GEOLOGICAL DEPLETION AND LOCATIONAL ADVANTAGE IN THE ANALYSIS OF MINERAL EXTRACTION PROGRAMMES – CASE OF COKING COAL IN INDIA

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ABSTRACT

The aim of the study is to evolve optimal production and linkage plans, to meet an exogenously specified, spatially distributed time profile of demands from a set of spatially dispersed coking coal bearing geological blocks. The plans are optimal in the sense of minimum discounted present value of the sum of production, washing and transport costs.

Focussing our attention on a geological block consisting of many coal seams, we work with it as if it was operated as one production complex. Geological depletion in each block is formalised by estimating a Block Level Cumulative Cost Function (BLCCF) based on the data on geological parameters of the coal seams. Marginal cost in each block increases with each tonne of coal extracted from a block. In general the rate at which marginal cost increases varies across blocks depending on the variation in geological complexity.

Using as inputs the BLCCF and a transportation network, connecting blocks and steel plants, an overall programming model is specified to minimise the discounted present value of production, washing and transportation

cost. The model is solved to obtain time profiles of production and linkages. The importance of integration of production and transportation costs is examined by performing sensitivity analysis with respect to transport costs, and comparing the results of a partial optimisation exercise with that of a total optimisation exercise. The model solution provides marginal cost functions for prime and medium coking coal and a set of shadow prices at all steel plants.

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Chapter - 1

INTRODUCTION

1.1 Issues and policy implications:

This study was motivated by two specific issues of importance for policy in the Indian Coking Coal industry.

(i) The need for a quantitative estimate of the rising costs of supply due to geological depletion in the industry as coking coal fields are worked and approach exhaustion.

(ii) In the context of these estimates of rising costs of extraction to investigate the inter-action between production and transportation costs and its bearing on the spatial invest-ment/supply pattern in the sector.

In the course of exploring the specific policy issues it was hoped to develop a methodology that could be used for other industries in the mining sector also.

One expects rising costs as the industry supplies increasing quantities of coking coal as the seams get depleted and one moves to other seems more difficult to mine. A quantitative estimate of this supply cost func-tion should be the starting point for policy such as pricing of coking coal, deciding on the volume of expenditures today on coke rate control in the blast furnace, allocation of funds for research on developing a substitute for coke in the blast furnace such as formed coke and lastly designing of an appropriate policy for the import of coking coal. While a quantitative estimate of the effects of depletion is important for policy making at an aggregate level an understanding of the magnitude of interaction between production and transportation costs has bearing on the spatial dimension of the industry and is useful for project identification, initiating tactical drilling programmes etc.

The two issues that are outlined above are interrelated. While optimal allocation of production across coal producing areas cannot be made unless one takes into account production cost differences arising out of intrinsic variation in geological complexity - no estimate of the industry level marginal cost increase is meaningful unless it refers to such an optimal development plan.

There have been some attempts in the past to study the phenomenon of geological depletion in coal fields. As brought out in the literature survey that follows, at the methodological level these studies have not satisfactorily translated geological depletion into usable supply functions and no empirical study along these lines exists for India. Moreover, these studies have not paid adequate attention to the locational advantage of different coal fields vis-a-vis each other. At the same time there has been considerable modelling effort which has focussed on the transportation of coal while making simplifying assumptions regarding varying rates of depletion in coal fields. The purpose of this study is to first estimate supply cost functions for different coal blocks and then construct a computable mathematical model which evolves optimal production strategies in different coal producing areas taking into consideration simultaneously varying rates at which geological depletion takes place and their relative distances from the demand points.

Depletion in this study is formalised by a 'cumulative' cost function which is empirically estimated at the level of a 'geological block' from data on geological complexity and the quality characteristics of balance reserves of coal. The cumulative cost function gives the undiscounted sum of costs incurred in extracting of coal from a block without making any explicit reference to the trajectory of annual output. It is possible to characterize the block level cumulative cost function which is independent of the rate of extraction under fairly reasonable assumptions. This greatly simplifies analysis. The transportation network is then juxtaposed on the producing centres and a quadratic programming model is specified to minimise the total systems cost. The model is solved at various levels of demands to suggest spatial production/linkage plans on the one hand and to provide an estimate of industry level rise in marginal costs on the other.

Throughout the model we assume the existence of a single decision maker who coordinates the production/ linkage decisions - a valid assumption since the entire coal industry is nationalised. However working with an exogenously specified location specific demand forecast implies an insensitiveness of the location decisions in the steel industry to coal costs. In estimating the cumulative cost functions we assume perfect knowledge about the lay disposition and quality of balance coal reserves. An assumption which is valid in the case of coal where unlike hydrocarbons dramatic discoveries are rare.

Although the model is specified and solved for the coking coal industry its thrust is towards modelling a broader problem of exploitation of spatially distinct resource pools. In particular, it would be applicable to the non-coking coal industry, and it is hoped to use it **later to** estimate supply cost function for it. Such a function should be valuable in formulating energy policy for the country.

1.2 Coking coal industry - performance and planning effort:

In October 1971, all coking coal mines with the exception of those owned by two major steel companies TISCO and IISCO were nationalised, this was to be followed by the nationalisation of non-coking coal mines also in

May 1973. Thus within a short span of 18 months, the entire mining industry come under public ownership.

Quite apart from the moral issues of ownership and management of the country's natural resources, the need for rational exploitation of the reserves with a view to their conservation, and the possibility of integrated planning leading to overall social benefit are some of the more compelling arguments for nationalisation.[1]

Table 1.2.1 and 1.2.2 show the growth of output of hot metal and coking coal in India. The coking coal is divided into Prime, Medium coking coal, and Blendable coal. This classification is done based on the calorific value, caking index, etc. Each steel plant is designed to accept a specific blend of Prime/Medium and Blendable coal. Table 1.2.2 reports the growth in the output of coking coal in each of these three categories. Not all the coal as mined can be fed directly into the blast furnace. Quality upgrading to bring down the ash % is done by coal washeries by a process of physical beneficiation resulting in weight loss.[2] Table 1.2.2 reports the annual input of raw coal from year 69/70 to year 74/75 and output of washed coal in the country. As is seen, in the year 74/75 the total

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1 S.Mohan Kumaramangalam (July, 1973). References are indicated as they occur by author and year. A complete bibliography is given at the end.

2 A detailed discussion is available in the appendix to Chapter 3.

Table - 1.2.1 : Yearly production of Hot Metal, Coking Coal in India.

All figures in M.Ts.

Year	Output of Hot metal	Coking coal in-put to steel plants*	Total Coking coal produc-tion	Prime Coking Coal produc-tion	Medium	Blend-able
69/70	7.23	10.52	19.79	13.50	4.60	1.49
71	6.86	10.27	20.52	14.64	4.68	1.23
72	6.59.	9.82	19.77	13.88	4.77	1.12
73	7.27	10.28	19.71	13.87	4.74	1.10
74	6.95	9.92	17.69	11.69	5.04	.96
75	7.58	11.08	20.52	12.00	7.60	.92

Source : Chari Committee.

*As charged to the coke ovens.

Table 1.2.2 : Total washed coal output and consumption by Steel Plants

Year	Prime		Medium		Total		Input to steel plants	Hot metal output
	Input	Output	Input	Output	Input	Output		
69/70	7.37	5.73	2.69	1.80	10.06	7.53	10.52	7.23
71	7.50	5.49	3.05	2.02	10.55	7.51	10.27	6.86
72	7.73	5.73	3.19	2.11	10.92	7.83	9.82	6.59
73	7.14	5.57	3.84	2.57	11.48	8.13	10.28	7.27
74	8.07	5.85	3.98	2.60	12.05	8.45	9.92	6.95
75	9.99	7.04	4.32	2.92	14.31	9.96	11.08	7.58

Source : Chari Committee.

output of washed Prime and Medium coking coal of 9.96 M.T. is less than the total requirement of 11.08 M.T., the deficit being made up by direct feed and blendable coal. Thus to support a hot metal production of 7.58 M.T. in the year 74/75 roughly 15 to 16 M.T. of coking coal needs to be produced.[1] The balance coking coal production in the country is utilised for making soft coke in the merchant coke ovens and other uses. Steel production in the country has remained virtually stagnant from 69/70 to 74/75. However, this picture will be drastically altered as more steel plants go into operation during the next decade.

The demand for coking coal for this study is considered exogenously specified and is obtained by applying technical coefficients to the projected hot metal output over time.[2] This also is not always a straight forward exercise partly because of flexibility in the proportion of different types of coking coal which go to make the blend and partly because the coke rate itself can be controlled by adopting various techniques in the blast furnace shop (at cost).

Thus the demand projections made by different working groups differ primarily due to their different assumptions regarding coke rate and the blend of prime/

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1 The overall yield in the washery is 9.96/14.31 = .696.

2 Adopting such a procedure implies insensitivity of decisions in the steel industry regarding output, spatial location etc. to costs in the coking coal industry.

medium coking coal fed into the blast furnace. Table 1.2.3 shows the demand projections by the end of the fifth five year plan (78/79) made by two different working groups. Table 1.2.4 contrasts the actual coke rate in 73/74 against that assumed for arriving at the demand by the two working groups.

To understand the magnitude of planning effort and costs, we shall study the demand projections made by the Chari Committee[1]. We shall rely on this study for the demand side of the model because it is the only in-depth study which has steel plantwise demand projections for a ten year horizon.

Table 1.2.4 shows plantwise projection of hot metal production upto 84/85. Of these, the first six are operating steel plants whose planned expansion has been taken into consideration. The last two namely Visakhapattanam Steel plant (VZP) and Vijayanagar Steel plant (VAP) are proposed to be initiated in the sixth plan and contribute to output only from 82/83. From the tragectory of hot metal, the trajectory of total coking coal required per year is obtained by using an aggregate coke rate, which is different for each steel plant. This is reported in Table 1.2.5. The coke rate that is used takes into consideration predetermined decisions in the plants, to reduce coke rate by using several available techniques, in the blast furnace.

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1 Ministry of Energy, (Department of Coal), "Report of the Committee to review plans for coal supplies to steel plants during the fifth and sixth plan periods", September, 1975, Chairman Shri K.S.R.Chari.

Table 1.2.3 : Demand for Coking Coal in 78/79.

Figures in (M.Tonnes)

Steel Plants	Task force on coal and Lignite*	Chari Committee**
Bhilai (BSP)	3.933	3.708
Rourkela (RSP)	2.830	2.867
Durgapur (DSP)	1.699	2.642
Bokaro (BOK)	5.041	4.895
TISCO (TIS)	2.990	2.630
IISCO (IIS)	1.861	2.110
VIJAYANAGAR (VAP)	0.0	0.0
VISHAKAPATTANAM (VZP)	0.0	0.0

* See task force Planning Commission (1972)

** Ministry of Energy (September, 1975).

Table 1.2.4 : Coke rate at various steel plants coke as charged to ovens.

M.T./M.T. of Pig Iron

Steel Plant	73/74 Actual	Task Force	Chari Committee
BHI	1.66	1.09	1.285
DSP	1.21	1.17	1.53
RSP	1.62	1.60	1.619
BOK	1.38	1.27	1.384
TISCO	1.19	1.53	2.009
IISCO	1.44	1.44	1.295

Table 1.2.5 : Yearwise production plan of Hot Metal Production upto 84/85 for each steel plant.

All figures in (M.T.)

YEAR	BHILAI	DSP	RSP	TISCO	IISCO	BOKARO	VAP	VZP	TOTAL
75/76	2.50	1.10	1.3	1.05	.88	1.36			9.14
77	2.70	1.35	1.60	1.85	.93	2.85			11.28
78	2.76	1.55	1.70	1.90	1.01	3.67			12.59
79	2.88	1.72	1.77	1.90	1.05	4.55			13.87
80	2.75	1.72	1.77	1.90	1.05	4.63			13.82
81	3.03	1.72	1.77	1.90	1.10	4.63			14.15
82	3.49	1.72	1.77	1.90	1.05	4.63			14.56
83	3.80	1.72	1.77	1.90	1.10	4.63	.80	.85	16.57
84	4.00	1.72	1.77	1.90	1.05	4.63	1.65	1.75	18.47
85	4.00	1.72	1.77	1.90	1.05	4.63	2.32	2.50	19.89

Source : Chari Committee

Table - 1.2.6 : Yearwise requirement of coking coal in '000 Tonnes (as charged to the ovens).

Year	BHILAI	DSP	RSP	TISCO	IISCO	BOKARO	VAP	VZP	TOTAL
75/76	3660	1970	2106	2323	1496	1973			13582
77	3614	2183	2591	2323	1843	3692			16246
78	3647	2432	2753	2630	1843	4755			18060
79	3708	2642	2867	2630	2110	4895			19852
80	3541	2642	2867	2630	2048	5921			19649
81	3901	2642	2867	2630	2048	5845			19933
82	4434	2642	2867	2630	2048	5845			20466
83	4827	2642	2867	2630	2048	5845	990	1052	22901
84	5082	2642	2867	2630	2048	5845	2042	2167	25323
85	5082	2642	2867	2630	2048	5845	2871	3094	27079

Source : Chari Committee.

From Table 1.2.5 we can therefore infer that the coking coal requirement increases from 11.08 M.T.P.A.[1] in 74/75 to 27.08 M.T.P.A. in 84/85, this implies an increase of 16 M.T.P.A. as an input to steel plants. However to support this level of clean coal input to the coke ovens, the production of raw coal must increase from roughly 16 M.T.P.A. to 39 M.T.P.A.[2] Thus, merely to support the increased activity in the steel sector the coking coal industry must more than double the annual output in the coming ten years, add 23 M.T.P.A. of capacity.[3] The total cumulative output over the ten years is about 200 M.T. The cost of exploitation being roughly Rs.100/t, this implies a total cost of Rs.2000 crores over a ten year horizon - the financial outlay being much greater.

The coking coal reserves are distributed in six coal fields[4] and 18 geological blocks, each of which differs in the geological complexity and quality characteristics of its coal seams. The average lead in the coal industry is 353 KMS,[5] the transport cost is roughly 20% of the total C.I.F.

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1 Million tonnes per annum.

2 This assumes an aggregate washery yield of .696.

3 This assumes that there is no excess capacity in the base year, which can be progressively used to augment annual output.

4 Appendix to the Planning Commission task force on Coal and Lignite (ICSL Statement).

5 See Table 4.1, Chapter 4.

to the plants and thus is expected to affect the location of production significantly.

Table 1.2.7 shows the balance life of some non renewable resources in India at anticipated consumption levels in 1988/89.

Table 1.2.7 : Balance life of known reserves at 1988/89 consumption levels.

Mineral	Balance life (years)
1. Coking Coal	44
2. Non Coking Coal	
(a) Without exports	168
(b) With exports	159
3. Iron Ore (Hameatite)	
(a) With exports	62
(b) Without exports	165
4. Magnetite	84
5. Limestone	475

Source : Fifth Five Year Plan, Planning Commission.

As seen from the table, the balance life of coking coal reserves of 44 years is short compared to that of other raw materials used in the iron and steel industry. Further, though it may be technically feasible to exploit the reserves upto the end of their life, the marginal cost of exploitation may increase dramatically as the coking coal fields are worked and approach exhaustion. Thus issues such as whether and if yes when to import coking coal and how much and when to invest in expensive research programmes to find a substitute for coke (like formed coke) cannot be evaluated unless a quantitative estimate of the geological deterioration and concommittent increase in costs is available.

Thus, the considerable financial outlay involved together with complex interrelated issues establishes the need to formalise the system in the form of a mathematical model to help the management of the industry in rational decision making.

Chapter - 2

PLANNING MODELS FOR THE COAL INDUSTRY

2.1 Choice and strategies for formalisation:

Positioning himself at any point in time, the planner in the coking coal industry is confronted with determining the spatial pattern of production and linkages that minimises the total systems cost given a location specific demand overtime.

The purpose of the study is to construct a computable mathematical model which can be used to study/highlight some important choices in the prime/medium coking coal supply while answering the question posed above. The form, structure and the size of the model are basically determined by the nature of choice it examines and the specific policy issues to which it addresses itself.

Consider a demand centre at 'A' and two coal producing areas at 'B' and 'C'.

```
A *-----------------------* B
  |
  |
C *
```

The optimal production and linkage pattern would depend upon two factors.

(a) the relative production cost at the two coal producing areas;

(b) the relative distance over which coal has to be transported.

The production decisions would not only be influenced by the production cost differences as they exist today but also be influenced by varying rates of depletion across coal producing areas. Similarly the industry level geological depletion computed is meaningful only if it is with reference to this optimal development programme of the coal industry.

The problem outlined above can in principle be studied within the context of sectoral planning models which are designed to bring out the benefits due to inter regional coordination of investment decisions. This line of modelling began with Chenery (1952) who studied the problem of capacity expansion overtime at a single location and has been developed to a great level of generality in Kendrik and Stoutjesdijk (1975). Excellent expositions are available in Manne (1967), Erlenkotter (1969), and Scherer et al (1975). Empirically these models have been used to study many process industries like Cement, caustic soda and nitrogenous fertilisers - (Manne 1967), Cotton textiles and man made fibres - (Kornai 1965), Steel - (Kendrik 1967), Electricity planning - (Gateley 1971) etc. However, the absence of such a model being applied to coal and other extractive industries is noticeable. All these models work with existing and proposed production centres which serve as candidate locations, to meet a time profile of demand growth. It is also required that costs for each option be supplied in advance. The basic production unit in the coal industry is a mine[1] which in India has been of about .6 M.t. per year capacity. Given the limitations of managerial ability and the state of technological development in the coal industry in India the maximum mine size is not expected to increase to more than around 1.5 m.t. per year. Thus for an expected expansion in the rate of production of about 23 m.t. per year we would need to select around 15 to 20 feasibility reports. To offer meaningful choices one would like to have 30 to 50 feasibility reports ready on the shelf to serve as candidate locations from which an optimal production programme could be evolved. Following this system of planning presents a number of difficulties. Firstly from the view point of implementation it is unlikely that these many projects reports are ready on the shelf to serve as candidate locations. Secondly constructing a planning model with these as candidate locations would present computational difficulties on account of large number of variables particularly if there are non-linearities in production costs.

Apart from the operational problems of implementation of the planning system and the mathematical complexity of the resulting planning model, there is also a problem of the logical sequence of planning. For an extractive industry such as coal, the identification of sites and preparation of feasibility reports (at not an inconsiderable cost) is

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1 This is an accounting unit the level at which costs are maintained.

the last stage in a well defined planning procedure. Project identification being a crucial problem in the Indian Coal industry (where do I commission the next feasibility report?). We cannot work within the frame work of existing sectoral planning models which essentially evaluate alternatives after the projects are identified. The correct planning strategy would then be to work with a macro model which allocates production targets to coal producing areas based of broad knowledge about their geological complexity and transport cost advantages. A detailed drilling programme within each of these against allocated targets then leads to specific investment proposals (sites) for new mines. Detailed information obtained in the second stage can be continuously fed back into the macro model which can be solved again to firm up the targets.

Thus a model for the coal planning problem which highlights the benefits due to interregional coordination of production decisions must be different in two respects from a conventional planning model in which specific projects are selected from a set of alternatives. Firstly a large coal producing area and not a single mine, is its fundamental unit of production and secondly in each of these areas the marginal cost of production may be expected to rise with each tonne that is extracted.[1] The first aspect is reflected in this study in its focus on a geological block

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1 Thus geological depletion should lead to increasing costs even for sustaining a given level of output.

consisting of several small coal producing units and virgin seams, and working with it as if it is being developed and operated as one production complex. The second aspect is taken into account by estimating a cumulative cost function for each block taking into consideration the geological complexity and quality characteristics of the coal seams contained in it. In a cumulative cost function, cost at the margin is a function of past extraction - or history of the block.

Working with a cumulative cost function typically allows one to take account of varying rates of marginal cost increases across production points and permit marginal cost increases even for sustaining a rate of production in a block. This comes closest to our notion of cost behaviour in exhaustable resource pools. Examples of such specification are found in the analysis of mineral extraction programmes. Weitzman, M.L. (1977), considers a problem of obtaining a fixed flow of output (Q) from 'N' resource pools over an infinite horizon. He derives an optimal policy for an arbitarily specified[1] cumulative cost functions for the resource pools.

If area specific marginal costs functions are thought of as supply functions then together with the transportation network, the model may be viewed as a spatial equilibrium

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1 Of course if costs are convex (upwards - non-decreasing extraction costs) then the optimal policy is to draw from the pool with lowest marginal cost.

problem of the Takayama and Judge (1971) variety - in which demands are price inelastic. However, the difference is that the supply functions in the coal industry cannot be be estimated from past behaviour of prices but must be estimated with reference to the geological complexity of balance reserves.

1.2 Review of literature on coal planning:

By depletion we refer to the phenomenon of rising marginal costs and suggest that it can be best represented by a cumulative cost function at the level of a geological block.[1] In an extractive industry costs in the future cannot be obtained by extrapolating costs in the past[2] because expanding or continuing production involves a continuous change in the profiles of the deposits. Thus we must estimate the impact of geological parameters on costs at the level of a single producing unit and combine this with information on balance reserves and organisation of production at the level of a geological block to predict the rate at which costs rise in the industry.

Johnston (1961) is eloquent as he reports failure to correlate cost of mining to the geological parameters of a mine by statistical cost analysis, so is Naganna, N., (1974) as he fails to find input structures that are stable across mines in India. While it is generally agreed that

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1 Hottling, H., (April 1931).

2 Lessourne (1955) (1960).

geological parameters significantly affect the cost of production the failure to find such correlation at the level of a mine is probably due to a failure to find proxy variables that reflect adequately the geological complexity of the mine.

We come acrss successes when costs involved in unit operations of mining like face work, underground transportation, etc. are correlated to the specific geological variable that influences it. Naganna (1974) reports correlation between costs of transport and the face to surface distance. Zambo, J., (1968), reports the relation -

$$K_B = B.q.^V L^W$$

where K_B = Operating cost per tonne

q = Capacity of the mine

L = Transportation distance from the face to shaft bottom.

If the mine operates with more than one face contributing to production q, then the 'L' is taken as the weighted average of distances.

$$L = \sum_i \frac{Li\ qi}{qi} = \sum_i \frac{Li\ qi}{q}$$

where,

qi = Production from the ith face, $q = \sum_i qi$

Li = Distance from the ith face to the shaft bottom.

While face to surace distance is stressed in some studies, the impact of seam thickness on cost of mining is stressed in others. Zimmermann (1977) estimates the impact of seam thickness on productivity as

$$\text{Log } \frac{Q}{S} = \text{Log } A + b \text{ Log th} + (C-1) \log S$$

where $\frac{Q}{S}$ = Productivity per section

th = Seam thickness

S = Number of sections

C = Dummary variable : 1 if S greater than 7

0 if S less than 7

The productivity (hence the seam thickness) is linked to cost using engineering estimates. He then combines the above relation with data on coal seams in different coal producing areas, and observed cost of new mines to obtain a cumulative cost function which basically indicates the behaviour of incremental cost as cumulative output builds up.

In his analysis he finds that depth has no impact on the cost of production. This is probably the result of the fact that he works with a sample of deep mines where intermine variation in depth is low. Similarly there is no mechanism in his model of production to allow for switching between strip mining and underground mining if the seam parameters are favourable. Also it is doubtful if increasing seam thickness would continuously lead to decrease in costs, because it is well known that for thick seams, if mined by underground methods the strata control problems leading to complex mining techniques,[1] result in high cost.

There have been very few studies which characterise a coal producing area (rather than a mine) in terms of its costs of production. Lesourne (1955) (1960) has worked in terms of a broad coal producing areas. He has computed costs of cutting down output from a coal producing area as savings resulting from shutting down the most unsatisfactory mine in the coal field. He observes that computing costs of expansion would involve constructing new units and consideration of varying geological structures - which is what we propose to do in the course of our study.

Location specific production costs, significant transport costs and spatially distributed demand - with these features coal industry is ideal for transportation modelling. If we assume constant marginal costs of production, at the level of a mine or a coal producing area then the minimum cost (production + transportation) production linkage programme can be computed using a linear programming formulation. Henderson (1958), for U.S.A., Upadhyaya (1964) and Chakravarty (1965) for India have computed models which fall in this category.

Federenko (1974) works with 230 operating and 60 projected pits and quarries. The production cost at each is specified. The model has quality specific demand

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1 Like sublevel caving.

from each demand centre which is satisfied by a minimum total cost model formulated as a large linear programming problem.

A slightly more elaborate model is that of Bohjle and Libbin (1977) in which demand is quality specific and capacity can be increased by adding new mines (each of a prespecified capacity) which are constructed at fixed cost which varies across regions depending upon mining type, size, seam thickness, over burden and within region by seam thickness and mining type. Depletion in this model is brought about by retiring 5% of the capacity every year. This may result in increased average cost of production overtime. However cost of expanding capacity are unchanging overtime as they assume that "all costs are held constant throughout the time horizon because, there is no basis for differential rates of change in costs by region". Exactly the opposite result is thrown up by Zimmermann (1977) as he reports widely varying rates of depletion across coal bearing areas in the U.S.[1]

As brought out from the literature survey there is no satisfactory method of estimating geological depletion in the coal industry. The problems of characterising coal producing areas in terms of their geological complexity

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1 "If output rates were to double, depletion would lead to an increase of 30 per cent in 20 years in North Appalachian high sulphur coal costs. Similarly it would take 42.4 years in the midwest for cost to increase 30% if output rates were doubled." Zimmermann (1977).

and using this to estimate and predict industry level depletion are not explored fully satisfactorily. In addition research effort in past has focussed on geological depletion and coal transportation in isolation without exploring the interactions between them. In our study we propose to analyse all these problems in an integrated framework . We characterise geological blocks by a cumulative cost function and use this to estimate depletion and suggest production transportation programmes for the coking coal industry over varying horizons. Unlike previous researchers we recognise and consider the impact of quality in determining depletion.

While the integrated model will be useful as a starting point from which further models of extractive industries may be constructed, it is hoped that the empirical results not available before will prove to be useful to policy makers in this area.

2.3 Plan of Study :

The aim of the study presented here is to evaluate alternate cumulative output strategies in 18 geological blocks of Prime/Medium coking coal within the framework of a quadratic programming model. This study proceeds in two distinct stages. Furst in Chapter 3 we estimate the cumulative cost and yield functions for all geological blocks. The regional setting of the coking coal industry is outlined in Chapter 4. The second stage begins in Chapter 5 where we juxtapose the transportation network and specify the quadratic programming model to minimise the total systems cost in a dynamic sense and explore some of its solution properties. Chapter 5 also presents model solutions to suggest optimal production trajectories for blocks in the Prime and Medium Coking coal industries in the absence of transport costs. For each, an estimate of the geological depletion is obtained as well as the long run marginal cost function.

In Chapter 6 model solutions are obtained when transport costs are considered and sensitivity of the solution to transport cost variation is studied. This is followed by computing the output profiles, marginal costs and linkages as the model is solved successfully for various time horizons.

Chapter - 3

BLOCK LEVEL CUMULATIVE COST FUNCTION

3.1 Specification of a Block Level Cumulative Cost function:

Several considerations, lead us to focus our attention on a geological block as a unit of production. A geological block would in general contain many coal seams, and although each of these could independently contribute to production, the geological block is thought of as being developed and operated as an integrated production complex. While specifying a cost function at the level of a geological block one is indeed looking for an index which adequately reflects interblock variation in geological complexity and quality characteristics to be used as inputs into a programming model. Starting from a configuration of seamwise balance coal reserves in a block, a rational production planner would organise the production within a block, in a manner which allows him to exploit coal seams which yield coal cheaply before moving on to more expensive ones. In this sènse the cost of mining a tonne of coal at the margin would be non-decreasing as increasing amounts of coal are extracted from a block. Thus a desirable specification for a block level cost function would be one in which the cost of mining a tonne depends not only upon the current production, but cumulative output from a block as well one would like to estimate a complete specification such as, (1) below:

$$c = f\,(q,\ qc,\ Q) \qquad \ldots (1)$$

where

q annual output from the block (m.t.)

qc Capacity at the level of a block (m.t./annum).

Q Total amount of coal that has already been produced from that block.

c Cost of mining q m.t. from the block with capacity qc and from where Q m. tonnes have been previously exploited.

Thus specification (1) is a short run cost function when qc is considered fixed in which the marginal cost may be expected to increase rapidly with q. Explicit inclusion of Q in the specification is to capture the impact of geological depletion on the production costs. The parameters associated with q and qc indicate the scale effects.[1] The main aim of this study is to focus attention on geological depletion at the somewhat aggregate level of a block. The above specification would be difficult to estimate and would lead to complicated programming formulation for the optimisation problem involving a group of geological blocks. We shall demonstrate that under certain reasonable assumptions on the technology it would be possible to adopt a specification which leads to considerable simplification while retaining the focus on geological depletion. This

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1 Inclusion of factor prices in the specification would bring out the substitution effects.

estimates the cumulative cost incurred as a function of cumulative output only, as follows:

$$c = f\ (Q) \qquad \ldots \quad (2)$$

The estimation of the block level cost function proceeds in two stages.

1. First, the total cost curve for each seam in the geological block is estimated.

2. Based on the ranking provided by (1) these are aggregated into a block level function.

3.2 Specification of the seam level total cost curve.

Coal mining consists of a sequence of activities, starting from drilling, blasting, transportation etc. conducted in that sequence. Costs of mining coal are thought of as the sum of costs incurred in performing each of these operations. Consider the general problem of performing any of these activities to yield R/ million tonnes (m.t.) in a year, when a capacity of R_c m.t./annum has been installed. Let the technology at the seam level be characterised by the following assumptions:

A1. Constant returns to scale.

