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Farm Efficiency under Cobb-Douglas Production Technology

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The authors aim to examine the allocative efficiency hypothesis in regard to West Bengal agriculture, rejected by Dey and Rudra after a period of some 18 years during which a transformation of the West Bengal agrarian economy is believed to have taken place, partly in response to use of modern agricultural inputs and partly due to institutional changes. It is interesting to examine the efficacy of the allocative efficiency model under the changing conditions of West Bengal agriculture.

I
Introduction

IN earlier communications to this journal, Dey and Rudra (1973 a, b) presented some results based on farm-level data of Farm Management Survey of West Bengal of a test carried out for the hypothesis of allocative efficiency of farmers under the assumption of Cobb-Douglas technology.[1] The authors considered the production function connecting value of output (O) with two input variables defined as labour input (L) and material inputs (M) both for crops taken individually and for crops taken together. They studied empirically the relation between the pairs of variables (M, L and O) and found that there is a strong association between the pairs of variables for all crops and no association for individual crops. The lack of association between the variables for individual crops with the derived test results prompted them to conclude that the hypothesis of allocative efficiency of farmers gets rejected at least in the case of West Bengal irrespective of any production technology.

The aim here is to examine the same allocative efficiency hypothesis in the same region after a period of about 18 years. During this period, some authors argue that a transformation of West Bengal agrarian economy has taken place in terms of its changes in production of major crops. This has been possible partly due to the widespread use of modern agricultural inputs and partly due to the institutional changes that occurred for the implementation of land reform programme and the establishment of the three-tier panchayat system in the 1980s in rural West Bengal [Saha and Swaminathan 1994]. It will be interesting to examine the efficacy of allocative efficiency model under the changing conditions of West Bengal agriculture.

II
Methodology and Data

Theoretically it is argued that if the farmers operate under a common Cobb-Douglas production function and if they optimally allocate material and labour inputs, then observed material inputs and labour input figures may be expected to fall along a straight line passing through the origin. Based on this theoretical result, Dey and Rudra (1973a) designed a test which has the following four fits for *all crops* as a whole:

$$H_0 : \bar{L}_r = \theta_1 \bar{M}_r$$

$$H_1 : \bar{L}_r = \theta_1 + \theta_2 \bar{M}_r$$

$$H_2 : \bar{L}_r = \theta_2 \bar{M}_r + \theta_3 \bar{M}_r^2$$

$$H_3 : \bar{L}_r = \theta_1 + \theta_2 \bar{M}_r + \theta_3 \bar{M}_r^2$$

where θ_1, θ_2 and θ_3 are constants; $\bar{M}_r$, per acre material inputs and $\bar{L}_r$, per acre labour input in farm r. Tests for null hypothesis (H_0) against the specific alternate hypothesis (H_1, H_2, H_3) may be carried out with view to find out what functional form might best explain the data and therefore to draw inferences on the hypothesis of profit maximisation under Cobb-Douglas production function.

Further, to examine the allocative efficiency hypothesis in a different way Dey and Rudra (1973b) considered the proportions in which the inputs are allocated among different crops and suggested some test criteria for individual crops as well.[2]

In this exercise we have examined the allocative efficiency model mainly on the basis of data of all crops. Since the data on output and inputs into the different crops are not available except for paddy, we have concentrated on the cropwise analysis of the major crop paddy for all varieties cultivated by the farmers of selected households. We have used Comprehensive Scheme for Studying Cost of Cultivation (CSSCC) data pertaining to the year 1989-90 for West Bengal only.[3]

