms in the basic equation have been neglected. The present calculation may show the drawback of an eddy-diffusivity approach to the closest downstrean station that does not take into account the time-dependent nature. As the mean velocity is considered to be steady, it would be more reasonable to adopt the corresponding eddy-diffusivity that is independent of time. It may be pointed out here that with this approach, taking into account time-independent nature in eddy-diffusity, we have a reasonably good agreement with the experimental results. However, for time-dependent case, we have some discrepancy in explaining all the experi-

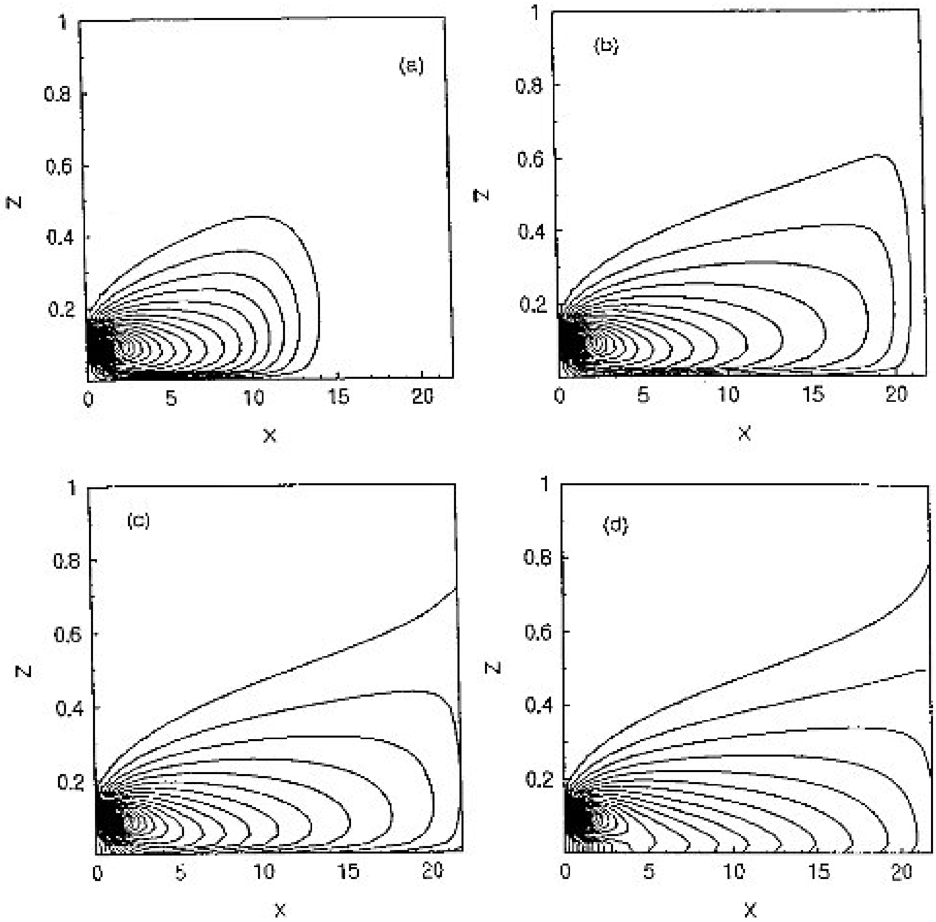

Figure 2. Contours of concentration of the slug injected at a height $z_p = 0{\cdot}11$, in the (x, z) plane at different non-dimensional time: (a) $t = 0{\cdot}2$, (b) $t = 0{\cdot}6$, (c) $t = 2{\cdot}0$, (d) $t = 10{\cdot}0$.

mental data in a uniform way. The excellent agreement between the results in Figure 1 except Figure 1(a) can be used as validation of the numerical scheme for the case of log-wake law velocity. The discrepancies at $x^*/h = 2{\cdot}5$ may be due to the formulation of the model and/or the neglect of the off-diagonal diffusion terms. However, a better representation of data at *only* the closest measuring station ($x^*/h = 2{\cdot}5$) from the source was achieved by Sullivan and Yip [7], incorporating some form of time-dependent eddy-diffusivity in the steady flow (Figure 1a).