A2. Short run marginal costs that are constant and do not depend upon the scale of operations.

A3. Fixed capacity R_c.

Then the short run total cost function can be written as

$$f\,(R, Rc) = aR + b.Rc \qquad 0 \leq R \leq Rc \qquad (1)$$
$$\infty \qquad \text{if} \quad R > Rc$$

where,

Rc = capacity

R = annual output

a = Short run marginal cost

b = Marginal capacity creation cost.

The long run total cost curve F(R) for any activity is obtained when the capacity is allowed to vary and the output R produced is equal to capacity Rc. In other words we assume that interperiod adjustments in output take place in a way that constantly permits perfect adjustment of capacity and design to the quantity produced in that year.[1] This may be written as

$$F(R) = f\,(R, Rc) = Ac.R, R \geq 0 \qquad (2)$$

where

Ac = (a + b) and R : rate of production

This situation is depicted in fig. (3.1) where the long run total cost curve is plotted as an envelope of the short run total cost curves for various values of Rc.

In the formulation above, the parameter 'a' is the short run marginal cost of producing R m.t. These are the

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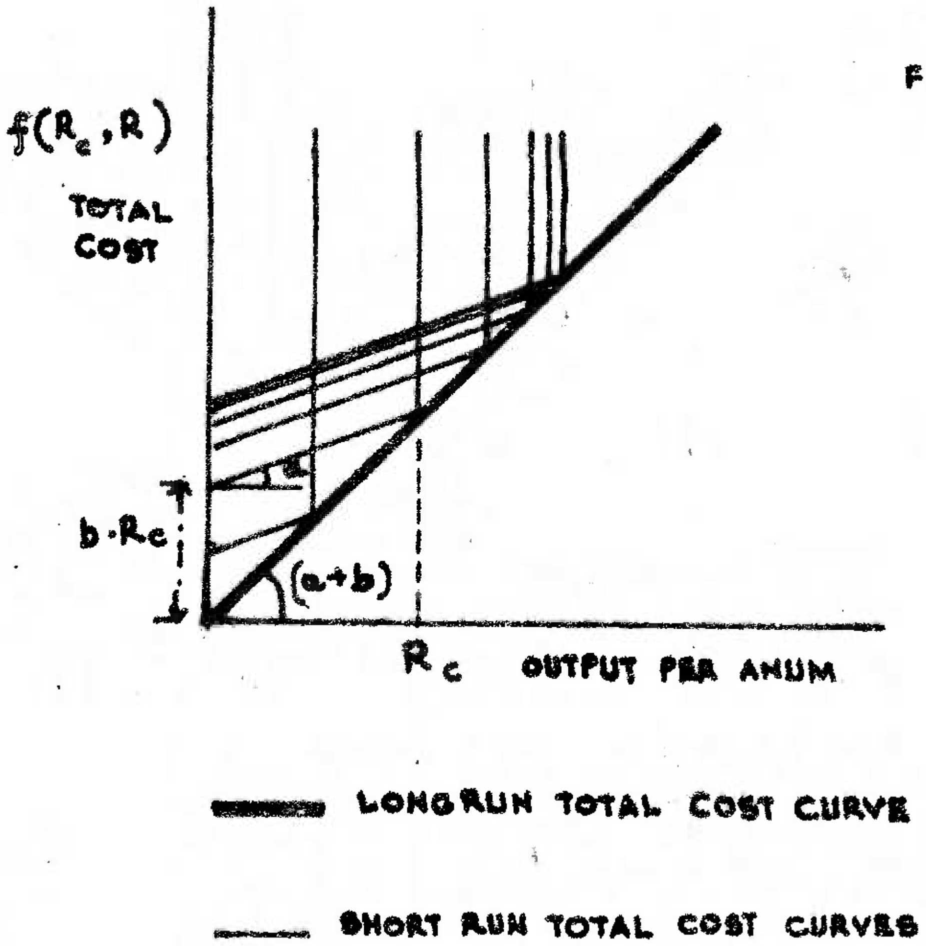
1 See Dre'ze (1964).

* Think of the cost of transporting R m.t./ in a year underground in a seam on which a belt conveyor of capacity Rc m.t/ annum has been installed; while no more than Rc may be transported in the short run, in the long run Rc may be adjusted to equal planned output in that year by replacing the motor by one with higher/ lower horsepower, changing the belting or removing the belt conveyor and installing one which matches the new planned capacity.

Fig: [3.1]



f(Rc, R)
TOTAL COST
b·Rc
(a+b)
Rc
OUTPUT PER ANUM
LONGRUN TOTAL COST CURVE
SHORT RUN TOTAL COST CURVES

expenditures that are directly related to the output R, such as labour, electricity, explosives etc. The parameter 'b' is interpreted as the expenditure that the production manager must charge himself in a year to avail himself of an additional unit of capacity. Assuming constant returns to scale, the total capital expenditure incurred in setting up a mining facility of R m.t/ annum is given by

$$K = k.R \qquad (3)$$

where

K = Total capital costs incurred instantaneously before the start of production.

R = Capacity in m.t/annum.

k = Unit capital cost (Rs./tonne)

It is assumed that the production manager discounts costs incurred in future at a rate of r%/annum.[1] If 'b' Rs/tonne is what the production manager charges himself annually for the use of capital equipment of value K = k.R. Rs, then the discounted present value of the capital charge over the life of the equipment should be equal to the total capital expenditure, K Rs.

If L is the life of the capital stock

$$\int_0^L b.R.e^{-rt}\, dt = K = k.R$$

$$b = k / \int_0^L e^{-rt}\, dt. \qquad (4)$$

Using capital charge as in (4) above implies that there

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1 The impact on policies due to the variation in r is explored later.

are no seam specific sunk costs. Thus we need to make the following assumption.

A4. The capital stock employed in exploiting a seam is amortised over its useful life and not the life of the coal seam(s) on which they are operated. Thus all items of capital stock can be costlessly shifted and employed elsewhere if they outlive the life of the coal seam.

Suppose there are 'M' activities involved in mining a tonne of coal, indexed by i, and the cost curve for ith of these is given by:

$$Fi\ (R) = Aci.\ R,\ Aci = (ai+bi)$$

The cost curve for the entire mining operation can be written as.

$$F(R) = \sum_{i=1}^{M} Aci.R = \sum_{i=1}^{M} (ai + bi)\ .\ R \qquad \ldots. \ (5)$$

thus not only is each of these activities carried out along the long run cost curve, but capacities in different stages in mining are consistent with each other.[1]

Quality considerations

The seam level total cost curve is designed to indicate, the relative desirability of selecting coal seams for exploitation vis-a-vis each other. The impact of interseam variation in quality is taken into considera-tion by specifying the cost function in terms of clean

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1 Thus in a single mine, producing q. m.t./annum the capacity of transport equipment, and drilling and blasting equipment are both assumed to be equal to q. m.t./annum.

(washed) coal. Steel plants are designed to accept coal of a specific quality (17% ash). Quality upgrading is done in coal washeries by a process of weight losing physical beneficiation. Although coal production and washing are distinct activities (carried out in that sequence), the seam level total cost curve is specified for a vertically integrated production/washing activity. Costs of mining and washing depend only upon the raw coal mined/washed however coal from different seams has different yields given by

$$q = y R^{1} \qquad \ldots \qquad (6)$$

where

R = quantity of raw coal processed in the washery.

q = quantity of clean coal obtained.

y = seam specific yield factor ($0 \le y \le 1$)

Thus if F_1 (R) and F_2 (R)[2] are the seam level total cost respectivel curves for the integrated production and washing activities for a seam which has a yield factor y, the cost function for the integrated production/washing activity in terms of clean coal may be written as -

$$F_1 (R) = AC_1 \cdot R$$

$$F_2 (R) = AC_2 \cdot R$$

$$F (R) = (AC_1 + AC_2) R \text{ , Since } q = y.R.$$

$$F (q) = \frac{AC_1 + AC_2}{y} \cdot q = AC. q \qquad \ldots (7)$$

---

1 Details at Appendix II where econometric analysis of washery yield is presented.

2 F_2(R) shall be the same for all seams since it depends only upon raw coal processed.

The final form in which the seam level cost curve is used is (7). The following properties of the seam level total cost curve need to be emphasised.

1. <u>It depends on the discount rate used:</u>

Since the cost curves $Fi(q)$ of each of the activities involved in mining coal and the yield function depend on the geological complexity and quality characteristics of the coal seam, the aggregated cost curve (the parameter AC) is seam specific. Similarly since the valuation of capital stock in use is contingent on the discount rate used, the seam level cost curve $F(q) = AC.q$, depends upon the discount rate used, changing 'r' the discount rate changes the shape of the cost curve.

2. <u>Cumulative costs are independent of rate of extraction.</u>

Consider the problem of obtaining a cumulative output Q from a single seam along a trajectory qt, of yearly outputs and $F(qt) = AC.\ qt.$ of costs. The undiscounted algebric sum of costs $C(Q)$, incurred in reaching Q are worked out as follows. If the life of the seam is $T(qt, Q)$

$$C(Q) = \int_{0}^{T(qt,Q)} AC.\ qt.\ dt$$

$$C(Q) = AC. \int_{0}^{T(qt.Q)} qt.dt.$$

$$= AC.Q \qquad \ldots \qquad (8)$$

Thus the total undiscounted algebric sum of costs incurred in reaching a cumulative output Q are independent of the time profile of yearly outputs along which it is reached.

3. Choice of extraction

Consider two seams with reserves Q_1 and Q_2 and cost functions, AC_1q_1 and $AC_2\ q_2$, and $AC_1 \leq AC_2$. Costs incurred in future on variable inputs as well as towards capital services are discounted at a rate r%/annum.[1] What is the optimal strategy of exploitation of coal seams in the sense of minimum discounted present costs? Since the cost incurred in obtaining Q_1 from seam one are AC_1Q_1 and AC_2Q_1 if obtained from seam two, for any trajectory of Q_1 seam one will be completely exhausted before initiating any production from seam two. The sum of costs incurred in obtaining an output $Q = Q_1 + Q_2$ will be $(AC_1Q_1 + AC_2Q_2)$ regardless of the time profile of Q.

3.3 Deriving the Block Level Cumulative Cost Function (BLCCF) from seam level cost curves

Focussing our attention on a geological block, which contains many coal seams, we shall study the problem of obtaining Q m.t. of coal from it in the sense of minimum discounted present costs as defined before.

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1 The discount rate 'r' needs to the same as that used in the construction of the BLCCF.

specifically;

Let there be N seams in the block indexed by i.

Let the reserves in the ith seam be $\bar{Q}i$

Let the total cost curve associated with the ith seam be

$F_i(q) = ACiqi$, $ACi = (ai + bi)$

The seams are ranked according to increasing cost per tonne as follows -

$$AC_1 \leq AC_2 \leq \ldots . ACi \leq ACi+1 \ldots \leq ACN,$$

with reserves $\bar{Q}_1$ $\bar{Q}_2$ ... $\bar{Q}i$ .... $\bar{Q}N$ of washed coal.

As indicated before the total undiscounted sum of costs incurred in exploiting the first seam is $TC_1 = AC_1 \bar{Q}_1$, the cost incurred in exploiting the first two seams would be $TC_1+TC_2 = AC_1\bar{Q}_1 + AC_2 \bar{Q}_2$., If the block is exploited in an increasing cost sequence of seams, the cost of exploiting the first J seams, CJ is given by.

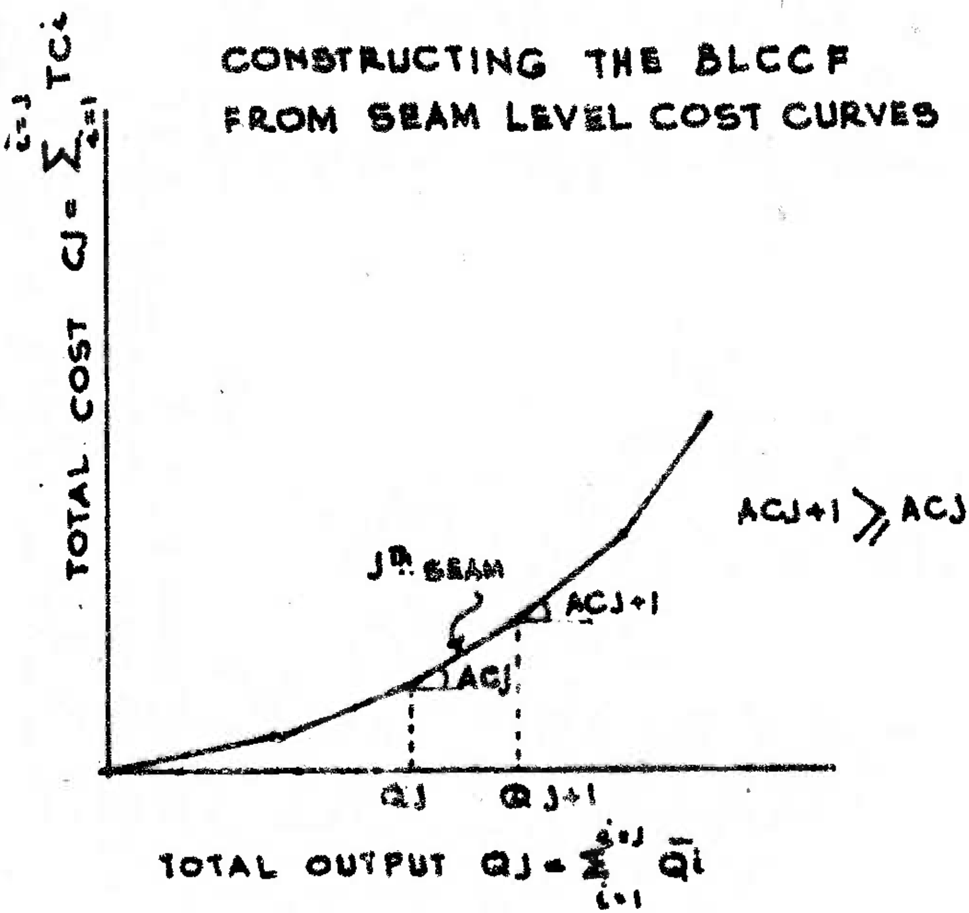
$$CJ = \sum_{i=1}^{i=J} TCi = \sum_{i=1}^{i=J} ACi\bar{Q}i$$

while the output is given by $QJ = \sum_{i=1}^{i=J} \bar{Q}i$

Figure 3.2 shows the variation of the total cost with output as seam after seam is exploited.

The profile of cumulative cost incurred as the cumulative output is obtained from any block thus consists of 'N' linear segements (each representing one seam) joining the points (CJ, Q_J) as generated above.

CONSTRUCTING THE BLCCF
FROM SEAM LEVEL COST CURVES fig.[3.2]



The mode of cost function development adopted in this study results in a cost function where the blocks are characterised by a set of points (C_J, Q_J) J = 1.....N (N, equals the number of seams in the block). However, we wish to go one step further in achieving economy of representation by approximating the piecewise linear function by a suitable polynomial - in this case a quadratic function

$$C = A1Q + A2Q^2 \qquad (9)$$

where C : Cumulative Cost (M. Rs.)

Q : Cumulative output (M. tonnes)

A1 and A2 : Constants.

A least square quadratic approximation to the piecewise linear function results in a geological block being characterised by just two parameters A1 and A2 rather than N points (C_J, Q_J). This economy we would argue more than compensates for a loss of information which is in any case not too large judging from the high R^2 with which the quadratic approximation fits the seatter. Secondly, the cost functions are ultimately to be used as inputs into a programming model where working with quadratic functions allows us to study the impact of parametric variations in an easily comprehensible form.

Along with the cost/quantity profile we could also obtain the following profile which indicates the total amount of raw coal that is processed to yield increasing amount of clean coal from a particular block if the seams were exhausted in an increasing cost sequence.

If $\bar{R}i$ = Total raw coal reserves in seam i.

$\bar{Q}i$ = Total clean coal produced from seam i.

$\bar{R}i = yi.\bar{Q}i$ where yi is the seam specific yield factor.

then compute

$$R_J = \sum_{i=1}^{i=J} \bar{R}i \qquad Q_J = \sum_{i=1}^{i=J} \bar{Q}i$$

The Block level yield function (BLYF) gives the total raw coal that needs to be processed to obtain a tonne of clean coal, the relation is specified as -

$$R = A3Q. \qquad (10)$$

where

R = Raw coal processed

Q = Clean coal obtained

With observations at (R_J, Q_J) $(J=1....N)$ for any block, the least square estimate for 'b' the block level yield factor can be ontained by

$$A3 = \sum_{j=1}^{N} \frac{RJ.\ QJ}{(Q_J)^2} \qquad \ldots \qquad (11)$$

The interblock variation in A3 is thus an aggregate index of the quality variation in its seams. for a programming formulation in terms of clean coal, the yield curve is useful to infer the amount of raw coal that needs to be handled at the level of a block to supply a given quantity of clean coal.

3.3 Data on coal reserves:

The most comprehensive information on coking coal reserves was available in the appendix to the Planning Commission task force report on Coal and Lignite.[1] This information was compiled by the Jharia coal survey laboratory based on 102 reports of the Geological Survey of India and the Indian Bureau of Mines (hereafter referred to as the JCSL Statement).

This statement reports coal reserves separately for prime and medium coking coals. The entire coal bearing area in the country is first divided into coal fields which are further divided into geological blocks. The production centres for prime and medium coking coal were considered to be geological blocks as delineated above. Of the total number of blocks reported, we excluded those which did not have significant (greater than five million tonnes) proved reserves and aggregated blocks which are adjescent to each other, so that finally each

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1 Planning Commission, "Report of the task force on Coal and Lignite", Appendix 1 to 4 (compiled by Jharia Coal Survey Laboratory, 1972).

block had sufficient resource base from which production could be obtained.

Table 3.3.1 shows the division of the coal field into various geological blocks separately for the prime and medium coking coal.[1]

Each geological block contains a number of coal seams. In each block reserves are reported seamwise. In each seam the reserves are divided into three categories - proved indicated and inferred.[2] In this study we shall be working only with proved reserves, which are within 200 meters of workings, out crops or borehole.

The proved reserves (in each seam) are further divided into four quality classes. The classification is as follows:

Class I	:	less than 17% ash.
Class II	:	17 to 24% ash.
Class III	:	25 to 35% ash.
Class IV	:	35 to 50% ash.

---

1 Coal seams which show prime or medium coking characteristics are distinct. Hence though it is likely that there is some overlapping of geological blocks in the geographical sense, the prime and medium coking coal blocks can be considered as conceptually distinct for the purpose of mathematical modelling.

2 For the classification see Vol. 88 GSI (1971).

Table - 3.3.1 CODE FOR SUPPLY CENTRES

Name	AREA AS SHOWN IN THE RESOURCE STATEMENT (BLOCK)
JHARIA (COAL FIELD) PRIME COKING	
JP1	Private lease holds including BCCL, TATA, IISCO Group of Collieries (Seams IX to XVIII). (The private leaseholds are now nationalised except TATA and IISCO Collieries).
JP2	Parbatpur Block.
JP3	Mahal Block.
JP4	Kapuria Block.
JP5	Monidih Joma Block.
JP6	Sudamdih Colliery
JHARIA (COAL FIELD) MEDIUM COKING	
JM1	Parbatpur block, Kapuria block with Kapuria extension and Deogara area west of Kapuria block.
JM2	Private leaseholds (Seams V to VIIIA) (Now nationalised).
JM3	Unleased area to the west of Kirkend blocks.
RANIGUNJ (COAL FIELD) MEDIUM COKING	
RM1	Chunch, Begunia, Rampur, Shympur (No.1 to 6), Laikdih and Kharbari.
EAST BOKARO (COAL FIELD) MEDIUM COKING	
EBM1	Swang, Govindpur and Uchitdih.
EBM2	Kathara, Jarangdih and Jhirki block.
EBM3	Chalkari, Kargali and Chapri block.

contd...

Name	AREA AS SHOWN IN THE RESOURCE STATEMENT (BLOCK)
WEST BOKARO (COAL FIELD) MEDIUM COKING	
WBM1	Private leaseholds (Seams 1 to 13) (Now nationalised).
WBM2	Unleased area (Seams 1 to 13).
NORTH KARANPURA (COAL FIELD) MEDIUM COKING	
NKM1	Badam, Rautpura block.
NKM2	Ronhe and Chano rikba blocks.
RAMGARH (COAL FIELD) MEDIUM COKING	
RGM1	Main basin (blocks 1, 2 and 3).
RGM2	Main basin (blocks 3, 4 and 5).
RGM3	Private leaseholds in Gobardhana area (now nationalised).

Source: Planning Commission, "Report of the Task Force on Coal and Lignite", Appendix 1-4, Compiled by Jharia Coal Survey Laboratory (1972).

The statement also reports proved reserves which are as yet unclassified, which we assume to be of class four (this is by the way of being prepared for the worst).

For each seam in addition to the data on quality (as indicated by the ash %) the following geological information is also reported.

(1) The thickness range of the coal seam.

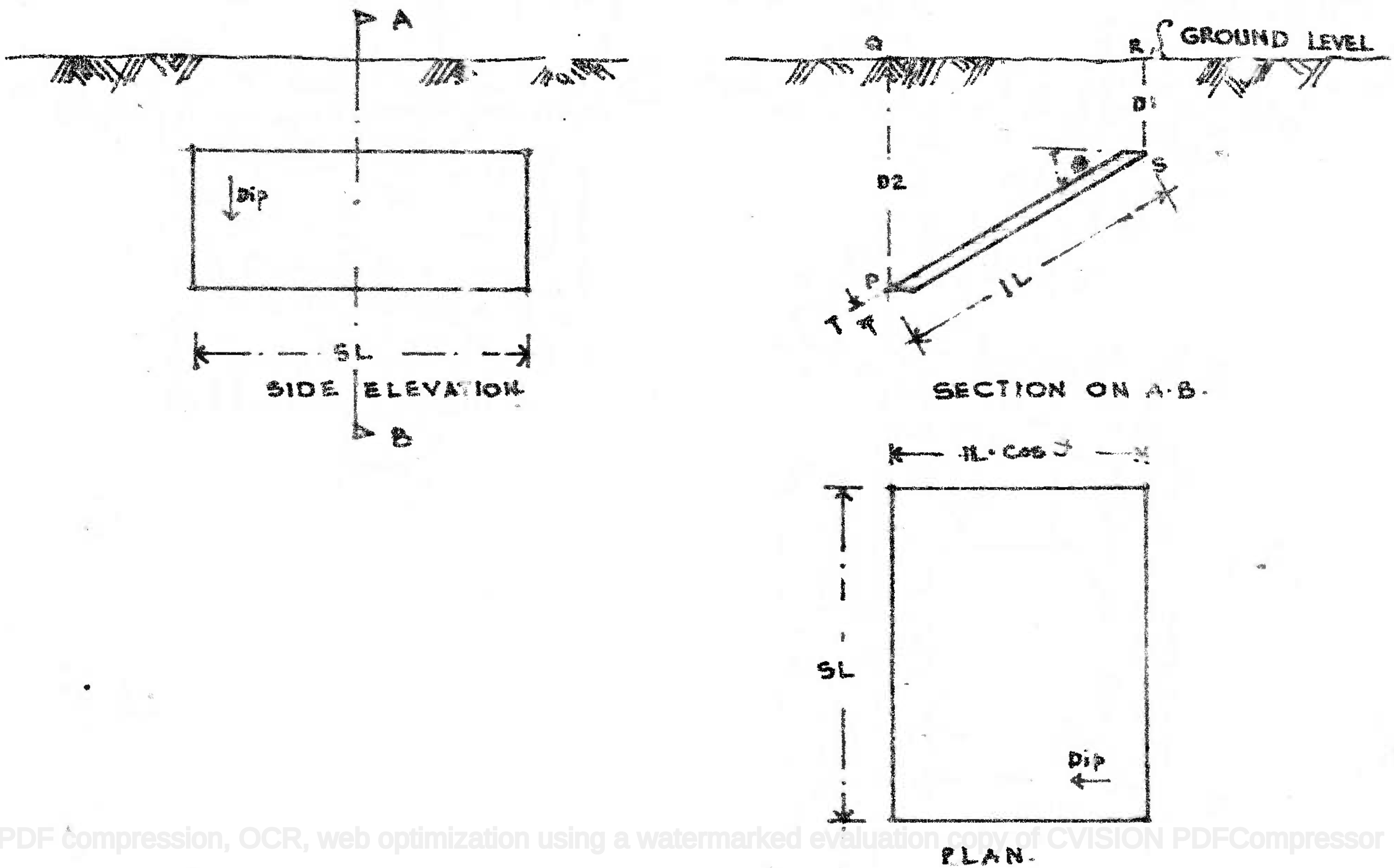
(2) Depth range of the coal seam.

To get a complete geological picture of the seam we need information on the dip, or the inclination of the coal seam. The JCSL statement did not give dips for each coal seam. Also it was not feasible to record dips of individual seams. However data on the average dip in a geological block was available in the geological maps obtained from the coal industry, the dip of a coal seam was assumed to be the average dip of the geological block that it belonged to.

Thus from the information reported it is possible to arrive at the geological profile of the coal seam as follows:

See Figure 3.3.

OUTLINE OF A MINE FIELD Fig. [3.3]



T = The average thickness of the seam.

D1 and D2 = The range of depth.

R = Total proved reserves in the seam.

Θ = The dip of the seam.

The inclined length IL = (D2 - D1) Cosec.Θ .

Volume of the coal in the seam

V = R/(SG : The specific gravity of coal*)

If SL is the strike length[1]

$$V = IL.\ T.\ SL$$

$$SL = V / IL.T = R / SG.IL.T.$$

$$= R / SG.(D2 - D1)\ \mathrm{Cosec}\,\Theta .T.$$

Thus from the information reported the geological profile of the seam may be arrived at.

3.4 Scheme for estimating the seam level total cost function

3.4.0 Cost parameters and the base year

The base year of the exercise is 74/75. As outlined in Section (3.2) unit cost for each activity are arrived at by adding up expenditure on variable inputs, labour and an element of amortised capital costs depending upon its life and the discount rate. All costs in the

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* The specific gravity of coal is taken to be 1.5 gram / cc i.e. see Vol.88 G.S.I. (1971).

1 For a detailed discussion of this See - Shevyakov L., "Mining of mineral deposits", Foreign languages publishing house, Moscow.

base case are expressed in terms of 74/75 market prices and the discount rate used is 12%. The sensitivity of the cost functions and the resulting production programme is studied by departing away from the base case in two directions.

Case A : The impact of increased scarcity of capital is studied by increasing the discount rate to 18%.

Case B : The Labour costs are re-estimated at a shadow wage rate, .714 times[1] the market wage rate the model is reworked by decreasing the wage costs by a factor of .714.

Given a coal seam the method of estimating the unit cost of production is schematically represented in Fig. (3.4). For each coal seam, we assume that the mining engineer choses that technique of production which allows him to work it at minimum cost per tonne, which includes the choice of whether the seam is to be worked by underground or open cast methods. In what follows follows we shall outline how given the geological profile of the seam thse are estimated.

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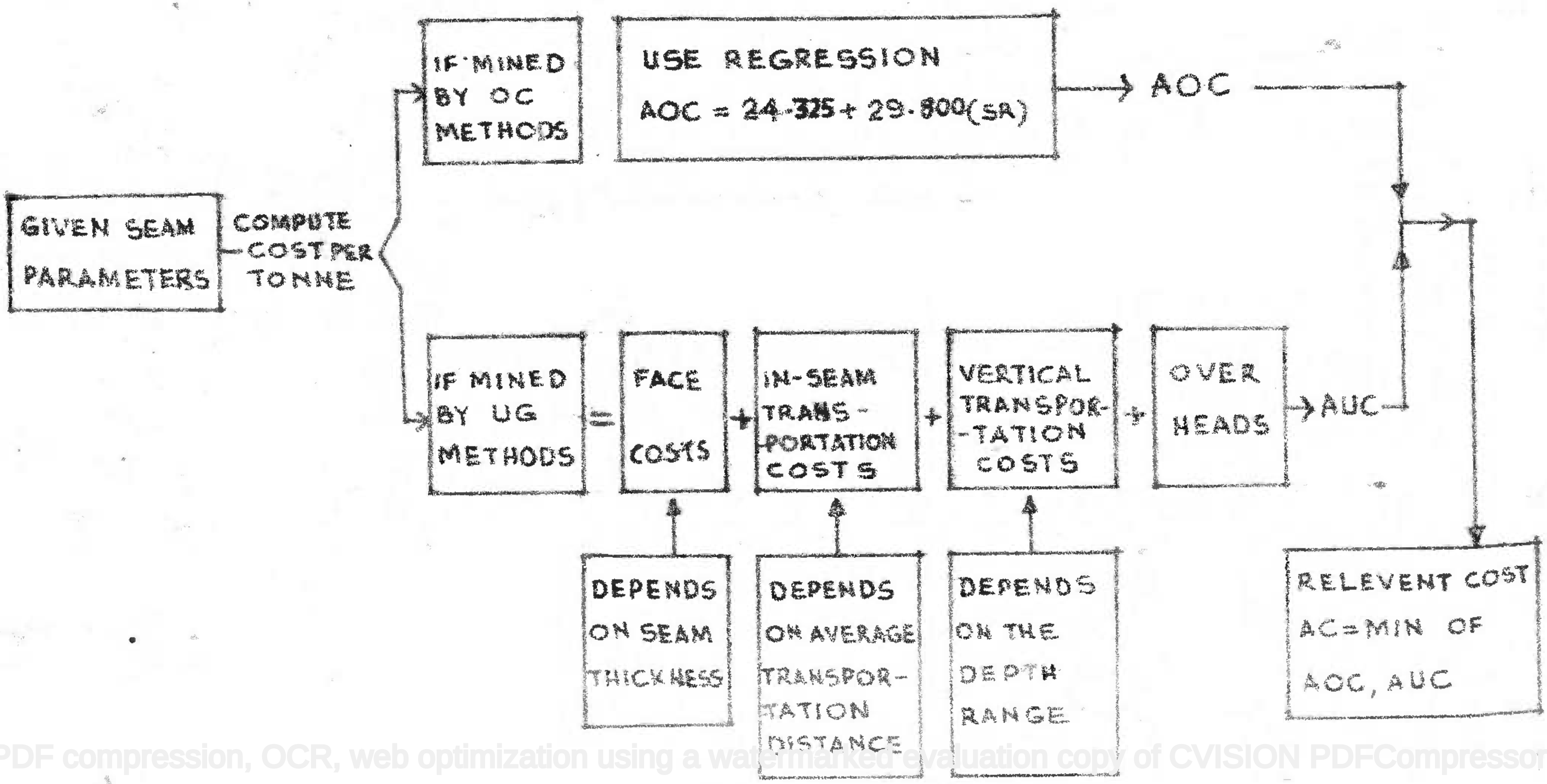
1 Planning Commission (1974) gives the ratio of social to market wage rate **for different** states. The rate used refers to that of unskilled urural labour for Bihar State. All the coal fields (except Raniganj) are in the Bihar State, and the mining industry draws its works force from the rural population, thus the ratio used may be considered an adequate approximation to the social cost of labour employed in the coal industry.

