CONTRIBUTIONS TO METHODOLOGY OF CONSTRUCTION OF CONSISTENT INDEX NUMBERS

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CONTENTS

INTRODUCTION

International variation in national product is a subject of considerable interest, and any meaningful comparison of national product or per capita national income has to be in terms of a common currency. An inter-temporal comparison for the same country at two points of time is more meaningful in real terms, or in other words when the currency unit is adjusted for change in the general price level between the two points of time. In the same manner, the level of output or of consumption of a group of persons belonging to a region or to a particular socio-economic group can be meaningfully compared with another such group, when an appropriate adjustment is made for the variations in the purchasing power of the monetary unit used in the two groups to obtain the measures of the aggregates at current prices. This results in the need for suitable index number formulae for price comparisons. Research in this line together with the availability of numerous index number formulae made the problem of choice of a system of index numbers, for a given comparison problem, more acute.

This has led to the formulation of some restrictions which are intutively justified, so that the index numbers not satisfying these restrictions may be discarded. These are termed as 'consistency tests' in the literature of index numbers. It is Irving Fisher who introduced the idea of consistency tests and brought them into prominence. We focus our attention on the circularity test which is the generalization of Fisher's time reversal test and this test gives the basic problem of the thesis.

Let us formulate the problem of multilateral price and quantity comparisons, given the price and quantity vectors concerning each country. This discussion can be made more general by replacing the countries by any set of well defined population groups. The groups may refer to different countries or different sections of the population of a country defined according to certain social and economic characteristics, and it may be convenient to talk in terms of groups from now on. The analysis works in all cases without loss of generality. Let I_{jk} represent the price index for the k-th group with the j-th group as base.

In this set-up, the circularity test requires that the price change between any two groups can be expressed as the product of price changes measured through a third group. Hence the test stipulates that $I_{jk} = I_{j\ell} . I_{\ell k}$ for all groups j, k and ℓ. This test makes use of the assumption that the price changes are multiplicative. The condition becomes

$I_{jk} = I_{j\lambda} + I_{\lambda k}$ in the case of additive set-up. These two tests are complementary in nature and index numbers satisfying one condition generate a system of index numbers satisfying the other condition.

The circularity test implies the existence of M positive numbers say π_j $(j = 1, \ldots, M)$ in the case where number of groups is M, so that any index I_{jk} is given by π_k / π_j. Here π_j can be interpreted as the general price level of group j, so that the ratio of these numbers gives the required index number. In any problem, one of the π_j's can be chosen to be unity since we are mainly interested in the ratios of these numbers. However, it might happen that one index number system leads to multiple sets of these numbers, and each set of numbers satisfying the circularity test. Hence the formulae which led to unique set of the ratios of these numbers is of importance. Index numbers with this property are termed as consistent and our problem is to obtain formulae which yield consistent comparisons. We may regard the formulae leading to indices satisfying circularity test but not consistent as semi-consistent methods. The virtue of a consistent method is that it guarantees unique ordering of the given price and quantity vectors and the ordering is complete and transitive. Transitivity is a major requirement in any practical problem.

Further the problem of obtaining numerous binary comparisons is absent in this case where the general price levels are determined simultaneously.

In the traditional literature on index number construction, we find that the circularity test is used in conjection with many other tests like factor reversal test, factor test, proportionality test, determinateness test and commensurability. test. If there the proportionality test is of some relevance to us. The test states that if all prices increase by a given proportion, then the index number should also increase by the same proportionate change. We make use of a weaker version of this test which may be called 'weak proportionality test'. This necessiates that if prices in one group are higher than prices in another, the index number for the latter group with the former as base should be greater than unity.

This thesis studies the methodology of construction of consistent index numbers for price comparisons among well defined groups of population. The results obtained here are useful for both inter-temporal and international comparison problems. We concentrate on the price comparisons only, for, every system of price indices has an implicit system of quantity index numbers. This is because of the condition that the product of price and quantity indices should yield the value ratios. In fact, there is a kind of duality between the price and quantity comparisons. The thesis consists of three chapters and the chapter-wise summary is given below.

About the Thesis :

The first chapter deals with the Geary-Khamis method of constructing consistent index numbers, which appears in Geary, R.C. (1958), Khamis, S.H. (1969, 1970a, 1970b). These index numbers are based on intuitively appealing concepts like 'exchange rate' of a currency and 'average price' of a commodity. The exchange rate is used in the same sense as the purchasing power of a currency which is the reciprocal of the general price level, defined earlier. Once this is defined, the average price of a commodity is obtained as the measure of central tendency of the prices of the commodity in various groups transformed into a comparable currency unit. The values of the unknowns are obtained as solutions of a system of linear homogeneous equations, given the price and quantity information. Khamis obtained some sufficient conditions which ensure the existence of meaningful solution for the underlying equations (positivity and uniqueness). However, sufficient conditions are only of limited use in any practical situation. We obtain necessary and sufficient conditions for the existence and uniqueness of positive solution to the above unknowns. We prove that meaningful solutions exist if and only if the set of countries cannot be divided into two nonempty subsets such that they do not have any commodity common in their consumption baskets. The result is proved by using the properties of non-negative, irreducible and dominant diagonal matrices and graphs. Apart from its virtue of easy verifiability in any practical problem, this condition implies the impossibility of price comparisons when the groups involved are not related in their expenditure patterns.

In the next section we propose a new system of consistent index numbers. These indices also make use of the concepts of exchange rate and average price. Observing that these are defined as arithmetic averages of a set of observations with some specified weights in the Geary-Khamis set-up, we make use of geometric mean and a different set of weights. Hence the solution of the unknowns emerges from a system of log-linear equations. The consistency of this system of equations is established and the conditions stated in the previous paragraph are shown to be necessary and sufficient in this case also. In the particular case where the number of countries involved are only two, these new index numbers turn out to be similar to those used by Kloek, T. and Theil, H. (1965). Further we show that if all prices move proportionately by a fixed amount then the indices show an increase by the same proportion.

The second chapter starts with a brief introduction to a few other index number formulae which are in use, like Braithwaite's (1970) fixed weight indices, Elteto, O. and Koves, P. index numbers (1965), and Yzeren's indices (1956). We provide an alternative derivation, which is simpler, to the Elteto-Koves index number formula. These index numbers minimize the logarithmic distance from the corresponding Fisher's ideal indices for binary comparisons.

The next section probes into the definition to average price that is used in Geary-Khamis method. Given the exchange rates, the average price is defined as the weighted arithmetic mean of the prices transformed into a common currency unit. The weights employed are the quantity shares. The problem is one of choosing a measure of central tendency. We choose the formula for the average which minimizes the sum of squares of deviations of the mean from the variables. The Geary-Khamis definition of average price is shown to be the result of one such minimization. Some interesting results are obtained by giving equal weights. In this it is proved that, in most of the cases, only trivial solution is possible. However, in this case, by neglecting one of the equations, we can obtain a unique, strict positive solution for the unknowns. This leads to multiplicity of solutions for unknowns, one for each group. By taking the geometric mean of these sets of values, we can arrive at one set of values, which yield a consistent system of index numbers. It is shown that, in the case where the number of countries is two, the index numbers given by the vectors correspond to the Laspeyres' and Paasche's indices. The indices related to the geometric mean of the vectors, obviously, coincide with Fisher's ideal indices. Thus this particular case provides an interesting generalization of these indices, in the direction of number of groups. On the otherhand the whole exercise establishes a relation between the traditional and more popular index numbers and the Geary-Khamis index numbers, all of them linked through the concepts of 'exchange rate' and 'average price'.

The final section considers the problem of the choice of a suitable formula in a given price comparison problem. The availability of numerous index number formulae satisfying the various consistency tests mentioned previously, makes the problem more acute. We propose an emperical criterion based on the simple idea of minimum sum of squares of deviations. This criterion may be used directly to obtain consistent indices, but the involved non-linearities make it difficult to apply. However, a slightly modified criterion, based on the minimum sum of squares of the deviations of the logarithms can be used for such purpose. In the case of binary comparisons, we have shown that the resulting index number formula coincides with the Kloek-Theil index number formula.

The last chapter deals with the problems that arise in the application of consistent system of index numbers for concrete price comparisons. Firstly, we consider the applicability of consistent index number formulae in comparing price vectors in heterogeneous groups. It can be seen that the binary comparisons are distorted due to the consistency restriction. We demonstrate that in some extreme cases, consistent index numbers fail to result in meaningful comparisons. For example, the index number may indicate a fall in general price level even in the case of increase in prices of all items. We show that the Geary-Khamis system falls under such category, by exhibiting an example and analysing the underlying linear equations. We prove a similar result in the case of Elteto-Koves system of index numbers.

A modification of these two systems has been suggested to yield better comparisons.

It is frequently noticed that while the aggregate expenditure can be exhaustively broken down by a detailed list of items, it may not be possible to get separate quantity and price information for all the items. But all the index number formulae presuppose the existence of price and quantity components of the expenditure data. In view of this we suitably adapt the various index number formulae. Another case of very high significance is when the lists of items of consumption of various countries are not identical. This occurs frequently in both inter-temporal and inter-country price comparisons. The reasons for this are many. Since the main aim of the price comparison between countries and groups is to obtain comparable average levels of living by deflating the aggregate consumption expenditure by the price indices, we seek to identify such characteristics of commodities which directly contribute to the level of living. Once these characteristics are obtained, the problem boils down to one of finding the corresponding price vectors. Similar problem is tackled by using Regression Methods in obtaining price indices for automobiles by Griliches, Zvi (1961).

In this section we use the Linear Programming technique to obtain the equilibrium price vector of the identified characteristics and discuss the properties of this method. It is observed that this method seems potentially to be superior to the Regression Method.

About the Data :

Most of the theoretical results are supported by numerical illusttations. In the absence of data relating to prices and quantities in various countries, the problems discussed in thesis for inter-group or inter-country comparisons are illustrated through Indian data. Apart from serving the purpose of illustration, the numerical exercises undertaken in the thesis have special significance. Through these examples we try to measure the relative price levels in various income groups in India. The income groups are defined in the following manner. Sample households are arranged in ascending order of their percapita incomes and groups are formed such that each particular group, from the bottom, gets a specified percentage of estimated population, the lowest decile group thus having the groups have been described as fractile groups by Mahalanobis P.C. These groups gives the size classes with which we are concerned. In some cases the Rural-Urban comparisons are also obtained.

The importance of price comparisons in various income groups is well recognized and the first formal study on this subject is by Ryotaro Iochi (1964) and there are many attempts by research workers in India and elsewhere to measure the price changes in groups of population belonging to different income brackets. The first reaction of anybody on this problem would be to question the proposition that people in different income groups pay different prices for the same commodity. At the outset the answer may seem to be 'no'. An intuitive feeling may be that people pay higher price for a commodity for its higher quality or other considerations. Against this is the argument that people in higher income groups pay a smaller price for the item, due to some economies of scale or the bulk of commodities they purchase. On the otherhand the market **imperfections** and the mentality of the upper income group people justify the higher prices they pay for the same item. The shopping habits and reluctance to find out the actual price of a commodity lead to higher price payments. Hence the problem of inter-fractile group price comparisons exists and is very important. The fact that expenditure pattern depends mainly on the income of the individuals, the impact of price change on two income groups would be different. This problem is considered in Ryotaro Iochi (1964) and the existence is demonstrated in the case of Japan. This proposition is too simple to need any special demonstration. The measurement of consumer price changes by income classes, temporal or spatial, is essential in major policy making.

We consider this problem in this thesis and attempt to obtain consistent price comparisons. At this stage a brief introduction to the type of data used, is warranted. Naturally the quality of the data infringes on the results and the consequent interpretation. Our study is based on the material on household budgets provided by the Indian National Sample Survey (N.S.S.). The N.S.S. is a socio-economic survey of all-India coverage carried out in the form of successive rounds, each round taking a few months to one year, for its completion. The survey collects various types of data and the enquiry on consumer expenditure has been a constant feature of all the rounds. We consider data from the 18th round of NSS which covers the period, February 1963 - January 1964.

Estimates for the distribution of population by size classes of monthly per capita total consumer expenditure are available. Rural and Urban sectors are also distinguished and the estimates are available for different states of India. The data are collected from sample households by interview method. Probability sampling is used in selecting households and stratified and multistage design is used. Further the sample consists of two independent and interpenetrating sub-samples providing equally valid estimates of population characteristics. The divergence between subsample estimates indicates the extent of uncertainty associated with the combined sample estimates. All the illustrations in the thesis relate to the combined sample estimates, since there is not much divergence in the corresponding sub-sample estimates. The sample size for Rurual and Urban areas in this round is given by the total number of households which are 21776 and 4296 respectively.

We concentrate on the price comparisons for the ten decile groups of population for both rural and urban areas, these groups being based on a ranking by per capita household consumer expenditure, each group containing ten percent of the estimated population.

For each item in the consumer's budget the entries for quantity and value of consumption are given. This gives an implicit price for each item. The whole data on this were subject to a close scrutiny. The main idea was to detect the extreme values in the implicit prices, arising presumably as a result of confusion regarding local and standard units of quantity. For our purposes, 56 items are chosen on the basis of number of reporting households and the divergence between subsample estimates of value and quantity. The price, quantity and expenditure data on the 56 items are presented in Appendix A. This information has been used for inter-regional price comparisons by Chatterjee, G. S. and Bhattacharya, N. (1969) and they obtained fairly good inter-regional comparisons in real terms using conventional methods. However, they have not considered the problem of consistent price comparisons.

Our findings by and large indicate that formulae obtained by us as well by others for consistent comparisons do not produce results that are far away from those obtained on the basis of Laspeyres, Paasche and other conventional formulae. Both show a slowly rising level of prices as we go up the expenditure groups ; also, rural price levels are systematically lower than the corresponding urban price levels in all the expenditure groups. Thus, the use of consistent index number systems give comparisons that are unique though in the neighbourhood of estimates based on pairwise comparisons using conventional methods.

CHAPTER ONE

A NEW CLASS OF CONSISTENT INDEX NUMBERS

The basic idea behind a consistent system of index numbers is the existence of unique set of real numbers indicating the general price level. This has resulted in the concept of 'exchange rate' or 'purchasing power' of a currency, which is defined as the reciprocal of the general price level. This together with the concept of average price of each item, over a number of groups, form the basis to consistent index number system by Geary, R.C. and Khamis, S. H. We study the properties of these index numbers and derive conditions for the existence and uniqueness of these index numbers. Probing deeper into the concepts used by them, we arrive at a new index number system which is consistent and possesses the properties of Geary-Khamis indices. We prepare the ground work needed for the generalization of Laspeyres and Paasche index numbers given in the next chapter.

Section 1.1 Geary-Khamis System of Index Numbers

Introduction : An interesting system of consistent index numbers which is both theoreticaly sound and practicable is provided by Geary, R.C. (1958) and Khamis, S.H. (1969, 1970a). Geary first proposed the basic system, working out details in the simplest case where number of countries is two. The difficult task of tackling the multilateral comparisons case and formalising the index number system was undertaken by Khamis, which brought the index number system into lime light. The comparisons are mainly based on concepts of 'exchange rate' of a currency and 'average price' of a commodity as explained earlier. These values are inter-related by the actual price and quantity data.

The index numbers emerge from the solution for these unknowns which are related by a set of linear equations. The price index for any given comparison is given by the ratio of the corresponding exchange rates. Khamis obtained some sufficient conditions for the existence of unique positive solution for these unknowns. Uniqueness guarantees that the resulting price comparisons are consistent. In the sections that follow, we obtain some necessary and sufficient conditions for the existence of meaningful solution, which are easily verifiable in any practical situation. In the finishing stages of this dissertation we came to know that Khamis (1970b) gives a set of necessary and sufficient conditions for the existence and uniqueness.

Definitions :

Let p_{ij} and q_{ij} ($i = 1,\ldots,N$; $j = 1,\ldots,M$) denote the price and quantity of the i-th commodity consumed in the j-th country. Given the exchange rate, R_j, for the j-th currency, the expenditure on a particular item can be transformed into a comparable currency unit, $R_i\ p_{ij}\ q_{ij}$ gives the expenditure on the i-th item by the j-th country in a common unit. Adding this over countries and dividing the sum by the total quantity of the i-th commodity consumed, we can obtain the average price of the i-th item. Hence, for all i

$$P_i = \frac{\sum_j R_j\ p_{ij}\ q_{ij}}{\sum_j q_{ij}} \qquad (1.1.1)$$

On the otherhand, given the average price level of each item, we can derive the definition for the exchange rate. $P_i\ q_{ij}$ shows the average expenditure to be incurred to acquire q_{ij} and $\sum_i P_i\ q_{ij}$ gives the average level expenditure, in a common currency unit, required for the commodity bundle used in the j-th country. On the contrary, $\sum_i p_{ij}\ q_{ij}$ is the actual expenditure on the same commodity bundle. So a ratio of these two values, gives the actual exchange rate. Hence, for all j,

$$R_j = \frac{\sum_i P_i \, q_{ij}}{\sum_i p_{ij} \, q_{ij}} \qquad (1.1.2)$$

In all, there are M+N equations in equal number of unknowns. The credit of formulating these equations goes to Geary. The final price index numbers are given by

$$I_{jk} = \frac{R_j}{R_k}$$

where I_{jk} represents the price index for the k-th country with the j-th country as base. Geary established that non-trivial solutions exist depending on an arbitrary parameter and obtained explicit solutions when M = 2. The index number in this case is shown to be

$$I_{12} = \frac{\sum_{i=1}^{N} p_{i2} \dfrac{q_{i2} \, q_{i1}}{q_{i2}+q_{i1}}}{\sum_{i=1}^{N} p_{i1} \dfrac{q_{i2} \, q_{i1}}{q_{i2}+q_{i1}}}$$

In contrast to the customary price index number formulae, we have the harmonic mean of the quantity vectors as weights.

Since the final solution of the equations (1.1.1) and (1.1.2) result in the required price comparisons, let us focus our attention towards the solution of this system of linear equations. Further we restrict our attention towards the exchange rate solutions. Substituting the values of P_i from (1.1.1) in (1.1.2), we have a system of M linear homogeneous equations in the M unknowns. Denote

$$e_{ij} = p_{ij}\, q_{ij}\ ;\ E_j = \sum_{i=1}^{N} e_{ij},\quad v_{ij} = e_{ij}/E_j$$

$$Q_i = \sum_{j=1}^{M} q_{ij}\ ;\ q^*_{ij} = q_{ij}/Q_i \quad \text{and} \quad R^*_j = R_j\, E_j.$$

By substitution and simplification, we get

$$BR = 0 \qquad (1.1.3)$$

where $B = ((b_{k\ell}))$ and $b_{k\ell} = \delta_{k\ell} - \sum_i q^*_{ik}\, v_{i\ell}$;

$\delta_{k\ell} = 1$ if $k = \ell$ and $= 0$ if $k \neq \ell$ and $R = [R^*_1, \ldots, R^*_M]'$

The equation system (1.1.1) and (1.1.2) is mathematically meaningful only if the respective denominators are non-vanishing. This necessiates the following fundamental assumption.

<u>Assumption</u> : For all i and j, i) $p_{ij} > 0$, ii) $Q_i > 0$ and iii) $E_j > 0$.

This assumption can be easily interpreted and imposes little restriction on the price and quantity data. The assumption requires that all prices are positive; each commodity is consumed in at least one country and each country consumes at least one item. This is satisfied in any practical problem.

For all discussions, it is enough to work with (1.1.3), since there is one-to-one correspondence between solutions (1.1.1) and (1.1.2) and of the system (1.1.3). Hence we consider only equation system (1.1.3).

A slight deviation, from the main task, towards an alternative formulation of the system of equations (1.1.3) is interesting. Since R_j is the exchange rate of the j-th currency, $R_j\ E_j$ gives the total expenditure of the j-th country in a common currency unit. Similarly, $\sum_{j=1}^{M} e_{ij}\ R_j$ represents the worth of the total money expenditure on the i-th commodity in all countries taken together. Assuming that quantity shares govern the shares of expenditure, $q^*_{ik} \sum_{j=1}^{M} e_{ij}\ R_j$ gives the share of the k-th country in $\sum_{j=1}^{M} e_{ij}\ R_j$. This can be done for for all commodities. The assumption implies that, for any k,

$$R_k\ E_k = \sum_{i=1}^{N} q^*_{ik} \sum_{j=1}^{M} e_{ij}\ R_j$$

$$= \sum_{i=1}^{N} q^*_{ik} \sum_{i=1}^{M} \frac{e_{ij}}{E_j}\ E_j\ R_j$$

$$R^*_k = \sum_{i=1}^{N} q^*_{ik} \sum_{j=1}^{M} v_{ij} R^*_j$$

Therefore

$$R^*_k = \sum_{j=1}^{M} R^*_j \sum_{i=1}^{N} q^*_{ik} v_{ij} = 0 \quad \text{for} \quad k = 1,\ldots,M.$$

These equations can be rewritten in the form

$$BR = 0$$

which coincides with equation system (1.1.3).

This way of formulating the system is simple, though it it difficult to justify the assumption involved in the formulation. However, one can do away with the concept of 'average price' of a commodity and still arrive at the basic system of equations.

Existence and uniqueness :

Let us consider the problem of the existence of a non-trivial solution for (1.1.3) which is unique.

Consider the equations

$$BR = 0.$$

The matrix B has the following properties.

i). The element $b_{k\lambda} \geq 0$ if $k = \lambda$ and $b_{k\lambda} \leq 0$ if $k \neq \lambda$ for all k and λ.

ii) Each column sum in B is zero. Cosider the λ-th column. We have

$$b_{\lambda} = \sum_{k=1}^{M} b_{k\lambda}$$

$$= (1 - \sum_{i=1}^{N} q_{i\lambda} v_{i\lambda}) - \sum_{\substack{k=1 \\ \neq\lambda}}^{M} \sum_{i=1}^{N} q^*_{ik} v_{i\lambda}$$

$$= 1 - \sum_{k=1}^{M} \sum_{i=1}^{N} q^*_{ik} v_{i\lambda}$$

$$= 0 \text{ (since } \sum_{k=1}^{M} q^*_{ik} = 1 \text{ and } \sum_{i=1}^{N} v_{i\lambda} = 1)$$

By property (ii) we have that B is singular. Let

$$C = B' = ((c_{k\lambda}))$$

where $c_{k\lambda} = \delta_{k\lambda} - \sum_{i=1}^{N} v_{ik} q^*_{i\lambda}$ and $\delta_{k\lambda} = 1$ if $k = \lambda$ and $= 0$ if $k \neq \lambda$.