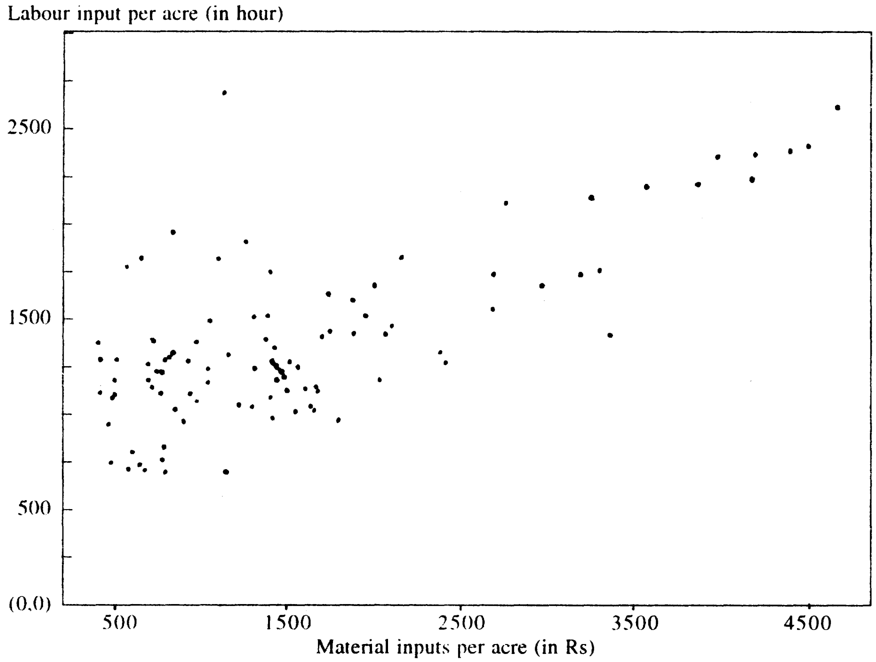
For the purpose of collecting CSSCC data in West Bengal, the entire state was divided into six agro-climatic zones based on cultivation practices and rainfall.[4] Varying incidence of these village specific factors such as soil fertility, irrigation facilities, etc, may alter the efficacy of allocative efficiency model. In order to take a view in this matter we have used in this exercise a sample of 103 households from three prosperous zones and 98 households from one relatively less prosperous zone of West Bengal.[5] Thus, we have divided our total sample, (i e, 201 households) into two sub-samples and analysed separately the relationships of the variables described above for each sub-sample on the same line as those carried out by Dey and Rudra (1973a, b). We may now present the results of our analysis in the following section.

TABLE 1: RESULTS OF REGRESSION COEFFICIENTS WITH THEIR STANDARD ERRORS AND R^2

Forms of the Fits	Developed Zone				Less Developed Zone			
	θ_1	θ_2	θ_3	R^2	θ_1	θ_2	θ_3	R^2
$H_1 : \bar{L}_r = \theta_1 + \theta_2 \bar{M}_r$	940.71	0.290	–	0.5285	990.47	0.165	–	0.4297
	(148.35)	(0.027)			(33.83)	(0.019)		
$H_3 : \bar{L}_r = \theta_1 + \theta_2 \bar{M}_r + \theta_3 \bar{M}_r^2$	1069.93	0.110	0.00004	0.5462	963.97	0.203	–0.000008	0.4325
	(80.97)	(0.095)	(0.00002)		(51.33)	(0.058)	(0.00004)	

Standard errors are in parenthesis.

DIAGRAM 1(a): DEVELOPED ZONE



III
Results

We present the scatter diagrams showing $\bar{L}_r$ and $\bar{M}_r$ for all crops for developed and less developed zones of West Bengal in Diagrams 1(a) and 1(b) respectively. The scatter for developed zone shows a positive relationship between L_r and M_r, but the observed points do not lie closely along any straight line passing through the origin. The scatter for less developed zone also shows similar relationship between the two variables.

As a measure of association, tests for null hypothesis against the specific alternatives (H_1 and H_3) were carried out with the help of the above mentioned data and the results were presented in Table 1. Table 1 shows that the estimate of intercept for an unconstrained straight line fit turns out to be 940.71, the 't' value (19.46) of which is significant at 1 per cent level. This means that our data do not give acceptable fit to a straight line passing

TABLE 2A: RESULTS OF ANALYSIS OF VARIANCE FOR $H_1 : \bar{L}_r = \theta_1 + \theta_2\bar{M}_r$

Source	Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	9358974	1	9358974	113.19	7.19	3244556.8	1	3244557	72.32	7.19
Deviation from regression	8350720	101	82680.40			4306963	96	44864.20		
Total	17709694	102				7551519.8	97			