Fig. [3.4] SCHEME OF ARRIVING AT THE PRODUCTION COST FOR A COAL SEAM



3.4.1 Estimating unit cost for underground mining:

In our analysis we work with an idealised model of an underground mine in which coal mining takes place in the following stages.

(1) Face work : Drilling, blasting coal, and transporting it along the face before it is put on a trunk transportation system.

(2) Transporting coal 'in-seam' by a trunk transportation system to the shaft bottom at the centre of the mining property.

(3) Transporting coal vertically through a shaft to the surface.

In a sense the three stages of mining can be identified with costs which depend upon three geological parameters of the coal seams. Face costs depend roughly on the seam thickness, in-seam transportation costs on the dimensions of the seam, and the vertical transport costs depend upon the range of depth. The total cost of mining is the sum of costs incurred in each of these operations. In what follows we shall outline in detail how each of these costs are estimated.[1]

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1 An attempt was made to statistically estimate the total cost function at the seam level (integrating all three stages) using geological parameters associated with a coal seam as explanatory variables. This analysis reported in Appendix3AIwas not successful.

A. Face Costs

It is assumed that a coal seam is mined by repeating a standardised coal production unit (a coal face) using a mining technique which depends on the thickness of the seam. Thus by face costs we mean the expenditure incurred in mining coal and transporting it to the out bye end within the mine before loading it on a trunk transportation system. These are the costs incurred in timbering, explosives, coal cutting machines, labour employed at the face and the face transportation system. The geological parameter which determines the face costs is the seam thickness. For thin seams, the workers have to work in a crouched position, the movement of men and material becomes difficult. In some cases, the roof has to be blasted to facilitate easy movement in galleries. Similarly for thick seams, the face costs are high because the strata control becomes difficult. Face costs vary across seam thickness because the mining engineer is forced to switch to inherently expensive, technically feasible mining techniques as he moves to thick or thin seams. In our idealised model of a coal mine, the mining technique depends entirely on the thickness class to which the coal seam belongs and there is no variation in costs within a thickness class. Thus cost variation due to factor productivity changes for the same mining technique practiced for varying thickness within the same thickness category are ruled out. Seams are divided into three thickness categories and technically feasible mining technologies for each of these are identified and presented in table (3.4.1). Table (3.4.2) presents capital and operating costs incurred in mining coal from a standardised[1] face using each of these mining techniques. Units costs for all the three cases considered are reported. As seen from table (3.4.1) for thick and thin seams, there is one dominant technology. For the seams of average thickness, purely from cost considerations, only the cheapest technique need be considered. Currently almost all the underground production comes from manual board and pillar, which is an established technique of mining. We shall assume that there are limitations to the rate at which the mechanised longwall technique, which is the cheapest, can be introduced. Also in many circumstances the three techniques may not be strictly competitive depending upon on the spot engineering considerations. For seams of average thickness, we shall use costs referring to

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1 A Standardised coal face is defined in Table (3.4.2).

Table 3.4.1

Feasible mining technologies for each thickness class.

Thickness Class	Seam thickness Range	Mining technology feasible
Thin seams	Less than 2 M.	Long wall with solid blasting.
Average thickness seams	2M to 4.5M	The technology adopted is a mix of - (a) Bord and pillar with manual loading. (b) Bord and pillar with mechanised loading. (c) Mechanised long wall.
Thick seams	More than 4.5M	French sublevel caving.

Source: Compiled from P.D.Nath (1975).

Table 3.4.2

Unit cost of mining coal from a standardised[1] face using different technologies.

Cost in Rs./tonne

Items of Cost	Bord and pillar (B.P.) manual loading	B.P. with mechanised loading	Mechanised longwall	Longwall L.W. with solid blasting	French sub-level car-ving
1. Labour (1)	22.72	14.70	15.89	15.45	17.47
2. Other operating costs (2)	12.81	11.90	6.76	18.46	20.22
3. Total operating costs (1)+(2)	35.53	26.00	22.65	33.91	37.69
4. Capital costs Rs./tonne	29.49	31.50	43.25	58.61	67.50
5. Life of equipment (years)	18	18	18	18	18
6. Capital charge at dis. rate					
(a) 12%	4.07	4.35	5.97	8.09	9.31
(b) 18%	5.57	5.95	8.17	11.07	12.75
7. Total costs for base case	39.60	30.35	28.62	42.00	47.00
8. Total costs for Case A 18% dis. rate	41.10	31.95	30.82	44.98	50.44

contd...

Table 3.4.2 (contd.)

Cost in Rs./tonne

Items of Cost	Bord and pillar (B.P.) manual loading	B.P. with mechanised loading	Mechanised longwall	Longwall L.W. with solid blasting	French sub-level carving
9. Total costs for case B shadow wage rate	33.10	26.31	24.07	37.58	42.00

Source: Compiled from Nath P.D. (1975)

1 Computations are reported with reference to a standardised coal face, with the following characteristics -

- For a long wall face, the face length is 150M and a production of 768 tonnes/day.
- The bord and pillar methods the pillar size is 21 M and a production of 450 tonnes/day.

an average technology, based on the projected share of each of these[1]. The industry projections for the share of each of these technologies is indicated in Table (3.4.3).

Table 3.4.3

Technology	Projected share (%) of each technology in the total output for years				
	76/77	78/79	83/84	85/86	Weighted average over ten years[2]
Manual Board and Pillar	100	76.82	33.85	22.49	60
Mechanised Board and Pillar	0	9.62	23.71	27.46	25
Mechanised long wall	0	13.56	42.44	50.05	15

Source: Project black diamond.

1 This may only be considered as approximation since the share resulting from the model may be at variation with the share used here.

2 The share is arrived at by using the information at the four points and by assuming a linear transition from one state to the other.

Thus the medium thickness seams are worked by a mix of technologies over the next ten years and the cost corresponds to the weighted average (weighted by the % share) of individual technologies, computed as follows:

$$\text{Cost per tonne} = 39.60 \times .60 + 30.35 \times .15 + 28.62 \times .25$$
$$= 35.47 \text{ Rs.}$$

The costs for each of these technologies are worked out at 18% discount rate (case 4) and shadow wage rate (case B); and the above proportions are used to arrive at the cost referring to an average technology.

B. <u>Cost of in-seam transport</u>

The cost of in-seam transport is the expenditure that is incurred in transporting coal from the face (outbye end) to the shaft bottom. These are costs incurred on men, and material, and the capital costs incurred in the installation of the trunk transportation system. The total in-seam transport cost therefore depends upon the length of haul, from the outbye end to the shaft bottom. Clearly in any given year, the cost of in-seam transport would depend upon the choice of faces to be worked - whether it is near or far away from the shaft bottom. We shall however compute the in-seam transport cost incurred per tonne on the basis of average distance over which a tonne of coal is transported over the life of the seam, thus inter-seam variation in transport cost is a result of inter-seam variation is haul-length which depends upon the dimensions of the seam.

If, CHT : Average in-seam transport cost over the life of the mine.

ITC : Cost / tonne-KM of carrying coal underground.

L : Average distance over the life time of the mine.

then CHT = ITC.L Rs./tonne.

Fig (3.5) shows the idealised mine design in which the shaft is sunk at the centre of the property. The average distance (L) over which a tonne of coal is transported is computed by -

$$L = \frac{1}{4} (SL\ 1 + SL\ 2) = XY+YZ$$

where

SL 1 is the total length along the strike

SL 2 is the length along the incline.

We assume that the underground transportation is carried out by means of a belt conveyor. Estimates of capital andoperating costs incurred in operating a belt conveyor system to carry 1 m.t./annum, underground are computed from engineering calculations.[1] We assume

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1 Mishra, B.C. (1976).

constant returns to scale and that costs vary linearly with distance. The capital costs are amortised over the life of the belt conveyor as discussed before. The estimates of cost of underground transport/tonne kilometre are reported in table (3.4.4).

Table (3.4.4)

Cost of underground transportation using belt conveyor system of length one kilometre.

Item		Cost Rs./tonne KM.
Labour (Rs./tonne)		2.4
Operating cost stores and power (Rs./tonne)		1.2
Cost of equipment (Rs./tonne)		13.67
Life of equipment (years)		6
Capital charge at discount rate	12%	6.56
(Rs./tonne)	18%	8.58
Total cost for base case (Rs./tonne)		10.16
Case A (18% dis. rate)		12.18
Case B Shadow wage rate		9.47

C. <u>Cost of vertical transport</u>

In the idealised model of the mine, coal is transported from the centre of the property to the surface through a vertical shaft. We are trying to

Fig. [3.5] IN SEAM TRANSPORTATION DISTANCE

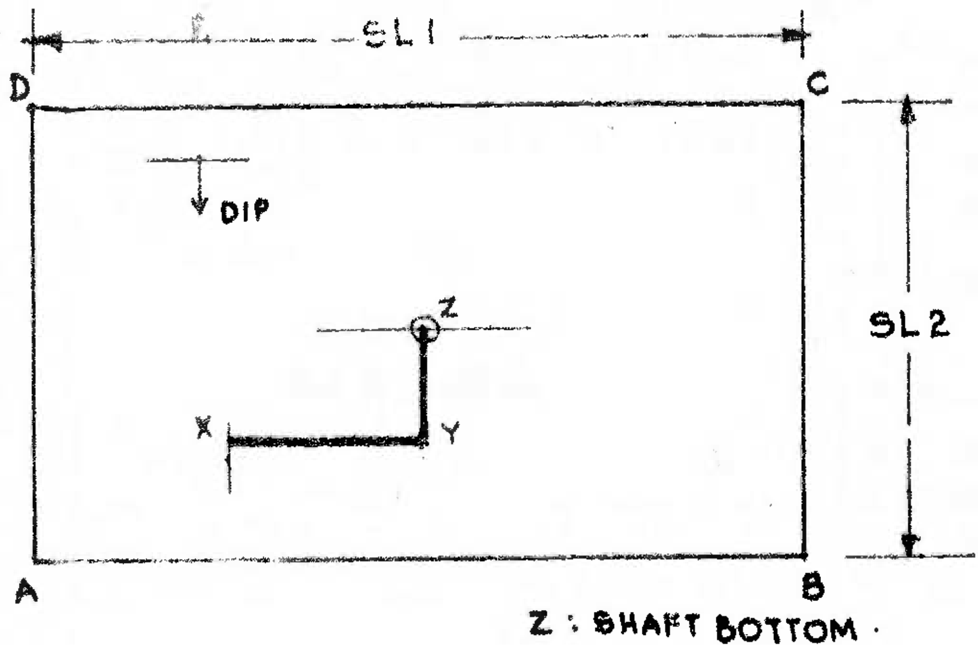
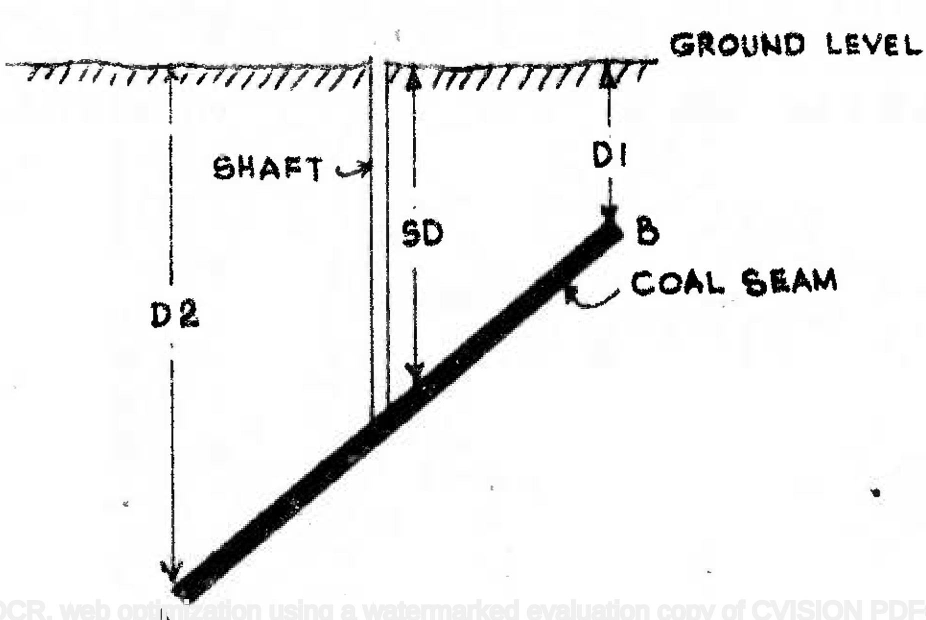


Fig [3.6] VERTICAL TRANSPORTATION DISTANCE.



identify expenses incurred in mining that depend upon the depth of operation these consists of haulage costs together with pumping and ventilation costs. In both cases we have engineering estimates available for costs incurred in handling 1 m.t./annum from a 100 M. deep shaft. We assume that all these costs are linearly proportional to depth and that there are constant returns to scale. Thus interseam variation in depth costs is a result of interseam variation in depth. Costs of vertical transport are estimated by -

$$CVT = VTC \cdot SD$$

where

CVT : Total cost of vertical transport.

VTC : Cost/tonne of servicing a seam 100 M.deep.

SD : Depth of the shaft.

As shown in figure (3.6) SD is = (D1 + D2)/2 where D1 and D2 are the range of depth of the coal seam.

Table (3.4.5) gives the engineering estimates of cost of carrying a tonne of coal to the surface from a coal to the surface from a coal seam 100 M. deep.[1]

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1 Costs incurred in construction of the opening of a 100 M. deep shaft to produce 1 M.t/annum are Rs.12.45 Million. These strictly would have to be considered as sunk non shiftable costs and thus the life over which those have to be amortised will depend upon the life of the seam which it will service. If the life of the seam is computed to be 10 years instead of 15 the capital costs would be understated to the extent of 0.37 Rs./tonne, which is only 2.46% of overall cost of vertical transport.

Table 3.4.5

Cost of vertical transport/tonne / 100 M. deep shaft.

Item	Haulage Cost	Pumping and ventilation costs	Total Costs
1. Wages & Salaries	3.66	1.74	5.40
2. Power and Stores	.96	1.01	1.97
3. Total operating Cost 1+2 (Rs./tonne)	4.62	2.75	7.37
4. Cost of constructing the opening and providing equipment Rs./tonne	36.44	12.86	49.30
5. Life (years)	15	10	-
Capital charge at dis. rate = 12%	5.36	2.27	7.63
18%	7.14	2.85	
Total cost for Base Case	9.98	5.02	15.00
Case A 18% dis. rate	11.76	5.60	17.36
Case B Shadow wage rate	8.93	4.25	13.18

Source: Mishra B.C. (1976)

D. Output and Administrative Overheads

In general only a fraction of the in situ reserves can be mined. In our study we have adopted a figure of .5 based on the judgement of mining engineers. H.L.Rhodes observes[1] "An accepted factor in Britain for translation of workable reserves (which are generally less than in-situ reserves) into minable reserves is .5" thus by applying 50% to the in-situ reserves we are being optimistic.

To the underground mining costs as discussed before, are added administrative overheads and surface handling costs. From our data on Jharia coal field[2], it is found that these are Rs.8.56 / tonne, to this are added surface handling charges of Rs.1.5/tonne. Thus the total overheads of Rs.10/tonne are added to the unit operations costs for underground mining.

3.4.2 Estimating unit costs if the seam was mined using open cast (C) techniques

In the case of open cast operations it was possible to summarise the geological attributes by means of a single variable namely the strip ratio. The strip ratio (also known as the coal overburden ratio expressed as M^3/T) indicates the amount of overburden, which must be

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1 Rhodes, H.L. (1972).

2 Appendix 3A1.

removed to win a tonne of coal. The strip ratio is found to be an adequate proxy for the geological complexity of the coal seam whereas no single proxy could be designed for UG mining.

Data were collected from 26 open cast workings of the central coal fields limited. The data gave, the strip ratio and cost per tonne for two halves of the year 74/75. These were pooled together to make in all 52 observations. An analysis of the cost data shows, that the average per tonne expenditure on capital (depreciation + interest) forms only about 2.54% of the average cost of mining. The capital charges in the industry are computed by using the straight line depreciation and charging a 10.5% interest on long term loans. The loan/equity ratio in the coal industry is 2:1. Thus the factor by which the gross fixed assets are multiplied by to arrive at the capital charge is computed as follows. If K is the capital charge per tonne, L is the life of open cast equipment, then, the depreciation and interest charge / tonne is as follows.

$$p = \left\{ \frac{K}{L} + g.k.r. \right\} = K \left\{ \frac{1}{L} + g.r. \right\}$$

where K/L is the depreciation

g is the loan equity ratio.

Thus the implicit capital recovery factor is $(\frac{1}{L} + g.r.)$, which for L = 12 years, g = .66 and r = 10.5% is equal to .153. This implies an economic discount rate of roughly 11% / annum (for a life of 12 years). This establishes compatibility with the way underground mining costs are computed. Cost per tonne at 18% discount rate are computed by increasing the capital charge for each colliery by a factor of .208/.153, (where .208 is the capital recovery factor for 18% discount rate for a life of 12 years). Similarly for case B, the wage costs are decreased for each collieary by a factor of .714.

The impact of the strip ratio on the average costs as defined above, was estimated by a linear regression. Table 3.4.7 reports the results of the regression analysis for all three cases considered.

Our aim is to use the relation to predict the cost per tonne on the basis of the strip ratio. The linear relationship fits with an R^2 of .45504. which is low, however, both the coefficients have small enough standard errors to make them significantly different from zero.

The addition of the term $(SR)^2$, improves the fit only marginally (R^2 increasing from .45 to .46) at the same time both the coefficients have low t values.

The log linear form fits with a lower R^2 (.326). Thus in our study it was decided to accept the linear relation as the most suitable underlying model. This relation is used in combination with the strip ratio to predict cost/tonne.

The strip ratio of the seam is arrived at as follows : See Fig. (3.3).

Volume of the overburden associated with the seam.

V = Volume of the trepezoid PQRS

$$V = \tfrac{1}{2}\,(D1 + D2) \cdot QR \cdot SL$$

$$QR = (D2 - D1) \cdot \mathrm{Tan}\,\theta$$

$$V = \tfrac{1}{2}\,(D2 + D1)\,(D2 - D1) \cdot SL / \mathrm{Tan}\,\theta$$ (Million Cubic Metres)

Reserves in the coal seam = R m.t.

Strip ratio = V/R = M^3 / tonne.

It is important to recognise at this stage, that the strip ratio reported in the data corresponds to the strip ratio in that half year, whereas the strip ratio computed from the seam profile corresponds to the strip ratio over the entire life of the mine. It is known that the half yearly strip ratio may fluctuate a great deal over the life of the mine, (about the average strip ratio for the entire life). We hope that in the cross section data these variations are cancelled out, the half yearly strip ratio being higher than the average strip ratio in some and lower than the average in others thereby reducing bais in prediction.

Table - 3.4.7

Regression analysis with strip ratio and cost/tonne for the base case, 12% discount rate and market wage rate.

(1)	Average Cost (AC) =	24.325	+	29.800	+	R^2 = .45504
		(9.917)		(4.612)		
		t = 2.452		t = 6.461		R = .6745

(2)	AC =	39.4167	+	16.4992 SR	+ 2.32205$(SR)^2$	
		(30.7809)		(14.604)	(2.4187)	R^2=.4651
		t = 1.2805		t = 1.129	t = .960	R =.6819

(3)	Log AC =	3.8875	+	.697639 Log SR	R^2 = .32673
		(4.4154)		(.1416)	
		t = .8804		t = 4.925	R = .5716

CASE A : At 18% discount rate.

(4)	AC=24.4832	+ 29.7794 SR	R^2 = .4559
	(9.892)	(4.6003)	
	2.4749	t = 6.473	R = .6752

CASE B : At shadow wage rate.

(5)	AC=21.8442	+ 21.6542 SR	R^2 = .4529
	(7.2366)	(3.3652)	
	t = 3.0185	t = 6.434	R = .6730

3.4.3 Cost of Washing and impact of quality

The reserves of coal in a seam are divided into four quality classes, depending on the ash content. Coal in each class when washed to a prespecified quality has a specific yield. We use an econometrically estimated relation to predict yeield in each of the quality classes. For this purpose the following relation was estimated based on data from 8 washeries.

$$L = 26.025537 + 2.80465\ RA - 3.582583\ CA^{1} \quad (1)$$

L = Loss of coal in the washery, yield = (100-L) %

RA = Ash in the Raw coal, as mined.

CA = Ash in the clean coal.

The ash in the raw coal was assumed to be washed down to 17%.[2] Given the data on the raw coal ash RA, and the quantity of coal in each quality class it was possible to arrive at an average yield of a coal seam.[3] The predicted yield in the four quality classes was: (predictions being made at the mid-point of the interval).

Quality (i)	Ash %	Yield % (ai) at 17% ash.
1	less than 17	100
2	17 to 24	76.6
3	25 to 34	51.0
4	35 to 50	20.0

Source : Using regression (1)

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1 For a detailed discussion see Appendix 3A2.

2 See Appendix 3A2.

3 This rules out the possibility of selective mining within a coal seam, nevertheless this may not always be a technically feasible option.

Table - 3.4.7

Unit cost of processing one tonne of raw coal

Item		Cost Rs./tonne
1. Wages and Salaries		1.38
2. Other operating costs		11.07
3. Total operating costs (1)+(2)		12.45
Capital Costs Rs./tonne.		57.05
Life of the washery (years)		20
Capital Charge at discount rate		
Rs./tonne	12%	7.64
	18%	10.67
Total cost for base case 12% discount rate		20.09
Case A 18% dis. rate		23.12
Case B at Shadow wage rate		19.69

Source : Feasibility Report for Nandan Washery.

If 'R' MT were the total minable reserves and Ri are the reserves in the i^{th} quality class.

$$R = \sum_i Ri$$

The clean coal output when the coal from this seam is washed can be computed as follows:

$$Q = \sum_i ai \quad Ri$$

where Q = Clean coal output

ai = Yield in the i^{th} quality class

∴ The average yield factor Y for a seam with a composition Ri (i=1, 2, 4) is computed as

$Y = \sum_i ai\ Ri / R$, which tells us the amount of clean coal obtained when a representative tonne of coal from this seam is washed. The cost of processing a tonne of raw coal is taken again from detailed engineering estimates of the cost of setting up a washery of capacity 1.2 M.t/annum. Assuming constant returns to scale, the cost of processing a tonne of raw coal is as set out in Table 3.4.7.

3.5 Data and results

Table (3.5.1)[1] shows the total Prime and Medium coking coal reserves in India by coal fields, each of which is divided into geological blocks. The reserves in each block are classified according to their

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1 Tables 3.5.1 to 3.5.10 are given at the end of section 3.5.

quality class and degree of uncertainty indicated by the proved, indicated and inferred reserves. The total proved reserves of prime and medium coking coal are 3645.29 mt. and 4616.67 m.t. respectively. For each of these geological blocks, the cumulative cost function and yield function is estimated. The interblock variation in geological complexity is captured for the base case i.e. at market prices (74/75) and 12% discount rate. Table (3.5.2) gives the summary of all the parameters used in constructing the BLCCF.

Table (3.5.3) shows data on a sample block JP3, the Mahal block of the Jharia coal field which contains in all 13 seams. To each seam in the block the estimation procedure outlined in section 3.3 is applied to arrive at the seam level total cost curve and the average yield of clean coal. These seams are then ranked and are reported in Table 3.5.4. Of the 13 seams four are mined using open cast technique, and the rest are mined using underground methods. The choice of mining a seam by underground or open cast methods is made separately for each seam depending upon its profile.

Table 3.5.5 shows the broad characteristics of the geological blocks. As the in-situ reserves are first translated into minable reserves and then are further translated into useful clean coal reserves washed down to 17% ash. The estimate of the useful coking coal reserves is considerably scaled down from 3645.29 M.T. to 1416.219 M.T. for prime coking coal, and from 4616.676 M.T. to 1157.806 M.T. in the case of medium coking coal. However our knowledge about the reserves must not only relate to the total availability but also to the cost structure of the deposit.

Table 3.5.6 gives us in each block, the proportion of the open cast to underground production - suggested by the model. This proportion is low in the Jharia coal field and becomes much higher as we move to outlaying fields like Ramgarh, North Karanpura and Bokaro. This seems to be the result of the large number of thick seams occurring in this area. The last column in table 3.5.6 indicates the maximum strip ratio from all the seams that are mined using the open cast method in other words it is the strip ratio that is associated with the most expensive open cast mine in that block. It is important to notice that this exhibits a considerable variation across coal fields and even within a coal field. Thus it seems difficult to suggest a cut-off strip ratio for the entire industry as the strip ratio above which UG mining is recommended. The choice of open cast vs underground has to be made separately for each seam.

The procedure outlined in Section (3.3) is applied to all the seams in a block and the set of points (CJ, QJ) defined by the following relations is obtained -

$$C_J = \sum_{i=1}^{i=J} ACi\bar{Q}i \quad , \quad Q_J = \sum_{i=1}^{i=J} \bar{Q}i$$

The BLCCF is obtained by approximating this set of points by the following polynomial.

$$C = A1Q + A2\ Q^2 + tu \qquad tu \sim (0,\ \sigma_u^2) \qquad (1)$$

However while fitting the form (1) by OLS, in some blocks there was a high degree of multicollinearity between Q and Q^2. Similarly it was found that residuals in (1) increased with Q, the cumulative output. The following alternative functional form was estimated

$$\frac{C}{Q} = A1 + A2Q + te, \; te \sim (0,\ \sigma_e^2) \qquad (2)$$

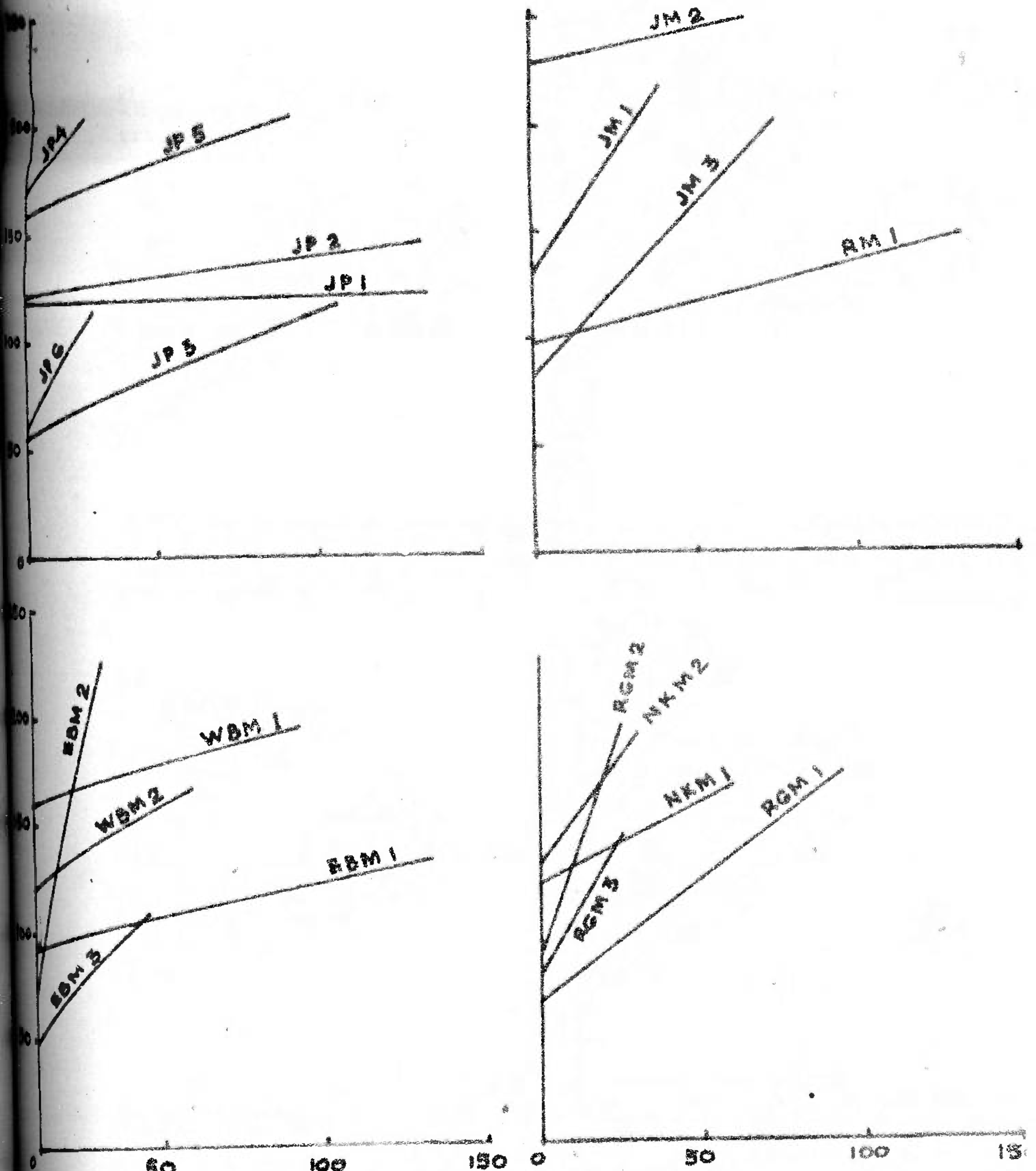
Thus if tu were not distributed on $(0,\ \sigma_u^2)$ but rather $tu = te.Q$ where $te \sim (0,\ \sigma_e^2)$ estimating (2) by OLS would remove hetroskedacity in (1).