So $c_{k\lambda} \geq 0$ for $k = \lambda$ and $c_{k\lambda} \leq 0$ for $k \neq \lambda$ and each row sum in C is zero and hence for all k

$$|c_{kk}| = \sum_{\substack{\lambda=1 \\ \neq k}}^{M} |c_{k\lambda}|$$

Also $c_{k\ell} = 0$ if and only if $c_{\ell k} = 0$. These properties follow by assumption 1.

We shall solve the system of equations (1.1.3) in the following manner. Setting one of the R_j^*'s equal to unity and obtain the solution for the rest of the R_j^*'s. Without loss of generality put $R_M^* = 1$. Then we have

$$\bar{B}\ \bar{R} = y \qquad (1.1.4)$$

where $\bar{B}$ is the matrix obtained by deleting the M-th row and the M-th column. $\bar{R}$ is obtained by deleting the M-th element of R and y is the last column of B without the last element. Look at systems (1.1.3) and (1.1.4). Since B is singular, there exists a non-trivial solution for the R_j^*'s. Should the solutions lead to consistent price comparisons, at least the ratios of R_j^*'s must be uniquely determined. This implies that $\bar{B}$ must be nonsingular and further $\bar{B}^{-1}y$ must be strictly positive to ensure the usefulness of the system. Hence we divert the attention to the problem of obtaining conditions which are necessary and sufficient for the existence of unique strict positive solutions and focus towards the system of equations (1.1.4). Let

Some pertinent definitions are given below.

Definition 1 : Let G be the graph with the countries as vertices and join vertices k and ℓ if there is a commodity i for which q_{ik} and $q_{i\ell}$ are positive. Call G the 'adjacent graph' of the given data.

Definition 2 : The graph G is said to be connected if we can pass from any vertex to any other vertex through an edge.

Definition 3 : We say that the i-th row of the matrix A is dominated by its diagonal if $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$.

Definition 4 : A matrix $A_{n \times n}$ is said to possess property P if the removal of any K rows $1 \leq K \leq n-1$, and the corresponding columns, leaves the matrix with at least one row dominated by its diagonal element.

Definition 5 : A matrix $A_{n \times n}$ is said to be dominant diagonal if there exist positive numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that for all i

$$\lambda_i |a_{ii}| > \sum_{j \neq i} \lambda_j |a_{ij}| \tag{1.1.5}$$

Definition 6 : We say that a matrix is a P-matrix if all the principal minors are positive.

Definition 7 : A non-negative matrix $A_{n\times n} = ((a_{ij}))$ is said to be indecomposable if there is no non-empty proper subset J of $\{1,\ldots,n\}$ such that $a_{ij} = 0$ for $i \notin J$ and $j \in J$.

Definition 8 : Let $A_{n\times n} = ((a_{ij})) \geqq 0$. Then let G^* be the graph adjoint to the matrix A with the vertex set $\{1,2,\ldots,n\}$ and join i-th vertex to the j-th vertex if there exists a chain of vertices $\{k_0, k_1, \ldots, k_\lambda\}$, $k_0 = i$ and $k_\lambda = j$ such that for any consecutive k_s and k_{s+1}, $a_{k_s k_{s+1}} > 0$.

Observe that the graph is a directed graph.

We will prove a sequence of lemmas tending to the final result yielding the necessary and sufficient conditions for the existence of unique positive solution to the system of equations (1.1.4).

Lemma 1 : A necessary and sufficient condition that the matrix C has property P is that the 'adjacent graph' G is connected.

Proof : Suppose first that G is connected. If the rows numbered $i_1,\ldots,i_k$ and the corresponding columns of C are deleted, let the new matrix be C_1. Now since G is connected, there is an edge joining one of $i_1,\ldots,i_k$ to some vertex λ outside. Then in the λ-th row of C there is a non-zero element in one of the columns $i_1,\ldots,i_k$ and since

$$|c_{\ell\ell}| = \sum_{j\neq\ell} |c_{\ell j}|$$

it follows that the corresponding row of C_1 is dominated by its diagonal element. This proves sufficiency.

Next let C have property P. If G is not connected, then the vertices can be partitioned into two non-empty, disjoint subsets $\{i_1,\ldots,i_k\}$ and $\{j_1,\ldots,j_\ell\}$ such that there is no edge joining an i and j. Now we have $c_{i_\alpha j_\beta} = 0$ for $\alpha = 1,\ldots,k$ and $\beta = 1,\ldots,\ell$. Thus the matrix obtained from C by deleting the rows numbered $i_1,\ldots,i_k$ and the corresponding columns has no row dominated by its diagonal element. This contradiction proves the necessity.

Since $A = \bar{B}'$, it is also the matrix obtained by deleting the last row and the last column of C. Then we have

<u>Lemma 2</u>: A is dominant diagonal if and only if A has property P and A has a row dominated by its diagonal element.

<u>Proof</u>: First let A be dominant diagonal. Then there exist positive numbers $\lambda_1,\ldots,\lambda_{M-1}$ satisfying equation (1.1.5) above. Suppose now that A does not have property P. Then there exist $i_1,\ldots,i_k$ with $1 \leq k \leq M-2$ such that the matrix obtained from A by deleting the rows numbered $i_1,\ldots,i_k$ and the corresponding columns does not have any row which is dominated by its

diagonal element.

Thus

$$|a_{ii}| = \sum_{j \neq i, i_1, \ldots, i_k} |a_{ij}| \quad \text{for } i \neq i_1, i_2, \ldots, i_k.$$

Let

$$\lambda_i = \min \{ \lambda_j : j \neq i_1, \ldots, i_k \}$$

Then we have

$$\sum_{j \neq i, i_1, \ldots, i_k} \lambda_j |a_{ij}| \geqq \sum_{j \neq i, i_1, \ldots, i_k} \lambda_i |a_{ij}| = \lambda_i |a_{ii}|$$

$$> \sum_{j \neq i, i_1, \ldots, i_k} \lambda_j |a_{ij}|$$

a contradiction which proves that A has property P. Since A is dominant diagonal there exists a row which is dominated by its diagonal element. This proves the only if part.

Conversely let A have property P and let a row be dominated by its diagonal element. Then we construct positive numbers $\lambda_1, \lambda_2, \ldots, \lambda_{M-1}$ satisfying (1.1.5) above.

Let $i_1, \ldots, i_k$ be the rows of A dominated by their diagonal elements. Then choose any numbers $\mu_{i_1}, \ldots, \mu_{i_k}$ such that

$$\frac{\sum_{j \neq i_\alpha} |a_{i_\alpha j}|}{|a_{i_\alpha i_\alpha}|} < \mu_{i_\alpha} < 1 \quad \text{for } \alpha = 1,\ldots,k.$$

Let $\mu^{(1)} = \max(\mu_{i_1},\ldots,\mu_{i_k})$ and define

$$\lambda_{i_1} = \lambda_{i_2} = \ldots = \lambda_{i_k} = \mu(1)$$

In our construction all the λ's will be less than unity. Hence for $\alpha = 1,\ldots,k$,

$$\lambda_{i_\alpha}|a_{i_\alpha i_\alpha}| = \mu^{(1)}|a_{i_\alpha i_\alpha}| \geq \mu_{i_\alpha}|a_{i_\alpha i_\alpha}|$$

$$> \sum_{j \neq i_\alpha} |a_{i_\alpha j}| \geq \sum_{j \neq i_\alpha} \lambda_j |a_{i_\alpha j}|$$

Now delete the rows numbered $i_1,\ldots,i_k$ and the corresponding columns from A and let $j_1,\ldots,i_\lambda$ be the rows which are dominated by their diagonal elements in the new matrix. Then choose numbers $\mu_{j_1},\ldots,\mu_{j_\lambda}$ such that for $\beta = 1,\ldots,\lambda$,

$$\frac{\sum_{j \neq j_\beta, i_1,\ldots,i_k} |a_{j_\beta j}| + \sum_{j = i_1,\ldots,i_k} \lambda_j |a_{j_\beta j}|}{|a_{j_\beta j_\beta}|} < \mu_{j_\beta} < 1$$

This choice is possible since

$$|a_{j_\beta j_\beta}| = \sum_{j \neq j_\beta, i_1, \ldots, i_k} |a_{j_\beta j}| + \sum_{j = i_1, i_2, \ldots, i_k} |a_{j_\beta j}|,$$

$$\sum_{j = i_1, i_2, \ldots, i_k} |a_{j_\beta j}| > 0 \quad \text{and} \quad \lambda_j < 1 \quad \text{for} \quad j = i_1, i_2, \ldots, i_k$$

Let $\mu^{(2)} = \max(\mu_{j_1}, \ldots, \mu_{j_\ell})$ and define

$$\lambda_{j_1} = \lambda_{j_2} = \ldots = \lambda_{j_\ell} = \mu^{(2)}$$

Then for $\beta = 1, \ldots, \ell$

$$\lambda_{j_\beta} |a_{j_\beta j_\beta}| = \mu^{(2)} |a_{j_\beta j_\beta}| \geq \mu_{j_\beta} |a_{j_\beta j_\beta}|$$

$$> \sum_{j \neq j_\beta, i_1, \ldots, i_k} |a_{j_\beta j}| + \sum_{j = i_1, \ldots, i_k} \lambda_j |a_{j_\beta j}|$$

$$\geq \sum_{j \neq j_\beta} \lambda_j |a_{j_\beta j}|$$

Now deleting the rows numbered $j_1, \ldots, j_\ell$ and the corresponding columns we get some rows which are dominated by their diagonal elements. The corresponding λ's can be defined similarly and the process is repeated until all rows are exhausted. It is evident that the resulting numbers $\lambda_1, \ldots, \lambda_{M-1}$ are less than unity and satisfy condition (1.1.5) above. This completes the proof of the theorem.

<u>Lemma 3</u> : A has property P and A has at least one row dominated by its diagonal element if and only if C has property P.

<u>Proof</u> : If part is trivial, for, the removal of rows numbered $i_1, \ldots, i_k$ and the corresponding columns of A is equivalent to the removal of the rows numbered $i_1, i_2, \ldots, i_k$, M and the corresponding columns from C.

To prove the only if part suppose that A has property P, A has a row dominated by its diagonal element and C does not have property P.

Let $i_1, \ldots, i_k$ be such that the removal of rows $i_1, \ldots, i_k$ and the corresponding columns from C gives a matrix with row dominated by its diagonal element. Then evidently $M \neq i_1, \ldots, i_k$. Let now $\{M, j_1, \ldots, j_\lambda\} = \{1, \ldots, M\} - \{i_1, \ldots, i_k\}$. Now by definition of $i_1, \ldots, i_k$, we have

$$c_{j_\alpha i_\beta} = 0 \quad \text{for} \quad \alpha = 1, \ldots, \lambda \quad \text{and} \quad \beta = 1, \ldots, k$$

$$c_{M i_\beta} = 0 \quad \text{for} \quad \beta = 1, \ldots, k.$$

But then $c_{i_\beta j_\alpha} = 0$ and $c_{i_\beta M} = 0$ and so the deletion of the rows and columns numbered M, $j_1, \ldots, j_\lambda$ in C leaves a matrix with no row dominated by its diagonal element, a contradiction to the hypothesis. This completes the proof of the theorem.

From lemmas 2 and 3 we get

Theorem 4 : A is dominant diagonal if and only if C has property P.

Theorem 5 : A matrix $A = (a_{ij})$ having a positive dominant diagonal is a P-matrix.

Proof : See Nikaido, H. (1969), pp. 386-87.

If A is a matrix whose typical element $a_{k\ell}$ and $a_{k\ell} = \rho\,\delta_{k\ell} - d_{k\ell}$ where $\delta_{k\ell} = 0$ for $k \neq \ell$ and $\delta_{k\ell} = 1$ for $k = \ell$, and $d_{k\ell} \leqq 0$ for $k \neq \ell$ and

$$Ax = y \tag{1.1.6}$$

is a system of equations, we have

Theorem 6 : The following statements are equivalent.

(i) System (1.1.6) has a set of non-negative solutions $x_i \geqq 0$ $(i = 1,\ldots,n)$ for some set of positive $y_i > 0$ $(i = 1,\ldots,n)$.

(ii) System (1.1.6) is solvable in the non-negative unknowns $x_i \geqq 0$ $(i = 1,\ldots,n)$ for any set of non-negative $y_i \geqq 0$ $(i = 1,\ldots,n)$.

(iii) All the principal minors are positive.

<u>Proof</u> : Nikaido, H. (1969), pp. 90-93.

As a result of the above theorem, we get

<u>Corollary 7</u> : If A is a P-matrix and of the form required by (1.1.6) then A^{-1} exists and is non-negative.

<u>Proof</u> : A^{-1} exists since A is a P-matrix. A^{-1} is non-negative follows from (ii) of the above theorem.

If A^{-1} exists and is non-negative we have that B^{-1} exists and is non-negative which assures that $\bar{R}$ is non-negative.

Our aim is to show that existence of A^{-1} implies that the graph G is connected. Let us prove the following.

<u>Lemma 8</u> : If A^{-1} exists then graph G is connected.

<u>Proof</u> : We have that A is non-singular. Now suppose that G is not connected. Then the vertices can be partitioned into non-empty subsets $\{i_1, \ldots, i_k\}$ and $\{j_1, \ldots, j_\ell\}$ such that there is no edge between these two sets. Observe that $\{M\}$ cannot be equal to either of the sets, for, the matrix A which is obtained by deleting the row and column numbered M from the matrix C, would not have any row dominated by its diagonal element. This implies that A is singular.

Further the fact that there are no edges between the sets $\{i_1,\ldots,i_k\}$ and $\{j_1,\ldots,j_\ell\}$ implies that A can be written in the form

$$\begin{matrix} A_1 & 0 \\ 0 & A_2 \end{matrix}$$

where one of the matrices A_1 and A_2 has all the row sums equal to zero which means that A is singular. This contradiction proves the theorem.

In view of the above theorem, we have that if $\bar{B}^{-1}$ exists, which means that A^{-1} exists, then G is connected.

<u>Theorem 9</u> : A is non-negative invertible if and only if the 'adjacent graph' G of the data is connected.

<u>Proof</u> : Result follows from the lemmas, theorems and corollaries proved before.

Theorem 9 implies that unique non-negative solution exists for R_j^* ($j = 1,\ldots,M-1$), given $R_M^* = 1$, since $A = B'$ is obtained by deleting the M-th row and M-th column of B_1'. This means that, under the above conditions, any (M-1) equations from the system (1.1.3) are linearly independant. Thus we are able to get conditions under which the ratios of R_j^*'s are uniquely determined. The ratios turn out to be same no matter what R_j^* is chosen to be unity. We shall derive conditions for the existence of positive solutions for R_j^*'s. We state two theorems, proofs of which appear in Nikaido, H. [1969], pp. 107-109.

<u>Theorem 10</u> : A matrix $A = ((a_{ij})) \geqq 0$ is indecomposable if and only if the 'adjoint graph' G^* of A is connected.

<u>Theorem 11</u> : If the matrix $(\rho I - A)$, $\rho > 0$ and $A \geqq 0$, is non-negatively invertible for an indecomposable A, its inverse $(\rho I - A)^{-1}$ is a positive matrix.

Let us now state and prove the following Main Theorem of this section.

<u>Main Theorem</u> : Unique positive solution for system (1.1.3) exists if and only if the 'adjacent graph' G of the data is connected.

<u>Proof</u> ; Necessity follows from Theorem 9. To prove the sufficiency let the 'adjacent graph' G of the data be connected. Then we can find at least two vertices whose removal would result in a connected graph (refer Berge, C. [1962]). Let k be one such vertex. Let $A = \bar{B}^t$, where $\bar{B}$ is obtained by deleting the k-th row and the k-th column. It is enough if we prove that A^{-1} is positive. Observe that A is a matrix of the form $(I-D)$ and $D \geqq 0$. Further the 'adjoint graph' G^* of this matrix is connected since it has the property $d_{ij} \neq 0$ if and only if

$d_{ji} = 0$. Hence the 'adjoint graph' of D is connected. By Theorems 10 and 11, we have that A^{-1} is positive. This completes the proof of the theorem.

Some interesting remarks are in order.

<u>Remark 1</u> : The condition that the 'adjacent graph' of the data is connected can be shown to be necessary and sufficient for the existence of unique strict positive solution for all the M + N unknowns in the M+N equations (1.1.1) and (1.1.2). If this condition is satisfied, unique positive solution for R_j's exists and by substituting these values in equations (1.1.1), we can obtain positive solution for the P_i's, by the assumption. The necessity of this condition is obvious.

<u>Remark 2</u> : This concerns with the interpretation of the condition obtained above. It is important to see what sort of restrictions are to be imposed on the price and quantity data so as to enable comparisons with this method. This condition translated into ordinary terms using definitions 3 and 4 mean the following. Price comparisons are possible under this method if and only if the set of countries cannot be divided into two non-empty, disjoint subgroups such that these two groups do not have any commodity common in their consumption baskets. This is a very meaningful condition and is satisfied

by almost all price comparison problems. This condition can also be justified intutively. For example, it is difficult to concieve the comparison of prices in two countries when they do not have any commodity which is commonly used.

At this stage it is legitimate to look at the sufficient conditions obtained by Khamis. First, observe that the condition obtained above is a stronger condition than what the assumption implies, in the sense that this condition implies the assumption but not vice versa. The conditions given by Khamis (1970a) are sufficient for the existence of unique positive solution for M+N-1 unknowns in terms of one unknown whose value is arbitrarily assigned. There are three conditions :

Condition A : Every country consumes at least two commodities and every commodity is consumed in two countries (i.e.) $q_{ij} > 0$ for at least two i, for each j and $q_{ij} > 0$ for at least two j for each i and the irreducibility of the corresponding matrix.

Condition B : Any two countries have a commodity common in consumption (i.e.) for all j and k, $q_{ij} > 0$ for at least one i and the irreducibility of the corresponding matrix.

Condition C : There is a country which consumes all the N commodities.

Our condition is weaker than condition C, which is the most straightforward. In virtue of the definitions 3 and 4, Condition B without irreducibility implies that the 'adjacent graph' of the data is connected and hence the matrix corresponding to that is irreducible. This condition can be observed to be stronger than the connectedness of the 'adjacent graph' of the data. Similar result can be observed regarding the Condition A also. We came to know that Khamis (1970b) proves that strict positive solution exists if and only if the matrix involved is irreducible, using results on sign-symmetric, irreducible and non-negative matrices.

Remark 3 : We bring out an interesting property of the Geary-Khamis method. Suppose that a commodity is consumed in only one of the countries under consideration. Then the comparisons resulting from this method are independant of this commodity. To prove that, suppose that the first item is consumed in the first country alone. Hence $q_{1j} > 0$ for only $j = 1$. Recall first equation in the equation system (1.1.3). It is

$$R_1 E_1 - \sum_{j=1}^{M} R_j E_j \sum_{i=1}^{N} q^*_{i1} v_{ij} = 0$$

Since $v_{ij} = 0$ for $j \neq 1$ and $q^*_{i1} = 1$, we have

$$R_1 E_1 - R_1 E_1 \, v_{11} - \sum_{j=1}^{M} R_j \sum_{i=2}^{N} q^*_{i1} \, e_{ij} = 0$$

$$R_1 (E_1 - e_{11}) - \sum_{j=1}^{M} R_j (E_j - e_{1j}) \sum_{i=2}^{N} q^*_{i1} \frac{e_{ij}}{(E_j - e_{1j})} = 0$$

$$R_1 \bar{E}_1 - \sum_{j=1}^{M} R_j \bar{E}_j \sum_{i=2}^{N} q^*_{i1} \, \bar{v}_{ij} = 0.$$

where $\bar{E}_k = E_k - e_{1k}$ and $\bar{v}_{ik} = e_{ik} / \bar{E}_k$ for all i and k.

Further the other $M-1$ equations can be shown to be independent of the first commodity. Hence the final solutions for R_j's are independant of this particular item. Geary-Khamis method ignores all items which are consumed in only one country. However similar result does not hold for more than one item.

Numerical Illustration :

Let us apply the Geary-Khamis method for inter-fractile group comparison of price in Rural India for the year 1963-64. For these purposes we use the price and quantity data for 56 items used in ten fractile groups. The basic data is presented in Appendix A. The table below gives the price indices for various groups with the 0-10 group as the base. The corresponding Laspeyres and Paasche index numbers for binary comparisons are also presented to show the relative performance of these indices.

Table

Fractile group	Geary-Khamis Index Numbers	Laspeyres' Indices	Paasche's Indices
(1)	(2)	(3)	(4)
0 - 10	100.00	100.00	100.00
10 - 20	103.66	103.61	103.44
20 - 30	106.31	106.27	105.88
30 - 40	106.38	106.59	106.15
40 - 50	107.85	108.33	107.77
50 - 60	109.62	110.25	107.31
60 - 70	109.57	109.69	109.62
70 - 80	110.78	111.41	110.80
80 - 90	111.37	110.90	111.55
90 -100	115.46	113.43	115.62

Generally the trend of the prices is increasing as income increases. This is consistent with the usual contentions. Another interesting feature reflected in the table is the fact that Geary-Khamis indices lie in the boundaries provided by Laspeyres' and Paasche's indices, whenever they provide the usual bounds. Khamis has shown that these indices necessarily lie in the range determined by Laspeyres' and Paasche's indices when the number of commodities involved is two (i.e.) N = 2. Some results which deal with more interesting aspects of this are presented in Section 2 of next chapter.

Section 1.2 A New System of Consistent Index Numbers

Introduction : In this section we derive a new system of consistent index numbers. These are also based on the concepts 'exchange rate' and 'average price', which formed the basis for Geary-Khamis method. These unknowns are interrelated by a system of log-linear equations. In the sub-sections below we establish the consistency of the equation system and also obtain the conditions for the existence and uniqueness of solution for the equation system. We prove that the condition given in the previous section serves a similar purpose in this case also.