For the results of unsteady dispersion, Figures 2(a–d) show the contours of iso-concentration drawn with an increment 0·01, the outermost contour having a value 0·01, when the slug of the solute is released from a point $z_s = 0{\cdot}11$ at a different non-dimensional time t (= 0·2, 0·6, 2·0, 10·0). As time increases, the solute disperses more and more in the longitudinal as well as in the vertical directions and it can also be observed from the figures that the concentration contours become more elongated in the longitudinal direction compared to the vertical direction, because dispersion due to longitudinal convection is much stronger than transverse diffusion. Variations of C with dimensionless time t are plotted in Figure 3 at downstream stations: (a) near the source and (b) away from the source. Figure 3 shows that, near the source,

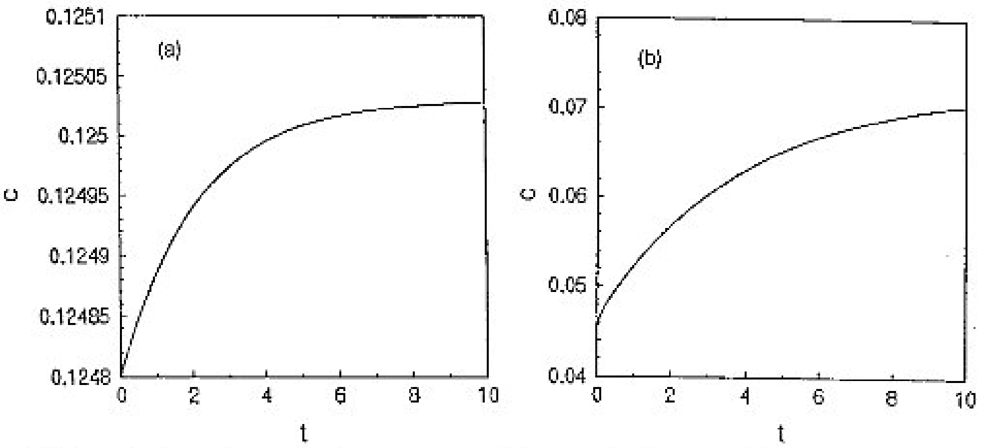

Figure 3. The variation of concentration C with non-dimensional time t at : (a) very near to the source ($x = 1{\cdot}0, z = 0{\cdot}175$), (b) away from the source ($x = 13{\cdot}5, z = 0{\cdot}175$).

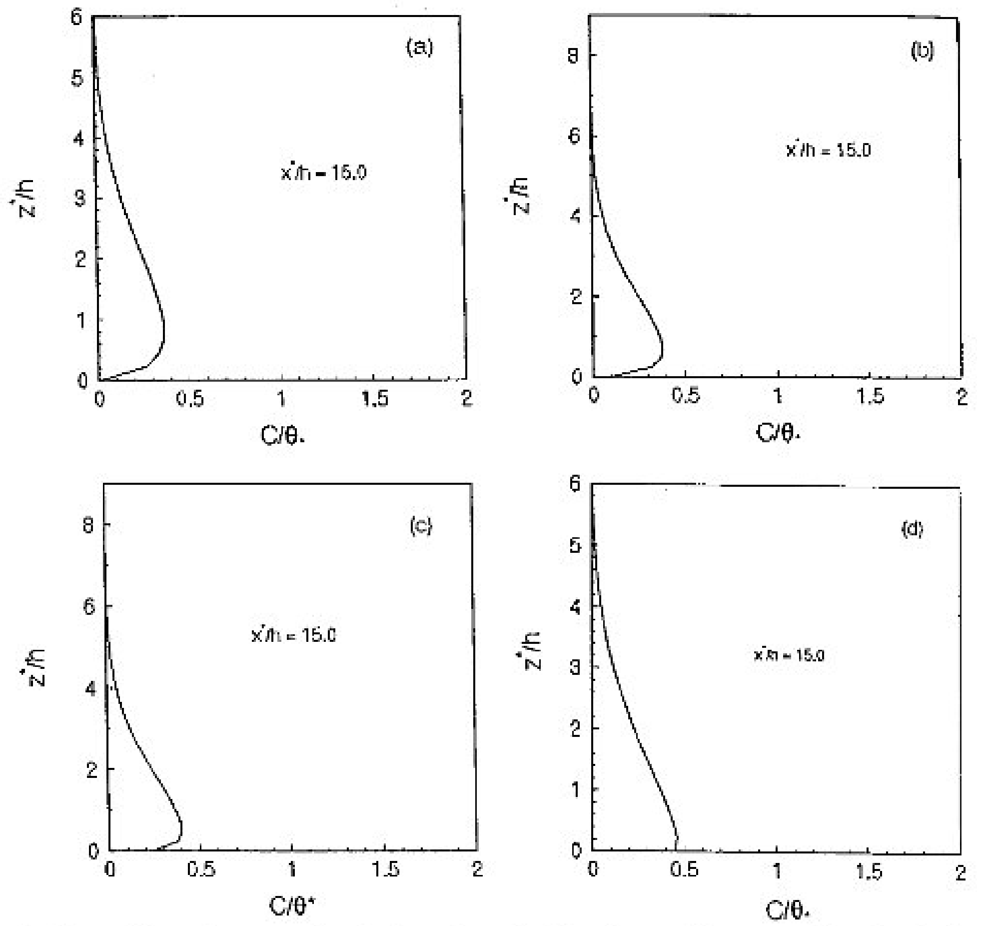

Figure 4. The variation of concentration C along the vertical direction at different non-dimensional time : (a) $t = 0{\cdot}2$, (b) $t = 0{\cdot}6$, (c) $t = 2{\cdot}0$, (d) $t = 10{\cdot}0$ at the downstream station $x^*/h = 15{\cdot}0$.