The yield factor A 3 is estimated by the seatter of Raw coal and Clean coal content in each seam (RJ, QJ), by the following

$$A3 = \frac{\sum_J RJ \cdot QJ}{\sum_J Q_J^2} \quad .. (3)$$

Estimates of A1, A2 are presented in Table 3.5.7, along with the R^2 and standard errors of the estimates. These, are summarised and reported together with the yield coefficients A 3 , in Table 3.5.8. The cumulative cost functions are represented by fig. 3.3 by plotting the marginal cost curves MC = A1 +2A 2.Q. A1 represents the marginal cost at Q = O and A2 shows how fast the marginal cost increases. The values of A1 and A2 are thus representative of the nature of geological deterioration at the level of a block. Blocks which have poor yield JM2, EBM2, have steeply rising marginal costs. Similarly the high cost blocks of North Karanpura NKM1, NKM2, also seem to be basically the result of poor quality coal seams in this area. (A3 = 1.889 and 1.851). However, it must be stressed that no generalisation seems to emerge. It does not seem possible to have an idea of the underlying economic importance by looking at falling quality or increasing geological complexity alone. It thus seems best to think in terms of a BLCCF

BLOCK LEVEL MARGINAL COST FUNCTIONS Fig. [3.7]



X- AXIS: CUMULATIVE CLEAN COAL OUTPUT (M.T.)
Y- AXIS: MARGINAL COST (RS./TONNE)

which summarises the simultaneous impact of these two dimensions of economic valuation of in-situ coal seams. The variation in the discount rate and the price of labour has an impact on the BLCCF that has to be traced from its impact on the seam level total cost curves. Depending upon whether, the seam is mined using OC or UG technology, the impact of making capital expensive or labour cheaper depends on the relative share of these components in overall costs of mining the seam. It is clear that seams in which capital intensive techniques are selected, (which have a higher share of capital costs) become more expensive as discount rate is increased to 18%. At the block level the impact can take the form of a change in ranking and change in the mode of mining. All these effects are summarised by observing the changes in A1 and A2; Table 3.5.9 reports the values of A1, A2 for the two cases considered. For an increase in capital costs the A1 and A2 increase uniformly over the base case whereas in the case of using the shadow wage rate they decrease uniformly over the base case. The increase in the capital costs increases the cost of open cast mining marginally and the decrease in labour cost decreases it significantly. Thus for both cases A and B, the model opts to exploit more seams by O/C methods than underground methods. Blocks in which these two technologies are competing would show decreased underground production in the two cases. Table 3.5.10 shows how the underground production falls in the two cases. As seen from the table 8 out of 20 blocks show no change which indicates that the geological structure of these blocks leads to mining technology, which is insensitive to the changes in the relative price of capital and labour.

Table - 3.5.1 COKING COAL RESERVES IN INDIA

Figures in Million Tonnes

Name	Proved Reserves Class 1	2	3	4	Unclassified	Total	Indicated	Inferred	Grand Total
PRIME COKING COAL									
JP1	607.99	1268.63	130.11	-	257.71	2264.414	1433.15	460.73	4158.32
JP2	247.60	457.96	-	-	-	705.64	-	-	705.64
JP3	89.48	133.80	41.57	-	-	264.68	102.56	-	367.41
JP4	-	74.36	-	-	-	74.36	-	-	74.36
JP5	-	244.51	-	-	-	244.51	-	-	244.51
JP6	29.81	21.79	30.95	-	-	91.55	-	-	91.55
TOTAL JHARIA	974.88	2201.05	211.63	-	257.71	3645.29	1535.76	460.73	5641.80
MEDIUM COKING COAL									
JM1	4.18	106.5	-	-	-	110.68	-	-	110.68
JM2	13.07	354.32	-	-	563.21	930.60	560.00	332.98	1823.58
JM3	3.87	37.2	245.16	67.28	34.88	388.39	1022.67	2993.93	4404.99
TOTAL JHARIA	21.12	498.02	245.16	67.28	598.09	1429.67	1582.67	3326.91	6339.19
RM1	262.76	132.32	15.94	-	282.96	693.98	593.05	796.08	2082.11
EBM1	5.60	120.91	-	-	361.66	488.17	106.96	-	595.13
EBM2	5.79	15.75	15.33	147.89	-	184.76	55.72	-	240.48
EBM3	31.53	68.864	19.77	5.48	6.095	131.79	228.18	-	359.97
TOTAL E.BO-KARO	42.92	205.524	35.10	153.37	367.756	804.72	390.86	-	1195.58

contd...

Table – 3.5.1 (Contd.)

Name	Proved Reserves Class 1	2	3	4	Unclassified	Total	Indicated	Inferred	Grand Total
WB1	23.30	53.41	345.41	.84	67.28	490.24	782.12	120.12	1392.48
WB2	–	67.61	253.28	3.8	29.73	354.42	1131.96	200.21	1686.59
TOTAL W. BOKARO	23.30	121.02	598.69	4.64	97.01	844.66	1914.08	320.33	3079.07
NKM1	.51	40.17	145.21	9.50	58.93	249.32	1636.42	405.83	2291.57
NKM2	–	2.1	88.79	11.89	49.07	151.85	723.07	149.73	1024.65
TOTAL N.KARAN-PURA	.51	42.27	234.00	21.39	103.0	401.17	2369.49	555.56	3316.22
RMG1	–	103.68	232.2	–	–	315.88	10.54	–	326.44
RMG2	–	11.6	27.036	–	12.09	50.726	187.43	33.88	272.036
RMG3	.27	49.53	14.49	–	6.58	75.87	73.86	37.24	186.97
TOTAL RAMGARH	.27	164.81	258.726	–	18.67	442.476	271.83	71.12	785.426
TOTAL M. COKING	350.88	1163.964	1387.616	246.68	1467.486	4616.676	7111.98	5070.00	16798.636
TOTAL PRIME+ MEDIUM	1325.76	3365.01	1599.246	246.68	1725.196	8261.966	8647.74	5530.73	22440.436

... 3.2 : SUMMARY OF THE PARAMETERS USED.

A. Underground Mining

Items of Cost	Face costs (Rs. / tonne / year)					In-seam Trans-port costs for belt-conveyor Rs/tonne/.KM	Vertical trans-port cost Rs/tonne/100 M.	Washing Cost Rs./tonne/year.
	Bord and Pillar (BP) manual loading	E.P. Mechanised loading	Long-wall (L.W.) mechanised	L.W. with solid blasting	French sub-level caving			
1. Labour	22.72	14.70	15.89	15.45	17.47	2.40	5.40	1.38
2. Other operating costs	12.81	11.90	6.76	18.46	20.22	1.20	1.97	11.07
3. Total operating costs (1)+(2)	35.53	26.00	22.65	33.91	37.69	3.60	7.37	12.45
4. Capital cost Rs./tonne	29.49	31.50	43.25	58.61	67.50	13.67	49.30	57.05
5. Life of equipment (yrs)	18	18	18	18	18	6	-	20
6. Capital charge at dis. rate								
a. 12%	4.07	4.35	5.97	8.09	9.31	6.56	7.63	7.64
b. 18%	5.57	5.95	8.17	11.07	12.75	8.58	9.99	10.67
7. Total costs for Base CASE (12% dis. rate)	39.60	30.35	28.62	42.00	47.00	10.16	15.00	20.09
8. CASE A 18% dis. rate	41.10	31.95	30.82	44.98	50.44	12.18	17.36	23.12
9. CASE B shadow wage rate	33.10	26.31	24.07	37.58	42.00	9.47	13.18	19.69

Source : Compiled from Nath P.D. (1975)
Mishra, B.C. (1976)
Wadhwa and Chakravarty (1975)

Table 3.5.2 (contd.)

B. Open Cast mining

BASE CASE : AC = 24.325 + 29.800. SR.

CASE A : 18% discount rate AC = 24.483 + 29.779. SR.

CASE B : Shadow Wage rate : AC = 21.844 + 21.654. SR.

Table - 3.5.3

DETAILED GEOLOGICAL DATA OF A SAMPLE BLOCK
(AS REPORTED IN THE JCSL REPORT)

All Figs in (Million Tonnes)

Name	Area Sq.Km.	Name of the Seam	Range of Thickness (M)	Range of Depth (M)	Provcd Reserves in-situ Class I	II	III	IV	Uncl-assi-fied	To-tal	Indi-ca-ted	In-fer-red	Grand Total
MAHAL BLOCK (JP3)	4.53	XVIII	1.3-3.8	0-609	-	9.35	-	-	-	9.35	3.06	-	12.41
		XVII	2.8-4.8	0.609	16.75	-	-	-	-	16.75	8.08	-	24.83
		XVII TOP	1.2-3.9	0-609	-	6.28	-	-	-	6.28	.05	-	7.13
		XVI BOTTOM	1.3-3.5	0-609	-	-	9.77	-	-	9.77	-	-	9.77
		XV COMBINED	16.8-17.77	0-709	33.84	-	-	-	-	33.84	6.41	-	40.25
		XV TOP	1.2-11.16	0-609	23.91	-	-	-	-	23.91	7.32	-	31.23
		XV BOTTOM	1.4-3.4	0.609	5.18	-	-	-	-	5.18	2.74	-	7.92
		XIV A	1.33-154.43	0-609	-	45.65	-	-	-	45.65	27.66	-	73.31
		XIV	2.96-7.30	0-609	-	16.25	-	-	-	16.25	9.64	-	25.89
		XIII	7.00-11.08	0-609	-	21.50	-	-	-	21.50	11.35	-	32.85
		XII	6.15-13.70	0-609	-	34.77	-	-	-	34.77	13.27	-	50.04
		XI	2.6-10.2	0-609	-	-	31.80	-	-	31.80	10.18	-	41.98
		X	1.4-5.00	0-609	9.80	-	-	-	-	9.80	-	-	9.80
		TOTAL	-	-	89.48	133.8	41.57			264.85	99.76		364.61

Source : JCSL Statement.

Table - 3.5.4 Seamwise costs of (JP3)

Name	(Minable) Gross Reserves (MT)	Clean Coal (Reserves) (MT)	Seam Thickness (M)	Transportation Distance (M)	Average Depth (M)	Strip Ratio M^3/T	Whether O/C or UG	Yield Factor	Average Cost (Rs/T)	Total Cumulative Output (M.T.)	Total Cumulative cost (M.Rs.)
XIV A	22.825	17.483	77.8	479.469	304.5	.244	OC	1.305	67.797	17.483	1185.36
XV Combined	16.92	16.92	17.285	486.407	304.5	1.103	OC	1.00	76.983	34.403	2487.92
XV Bottom	2.59	2.59	2.40	468.200	304.5	7.948	UG	1.00	115.798	36.903	2787.85
X	4.90	4.90	3.65	483.252	304.5	5.226	UG	1.00	115.847	41.893	33[illegible].50
XVII	8.375	8.375	3.80	463.473	304.5	5.019	UG	1.00	116.009	50.268	4327.08
XVI Bottom	11.955	11.955	6.18	481.362	304.5	3.086	UG	1.00	127.488	62.223	5851.20
XII	17.40	13.317	9.925	465.350	304.5	1.9[illegible]2	OC	1.306	131.670	75.540	7604.65
XIII	10.75	8.235	9.040	450.635	304.5	2.110	OC	1.305	138.836	83.775	8747.90
XVII TOP	3.14	2.405	2.550	474.798	304.5	7.480	UG	1.305	151.210	86.180	9111.60
XVIII	4.675	3.581	2.550	476.410	304.5	7.480	UG	1.404	151.357	89.761	9653.62
XIV	8.125	6.224	5.130	477.939	304.5	8.718	UG	1.213	166.348	95.985	10688.93
XVI Bottom	4.885	2.491	2.400	491.656	304.5	7.948	UG	1.038	227.406	98.476	11255.48
XI	15.90	8.109	6.400	470.279	304.5	2.980	UG	1.267	250.179	106.585	13284.18

Source : JCSL Statement.

Table - 3.5.5 : BROAD CHARACTERISTICS OF GEOLOGICAL BLOCKS

Name of the Block	Area (Sq.Km)	No. of Seams	Dip. (Degrees)	Miniable Reserves (M.T.)	Miniable Reserves at 17% Ash (M.T.)
JP1	NA*	18	25	1132.22	848.829
JP2	25.30	14	22.5	350.32	313.074
JP3	4.53	13	20	132.425	106.585
JP4	8.46	15	25	37.180	20.647
JP5	14.13	10	20	122.255	93.647
JP6	5.14	7	35	45.775	33.437
Total Prime Coking:					1416.219
JM1	NA	13	22.5	55.340	42.870
JM2	NA	7	20	452.230	149.064
JM3	NA	11	25	194.195	83.148
RM1	NA	15	20	346.990	214.41
EBM1	19.29	27	14	244.085	141.331
EBM2	17.17	13	15.5	92.38	27.625
EBM3	NA	15	11.5	65.898	46.047
WBM1	NA	13	10	245.120	126.990
WBM2	NA	13	17.5	177.21	93.834
NKM1	64.0	14	20	124.66	59.011
NKM2	35.8	11	20	75.925	29.541
RGM1	NA	17	15	157.980	93.840
RGM2	NA	22	10	50.49	25.363
RGM3	NA	26	10	37.935	24.732
Total Medium Coking					1157.806
Total (Prime+Medium Coking)				4130.983	2574.025

Source: Model Output.

*This area is not reported in the JCSL Statement.

Table - 3.5.6 : PROPORTION OF OPEN CAST/UNDERGROUND PRODUCTION IN EACH BLOCK.

Name	Minable Reserves (MT)	Production Open cast (MT)	Production Underground (MT)	OC/ TOTAL	Maximum Strip Ratio $\frac{M^3}{T}$
JP1	1132.22	427.145	1705.075	.3772	2.874
JP2	350.32	140.805	209.515	.4019	4.64
JP3	132.425	67.88	64.54	.5125	2.110
JP4	37.18	0	37.18	0	—
JP5	122.255	0	122.255	0	—
JP6	45.775	39.33	6.445	.8592	2.061
JM1	55.34	1. 26	54.08	.0227	—
JM2	452.23	298.27	153.96	.659	1.107
JM3	194.195	134.955	59.240	.694	1.801
RM1	346.99	145.695	201.295	.4198	2.225
EBM1	244.085	202.815	41.274	.8309	2.167
EBM2	92.38	76.72	15.66	.830	.798
EBM3	65.898	60.93	4.958	.9246	2.662
WBM1	245.12	151.78	93.335	.6192	2.805
WBM2	177.21	96.67	80.54	.5463	2.726
NKM1	124.66	120.86	3.795	.9695	1.592
NKM2	75.925	72.44	3.485	.954	1.546
RGM1	157.980	120.345	37.635	.7617	2.585
RGM2	50.495	10.165	40.325	.2013	2.448
RGM3	37.935	29.265	8.678	.771	2.880

Source: Model Output.

Table - 3.5.7 : RESULTS OF THE REGRESSION
$\Lambda C = \Lambda 1 + \Lambda 2O$ (BLOCKWISE)

Name	Λ_1	Λ_2	R^2
JP1	117.81437 (.8484) t = 138.85	.04435 (.00149) t = 30.591	.9831 (18)*
JP2	120.956 (3.781) t = 31.98	.2048 (.02114) t = 9.689	.886 (14)
JP3	54.029 (1.6815) t = 32.1304	.6161 (.023) t = 26.79	.9849 (13)
JP4	169.3396 (5.5814) t = 30.3396	1.5329 (.2966) t = 5.167	.8990 (5)
JP5	158.7934 (3.6248) t = 43.8089	.5152 (.6275) t = 8.211	.8939 (10)
JP6	57.9968 (4.1201) t = 14.0764	1.9234 (.1655) t = 11.621	.9643 (17)
JM1	129.6827 (14.7383) t = 8.798	2.2577 (.5308) t = 4.253	.62186 (13)
JM2	227.9495 (4.2588) t = 53.5236	.3161 (.0347) t = 9.103	.9431 (7)
JM3	82.4738 (22.1358) t = 3.7251	1.5711 (.361549) t = 4.345	.6772 (11)
RM1	98.7926 (6.5306) t = 15.1276	.3768 (.03850) t = 9.86	.8804 (15)
EBM1	91.4547 (7.427) t = 12.314	.3025 (.07836) t = 3.860	.61172 (27)
EBM2	68.478 (5.4811) t = 12.4934	5.2697 (.4114) t = 12.809	.9371 (13)
EBM3	49.7310 (2.8111) t = 17.690	1.2760 (.0921) t = 13.854	.9357 (15)

*Figures in bracket indicate the number of seams (observations).

Table 3.5.7 (contd.)

Name	A1	A2	R^2
WBM1	156.044 (1.675) t = 93.12	.4173 (.01864) t = 22.376	.9785 (13)
WBM2	160.6697 (2.7598) t = 58.2162	.5657 (.05081) t = 11.133	.9184 (13)
NKM1	118.6098 (1.6438) t = 72.1524	.8381 (.03307) t = 25.338	.9816 (14)
NKM2	129.5736 (1.795) t = 72.1626	2.0469 (.08865) t = 23.088	.9833 (11)
RGM1	53.8153 (5.6001) t = 9.609	1.1533 (.09839) t = 11.721	.8729 (17)
RGM2	94.0639 (2.85) t = 33.004	4.8590 (.1919) t = 25.319	.9697 (22)
RGM3	80.8579 (1.9818) t = 40.7988	2.4943 (.1109) t = 22.489	.9546 (26)

Source: Model Output.

Table – 3.5.8 : COEFFICIENTS OF BLOCK LEVEL CUMULATIVE COST-FUNCTIONS/YIELD FUNCTIONS

Name	Minable Reserves (M.T.)	Clean Coal at 17% ash (M.T.)	Yield Factor A3	A1	A2
JP1	1132.20	848.829	1.3096	117.8143	.04435
JP2	350.20	313.074	1.0840	120.9560	.2048
JP3	132.425	106.585	1.1835	54.0290	.6161
JP4	37.18	20.647	1.3055	169.3396	1.5329
JP5	122.25	93.647	1.3055	158.7934	.5152
JP6	45.77	33.437	1.3660	57.9968	1.9234
JM1	55.34	44.87	1.2966	129.6827	2.2578
JM2	452.23	149.064	3.027	227.9495	.3161
JM3	194.195	83.148	2.2160	82.4738	1.5711
RM1	346.990	214.41	1.673	98.7927	.3768
EBM1	244.085	141.331	1.677	91.4547	.3025
EBM2	92.38	27.625	3.327	68.4780	5.2693
EBM3	65.898	46.047	1.385	49.7310	1.2760
WBM1	245.120	126.990	1.889	156.044	.4173
WBM2	177.21	93.834	1.851	160.6697	.5657
NKM1	124.66	59.011	2.135	118.6098	.8381
NKM2	75.925	29.541	2.599	129.5736	2.0469
RGM1	157.980	93.840	1.680	53.8153	1.1534
RGM2	50.49	25.363	2.010	94.064	4.8590
RGM3	37.935	24.732	1.497	80.8579	2.4943

Source: MODEL OUTPUT.

Table - 3.5.9 : COEFFICIENTS OF THE BLCCF FOR THE THREE CASES CONSIDERED.

NAME OF THE BLOCK	BASE CASE		CASE A 18% DIS. RATE		CASE B. SHADOW WAGE RATE	
	A1	A2	A1	A2	A1	A2
JP1	117.814	.044	121.053	.055	97.033	.041
JP2	120.956	.205	131.628	.217	104.749	.157
JP3	54.029	.616	56.496	.679	49.063	.506
JP4	169.339	1.533	189.084	1.782	151.029	1.431
JP5	153.793	.515	177.080	.595	143.731	.455
JP6	57.997	1.923	60.166	2.047	49.076	1.628
JM1	129.683	2.258	144.307	2.586	116.803	2.037
JM2	227.949	.316	252.654	.272	204.634	.210
JM3	82.474	1.571	89.786	1.645	74.449	1.304
RM1	98.793	.377	113.193	.387	91.890	.306
EBM1	91.455	.302	96.311	.354	79.039	.290
EBM2	68.478	5.269	80.450	5.265	64.020	4.523
EBM3	49.731	1.276	52.853	1.272	46.836	.961
WBM1	156.044	.417	160.390	.507	128.604	.396
WBM2	160.669	.566	155.935	.864	124.386	.688
NKM1	118.610	.838	113.829	.886	102.032	.691
NKM2	129.573	2.0470	133.867	2.259	112.531	1.740
RGM1	53.815	1.153	57.296	1.206	49.276	.936
RGM2	94.064	4.859	99.831	5.477	82.681	4.374
RGM3	80.858	2.494	83.955	2.718	71.154	2.014

Source : Model Output.

Table - 3.5.10 : TOTAL UNDERGROUND PRODUCTION FOR ALL THREE CASES.

(Figs. in million tonnes)

Name	BASE CASE	CASE A	CASE B
JP1	705.08	337.42	337.42
JP2	209.51	134.26	111.20
JP3	64.54	36.69	36.69
JP4	37.18	*/1	*
JP5	122.25	*	*
JP6	6.44	*	*
JM1	54.08	*	*
JM2	153.98	108.66	70.62
JM3	59.24	*	*
RM1	201.29	*	*
EBM1	41.26	*	*
EBM2	15.66	13.16	13.16
EBM3	4.97	4.62	4.62
WBM1	93.33	93.33	49.03
WBM2	98.26	86.30	65.83
NKM1	8.84	8.36	8.36
NKM2	3.49	1.37	1.37
RGM1	37.63	*	*
RGM2	40.32	39.36	36.51
RGM3	7.50	2.91	2.91

/1 * Implies that the value is the same as the base case.

APPENDIX 3A1

Econometric analysis of average costs at the seam level:

Mining costs are noted for the great hetrogenety that they exhibit[1]. This is the result of a multiplicity of geological factors which have bearing on the average costs and vary a great deal from mine to mine. The varying geological conditions in turn also lead to large intermine variation in the input structures. This fact was brought out in a series of studies conducted by N.Naganna[2] In his study he is led to conclude "It is inferred that stable input structures, stable over mines is an impossible proposition".

To understand the nature of mining costs data was collected from 82 working collieries of the Bharat Coking coal limited. All the collieries operated in the Jharia coal field (Bihar) and account for the entire prime coking coal output in the country. The data gave for each colliery expenditure on various inputs for the year 1974/75. The costs under various heads were grouped as follows:

Variable	Expenditure
LABR	Wages + Fringe benefits.
VAIN	Explosives + Timber + Stores.

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1 Johnston (1961) gives a rather detailed discussion of all the diverse factors that affect the cost per tonne in coal mining.

2 Naganna N. (1975), (1974).

Variable	Expenditure
ENER	Electricity + Coal for boiler.
OVHD	Royalty + Sand transport + Coal transport + Overheads.
OBR	Expenditure on overburden removal.
CAPC	Depreciation + interest + repairs and maintenance.
TC	Total Cost
OTPT	Output

It was found that the average cost of collieries in the Jharia coal field exhibited a great deal of hetrogenety varying from Rs.44.95 to Rs.122.71. The mean of the average cost was 70.26 and the coefficient of variation was .22.

Table 3A 1.1 shows the average per tonne expenditure on various inputs. It is found that labour costs amount to as much as 69.14% of the total cost per tonne. The administrative costs are on an average Rs.8.56 per tonne.

It was also found that there was no relation between the average costs and the size of the mine. The regression of total cost on output and $(\text{output})^2$ gave zero coefficient for the square of the output.

$$TC = \underset{(2803.4)}{3125.64} + \underset{(.01240)}{0.052389}\,(OTPT) + 0.00\,(OTPT)^2$$

$$R^2 = .6534$$

$$R = .8087$$

Table - 3A 1.1

Expenditure on various items of input for the 82 collieries

Units: Output: '000 tonnes
Expenditure : '000 Rs.

VARIABLE	MEAN OF THE VARIABLE	STANDARD DIVIATION	COFF OF VARIATION
OUTPUT	209.10	107.10	.51
LABR	11075.35	7023.13	.63
VAIN	249.14	236.64	.94
ENERG	780.23	185.17	.74
OVHD	1791.62	1179.13	.65
OBR	67.48	105.50	1.56
CAPC	2053.08	8846.15	4.30
TOTAL COST	16016.9	7380.57	.51

PER TONNE EXPENDITURE ON VARIOUS INPUTS FOR THE INDUSTRY*

ITEM	EXPENDITURE PER TONNE (Rs.)	PERCENTAGE
LABR	52.96	69.14
VAIN	1.19	1.56
ENER	3.73	4.87
OVHD	8.56	11.17
OBR	.32	.42
CAPC	9.81	12.81
TOTAL COST	76.59	100

* MEAN EXPENDITURE ON THE INPUT/MEAN OUTPUT.

$$TC = \underset{(8064.53)}{2579.8340} + \underset{(.02769)}{0.061}\,(OTPT) + 0.0\,(OTPT)^2 + 0.0\,(OTPT)^3$$

$$R^2 = .6540$$

$$R = .8087$$

This finding is in line with the earlier work in this area.[1]

If observed hetrogenity was basically the result of varying geological conditions under which the mine operated, then it must in principle be possible to statistically estimate their impact through a properly specified production function, which includes the geological variables in addition to other inputs as variables explaining the output.

At the time this study was undertaken data on detailed geological information and cost/output, was not simultaneously available for a reasonably large sample of mines. So we had to rely on a set of project reports which were obtained from the coal industry. Twenty one project reports in all were available, in which detailed geological description of the mine was available along with data on output, inputs, and unit cost calculations. All the cost calculations were done with market prices for the year 1974/75.

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1 Johnston (1961), Naganna (1974), (1975).

From the available geological information three variables representing the geological complexity of the mine were constructed, these were.

(1) Dip: (D): This gave the inclination (in degrees) of the coal seam on which the mine operated.

(2) Seam thickness (Th.): This gave the thickness of the seam on which the mine operated.

(3) Number of seams (N): This gave the number of seams on which the mine carried out its operations. For mines which operated on more than one seam, the seam thickness and dip were values of average of all the seams.

In addition to the above, data on the following variables was also available.

(4) O : Output (Million tonnes/year).

(5) K : Gross fixed assets used as a proxy for capital stock (lakhs of Rs.).

(6) L : Total number of workers to be used as a proxy for labour.

(7) (O/C) : Average cost (Rs./tonne).

(8) EMS : Earnings per man shift, this gave the total wage bill divided by the total number of workers. This was used as a proxy for average wage rate.

The data on cost, input, output etc. are basically generated by doing detailed engineering calculations with reference to a particular design of the mine.

The idea of the analysis that follows is to try and estimate the underlying production / cost function econometrically so that it can be used to predict the cost of mining once the geological profile of the seam is specified.

We specify the production function as follows:

$$O = AK^{a1}L^{a2}e^{a3D} \; th^{a4} \; N^{a5} \quad .... \quad (1)$$

which is basically a cobb-Douglous production function in which the geological variable appear in a multiplicative form. The 'dip' variable (D) is introduced as e^{a3D}, since in many mines, the dip went to zero and a multiplicative form would imply that the output also was zero.

We shall assume, that the mine manager is supplied with information on the geological profile of the seam, and he is asked to produce a given output which he attempts to do at minimum cost. That is he minimises,

Total Cost $C = WL + rk$

W = Wage rate

r = rental

Subject to 1 $O = \bar{O}$

$$O = AK^{a1} \, L^{a2} \, e^{a3D} \, th^{a4} \, N^{a5}$$

and a set of given geological parameters. Under the above set of assumptions it can be shown that [1] the cost function has the following form.

$$C = B\, O^{b1}\, W^{b2}\, r^{b3}\, e^{b4D}\, th^{b5}\, N^{b6}$$

Since the entire coal industry is nationalised, the rental 'r' can be assumed to be the same for all mines, however, the wage rate will differ from mine to mine, depending upon the region in which it is located. Thus the form of the cost function that was ~~fi~~ fitted was

$$(C/O) = BO^{b1}\, (EMS)^{b2}\, e^{b3D}\, th^{b4}\, N^{b5}$$

Where

(EMS) : The earnings per man shift which was used as a proxy for wage rate.

Results:

Both the production and cost functions were fitted with and without the geological variables. The results are reported in table 3.4.2.