Definitions : The underlying relations between the exchange rates and the average prices along with the price and quantity are given below.

$$R_j = \prod_{i=1}^{N} \left(\frac{P_i}{p_{ij}}\right)^{v_{ij}} \quad \text{for} \quad j=1,\ldots,M \tag{1.2.1}$$

$$P_i = \prod_{j=1}^{M} [R_j \, p_{ij}]^{v^*_{ij}} \quad \text{for all} \quad i = 1,\ldots,N \tag{1.2.2}$$

where $v^*_{ij} = \dfrac{v_{ij}}{\sum_j v_{ij}}$. v^*_{ij} shows the share of the j-th country in the total share of values on the i-th commodity. Note that the basic assumption of the previous section is essential in formulating the above non-linear equations.

To justify the above equation system, we shall examine the Geary-Khamis' definition of these concepts more closely. Let us take-up the definition of average price in equations (1.1.1). P_i is a weighted arithmetic mean of the values $R_j\, p_{ij}$ $(j = 1,\ldots,M)$, the weights were chosen to be the quantity shares. This is evident by the following equations which are obtained by reformulating (1.1.1) and (1.1.2) we have

$$R_j = \frac{\sum_{i=1}^{N} P_i\, q_{ij}}{\sum_{i=1}^{N} p_{ij}\, q_{ij}} = \sum_{i=1}^{N} \frac{P_i}{p_{ij}}\, v_{ij} \qquad \text{for} \quad j = 1,\ldots,M$$

$$P_i = \frac{\sum_{i=1}^{M} R_j\, p_{ij}\, q_{ij}}{\sum_{j=1}^{M} q_{ij}} = \sum_{j=1}^{M} R_j\, p_{ij}\, q^{*}_{ij} \qquad \text{for } i = 1,\ldots,N.$$

So the definition of P_i in the Geary-Khamis set up is only one possible way of solving the problem of obtaining a formula for central tendency given the values $R_j\, p_{ij}$ $(j = 1,\ldots,M)$.

Let us restrict the range of the formulae in the following way. Since P_i is an average, choose the value which minimizes the sum of squares of deviations of the average from the original values. Hence P_i $(i = 1,\ldots,N)$ should minimize

$$\sum_{i=1}^{N} \sum_{j=1}^{M} (R_j\, p_{ij} - P_i)^2 .$$

Minimization of weighted sum of squares of deviations may be deemed to be superior to an unweighted minimization (or equal weights). Further the weights must depend on the actual price and quantity data. We consider two weighting schemes other than the equal weights. One scheme gives weights equal to the quantity shares in which case the minimand is given by

$\sum_{i=1}^{N} \sum_{j=1}^{M} (R_j\, p_{ij} - P_i)^2\, q^*_{ij}$, and the other has weights corresponding to the value shares and the minimand turns out to be

$\sum_{i=1}^{N} \sum_{j=1}^{M} (R_j\, p_{ij} - P_i)^2\, v_{ij}$. Minimizing these with respect to P_i, we have, for all i

$$P_i^{(1)} = \frac{1}{M} \sum_{j=1}^{M} R_j\, p_{ij} \qquad (1.2.3)$$

$$P_i^{(2)} = \sum_{j=1}^{M} R_j\, p_{ij}\, v^*_{ij} \qquad (1.2.4)$$

$$P_i^{(3)} = \sum_{j=1}^{M} R_j\, p_{ij}\, q^*_{ij} \qquad (1.2.5)$$

Equations (1.2.3) correspond to the minimization with equal weights.

System of equations corresponding to (1.2.5) and (1.1.2) give the Geary-Khamis indices. It is interesting to note that (1.1.2) together with either of (1.2.3) and (1.2.4) fail to give unique positive solution which is essential. The proof of this appears in Section 2 of next chapter. However, a few more interesting results emerge from the above situations which are also discussed there.

In light of this position, let us use geometric mean instead of the arithmatic mean. As it will be seen, this works out nicely. The modified equation systems corresponding to (1.2.3) and (1.2.5) along with (1.1.2) do not yield unique solutions, leaving alone the system of equations corresponding to (1.2.4) and (1.1.2) which coincides with the one proposed by us in (1.2.1) and (1.2.2). Let us proceed to the problem of existence and uniqueness.

<u>Existence and Uniqueness</u> :

Transforming the non-linear equations (1.2.1) and (1.2.2) by natural logarithms, we have

$$R_j^* = \sum_{i=1}^{N} (P_i^* - \log p_{ij}) \, v_{ij} \qquad \text{for } j = 1, \ldots, M$$

$$P_i^* = \sum_{j=1}^{M} (R_j^* + \log p_{ij})\, v_{ij}^* \qquad \text{for } i = 1,\ldots,N$$

where $R_j^* = \log R_j$ and $P_i^* = \log P_i$.

This is a system $M + N$ linear equations in $M + N$ unknowns (R_j^*'s and P_i^*'s). Since our main interest centers round the price comparisons, let us consider the solution of R_j^*'s. Substituting the values of P_i^*'s in and simplifying, we get for $k = 1,\ldots,M$

$$R_k^* = \sum_{j=1}^{M} \sum_{i=1}^{N} R_j^* v_{ij}^* v_{ik} + \sum_{j=1}^{M} \sum_{i=1}^{N} (\log p_{ij} - \log p_{ik}) v_{ik} v_{ij}^*$$

This gives a set of M linear non-homogeneous equations in M unknowns as against linear homogeneous equations in the previous section. The equations are given by

$$DR^* = b \tag{1.2.6}$$

where $D = ((d_{jk}))$; $d_{jk} = \delta_{jk} - \sum_i v_{ik}^* v_{ij}$; $\delta_{jk} = 1$ if $j = k$ and $= 0$ if $j \neq k$; $R^* = [R_1^*,\ldots,R_M^*]'$ and $b_k = \sum_j \sum_i (\log p_{ij} - \log p_{ik})\, v_{ik} v_{ij}^*$.

Linear equations (1.2.6) have some nice properties.

Property (i) : Each row sum is zero. The k-th row sum is given by

$$1 - \sum_j \sum_i v_{ik} v^*_{ij} = 1 - \sum_i v_{ik} \sum_j v^*_{ij} = 0$$

since $\sum_i v_{ik} = 1$ and $\sum_j v^*_{ij} = 1$.

Property (ii) : The matrix D is symmetric, for, $d_{kj} = \sum_i v^*_{ij} v_{ik} = \sum_i v_{ij} v^*_{ik} = d_{jk}$. Consequently each column sum is zero.

Property (iii) : The column vector b adds upto zero. The sum of the elements is given by

$$\sum_{k=1}^{M} b_k = \sum_{k=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{M} (\log p_{ik} - \log p_{ij}) v_{ik} v^*_{ij}$$

$$= \sum_i \sum_k \sum_j (\log p_{ik} - \log p_{ij}) v_{ik} v^*_{ij}$$

$$= 0$$

since $v_{ik} v^*_{ij} = v^*_{ik} v_{ij}$.

From properties (i) and (ii) it follows that the matrix D is singular. Hence unique solution is not possible for R^*_j's. However, our interest is to determine the ratios of R^*_j's uniquely. That is to say, R^*_j's must be uniquely determined in terms of one of the R^*_j's. For this it is necessary that D is of rank M-1. Before proceeding to obtain the conditions for this, we prove the consistency of the equations (1.2.6) which is to be established. This was not necessary in the previous case since (1.1.3) is a system of homogeneous linear equations. Given that rank of D is M-1, we show that the rank of the augmented matrix (D : b) is also M-1 consider the rows of (D : b). By properties (ii) and (iii), we know that all columns add upto zero. Hence the Rank (D : b) $\leq$ M-1. Since Rank(D) = M-1, we have Rank (D : b) = M-1. This establishes that (1.2.6) is a consistent system of equations.

Let us take up the remaining problem. Set, without loss of generality, $R^*_m = 1$. We have a new system of equations

$$\bar{D}\,\bar{R}^* = \bar{b} + y \qquad (1.2.7)$$

where $\bar{D}$ is obtained by deleting the last row and last column of D. $\bar{R}^*$, b and y are obtained by deleting, the last element of R^*, b and the last column of D respectively. Our problem boils down to one of obtaining necessary and sufficient conditions for the existence of $\bar{D}^{-1}$. In fact we are not concerned about the non-negativity of $\bar{D}^{-1}$ since our main interest is in the positive R_j's. Since $R_j = e^{R^*_j}$, R_j is positive in any case.

Under definitions (3) and (4) of the previous section, we prove the main result.

Theorem : Unique solution for system (1.2.6) exists if and only if the 'adjacent graph' G of the data is connected.

Proof : The proof is complete once the formal similarity between the matrix B in system (1.1.3) and matrix D in system (1.2.6). Consider the typical elements b_{jk} and d_{jk} of the matrices B and D respectively. We have $b_{jk} > 0$ and $d_{jk} > 0$ for $j = k$ and $b_{jk} \leq 0$ and $d_{jk} \leq 0$ for $j \neq k$. Also

$$b_{jk} = \delta_{jk} - \sum_{i=1}^{N} q^*_{ij} v_{ik}$$

$$d_{jk} = \delta_{jk} - \sum_{i=1}^{N} v^*_{ij} v_{ik}$$

where δ_{jk} is the Kronecker's delta. Further observe that $q^*_{ij} = 0$ if and only if $v^*_{ij} = 0$. This implies that $b_{jk} = 0$ if and only if $d_{jk} = 0$. Further columns in each of these matrices add up to zero. Hence the structure of the two matrices is same.

The proof follows by repeating the steps involved in the proof the Main Theorem in the previous section. It is interesting to note that the conditions which are necessary and sufficient for meaningful comparisons in both Geary-Khamis' method and our method are same. A few observations on this method are interesting.

<u>Remark 1</u> : Consider the binary comparisons problem. Here $M=2$. We have the price index for country 2 with country 1 as base given by

$$I_{12} = \prod_{i=1}^{N} \left(\frac{p_{i2}}{p_{i1}}\right)^{\frac{v_{i1}\, v^*_{i2}}{\sum_i v_{i1} v^*_{i2}}}$$

This formula is very much similar to the one used by Klock-Theil (1965) for international comparisons. The formula is given by

$$I_{12} = \prod_{i=1}^{N} \left(\frac{p_{i2}}{p_{i1}}\right)^{\frac{v_{i1}+v_{i2}}{2}} = \prod_{i=1}^{N} \left(\frac{p_{i2}}{p_{i1}}\right)^{\frac{v_{i1}+v_{i2}}{\sum_i (v_{i1}+v_{i2})}}$$

Formula used by Klock-Theil is obtained by the weighted geometric mean of the price relatives, the weights given by the arithmetic average of the expenditure shares of the two countries. On the other hand our formula uses the harmonic mean of the expenditure shares of the two countries.

<u>Remark 2</u> : It can be shown that when $p_1 = p_2 = \dots = p_M$ then $R_j = 1$ for all j. This means that all index numbers are equal to unity showing no price change. In fact a stronger result is proved below that if $p_j = \lambda_j\, p_M$ and $\lambda_j > 0$, for all j then

$R_j = 1/\lambda_j$. However $\lambda_j \leq 0$ is ruled out by the basic assumption.

Proof : Let $R^*_M = 1$. Then we have by equations (1.2.7)

$$\bar{D}\,\bar{R}^* = \bar{b} + y$$

Solution for $\bar{R}^*$ is given by

$$\bar{R}^* = \bar{D}^{-1}\,\bar{b} + \bar{D}^{-1}\,y \qquad (1.2.8)$$

Look at the right hand side of equations 1.2.8. Let $\bar{D}^{-1} = ((D_{jk}))$. Consider the first element of $\bar{D}^{1}\,y$, we have

$$D_{11}\,y_1 + \ldots + D_{1\,M-1}\,y_{M-1}$$

$$= D_{11}\sum_i v_{i1}\,v^*_{iM} + \ldots + D_{1M-1}\sum_i v_{i\,M-1}\,v^*_{iM}$$

$$= D_{11}\sum_{k=1}^{M-1} d_{1k} + \ldots + D_{1M-1}\sum_{k=1}^{M} d_{M-1k}$$

$$= 1$$

by property (i) of system (1.2.6). Further consider the first element of $\bar{D}^{-1}\,\bar{b}$. We have,

$$R^*_1 = D_{11}\Big[\sum_{j=1}^{M}\sum_i (\log p_{ij} - \log p_{i1})v_{i1}\,v^*_{ij}\Big] + \ldots +$$

$$D_{1M-1}\Big[\sum_{j=1}^{M}\sum_i (\log p_{ij} - \log p_{iM-1})\,v_{iM-1}\,v^*_{ij}\Big] + 1$$

Since $p_{ij} = \lambda_j p_{iM}$, we have

$$R_1^* = D_{11}[\sum_j (\log \lambda_j - \log \lambda_1)\sum_i v_{i1} v_{ij}^*] + \dots + D_{1M-1}[\sum_j (\log \lambda_j - \log \lambda_{M-1}) \sum_i v_{iM-1} v_{ij}^*] + 1$$

$$= 1 - \log \lambda_1 [D_{11}(1 - \sum_i v_{i1} v_{i1}^*) - \dots - D_{1M-1} \sum_i v_{iM-1} v_{i1}^*] - \dots$$

$$- \log \lambda_{M-1}[-D_{11} \sum_i v_{i1} v_{iM}^* - \dots + D_{1M-1} \sum v_{iM-1} v_{iM-1}^*].$$

By property (i) of (1.2.6) and $\sum_i v_{ik} v_{ij}^* = \sum_i v_{ij} v_{ik}^*$ for all j,k,

$$R_1^* = 1 - \log \lambda_1 [D_{11} d_{11} + \dots + D_{1M-1} d_{M-11}] - \dots$$

$$- \log \lambda_{M-1} [D_{11} d_{M-11} + \dots + D_{1M-1} d_{M-1M-1}]$$

$$= 1 - \log \lambda_1 .$$

This is true for any of the R_j^*'s. Hence

$$R_j^* = 1 - \log \lambda_j \qquad \text{for } j = 1, \dots, M-1$$

$$R_M^* = 1.$$

Setting $R_M = 1$; we have

$$R_j = 1/\lambda_j \qquad \text{for } j = 1, \dots, M-1$$

$$R_M = 1$$

Hence the result.

However, it does not seem to be true that $p_j > p_k$ implies that $R_j > R_k$. But it seems that the values of R_j are less than their Geary-Khamis analogues. This may be due to the use of geometric mean in the formulae for R_j's and P_j's which irons out the effect of the extreme cases to some extent. These are discussed in the section that follows.

Numerical Illustration :

In the following table we present the index numbers obtained by the new method discussed in this section along with the Geary-Khamis index numbers for the ten size classes of Rural India. The indices are presented with group 0-10 as base. The table gives the Laspeyres and Paasche index numbers also.

Table

Group	New system	Geary-Khamis	Laspeyres	Paasche
(1)	(2)	(3)	(4)	(5)
0 - 10	100.00	100.00	100.00	100.00
10 - 20	103.62	103.66	103.61	103.44
20 - 30	106.06	106.31	106.27	105.88
30 - 40	106.25	106.38	106.59	106.15
40 - 50	107.82	107.85	108.33	107.77
50 - 60	108.59	109.62	110.25	107.31
60 - 70	109.58	109.57	109.69	109.62
70 - 80	110.78	110.78	111.41	110.80
80 - 90	111.47	111.37	110.90	111.55
90 -100	115.73	115.46	113.43	115.62

The new system of index numbers also show a steady rise in the price level as the income increases. These index numbers are generally smaller than the Geary-Khamis indices as expected and lie in the range given by the Paasche and Laspeyres indices. This system of index numbers is expected to do well in the case of more heterogeneous groups.

CHAPTER TWO

OTHER CONSISTENT INDEX NUMBERS AND PROBLEM OF CHOICE

Apart from the Geary-Khamis system of index numbers, several other formulae yield consistent comparisons. The index numbers used by Economic Commission for Latin America (ECLA) in comparing prices and quantities among Latin American countries, and Elteto, O.-Koves, P. (1965) are considered in the first section of this chapter. We derive some properties of these index numbers and obtain price comparisons for the fractile groups of Rural India. Both the index number systems are based on the concept of general price level in each group and thus related to the indices considered in the previous chapter in a wider set-up. It is shown that the concept of exchange rate of a currency and average price of a commodity can be used to derive Laspeyres' and Paasche's index numbers. An attempt has been made to generalize these index numbers to multilateral comparisons, which also leads to a system of generalized Fisher's ideal indices satisfying the circularity test. This mildly suggests that all the index number systems in use could be derived using these concepts of exchange rate and average price, in the sense that for each index number system, there exists a definition for the exchange rate and average price resulting in the required index numbers. Finally, the problem of choice of a suitable index number for the comparisons in a concrete problem, is considered in the final section. An empirical criterion has been proposed for this purpose and the relative efficiencies of the index number systems are studied.

Section 2.1 Other Consistent Index Numbers

ECLA Index Numbers :

The simplest of all and also an intutively appealing index number system is one used by United Nations Economic Commission for Latin America (ECLA) in comparing the general price levels in Latin American countries. This system of index numbers is proposed to serve the purpose of price comparison in the absence of adequate computational facilities which prevent the use of a highly sophisticated index number system. For example, the Geary-Khamis method and the new system considered in Chapter I need good computational facilities. As against the standard formulae like Laspeyres' and Paasche's for price comparisons which make use of only one of the quantity vectors for weighting, this formula makes use of the average of the quantity vectors for the purpose of weighting the price relatives. This method is described in Braithwaite, S.N. [1970].

All the binary comparisons must make use of the average consumption pattern which is given by

$$\bar{q}_i = \frac{\sum_{j=1}^{M} q_{ij}}{M} \qquad \text{for } i = 1,\ldots,N$$

So $\bar{q}_i$ may be considered as an average requirement vector so that the ratio of the expenditures, obtained by evaluating the quantity vector at the respective price levels, would give the corresponding price index number. Hence I_{jk}, for $j,k=1,\ldots,M$ is given by

$$I_{jk} = \frac{\sum_{i=1}^{N} p_{ik} \bar{q}_i}{\sum_{i=1}^{N} p_{ij} \bar{q}_i}$$

In fact the general price level π_j $(j = 1,\ldots,M)$ when $\pi_1 = 1$ is given by

$$\pi_j = \frac{\sum_{i=1}^{N} p_{ij} \bar{q}_i}{\sum_{i=1}^{N} p_{i1} \bar{q}_i}$$

so that $I_{jk} = \pi_k / \pi_j$.

Obviously, the ECLA indices form a consistent index number system. However, the consistency is achieved only by using an unrealistic assumption that $\bar{q}_i$ is a representative consumption vector for all the groups involved in the problem. But this formula works well in the case where the consumption patterns are not very heterogeneous. In fact, this is the situation in Latin America where this formula yielded good results. An interesting property of these indices is that they fall in the interval obtained by their Laspeyres and Paasche counterparts when $M = 2$. Consider I_{12}. We have

$$I_{12} = \frac{\sum_i p_{i2}\, \bar{q}_i}{\sum_i p_{i1}\, \bar{q}_i} = \frac{\sum_i p_{i2}\, (q_{i1} + q_{i2})}{\sum_i p_{i1}\, (q_{i1} + q_{i2})}$$

$$= \frac{\sum_i p_{i1}\, q_{i1}}{\sum_i p_{i1}q_{i1} + \sum_i p_{i1}q_{i2}} \cdot \frac{\sum_i p_{i2}q_{i1}}{\sum_i p_{i1}q_{i1}} + \frac{\sum_i p_{i1}q_{i2}}{\sum_i p_{i1}q_{i1} + \sum_i p_{i1}q_{i2}} \cdot \frac{\sum_i p_{i2}\, q_{i2}}{\sum_i p_{i1}\, q_{i2}}$$

This is nothing but a weighted average of the Laspeyres and Paasche index numbers. Hence the result. However the same result need not hold when $M > 2$.

We now present some inter-fractile group price comparisons for the Rural India based on the data presented in the Appendix. The following table shows the ECLA indices for various groups with group 0 - 10 as the base and also the standard indices for the purpose of comparison.

Table

Group	ECLA	Laspeyres'	Paasche
0- 10	100.00	100.00	100.00
10- 20	103.16	103.61	103.44
20--30	105.75	106.27	105.88
30- 40	106.00	106.59	106.15
40- 50	107.63	108.53	107.77
50- 60	110.36	110.25	107.31
60- 70	109.55	109.69	109.62
70- 80	110.90	111.41	110.80
80- 90	111.23	110.90	111.55
90-100	114.62	113.43	115.63

The table shows the peculiar behaviour of the ECLA indices. While the indices are less than their Laspeyres and Paasche counterparts for the groups 0-10 to 40-50; there is a sudden jump in the value of the index from the next fractile group.

This may be due to the choice of the weights. On the other hand the property that it lies in the bounds of these two indices when $M=2$, does not hold good in this case.

Elteto-Koves Index Numbers

In a novel attempt to construct a consistent system of index numbers, Elteto, O.-Koves, P. (1965) made use of the Fisher's ideal indices. Their aim was to generate a consistent system with some properties from an inconsistent system. The index number can be defined in the following way. Consider I_{jk}. This measures the price change from j-th country to the k-th. This can also be measured through a third country. That is, we express I_{jk} as a product of two numbers $I_{j\lambda}$ and $I_{\lambda k}$, for some λ. If we denote the Fisher's ideal indices by F_{jk}, we have $F_{j\lambda} \cdot F_{\lambda k}$ (for $\lambda = 1, \ldots, M$) as a proxy for I_{jk}. There are M such possible values and the required index is defined to be the geometric mean of these M numbers. Hence we have

$$I_{jk} = \left[F_{jk}^{(1)} \ldots F_{jk}^{(M)} \right]^{\frac{1}{M}} \qquad (2.1.1)$$

where $F_{jk}^{(\lambda)} = F_{j\lambda} \cdot F_{\lambda k}$, for $\lambda = 1, \ldots, M$.