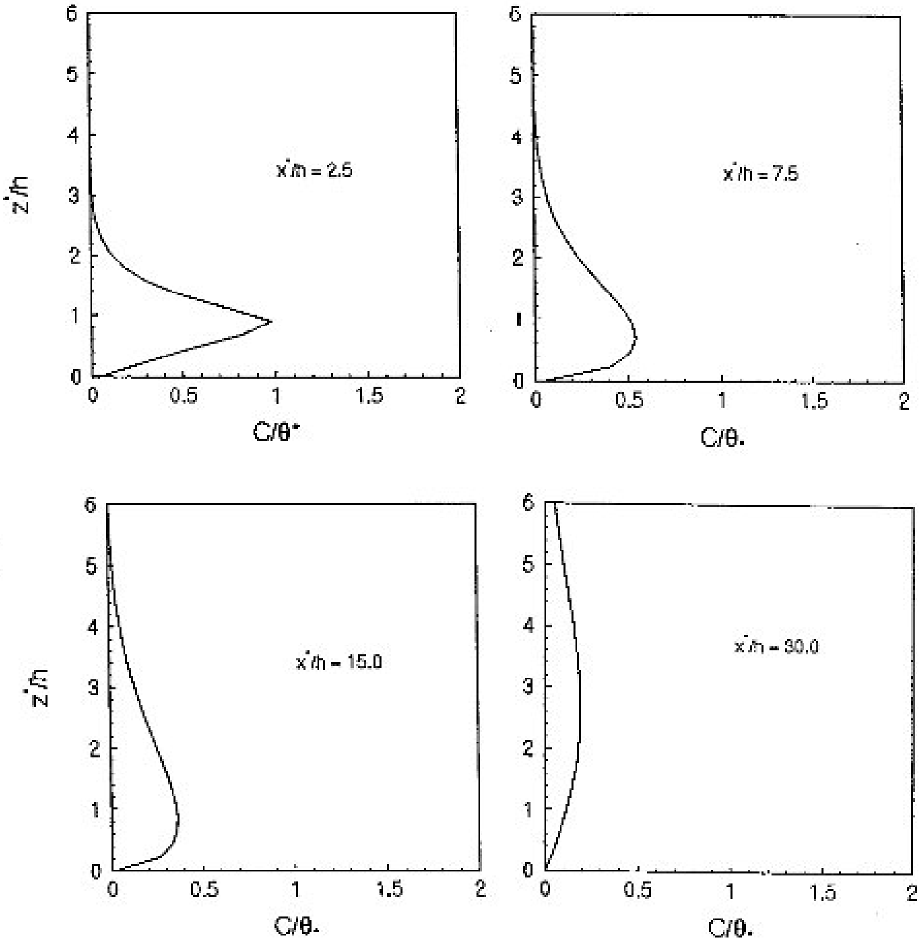

Figure 5. The variation of concentration C along the vertical direction at different downstream distances (x^*/h) at time $t = 0{\cdot}2$.

C increases rapidly with dimensionless time t ($0 \leq t \leq 4$) and asymptotically reaches to a steady state at time $t \approx 7{\cdot}0$. Far from the source, C also increases rapidly and asymptotically reaches a steady state after a certain time $t = 10$, but the amount of concentration is much less than that of near-source. Figure 4 shows the vertical variations of C at a fixed downstream station ($x^*/h = 15{\cdot}0$) for different dimensionless times ($t = 0{\cdot}2,\ 0{\cdot}6,\ 2{\cdot}0,\ 10{\cdot}0$). It is interesting to note that, as the time of dispersion increases, the vertical distribution of C approaches a steady-state at $t = 10{\cdot}0$ (Figure 4d), which is comparable with Figure 1(c). The normalised distributions C/θ_* are plotted against z^*/h for an initial time $t = 0{\cdot}2$ in Figure 5 for various downstream stations. It is observed from these figures that, for a given initial time, the mean concentration distribution disperses across the vertical distance and tends to become flat, reducing the concentration value as the distance increases in the downstream direction.

5. Conclusions

A numerical solution to the convection-diffusion equation has been obtained for solute dispersion in an open-channel flow with a modified 'log-wake law' and the corresponding eddy-diffusivity due to Nezu and Nakagawa [9]. The concentration values obtained from the proposed velocity profile and the corresponding eddy-diffusivity fit well with the experimental data of Raupach and Legg [4] and those obtained by the solution scheme (Sullivan and Yip [5]), except for the closest downstream measuring station.

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