The cobb douglous production function fits with an R^2 of .5257 (low) and exhibits slight dis-economies of scale, (a1+a2) = .8767. However both the coefficients a1 and a2 have high standard errors and are not significant at 5% level. Similarly the hypothesis of constant returns to scale i.e. a1+a2 = 1 cannot be rejected

---

1 Nerlove Marc (1965). It may be noted that if the production function is of the form g (K,L). h (D~~M~~, th,N) where g (K,L) is homogeneous (not necessarily of degree 1), the cost minimising K/L will depend only on w/r and will be independent of D, th, N, the Cobb-Douglous form being a special case of this.

at 5% level.

It is quite surprising that the cost function fits quite well (given the poor fit of the production function). However most of the variation in the average cost seems to be explained by the variation in the earnings per manshift (wage rate).

Unfortunately when the geological variables are introduced, the fit does not improve considerably both for the production and cost functions. Also, in both the cases the coefficients of the geological variables turn out to be insignificant at all levels. Similarly the signs of some elasticities are not consistent, for instance, the elasticity corresponding to dip of the seam (D) has a positive sign a3 = .016 in the production function and is again positive in the cost function a3 = .0035 implying that the dip has a positive impact on production and cost simultaneously. Thus from the data at hand it does not seem possible to predict the cost per tonne given the geological description of the mine. Clearly this is because, the number and the kind of geological variables that are used, do not adequately represent, the geological complexity of the mine.[1] It is also likely that, different geological variables affect

---

1 Constructing proper proxy variables, to represent the geological complexity of the mine seems to be a major problem in the work done in this area. Sinha (1974) reports similar analysis done with data on two coal fields in U.S.S.R. The following regressions are reported.

contd..

Table - 3A 1.2

Regressions showing the impact of geological variables on output and cost per tonne (21 observations).

....

$$
\begin{array}{llllll}
(1) & \mathrm{Ln}\ (O) = 7.0287 & + .2320\ \mathrm{Ln}(K) & + .6447\ \mathrm{Ln}(L) & & R^2 = .5257 \\
 & \quad (43346.58) & (.1585) & (.3631) & & \\
 & \quad t = .0001 & t = 1.463 & t = 1.775 & & R = .7250
\end{array}
$$

$$
\begin{array}{lllll}
(2) & \mathrm{Ln}\ (O) = 6.6398 & + .173840\ \mathrm{Ln}(K) & + .7850\ \mathrm{Ln}(L) & + .016\ D \\
 & \quad (46806.66) & (.1916) & (.4544) & (.0247) \\
 & \quad t = .0001 & t = .907 & t = 1.727 & t = .646 \\
 & - .2115\ \mathrm{Ln}(Th) & - .200\ \mathrm{Ln}\ (N) & R^2 = .5923 & \\
 & (.2086) & (.1876) & R = .7696 & \\
 & t = 1.013 & t = 1.067 & &
\end{array}
$$

$$
\begin{array}{lllll}
(3) & \mathrm{Ln}(C/O) = 2.0964 & - .00745\ \mathrm{Ln}(O) & + .6125\ \mathrm{Ln}(EMS) & R^2 = .9049 \\
 & \quad (.7886) & (.02156) & (.0528) & R = .9512 \\
 & \quad t = 2.7635 & t = .345 & t = 12.755 &
\end{array}
$$

$$
\begin{array}{lllll}
(4) & \mathrm{Ln}(C/O) = 1.9762 & - .003199\ \mathrm{Ln}(O) & + .6125\ \mathrm{Ln}(EMS) & + .003512D \\
 & \quad (.7658) & (.02251) & (.0528) & (.002901) \\
 & \quad t = 2.5805 & 5 = .142 & t = 11.591 & t = 1.210 \\
 & + .0364\ \mathrm{Ln}(Th) & + .009738\ \mathrm{Ln}(N) & & \\
 & (.02495) & (.023391) & R^2 = .924703 & \\
 & t = 1.461 & t = .416 & R = .96164 &
\end{array}
$$

1* (contd.)

For the Donbass Coal field U.S.S.R.

$$(1)\quad Y = 165.28 + .110\,D - 10.46\,q - .105H + 4.572L + 1.48n$$

$$R = .579$$

For Kuzbass coal field U.S.S.R.

$$(2)\quad Y = 165.96 - .116\,D + 42.76q + .114H + 1.97L + 10.72n$$

$$R = .536$$

Where

- Y = Yearly planned output.
- D = Planned production in the mine.
- q = Category of cossiness.
- L = Average supported length of the roadways.
- H = Maximum depth of the workable seam.
- n = Numberof faces to be worked simultaneously.

In the above relation it must be noted that the R^2 is quite low (only R is reported) at the same time since standard errors are not reported it is difficult to say whether coefficients were significant.

The following relation estimated in Poland (See ECE Sub-Committee on mining problems 1971) uses a slightly different set of variables.

$$PO = 689.02 - .259\,Qe + .0005\,QO^2 - .665.\,QO + .00112\,Ws^2 - .94\,Ws^2 - 3.065\,WR^2 + 7.159\,EO.$$

Where PO = Manshifts/1000 tonnes in a district, Qe ; Daily face tonneage, QO : Daily production from a district, Ws: Daily tonnage/100 M of conveyor in the district, WR : Ratio of the total length of the conveyor, to the weighted length by tonnage, E.O.: Rate of total production obtained in prepara-tory work. This relation attempts to estimate the impact of the geological variables on productivity for possibly linking it with cost. Here again since R^2 and standard errors are not reported, the quality of the regression is difficult to judge.

the costs at different stages, and thus making it difficult to construct proxy variables which adequately reflect the impact of geology on total costs. Thus it is likely that coal mining, which takes place in stages must be represented by a production/cost function for each stage, in which the impact of geological variables could be estimated. In the absence of good statistical data in this area we shall rely upon engineering estimates as a basis of arriving at cost per tonne of setting up a mining facility and obtaining a tonne of coal from a seam - given its geological profile.

APPENDIX 3A 2

Washing of Coking Coal:

The aim of washing coal is to reduce the ash content. Coal washing is a process of physical beneficiation in which raw coal is crushed and with the help of a heavy media separator partitioned into high ash (which being heavier are 'sinks') and low ash coal (which being lighter are 'floats').

Washing is a method of reducing the physical impurities in coal by a simple (cheap) technique so that ultimately these do not have to be slagged out in the blast furnace.[1]

In principle the yield of clean coal in a washery is governed by the washability characteristics of coal, which is an indicator of how finely the physical impurities (shale) are mixed in the matrix of coal. Apart from the intrinsic washability difference for any given washery the yield would depend upon:

(1) Raw coal ash : As the ash in the raw coal input increases cetris peribus the yield in the washery falls.

---

1 Washing belongs to the family of techniques, (other being palletisation, sintering etc.) which are oriented towards carrying out the easy reactions outside the blast furnace, so that the productivity of the blast furnace is improved.

(2) Clean Coal ash : As the ash in the clean coal is allowed to increase the yield would tend to increase.

(3) The specific gravity of the heavy media separator: As the density at which we maintain the heavy media (called the cut off density) is increased the yield in the washery increases.

(4) As the size to which the coal is ground becomes smaller, the yield tends to increase.

For the purpose of our model, we are interested in arriving at a reasonable estimate of the yield in the four quality classes as delineated by G.S.I.

Class	Ash %
1	less than 17
2	17 to 24
3	24 to 35
4	35 to 45

Given the ash in the raw coal the yield would depend upon the ash level to which we plan to wash it.

The problem of to what ash % the coal should be washed in the washeries before supplying to the steel plant has been a subject matter of a great deal of debate between executives of the steel and the coal industries. In a situation where, the coal washeries are owned and operated by the coal sector it is very interesting to note that the benefits of washing coal are realised in the steel industry (in the blast furnace shop). whereas the cost of washing is borne by the coal industry. The technical Committee on Coal Washery[1] observed "the experts on steel making have preferred to retain the ash content of clean coal strictly within 17 to 18%. In their opinion blast furnace coke has to contain ash of about 23%. For an increase or decrease of coke ash above the level of 23% the production increases or decreases by 2% and the coke rate increases or decreases by 2 to 3%." Thus any cost benefit analysis performed would have to compare these benefits tothe cost borne by the coal industry. The allowable ash in the clean coal would affect the systems cost of supplying coal in two ways. Firstly an increase/ decrease in the allowable ash would decrease/increase the yield and thus push up/down the cost of producing a ton of clean coal. At the same time since, we now have lesser weight per unit of calorific value supplied to the steel plant there is a benefit conferred on the industry by the way of decreased systems cost of transport. In addition to this, there may be interdependence in the production - transportation costs leading to a change in linkages and hence change the overall systems costs.

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1 Technical Committee on Coal Washeries (1972).

Both coal and steel are nationalised industries, hence the issue of the optimum ash in the clean coal has to be solved by mutual discussion for if they were operating in a competitive market at the margin, the price differential between the clean and raw coal would be equal to the net benefits conferred on the steel industry. However there is a consensus now among the executives in the two industries that the average ash level of in clean coal under the present Indian conditions has to be maintained between 17 to 18%.[1]

For our analysis then we needto know, what the yield of clean coal would be if the coal in various quality classes was washed down to 17% ash. In other words we need a usable relation between the yield, clean coal ash and the raw coal ash to be used for prediction. Such a relation is estimated using available washery level data on yield and raw coal ash and clean coal ash.

Data and Results:

The data on the washery operation was taken from the report of the technical committee on coal washeries.[2]

---

1 See "The Report on Technical Committee on Coal Washeries (1972)."

2 See "Report of Technical Committee on Coal Washeries (1972)."

From the data available the following variables were constructed:

(1) Ash in raw coal : RA

(2) Ash in clean coal : CA

(3) Ash difference : DA

(4) Yield : Y

(5) Loss = 100-Y : L

Observations on the above variables were available for 8 working washeries for two years 69-70 and 70-71. The data gave monthly figures for raw coal ash, clean coal ash and yield. The data for all the washeries was pooled together to make in all 187 observations.

Since the data came from working washeries which did not accept high ash coal and were obliged to wash the coal down to a specific ash %, the data showed low variability in which the raw coal ash varied from 28 to 20% and the clean coal ash varied only from 15 to 20%. Hence we would expect the prediction outside this range (for quality of ash % = 40.5) to have large prediction errors.

The following relations, were specified -

$$L = f\ (R_A\ ,\ C_A) + t \qquad \ldots\ldots\ (1)$$

$$L = f\ (DA) + t \qquad \ldots\ldots\ (2)$$

We shall with the help of the relations as specified above be in a position to predict what the washery yield would be, if coal of the various qualities (raw coal ash) was washed down to a stipulated ash % (in our case 17%).

The table 3A2.2 reports the results of the regressions fitted to the data, Table 3A21 reports the prediction at the mid point of the three quality classes with value of C_A at 17%. The choice of appropriate relation to accept is a matter of judgement, depending upon the multiple correlation coefficients and the standard errors of the estimate. The quadratic form fits best with an R^2 of .5487 however it is found that for the range of prediction under consideration, the loss decreases above some value of RA[1], hence it is not accepted. Regression (3) which gives a slightly better R^2 than regression (1) has low 't' values of the coefficients. Thus relation (1) was found to be the most suitable one to make point predictions at the mid point of the interval of the R_A. Fixing the C_A at 17% the relation

---

1 The relation $L = -155.765+12.47\ RA - .19293\ RA^2$ (at $C_A = 17$) has maxima ($\frac{\partial L}{\partial RA} = 0$) at $R_A = (12.47 / .3856) = 32.34$ which means that loss decreases after $R_A > 32.34$ which is technically not possible, thus this specification seems to lead to a very large prediction error.

Table - 3A 2.1

Mid-point predictions of Loss in various quality classes with coal washed down to 17% ash.

Fig. Loss in %

Regression No.	Quality class 2 RA = 20.5 DA = 3.5	3 RA = 29.5 CA = 12.5	4 RA = 40.5 C_A = 23.5
1	23.34	48.9	80.14
2	18.84	44.28	32.9*
3	23.12	52.212	129.69**
4	20.70	44.60	65.30

* Loss decreasing : has too large a prediction error.

** Loss more the 100% this also has too large a prediction error.

The equations at C_A = 17% are

(1) $L = -34.88 + 2.84\ R_A$

(2) $L = -155.765 + 12.47\ R_A - .19283\ R_A^2$

(3) $L = 20.752 + .1823\ D_A + .1906\ DA^2$

(4) $\mathrm{Log}_e L = 2.273 + .6038_e\ \mathrm{Log}_e\ D_A.$

Table - 3A 2.2

Relations between washery yield and quality of coal:

(1) $L = 26.0255 + 2.8465\ R_A - 3.582583\ C_A$

(.2476)	(.5955)	R^2 = .4708
t = 11.49	t = 6.015	R = .6817

(2) $L = 512\text{-}816 + 18.26698\ R_A - 80.2411\ C_A - .1928\ RA^2 + 2.406\ CA^2$

(5.92246)	17.792	(.1327)	(.4536)
t = 3.084	t = 4.509	t = 1.452	t=5.304

$- .34086\ R_A\ C_A$

(.27883)	R^2 = .5487
t = 1.222	R = .7407

(3) $L = 20.752 + .1823\ DA + .1906\ DA^2$

(1.545)	(.1064)	R^2 = .4757
t = .177	(1.790)	R = .68975

(4) $\text{Log } L = 2.2773 + .6038 \text{ Log } D_A$

(.063289)	R^2 = .3297
t = 9.54	R = .5742

has the form

$$L = -34.88 + 2.84\, R_A \quad (L = 26.025 + 2.8465\, R_A - 3.582\, C_A)$$

From the above relation it is observed that:

$$\frac{\partial L}{\partial R_A} = 2.845 \qquad \ldots. \quad (1)$$

$$\frac{\partial L}{\partial C_A} = -3.582 \qquad \ldots. \quad (2)$$

which means that an increase of 1% in the raw coal ash supplied to the washery results in a loss of yield to the extent of 2.845% whereas a relaxation of the clean coal ash to which the coal may be washed down to increases the yield by 3.582%.[1] Alternatively keeping the yield constant a specification of C_A lower by 1% would lead to decrease in the acceptable R_A by 1.259%.[2]

---

1 Thus it seems that the control of the clean coal ash in the washery seems to be more crucial (3.582/2.845 = 1.259) from the point of view of yield control.

2 Some form of elasticity of substitution.

Chapter - 4

REGIONAL SETTING AND THE TRANSPORTATION NETWORK

The aim of the model is to explore choices between theproduction and transportation costs, with a view to minimise the total cost of supplying coal to the steel plants. Having developed the block level cumulative cost functions we must now juxtapose the transportation network over the production and demand points to solve the model. While abstracting from the actual production/transportation system our aim shall be to maintain a high level of dis-aggregation, while keeping the variables within manageable limits.

We shall assume that the only mode of transport is railways. While number of possible choices exist for alternative modes of transport for coal at various stages, there is evidence that with the possible exception of specific cases, where the terrain is difficult or the length of haul is very small for the industry to opt for an ariel ropeway, truck, or possibly a belt conveyor, railways offer the most economical means of transport.

Conceptually the transportation system is viewed as follows:

BLOCK ⟶ WASHERY ⟶ STEEL PLANT

Raw coal is produced in a geological block, washed at a washery and then supplied to the steel plant. In practice the raw coal produced at a number of collieries within a block is collected by coal pilots (which are short haul trains) and is then brought to a central loading point where long distance trains are formed marked for various destinations. The actual transportation system was reconciled with the abstract system used in the model as follows:

(a) An aggregated transportation network was constructed which connected the eight steel plants with loading points in the various coal fields.

(b) Washeries which were less than ten kilometres away from each other were aggregated and identified as washing capacities.

(c) Each washing capacity and gelogical block could be identified with a loading point on the aggregated railway network. In doing this it was noticed that many a times it was possible to identify a washing capacity, and a geological block with the same loading point on the railway network.

(d) As a next step distance between pairs of nodes on our aggregated transportation network (loading points) was computed using the detailed railway maps given in the "Study report on coal transport planning".[1]

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1 Ministry of Railways, "Study report on coal transport planning for fifth five year plan", part 1, 2, 3, 4, Chairman - Shri Shahid Ali Khan (Sept. 1973).

Having obtained the distance between two points, we arrived at the cost of hauling a ton of coal by multiplying the distance by the revenue earning per tonne kilometre of the Railways. Following such a procedure implies that the average cost of transportation remains constant over distance as well as the quantity transported. This in some sense implies "constant returns to scale with respect to distance as well as quantity transported. We had to adopt such a procedure because no firm estimates of economies (or diseconomies) of scale with respect to distance and the quantity transported were available.

Table 4.1 shows features of operations of various units of Indian Railways. The table gives figures aggregated for both coking and non coking coals. Most of the coking coal is transported by South Eastern and Eastern Railways. We shall use the revenue earnings for the total broad gauge .. 7.19 paise/tonne Km. as the average cost of carrying a tonne-km. of coal.

While considering the transport cost we are only considering the distance between themajor loading points associated with each phase of transport of coal. We are thus implicitly neglecting the cost of collection of coal from various collieries beforebeing transported in bulk.

Table - 4.1

Revenue earnings for (broad gauge) rail transport of coal.

All Figs. in ('000)

Railway	Tonnes carried	Net tonne Kilometres	Earnings (Rs.)	Average Lead	Average rate (paise)
Central	120	19058	1314	159	6.89
Eastern	5158	355750	40287	69	11.3
N. Frontier		47	3	356	6.38
Southern	135	37766	1191	280	3.15
S.Central	160	98913	3887	618	3.93
S.Eastern	10194	3863580	267800	379	6.93
Western		55	2	865	4.28
Overall	12408	4375169	314484	353	7.19

Source : Ministry of Railways.

Supplement to the Indian Railways reports and accounts 1974-75 statistical statements (Railway board).

We are assuming that this cost is a small portion of the total cost, also since it is common to all supply points does not significantly affect the choice at hand in any significant manner.

Table 4.2 shows the base year distribution of washing capacity in the industry. The total washing capacity (for both prime and medium coking coal) is 12.303 M.T./year, which is less than the total demand in 1975-76 for prime and medium coking coal of 12.365 M.T. which is made up by using some raw prime coking coal. Most washeries are pithead washeries however the Durgapur group of washeries operates at the Durgapur steel plant and is fed by coal from Raniganj coal field. All the washeries operating in the Jharia coal field are prime coking coal washeries and those outside are medium coking coal washeries.

Table 4.3 shows the loading points and the cost of transporting a tonne of coal from each of these to various steel plants computed at 7.19 paise/tonne kilometre. Table 4.4 indicates geological blocks associated loading points and nearest washing capacity. Blocks in Ramgarh and North Karanpura have no washing capacity at the pithead, thus any increase in production would suggest a creation of washing capacity. The map given shows the location of various coal fields washeries and the railway network that connects them to various steel plants.

Table - 4.2

Distribution of washing capacities in the base year 1975-76

Washery group	Associated field	Capacity (output of clean coal) M.T./year
Patherdih group consisting of Bhojudih	Jharia	4.94
Patherdih-Lodna		
Jamadoba		
Dugdha group		
(Dugdha I and II)	Jharia	2.17
Durgapur (DPL and DSP)	This operates at Durgapur Steel Plant and not at a pithead	1.53
Kargali	East Bokaro	1.71
Kathara	East Bokaro	1.7
Swang	East Bokaro	.448
W.Bokaro	W.Bokaro	.305
Total capacity for Washing		12.303

Table - 4.3 <u>Transport Costs to Steel Plants at 7.19 paise/tonne-KM</u>

Figures in Rs./tonne.

Steel Plant / Loading Point	Bhilai	Durgapur	Rourkela	TISCO	IISCO	Bokaro	Hospet	VZP
Patherdih*	53.66	8.33	26.19	11.53	4.30	5.84	145.59	69.19
Dugdha*	53.48	10.58	21.20	11.86	6.53	.92	145.44	68.44
Ranigunj	62.85	1.72	27.46	17.46	2.30	9.79	151.56	75.13
Durgapur*	64.57	0.00	27.45	19.83	4.025	11.51	153.97	77.54
Swang*	54.54	12.44	22.25	13.72	8.40	2.79	146.50	69.90
Kathara*	54.95	12.03	22.67	13.31	7.99	2.38	146.92	69.49
Kargali*	55.39	11.59	22.22	12.87	7.55	1.51	145.96	69.45
Kedla*	55.43	14.24	23.14	15.02	10.06	4.47	148.96	70.38
W.Bokaro	54.49	15.03	22.21	16.31	11.00	5.39	149.90	69.44
Patratu	54.44	18.05	22.16	19.71	14.024	8.40	146.40	69.40
Ramgarh	53.26	16.82	20.98	18.64	12.79	7.23	145.23	68.23

*These loading points have associated washing capacities.

REGIONAL SETTING OF THE COKING COAL INDUSTRY

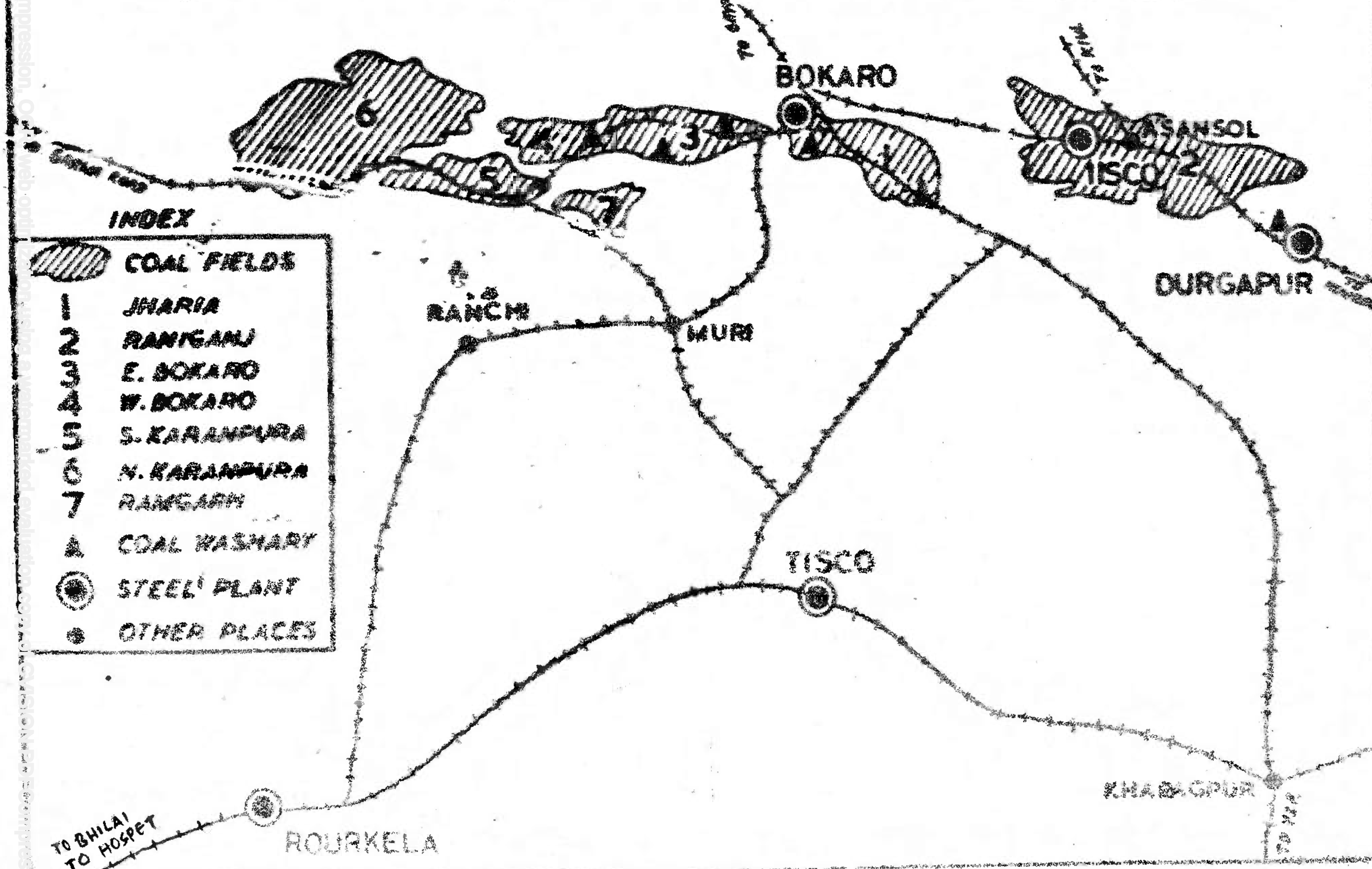


Table - 4.4

Geological Blocks and associated loading points for the medium coking coal industry.

Geological Block	Nearest Loading Point	Nearest washing capacity in the base year 75/76 (Name)
JM1	Patherdih	Fatherdih group of washeries.
JM2	Patherdih	Patherdih group of washeries.
JM3	Dugdha	Dugdha I and II group of washeries.
RM1	Raniganj	No pithead washery. Nearest at Durgapur.
EBM1	Swang	Swang washery.
EBM2	Kathara	Kathara Washery.
EBM3	Kargali	Kargali washery.
WBM1	W.Bokaro	W.Bokaro washery.
WBM2	Kedla	Kedla washery.
NKM1	Patratu	
NKM2	Patratu	
RGM1	Ramgarh	No pithead washery - washing capacity creation to be suggested by the optimisation model.
RGM2	Ramgarh	
RGM3	Ramgarh	

Chapter - 5

THE MODEL AND ITS SOLUTIONS WITHOUT TRANSPORT COSTS

5.1 Specification of the model

The objective of the model is to evolve a minimum cost production/linkage programme in the coal industry to meet an exogenously specified spatially distributed bill of demands over a finite time horizon. The base year of the model is 74/75, production plans are sought to be evolved over a time horizon T. The time profile of demand is exogenously specified at each steel plant. Each steel plant is designed to accept a pre-specified mix of prime and medium coking coal. Similarly cost functions are evolved separately for prime and medium coking coal for each coal block. Thus there being no substitution possibility in the end use and in the production, the prime and medium coking coal industry virtually function as two sub-industries without any inter-dependence. An overall model is thus specified to be used separately for analysing choices in prime and medium coking coal industries.

Production centres for both prime and medium coking coal are geological blocks. Cumulative cost functions, yield functions for each of these, are estimated using the procedure outlined in the previous Chapters.

The cost function gives the undiscounted cumulative cost incurred as cumulative output of clean coal builds up from a geological block. The cost function for the ith block is estimated as

$$Ci \;=\; \Lambda 1i\; Qi + \Lambda 2i\; Qi^2 \qquad \ldots (1)$$

where

Ci = undiscounted cumulated cost

Qi = Cumulated output

The yield function for the block is specified and estimated as

$$Ri \;=\; \Lambda 3i\; Qi \qquad \ldots. (2)$$

where

Ri = Total quantity of raw coal processed.

Qi = Total quantity of clean coal obtained.

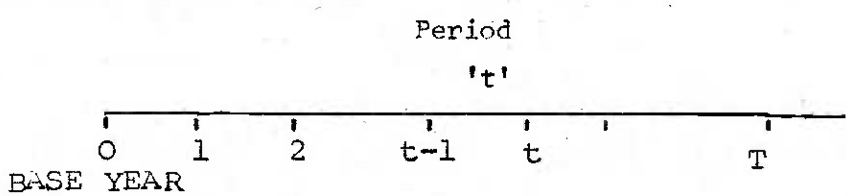
If $Wi = (\Lambda 3i - 1)$ then WiQi would indicate the quantity of extra weight that is associated with a tonne of coal before it is washed in a washery.

The cost function refers to cumulative output without making any specific reference to the time profile of that output. Costs are associated with the output and are discounted as they are incurred in time. In this sense the model is designed to yield time profiles of output optimal in the sense of minimum discounted costs. The transportation network described in the previous Chapter is juxtaposed over the production and demand centres. With these as inputs a model is specified which minimises the discounted sum of the total systems cost of meeting an exogenously specified spatially distributed bill of demands. It is a multi-period programming model with a quadratic objective function. The symbols used in describing the mathematical relations of the model are given in Table 5.1.1. The following assumptions are made while formulating the dynamic model.

MODEL (1)

1. In the system there is no provision for storage of coal. Thus no coal can be stored during any period to be carried over for supply in the next period.

2. The base year is denoted by the time subscript t = 0, as shown below, the period 't' is the interval that passes between time point t-1 and t as shown in the figure below:



Costs incurred during any period are discounted by the discount factor corresponding to the middle of that period.

3. We have as the initial conditions $\bar{D}(0) = 0$ and $xijk0 = 0$ i.e. the cumulated values for demand and supply variables are zero.

<u>Constraints</u>

1. Supply = Demand

The model is specified in terms of cumulated values of output and demands. Since there is no provision for storage in the model, the demand at each steel plant in each period must be supplied from output during that period, thus

$$1(A): \quad \sum_i \sum_j Xijkt - Xijk(t-1) = \bar{D}kt - \bar{D}kt{-}1$$

For $k = 1, 2 \ldots\ldots M$ and $t = 1, 2, \ldots\ldots T$

With $\bar{D}k(0)=0$ and $Xijk(0) = 0$,[1] Consider the following system of equations (1B) obtained by adding up 't' equations of system (1A) at a time

$$1(B): \sum_i \sum_j Xijkt = \bar{D}kt \qquad k = 1, 2\ldots.M \text{ and } t = 1, 2\ldots.T.$$

It can be seen that any solution Xijkt which satisfies the equation system (1B) also satisfies (1A) and vice-versa because each of the systems can be generated from the other by adding or substracting successive equations. Hence the specification (1B) is used in the model, which demands the equality of demands and supplies, cumulated upto

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1 Since we assume that capacities created can be shifted across seams and blocks at negligible costs we do not take into consideration the base year inherited distribution of capacities and their impact on production plans.