TABLE 2B: RESULTS OF ANALYSIS OF VARIANCE FOR $H_3 : \bar{L}_r = \theta_1 + \theta_2\bar{M}_r + \theta_3\bar{M}_r^2$

Source	Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	9358974	1	9358974	116.44	7.19	3244556.8	1	3244556.7	71.92	7.19
Addition due to quadratic term	313245	1	313245	3.90	7.19	21355.3	1	21355.3	0.47	
Residual	8037475	100	80374.75			4285607.7	95	45111.66		
Total	17709694	102				7551519.8	97			

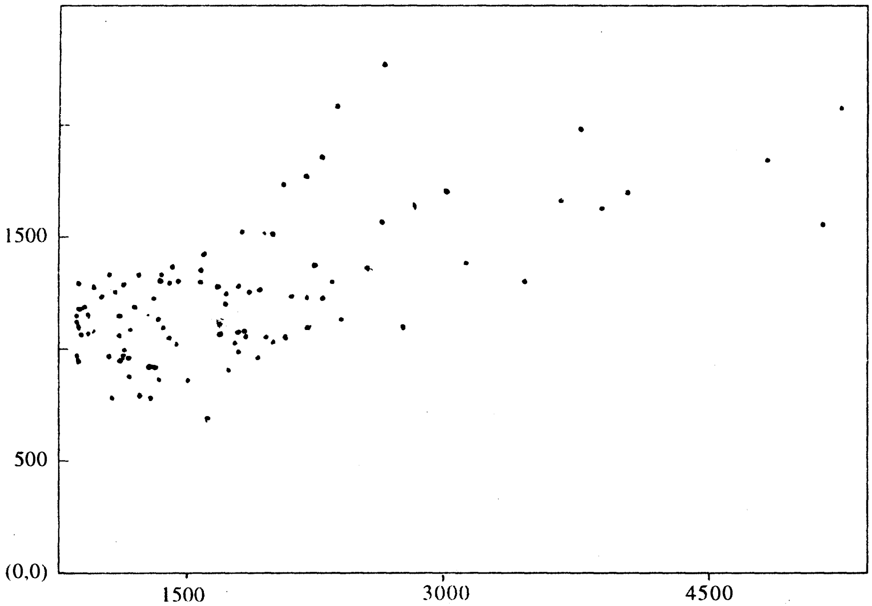
TABLE 3A: RESULTS OF ANALYSIS OF VARIANCE FOR $\bar{L}_r = \theta_1 + \theta_2\bar{O}_r$

Source	Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	11550216	1	11550216	189.39	7.19	2721860.5	1	2721860.5	54.10	7.19
Deviation from regression	6159478	101	60984.93			4829660	96	50308.96		
Total	17709694	102				7551520.5	97			

TABLE 3B: RESULTS OF ANALYSIS OF VARIANCE FOR $\bar{L}_r = \theta_1 + \theta_2\bar{O}_r + \theta_3\bar{O}_r^2$

Source	Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	11550216	1	11550216	189.24	7.19	2721860.5	1	2721860.5	54.10	7.19
Addition due to quadratic term	55856	1	55856	0.92	7.19	92577.2	1	92577.2	1.86	7.19
Residual	6103622	100	61036.22			4737082.8	95	49864.03		
Total	17709694	102				7551520.5	97			

DIAGRAM 1(b): LESS DEVELOPED ZONE



through the origin for the developed zone. Hence if the nature of production function is assumed to be a Cobb-Douglas type, our results do not support the validity of profit maximisation hypothesis. Regression results for the less developed zone, presented in the same table, also provide the same picture.

The estimates of parameters for quadratic fit, presented in Table 1, show that the use of labour input increases (θ_3 = 0.00004) at a faster rate than that of material inputs in the developed zone and the value of R^2 is marginally higher than that of linear fit. However, considering the values of other parameters and their standard errors, our data can be better fitted by a straight line with a positive intercept on the L axis than that of a quadratic fit not only for the developed zone but for the less developed zone as well. This can be verified by the results of Analysis of Variance exercises presented in Tables 2A and 2B.