These indices make use of Fisher's ideal indices in an interesting way. In fact, we prove the following nice property of these indices. An intutively obvious extension of (2.1.1) is to see what happens if F_{jk} is measured as the product of three numbers $F_{j\lambda} \cdot F_{\lambda m}$ & F_{mk} (for $\lambda, m = 1, \ldots, M$). This is to measure the price change from j to k as a product of price change from j to λ, λ to m and m to k. The required index is defined as the geometric mean values corresponding to all the possibilities. Hence

$$I_{jk} = \left[\prod_{\lambda, m=1}^{M} F_{jk}^{\lambda m} \right]^{\frac{1}{M^2}} \qquad (2.1.2)$$

where $F_{jk}^{\lambda m} = F_{j\lambda} \cdot F_{\lambda m} \cdot F_{mk}$, for $\lambda, m = 1, \ldots, M$. Here we consider pairs (λ, m) as well as (m, λ) since $F_{jk}^{\lambda m}$ need not be the same as $F_{jk}^{m\lambda}$. The interesting conclusion is that (2.1.2) gives the same indices as (2.1.1). To prove this it is enough if we show that the terms in the bracket in (2.1.2) consist of F_{jk}^{λ} ($\lambda = 1, \ldots, M$), each M times and nothing else. Consider the following matrix.

$$F = \begin{bmatrix} F_{jk}^{11} & \cdots & F_{jk}^{1M} \\ F_{jk}^{21} & \cdots & F_{jk}^{2M} \\ F_{jk}^{M1} & \cdots & F_{jk}^{MM} \end{bmatrix} .$$

Equation (2.1.2) is just the M^2-th root of the product of all the elements in the matrix F. The product of the first row and first column yields by the property of Fisher's Ideal indices $[F_{j1} \cdot F_{1k}]^M \left(\prod_{\lambda = 2}^{M} F_{j\lambda} F_{\lambda k} \right)$, product of the second row and second column, excluding the elements already considered, is $[F_{j2} \cdot F_{2k}]^{M-1} \left(\prod_{\lambda = 3}^{M} F_{j\lambda} F_{\lambda k} \right)$. Similarly finding out the product of all the rows and columns, the over all product can be identified as the required term. In fact a similar result can be shown to be true when the price change from j to k is measured through more than two countries. These idices are unique in this particular sense.

They also possess another beautiful property. Let us consider the following problem. Given the Fisher's index numbers for binary comparisons, obtain a consistent system which is minimum distant from the given indices. Suppose we look for an index number system, logafthm of which is minimum distant from the logarithm of the corresponding Fishers's ideal indices (i.e.) what is the system $I_{jk}(j,k = 1,\ldots,M)$ for which

$$\sum_j \sum_k (\log I_{jk} - \log F_{jk})^2 \qquad (2.1.3)$$

is minimum subject to the conditions of consistency like $I_{jj}=1$ and $I_{jk} \cdot I_{k\lambda} = I_{j\lambda}$ for all j,k and λ. Elteto-Koves prove that their indices possess this property. They derive these indices from (2.1.3). But we shall give a simple and more straightforward derivation using a result of Kloek, T. and Theil, H. (1965). Kloek and Theil consider the problem of obtaining an additive consistent system, which is discussed in the introduction, from a given system of index numbers. Existence of additive consistent system implies the existence of numbers $(r_1, r_2, \ldots, r_m)$ such that the index is given by the difference of the corresponding numbers. If P is the matrix of price indices given, we have

$$P = r\,i' - i\,r' + U$$

where $r = [r_1,\ldots,r_M]'$ and $i' = [1,\ldots,1]$ and U is showing the discrepancy between the consistent system and the given system P. So the problem boils down to one of finding out r such that trace (U'U) is minimum. Since we are interested in a consistent system, we can impose an additional restriction of the following type

$$r'i = 0$$

used by Kloek and Theil and the solution turns out to be

$$r_j = \frac{1}{M} \sum_{\ell=1}^{M} \pi_{j\ell}$$

Since the problem of Elteto-Koves is equivalent to one of minimizing trace (U'U) where P is the matrix log F_{jk}'s, we have

$$r_j = \frac{1}{M} \sum_{\ell=1}^{M} \log F_{j\ell}$$

and, $$\pi_j = \left[\prod_{\ell=1}^{M} F_{j\ell} \right]^{\frac{1}{M}} \qquad \text{for } j = 1,\ldots,M$$

since $r_j = \log \pi_j$.

Hence the Elteto-Koves indices are given by

$$I_{jk} = \frac{\pi_j}{\pi_k} = \left[\frac{\prod_{\ell=1}^{M} F_{j\ell}}{\prod_{\ell=1}^{M} F_{k\ell}} \right]^{\frac{1}{M}}$$

$$= \left[\prod_{i=1}^{M} F_{j\ell} \prod_{i=1}^{M} F_{\ell k} \right]^{\frac{1}{M}}$$

$$= \left[\prod_{i=1}^{M} F_{jk}^{\ell} \right]^{\frac{1}{M}}$$

This coincides with the definition in (2.1.1). This derivation of the indices is much simpler and more appealing than one given by Elteto-Koves which is lengthy. Further this derivation makes use of the relationship between the additive and multiplicative consistent system.

However it remains to be seen, as to what happens if we undertake minimization of the Euclidean distance between the Fisher's ideal indices and the consistent system. Here the problem turns out to be one of obtaining I_{jk} (j, k=1,...,M) which minimize

$$\sum_j \sum_k (I_{jk} - F_{jk})^2$$

In general this problem turns out to be intractible due to the involved non-linearities and the resulting indices may not be easily computable. Further use of an iterative method to obtain I_{jk}'s defeat the purpose of the consistent systems. As a passing remark we observe that Elteto-Koves system does not minimize the Euclidean distance. Counter examples can be constructed to this effect.

Some Price Comparisons

Before presenting some of the price comparisons in Rural India, we consider the following. Elteto-Koves type of indices can be constructed using any formula for index numbers which satisfies the time reversal test. We already have one such at our disposal, the Geary-Khamis system index for binary comparisons. There is no sanctity in using Fisher's ideal indices for the construction of consistent indices. Under these circumstances it is better to use index number formulae which have a sound theoretical basis. We present in the following table, indices for inter-fractile price comparisons using the Elteto-Koves indices based on both Fisher's and Geary-Khamis binary indices. The Fisher's and Geary-Khamis' index numbers for binary comparison are also presented side by side.

Table

Group	Fisher's	Elteto-Koves (Fisher)	Elteto-Koves (Geary-Khamis)	Geary-Khamis
(1)	(2)	(3)	(4)	(5)
0 - 10	100.00	100.00	100.00	100.00
10 - 20	103.52	103.49	103.62	103.54
20 - 30	106.08	105.97	106.11	106.13
30 - 40	106.37	106.18	106.38	106.46
40 - 50	108.05	107.83	107.96	108.19
50 - 60	108.77	108.92	108.30	108.49
60 - 70	109.65	109.62	109.71	109.71
70 - 80	110.10	110.90	111.07	111.41
80 - 90	111.23	111.37	111.36	111.20
90 -100	114.53	115.05	114.80	114.15

As expected the Elteto-Koves indices based on Fisher's ideal and Geary-Khamis indices move closely with the index numbers providing their base. Relative efficiencies of these indices according to an empirical criterion are studied in the last section of this chapter.

This section may be closed with a warning that the index numbers discussed in this section leave out numerous other possible consistent systems of index numbers. At this stage index numbers in Yzerene, J.Van (1956) may referred due to their relevance. Yzeren's method makes use of the concept of exchange rate and gives an interesting generalization of Fisher's index numbers. The index numbers are shown to emerge from a system of linear equations and the indices in any practical situation are derived by an iterative procedure. The convergence of the iterative procedure and uniqueness of the solutions are established in the Yzerene (1956). However, iterative procedures lead to multiple sets of index numbers depending on the stage at which the procedure is terminated.

Another attempt made in this line is by Mukerji, V. and Mukerji, K. (1969), to construct index numbers in a stochastic set-up. Assuming that the probability-distributions of price and quantity vectors are known, the current level of living index and current cost of living index are defined as linear functions of price and quantity vectors. Then the problem boilsdown to one of estimating the coefficients of the linear functions. The coefficients are obtained from the normal equations of a least squares procedure minimizing the variance of the difference of product of these indices and the total expenditure subject to the condition that the expected value of the product of these indices equals the total expenditure. The normal equations are non-linear in the unknowns and an iterative procedure is suggested to estimate the coefficients. Once the coefficients are estimated, they form a basis for consistent price and quantity comparisons. But no empirical results based on this method have been reported so far.

We end this discussion with a reference to the index numbers used by Klock and Theil (1965) for binary comparisons of prices. They obtained an index number formula in the additive set-up satisfying the time reversal test and some demand theoretic interpretation is also furnished. These index numbers are found to yield good price comparisons. The next section suggests some possible extensions of these index numbers to the case of multilateral comparisons. We also provide another consistent system of index numbers by generalizing the Fisher's index numbers.

Section 2 : Generalized Laspeyers and Paasche Index Numbers.

A first glance at the basic equations defining the 'Exchange Rate' and the 'average price' used in Geary-Khamis method, gives an impression that the given definitions are logical implications of the concepts. However, in Section 1.2 we observed that it is only one of many other possible definitions and one can derive alternative systems of index numbers using the same concepts. In this section we observe that even the widely used Laspeyres and Paasche indices can be obtained using these two concepts and thus establishing the importance of this approach to index number construction. In fact, this result suggests the possibility of interpreting any index number system in terms of these two concepts (i.e.) we can provide a system of equations emerging from a definition of the concepts which result in the required system of index numbers. This in a sense provides an unified approach to the problem of construction of index numbers.

Let us recall the discussion on the definition of average price in Section 1.2. We observe that, for the i-th commodity, P_i is defined as an average of numbers $R_j p_{ij}$ $(j=1,\ldots,M)$. Further we obtained three possible definitions of P_i, by minimising $\sum_i \sum_j (P_i - R_j p_{ij})^2 w_{ij}$, w_{ij} being a system of weights, ranging over equal $(\frac{1}{M})$, quantity share (q^*_{ij}) and share of expenditure shares (v^*_{ij}). The definitions are, for $i=1,\ldots,N$,

$$P_i^{(1)} = \frac{1}{M} \sum_j R_j \, p_{ij}$$

$$P_j^{(2)} = \sum_j R_j \, p_{ij} \, q^*_{ij}$$

$$P_i^{(3)} = \sum_j R_i \, p_{ij} \, v^*_{ij}$$

As observed already, $P_i^{(2)}$ coupled with definition of R_j in (1.1.2) gives the Geary-Khamis method. Consider $P_i^{(1)}$ and R_j which lead to an interesting system of equations. We have,

$$P_i = \frac{1}{M} \sum_j R_j \, p_{ij}$$

$$R_j = \frac{\sum_i P_i \, q_{ij}}{\sum_i p_{ij} \, q_{ij}} \qquad (2.2.1)$$

By substituting the values of P_i in R_j and simplifying we obtain

$$BR = 0 \qquad (2.2.2)$$

where $B = ((b_{k\ell}))$; $b_{k\ell} = \delta_{k\ell} - \frac{1}{M} \frac{\sum_i p_{i\ell} \, q_{ik}}{\sum_i p_{i\ell} \, q_{i\ell}}$

and $\delta_{k\lambda}$ is as defined earlier. Further $R = [R_1, \ldots, R_M]'$. The main difficulty with the above system of equations is the non-singularity of the matrix B. It will be shown that in general matrix B is non-singular. In the rare case, when B is singular, the condition that all prices are positive in all countries, $p_{ij} > 0$ for all i and j, and that each country consumes at least one commodity, $q_{ik} > 0$ for at least one i for each k, which is necessary for defining (2.2.1) is also sufficient for the existence of unique positive solution, in terms of one of the R_j's. The proof is simple and hence omitted.

Let us focus our attention to the case where M = 2. We have

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2}\frac{\Sigma p_{i2} q_{i1}}{\Sigma p_{i1} q_{i1}} \\ -\frac{1}{2}\frac{\Sigma p_{i1} q_{i2}}{\Sigma p_{i2} q_{i2}} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The determinant vanishes only if $\frac{\Sigma p_{i2} q_{i1}}{\Sigma p_{i1} q_{i1}} \cdot \frac{\Sigma p_{i1} q_{i2}}{\Sigma p_{i2} q_{i2}} = 1$

which implies that the Laspeyres and Paasche formulae for I_{12} must give the same value. In general this is not true. This means that only trivial solution for the equations is possible in most of the cases. However, interesting results emerge from the cases obtained by ignoring one restriction. In the above case, suppresing one of the restrictions, we have one equation in two unknowns (R_1 and R_2). Hence the ratios are uniquely determined.

Values of R_j's, obtained by first equation are

$$R_1 = 1 \quad \text{and} \quad R_2 = \frac{\sum_i p_{i1} q_{i1}}{\sum_i p_{i2} q_{i1}}$$

and the values obtained by second equation are

$$R_1 = 1 \quad \text{and} \quad R_2 = \frac{\sum_i p_{i1} p_{i2}}{\sum_i p_{i2} q_{i2}}$$

Hence I_{12} turns out to be $\dfrac{\Sigma\, p_{i2}\, q_{i1}}{\Sigma\, p_{i1}\, q_{i1}}$ in the first case and $\dfrac{\sum_i p_{i2}\, q_{i2}}{\sum_i p_{i1}\, q_{i2}}$ in the second case, which coincide with the Laspeyres and Paasche indices for I_{12} respectively. The similarity is complete when we observe that the first set of values of R_1 and R_2 are obtained on p_1, p_2 and q_1 only. Using the usual interpretation of these formulae rest of the index numbers can be defined similarly. Having obtained two sets of solutions for the exchange rates, we may combine them to yield price comparisons satisfying circularity test. The geometric mean of the two sets, yields

$$R_1 = 1 \quad \text{and} \quad R_2 = \left[\frac{\sum_i p_{i1}\, q_{i1}}{\sum_i p_{i2}\, q_{i1}} \cdot \frac{\sum_i p_{i1}\, q_{i2}}{\sum_i p_{i2}\, q_{i2}} \right]^{\frac{1}{2}}$$

and the indices resulting from the above R_1 and R_2 coincide with Fisher's ideal index numbers.

Preceeding discussion establishes a link between the well-known index number formulae and the concepts of average price and exchange rate, using the system of equations in the case $M = 2$. This opens up a possibility of generalizing these index numbers for binary comparisons to multilateral comparisons. There has been no attempt to extend the idea of Laspeyres and Paasche index numbers to the case where $M > 3$. Laspeyres price index is defined as the base quantity weighted average of the prices whereas the Paasche price index is defined current quantity weighted average of the prices. This can also be interpreted in a slightly different fashion which makes the generalization possible. Laspeyres price index is defined by ignoring the current quantity vector and conversely for the Paasche index. Look at the system of equations (2.2.2). We have observed that, in general, only trivial solution is possible for the vector R. However, suppressing one of the equations, we have only $M - 1$ equations in M unknowns. By fixing one of the unknowns, rest of the variables can be solved uniquely, under the condition stated already. Let R_j^k $(j = 1, \ldots, M)$ be the solution for the unknowns, resulted from the commission of the k-th equation, which implies that the corresponding quantity vector is neglected in the price comparisons. Given the M sets of solutions for R_j's we shall define the required index numbers in the following manner. The generalized Laspeyres price index I_{jk}, for the k-th group with j-th group as base, is defined as the ratio of the corresponding exchange rates obtained by ignoring the k-th restriction. Hence the index number is given by R_j^k / R_k^k. Similarly generalized Paasche price index is given by the ratio R_j^j / R_k^j. The index numbers from the geometric mean of the sets of values of R_j's, give the generalized Fishers ideal index numbers. The actual index numbers are given by the ratio of the exchange rates R_j which are given by

$$R_j = \left[\prod_{k=1}^{M} R_j^k \right]^{\frac{1}{M}}$$

and this yields index numbers satisfying circularity test.

The generalized Laspeyres, Paasche and Fisher index numbers coincide with their counterparts for binary comparisons and also possess similar properties. For example, the result that Laspeyres index I_{jk} equals the reciprocal of Paasche index for I_{kj} also holds in this particular case. Further, suppose that the Laspeyres and Paasche index numbers for binary comparisons happen to form a consistent index number system, then it can be shown that the generalized index numbers coincide with their binary comparison counterparts. This results sounds similar to a result of Yzeren (1956) which shows that the generalized Fisher's index numbers obtained by him coincide with their binary counterparts whenever they are consistent. Incidentally, we have obtained an alternative generalization of Fisher's index numbers. But the difference that the generalized Fisher's indices given here are not given by the geometric mean of the generalized Laspeyres and Paasche indices is to be noted.

Emperical Results :

Based on the data on price and quantity in Appendix A relating to the Rural India, we have obtained ten sets of exchange rates corresponding to equation systems obtained by deleting the equations concerning the size classes 0-10 to 90-100. The following table gives the exchange rates, the first ten columns give the exchange rates obtained by deleting the equations corresponding to the size classes and the last column gives the geometric mean of the exchange rates in the previous columns.

Table

Group	Exchange rates obtained by omitting equation relating to group										Geometric mean of cols. 1 to 10
	0 -10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0 - 10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10 - 20	0.9740	0.9550	0.9640	0.9639	0.9640	0.9639	0.9640	0.9639	0.9640	0.9641	0.9641
20 - 30	0.9523	0.9425	0.9357	0.9425	0.9424	0.9425	0.9425	0.9424	0.9424	0.9411	0.9447
30 - 40	0.9510	0.9412	0.9412	0.9316	0.9412	0.9412	0.9412	0.9411	0.9411	0.9266	0.9422
40 - 50	0.9363	0.9267	0.9265	0.9267	0.9172	0.9267	0.9267	0.9267	0.9267	0.9266	0.9267
50 - 60	0.9387	0.9290	0.9291	0.9290	0.9290	0.9197	0.9291	0.9290	0.9290	0.9290	0.9291
60 - 70	0.9211	0.9107	0.9118	0.9117	0.8205	0.9118	0.9023	0.9117	0.9118	0.9117	0.9021
70 - 80	0.9116	0.9022	0.9022	0.9022	0.9022	0.9022	0.9022	0.8929	0.9023	0.9023	0.9022
80 - 90	0.9052	0.8955	0.8955	0.8956	0.8956	0.8958	0.8951	0.8955	0.8866	0.8954	0.8956
90 -100	0.8726	0.8636	0.8635	0.8635	0.8635	0.8635	0.8635	0.8636	0.8636	0.8546	0.8635

The exchange rates presented in the above table do not show marked differences in the solutions obtained by various equation systems. In many cases the vectors of exchange rates are pretty close. However the exchange rates are different from each other establishing the non-singularity of the matrix involved. Another interesting feature of the table are the values of the exchange rates concerning the groups, which are excluded from the system of equations. For example, the values of the exchange rates of groups 10-20 to 90-100, with the exception group 60 - 70, assumed minimum value in the corresponding rows. By the definition of the generalized Laspeyres and Paasche indices, this suggests that the generalized Paasche indices are tending to be smaller than their Laspeyres counterparts, which is in general true in the case of binary comparisons. The following table showing these index numbers along with their binary comparison indices also supports the above phenomena.

Table

Group	Laspeyres	Generalized Laspeyres	Paasche	Generalized Paasche	Fisher	Generalized Fisher
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0- 10	100.00	100.00	100.00	100.00	100.00	100.00
10- 20	103.61	104.71	103.54	102.66	103.52	103.73
20- 30	106.27	106.87	106.13	105.01	106.08	105.85
30- 40	106.59	107.34	106.46	105.15	106.37	106.14
40- 50	108.33	109.03	108.19	106.80	108.05	107.91
50- 60	110.25	108.73	108.49	106.53	108.77	107.64
60- 70	109.69	110.83	109.71	108.57	109.65	110.86
70- 80	111.41	111.99	111.41	109.70	111.10	110.84
80- 90	110.90	112.79	111.20	110.47	111.23	111.66
90-100	113.44	117.01	114.15	114.60	114.53	115.80

From the table it can be seen that the generalized Laspeyres index numbers are generally higher than the Laspeyres indices whereas the generalized Paasche indices are below the Paasche index numbers. Further the generalized Paasche indices are lower than the generalized Laspeyres indices which is consistent with the usual notion that Paasche and Laspeyres indices provide the lower and upper bounds respectively. Fisher indices for binary comparisons are, in general, higher than the generalized Fisher index numbers.

The fact that the Laspeyres and Paasche index numbers can be derived through the use of notion of exchange rate and average price suggests that most of the index numbers can be brought under this framework. Considerable research has to be undertaken to interpret various systems of index numbers using these two concepts.

Other Possible Systems :

We are still left with some more possible systems which correspond to the definitions of the 'average price' and 'exchange rates' pointed out in the previous discussions. Our main objective in this case is to establish the non-existence of unique positive solutions in general. For the sake of simplicity, we restrict the analysis to the case M=2. Following is the analysis given system-wise.

System I : Definitions involved are

$$P_i = \sum_{j=1}^{2} R_j \, p_{ij} \, v^*_{ij} \qquad \text{for all } i$$

$$R_j = \frac{\sum_i P_i \, q_{ij}}{\sum_i p_{ij} \, q_{ij}} \qquad \text{for all } j$$

This results in a system of homogeneous equations given by

$$\begin{bmatrix} 1 - \dfrac{\sum_i p_{i1} v^*_{i1} q_{i1}}{E_1} & -\dfrac{\sum_i p_{i2} v^*_{i2} q_{i1}}{E_1} \\ -\dfrac{\sum_i p_{i1} v^*_{i1} q_{i2}}{E_2} & 1 - \dfrac{\sum_i p_{i2} v^*_{i2} q_{i2}}{E_2} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the determinant need not vanish, and this results in a trivial solution for R_1 and R_2. However, analysis similar to that one in last discussion may be carried out in this case also.

So far, we concentrated on the definitions based on arithmetic mean. Let us switch on to the geometric mean formulae. In this, a particular case has already been tackled in Chapter I.