Table - 5.1.1 **LIST OF SYMBOLS**

1. Indices

i	:	Geological block	i = 1,2,......N
j	:	Washing capacity	j = 1,2,......P
k	:	Steel plant	k = 1,2,......M
t	:	time period	t = 1,2,......T

2. Variables to be determined

Xijkt : The coal produced in the geological block 'i' washed at washery 'j', supplied to steel plant 'k' cumulated upto the end of time period 't'.

3. Exogenous Variables

$\bar{D}$kt : Demand for prime/medium coking coal at steel plant 'k', cumulated upto the end of time period 't'.

4. Parameters

Cijk : The cost of transporting a tonne of coal from production centre 'i' to washery 'j' and to steel plant 'k'

Cij : Cost of transporting a tonne of coal from the production centre 'i' to washery 'j'

A1i, A2i : Estimated coefficients of the block level cumulative cost function.

A3i : Estimated coefficients of the yield function.

Wi : (A3i-1)

Pt, Ft : Discount factors.

time t.

2. Definition of Block output

$$\sum_{j} \sum_{K} Xijkt = Qit$$

3. Non-Negativity

$$Xijkt \geq 0$$

<u>Objective function</u>

Cost incurred in meeting the exogenously specified demand profile are discounted by the discount factor corresponding to the middle of that period. The objective of the model is to minimise the discounted sum of costs. The total undiscounted sum of costs upto the time t are given by

$$U(t) = \sum_{i} \sum_{j} \sum_{k} Cijk. \; Xijkt$$: The total cumulated cost of transporting Xijkt

$$+ \sum_{i} (\Lambda 1i \; Qit + \Lambda 2iQit^2)$$: The total cost of producing and washing Xijkt m.t of coal.

$$+ \sum_{i} \sum_{j} \sum_{k} Cij. \; Wi. \; Xijkt$$: The total cost of carrying the associated excess weight before it is washed in the washery.

If U(t) are the total undiscounted sum of costs incurred upto time 't' the costs during the period t can be written as

$$U(t) - U(t-1) \quad \text{where}$$

$$U(t) = \sum_{i} \sum_{j} \sum_{k} (Cijk + WiCij) \; Xijkt + \sum_{i} (\Lambda 1iQit + \Lambda 2i(Qit)^2)$$

The objective function is given by the total discounted costs. The cost functions constructed in the study are with reference to specific discount rate that which is used in amortising the capital costs. The discount rate used in the objective function must be the same as that used in the cost function development in order to maintain consistency. If Ft is the discount factor associated with the period t the discounted costs add up to Z,

$$Z = \sum_{t=1}^{t=T} Ft.(U(t) - U(t-1))$$

Where $Ft = 1/(1+r)^t$, r is the discount rate (12%) in the base case). This may be rewritten as -

$$Z = \sum_{t=1}^{t=T} Pt.\ U(t) \quad \text{as } Xijko=0 \text{ and } \quad U(0) = 0,$$

and
$$P1 = F1 - F2$$
$$P2 = F2 - F3$$
$$Pt = Ft - Ft+1$$
$$PT = F_T$$

MODEL (1) can now be written in short as -

Minimise $Z = \sum_{t=1}^{t=T} Pt.\ U(t)$ Subject to constraints 1B, 2 and 3.

Constraints 1B, and 2 are linear and the objective function is quadratic hence the problem may be formulated as a quadratic programming problem. To do this we define the following vectors

Xi.t, the Vector[1] of dispatches from the ith geological block upto time t.

$X'i.t$ = (Xi11t, Xi12t.... XiPMt) (1 X PM)

X' [2] the vector of (dispatches) Xi.t's for all blocks and all time periods (t=1,2.....T).

X' = (X1.1, X2.1....XN.1, X1.2, X2.2...XN.2..X1.T, X2.T..XN.T)

~~where~~ c is the vector defined by Pt (Cijk+tWiCij+Λ1i)

and D is block diagonal consisting of (NT) square blocks along the diagonal, one for each (i,t), and each block consisting of PM rows/columns. each element of the (i,t)th block is (Λ2i.Pt) to give

$$X' D X = \sum_{i} \sum_{t} Pt.\ \Lambda 2i\ (Qit)^2$$

Where

Qit = e'Xi.t, e' is the unit vector (1,1,....1) of dimension (1 X PM)

---

1 All vectors are column vectors, X' indicates the transpose of X.

2 X is the vector generated by elements Xijkt by running through the indices k, j, i and t in that sequence.

Let Lkt be the shadow price associated with the kt^{th} constraint in set 1(B), the Lagrangian for the optimisation problem may be written as follows:

$$W = \sum_i \sum_j \sum_k \sum_t \; Pt\,(Cijk + WiCij)\,Xijkt$$

$$+ \; Pt\,(A1iQit + A2i(Qit)^2)$$

$$- \; Lkt\,(\sum_i \sum_j Xijkt - Dkt)$$

The necessary and sufficient conditions for local optimality are

4. $\dfrac{\partial W}{\partial Xijkt} \geq 0$

5. $\dfrac{\partial W}{\partial Xijkt} \cdot Xijkt = 0$

6 $\dfrac{\partial W}{\partial Lkt} \geq 0$

7 $\dfrac{\partial W}{\partial Lkt} \cdot Lkt = 0$

If we can find an (X*, L*) which satisfies the above conditions, then the global optimality of X* is ensured if the objective function is Convex[1], which would be the case if the 'D' matrix is positive semi definite. In our problem, since the marginal cost in each block is rising, convexity is ensured, this can be seen as follows:

The objective function can be written as the sum of a linear and non linear part. It is enough if we

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1 Mangasarian (1969)

show that the non-linear part of the objective function is convex.

The non linear part is given by,[1]

$$\sum_{i=1}^{N} \sum_{t=1}^{T} Pt.\ \Lambda 2i(Qit)^2 \quad (Qit= \sum_j \sum_k Xijkt = e' \cdot Xi.t\ )$$

This is the sum of NXT functions, the i t^th of which, defined over a subset of variables Xi.t can be written as a quadratic form as follows.

$$PtA2i(Qit)^2 = (e'\ Xi.t) \quad (e'\ Xi.t).\ Pt.\Lambda 2i$$

$$= Pt.A2i\ (X'i.t\ (\ e\ e'\ )\ Xi.)$$

thus the i t^th quadratic form will be non negative if Pt A2i $\geq$ 0, which is ensured if A2i $\geq$ 0 or the marginal cost in each block is increasing. This being the case, the objective function is a sum of N convex functions and hence convex which ensures that any point (X*L*) which satisfies (1) to (4) is a global optimum.

Substituting Qit = $\sum_j \sum_k$ Xijkt into the lagrangian conditions (6) and (7) imply.

8. Lkt $\leq$ Pt. ((Cijk+WiCij) + Ali+2Λ2i($\sum_j \sum_k$ Xijkt))

alternatively Lkt/Pt $\leq$ (Cijk+WiCij) + Λli+2Λ2i($\sum_j \sum_k$ Xijkt))

If the above relation holds with strict inequality

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1 The vector e' is a row vector (1, 1, 1.....1) of dimension (P X M)

then $Xijkt = 0$, and if $Xijkt > 0$ then the relation holds with strict equality.

Condition 6 implies.

$$9 \quad \sum_i \sum_j Xijkt = \bar{D}kt.$$

It can be seen from 8 and 9, that there are no relations which connect shadow prices across time periods. Thus in effect if costs in each period are discounted, the geological blocks are constrained to meet the demand as it arises, the solution which minimises total discounted cost over a finite horizon in this 'dynamic' set up is identical to one obtained by solving the model period by period. This decomposability of the model is the outcome of the convexity of the cost functions, together with the possibility in the model of costless scaling up or down of output from a block from period to period.

Thus we have shown that the optimal production transportation programme to meet a trajectory $\bar{D}kt$ can be arrived at by solving for $\bar{D}kt$ sequentially period by period. In other words, there is one (undominated) strategy of meeting a cumulated demand configuration $\bar{D}K$ independent of the time at which it is realised and this is given by solving the static quadratic programming problem written by dropping the subscript 't', as follows:

MODEL (2)

$$\min_{X_{ijk}} U = \sum_i \sum_j \sum_k (C_{ijk} + W_i C_{ij}) X_{ijk} + \sum_i (A1_i Q_i + A2_i (Q_i)^2) \qquad ..10$$

Subject to.

11. $\sum_i \sum_j X_{ijk} = \bar{D}_k$

12. $\sum_j \sum_k X_{ijk} = Q_i$

13. $X_{ijk} \geq 0.$

Here $\bar{D}_k$, the demand at the steel plant k, and X_{ijk} the supply variables, both cumulated over a fixed horizon T for which the model is sought to be solved. Thus Model (1) which traces the output over a number of periods, can be solved by successively solving Model (2) period by period.

As before the conditions for optimality in Model (2) are given by relations 14 to 17 below.

14. $\dfrac{\partial U}{\partial X_{ijk}} \geq 0$

15. $\dfrac{\partial U}{\partial X_{ijk}} . X_{ijk} = 0$

16. $\dfrac{\partial U}{\partial L_k} \geq 0$

17. $\dfrac{\partial U}{\partial L_k} . L_k = 0$

The conditions 14, 15 for a one period static model are -

$$Lk \leq (Cijk + Wi\ Cij) * A1i + 2A2i \left(\sum_j \sum_k Xijk \right)$$

If the above relation holds with strict inequalty then Xijk = 0 and if Xijk > 0 then the relation holding with strict equality. Here A1i +2A2i Qi is the marginal cost of production in the i^{th} block, Cij + Wi Cij is the unit cost of transport. Thus if Xijk > 0 then the marginal cost of production + the unit cost of transport (which is marginal cost of transporting Xijk and the associated excess weight), should be equal to the shadow price L_K.

Xijk's are all expressed in terms of tonnes of clean coal that are transported from the i^{th} block, to the j^{th} washery and then to the k^{th} steel plant. Each tonne of clean coal that is thus transported has associated with it an excess weight before passing through the washery stage. The yield function (which is block specific) is used to estimate the excess burden carrying cost as: $\sum_i \sum_j \sum_k$ Cij Wi Xijk. In principle the above formulation allows for coal from any block to be processed at any washery. However, the model specification implies that it would not pay to wash coal at any location other than at the pithead (where Cij = 0) and incur the excess burden carrying cost of $- \sum_i \sum_j \sum_k$ Wi Cij Xijk. The model has no explicit activities for washery capacity creation, thus it is assumed that necessary washing capacity will be created at the pithead to sustain the level of clean coal output from each block. This being the case we could have specified the model entirely in terms of geological blocks and steel plants only. However the present specification is maintained to take into account the fact that the Ranigunj coal is committed tobe fed into the existing DSP washery which operates at the Durgapur steel plant. Hence only for this linkage, (Ranigunj-Durgapur), the model has excess burden carrying costs.

5.2 Demand

The demand for coking coal by steel plants is exogenously specified. All the demand figures for this study are adopted from the Chari Committee report.[1] As explained in Chapter 1, the demand for coking coal at each steel plant is estimated by multiplying the hot metal production by the prespecified coke rate.

This aggregate demand is split into prime/medium and blendable coal again using a specified blend different

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1 Department of Coal: Report of the Committee to review plans for coal supplies to steel plants during the fifth and sixth plan periods (Sept.1975).

Table - 5.2.1

BLEND OF THE PRIME/MEDIUM/BLENDABLE COAL IN VARIOUS STEEL PLANTS

(Figs in % of Total)

	PRIME	MEDIUM	BLENDABLE	
BHILAI	55	38	7	
DSP	50	35	15	
RSP	50	40	10	
TISCO	65	25	10	UPTO 78/79
	63	27	10	IN 78/79
	52	38	10	80/81 ONWARDS
IISCO	65	25	10	
VAP	55	45	-	
VZP	55	45	-	

Table – 5.2.2 : THE TIME PROFILE OF YEARLY DEMANDS FOR EACH STEEL PLANT FOR PRIME AND MEDIUM COKING COAL

All figures in '000 tonnes.

Year	Type of Coal	BHILAI (BSP)	DURGAPUR (DSP)	ROURKELA (RSP)	TISCO (TISCO)	IISCO (IISCO)	BOKARO (BOK)	WALTAIR (VZP)	HOSPET (VAP)	TOTAL DEMAND
75/76	PRIME (P)	1940	965	1080	1394	975	1020	-	-	7374
	MEDIUM (M)	1391	712	1000	697	374	953	-	-	4971
76/77	P	1988	1092	1296	1510	1198	2031			9115
	M	1373	764	1000	581	461	1661			5840
77/78	P	2006	1216	1377	1710	1198	2615			10122
	M	1385	851	1001	657	461	2140			6495
78/79	P	2039	1321	1433	1710	1372	3242			11217
	M	1409	925	1147	657	527	2653			7318
79/80	P	1947	1321	1433	1647	1331	3257			10936
	M	1346	825	1147	720	512	2664			7314
80/81	P	2146	1321	1433	1367	1331	3215			10813
	M	1482	925	1147	1000	512	2630			7696
81/82	P	2705	1321	1433	1367	1331	3215			11372
	M	1419	925	1147	1000	512	2630			7633
82/83	P	2655	1321	1433	1367	1331	3215	579	545	12466
	M	1834	925	1147	1000	512	2630	475	445	8966
83/84	P	3100	1321	1433	1367	1331	3215	1192	1123	11082
	M	1627	925	1147	1000	512	2630	1392	1292	10528
84/85	P	3100	1321	1433	1367	1331	3215	1702	1579	15048
	M	1627	925	1147	1000	512	2630	1392	1292	10525
TOTAL CUMULATED	P	23626	12520	13784	14806	12729	34670	3473	3247	11885
	M	14893	8802	11030	8312	4895	22721	3259	3029	76941

Source: Committee for the Supply of Coking Coal to steel plants (1975).

Table - 5.2.3 :TIME PROFILE OF CUMULATIVE DEMANDS AT EACH STEEL PLANT.

FOR MEDIUM COKING COAL FIGS IN M.T. AS CHARGED TO THE OVENS

YEAR.	BHILAI (BHI)	DURGAPUR (DSP)	ROURKELA (RSP)	TISCO (TISCO)	IISCO (IISCO)	BOKARO (BOK)	WALTAIR (VZP)	HOSPET (VAP)
75/76	1.391	.712	1.00	.697	.374	.953		
76/77	2.764	1.476	2.00	1.278	.835	2.614		
77/78	4.419	2.327	3.001	1.935	1.296	4.750		
78/79	5.558	3.252	4.148	2.592	1.823	7.403		
79/80	6.904	4.177	5.295	3.312	2.335	10.067		
80/81	8.386	5.102	6.442	4.312	2.847	12.697		
81/82	9.805	6.027	7.589	5.312	3.359	15.327		
82/83	11.639	6.952	8.736	6.312	3.871	17.957	.475	.445
83/84	13.266	7.877	9.883	7.312	4.383	20.587	1.867	1.737
84/85	14.893	8.802	11.030	8.312	4.895	22.217	3.259	3.029

Source : CHARI COMMITTEE.

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for each steel plant and changing through time. Table 5.2.1 shows the blend that is technically accepted at each steel plant. The aggregate coking coal requirement as obtained in Chapter 1, is split up into prime/ medium and blendable using these ratios. Table 5.2.2 shows the yearwise requirement of prime and medium coking coal. Table 5.2.3 reports the demand for medium coking coal cumulated upto each time period for ten years at the eight steel plants. Since the model is specified in terms of cumulated production, the cumulated demand figures are used to solve the model for various time horizons.

5.3 Model solutions for Prime/Medium Coking Coal without transport costs:

Having specified the overall model we shall first explore the production profiles and rates of increase of costs without considering the spatial dimension of the industry. This is done to observe the trade off between production and transport costs. Further in the case of Prime Coking Coal all the geological blocks occur in the Jharia Coal field thus in this specific case there is no trade off between production and transportation costs. In the absence of transport costs the model (2) reduces to

<u>Problem</u> (1)

$$\text{Minimise} \quad \sum_{i=1}^{N} (A1iQi + A2iQi^{2})$$

subject to

$$\sum_{i=1}^{N} Qi = D \; \ldots\ldots \; (1)$$

$$Qi \geq 0 \qquad (2)$$

The model can be used to draw inference about the patterns of cumulative output in various blocks as they are jointly constrained to supply increasing amounts of total demand of clean coal - $D = \bar{D}K$. It is also feasible to compute the rate at which the marginal and average cost would increase as increasing amounts of coal are drawn out. This would be an increase that would be purely attributed to the geological depletion in the industry at constant base year prices.

Although problem (1) is a quadratic programming problem its intrinsic simple structure makes it amenable to simpler solution procedures.

The optimality conditions are the following:

If 'L' is the shadow price of constraint (1) and (L* Q*) is an optimal solution then

$$L^* \leq A1i + 2A2iQi^*, \text{ for all 'i'} \qquad (3)$$

$$Q_i^* > 0, \text{ implies } L^* = A1i + 2A2iQi^* \qquad (4)$$

$$L^* < A1i + 2A2i\, Qi^* \text{ implies } Qi^* = 0 \qquad (5)$$

since $A2i \geq 0$ for all 'i'

$$L^* < A1i \text{ implies } Qi^* = 0 \qquad (5)$$

$$Qi^* = D \qquad (6)$$

Using conditions (3), (4), (5) and (6) Problem (1) can be formulated as a linear programming problem as follows.

<u>Problem (2)</u> Maximise L

$$\text{Subject to } L \leq A1i + 2A2iQi \quad i=1, 2, \ldots N \qquad (7)$$

$$\sum_i Qi = D \qquad (8)$$

$$Qi, L, \geq 0. \qquad (9)$$

It can be shown that any solution (Q^*, L^*) of the linear programming <u>problem (2)</u> is also a solution to the quadratic programming <u>problem (1)</u>. Observe that the solution to L.P. problem (2) satisfies conditions 3 and 6. It also satisfies the complimentarity conditions (4) and (5).

$$L^* = A1i + 2A2i\, Qi^* \text{ if } Qi^* > 0 \qquad (4)$$

$$\text{if } L^* < A1i \text{ then } Qi^* = 0 \qquad (5)$$

This can be seen as follows:

Suppose an optimal solution (Q^*, L^*) of the LP problem (2) violates conditions (4) and (5)[1] specifically consider the following situation

$Qi^* > 0$ for i belonging to an index set (I^*) subset of (N)

$$\sum_{i \varepsilon I^*} Qi^* = D$$

and there is atleast one $i = i^+$. belonging (I^*) where

$$L^* < A1i^+ + 2A2i\ Q^*_{i+} \quad \text{and } Q^*_{i+} > 0$$

and $L^* = A1i + 2A2i\ Qi^*$ for all i's other than i^+

then in such a case it should be possible to reallocate $Q^*_{i^+}$ across other blocks in (I^*) (maintain $\sum_i Qi^* = D$)

and bring about an increase in L^*. This violates the original contention that (L^*, Q^*) was optimal. Therefore any solution to the LP problem (2) must satisfy the Complimentary slackness conditions (5) and (4), hence must also be a solution of the Q.P. problem (1).[2]

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1 If conditions (4) and (5) are satisfied then the LP solution is also a solution to QP problem (1) and there is nothing to prove.

2 The author is grateful to Larry Westphal (IBRD) for pointing this out in a private communication.

Although the possibility of solving the problem (1) as an L.P. is a great simplification in itself, the structure of the problem leads to an even simpler dual based enumeration procedure to answer the two questions that are posed.

- Given any aggregate demand D, what are the contributions to production from each block that minimise the total cost?

- How do marginal cost and average cost behave as increasing quantities of coal are supplied by the industry?

In solving this problem we shall also insist that if Qi^* are the total reserves in the i^{th} block no more than Qi be drawn out from it

$$\text{i.e. } Qi \leq Qi^* \quad i=1, 2\ldots\ldots N \quad (10)$$

The quadratic programming problem (1) is solved using the following algorithm.

Step (1) Chose 'L' atleast as large as (but not equal to) the smallest $A1i$ - marginal cost at zero output.

Step (2) : Clearly all blocks with $A1i > L$ would not contribute to the output and $Qi = 0$.

All blocks with $A1i < L$ would contribute to the output and the following equality would hold : $L = A1i + 2A2iQi$

$$Qi = (L - A1i)/2A2i$$

if computed $Qi > Qi^*$ (total reserves) for any 'i'

set
then $Qi = Qi^*$

Compute cost of production $TCi = A1iQi + A2iQi^2$

Step (3) : Compute $D = \sum_i Qi$: total output

Total cost : $\sum_i TCi = TC$

Average cost = TC/D.

Step (4) : Increase L by a small quantity and go back to step (2).

Thus varying 'L' over a large enough range it was possible to generate the schedule for total output, average costs, etc. as a function of L which is marginal cost at the output D.

From the above procedure, given any prespecified aggregate output D*, the Qi's can be obtained to any desired degree of accuracy by varying L by small quantities until D (L) = D*[1].

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1 Alternatively, it should be possible to guess from the schedule the set of blocks (I*) which would have output > 0 for an aggregate demand D* knowing this it should be possible to solve for (Qi*), the block level outputs by solving the following simeutaneous equations.

$$\sum Qi = D^*$$

and $\quad A1i + 2A2iQi = L \quad$ for $i \in (I^*)$.

5.3 Results

Starting with cumulative cost curves with parameters for the base case the results of applying the procedure outlined above to six blocks of Prime Coking Coal and 14 blocks of medium coking coal are reported in Tables 5.3.1 and 5.3.2. Starting from a shadow price (Marginal cost) = Min $\Lambda li + e$, $e > 0$ the average, total cost and total output are computed as marginal cost is systematically increased. Tables 5.3.3 and 5.3.4 indicate the contribution from each block as increasing quantities of coal are jointly supplied by all the blocks. The entire exercise is repeated for cost functions estimated for 18% discount rate (Case A) and shadow wage rate (Case B) Fig. 5.1 Charts the movement of average and marginal cost as cumulative output is built-up for all three cases.

As is evident from the graphs, the marginal costs are continuously rising, however, they are more steeply rising at first and tend to rise less steeply after three to four hundred million tonnes are mined out. Thus by the time half the total (1416.12) reserves of Prime Coking Coal are mined out the marginal cost increases to about 2.5 times the present value, from 55 to 153 Rs./tonne.[1]

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1 The marginal costs here are 'marginal' with respect to cumulative output, they are the average costs for the last tonne extracted when say 'D' m.t. of cumulative output has been built-up. The average costs are also average with respect to cumulative output.

Similarly it is found that the marginal cost curve for the medium coking coal **sector** lies below that of the Prime Coking Coal sector, which is quite reasonable considering the fact that the former is less depleted than the later. However, the marginal cost in the Medium Coking Coal industry seems to rise more steeply with respect to cumulative output than the prime coking coal. This larger % increase in marginal cost in the Medium Coking Coal sector than the Prime Coking Coal sector for an equal quantity of coal extracted seems to be the result of faster deteriorating quality, an increasing reliance upon seams which give low yield. The schedules generated, can be used as in Table 5.3.1 to 5.3.4, from where it is possible to 'read off' the values of average marginal cost, and blockwise contribution for a given level of output. However our understanding of the behaviour of these parameters, and their variation with respect to cumulative output would be increased/simplified if the schedules could be approximated by continuous functions. These functions would also be easier to work with as supply functions for the coal industry as inputs to further models constructed to analyse price policy, interfuel substitution etc etc. Table 5.3.5 indicates the results of least square regression for the cost schedules generated for Prime and Medium Coking coal for all three cases. The marginal cost schedules can be thought of as long run supply schedules. If coal price is set equal to the marginal cost the marginal cost curve indicates how coal price needs to be adjusted (upwards) over times in response to geological depletion. Fig. 5.2 shows how the change in discount rate and valuing labour at shadow wage rate shifts the marginal cost curve for both Prime and Medium coking coal. Table 5.3.5 can be used to study the impact of these variation on the least square regression.

The schedules obtained in Table 5.3.1 to 5.3.4 estimate themarginal cost as a function of cumulative output. As shown before the production programme is optimal for any time frame of demand growth. The marginal cost at any time point in future depends only upon cumulative output upto then and not on the rate at which it is built up. Here we make an attempt to predict the time profile of marginal costs for varying rates at which coal is produced.

Taking 10 year periods as a unit of time, let the time profile of cumulative outputs be represented by

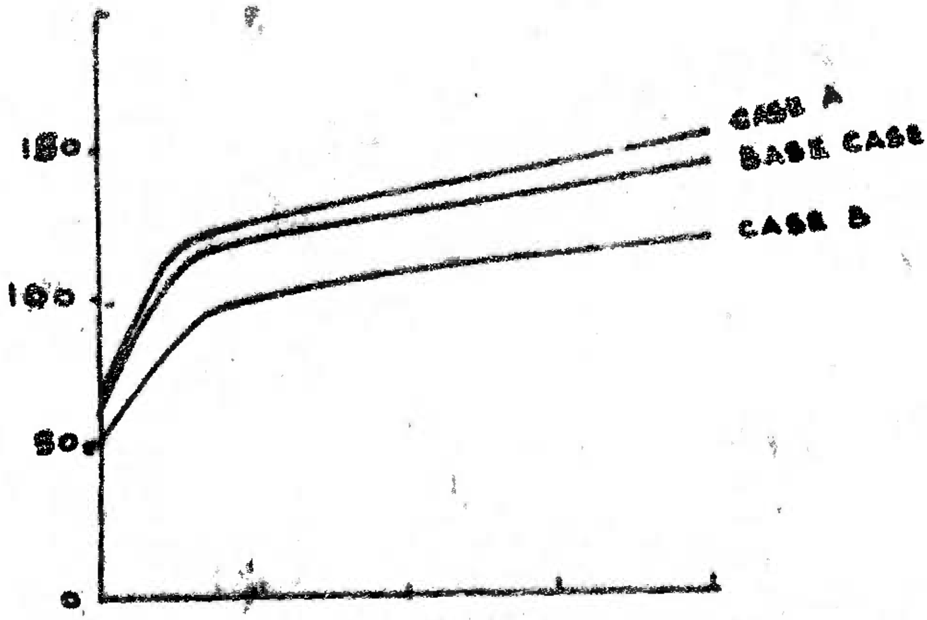
$$Qt = Qot^{b} \qquad (1)$$

where

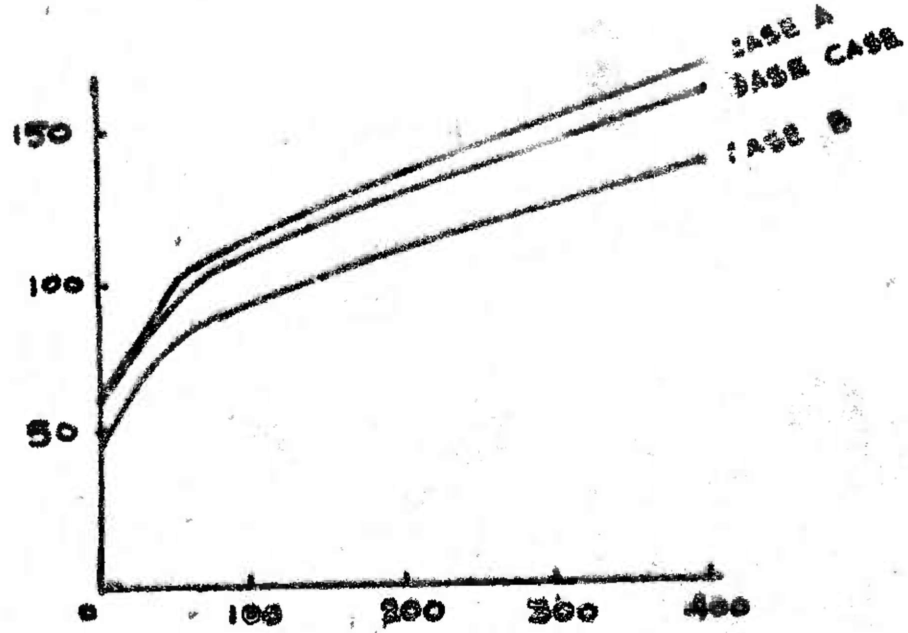
Qt : Cumulative output option time t.