Results presented in Tables 2A and 2B show that hypothesis H_1 explains the data significantly better than hypothesis H_3 both for developed and less developed zones. In fact, observed 'F' value in case of linear fit is significant at 1 per cent point whereas in case of quadratic fit the observed 'F' value is much lower than the expected value at 1 per cent level. These findings once again confirm the strong linear association between labour input and material inputs for both the zones. However, such strong relationship does not necessarily imply that the farmers are efficient resource allocators under the Cobb-Douglas technology for the reason that the intercept of the linear fit differs significantly from zero.

Now with the expectation that strong association between $\bar{L}_r$ and $\bar{M}_r$ for all crops should give similar association between $\bar{L}_r$ and $\bar{O}_r$ as well as $\bar{M}_r$ and $\bar{O}_r$ we fitted both linear and quadratic regression curves between the variables of each pair both for developed and less developed zones of West Bengal. We find once again that linear curve with positive intercept gives relatively better fit than the quadratic curve. The fitted linear equations are presented below.

For developed zone:

$$E(\bar{L}_r) = 342.04 + 0.10\bar{O}_r \; (R^2 = 0.6520)$$

$$E(\bar{M}_r) = -1063.70 + 0.26\bar{O}_r \; (R^2 = 0.6302)$$

For less-developed zone:

$$E(\bar{L}_r) = 826.13 + 0.04\bar{O}_r \; (R^2 = 0.3604)$$

$$E(\bar{L}_r) = -865.08 + 0.24\bar{O}_r \; (R^2 = 0.7483)$$

Results of Analysis of Variance for two fits are presented in Tables 3A and 3B respectively. Table 3A confirms the strong linear association between $\bar{L}_r$ and $\bar{O}_r$ both for developed and less developed zones and Table 3B rules out the significance of quadratic fit. Similar results are obtained in studying the association between $\bar{M}_r$ and O_r with respect to the two fits (Tables 4A and 4B).

Thus, our results suggest that there exist some strong association among the three variables for all crops in case of developed zone and in case of less developed zone, though the estimate of R^2 between $\bar{L}_r$ and $\bar{O}_r$ is quite small (0.36), it is as high as 0.75 between $\bar{M}_r$ and $\bar{O}_r$. These findings are more or less similar to the findings of

TABLE 5A: CORRELATION COEFFICIENTS BETWEEN PER ACRE OUTPUT AND PER ACRE INPUT FOR PADDY

Per Acre Input	Per Acre Output: Developed Zone	Less Developed Zone
Material Input	0.7071	0.7099
Labour Input	0.7676	0.4879

TABLE 4A: RESULTS OF ANALYSIS OF VARIANCE FOR $\bar{M}_r = \theta_1 + \theta_2\bar{O}_r$

Source	Developed Zone: SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone: SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	69957208	1	69957208	172.15	7.19	89055416	1	89055416	285.43	7.19
Deviation from regression	41044760	101	406383.8			29952708	96	312007.4		
Total	111001968	102				119008124	97			

TABLE 4B: RESULTS OF ANALYSIS OF VARIANCE FOR $\bar{M}_r = \theta_1 + \theta_2\bar{O}_r + \theta_3\bar{O}_r^2$

Source	Developed Zone: SS	df	Mean SS	Obs F	F at 1 Per Cent	Less Developed Zone: SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	69957208	1	69957208	178.51	7.19	89055416	1	89055416	282.54	7.19
Addition due to quadratic term	1855943	1	1855943	4.74	7.19	8931	1	8931	0.028	7.19
Residual	39188817	100	391888.17			29943777	95	315197.66		
Total	111001968	102				119008124	97			

TABLE 5B: ANALYSIS OF VARIANCE FOR LINEAR REGRESSION BETWEEN LABOUR INPUT AND OUTPUT FOR PADDY

Source	Developed Zone					Less Developed Zone				
	SS	df	Mean SS	Obs F	F at 1 Per Cent	SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	8457577	1	8457577	144.84	7.19	1510297.3	1	1510297	29.99	7.19
Residual	5897573	101	58391.81			4835242	96	50367.11		
Total	14355150	102				6345539	97			

TABLE 5C: ANALYSIS OF VARIANCE FOR LINEAR REGRESSION BETWEEN MATERIAL INPUT AND OUTPUT FOR PADDY

Source	Developed Zone					Less Developed Zone				
	SS	df	Mean SS	Obs F	F at 1 Per Cent	SS	df	Mean SS	Obs F	F at 1 Per Cent
Linear regression	87938072	1	87938072	101.00	7.19	21729642	1	21729642	97.53	7.19
Residual	87941944	101	870712.30			21389636	96	222808.7		
Total	175880016	102				43119278	97			

Dey and Rudra (1973b) who observed a strong association among these variables.