<u>System II</u> : Definitions involved are

$$R_j = \prod_{i=1}^{N} \left(\frac{P_i}{p_{ij}} \right)^{v_{ij}} \qquad \text{for all } j$$

$$P_i = \left[\prod_{j=1}^{M} R_j \, p_{ij} \right]^{\frac{1}{M}} \qquad \text{for all } i$$

Since these equations are log-linear, transforming the unknowns R_j and P_i suitably, we have

$$R_j^* = \sum_i (P_i^* - \log p_{ij}) v_{ij}$$

$$P_i^* = \frac{1}{M} \sum_j (R_j^* + \log p_{ij})..$$

This yields a system of linear non-homogeneous equations. We have

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} R_1^* \\ R_2^* \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \sum_{j=1}^{2} \sum_i (\log p_{ij} - \log p_{i1}) v_{i1} \\ \frac{1}{2} \sum_{j=1}^{2} \sum_i (\log p_{ij} - \log p_{i2}) v_{i2} \end{bmatrix}$$

Though the matrix is singular in this case, the system of equations in general fail to be consistent. This is evident by following solutions based on different equations. Solutions based on first equation are

$$R_1 = 1 \quad \text{and} \quad R_2 = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i2}}\right)^{v_{i1}}$$

whereas the solutions based on the second equation are

$$R_1 = 1 \quad \text{and} \quad R_2 = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i2}}\right)^{v_{i2}}$$

These two sets of values in general do not coincide, which accounts for the inconsistency of the above equation system. An interesting feature here is the possibility of analysis similar to the one presented above. Geometric mean of the above two vectors give us

$$R_1 = 1 \quad \text{and} \quad R_2 = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i2}}\right)^{\frac{v_{i1} + v_{i2}}{2}}$$

and this gives the index numbers which coincide with those used by Kloek and Theil (1965) for international comparisons of prices. Tracing the steps of the previous analysis, we can obtain a generalization of these indices for the case where $M > 2$. Details are omitted due to their mechanical nature.

<u>System III</u> : Definitions pertaining to this are

$$R_j = \prod_{i=1}^{N} \left(\frac{P_i}{p_{ij}}\right)^{v_{ij}}$$

$$P_i = \prod_{j=1}^{M} (R_j \, p_{ij})^{v^*_{ij}}$$

The relevant system of equations can be shown to be

$$\begin{bmatrix} 1 - \sum\limits_i v^*_{i1} v_{i1} & - \sum v^*_{i2} v_{i1} \\ - \sum v^*_{i1} v_{i2} & 1 - \sum v^*_{i2} v_{i2} \end{bmatrix} \begin{bmatrix} R^*_1 \\ R^*_2 \end{bmatrix} = \begin{bmatrix} \sum\limits_j \sum\limits_i (\log p_{ij} - \log p_{i1}) v^*_{ij} v_{i1} \\ \sum\limits_j \sum\limits_i (\log p_{ij} - \log p_{i2}) v^*_{ij} v_{i2} \end{bmatrix}$$

Though the determinant of the matrix vanishes, the system can be seen to be inconsistent by looking at the right hand side of the equations. The explicit values are not furnished, since they are easy to derive.

This takes care of most of the possible systems of equations based on several plausible definitions of average price and exchange rate. We stuck to the expenditure weighted average of $\frac{P_i}{p_{ij}}$ $(i = 1, \ldots, M)$ for R_j since it is customary to use the expenditure ratios to weigh the price relatives. However, the possibility of other definitions of these concepts can be considered, but it may be expected that in many cases unique solutions do not exist.

We end this section with a reference to the usefulness of the concepts — exchange rate of a currency and average price of a commodity — which are able to contain many well established index number formulae as special cases. Further this lends a new flavour to the Geary-Khamis indices and the indices proposed in Section 1.2. It may reasonably be expected that further studies on these concepts would result in more interesting and fruitful results.

Section 2.3 : Problem of choice of a consistent system.

It is indeed paradoxical to consider the problem of choice of consistent index numbers to compare the prices and quantities. As mentioned in the introduction, the consistency tests came into existence to solve the problem of selection of a suitable index number in a given situation. But the preceeding discussion shows that the problem still haunts us, despite the imposed restrictions. We have mentioned a number of formulae which satisfy the circularity test. These index numbers also possess the property of being independent of units of measurement and satisfy proportionality test. It is likely that the same situation prevails even after the introduction of a few more tests. We shall provide an empirical procedure, which in a given concrete situation enables us to solve the problem of choice. We discuss this criterion in a general set-up and the criterion is shown to be more simplified in the case of consistent index numbers.

Tracing back to the fundamentals, a price index is a real number which indicates the general price level in a particular group when compared to prices in another group. Hence the product of the price of a given item in the base group and the price index gives us an estimated level of price of the item in the given group.

Let j-th group be the base. So for the i-th item, $p_{ij} \cdot I_{jk}$ gives the estimate price of the item in the k-th group. For a reasonably good system of index numbers, one may expect that the deviations of the expected values from the original values are not large. This suggests a criterion based on the minimum sum of squares of deviations of the estimated prices from the actual prices for the choice of a system of index numbers. We choose an index number system for which the distance between actual and the estimated price vectors is minimum. In symbols, we choose the system of index numbers I_{jk} $(j,k = 1,\ldots,M)$ for which

$$\Delta = \sum_{j=1}^{M} \sum_{k=1}^{M} \sum_{i=1}^{N} (p_{ik} - p_{ij}\, I_{jk})^2 \qquad (2.3.1)$$

is minimum. This is similar to the familiar least squares criterion. Alternatively, we can insist on weighted sum of squares instead of the unweighted sum Δ . In this case, the distance function turns out to be

$$\Delta^* = \sum_{j=1}^{M} \sum_{k=1}^{M} \sum_{i=1}^{N} (p_{ik} - p_{ij} I_{jk})^2 \; w_{ijk} .$$

It can be seen that this criterion can be used even as a basis for the construction of an index number formula. When we need a consistent system, we can minimize Δ or Δ^* with respect to I_{jk} subject to the conditions of consistency (e.g. $I_{jk} . I_{k\lambda} = I_{j\lambda}$ for all j,k and λ). The required indices can be obtained from the solution of the system of simultaneous equations, derived from the first order conditions of minimization. But in this case, these equations would be difficult to solve due to the involved non-linearities. This may necessiate the use of some iterative methods, similar to those used in likelihood estimation, which do not serve the real purpose of obtaining unique system of index numbers.

The problem of the non-linearities can be resolved by changing the minimand from the ordinary distance between the observed and expected price vectors to the distance between the logarithms of these vectors. In this case we minimize

$$\Delta^{**} = \sum_j \sum_k \sum_i (\log p_{ij} - \log p_{ik} I_{kj})^2$$

Let us consider the particular case where $M = 2$. Further consider the weighted sum of squares of deviations, weights depending on the expenditure shares. Hence we minimize

$$\Delta^{***} = \sum_i (\log p_{i2} - \log p_{i1} - \log I_{12})^2 v_{i2} + \sum_i (\log p_{i1} - \log p_{i2} - \log I_{21})^2 v_{i1}$$

Imposing the consistency restriction $I_{12} \cdot I_{21} = 1$, the first order condition $d\,\Delta^{***} / d \log I_{12} = 0$, after simplification yields us

$$\log I_{12} = \sum_i \frac{v_{i1} + v_{i2}}{2} (\log p_{i2} - \log p_{i1})$$

and $$I_{12} = \prod_{i=1}^{N} \left(\frac{p_{i2}}{p_{i1}}\right)^{\frac{v_{i2}+v_{i1}}{2}}$$

This formula precisely coincides with Theil-Kloek (1965) index numbers for international comparisons of prices. In the same lines, we can obtain consistent index numbers for multilateral comparisons. This incidentally leads to a generalization of the Theil-Kloek indices. These indices turn out to be different from the consistent indices obtained by Theil-Kloek according to the criterion of minimum distance from the binary comparisons.

The criteria based on Δ and Δ^* can be more simplified in the case of consistent system of index numbers. Consider the M numbers π_j $(j = 1,\ldots,M)$, representing general price levels implied by the consistency. Let p_j be the price vector of the j-th country. Then $p_j \cdot \frac{1}{\pi_j}$ represents the price vector adjusted to the over all price change. In an ideal case, these adjusted price vectors must be *very close*. In view of this, since $R_j = 1/\pi_j$, we choose an index number system for which

$$\sum_{j=1}^{M} \sum_{k=j+1}^{M} \sum_{i=1}^{N} (R_j \, p_{ij} - R_k \, p_{ik})^2$$

is the least. Similar functions can be considered for the choice, based on different weighting schemes.

It is not out of place to mention the possibility of criterion based on price and quantity indices. For example, we may be interested in choosing price and quantity indices for binary comparisons such that they satisfy the factor test. This criterion can be made more complicated by introducing the time reversal test into the picture and analysis similar to the above follows. But it is difficult to show that a particular type of indices yield minimum sum of squares of deviations in general. However Δ^{**} and Δ^{***} lead to specific index number formulae, serving the main purpose.

We have attempted here to find out the relative efficiencies of the systems of index numbers discussed in the previous sections, and we have considered the following index numbers for the purpose. Laspeyres, Paasche, Fisher, Geary-Khamis, new system, ECLA and Elteto-Koves, based on Fisher and Geary-Khamis binary comparisons. We have used the criterion Δ given in (2.3.1) to test the efficiencies, based on the price and quantity data regarding the ten size classes of Rural India. The value of Δ is presented below for the different index numbers.

Δ (Laspeyres)	=	58.716649
Δ (Paasche)	=	58.761452
Δ [Geary-Khamis] (binary)	=	57.956754
Δ (Fisher)	=	58.026898
Δ (ECLA)	=	58.0004470
Δ [Elteto-Koves] (Fisher)	=	57.989985
Δ [Elteto-Koves] (Geary-Khamis)	=	57.914724

Δ (Geary-Khamis) = 58.217281

Δ (New System) = 58.702704

Strict adherence to the criterion suggests the use of Elteto-Koves based on Geary-Khamis indices for binary comparisons, for the purposes of price comparisons. However the value of Δ shows that there is little difference in the values in spite of the fact that Δ, in our calculations, depends on a fairly large number of differences between observed and expected prices. This type of behaviour of Δ over various systems of index numbers can be attributed to the type of data we have considered. A scrutiny suggests that the data used by us exhibit a systematic change from one group to another, there being a steady rise in the price and quantity elements, as we move up from lower to higher expenditure groups. Incidentally this also explains, to some extent the good performance of ECLA index numbers here. However in a situation where data change widely from one group to another, the value of Δ is likely to differ widely and enable us to choose an index number system from a number of possible systems. In the case of very heterogeneous groups, it is likely that consistent systems come out to be slightly inferior to the indices for binary comparisons. Some other aspects of this problem are considered again in the next chapter.

CHAPTER THREE

PROBLEMS OF CONSISTENT INTER-GROUP PRICE COMPARISONS

Any discussion on the problem of international or inter-group price comparison is incomplete if the difficulties regarding data are left untouched. The results obtained in this chapter show that just the availability of a index number formulae yielding consistent comparisons does not take us anywhere near the actual price comparisons. Application of such methods to yield good results, in general, springs many problems. It is undesirable to calculate index numbers using any of the formulae discussed in the two chapters in the case of countries or group that are relatively heterogeneous in the nature of their consumption patterns. The repercussions of such an application are demonstrated in the first section of this chapter. Further, the set up of full information, which is implicitly assumed in discussions so far, has to be waived, at least partially, because we may not get all the essential information. Several types of difficulties can be summerized as follows :

(i) It is frequently noticed that while the expenditure can be exhaustively broken down by a detailed list of items, it may not be possible to get separate quantity and price information for all the items. In view of this, attempts are sometimes made to obtain the expenditure pattern in an exhaustive way, and to obtain prices of some items in an expenditure category, for which exact specifications are provided. We thus get reasonably comparable prices, but they govern only a part of the expenditure category.

(ii) There are cases where the lists of expenditure categories are identical, but some items in the list, for which price and quantity details are available, have different specifications.

(iii) There may be cases when the lists themselves are not identical in the sense that they intersect only in respect of some of the items, the data in respect of expenditure, price and quantity being assumed to be available for all the common items. Thus, when M groups are being compared there could be $n (n < M)$ items common to all countries; also between any two countries j and k there are n_{jk} $(n_{jk} \geq n)$ common items.

Confronted with a situation like this, it becomes necessary to adapt the index number systems so that we can make best possible use of all available information, as well as information that can be collected within a reasonable time period. The problems are discussed in the light of the work done under the International Price Comparisons Project of the United Nations.

We shall consider these problems in what follows inthe same order.

Section 3.1 Price Comparisons in Heterogeneous Groups

The attraction for consistent systems for inter-country comparisons considered previously is obvious. The comparisons are based on the entire available evidence pertaining to the case and are unique. The main difficulty with consistent systems is that they do not satisfy some obviously desirable properties of index numbers. The use of consistent system distorts the binary comparisons implicit in the exercise. This is due to the stipulation of the condition of consistency. Each index for binary comparison, in addition to the measurement of price change, has to satisfy the circularity test. This means that the actual value depends on indices for other binary comparisons. Hence this index is also a function of the price and quantity vectors of other countries. So, between any two countries (A, B) out of M countries, a simple binary comparison has to be better than any comparison using information about other groups. Likewise a binary comparison is also the best between B and C. But apart from the question of consistency, it is not obvious that these two binary comparisons when linked provide the best possible comparisons between the countries (A, B, C) and nothing better could be done.

We obtain more concrete conclusions regarding this matter presently. In general, the price indices must posses the following property. If in two countries under comparison, the price vector in one country is uniformly greater than the other, the corresponding index should reflect the same. Essentially, if $p_k > p_j$ for countries j and k then I_{jk} must be greater than unity for any meaningful index number. This can be justified by the usual consumer behaviour approach to the index number problem. As stated in the introduction, a price index number is the ratio of the minimum expenditures necessary to keep an individual on the same level of indifference before and after a price change. If the prices are all uniformly higher, the new budget surface lies entirely below the old one which implies that the overall expenditure has to be increased to maintain the same utility level. This necessiates the index to be greater than unity. This particular criterion for the goodness of an index number system turns out to be a weaker version of the 'proportionality consistency' discussed by Swamy, S.(1965). Proportionality consistency implies that a fixed percentage change in the prices of all commodities should lead to a corresponding change in the index number and we may describe the property considered above as 'weak proportional consistency'. We observed that consistency effects the binary comparisons and now we shall prove that many of the consistent systems discussed above do not possess the property of 'weak proportionality consistency'. However this will be seen to hold in only some extreme cases which warrant a detailed study on consistent comparisons in heterogeneous groups, heterogeneous in the sense of the extreme nature of the price and quantity vectors. First we shall prove that the Geary-Khamis indices do not possess this property followed by a similar result with respect to the Elteto-Koves indices.

<u>Theorem</u> : The Geary-Khamis system of index numbers fails to satisfy the 'weak proportionality consistency' for $M \geq 3$.

<u>Proof</u> : For $M=2$, the indices possess the property, since we have

$$R_1 = 1$$

$$R_2 = \frac{\sum_i p_{i1} \frac{q_{i2}\, q_{i1}}{q_{i2}+q_{i1}}}{\sum_i p_{i2} \frac{q_{i2}\, q_{i1}}{q_{i2}+q_{i1}}}$$

We shall prove the theorem by exhibiting a counter example. Let us consider the case when $N = 3$ and $M = 4$ and the price and quantity data are given in the following table.

Items	Groups 1	2	3	4
	Prices			
1	5	5	6	7
2	2	3	2	4
3	1	2	2	15
	Quantities			
1	10	8	11	10
2	7	6	8	5
3	0	5	15	10

In the above example, $p_1 \leq p_2 \leq p_3$. The solution of the R_j's turns out to be $R_1 = 1.000$, $R_2 = 1.0301$, $R_3 = 1.1501$ and $R_4 = 0.3997$. Hence indices with the first group as base turn out to be $I_{11} = 1.0000$, $I_{12} = 0.9708$, $I_{13} = 0.9049$ and $I_{14} = 2.7521$. The values of I_{12} and I_{13} are less than unity indicating a fall in the general price level, and violating the required property. This proves the theorem, but detailed discussion on the nature of equations basic to this system would be more educative and serves as a general argument in support of this theorem.

Consider equation (1.1.4). Putting $R_M^* = 1$, we have

$$\bar{B}\,\bar{R} = y$$

where $\bar{B}$, $\bar{R}$ and y are as defined earlier. Assuming conditions for the existence obtained for unique positive solution, we have

$$\bar{R} = \bar{B}^{-1} y.$$

Observe that $\bar{B}$ is a matrix of the form (I-A) where A is a Leontief type matrix. Denote $\bar{B}^{-1}$ by $D = ((D_{jk}))$. Then consider the solution for R_1^* and R_2^*

$$R_1^* = E_1 R_1 = D_{11} \sum_i q_{i1}^* v_{iM} + \ldots + D_{1M-1} \sum_i q_{iM-1}^* v_{iM}$$

$$R_2^* = E_2 R_2 = D_{21} \sum_i q_{i1}^* v_{iM} + \ldots + D_{2M-1} \sum_i q_{iM-1}^* v_{iM}$$

By putting $R_M = 1$ we have

$$R_1 = \frac{1}{E_1} [D_{11} \sum_i q_{i1}^* e_{iM} + \ldots + D_{1M-1} \sum_i q_{iM-1}^* e_{iM}]$$

$$R_2 = \frac{1}{E_2} [D_{21} \sum_i q_{i1}^* e_{iM} + \ldots + D_{2M-1} \sum_i q_{iM-1}^* e_{iM}]$$

Since D_{ij}'s, E_1 and E_2 are independent of p_{iM} for $i=1,\ldots,N$, p_{iM}'s can be chosen so as to make R_1 greater than R_2 and vice versa under very mild conditions. Observe that D_{11} and D_{22} are greater than 1 and rest of the elements are quite small, in the above equations. This is due to the nature of the matrix $\bar{B}$ as pointed out earlier. If $q_{i1}^* > q_{i2}^*$ for some i, then the corresponding p_{iM}'s can be chosen to make $R_1 > R_2$. Similarly if $q^*_{i2} > q^*_{i1}$ for some i, we can make $R_2 > R_1$. This argument fails in the case where $q^*_{i1} = q^*_{i2}$ for all i. This means that the quantity vectors are same in both the countries 1 and 2 which is in general not true. Thus the discussion establishes why these indices do not satisfy the 'weak proportionality consistency'. A similar result is expected in the case of the new system of index numbers discussed in Section 1.2. Focussing our attention to the remarks at the end of that section, we observe that the indices fail to satisfy the property only in more extreme cases. This may be due to the logarithmic transformation involved in the basic system of equations. Let us go on to prove a similar theorem regarding Elteto-Koves indices.

Theorem : Elteto-Koves system of index numbers does not satisfy 'weak proportionality consistency' for $M \geq 3$.

Proof : This satisfies the above property for $M = 2$ since the indices coincide with the Fisher's ideal indices. Let $M = 3$ and further $0 < p_1 < p_2 < p_3$. Elteto-Koves index I_{12} is defined by

$$I_{12} = \left[F^{(1)}_{12} \; F^{(2)}_{12} \; F^{(3)}_{12} \right]^{\frac{1}{3}}$$

Since F_{jk} is Fisher's ideal index number, we have

$$I_{12} = \left[F_{12}^2 \; \frac{F_{13}}{F_{23}} \right]^{\frac{1}{3}}$$

Since $p_1 < p_2$, we have $F_{12} > 1$ and hence $F_{12}^2 > 1$. Let p_1 and p_2 be two vectors such that F_{12} is marginally greater than unity. We shall establish the possibility of the choice of p_3, q_1, q_2 and q_3 satisfying $F_{23} > F_{12}^2 \, F_{13}$ which implies that $I_{12} < 1$ and hence the theorem. We prove that vectors p_3, q_1, q_2 and q_3 can be chosen so as to make F_{23} exceed F_{13} by a very big quantity. We have

$$F_{23} = \left[\frac{\sum_i p_{i3} \, q_{i2}}{\sum_i p_{i2} \, q_{i2}} \cdot \frac{\sum_i p_{i3} \, q_{i3}}{\sum_i p_{i2} \, q_{i3}} \right]^{\frac{1}{2}}$$

$$F_{13} = \left[\frac{\sum_i p_{i3} \, q_{i1}}{\sum_i p_{i1} \, q_{i1}} \cdot \frac{\sum_i p_{i3} \, q_{i3}}{\sum_i p_{i1} \, q_{i3}} \right]^{\frac{1}{2}}$$

Hence

$$F_{23}^2 = \left[\sum_{i=1}^{N} \frac{p_{i3}}{p_{i2}} \, w_i^* \quad \sum_{i=1}^{N} \frac{p_{i3}}{p_{i2}} \, w_i^{**} \right]$$

$$F_{13}^2 = \left[\sum_{i=1}^{N} \frac{p_{i3}}{p_{i1}} \, w_i \quad \sum_{i=1}^{N} \frac{p_{i3}}{p_{i1}} \, \bar{w}_i^{**} \right]$$

where $w_i = \frac{p_{i1}\, q_{i1}}{\sum_i p_{i1}\, q_{i1}}$, $w_i^* = \frac{p_{i2}\, q_{i2}}{\sum_i p_{i2}\, q_{i2}}$, $w_i^{**} = \frac{p_{i2}\, q_{i3}}{\sum_i p_{i2}\, q_{i3}}$

and $\bar{w}_i^{**} = \frac{p_{i1}\, q_{i3}}{\sum_i p_{i1}\, q_{i3}}$.

$$F_{13}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{p_{i3}}{p_{i1}} \cdot \frac{p_{j3}}{p_{j1}} \cdot w_i\, \bar{w}_j^{**}$$

Now, let q_1 and q_3 be any two non-negative vectors such that $w_i . \bar{w}_j^{**} \neq 0$ only for $i = j = k$. Hence

$$F_{13} = \left[\frac{p_{k3}^2}{p_{k1}^2} \cdot w_k \cdot \bar{w}_k^{**} \right]^{\frac{1}{2}}$$

$$F_{23} = \left[\sum_i \sum_j \frac{p_{i3}}{p_{i2}} \cdot \frac{p_{j3}}{p_{j2}}\; w_i^* \cdot w_j^{**} \right]^{\frac{1}{2}}$$

Choose q_2 and q_3 such that $w_i^* . w_j^{**} \neq 0$ for only $i = j = k$ and $i = j = \lambda$ and $w_k^* = w_k^{**}$ and $w_\lambda^* = w_\lambda^{**}$. It is easy to check that such a choice of q_1, q_2 and q_3 is possible. Therefore

$$F_{23} = \frac{p_{k3}}{p_{k2}} \cdot w_k^* + \frac{p_{\lambda 3}}{p_{\lambda 2}}\, w_\lambda^*$$

$$F_{13} = \frac{p_{k3}}{p_{k1}} \left(w_k . \bar{w}_k^{**}\right)^{\frac{1}{2}}$$

Given p_3 and p_1, F_{13} entirely depends on q_1 since $\bar{w}_k^{**}$ is already given due to the choice of q_2 and q_3. We can choose q_1 such that w_k is small enough to make $F_{13} < F_{23}$ and as small as we need. Hence the theorem.