Qo : Cumulative output for the first ten years (t=1)

MARGINAL COST CURVES Fig: [5.1]



PRIME COKING COAL

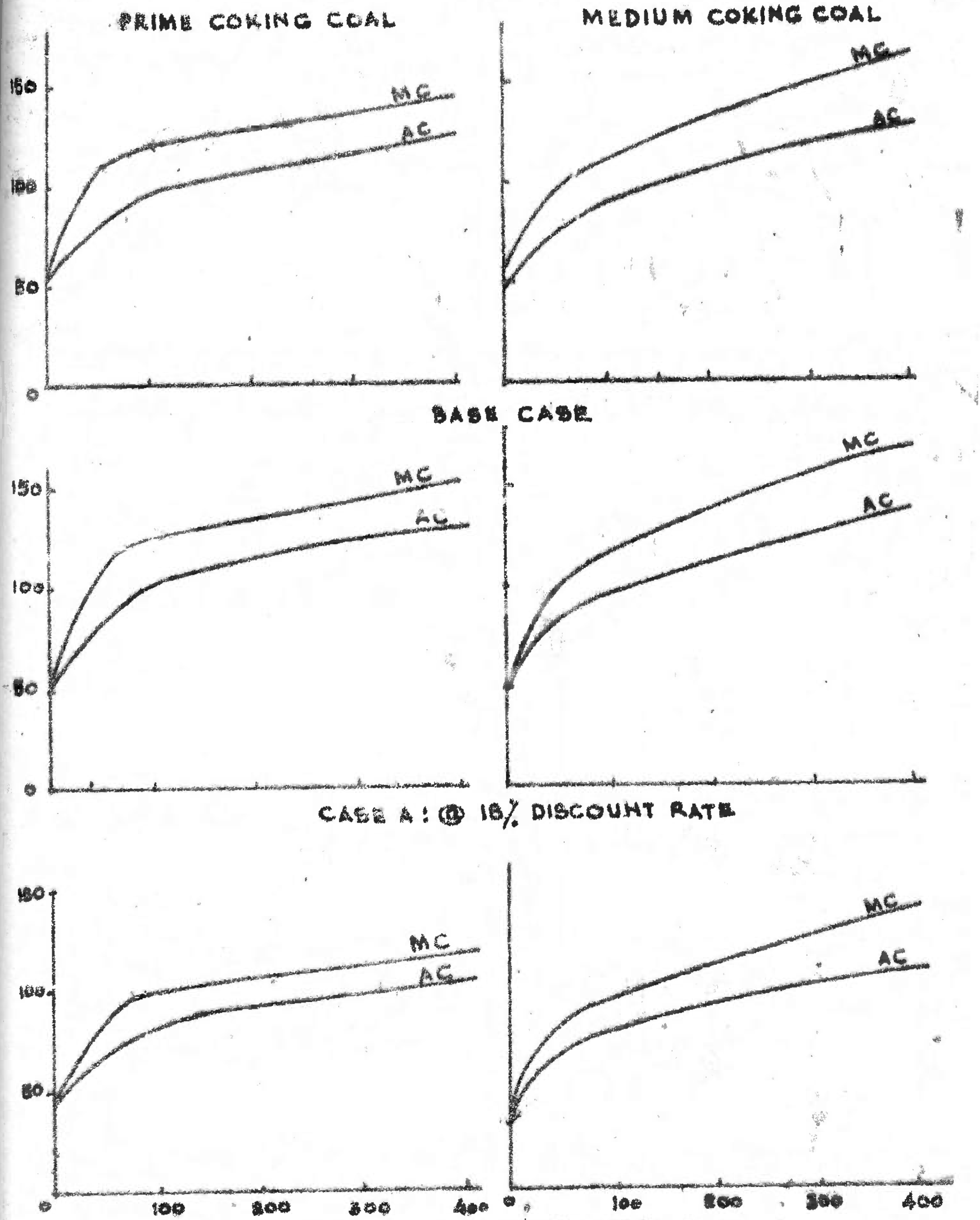


MEDIUM COKING COAL

X - AXIS : CUMULATIVE CLEAN COAL OUTPUT (M.T.)

Y - AXIS : MARGINAL COST (Rs./TONNE)

MARGINAL (M.C) AND AVERAGE (AC) COST FUNCTIONS Fig. [5.2]



CASE B : (ii) SHADOW WAGE RATE.

X- AXIS : CUMULATIVE CLEAN COAL OUTPUT (M.T.)

Y- AXIS : MARGINAL (MC) AND AVERAGE COST (AC) (Rs/TONNE) b : rate of growth of cumulative output[1]

t : time period.

The marginal cost profile is estimated as

$$Mct = A\ (Qt)^{a} \qquad (2)$$

Substituting (1) and (2)

$$Mct = A\ Qo^{a}\ t^{a.b}$$

Table 5.3.6 shows the values of marginal cost at the end of various time period for various values of growth of output b. In both cases marginal costs increase as we increase the rate of depletion b. For a given rate of exploitation b, marginal costs in the Prime Coking Coal sector increase by greater absolute amounts overtime than the Medium Coking Coal sector because it is being depleted on a larger scale (Qo = 118.85 against Qo = 76.941). For 5% annual increase in demand, over the next 50 years the marginal cost in the Prime Coking coal sector increases by 40% and that in the medium coking coal sector by 38%. over the base year.

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1 Roughly if the output per time period grows at X% the cumulative output grows at (100+ x)%.

Table 5.3.1 : INDUSTRY LEVEL COST CURVE - PRIME COKING COAL

Output M.T.	Average Cost Rs./Tonne	Marginal Cost Rs./Tonne	Total Cost M. Rs.	No. of Blocks Contributing	Sl. No.
.78	54.51	55	42.95	1	1
2.41	55.51	57	133.85	1	2
4.29	56.63	59	243.24	2	3
6.43	57.75	61	371.83	2	4
8.58	58.81	63	504.69	2	5
10.72	59.85	65	641.85	2	6
12.86	60.87	67	783.29	2	7
15.01	61.89	69	929.01	2	8
17.15	62.90	71	1079.02	2	9
19.29	63.91	73	1233.32	2	10
21.43	64.92	75	1391.91	2	11
23.58	65.93	77	1354.78	2	12
25.72	66.93	79	1721.93	2	13
27.86	67.94	81	1894.47	3	14
30.01	68.94	83	2069.10	2	15
32.15	69.94	85	2249.12	2	16
34.29	70.95	87	2433.42	2	17
36.44	71.95	89	2622.00	2	18
38.58	72.95	91	2814.87	2	19
40.72	73.95	93	3012.03	2	20
42.86	74.95	95	3213.48	2	21
45.01	75.96	97	3419.21	2	22
47.15	76.96	99	3629.22	2	23
49.29	77.96	101	3843.53	2	24
51.44	78.96	103	4062.12	2	25
53.58	79.96	105	4284.99	2	26
55.72	80.96	107	4512.15	2	27
57.87	81.96	109	4743.60	2	28
60.01	82.96	111	4979.33	2	29
62.15	83.97	113	5219.35	2	30
64.29	84.97	115	5463.65	2	31
66.44	85.97	117	5712.25	2	32
81.95	92.10	119	7547.93	3	33
106.75	98.58	121	10523.84	4	34
136.32	103.66	123	14131.84	4	35
165.89	107.28	125	17798.98	4	36
195.47	110.11	127	21525.28	4	37
225.04	112.46	129	25310.72	4	38
254.62	114.50	131	29155.30	4	39
284.19	116.32	133	33059.04	4	40
313.76	117.99	135	37021.92	4	41
343.34	119.54	137	41043.95	4	42
372.91	121.00	139	45125.13	4	43
402.48	122.40	141	49265.45	4	44
432.06	123.74	143	53464.93	4	45
461.63	125.04	145	57723.55	4	46
491.21	126.30	147	62041.32	4	47
520.78	127.53	149	66418.23	4	48
550.35	128.74	151	70854.30	4	49
579.93	129.92	153	75349.51	4	50

Table 5.3.2 : INDUSTRY LEVEL COST CURVE - MEDIUM COKING COAL.

Output M.T.	Average Cost Rs./Tonne	Marginal Cost Rs./Tonne	Total Cost M. Rs.	No. of Blocks Contributing	Sl. No.
1.28	51.36	53	65.79	1	1
2.57	52.77	55	136.05	2	2
4.22	54.03	57	228.49	2	3
5.87	55.14	59	324.23	2	4
7.53	56.21	61	423.28	2	5
9.18	57.25	63	525.62	2	6
10.83	58.27	65	631.26	2	7
12.48	59.30	67	740.21	2	8
14.18	60.34	69	855.86	3	9
16.02	61.45	71	984.70	3	10
17.86	62.54	73	1117.21	3	11
19.70	63.61	75	1253.41	3	12
21.54	64.66	77	1393.28	3	13
23.38	65.71	79	1536.84	3	14
25.25	66.77	81	1686.51	4	15
27.66	68.10	83	1884.16	5	16
30.54	69.60	85	2125.90	5	17
33.42	71.01	87	2373.40	5	18
36.29	72.36	89	2626.66	5	19
39.17	73.65	91	2885.67	5	20
44.60	75.90	93	3386.00	6	21
50.88	78.13	95	3976.37	7	22
57.27	80.13	97	4589.76	7	23
63.94	81.99	99	5243.13	8	24
72.98	84.22	101	6147.48	8	25
82.02	86.18	103	7069.90	8	26
91.07	87.95	105	8010.42	8	27
100.11	89.58	107	8969.02	8	28
109.15	91.11	109	9945.71	8	29
118.20	92.55	111	10940.48	8	30
127.24	93.93	113	11953.34	8	31
136.28	95.27	115	12984.29	8	32
145.33	96.55	117	14033.33	8	33
154.60	97.84	119	15128.10	9	34
164.84	99.22	121	16356.49	9	35
175.08	100.55	123	17605.35	9	36
185.31	101.84	125	18874.68	9	37
195.55	103.11	127	20164.49	9	38
205.79	104.35	129	21474.77	9	39
216.66	105.64	131	22888.94	11	40
227.83	106.93	133	24363.12	11	41

Contd...

Table 5.3.3 : PRIME COKING COAL.

Marginal Cost Rs./T	Total Cost M.Rs.	Total Output M.T.	BLOCK WISE CONTRIBUTION JP3	JP6	JP 1	JP 2
55	42.95	.78	.78			
57	133.85	2.41	2.41			
59	243.24	4.29	4.03	.26		
61	371.83	6.43	5.65	.78		
63	504.69	8.58	7.28	1.30		
65	641.85	10.72	8.90	1.82		
67	783.29	12.86	10.52	2.34		
69	929.01	15.01	12.12	2.86		
71	1079.02	17.15	13.77	3.38		
73	1233.32	19.29	15.39	3.90		
75	1391.91	21.43	17.01	4.42		
77	1554.78	23.58	18.64	4.94		
79	1721.93	25.72	20.26	5.45		
81	1893.37	27.86	21.88	5.97		
83	2069.10	30.01	23.51	6.49		
85	2249.12	32.15	25.13	7.01		
87	2433.42	34.29	26.75	7.53		
89	2622.00	36.44	28.38	8.05		
91	2814.87	38.58	30.00	8.57		
93	3012.03	40.72	31.62	9.099		
95	3213.48	42.86	33.25	9.61		
97	3419.21	45.01	34.87	10.13		
99	3629.22	47.15	36.49	10.65		
101	3843.53	49.29	38.11	11.17		
103	4062.12	51.44	39.74	11.69		
105	4284.99	53.58	41.36	12.21		
107	4512.15	55.72	42.98	12.73		
109	4743.60	57.87	44.61	13.25		
111	4979.33	60.01	46.23	13.77		
113	5219.35	62.15	47.85	14.29		
115	5463.65	64.29	49.48	14.81		
117	5712.25	66.44	51.10	15.33		
119	7560.93	82.06	52.72	15.85	13.47	
121	10556.06	107.02	54.35	16.37	36.20	.08
123	14080.59	135.91	55.97	16.89	58.93	4.10
125	17662.90	164.80	57.59	17.41	81.65	8.12
127	21302.99	193.69	59.22	17.93	104.38	12.14
129	25000.85	222.588	60.84	18.45	127.11	16.16
131	28756.50	251.46	62.46	18.97	149.84	20.18
133	32569.93	280.35	64.08	19.49	172.56	24.20
135	36441.13	309,24	65.71	20.01	195.29	28.22
137	40370.11	338.13	67.33	20.53	218.02	32.24
139	44356.88	367.02	68.95	21.05	240.75	36.26

contd..

Marginal Cost Rs./T	Total Cost M.Rs.	Total Output M.T.	EBM3	RGM1	EBM2	RGM3	JM3	EBM1	RGM2	RM1
50	5.25	.10	.10							
52	45.22	.88	.88							
54	91.07	1.75	1.67	.08						
56	181.86	3.40	2.45	.94						
58	275.95	5.05	3.24	1.81						
60	373.34	6.70	4.02	2.68						
62	474.03	8.35	4.80	3.54						
64	578.03	10.00	5.59	4.41						
66	685.32	11.65	6.37	5.28						
68	795.92	13.30	7.15	6.14						
70	919.82	15.10	7.94	7.01	.14					
72	1050.49	16.94	8.72	7.88	.33					
74	1184.85	18.78	9.50	8.75	.52					
76	1322.88	20.62	10.29	9.61	.71					
78	1464.60	22.46	11.07	10.48	.90					
80	1610.00	24.30	11.86	11.35	1.09					
82	1777.85	26.37	12.64	12.21	1.28	.23				
84	2004.31	29.10	13.42	13.08	1.47	.63	.48			
86	2248.93	31.98	14.21	13.95	1.66	1.03	1.12			
88	2499.31	34.85	14.99	14.81	1.85	1.43	1.75			
90	2755.44	37.73	15.77	15.68	2.04	1.83	2.39			
92	3100.01	41.51	16.56	16.55	2.23	2.23	3.03	.90		
94	3675.09	47.69	17.34	17.42	2.42	2.63	3.66	4.20		
96	4281.47	54.08	18.13	18.28	2.61	3.03	4.30	7.51	.19	
98	4901.25	60.47	18.91	19.15	2.80	3.43	4.94	10.81	.40	
100	5693.04	68.46	19.69	20.02	2.99	3.83	5.57	14.12	.61	1.60
102	6606.43	77.50	20.48	20.88	3.18	4.23	6.21	17.43	.81	4.25
104	7537.90	86.55	21.26	21.75	3.37	4.64	6.85	20.73	1.02	6.90
106	8487.46	95.59	22.04	22.62	3.56	5.04	7.48	24.04	1.22	9.56
108	9455.10	104.63	22.83	23.48	3.75	5.44	8.12	27.34	1.43	12.21
110	10440.83	113.68	23.61	24.35	3.93	5.84	8.76	30.65	1.63	14.87

contd...

Table 5.3.4 (Contd.)

Marginal Cost Rs/T	Total Cost M.Rs.	Total Output M.T.	EBM3	RGM1	EBM2	RGM3	JM3	EBM1	RGM2	RM1	NKM1	NKM2	JM1
112	11444.65	122.72	24.40	25.22	4.12	6.24	9.39	33.95	1.84	17.52			
114	12466.56	131.76	25.18	26.09	4.31	6.64	10.03	37.26	2.05	20.17			
116	13506.55	140.81	25.96	26.95	4.50	7.04	10.66	40.57	2.25	22.83			
118	14564.62	149.85	26.75	27.82	4.69	7.44	11.30	43.87	2.46	25.48			
120	15739.74	159.72	27.53	28.69	4.88	7.84	11.94	47.18	2.66	28.14	.82		
122	16978.36	169.96	28.31	29.55	5.07	8.24	12.57	50.48	2.87	30.79	2.02		
124	18237.46	180.20	29.10	30.42	5.26	8.64	13.21	53.79	3.08	33.44	3.21		
126	19517.03	190.43	29.88	31.29	5.45	9.05	13.85	57.09	3.28	36.10	4.40		
128	20817.07	200.67	30.66	32.15	5.64	9.45	14.48	60.40	3.49	38.75	5.60		
130	22160.23	211.08	31.45	33.02	5.83	9.85	15.12	63.71	3.69	41.41	6.79	.10	.07
132	23623.24	222.25	32.23	33.89	6.02	10.25	15.76	67.01	3.90	44.06	7.98	.59	.51
134	25108.59	233.42	33.02	34.76	6.21	10.65	16.39	70.32	4.10	46.71	9.18	1.08	.95
136	26616.27	244.58	33.80	35.62	6.40	11.05	17.03	73.62	4.31	49.37	10.37	1.56	1.39
138	28146.29	255.75	34.58	36.49	6.59	11.45	17.67	76.93	4.52	52.02	11.56	2.05	1.84
140	29698.64	266.92	35.37	37.36	6.78	11.85	18.30	80.24	4.72	54.68	12.76	2.54	2.28
142	31273.33	278.09	36.15	38.22	6.97	12.25	18.94	83.54	4.93	57.33	13.95	3.03	2.72
144	32870.36	289.26	36.93	39.09	7.16	12.65	19.58	86.85	5.13	59.98	15.14	3.52	3.17
146	34489.72	300.42	37.72	39.96	7.35	13.05	20.21	90.15	5.34	62.64	16.34	4.01	3.61
148	36131.42	311.59	38.50	40.82	7.54	13.46	20.85	93.46	5.55	65.29	17.53	4.50	4.05
150	37795.45	322.76	39.29	41.69	7.73	13.86	21.49	96.76	5.75	67.95	18.72	4.98	4.49
152	39481.82	333.93	40.07	42.56	7.92	14.26	22.12	100.07	5.96	70.60	19.91	5.47	4.94
154	41190.53	345.10	40.85	43.43	8.11	14.66	22.76	103.38	6.16	73.25	21.11	5.96	5.38
156	42921.57	356.26	41.64	44.29	8.30	15.06	23.39	106.68	6.37	75.91	22.30	6.45	5.82
158	44674.95	367.43	42.42	45.16	8.49	15.46	24.03	109.99	6.57	78.56	23.49	6.94	6.27
160	46450.66	378.60	43.20	46.03	8.68	15.86	24.67	113.29	6.78	81.21	24.69	7.43	6.71

Contd..

Table 5.3.4 (Contd.)

Marginal Cost Rs/T	Total Cost M.Rs.	Total Output M.T.	EBM3	RGM1	EBM2	RGM3	JM3	EBM1	RGM2	RM1	NKM1	NKM2	JM1	WBM2	WBM1
162	49573.25	398.08	43.99	46.89	8.87	16.26	25.30	116.60	6.99	83.87	25.88	7.92	7.15	1.17	7.13
164	52072.38	413.41	44.77	47.76	9.06	16.66	25.94	119.90	7.19	86.52	27.07	8.40	7.59	2.94	9.53
166	54602.18	428.74	45.55	48.63	9.25	17.06	26.58	123.21	7.40	89.18	28.27	8.89	8.04	4.71	11.92
168	57112.91	443.78	46.04	49.49	9.44	17.46	27.21	126.52	7.60	91.83	29.46	9.38	8.48	6.47	14.32
170	59571.59	458.33	56.04	50.36	9.63	17.87	27.85	129.82	7.81	94.48	30.65	9.87	8.92	8.24	16.72
172	62059.37	472.88	46.04	51.23	9.82	18.27	28.49	133.13	8.01	97.14	31.85	10.30	9.37	[illegible]	19.11
174	64576.24	487.42	46.04	52.10	10.01	18.67	29.12	136.43	8.22	99.79	33.04	10.05	9.81	11.78	21.51
176	67122.21	501.97	46.04	52.96	10.20	19.07	29.76	139.74	8.43	102.45	34.23	11.34	10.25	13.54	23.91
178	69392.17	514.80	46.04	53.83	10.39	19.47	30.40	141.33	8.63	105.10	35.43	11.82	10.70	15.31	26.30
180	71404.60	526.05	56.04	54.70	10.58	19.87	31.03	141.33	8.84	107.75	36.62	12.31	11.14	17.08	28.70
182	73439.51	537.29	46.04	55.56	10.77	20.27	31.67	141.33	9.04	110.41	37.81	12.80	11.58	18.85	31.09
184	75496.91	548.53	46.04	56.43	10.96	20.67	32.31	141.33	9.25	113.06	39.01	13.29	12.02	20.62	33.49
186	77576.79	559.77	46.04	57.30	11.15	21.07	32.94	141.33	9.46	115.72	40.20	13.78	12.47	22.38	35.89
188	79679.16	571.02	46.04	58.16	11.34	21.47	33.58	141.33	9.56	118.37	41.39	14.27	12.91	24.15	38.28

Table - 5.3.5

Results of the regressions fitted to the cost schedule generated by the Quadratic programming models for capital costs at 18% discount rate ((Case A)

Prime Coking Coal

1.	Log (Cost	= 3.8024 + (179.76) t = .0211	1.1666 Log(O)* (.00839) t = 138995	R^2 = .9975 R = .9987
2.	Log (margi- nal cost)	= 3.8967 + (.9527) t=4.09	.1871 Log (O) (.00798) t = 23.455	R^2 = .919 R = .959
3.	Average cost	= 69.994 + (1.6019) t = 43.6612	.1693 (O) (.0093) t=18.031	R^2 = .8713 R = .9334
4.	Marginal cost	= 85.0875 + (2.7945) t=30.4473	.1901 (O) (.01638) t=11.602	R^2 = .7371 R = .8585

Medium Coking Coal

1.	Log (cost)	= 3.9059 + (156.798) t = .0249	1.1372 Log (O) (.0086) t =131.533	R^2 = .9972 R = .998
2.	Log (Margi- nal cost)	= 3.9553 + (1.0838) t=3.6493	.1698 Log (O) (.00867) t=19.661	
3.	Average cost	= 63.941 + (1.2125)	.297 (O) (.010)	R^2 = .9227 R = .9606
4.	Marginal cost	= 73.114 + (1.9066) t=38.3467	.3286 (O) (.0157) 5=20.878	R^2 = .9008 R = .9491

*(O) is the Cumulative output from all blocks.

Table - 5.3.5

Results of the regressions fitted to the cost schedule generated by the quadratic programming model for Labour valued at shadow wage rate (.714 times the market wage rate, Case B).

Prime Coking Coal

1. Log (cost)	= 3.5811 (114.502) t=.0312	+	1.1737 Log (o)* (.0005029) t=233.372	R^2 = .999 R = .999
2. Log (Marginal cost)	= 3.6832 (4.338) t=5.999	+	.1880 Log (o) (.0048) t=38.445	R^2 = .968 R = .984
3. Average cost	= 63.2317 (1.647) t=38.390	+	.0777 (.004327) t=17.962	R^2 = .8705 R = .9330
4. Marginal cost	= 74.539 (2.185) t=34.099	+	.09498 (.005743) t=16.538	R^2 = .8507 R = .9223

Medium Coking Coal

1. Log (cost)	= 3.7325 (237.837) t=.0156	+	1.1438 Log (o) (.00828) t=138.022	R^2 = .9974 R = .9987
2. Log (Marginal cost)	= 3.768 (1.0869) t=3.4664	+	.1779 Log (o) (.00868) t=20.500	R^2 = .8974 R = .9473
3. Average cost	= 57.9072 (1.1663) t=49.6484	+	.1424 (o) (.00578) t=24.644	R^2 = .9267 R = .9626
4. Marginal cost	= 65.662 (1.673) t =39.2252	+	.2038(o) (.00829) t=24.572	R^2 = .926 R = .9624

*(o) is the cumulative output from all blocks.

Table - 5.3.5

Results of the regressions fitted to the cost schedule generated by the Quadratic programming problem for base case.

<u>Prime Coking Coal</u>

1. Log (Cost)	= 3.7075	+ 1.1771 Log (o)*	R^2 = .9982	
	(143.3996)	(.007011)	R = .9991	
	t = .0258	t = 166.94		
2. Log (Marginal cost)	= 3.8205	+ .1922 log (o)	R^2 = .9384	
	(.9166)	(.007107)	R = .9687	
	t=4.1677	t=27.057		
3. Average cost	= 68.5117	+ .1323 (o)	R^2 = .8627	
	(1.6747)	(.007619)	R = .9288	
	t=40.9074	t=17.372		
4. Marginal cost	= 83.5888	+ .14614 (o)	R^2 = .7385	
	(2.7584)	(.01255)	R = .8593	
	t=30.3024	t=11.644		

<u>Medium Coking Coal</u>

1. Log (cost)	= 3.6749	+ 1.1796 Log (o)	R^2 = .99913	
	(82.507)	(.005072)	R = .99956	
	t=.045	t=232.539		
2. Log (Marginal cost)	= 3.7162	+ .2137 Log (o)	R^2 = .9801	
	(.5065)	(.004391)	R = .9900	
	t=7.3358	t=48.666		
3. Average cost	= 62.357	+ .1945 (o)	R^2 = .9248	
	(1.218)	(.008)	R = .9900	
	t=51.1642	t=24.311		
4. Marginal cost	= 71.4878	+ .2706 (o)	R^2 = .9229	
	(.001128)	(.01128)	R = .9606	
	t = 23.794	t=23.794		

*(o) = Cumulative Clean Coal output from all the blocks.

Table - 5.3.6

Marginal costs in the Industry at various points in time

PRIME COKING COAL

LOG_e (MARGINAL COST) = 3.8205 + .1922 log (cumulative output)

Qo = 118.85 M.T. $Qt = Qot^b$ (t = 1 implies 10 years)

Figures in Rs./Tonne.

Marginal Costs at the end of		Value of Marginal Cost for values of b.		
t	Years	1.05	1.10	1.20
1	10	114.50	114.50	114.50
2	20	131.40	132.2	133.90
3	30	142.60	144.20	147.10
4	40	151.10	152.80	157.20
5	50	158.0	160.50	165.4

Source : Model output (text)

MEDIUM COKING COAL

Log_e (MC) = 3.7162 + .2131 Log_e (Cumulative output)

Qo = 76.94 M.T. $Qt = Qot^b$

Figures in Rs./Tonne

Marginal cost at the end of		Value of Marginal cost for the following values of 'b'		
t	Years	1.05	1.10	1.20
1	10	104.0	104.0	104.0
2	20	112.35	122.35	124.20
3	30	132.1	134.60	137.80
4	40	131.25	144.0	148.35
5	50	143.45	151.80	151.10

Source : Model output (text).

Chapter - 6

MODEL SOLUTIONS WITH TRANSPORT COST

6.1 Model solutions for medium coking coal:

In the case of medium coking coal, the supply centres being distributed in space, the production and linkage programme has to be simultaneously determined. As shown before, the production programmes optimal in the sense of the dynamic model (1) can be obtained by solving model (2) successively for each period. As a first step we solve the model (2) for one period and study the behaviour of the underlying production/linkage programme in response to variation in some parameters. The model specified was solved with the following parameters.

The time horizon was fixed at ten years (75/76) to (84/85). The demand trajectory was cumulated over the entire period.

There are in all 14 blocks which could contribute to the medium coking coal production. In total 8 steel plants were considered of which the two steel plants would operate only from 82/83.

In order to keep the variables within manageable limits judgement was used to discard blocks which would not contribute to the optimal programme. Each of the blocks was associated with a loading point/washery capacity in the network. The coal from Ranigunj was however linked to the Durgapur washery; this involved the excess burden carrying cost from the loading point Ranigunj to the Durgapur Washery for Ranigunj Coal field (Block PM1).

A3i : 1.673, C

Cij (Raniganj - DSP) = 1.722 Rs.

(WiCij) = .673 x 1.722 = 1.1589 Rs/tonne of clean coal supplied.

In the final form the model solved had 80 variables and 8 constraints. The quadratic programming problem was solved using LEMKE's algorithm[1], on IBM 1620. A problem of this size took 64 iterations and a total CPU time of one hour and 45 minutes.

RESULTS

The results of the 1st run are reported in Table 6.1.2. For the purpose of simplicity only geological blocks which contribute to production are reported. The table shows production, linkages and the shadow price at each steel plant. As we have observed before, there can be more than one patternof production/linkages corresponding to the same value of the objective function, however, no other programme can lead to the value strictly less than reported. Thus what we report may be only one of the many possible optimal programmes. The optimality of the solution can be checked as follows. (See Table - 6.1.2)

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1 See Ravindran (1972).

Table - 6.1.1

MODEL : Run for medium coking coal with transport costs at 7.19 paise/tonne-KM.

- Base year 75/76
- Target year 84/85, Horizon: ten years.
- No. of variables 64, No. of constraints 8.

time taken 1 hour 30 minutes, IBM 1620, No. of iterations: 37.

Quadratic Programming, Lemke's algorithm.

	Value	% of total	Average (/tonne)
Total supply of coal to 8 steel plants (M.T.)	76.941	-	-
Total cost - value of the objective function at the optimal point (M.Rs.)	8463.101	-	109.99
Cost of production only (M.Rs.)	6574.308	77.68	84.633
Cost of transportation only (M.Rs.)	1888.7931	22.32	24.55
Lead (average kilometers)	341.427		
(average tonne-kilometers)	26269.723		

Source : Model Output.

ble – 6.1.2 : TABLE SHOWING OPTIMAL PRODUCTION TRANSPORTATION PROGRAMME UPTO 84/85.

STEEL PLANTS / OLOGICAL OCKS	BHI	DSP	RSP	TIS	IIS	BOK	VAP	VZP	Qi	A1i	A2i	MCi(Qi)= A1i+ 2A2iQi
JM3					3.475	2.618			6.094	82.473	1.5711	101.623
RM1		8.802							8.802	101.6727	.3768	108.[illegible]
EBM1			6.987	6.725					13.713	91.4547	.3025	99.751
EBM2				1.587	1.419				3.006	68.478	5.269	100.159
EBM3						20.102			20.102	49.731	1.276	101.318
RGM1	10.133		4.042				3.029	3.259	20.463	53.815	1.153	101.021
RGM2	.716								.7159	94.064	4.859	101.021
RGM3	4.043								4.043	80.850	2.4943	101.021
MAND (M.T.)	14.893	8.802	11.03	8.312	4.895	22.721	3.029	3.259				
ADOW PRICE AT E STEEL PLANT (LK)	154.289	108.307	122.009	113.474	108.157	102.549	246.252	169.250				

Firstly we observe that demand at all the steel plants is met with strict equality resulting in a positive shadow price at each steel plant.

Secondly, we need to check

$L_K \leq$ (Cijk + Wi Cij) + Λ1i +2Λ2i.Qi, for all i, j and K's.

The Rourkela Steel ~~plants~~ plant is supplied coal from two blocks RGMI and EBMI, hence the marginal cost of supplying coal to Rourkela from these two centres must be equal to to the shadow price at Rourkela. The marginal cost of supply from all candidate centres must be strictly less than the shadow price.

1. Shadow price at Rourkela (122.009) =

Marginal cost at RGM 1 (which supplies coal to it) :

(101.021)

\+ Transport cost from RGM1 to Rourkela : 20.98

122.009 = 101.021 + 20.98

which is again equal to

Marginal cost of production at EBM1 (which supplies coal to it) (99.751) + Transport cost of EBM1 to Rourkela = 22.25

122.009 = 99.751 + 22.25

For all the blocks which do not supply coal to Rourkela, the value, Mci (Qi) = (Λ1i + 2Λ2iQi) + Cijk is greater than 122.009 - which is indeed why they are not supplying coal to Rourkela.