Now if we want to consider not merely the combination in which different resources are used but also the proportions in which they are allocated among different crops, one should consider output and inputs separately into different crops. We have already mentioned that our data at the individual crop level are available only for paddy. We have thus calculated product moment correlation coefficient between the different relevant pairs of three variables to examine the nature of association between the variables only for paddy. Interestingly, our results show significant linear relationship between the relevant pairs of three variables for paddy both for developed and less developed zones of West Bengal (Tables 5A, 5B, 5C). It may be noted in this context that Dey and Rudra (1973b) observed the complete absence of any association between these variables at the crop-level analysis.

It is quite clear from the above analysis that our findings based on crop-level data do not agree with the findings made by Dey and Rudra (1973b) who suggested no association between labour input (per acre) and output (per acre) and material input (per acre) and output (per acre) for individual crops. We also observe that the relationships between L, M and O can best be represented by the linear functional form with positive intercept than the quadratic type. Obviously, such kind of linear relationship does not necessarily accept the hypothesis of allocative efficiency under the assumption of Cobb-Douglas technology.

IV
Concluding Observations

We may conclude by reiterating that the study done by Dey and Rudra ruled out the confirmation of the hypothesis of allocative efficiency of Indian farmers under a Cobb-Douglas technology. The present study, by and large, supports their conclusions. However, our study does not agree with their view that no association between labour input, material inputs and output exists at the individual crop-level. Our study indicates the existence of linear relation between these variables both for developed and less developed zones of West Bengal. In fact, judging by the results of our analysis it seems to us that the distinction between developed and less developed zones might not be very meaningful in terms of the fitted results obtained in our study. Similarly, no conclusion can be drawn from the findings of our study regarding the effects of new agricultural technology as well as institutional reforms taking place in rural West Bengal in the recent decades on the production function frontier. However, on the basis of the existence of linear relationship between labour input, material inputs and output, it may be suggested that there should exist some production function frontier which seem to be appropriate in the context of understanding both institutional and technological reality prevailing in West Bengal agriculture.

Notes

1 Dey and Rudra (1973a, b) used farm level disaggregated data collected from 142 farms belonging to 15 villages of Hooghly district of West Bengal for agricultural year 1970-71.

2 The test criteria derived by Rudra (1973b) for individual crops are as follows:

$$H_1 : \frac{M^i}{O^i} = \theta_1 \left(\frac{M^j}{O^j}\right)$$

$$H_2 : \frac{M^i}{L^i} = \theta_2 \left(\frac{M^j}{L^j}\right)$$

$$H_3 : \frac{L^i}{O^i} = \theta_3 \left(\frac{L^j}{O^j}\right)$$

where θ_1, θ_2, and θ_3 are constants; M, material inputs; L, labour input; O, output and i and j stand for crops.

3 Data used in this paper were collected by the CSSCC from different parts of West Bengal rather than from a particular district as was done by Dey and Rudra (1973a, b). We find that the crop combination, as considered by Dey and Rudra, is quite different from one region to another in our sample. However, paddy cultivation is common for all the farmers and for all the regions under study.

4 Details of sample design and selection of households, followed by the CSSCC, have been given in Chattopadhyay and Sengupta (1997).

5 The CSSCC selected 600 farming households from six zones of West Bengal. Zones were classified in terms of soil fertility, irrigation facilities and other factors [see, for details, Chattopadhyay and Sengupta 1997]. We have, in this study, selected 103 households from three prosperous zones of West Bengal, viz, Old Alluvial, New Alluvial and Saline zones and 98 households from one relatively less prosperous zone, viz, Red Laterite.

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