This result would be more difficult to establish in the case where p_3 is unrestricted. Using the above as basis, it is possible to construct counter examples which establish the theorem.

Though the above results raise doubts on the utility of the consistent systems in actual practice, a close look at the proofs given above shows that the occurence of such a situation is limited to the extreme cases where the price and quantity vectors of the M countries are very heterogeneous. We explore the possibility of applying consistent systems in a more meaningful fashion than a straightforward application of the formulae.

In a general problem of inter-country comparison of prices, the set of countries might include a few highly developed economies and a few developing economies. In such a case the consumption patterns of these two subgroups would be widely divergent. A similar situation may arise in the case where prices in a few regions of a country are compared over time. Comparison of prices in a developing region, within a country, ten years back and a highly developed region today would again result in such a problem. This problem may be termed as one of multiple classification and is considered below.

Suppose that we have M size classes each for urban and rural areas, then the comparison between the 2M groups can be made in different ways given a formula apriori. On the other hand, we may for example, consider the urban and rural areas as two large groups having some special characteristics. In the case of countries, we can group them into categories having similar consumptions patterns. In our example, we may apply Geary-Khamis or similar approaches within the larger groups and link the two groups through some selected size classes or countries, one chosen from each group. The resulting comparisons are semi-consistent and different from those obtained when all the groups or countries are treated together. The use of prior knowledge in selection of relatively homogenous larger groups and of particular elements of groups for purpose of linking, again from homogeneity considerations, may give better results than a straightforward application of Geary-Khamis or allied methods, to all groups. Finally, the amount of computation necessary for obtaining the final results will be much smaller in this case. In the rural-urban comparison problem, the largest matrices to be inverted reduce to size $M \times M$ as against the inversion of a matrix of dimension $2M \times 2M$ in the case of a straightforward application of the Geary-Khamis method.

Let us formalise the method and apply in the problem of price comparison for the 10 size classes in rural and urban areas. Instead of applying Geary-Khamis method (chosen for illustrative purposes) to all the 20 size classes, we may compare the rural size classes with the corresponding urban size classes and link then by using relative purchasing powers of size classes with the rural (or urban) area alone. Let us call comparisons within the rural and urban areas as vertical and between urban and rural areas for the same size class as horizontal. This notation may be used for its simplicity.

The method can be described in a more general set up. For each size class j, there can be more than two groups. Let there be s groups and t size classes. Given this situation, there are three possible methods of applying any system of consistent index numbers. For example consider Geary-Khamis method.

Method 1 : Apply Geary-Khamis method for $s \times t$ groups.

Method 2 : For each size class j, obtain R_{ij} $(i = 1,\ldots,s)$ by horizontal comparisons. Then fix a group i and make vertical comparison. For example fix the first group for vertical comparisons. This yields r_{1j} $(j = 1,2,\ldots,t)$. So in all, we have

$$\begin{matrix} R_{11} & R_{21} & \cdots & R_{s1} \\ R_{12} & R_{22} & \cdots & R_{s2} \\ \vdots & & & \\ R_{1t} & R_{2t} & \cdots & R_{st} \end{matrix}$$

through horizontal comparisons and $r_{11}, r_{12}, \ldots, r_{1t}$ through vertical comparisons. Using the vertical comparisons as link between the independent horizontal comparisons, we define price indices in the following manner. Consider two points for comparison, say j_1-th size class in i_1-th group and j_2-th size class in i_2-th group. We measure the price change from i_1-th group in j_1-th size class to the first group in j_1, from the first group in j_1 to the first group in j_2 and from the first group in j_2 to i_2-th group in j_2, and define the price index number for j_2-th size class in i_2-th group with j_1-th size class in i_1-th group as base

$$I_{j_1 j_2, i_1 i_2} = \frac{R_{i_1 j_1}}{R_{1\,j_1}} \cdot \frac{r_{1\,j_1}}{r_{1\,j_2}} \cdot \frac{R_{1\,j_2}}{R_{i_2 j_2}}$$

This definition involves implicit assumption that the price change is multiplicative. But this assumption is justified in the light of consistency tests like the time reversal and circularity tests. It can be verified that the above definition yields a system of index numbers satisfying circularity test.

Method 3 : We obtain purchasing power for each group i, r_{ij} (j = 1,...,t) by vertical comparison and for j-th class, R_{ij} (i = 1,...,s) by horizontal comparison and define index numbers in exactly the same way as done above.

As a result of the previous conclusions, method 1 is not desirable. Let us illustrate the application of method 2 to our problem of rural-urban price comparisons in 10 size classes. The results obtained by using the methods 1 and 2 are presented in the table below.

Table : Exchange rates obtained by Geary-Khamis method and method 2

size classes	horizontal rural	horizontal urban	vertical rural	Geary-Khamis rural	Geary-Khamis urban	Geary-Khamis urban with rural = 1.0000
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0 - 10	1.0000	0.8965	1.0000	1.0000	0.8880	0.8880
10 - 20	1.0000	0.8692	0.9647	0.9627	0.8508	0.8838
20 - 30	1.0000	0.8661	0.9407	0.9379	0.8468	0.9029
30 - 40	1.0000	0.8610	0.9401	0.9376	0.8260	0.8810
40 - 50	1.0000	0.8717	0.9272	0.9245	0.8305	0.8985
50 - 60	1.0000	0.8532	0.9123	0.9469	0.8086	0.8539
60 - 70	1.0000	0.8486	0.9126	0.9109	0.7863	0.8632
70 - 80	1.0000	0.8593	0.9026	0.9008	0.7807	0.8667
80 - 90	1.0000	0.8216	0.8979	0.8966	0.7432	0.8289
90 -100	1.0000	0.7668	0.8661	0.8644	0.7077	0.8187

Column (7) shows the implicit horizontal comparisons in method 1. Comparison of columns (3) and (7) shows that the urban-rural price changes obtained from the two methods are widely different. This picture can be brought out more clearly by presenting a 20 × 20 matrix of index numbers with each of the 10 rural and 10 urban size classes as base, furnishing the figures obtained by the two methods one below another in each cell. To save space, we present, in the table below, only the figures for rural and urban size classes 1, 5, 6 and 10, the latter four being respectively denoted by 11, 15, 16 and 20. The first line in each cell is based on Geary-Khamis approach (method 1) while the second is obtained by method 2.

Table : Price index numbers for comparison between size classes.

	1	5	6	10	11	15	16	20
1	1.0000	1.0817	1.0561	1.1569	1.1261	1.2041	1.2367	1.4130
	1.0000	1.0785	1.0962	1.1546	1.1154	1.1472	1.1721	1.3041
5		1.0000	0.9763	1.0695	1.0411	1.1132	1.1433	1.3063
		1.0000	1.0163	1.0705	1.0342	1.0637	1.0867	1.2092
6			1.0000	1.0954	1.0663	1.1402	1.1710	1.3380
			1.0000	1.0533	1.0176	1.0466	1.0693	1.1897
10				1.0000	0.9734	1.0408	1.0690	1.2214
				1.0000	0.9660	0.9936	1.0151	1.1295
11					1.0000	1.0692	1.0982	1.2548
					1.0000	1.0284	1.0507	1.1691
15						1.0000	1.0271	1.1735
						1.0000	1.0217	1.1368
16							1.0000	1.1426
							1.0000	1.1127
20								1.0000
								1.0000

From the table above it can be observed that there is significant difference between the two sets of index numbers. An interesting feature of the indices from method 2 is their movement with respect to extreme groups. The indices for groups 11,15,16,20 with the rest as their bases, are uniformly lower than that of their counter-parts from method 1. This suggests that method 2 seems to control the extra-ordinary behaviour of the Geary-Khamis method with respect to extreme groups to some extent. Entries in the intersection of column belonging to the 6th group and row belonging to 5th group clearly brings out the usefulness of method 2. First hand examination of data on prices and quantities suggests that I_{56} must be greater than unity. But however while according to method 1, I_{56} is 0.9763, which is undesirable, it is 1.0163 by method 2.

We shall now present the indices obtained by method 1, method 2 and those given by binary comparisons so that the relative performances can be judged on that basis. Method 2 results in indices which to be nearer to the binary comparisons, even though it is not true in all cases.

Group	Method 2	Geary-Khamis (binary)	Method 1
0 - 10	100.00	100.00	100.00
10 - 20	103.66	103.54	103.87
20 - 30	106.30	106.13	106.62
30 - 40	106.37	106.46	106.66
40 - 50	107.85	108.19	108.17
50 - 60	109.61	108.49	105.61
60 - 70	109.58	109.71	109.78
70 - 80	110.79	111.41	111.01
80 - 90	111.37	111.20	111.53
90 - 100	113.46	114.15	115.62

Two problems crop up in the use of method 2, which otherwise seems to yield better results than method 1 in general. The first problem is the emergence of multiple sets of consistent systems through method 2. Given R_{ij} $(j = 1,\ldots,t)$ for $i=1,\ldots,s$ by horizontal comparisons, these can be linked up by any of the vertical comparisons based on various size classes, each giving one set of consistent indices. The second problem is the absence of well defined groups. In our example we had fairly homogeneous groups based on well defined economic characteristics. But this is not always the situation in practice. In inter-country price comparison problem, there is usually no apriori classification of the countries into various groups. While we suggest no concrete method for grouping these countries into relatively homogeneous groups, an examination of the price and quantity vectors and application of a standard clustering method, usually based on the minimum distance criterion, may yield clusters on the basis of which method 2 may be applied. Application of this method may yield two broad groups of countries or several groups of countries according to stages of development. In such a case comparisons may be undertaken for the two groups separately and linking these to yield over all price comparisons may be done by comparing prices in the most advanced of the developing economies and the least developed in the well developed economies. This procedure in general yields better results than a straightforward application of any of the consistent methods.

Section 3.2 Non-availability of Price-quantity Breakdown

All the index number formulae, consistent or otherwise, presuppose the existence of price and quantity information with respect to all items. Mere expenditure data is not used by any of these index numbers. The problem occurs generally in the following way. Out of all items consumed, we may take major items of consumption and leave out relatively unimportant items. On the other hand there may be some items for which the concept of quantity cannot be well defined. This is particularly true in the case of services, which include a variety of them like education for which proper definition of quantity is by itself a research problem. This particular problem was first considered by Mukherjee (1969) and brought to the notice of the author. The following discussion appears in Prasada Rao (1970).

In such a case, the simplest way is to ignore those items, which is equivalent to the assumption that prices of these items have the same trend as the general price level based on the items for which the price and quantity decomposition is available. We obtain a more satisfactory solution to this problem. The traditional way of accounting for the expenditure of the item is to distribute this expenditure over the other items according to some criterion. For example, we may distribute this expenditure over other items according to their value shares. Similar exercises were undertaken by studies on consumers prices in India by G.S.Chatterjee and N. Bhattacharya (1969). We will explore the possibility of introducing similar ideas into the construction of consistent index numbers. For the sake of analysis, we restrict ourselves to Geary-Khamis index numbers which however holds for any consistent index number system.

Without loss of generality we may combine all items for which the data is missing and call it 'composite commodity'. Suppose we start with a grouping of the commodities, say K groups represented by the sets $A_1, A_2, \ldots, A_k$ such that

$\bigcup_{i=1}^{k} A_i = A$ and $A_i \bigcap A_j = \emptyset$ for $i \neq j$, where A is the set of all commodities. The expenditure on the 'composite commodity' can be split up into the expenditure related to various groups. In some cases, there could be expenditure which is not related to any of the commodity groups. For example, in our data on Rural India's consumption pattern, price and quantity components for the service items are absent and these items form a separate group by themselves and hence cannot be related to any other group. Empirical details are given later.

Distribution of the expenditure on 'composite commodity' to various commodity groups, has the implicit assumption that the exchange rate of currency relating to a particular group governs the share of expenditure attributed to that group. If we want to set up Geary-Khamis system of equations similar to (1.1.1) and (1.1.2) for our purpose, we need to define some new variables. Let R_j^k $(k = 1,\ldots,K\ ,\ j = 1,\ldots,M)$ be the exchange rate of currency in j-th group, calculated on the basis of k-th commodity group exclusively. So for the k-th commodity group, we have

$$R_j^k = \frac{\sum_{i \varepsilon A_k} P_i^k q_{ij}}{\sum_{i \varepsilon A_k} p_{ij} q_{ij}} \quad \text{for} \quad j = 1,\ldots,M$$

where P_i^k represents the average price of i-th commodity based on only R_j^k's and is given by

$$P_i^k = \frac{\sum_{j=1}^{M} R_j^k e_{ij}}{\sum_{j=1}^{M} q_{ij}} \quad \text{for all} \quad i \varepsilon A_k.$$

The above definitions of R_j^k and P_i^k for all j and $i \varepsilon A_k$ form a complete system of equations. Hence the necessary and sufficient conditions derived in chapter 1 should be satisfied by the data with respect to each group. This imposes an additional restriction. However this condition is satisfied in general, for standard classification of the commodities. If this system is solvable for all k, then our problem is one of arriving at over all purchasing power for each j. Easiest way of doing this is to get a weighted average of R_j^k's, for each j, and the weights are given by the value ratios. Hence

$$R_j = \sum_{k=1}^{K} R_j^k \frac{E_j^k}{E_j}$$

for all j and $E_j^k = \sum_{i \varepsilon A_k} p_{ij} q_{ij}$. We will try to justify this definition in a more objective fashion. Consider the definition of R_j. We have

$$R_j = \frac{\sum_{i=1}^{N} P_i q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}}$$

$$= \frac{\sum_{k=1}^{K} \sum_{i \varepsilon A_k} P_i q_{ij}}{E_j}$$

$$= \sum_{k=1}^{K} \frac{\sum_{i \varepsilon A_k} P_i q_{ij}}{\sum_{i \varepsilon A_k} p_{ij} q_{ij}} \cdot \frac{E_j^k}{E_j}$$

But for the P_i's in these equations, the value of R_j is same as one given above. This indicates that over all R_j differs from one kind of distribution of expenditure on the 'composite commodity' over commodity groups to the other.

Let us illustrate this by considering a numerical example, based on the data for rural India already described. Apart from the 56 items of consumption for which separate price and quantity information are available, we have also taken a residual group giving only expenditure data. The following table shows the expenditure on 56 items and the residual expenditure for which the price and quantity information is absent.

Table : Consumption expenditure by size classes of households, Rural India, 1963-64 in Rs. per capita per month.

size classes	expenditure on 56 items	expenditure on residual items	total expenditure
0 - 10	6.65	1.69	8.34
10 - 20	9.34	2.15	11.49
20 - 30	11.11	2.58	13.69
30 - 40	12.72	2.75	15.47
40 - 50	14.11	3.36	17.47
50 - 60	15.89	3.92	19.81
60 - 70	17.72	4.86	22.58
70 - 80	20.21	6.05	26.26
80 - 90	24.00	8.23	32.23
90 -100	35.56	21.25	55.81

To consider the residual item, we grouped the 56 items into three groups, (i) Food (ii) Fuel and light, and (iii) clothing. Then the residual expenditure is distributed over these commodity groups so that we may assume that the average price levels in a group governs the corresponding residual expenditure. Rest of the residual expenditure is supposed to correspond to the service items and there is no justification in distributing it over the three groups considered. So we assume that the price corresponding to this has the same trend as the overall level in these three groups taken together. The following table shows the distribution of actual and residual expenditure over the commodity groups.

Table

Rs. per capita per month

factile group	food		fuel & light		clothing		remaining residual (services)	Total
	Actual	residual allocated	actual	residual allocated	actual	residual allocated		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0 - 10	5.9014	1.1571	0.6294	0.1471	0.1240	0.0701	0.3109	8.34
10 - 20	8.3133	1.3843	0.7925	0.1573	0.2315	0.1139	0.4972	11.49
20 - 30	9.7736	1.5251	0.9873	0.1943	0.3470	0.1676	0.6951	13.69
30 - 40	11.3469	1.4039	0.9524	0.1661	0.4256	0.2464	0.9287	15.47
40 - 50	12.4386	1.6410	1.0725	0.2554	0.5955	0.3253	1.1417	17.47
50 - 60	13.9579	1.8842	1.1405	0.2442	0.7962	0.3967	1.3903	19.81
60 - 70	15.4496	2.1513	1.2103	0.3590	1.0574	0.5547	1.7952	22.58
70 - 80	17.5090	2.4357	1.3396	0.3436	1.3569	0.7811	2.4941	26.26
80 - 90	20.2704	2.8377	1.5809	0.3650	2.1437	1.2490	3.7833	32.23
90 -100	28.0068	4.7296	2.1220	0.5778	4.4351	2.7941	13.1446	55.81

Now we present the R_j^k's calculated for the three groups and the overall purchasing power R_j obtained by computing the weighted average, along with the R_j's considering only 56 items, taking the exchange rate of currency in the decile group 0-10 to be unity.

Table

Exchange rates for currency

fractile group	food	fuel and light	clothing	weighted average	based on 56 items
(1)	(2)	(3)	(4)	(5)	(6)
0 - 10	1.000000	1.000000	1.000000	1.000000	1.000000
10 - 20	0.970153	0.908822	0.991014	0.965509	0.964692
20 - 30	0.948358	0.868573	0.932449	0.940475	0.940648
30 - 40	0.945371	0.872896	0.956361	0.933310	0.940051
40 - 50	0.934769	0.848098	0.917623	0.926756	0.927221
50 - 60	0.919917	0.843338	0.974110	0.911747	0.912272
60 - 70	0.921276	0.840770	0.857001	0.910210	0.912611
70 - 80	0.912252	0.807593	0.857143	0.899908	0.902635
80 - 90	0.911300	0.826575	0.793610	0.891464	0.897935
90 -100	0.893130	0.763937	0.713271	0.854490	0.866122

With the help of the above tables and discussion we are in a position to present the price index numbers based on Geary-Khamis method using only 56 items, adapted method for residual item, and Laspeyres' and Paasche's indices for the purposes of comparison. We give the indices with the first decile group as base.

Table : Adapted Geary-Khamis, Geary-Khamis, Laspeyres and Paasche index numbers for size classes of Indian rural population, 1963-64

size classes	Geary-Khamis adapted	Geary-Khamis 56-items	Laspeyres	Paasche
(1)	(2)	(3)	(4)	(5)
0 - 10	100.00	100.00	100.00	100.00
10 - 20	103.57	103.66	103.61	103.44
20 - 30	106.33	106.31	106.27	105.88
30 - 40	107.14	106.38	106.59	106.15
40 - 50	107.90	107.85	108.33	107.77
50 - 60	109.68	109.62	110.25	107.31
60 - 70	109.86	109.57	109.69	109.62
70 - 80	111.12	110.78	111.41	110.80
80 - 90	112.18	111.37	110.90	111.55
90 -100	117.03	115.46	113.43	115.62

When use is made of the part of the expenditure information on the residual items, we do get an index number system which is different from the one based on only 56 items. The above table shows that the method is useful when the expenditure on the 'composite commodity' is high. In the groups 70-80, 80-90 and 90-100 there is considerable difference.

We shall end this section with an observation on the adapted method discussed. Instead of obtaining the overall exchange rates by taking arithmetic average of the exchange rates of the commodity, we could have followed the following procedure which has interesting applications in other branches like national accounts.

Start with the commodity group-wise exchange rates, R_j^k. At this stage, the problem in a nut shell is one of obtaining an overall index number, given the groupwise indices. This can be deemed to be similar to the problem of index number construction. The expenditures deflated by R_j^k would be comparable over groups and can be considered as quantities. Further the prices are given by the reciprocals of exchange rates. Hence the new set of price and quantity data, p_{ij}, q_{ij} ($i = 1, \ldots, k$ and $j = 1, \ldots, M$) are given by

$$p_{ij} = 1/R_j^i$$

$$q_{ij} = R_j^i . e_{ij} .$$

We can simply apply the Geary-Khamis method to this problem and obtain over all exchange rates and the corresponding index numbers. This method is superior to the earlier method on two grounds. Firstly, there is no arbitrariness involved in the method of obtaining over all R_j's. Secondly, this method results in a unique set of R_j's and hence unique price comparisons, whereas the method discussed earlier yields different sets of R_j's depending on the R_j^k's. For example instead of $R_1^1 = 1$ in the numerical example, if $R_2^1 = 1$, we get an entirely new set of price comparisons. However, each of these sets yield a consistent system of index numbers.

Situations similar to that of the previous one are not uncommon. An interesting problem appears in the National Income accounting. In a standard national accounting frame work, the current and constant price values of the national income are available. Further sectorwise classification of the national income is also available. Usually in this case, the current price figures andthe sectoral deflators satisfy the consistency conditions, we can derive the implicit exchange rates. Suppose we have national income figures for time periods $t = 1, \ldots, T$ and sectors $j = 1, \ldots, M$. Hence we have sectoral exchange rates R_t^j. The idea is to get an overall deflator for each time period t. In usual practice, a weighted average of R_t^j, similar to one used by us, is obtained. However more meaningful deflators can be obtained by treating these variables in the same manner as above and get consistent national income deflators.

Section 3.3 : Problem of Non-identical Lists

Most intricate of all the problems that crop up in a price comparisons exercise is the presence of non-identical commodity lists for two or more groups. This may spring up in a number of ways. In the simplest case, there are some items common to all groups and a few others specific to a subset of the countries. In this case any of the standard methods can be applied to yield fairly good comparisons when the common items form a large part of the budget. However in many cases though the commodities in various lists of items are same on paper they differ in quality. This puts in doubt the validity of the straightforward application of any method of index numbers. In this case, the magnitude of the price index does not measure the actual price change. Both the price differences and the quality differences contribute to the value of the index. In many situations, it happens that people pay higher price due to the superior quality of the product and this has to be accounted for in the computation of price index numbers. But no traditional method takes this aspect into consideration.

This problem is more acute in the case of comparisons among dissimilar countries. Suppose that we are comparing the prices in developed country like United States and a developing country like India. The problem of quality variation occurs in respect of most of the items. In this connection, it may be of high relevance to discuss the procedure followed in the International Price Comparisons Project now in progress under the auspices of the United Nations and a preliminary report of Unit Nations(1969) is available to us. The procedure followed here is to obtain expenditure vectors based on an identical list of items (not necessarily of the same quality)for a pair of countries A and a fair amount of details, and then to choose some items with well defined specifications for which prices are available in both the countries to obtain the price relative, which governs the total expenditure of the category. This material is sufficient for comparing countries A and B. When A and C are compared, the list of expenditure items may however differ. In this way all countries are compared with A, providing a system of index numbers satisfying circularity test. The comparison between B and C is obtained through comparisons (A,C) and (A,B). However a different system emerges if the procedure is repeated by replacing the common country A by any other country. One difficulty with the above method is the assumption that prices of some items within a common expenditure category meaningfully represent the prices of other items entering the category, in the chosen pair of countries. But the items for which specification prices are obtained may not contribute a large share of expenditure with in a category. The validity of this procedure in practice has to be tested. The investigation involves collection of fresh data and we are not aware of the final results in this matter.

Some interesting statistical aspects of such a problem are studied by Summers, R. (1971), a preliminary draft of which is available to us. A stochastic formulation of the problem is used to make maximum use of the available data. This method is designed to use the price information on the items for which price data are available in some countries and missing in other countries.

However this does not consider the explicit problem of non-identical lists and where the items in the lists are of various qualities. The only method known so far which considers this problem explicitly is the Regression Method used in Griliches, Zvi (1961) and Kravis, I.B. and Lipsey, R.E.(1969), designed to measure the price changes in the automobiles. The basic assumption here is that the price paid for a unit quantity of a particular commodity is due to some measurable characteristics possessed by, the commodity. Thus, the price of beer depends more on the quantity of alcohol contained in it than on anything else. The price of a motor car depends much on the length, weight and horse power. Similarly one could think of such measurable characteristics of various other commodities governing prices of unit quantities. Such hypotheses can be tested by regression analysis where price is the dependent variable and various characteristics independent variables. When the analysis yields two regression equations in the two countries with high values of multiple correlations, these equations can be used in the following manner. For example, the dollar prices of Indian and American cars of average specifications obtained from American equation gives an index of quality and the actual price relative adjusted by the quality index gives a quality adjusted price relative between dollar and rupee for motor cars. This procedure thus enables us to obtain price ratios for cars even when American cars are rarely sold in India and Indian cars are not available in U.S.A. Similarly, the Indian equation will yield another price relation between dollar and rupee. This procedure can be extensively used for other items for inter-country comparison as well as the inter-group comparisons of the type considered in our empirical studies. But the basic problem here is of identifying measurable characteristics of this type for different countries which give reasonably good regression equations. How far this procedure would succeed can only be judged when many more attempts have been made. This opens up a vast field of empirical analysis and calls for enormous amount of new data. It is also not obvious that the regression approach should always lead to reasonable price relatives. Data may not give a good fit and the multiple correlation coefficients may turn out to be small. But the procedure suggests that it is fruitful to go beyond the quantities measured in the customary units and try to find some basic characteristics which make the commodity important and reflect the relative qualities of two items which have the same label. However, such characteristics need not be the only factors that influence the prices. We try to exploit this idea and propose a new method to take this into consideration, which we call 'programming approach' since it makes use of Linear Programming technique.

Programming Approach : Since one aim of the price comparison between countries is to obtain comparable average levels of living by deflating the aggregate expenditures by price indices obtained, we are permitted to seek such characteristics of commodities which directly contribute to the level of living, irrespective of the fact whether they have a close influence on the price or not. In other words, from the available quantity vectors, which may have some common and some non-overlapping items for a number of countries, we try to proceed to a basic human requirement vector having identical components for all countries. In food, for example, the human requirements can be taken to be represented by calorific contents and basic nutrients. While not much attention has been paid to the subject, it is conceivable that for other items of expenditure also, it would be possible to identify variables representing, in some sense, the basic human requirements. Once these variables are obtained, the problem is to find the prices implicit in the actual price and quantity vectors, so that inter-country comparison of purchasing power of money could be based on the prices and quantities of these new variables.

Hence we associate a basic human requirement vector b_j with q_j for country j. The consumers spend $p_j' q_j$ to obtain b_j. This implies that the individual pays a price for each of the elements of b_j indirectly. If we are able to obtain these prices (say y_j), then we have the vectors y_j, $b_j (j = 1, 2, \ldots, M)$ for the M countries. From this we can obtain a consistent system of index numbers using one of the methods already discussed. This problem can be solved by using linear programming in a way which avoids the difficulties associated with its application in index number problems.

Usually, linear programming method is applied to obtain the minimum expenditure necessary to obtain the base country requirement vector. The required index is defined as the ratio of this expenditure to the expenditure in the base country. However, in this case, the index for a country with itself as base may be different from unity (i.e., $I_{jj} \neq 1$). This difficulty has been resolved in a paper by Balintfy, J.L., Nester, S. and Wasserman, W. (1970). Here optimal expenditures corresponding to different countries, necessary for obtaining a fixed requirement vector, have been computed. The requirement vector is decided in advance on other grounds. Then the index numbers are defined by respective ratios of optimal expenditures. But the method is not fully satisfactory because of the arbitrary nature of the requirement vector and does not use the available quantity data.

These difficulties can be overcome in the approach discussed below. Given p_j and q_j (order $n \times 1$) for the j-th group, we obtain the corresponding nutrient vector b_j (order $m \times 1$), $b_j = A_j q_j$, where A_j is the transformation matrix for the j-th country showing the contribution of one unit of each item to the basic requirements. Suffix j to the matrix A shows that the same item in different countries may cater to different level of requirements. This accounts for quality change in the items.

The implicit assumption is that superior quality products are considered superior because of their contribution to satisfaction. Hence $p_j' q_j$ can be viewed as the amount spent to acquire the vector b_j. If the individual is taken to be a cost minimizer, he should have spent the amount equal to

$$\begin{aligned} &\text{minimum} \quad p_j' q \\ &\text{subject to} \quad A_j q \geqq b_j \\ &\qquad\qquad q \geqq 0 \end{aligned} \qquad (3.3.1)$$

This is true for all j. The above problem has a non-empty set of feasible solutions, since q_j is feasible for (3.2.1). Further $p_j' q_j$ provides an upper bound for the problem. We shall assume that the problem is solvable for all j which is essential. We next consider the dual of the primal problem. It can be written as

$$\begin{aligned} &\text{maximize} \quad y_j' b_j \\ &\text{subject to} \quad A_j' y_j \leqq p_j \\ &\qquad\qquad y_j \geqq 0. \end{aligned} \qquad (3.3.2)$$

This problem is solvable by the virtue of the fundamental duality theorem. The solution y_j^* has the conventional interpretation that they represent the ''shadow prices'' or ''equilibrium'' prices of various elements of b_j. This enables us to find a price for each of the elements of b_j and further we have a legitimate claim for calling y_j^* as price vector associated with b_j. Given these we can apply any procedure for constructing the price index numbers using y_j^* and b_j instead of p_j and q_j. Two observations about the index numbers from our approach may be noted, when A_j is same for all j.

(i) If $p_2 > p_1$, then the Paasche index based on the transformed variables is greater than unity. Let y_2^* and y_1^* represent the shadow prices of b_2 and b_1 respectively. Then by definition of Paasche index $I_{12} = y_2^{*\prime} b_2 / y_1^{*\prime} b_2$. Observe that y_1^* is a feasible solution for the linear programming problem for country 2, for $A' y_1^* \leq p_1 \leq p_2$. Further $p_2 > p_1$ implies that $p_2 \geqq (1+\lambda)\, p_1$, for some $\lambda > 0$. Hence $y_1^*(1+\lambda)$ is feasible for the problem of country (2). So, $y_2^{*\prime} b_2 \geqq y_1^{*\prime} (1+\lambda)\, b_2 > y_1^{*\prime} b_2$. Hence $I_{12} > 1$.

(ii) If $p_2 > p_1$ and b_2 is of the for kb_1, for $k > 0$, then the Laspeyres index $y^{*\prime} b_1 / y_1^{*\prime} b_1$ is greater than unity. Proof is trivial. This result is weaker than the one in the previous observation.

Comparing this method with those derived by Balintfy and others (1970), we find certain advantages. We are able to use the dual of linear programme set-up to obtain economically meaningful ''prices'' of the requirements that are implicit in the price and quantity data. Thus we make full utilization of the available data and remove the anamolies in the previous attempts. On the other hand, we face the problem of zero prices for some of the variables which makes it difficult to use standard formulae for price comparisons. To tackle this problem, we change the dual slightly and replace the constraints $y_j \geqq 0$ by $y_j \geqq \varepsilon$ where ε is a positive vector with entries very near zero. Since b_j is positive this does not offer any additional difficulties. Assuming the solvability of this problem, we can apply the standard methods to compute the price indices.

Formidable difficulty still attahces to the procedure, particularly when it has to be applied to aggregate final expenditure considerable researches supplemented by extensive surveys are needed to identify the elements of b_j and the computation of matrices A_j. Obviously, we need a lot of information other than price and quantity data to solve the problem. But on the positive side, apart from the advantages already pointed out, the method helps us to work with an identical list of basic requirements for all groups, and their associated prices, and there is little arbitrariness in subsequent stages. It might so happen that two basic requirement vectors do not have common positive elements, which is a very hypothetical case. In this situation there does not seem to be any meaning attached to the price comparisons and the project may be abandoned in such circumstances. Contrasted with the dependence on the information on the budget pattern and specification prices in respect of a small part of the total expenditure, in the case of comparison of two dissimilar countries, here we depend on the entire price and quantity information, a lot of subsidiary data and some clearly stated assumptions.

Numerical Illustration :

Confronted with difficulties about data, we illustrate the method by a simple example drawn from the statistical information already considered. The comparison relates to food items only and covers only four groups, rural 80-90 and 90-100 and urban 80-90 and 90-100. A single transformation matrix based on a paper by Chatterjee, G.S., Sarkar, D. and Paul, G. (1965) has been used for all the groups and the nutrients distinguished are protein, fat, carbohydrate, calcium and iron. Also, we have considered only seven major food items, rice, wheat ata, milk, ghee, potato, meat and sugar in this example. The price and quantity vectors relating to these items are given in Appendix A. The following table shows the transformation matrix.

Table : Transformation Matrix

Item	Protein (gms.)	Fat (gms.)	Carbohydrate (gms.)	Calcium (mg.)	Iron (mg.)
(1)	(2)	(3)	(4)	(5)	(6)
Rice	65.8	3.3	724.0	98.7	9.9
Wheat ata	111.9	16.5	674.7	362.0	65.8
Milk	32.8	32.9	46.1	1118.9	1.0
Ghee	0.0	934.6	46.1	0.0	0.0
Potato	16.8	0.0	181.8	56.0	5.6
Meat	120.4	132.1	0.0	74.5	13.7
Sugar	0.0	0.0	921.5	0.0	0.0

A typical element a_{ij} shows the amount of j-th nutrient in standard units contained in one seer (=.933 kg.) of i-th food item. On the basis of information given above, and using the solution of the dual problem, we obtain the price and quantity vectors of the nutrients. It is possible to build up price index numbers on the basis of these transformed variables. In the table below, we present the Laspeyres, Paasche and Fisher's ideal index numbers based on the transformed variables as well as on the original variables side by side. Here, rural groups 80-90 and 90-100 are represented by (1) and (2) respectively while the corresponding urban groups by (3) and (4) respectively.

Table : Price index numbers based on original and transformed variables

groups	original variables groups (1)	(2)	(3)	(4)	transformed variables groups (1)	(2)	(3)	(4)
				Laspeyres				
(1)	1.0000	1.0239	1.2263	1.2526	1.0000	1.0106	1.1609	1.1668
(2)		1.0000	1.1923	1.2230		1.0000	1.1611	1.1740
(3)			1.0000	1.0336			1.0000	1.0155
(4)				1.0000				1.0000
				Paasche				
(1)	1.0000	1.0241	1.2096	1.2538	1.0000	1.0126	1.1893	1.2208
(2)		1.0000	1.1821	1.2244		1.0000	1.1734	1.2018
(3)			1.0000	1.0365			1.0000	1.0197
(4)				1.0000				1.0000
				Fisher's ideal				
(1)	1.000	1.0240	1.2179	1.2532	1.0000	1.0116	1.1750	1.1935
(2)		1.0000	1.1850	1.2237		1.0000	1.1672	1.1878
(3)			1.0000	1.0350			1.0000	1.0176
(4)				1.0000				1.0000

The important thing to observe about the table is that while calculations on original variables produce index numbers showing a somewhat higher inter-group price variation, the pattern of inter-group variation depicted by the two systems is similar. This suggests that the linear programming approach used here can give meaningful comparison of inter-group price variations.

While the example considered above gives encouraging evidence about the applicability of the method, the limited nature of the example should be born in mind. In the absence of different transformation matrices corresponding to each group the above illustration does not take into account the quality differences, which actually is the main purpose of the above method. Again, the lists of items are not different though the consumption vectors show considerable variation among the groups considered. Further, the method may lead to better results in the presence of constraints of different nature, depicting the food habits in the groups. However the limited availability of data prevents us from undertaking any such exercise.

We end this section with a note on the applicability of this method to general price index numbers. While the approach may work smoothly in the case of food price index number, considerable difficulty attaches to other items of expenditure. For example, it may be difficult to identify some requirement vector in respect to other items like clothing. However, considerable research supported by efforts to collect new information in these lines may yield better price comparisons between dissimilar countries or groups where the problem of non-identical lists crops up.

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APPENDIX - A

Basic Data : Rural India

Size class		Rice	Muri	Wheat Ata	Jowar Ata	Bajra Ata	Maize Ata	Ragi Ata	Gram Ata	Milk
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0 - 10	P	.5370	.8806	.4189	.3130	.3706	.3135	.3342	.3674	.4739
	Q	4.3411	.0112	.9014	2.0084	.5084	1.0894	.7282	.2771	.2548
10 - 20	P	.5608	.9230	.4256	.3228	.3584	.3217	.3306	.3778	.4750
	Q	5.7946	.0266	1.4888	2.6305	.5587	1.0794	.8490	.3443	.5302
20- 30	P	.5796	1.0383	.4310	.3361	.3470	.3273	.3252	.4005	.4983
	Q	6.4753	.0514	1.9614	2.1793	.7736	1.1639	.9408	.3314	.7479
30 - 40	P	.5772	.8159	.4353	.3273	.3788	.3363	.3282	.3945	.4938
	Q	7.3784	.0605	2.2050	2.5516	1.0949	1.0974	.8175	.3756	1.0306
40 - 50	P	.5850	.9495	.4348	.3422	.9217	.3418	.3189	.3898	.4974
	Q	8.2323	.0935	2.5949	2.1037	.3876	.8949	.7947	.4544	1.2380
50 - 60	P	.5251	.9721	.4447	.3283	1.1029	.3261	.3361	.4067	.5085
	Q	8.8006	.0987	2.8095	2.2170	.3936	1.0578	.7411	.4165	1.5075
60 - 70	P	.5899	.9380	.4370	.3363	.3964	.3404	.3416	.3869	.5045
	Q	9.6528	.1727	3.0776	1.9963	1.1881	.9914	.7344	.5344	1.9640
70 - 80	P	.5986	.8293	.4370	.3405	.3972	.3396	.3406	.3965	.5033
	Q	10.1678	.2241	3.6950	1.7544	1.4846	.9116	.5846	.6088	2.5281
80 - 90	P	.6018	.9834	.4376	.3334	.3879	.3363	.3327	.3976	.5127
	Q	10.9690	.3212	4.4763	1.8766	1.2768	.6780	.5898	.7068	3.2074
90 -100	P	.6176	1.0270	.4455	.3224	.4028	.3281	.3173	.4092	.5249
	Q	12.0108	.2865	6.7904	1.7053	1.7246	.8533	.6491	.9606	5.5238

P represents the price in rupees per unit quantity and Q represents the total quantity.

Basic Data : Rural India (Continued)

Size class		Ghee	Arhar, Tur	Gram	Moong	Masur	Urd	Vanaspati	Mustard oil	Coco-anut oil
		(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0- 10	P	5.6890	.7017	.5222	.6260	.5756	.6350	2.9881	2.6037	2.6389
	Q	.0053	.2474	.0614	.0435	.0494	.0781	.0023	.0382	.0042
10 - 20	P	5.7695	.7226	.5439	.6405	.6097	.6553	3.0279	2.5981	2.7644
	Q	.0131	.3702	.0923	.0758	.0666	.1119	.0043	.0536	.0044
20 - 30	P	5.7991	.7464	.5606	.6630	.6481	.6815	3.1552	2.5686	2.6993
	Q	.0248	.4155	.1012	.0925	.0841	.1172	.0065	.0699	.0065
30 - 40	P	6.0277	.7345	.5726	.6610	.6341	.6924	2.9751	2.6138	3.0243
	Q	.0327	.5032	.1328	.1204	.1010	.1486	.0066	.0793	.0057
40 - 50	P	6.0875	.7520	.5669	.6666	.6065	.7099	2.9628	2.5904	2.9915
	Q	.0432	.4723	.1328	.1452	.1387	.1592	.011[illegible]	.0916	.0062
50 - 60	P	6.1145	.7545	.5771	.6863	.6711	.7150	3.0316	2.6104	3.0309
	Q	.0516	.5549	.1622	.1521	.1498	.1631	.0139	.1063	.0069
60 - 70	P	6.1421	.7702	.5885	.6856	.6769	.7156	3.0150	2.5837	2.8567
	Q	.0714	.5968	.1557	.1570	.1640	.1962	.0168	.1201	.0097
70 - 80	P	6.2377	.7577	.5742	.7005	.7007	.7212	3.0411	2.6022	3.0861
	Q	.1055	.6864	.1645	.2234	.1718	.2259	.0201	.1381	.0037
80 - 90	P	6.1455	.7630	.5841	.7009	.6878	.7409	3.13[illegible]6	2.6021	2.8389
	Q	.1407	.8269	.2049	.2689	.2092	.2807	.0316	.1690	.0146
90 -100	P	6.3645	.7598	.5685	.6946	.6902	.7764	2.9699	2.6016	2.7102
	Q	.2959	1.0694	.3413	.3501	.2855	.3768	.0692	.2119	.0220

Basic Data : Rural India (Continued)

Size class		Gingelly oil	Groundnut oil	Potato	Onion	Tomato	Brinjal	Banana	Cocoanut	Ground nut
		(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
0 - 10	P	2.2204	1.9645	.2726	.2325	.2441	.2339	.0387	.2670	.4311
	Q	.0078	.0374	.1987	.1879	.0228	.1249	.0582	.0866	.0067
10 - 20	P	2.2256	1.9679	.2849	.2482	.2414	.2526	.0312	.2794	.5459
	Q	.0105	.0549	.3060	.2805	.0365	.1838	.1722	.1139	.0113
20 - 30	P	2.3259	1.9570	.2977	.2558	.2720	.2450	.0320	.2785	.4789
	Q	.0137	.0690	.3756	.3000	.0430	.2378	.2787	.1868	.0229
30 - 40	P	2.2724	1.9334	.3018	.2535	.2724	.2558	.0290	.2765	.3709
	Q	.0182	.0847	.4506	.3553	.0469	.2649	.3181	.1804	.0312
40 - 50	P	2.3526	1.9625	.3139	.2463	.2656	.2475	.0331	.2771	.4914
	Q	.0224	.0891	.5221	.4329	.0635	.3020	.4609	.2526	.0155
50 - 60	P	2.3639	1.9478	.3256	.2638	.2866	.2709	.0333	.2751	.5750
	Q	.0197	.0971	.5862	.4342	.0804	.3142	.6545	.2395	.0273
60 - 70	P	2.3779	1.9366	.3321	.2725	.3040	.2577	.0348	.2779	.5472
	Q	.0202	.1094	.6983	.4316	.0913	.3829	.8983	.2892	.0395
70 - 80	P	2.3923	1.9611	.3305	.2650	.3080	.2696	.0320	.2923	.4925
	Q	.0252	.1155	.7906	.4979	.1088	.4418	1.1857	.3234	.0309
80 - 90	P	2.3196	1.9096	.3368	.2633	.3027	.2705	.0311	.2742	.4813
	Q	.0359	.1294	.9902	.5844	.1332	.4833	1.7655	.4846	.0526
90 -100	P	2.3501	1.8900	.3400	.2727	.3501	.2683	.0378	.2699	.5581
	Q	.0622	.1790	1.4058	.8009	.1934	.6127	3.3466	.6600	.0617

Basic Data : Rural India (Continued)

Size class		Goat meat	Mutton	Egg	Fish (fresh)	Fish (dry)	Sugar	Gur (cane)	Sea-salt	Turmeric
		(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
0 - 10	P	2.2324	2.3378	.1120	.8155	1.1005	1.1974	.7101	.2250	.0230
	Q	.0189	.0042	.0221	.0700	.0170	.0276	.1460	.3442	1.2281
10 - 20	P	2.3036	2.4228	.1169	.9367	1.2250	1.2016	.7525	.1131	.0233
	Q	.0276	.0072	.0555	.0834	.0219	.0601	.2354	.3943	1.6700
20 - 30	P	2.3252	2.2773	.1238	.9395	1.2100	1.2243	.7307	.1156	.0232
	Q	.0374	.0120	.0762	.1138	.0205	.0834	.2909	.4100	1.9946
30 - 40	P	2.2478	2.3661	.1162	1.0156	1.2520	1.2168	.7912	.1143	.0234
	Q	.0464	.0143	.1220	.1248	.0241	.1029	.3375	.4358	2.2842
40 - 50	P	2.3885	2.3418	.1195	1.1335	1.4515	1.2401	.7890	.1205	.0241
	Q	.0555	.0143	.1583	.1472	.0205	.1408	.4066	.4505	2.4717
50 - 60	P	2.4371	2.4814	.1212	1.1127	1.3216	1.2498	.7913	.1196	.0243
	Q	.0746	.0156	.1898	.1877	.0297	.1528	.5018	.4802	2.7007
60 - 70	P	2.4686	2.6244	.1269	1.2291	1.3232	1.2534	.7827	.1200	.0349
	Q	.0840	.0145	.2196	.1928	.0350	.2020	.5490	.5009	3.4082
70 - 80	P	2.4867	2.2053	.1255	1.3037	1.6527	1.2574	.7980	.1233	.0234
	Q	.0901	.0249	.2739	.2120	.0545	.2548	.7140	.5000	3.6242
80 - 90	P	2.4881	2.6199	.1253	1.3746	1.3463	1.2377	.8019	.1210	.0239
	Q	.1427	.0294	.4199	.2775	.0338	.3462	.8291	.5705	3.9298
90 -100	P	2.4642	2.7238	.1338	1.4493	1.4100	1.2745	.7810	.1212	.0251
	Q	.2022	.0344	.8492	.3357	.0370	.6833	1.3297	.6694	3.3550

Basic Data : Rural India (Continued)

Size class		Pepper, dry chillies	Green chill-ies	Tamarind	Tea(cups)	Tea(leaf)	Coffee (powder)	Pan(leaf)	Pan (finished)	Supari	Biri
		(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)
0- 10	P	.0325	.0087	.0090	.0757	2.6061	2.5302	.0030	.0325	.0582	.0061
	Q	5.3321	2.2521	3.1452	.3816	.0094	.0011	6.1066	.1259	.4777	11.8120
10- 20	P	.0325	.0083	.0089	.0752	2.8492	2.6117	.0030	.0314	.0567	.0060
	Q	7.2431	2.7654	4.1317	.6111	.0164	.0017	8.3966	.1571	.7428	16.3966
20- 30	P	.0321	.0082	.0090	.0748	2.9341	2.6045	.0030	.0300	.0592	.0062
	Q	8.0699	3.0841	4.3953	.7875	.0213	.0036	10.1021	.1971	.8729	20.4336
30- 40	P	.0330	.0086	.0099	.0787	3.1435	2.5809	.0031	.0353	.0550	.0062
	Q	8.4338	3.6160	4.6040	.8162	.0252	.0042	11.1088	.1456	1.0156	23.0655
40- 50	P	.0331	.0086	.0098	.0770	2.9397	2.5553	.0033	.0396	.0572	.0062
	Q	8.8061	3.8536	4.6756	.9211	.0338	.0063	12.2237	.2661	1.3047	28.0363
50- 60	P	.0338	.0084	.0098	.0781	2.9789	2.5607	.0032	.0361	.0806	.0062
	Q	9.6161	4.5412	5.0208	1.2467	.0415	.0063	15.2568	.4086	1.8554	29.4893
60 -70	P	.0334	.0088	.0099	.0786	3.0279	2.5086	.0034	.0334	.0566	.0063
	Q	9.8324	4.5782	5.2006	1.2559	.0450	.0098	16.1796	.4625	1.9373	34.0533
70- 80	P	.0343	.0090	.0099	.0808	3.0604	2.4582	.0037	.0335	.0568	.0075
	Q	10.4872	5.0279	1.3774	1.5896	.0560	.0111	18.2123	.4855	2.7234	37.2185
80- 90	P	.0332	.0090	.0106	.0834	3.1730	2.6877	.0039	.0335	.0552	.0064
	Q	11.9841	5.8869	5.7407	2.1215	.0695	.0178	22.2623	.7828	2.8366	44.8001
90-100	P	.0339	.0092	.0102	.0901	3.2293	2.8626	.0042	.0351	.0686	.0064
	Q	15.0713	7.0436	7.9768	2.8671	.1103	.0310	31.0298	2.7708	4.0769	56.8591

Basic Data : Rural India (Continued)

Size class		Cigarette	Leaf tobacco	Hookah tobacco	Coke coal	Firewood	Kerosene	Matches	Dhuti	Sari	Clothes for shirting
		(46)	(47)	(48)	(50)	(51)	(52)	(53)	(54)	(55)	(56)
0 - 10	P	.0227	3.2144	.8136	.0403	.0280	.6000	.0012	1.0426	1.0932	1.2405
	Q	.0304	.0270	.0296	.1297	17.4124	.1954	15.2013	.0390	.0419	.0302
10 - 20	P	.0200	3.2018	1.1178	.0546	.0315	.5862	.0012	1.0228	1.0818	1.3119
	Q	.1680	.0345	.0313	.2082	19.1070	.2557	22.3453	.0719	.0732	.0605
20 - 30	P	.0201	3.3945	.9722	.0390	.0334	.5927	.0012	1.0779	1.1820	1.3640
	Q	.2657	.0322	.0417	.2379	21.6226	.3702	27.9114	.0902	.0911	.1041
30 - 40	P	.0203	3.2113	1.1159	.0362	.0334	.5887	.0012	1.0283	1.1718	1.3323
	Q	.2953	.0364	.0448	.4272	21.3687	.3123	30.0295	.1116	.1116	.1352
40 - 50	P	.0176	3.1287	1.1297	.0404	.0346	.5885	.0012	1.0972	1.2326	1.3507
	Q	.4907	.0349	.0502	.4103	23.4104	.3414	34.5003	.1576	.1446	.1808
50 - 60	P	.0204	3.1635	1.0992	.0452	.0348	.5869	.0012	1.1128	1.2551	1.4925
	Q	.5062	.0383	.0521	.4587	24.1964	.3874	38.6849	.1919	.1964	.2252
60 - 70	P	.0204	3.0293	1.1429	.0484	.0348	.5902	.0012	1.1352	1.3263	1.4813
	Q	.5864	.0458	.0573	.4892	25.5646	.4083	41.2268	.2240	.2507	.3175
70 - 80	P	.0208	3.2196	1.3788	.0498	.0365	.5965	.0013	1.1527	1.2733	1.5033
	Q	1.3044	.0483	.0669	.5926	26.5753	.4818	49.2133	.2842	.2948	.4349
80 - 90	P	.0221	3.0678	1.2927	.0433	.0360	.5773	.0012	1.2393	1.4340	1.5878
	Q	1.9505	.0516	.0737	.8027	31.6839	.5693	59.1258	.4509	.4241	.6152
90 -100	P	.0241	2.9940	1.4260	.0521	.0400	.5827	.0013	1.3100	1.6783	1.7627
	Q	4.4067	.0616	.0978	.9733	37.4643	.8024	79.0382	.8211	.7704	1.1725

APPENDIX A

Basic Data : Urban India

Size class		Rice	Muri	Wheat Ata	Jowar Ata	Bajra Ata	Maise Ata	Ragi Ata	Gram Ata	Milk
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0 - 10	P	.5910	1.3270	.4430	.3444	.4011	.3432	.3006	.4031	.5679
	Q	3.2366	.0095	2.9165	2.7180	.7586	.3879	.3384	.2980	.7919
10 - 20	P	.6191	.9813	.4516	.3460	.4186	.3798	.3909	.4368	.5882
	Q	5.4860	.0291	3.3441	1.7757	.4095	.2163	.1906	.2619	1.1382
20 - 30	P	.6294	.9191	.4610	.3687	.4236	.3467	.3273	.4263	.6068
	Q	5.4375	.0288	3.7762	1.6614	.5544	.3620	.3025	.1931	1.4772
30 - 40	P	.6421	1.1088	.4588	.4104	.4106	.4016	.3529	.4235	.5980
	Q	5.7123	.0088	4.5310	1.0616	.8102	.0724	.3633	.3017	2.0957
40 - 50	P	.6296	1.0150	.4733	.4049	.4319	.3836	.2589	.4752	.5930
	Q	6.4288	.0449	4.3794	1.4701	.5990	.1439	2811	.1335	1.9525
50 - 60	P	.6609	1.2733	.4730	.3563	.4496	.3434	.3323	.3812	.6518
	Q	5.8746	.0402	5.1355	1.2829	.5426	.0229	.2172	.2750	2.5354
60 - 70	P	.6684	1.0499	.4800	.3809	.4030	.4398	.3832	.4496	.7091
	Q	7.2838	.0761	4.8795	.7525	.2001	.0422	.2371	.3677	3.1118
70 - 80	P	.6816	1.1639	.4788	.3917	.4503	.3673	.3764	.4427	.6629
	Q	7.3203	.0531	5.6469	.5812	.5018	.0853	.1073	.2568	3.6502
80 - 90	P	.7449	1.2887	.5056	.4280	.5226	.3979	.3469	.4755	.7226
	Q	6.7060	.0518	5.5470	.4133	.2336	.0717	.1254	.1506	4.9414
90 -100	P	.7495	1.2200	.5110	.3695	.4987	.4227	.4142	.4327	.7729
	Q	6.8922	.0596	5.1152	.3226	.3008	.0322	.0551	.1811	7.0146

P represents the price in rupees per unit quantity and Q represents the total quantity.

Basic Data : Urban India (Continued)

Size class		Ghee	Arhar, Tur	Gram	Moong	Masur	Urd	Vanaspati	Mustard oil	Coco-anut oil
		(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0 - 10	P	6.0860	.7891	.5672	.6840	.6871	.7741	2.9992	2.4131	3.0152
	Q	.0192	.3596	.0684	.0889	.0477	.0752	.0207	.0460	.
10 - 20	P	5.9987	.7854	.6046	.6729	.6895	.8532	2.9898	2.3502	2.8072
	Q	.0267	.4548	.1093	.1239	.0737	.1227	.0429	.0686	.0060
20 - 30	P	6.1950	.8030	.6240	.7226	.7043	.8001	2.9618	2.4198	2.6935
	Q	.0460	.4709	.1131	.1421	.0879	.1286	.0446	.0653	.0068
30 - 40	P	6.0798	.8496	.6543	.7301	.7334	.7992	2.9980	2.4468	2.6192
	Q	.0611	.5438	.1686	.1462	.0999	.1350	.0905	.0690	.0044
40 - 50	P	5.8708	.8193	.6334	.7216	.7554	.8241	3.0114	2.4480	3.2672
	Q	.0853	.5144	.1869	.1885	.1093	.1160	.0715	.0816	.0101
50 - 60	P	6.4687	.8454	.5905	.7125	.7290	.8546	2.9721	2.4628	2.2301
	Q	.0877	.5714	.1704	.2177	.1009	.1262	.1153	.1041	.0075
60 - 70	P	6.6741	.8307	.6048	.7263	.7271	.8527	3.0355	2.4588	2.7725
	Q	.1174	.6407	.1722	.2108	.0974	.1588	.1040	.1240	.0094
70 - 80	P	6.6628	.8403	.6060	.7551	.7759	.7525	3.0409	2.4440	2.9192
	Q	.1580	.7527	.2010	.2848	.1306	.1773	.1356	.1709	.0107
80 - 90	P	6.6958	.8559	.6080	.7671	.7587	.8850	3.0608	2.4409	2.7430
	Q	.2340	.6969	.2452	.3404	.1406	.2030	.1689	.1930	.0277
90 -100	P	6.7578	.8759	.6251	.7689	.7744	.8738	2.9019	2.3935	2.8386
	Q	.4189	.7630	.2564	.3161	.1532	.2568	.2584	.2345	.0495

Basic Data : Urban India (Continued)

Size class		Gingelly oil	Groundnut oil	Potato	Onion	Tomato	Brinjal	Banana	Cocoanut	Groundnut
		(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
0 - 10	P	2.0502	1.8515	.3183	.2455	.2540	.2631	.0380	.2839	.6031
	Q	.0092	.0900	.3236	.2854	.0591	.1882	.0878	.1343	.0091
10 - 20	P	2.2890	1.8459	.3814	.2796	.3687	.3348	.0361	.2814	.8350
	Q	.0222	.1202	.5218	.3748	.0854	.2446	.4609	.2083	.0053
20 - 30	P	2.0237	1.8367	.3915	.2567	.3968	.3202	.0372	.2369	.8912
	Q	.0368	.1476	.5112	.4615	.1044	.2587	.7987	.1945	.0070
30 - 40	P	2.2207	1.8790	.3917	.2764	.3903	.2898	.0409	.2714	.7956
	Q	.0321	.1604	.6186	.4288	.1538	.3412	.7788	.1881	.0120
40 - 50	P	2.1538	1.8485	.4227	.2714	.4007	.3092	.0374	.2676	.6128
	Q	.0304	.2413	.5705	.4428	.1327	.3921	1.1058	.3483	.0137
50 - 60	P	2.3006	1.9039	.4049	.2704	.3747	.3275	.0414	.2935	.8547
	Q	.0372	.2278	.7256	.5775	.1863	.3328	1.7985	.2564	.0117
60 - 70	P	2.4450	1.8714	.4137	.2941	.3734	3222	.0419	.2741	.8784
	Q	.0567	.1935	.8274	.5460	.2827	.5101	2.0166	.3804	.0191
70 - 80	P	2.1632	1.9537	.4255	.3167	.4248	.3576	.0419	.3043	.9083
	Q	.0508	.2379	1.1334	.6293	.2697	.4613	3.5503	.3648	.0237
80 - 90	P	2.3318	2.0001	.4214	.3092	.4816	.3786	.0452	.2969	.8952
	Q	.0517	.2555	1.4060	.7352	.3778	.5150	4.3364	.5785	.0243
90 -100	P	2.2544	1.9692	.4302	.3304	.5113	.4093	.0461	.3396	.9186
	Q	.0874	.2769	1.6795	.7666	.6562	.5759	9.4329	.8750	.0316

Basic Data : Urban India (Continued)

Size class		Goat meat (28)	Mutton (29)	Egg (30)	Fish (fresh) (31)	Fish (dry) (32)	Sugar (33)	Gur (cane) (34)	Sea-salt (35)	Turmeric (36)
10 - 10	P	2.5523	2.5989	.1079	.8823	1.4972	1.1750	.8081	.1074	.0256
	Q	.0203	.0231	.0594	.0530	.0113	.2032	.1660	.2965	1.7930
10 - 20	P	2.3841	2.7103	.1204	1.0642	1.2042	1.1761	.8459	.1172	.0256
	Q	.0631	.0276	.1460	.0634	.0127	.3048	.1701	.3426	1.9318
20 - 30	P	2.5349	2.8129	.1836	.9708	1.2195	1.1842	.8625	.1073	.0254
	Q	.0689	.0266	.1454	.0916	.0191	.3886	.1893	.3690	2.1575
30 - 40	P	2.5897	2.6406	.1672	1.1992	1.2110	1.1838	.8672	.1079	.0257
	Q	.0915	.0337	.2030	.1202	.0210	.4676	.2145	.3747	2.2285
40 - 50	P	2.6189	2.9344	.1385	1.2886	1.6542	1.1830	.8420	.1122	.0245
	Q	.1223	.0394	.3527	.1336	.0208	.5203	.2612	.3953	2.1948
50 - 60	P	2.7112	2.5943	.1619	1.7167	.8548	1.1689	.9503	.1172	.0255
	Q	.1347	.0619	.2844	.1421	.0253	.6591	.2769	.4078	2.7505
60 - 70	P	2.7639	2.9437	.1685	1.6338	1.3859	1.1867	.8579	.1126	.026
	Q	.1532	.0399	.3177	.2372	.0171	.7210	.2347	.4759	2.751
70 - 80	P	2.9841	2.6366	.1697	2.1721	2.0629	1.1924	.9107	.1280	.024
	Q	.1793	.0737	.7877	.2401	.0148	.8666	.3315	.4521	3.626
80 - 90	P	2.9019	3.2350	.1794	2.0201	1.8537	1.1971	.9039	.1272	.0257
	Q	.2413	.0662	1.1590	.3564	.0107	1.0568	.2937	.4636	3.960
90 -100	P	2.9817	3.2823	.1830	2.1796	1.6156	1.3508	.9647	.1325	.024
	Q	.2796	.1107	2.3499	.4211	.0266	1.3509	.2882	.4897	4.773

Basic Data : Urban India (Continued)

Size class		Pepper, dry chillies	Green chil-lies	Tamarind	Tea (cups)	Tea (leaf)	Coffee (powder)	Pan (leaf)	Pan (finis-hed)	Supari	Biri
		(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)
0 - 10	P	.0319	.0076	.0096	.0826	2.5354	2.7184	.0038	.0350	.0786	.006
	Q	6.8727	2.6758	3.1483	1.3770	.0282	.0058	8.4395	.3156	.4930	19.166
10 - 20	P	.0324	.0079	.0114	.0789	3.0654	2.9356	.0037	.0339	.0851	.006
	Q	8.2382	3.7650	6.1060	2.8171	.0342	.0081	9.9578	1.1525	.6610	25.20
20 - 30	P	.0341	.0080	.0110	.0831	2.9910	2.8512	.0045	.0368	.0659	.006
	Q	8.8464	3.5058	5.3207	2.6327	.0521	.0088	11.4796	1.0340	.8850	30.559
30 - 40	P	.0329	.0080	.0107	.0930	2.9863	2.9400	.0036	.0436	.0823	.006
	Q	8.7661	4.6729	6.5200	2.6380	.0597	.0142	13.3725	1.0363	1.0407	34.328
40 - 50	P	.0307	.0083	.0112	.0828	3.1124	2.5419	.0049	.0431	.0719	.006
	Q	9.4871	5.6872	5.9254	4.1720	.0655	.0218	13.1088	1.2272	1.4086	33.140
50 - 60	P	.0347	.0088	.0105	.0922	3.2753	2.6953	.0043	.0400	.0816	.006
	Q	9.5742	6.3946	5.3665	4.2383	.0927	.0153	16.4652	2.1060	1.2447	38.969
60 - 70	P	.0333	.0089	.0103	.0956	3.2387	2.9050	.0037	.0400	.0758	.006
	Q	10.3322	7.4457	8.4098	5.6282	.0914	.0325	22.4096	1.9177	1.6384	38.922
70 - 80	P	.0339	.0098	.0113	.0871	3.1057	2.6966	.0052	.0354	.0708	.006
	Q	10.6148	6.9183	5.9513	6.8341	.1191	.0324	18.4300	3.6845	1.5918	48.578
80 - 90	P	.0342	.0109	.0109	.0920	3.2931	2.8768	.0056	.0438	.0825	.0070
	Q	10.5435	7.7385	6.8692	10.6235	.1397	.0510	17.7434	4.7017	1.7300	55.015
90 -100	P	.0353	.0099	.0110	.0986	3.4024	2.8712	.0057	.0474	.0821	.0069
	Q	10.4174	10.0959	11.2500	13.9674	.2003	.0964	27.4767	6.8437	2.9934	41.305

Basic Data : Urban India (Continued)

Size class		Cigarette	Leaf tobacco	Hookah tobacco	Coke coal	Firewood	Kerosene	Matches	Dhuti	Sari	Clothes for shirting
		(46)	(47)	(48)	(50)	(51)	(52)	(53)	(54)	(55)	(56)
0 - 10	P	.0222	3.5335	.7528	.0376	.0467	.5603	.0012	1.1895	1.0065	1.2957
	Q	.6960	.0081	.0203	1.2363	13.0575	.2732	26.2713	.0077	.0150	.0391
10 - 20	P	.0197	3.8184	1.1736	.0650	.0523	.5642	.0012	1.2091	1.6426	1.3379
	Q	1.3110	.0087	.0234	1.2922	13.7205	.3492	40.6375	.0118	.0204	.0842
20 - 30	P	.0202	3.8616	1.1369	.0324	.0505	.5620	.0012	1.1172	1.3593	1.4224
	Q	1.6448	.0089	.0194	2.0730	17.1244	.4198	48.0799	.0379	.0747	.1521
30 - 40	P	.0216	3.6089	1.3017	.0761	.0531	.5466	.0013	1.1261	1.7201	1.5863
	Q	2.5794	.0112	.0179	1.3249	16.6324	.5141	49.6505	.0633	.0750	.1716
40 - 50	P	.0206	3.8845	1.1609	.0664	.0560	.5302	.0012	1.2414	1.6208	1.5264
	Q	3.1533	.0174	.0114	1.9556	16.8540	.7074	63.4471	.0504	.1234	.2691
50 - 60	P	.0257	4.1770	1.4147	.0732	.0580	.5300	.0012	1.1115	1.7668	1.3050
	Q	6.4068	.0059	.0172	2.4566	16.0895	.7355	65.8022	.0624	.0870	.4218
60 - 70	P	.0198	3.7800	1.1328	.0715	.0552	.5523	.0012	1.3701	1.7580	1.8065
	Q	4.9259	.0101	.0216	3.2817	18.1588	.8140	72.4205	.1196	.1152	.3241
70 - 80	P	.0233	4.1165	1.0874	.0682	.0560	.5176	.0013	1.3742	1.9756	1.8442
	Q	7.2455	.0152	.0209	3.9651	15.9246	1.0195	88.0283	.1314	.2211	.4629
80 - 90	P	.0220	4.1854	1.6755	.0654	.0579	.5209	.0012	1.4863	1.9043	2.0155
	Q	11.2355	.0152	.0077	5.4121	15.5189	1.2376	102.0678	.2168	.3270	.7355
90 -100	P	.0306	4.3778	1.8555	.0776	.0525	.5228	.0012	1.5906	2.5307	2.3841
	Q	24.3745	.0119	.0161	5.2917	17.6907	1.6922	123.3718	.3275	.5161	1.0171