In the case of Raniganj coal field the excess burden carrying cost has to be considered while checking optimality. Raniganj supplies DSP, thus (CijK + Wi Cij) + Λli + 2Λ2i Qi, is equal to.

1.722 + (.673 x 1.722) + (98.7927 + 2 x .3768 x 8.802)

= (2.881 + 98.7927) + 6.633

= 101.6737 + 6.633 = 108.307

which is equal to the shadow price of the DSP steel plant.

The total cost (Production + washing + transportation) of meeting the cumulative demand of 76.941 upto 84/85 is 8463.10160 Million Rupees.

Of the total cost of 8463.10160 M. Rs.

1888.7931 M. Rs. are incurred on account of transportation

6574.308 M. Rs. are incurred in production and washing

thus transportation accounts for 22.318% of the total cost and production washing accounts for 77.682% of the total cost.

The average cost of production defined as total cost/total output is computed to be

8463.10160 / 76.941 = 109.99

Similarly the average cost of transportation can be computed as

1888.7931 / 76.941 = 24.54859

From here we can compute the average lead in the industry corresponding to the optimal programme, as

24.54859/.0719 = 341.42 KMS

thus is slightly lower than the overall lead of 353 KMS reported in the Table - 5.1, as computed by the Ministry of Railways.[1]

Since the shadow price at each steel plant is different there is no one industry level marginal cost like in the prime coking coal industry, in fact the marginal cost would depend upon the spatial configuration of demand growth. However as a proxy the weighted average of marginal cost is computed to be $\sum_K L_K D_K / \sum_K D_K$. This is found to be Rs.114.90 per tonne.

Table - 6.1.3 shows the field-wise output of the clean coal and the associated raw coal that will have to be mined over the ten year horizon as suggested by the optimisation model.

Table - 6.1.3

Field	(Qi) Clean Coal	([illegible]Qi) Raw Coal Mined
Jharia	6.094	13.054
Raniganj	8.802	14.725
E.Bokaro	36.822	60.8389
W.Bokaro	-	-
N.Karanpura	-	-
Ramgarh	25.219	41.860
Total	76.941	130.4779

Source : Model output.

1 See Supplement to the Indian railways reports and accounts (1974-75).

From the table it is quite clear that over the next ten years from the point of view of the production, washing and transportation cost, the crucial coal fields are Ramgarh and E.Bokaro, contributing 25.219 M.T. (32.77%) and 36.822 M.T. (47.85%) respectively. The exact time profile of output in these coal fields is obtained by solving the model for sequentially for varying time horizons. The model suggests output of medium coking coal in Jharia. No contribution from West Bokaro and North Karanpura is suggested.

The washery capacities that are suggested by the model may be analysed as follows. A washery capacity is normally represented by the amount of raw coal that can be processed. From the knowledge of the amount of clean coal produced at each block and the block level yield function it is possible to estimate the washing capacity required at each location. The figures shown in the table 6.1.4 are washing capacities for medium coking coal only and indicate the quantities that are cumulated over ten years 1975-76 to 1984-85.

Table 6.1.4 - Field-wise washing capacity requirements

Field	Location washed at	Existing* capacity (M.T.) of Medium coking coal washery	Quantity of clean coal produced.Qi (M.T)	Quantity of raw coal processed A3iQi M.T.	Optimal suggested increase over the next ten years
Jharia	Dugdha	0.0	6.094	13.504	13.504
Ranigunj	Durgapur	22.00	8.802	14.725	-
E.Bokaro	Swang Kathara Kargali	58.5	36.822	60.8389	2.338
Ramgarh	Pithead	0.0	25.22	41.860	41.860

*By existing capacity we mean the existing operable capacity per year multiplied by ten and thus indicate the amount of coal that can be washed over the next ten years.

As table 6.1.4 indicates the model suggests setting up capacities for washing medium coking coal in Jharia and Ramgarh. Capacities adequate to wash a total of 13.504 and 41.86 M.T. of raw medium coking coal over the entire time horizon of ten years need to be set up in Jharia and Ramgarh. The model also suggests a small increase in the capacity at East Bokaro to be able to handle an additional 2.34 million tonnes. The model does not suggest any production in the West Bokaro coal field due to the high cost and poor yield in its blocks. Thus an existing capacity in this field of about .5 M.T. remains unutilised.

The shadow price L_K at the K^{th} steel plant can be interpreted as the marginal cost of supplying clean coal to that steel plant. It shows the impact on the total systems cost of increasing or decreasing the demand at the K^{th} steel plant. This is a valuable information for any study which attempts to make an explicit cost benefit analysis of controlling the coke rate at each steel plant. The table 6.1.5 ranks the various steel plants according to its relative coal cost as indicated by the shadow price.

Table - 6.1.5

Steel Plant	Shadow price
Bokaro	102.549
IISCO	108.157
Durgapur	108.307
TISCO	113.474
Rourkela	122.009
Bhilai	154.289
VZP	169.250
VAP	246.252

The costliest steel plants from the point of view of the C.I.F. coal supply cost are the steel plants in the south, namely VAP and VZP. The large transport cost associated with each of these plants explains this large shadow price. The best steel plant from the point of the coal cost is Bokaro, which is understandable since it is very close to the East Bokaro coal field, a cheap source of medium coking coal. The model was rerun with the cost functions estimated with capital evaluated at 18% discount rate (Case A) and labour valued at shadow wage rate (Case B) with the transport cost (freight charge) remaining the same, at 7.19 paise/tonne KM. The resulting production linkage programmes for Case A and B are reported in Table 6A.1 to 6A.4 in appendix. As observed in Chapter five, in Case A the cost functions shift upwards and in Case B they shift downwards relative to the base case. Thus in case A, since the production costs are increased relative to the transport cost the model responds by transporting coal over longer distances from now relatively cheaper sources, exactly opposite being the Case for Case B. Table 6.1.6 below shows the impact on lead and transport cost for the three cases.

Table 6.1.6 - Cost structure for Case A and B.

	Base Case	Case A	Case B
Transport cost as a % of total cost	22.32	21.40 (4.12)*	24.57 (10.08)
Lead - KM	341.427	344.86 (1.0)	336.51 (1.44)

*Figures in bracket show change over the base case.

It is found that movement to Case A and B, which in effect makes the Capital costs to increase relative to the wage costs, although it changes the magnitude of shadow prices at the eight steel plants, has no impact on their ranking as indicated in table 6.1.5.

Table 6.1.7 shows the impact on the output from different fields in response to the change in the relative price of capital.

Table - 6.1.7

Field	Base Case	Case A	Case B
Jharia	6.094	5.675 (6.87)*	5.426 (10.96)
Ranigunj	8.802	6.524 (25.88)	8.802 (0.0)
E.Bokaro	36.822	38.65 (4.96)	37.236 (.1.12)
Ramgarh	25.219	26.089 (3.45)	25.475 (1.01)

*Figures in bracket indicate % change over the base case.

It must be noted that the nature of the programming model - the positive semi definiteness of the quadratic form matrix, implies that there is a possibility of more than one optimal production linkage programmes corresponding to the same value of the objective function. Keeping this in view no firm conclusions like the geological structure of a certain coal field leads to more capital intensive techniques can be drawn.

6.2 Model solutions for alternate time horizons

The model is solved for spatial configuration of demands arising at different time horizons. This is expected to provide the following information.

- The model solution computes production linkage programmes as they are adjusted to varying spatial demand configurations as they are encountered in time.
- The increase in the average total costs of supply would not only account for geological depletion but would also reflect changes in the spatial configuration of demand as it moves away or closer to the supply centres over time.
- The result of production programme obtained at each time horizon considering transport cost is compared to one obtained without considering transport costs to throw further light on the degree of flexibility available in the underlying system.

The location specific demand projections are available only for ten years. Beyond which ~~it~~ they would depend on decisions to locate new steel plants, and expansion decisions in the currently operating steel plants. No information on these is available beyond ten years. However coal plans are computed for a steel production that is maintained at the level of 84/85 over the following decade. The resulting geological depletion can be interpreted as that arising due to sustaining of hot metal production plans initiated upto 84/85.

Table 6A.5 to 6A.10 in Appendix show the production linkage programme obtained by solving the model for various time horizons. Table 6.2.1 shows the behaviour of the average cost of production/washing, average cost of transportation only, and the industry wide average cost of supply as the model is solved for demands arising at various time horizons. Since the marginal cost at each steel plant is different, the table reports the weighted average as a proxy for the industry level marginal cost 'L'.

$$L = \sum_K D_K \cdot L_K \bigg/ \sum_K D_K$$

where L_K = Shadow price at each steel plant.

D_K = Demand at each steel plant.

Table - 6.2.1

Horizon	77/78	80/81	84/85	94/95
Total coal supplied (M.T)	17.458	39.786	76.941	182.182
Average total cost (Rs./T)	82.64	92.63	109.99	135.229
Average cost of production only	65.56	74.792	85.44	102.009
Average cost of Transport only	17.089	17.84	24.55	33.22
Average lead (kilometers)	237.68	248.22	341.427	462.03
Average marginal cost 'L' (Rs./T)	93.08	108.46	114.90	158.89

The increase in the industry wide average cost during the first six years of 12.08% is largely accounted for by an increase in the production cost of 14.08% together with a small increase in the lead of 4.39%.

However between 80/81 and 84/85 increase of 18.74% in the average cost of supply comes about as a result of a 37.55% increase in the cost of transportation, which is due to the southern steel plants of Vishakhapattanam and Vijayanagar going into production. Thus due to this phenomenon of spatial movement of demand away from the supply centres, average cost increases computed considering only production costs may be baised downwards.

Table 6.2.2 reports blockwise contribution at various time horizons with and without considering the transportation network. This is an indicator of the role transport cost plays in arriving at the optimal production programme. For instance considering only production costs, contribution from Ranigunj is low - 0 M.T., 3.5 M.T. but this block contributes 2.38 and 8.802 M.T. as transport costs are considered because it is favourably placed with respect to Durgapur steel plant. However over a twenty year horizon the picture reverses as transportation network stabilises, Ranigunj contributing more if only production costs are considered since its marginal cost rises at a slow rate.

Table 6.2.3 shows the average cost of production and washing with and without considering transport cost.

Table - 6.2.2 : BLOCKWISE CONTRIBUTION TO THE TOTAL CUMULATED DEMAND WITH AND WITHOUT CONSIDERING TRANSPORT COSTS

Year	Cumulated Demand	JM3	RM1	EBM1	EBM2	EBM3	RGM1	RGM2	RGM3	NKM1
76/78	17.458				.5437 (.3877)*	9.807 (8.947)	7.15 (8.123)			
80/81	39.786	3.314 (3.789)	2.3834 (0.0)		2.1784 (2.098)	16.515 (16.011)	14.230 (15.934)		1.162 (1.952)	
83/84	76.941	6.094 (8.850)	8.802 (3.504)	13.713 (15.682)	3.006 (3.127)	20.102 (20.259)	20.463 (20.631)	.7159 (.758)	4.043 (4.126)	
93/94	182.182	14.251 (18.909)	28.067 (32.097)	56.074 (51.298)	5.438 (5.172)	30.0145 (28.703)	31.573 (29.967)	3.353 (2.975)	9.181 (8.445)	4.097 (2.613)

*FIGURES IN BRACKET SHOW PRODUCTION PROGRAMMES WITHOUT CONSIDERING TRANSPORT COSTS.

The difference between the two values indicates the extent of choice available in the underlying system between the production and transport cost.

Table - 6.2.3 : Average cost of Production/ Washing of only.

Figs. in Rs./Tonne

Year	Considering production cost only	Considering production and transport cost
77/78	62.30	65.56
80/81	73.97	74.79
84/85	85.26	85.44
94/95	100.89	102.009

The average cost of production only is uniformly higher when transport costs are considered for then it may be optimal to exploit expensive blocks near the demand centres and bring about a saving in the overall cost. The difference between the average costs is wider in the beginning when wider differences in marginal costs across blocks exist, the difference narrows as marginal costs are progressively equalised by an optimal extraction programme. For the horizon 94/95 the difference again increases because, the spatial dispersion of demand makes the transport cost more important and provides scope for substitution of expensive block which are closer to the demand centre.

6.3 Model sensitivity to transport cost variation

The optimal solution obtained in Table 6.1.2 shows a production linkage pattern that minimises the total systems cost. However it is important for us to examine the role played by the transport cost vis-a-vis the production cost in arriving at the optimal pattern of supply. In other words we must try and quantify the trade off mentioned in the introduction.

One way to do this would be to increase the unit cost of transportation (Rs./tonne-KM) keeping the (production/washing) cost functions unchanged. Table 7A.11 in appendix reports the result of solving the model with the transport cost doubled. The model now reallocates production across blocks and shifts linkages, substituting supply from cheap sources near demand centres by expensive sources far away, thereby saving on transport cost which has now become relatively more important. The extent of saving on transport cost clearly depends upon degree of choice available in the underlying system between production and transport cost. Greater the possibilities of substitution and relinking greater would be the scope for reduction in transport cost and greater would be the drop in average lead in the system.

The production and linkages that are suggested by the optimisation model seem to be insensitive. The transport cost increases by less than double to the extent of 3770 against 3777 M. Rs. (if the production/transportation programme was unchanged). The decrease in average lead brought about is less than 1 KM (49 M. tonne KM). The minor changes in linkages observed are as follows.

The IISCO steel plant was earlier supplied its requirement of 4.895 M.T., partly from Jharia (JM3) to the extent of 3.475 M.T. and partly from Kathara in East Bokaro coal field (EBM2) - 1.419 M.T. The transport costs from these supply centres are Rs.6.53 and Rs.7.99/tonne kilometre each. When the transport costs are doubled all the 4.895 M.T. of demand is met from Jharia (JM3) which has relatively less transport cost.

The reallocation also induces changes like the following. The shipment of 2.618 M.T. of coal from Jharia (JM3) to Bokaro with transport cost of .92 Rs./tonne is reduced to 1.589 M.T. and are substituted by supply from blocks in East Bokaro coal field which have higher transport costs of Rs.2.38 and Rs.2.79/tonne. Thus the direction of change of any individual reallocation cannot be predicted but must emerge from the overall production transportation model.

The picture that emerges is that each steel plant is favourably placed from the point of view of transport cost with respect to some coal field. Each coal field has some blocks which are 'competitive' in the production/washing cost sense. Blocks competitive from the production cost point of view, located in a competitive coal field are linked to the favourably placed steel plant. This structure of the system makes the emerging coal field/steel plant linkages somewhat insensitive however the split of production across geological blocks within a field is determined by the quadratic cost functions.

The importance or otherwise of integration of production and transportation can also be studied by a two stage partial optimisation exercise, done as follows. In the first stage we assume that the planner minimises production washing cost only to supply the aggregate quantity of coal without considering the transport cost. In the second stage the contribution from each block to the output is given as a datum. Using these as capacities of supply centres the model is solved minimising transport cost to meet the spatial distribution of demands to determine linkages. Table 7A.13 in appendix reports the results of solving the transportation problem with capacities as determined by minimising production cost only for the three time horizons 77/78, 80/81 and 84/85. These can be compared with the corresponding production linkage programme emerging out of the to al optimisation exercise. As seen in the table 6.3.1, the cost of production only in the case of partial optimisation is lower than the total optimisation exercise, but this is more that offset by a large decrease in transport cost. If transport cost considerations affected the production programme greatly, then the total cost for the partial optimisation exercise would have been substantially higher. In our case, the total cost of partial optimisation is higher than the total optimisation, by .196% if the exercise is carried out for a three year horizon. This increases to .51% for a ten year horizon. During the period between 80/81 and 84/85, the industry undergoes a spatial dispersion as a result of the southern steel plants going into operation. Thus when solved for the year, 80/81 (base year 75/76), the partial optimisation results in total cost higher by .25% and when solved over ten year horizon upto 84/85, the total cost is higher by .51%. Thus, the spatial dispersion of the industry induces result in larger gains due to integration of production and transportation cost. The total increase of .51% is not a significant gain in % terms but in absolute terms integration of production and transport cost implies a gain of about 44 million rupees. This is expected to increase if, after 1984/85, there is a spatial dispersion of steel plants. Similarly it is hoped that the methodology evolved can be used for non-coking coal industry where due to a large dispersion of coal fields and power plants the integration may lead to greater gains.

Table - 6.3.1 : Structure of costs with total optimisation compared with that of partial optimisation.

ime orizon	Total coal supplied (M.tonnes)	With total optimisation (Million rupees) Cost of production only CPR	Cost of Trans-portation only CTR	Total Cost TC.	With partial optimisation (Million Rs.) CPR	CTR	TC	Difference (Total partial) in absolute terms (M.Rs.) CPR	CTR	TC	% terms CPR	CTR	TC
/78	17.458	1144.55	298.34	1442.90	1087.67	358.05	1445.72	56.88	-59.71	2.83	4.96	20.01	.196
0/81	39.786	2975.69	710.04	3685.74	2942.91	750.22	3693.13	32.78	-40.18	7.40	1.10	5.65	.25
4/85	76.941	6574.31	1888.79	8463.10	6560.52	1946.27	8506.79	13.79	-57.48	43.69	.21	3.04	.51

Table - 6A.1

MODEL : Run for medium coking coking with capital costs at 18% discount rate

Base year 75/76

Target year 84/85 Hrizon : ten years

No. of variables 64. No. of constraints 8.

Time taken: 1 hour 35 minutes IBM 1620

No. of iterations 42.

Quadratic programming, Lemke's algorithm.

	Value	% of Total	Average (tonne)
Total supply of coal to 8 steel plants (M.t.)	76.941		
Total cost-value of the objective function at the optimal point (N.Rs.)	8899.5254		115.6668
Cost of production washing only (M.Rs.)	6991.754	78.60	90.871
Cost of transportation only (M. Rs.)	1907.771	21.40	24.795
Lead Total tonne kilometres	344.857		
transported (M)	26533.671		

Table 6A.2 – Solution with capital valued at 18% discount rate.

STEEL PLANTS / GEOLOGICAL BLOCKS	BHI	DSP	RSP	TIS	IIS	BOK	VAP	VSP	Qi	A1i	A2i	MCiQi=A1i + 2A2iQi
JM3					4.5782	1.0974			5.6756	89.786	1.645	108.4587
RM1		6.5241							6.5241	113.193	.387	118.242
EBM1		2.2787	6.1213	6.109					14.509	96.311	.354	106.583
EBM2				2.202	.3185				2.520	80.450	5.265	106.990
EBM3						21.6238			21.6238	52.853	1.272	107.8639
RGM1	9.7641		4.909				3.029	3.259	20.961	57.296	1.206	107.854
RGM2	.7324								.7324	99.831	5.477	107.854
RGM3	4.396								4.396	83.955	2.718	107.852
DEMAND	14.893	8.802	11.03	8.312	4.895	22.721	3.029	3.259	76.941			
SHADOW PRICE L_K	161.114	119.024	128.834	120.304	114.983	109.374	253.840	676.074				

Table - 6A.3

MODEL : Run for medium coking coal with cost functions evaluated for shadow wage rate

- Base year 75/76
- Target year 84/85 Horizon : ten years.
- No. of variables 64. No. of constraints 8.

Time taken 1 hour 6 minutes IBM 1620

No. of iterations 21

Quadratic programming Lemke's algorithm.

	Value	% of total	Average Rs./tonne.
Total supply of coal to 8 steel plants (M.T.)	76.941		
Total cost value of the objective function at the optimal point (M.Rs.)	7698.138		100.05
Cost of production only (M. Rs.)	5806.511	75.43	75.46
Cost of transportation only (M. Rs.)	1891.627	24.57	24.58
Lead (average kilometres)	336.516		
(average tonne-kilometres)	25891.890		

Table 6A.4 - Solution with labour valued at shadow wage rate (.714 times the market wage rate)

STEEL PLANTS / BLOCKS	BHI	DSP	RSP	TIS	IIS	BOK	VAP	VZP	Qi	A1i	A2i	Mci(Qi)=A1i +2 A2i(Qi) Rs
JM3					4.895	.5308			5.4258	71.449	1.304	85.60
RM1		8.802							8.802	92.672	.3069	98.074
EBM1			6.735	5.756		.7679			13.258	79.039	.2900	86.728
EBM2				2.5557					2.5557	64.02	4.523	87.138
EBM3						21.4221			21.4221	46.836	.9610	88.01
RGM1	13.3619		4.2947				3.029		20.6856	49.276	.9360	87.79
RGM2	.6079								.6079	82.681	4.374	87.99
RGM3	.9231							3.259	4.182	71.154	2.014	87.99
DEMAND	14.893	8.802	11.03	8.312	4.895	22.721	3.029	3.259	76.941			
SHADOW PRICE L_K	141.259	98.074	108.979	100449	95.129	89.519	233.229	156.215				

Table - 6A .5

MODEL : Run for Medium Coking Coal.

- Base year - 75/76.
- Target year 77/78 ~~Horizon~~ 3 years from the base year.
- No. of variables 36, No. of constraints 6.

time taken 45 minutes. IBM 1620. No. of iterations 18.

Quadratic Programming Lemke's algorithm.

	Value	% of total	Average /tonne
Total supply of coal to the six steel plants (M.T.)	17.458		
Total cost - value of the objective function at the optimal point (M.Rs.)	1442.9		82.64
Cost of production only (M.Rs.)	1144.556	79.32	65.56
Cost of transportation only (M.Rs.)	298.344	20.68	17.089
Lead - Kilometers	237.680		
Tonne - Kilometers	4149.43		

Table - 6A.6 MODEL RUN WITH DEMANDS UPTO YEAR 77/78

STEEL PLANT GEOLOGICAL BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi	A1i	A2i	MCi(Qi)= A1i+2A2iQi
JM3									0	82.474	1.5711	82.474
DM1									0	101.6737	.3768	101.6737
EBM1									0	91.4547	.3025	91.4547
EBM2				.5437					.5437	68.478	5.2693	74.207
EBM3		2.327		1.3913	1.296	4.75			8.807	49.731	1.276	74.758
RGM1	4.149		3.001						7.15	53.8153	1.1534	70.308
RGM2									0	94.064	4.859	94.064
RGM3									0	80.85	2.4943	80.850
DEMAND (M.T.)	4.149	2.327	3.001	1.935	1.296	4.750	0.0	0.0				
SHADOW PRICE (L.K.)	123.577	86.24	91.29	87.52	82.20	76.167	0	0				

Table -6A.7

MODEL : Run for Medium Coking Coal with demands at six years from the base year.

- Base year 75/76.
- Target year 80/81 Horizon - six years.
- No. of variables 36, No. of constraints 6.

time taken 45 minutes, IBM 1620, No. of iterations: 24.

	Value	% of total	Average /tonne.
Total supply of coal to the six steel plants (M.T)	39,786		
Total cost - Value of the objective function at the optimal point (M.Rs.)	3685.7377	-	92.63
Cost of production only (M.Rs)	2975.69	80.74	74.792
Cost of transportation only (M.Rs.)	710.04	19.26	17.84
Lead - tonne kilometer	9875.38		
kilometers	248.212		

Table - 6A.8 MODEL RUN WITH DEMANDS UPTO 80/81

STEEL PLANT GEOLOGICAL BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi	A1i	A2i	MCi(Qi) = A1i+2A2iQi
JM3		.4676			2.847				3.314	82.4738	1.5711	92.887
RM1		2.3834							2.3834	101.6737	.3768	103.469
EBM1									0	91.4547	.3025	91.4547
EBM2				2.1784					2.1784	68.478	5.2693	91.435
EBM3		1.6844		2.1335		12.697			16.515	49.731	1.276	91.877
RGM1	8.386	.5664	5.278						14.230	53.8153	1.1534	86.641
RGM2									0	94.064	4.859	94.064
RGM3			1.162						1.162	80.850	2.4943	86.641
DEMAND	8.386	5.102	6.442	4.312	2.847	12.697						
SHADOW PRICE	139.914	103.469	107.635	104.750	99.422	93.394						

Table - 6A.9

MODEL : Run with demands cumulated for a 20 year horizon. Demands assumed to stabilise after ten years (84/85) and coal supply only to sustain a steel production level reached in 84/85.

- Base year 75/76.
- Target year 94/95, Horizon 20 years.

No. of variables 72, No. of constraints 8.

Time taken 2 hours. No. of iterations 36.

	Value	% of total	Average /tonne
Total coal supplied (M.T.)	182.182	-	
Total cost - Value of the objective function at the optimal point (M.Rs.)	24636.4230	-	135.229
Cost of production and washing only (M.Rs.)	18584.278	75.44	102.009
Cost of transportation only (M. Rs.)	6052.1452	24.56	33.220
Lead - Tonne-Kilometres	84174.48		
Kilometres	462.03		

Table - 6A.10 MODEL RUN WITH DEMANDS UPTO 94/95

STEEL PLANT GEOLOGICAL BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi	A1i	A2i	MCi(Qi) = A1i+2A2iQi
JM3						14.251			14.251	82.474	1.5711	127.253
RM1		18.052			10.015				28.067	101.6737	.3768	122.825
EBM1			22.50	17.489			16.085		56.074	91.4547	.3025	125.379
EBM2				.822		4.626			5.438	68.478	5.2693	125.787
EDM3						30.0145			30.0145	49.731	1.276	126.328
RGM1	14.531						.0934	16.949	31.573	53.8153	1.1534	126.649
RGM2	3.353								3.353	94.064	4.8590	126.649
RGM3	9.181								9.181	80.850	2.4943	126.649
NKM1	4.097								4.097	118.6098	.8381	125.477
DEMAND	31.163	18.052	22.50	18.312	10.015	49.012	16.179	16.949				
SHADOW PRICE (L.K.)	179.918	122.825	147.639	139.103	126.850	128.178	271.881	194.880				

Table - 6A.11

MODEL : Run for Medium Coking Coal with the transport cost doubled @ 14.38 Paise/tonne KM.

- Base year 75/76.
- Target year 84/85, Horizon; ten years.
- No. of variables 64, No. of constraints 8.
- time taken 1 hour 35 minutes, IBM 1620, No. of iterations: 37.

Quadratic programming, Lemke's algorithm.

	Value	% of total	Average /tonne
Total supply of coal to 8 steel plants (M.T)	76.941		
Total Cost - Value of the objective function at the optimal point (M.Rs.)	10364.034		134.70
Cost of production only(M.Rs)	6593.530	63.62	85.696
Cost of transportation only (M.Rs.)	3770.5046	26.38	49.0972
Lead - (average kilometers)	340.786		
- (average tonne-kilometers)	26220.445		

Table -6A.12 MODEL RUN WITH TRANSPORT COST DOUBLED

STEEL PLANT GEOLOGICAL BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi	Ali	A2i	MCi(Qi)= Ali+2A2iQi
JM3					4.895	1.589			6.484	82.4738	1.5711	102.8478
RM1		8.802							8.802	104.5547	.3768	111.1878
EBM1			6.530	6.1105					12.640	91.4547	.3025	99.1022
EBM2				2.2014		.782			2.9835	68.478	5.2693	99.920
EBM3						20.345			20.349	49.731	1.276	101.6639
RGM1	9.9451		4.4996				3.029	3.259	20.632	53.8153	1.1534	101.642
RGM2	.7798								.7798	94.064	4.859	101.642
RGM3	4.1679								4.168	80.850	2.4943	101.642
DEMAND (M.T.)	14.893	8.802	11.03	8.312	4.895	22.721	3.029	3.259	-			
SHADOW PRICE (L.K.)	208.179	111.187	143.619	126.549	115.914	104.699	392.105	238.101	-			

Table - 6A.13 : Production/transportation programme for the partial optimisation exercise, Qi's are determined by considering production cost only.

Time horizon : 83/84 Total Cost of Transport 1946.27 M.Rs.

STEEL PLANT BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi
JM3				1.494	4.895	2.462			8.851
RM1		3.505							3.505
EBM1		2.169	3.667	6.818			3.029		15.683
EBM2		3.128							3.128
EBM3						20.259			20.259
RGM1	14.893		2.479					3.259	20.631
RGM2			.758						.758
RGM3			4.126						4.126
DEMAND	14.893	8.802	11.030	8.312	4.895	22.721	3.029	3.259	

Time Horizon : 80/81 Cost of Transport 750.22 M.Rs.

STEEL PLANT BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi
JM3				.942	2.847				3.789
EBM2		2.043		.055					2.098
EBM3				3.315		12.697			16.012
RGM1	8.386	1.107	6.442						15.935
RGM3		1.952							1.952
DEMAND	8.386	5.102	6.442	4.312	2.847	12.697			

Time Horizon : 77/78 Cost of Transport 358.056. M.Rs.

STEEL PLANT BLOCK	BHI	DSP	RSP	TISCO	IISCO	BOK	VAP	VZP	Qi
EBM2		.387							.387
EBM3		.967		1.935	1.296	4.750			8.948
RGM1	4.149	.973	3.001						8.123
DEMAND	4.149	2.327	3.001	1.935	1.296	4.750			

Chapter - 7

AN OVERVIEW AND CONCLUSIONS

7.1 The Approach

In this study we have presented a methodology which analyses mineral extraction programmes keeping in mind notions of geological depletion and locational advantage vis-a-vis demand centres in an integrated analytical framework. The methodology, though applicable to mineral extraction programmes in general has been developed and applied to the coking coal industry. Methodologically as brought out in the literature survey, there is no single satisfactory index of geological depletion in a pool of exhaustible resource which conforms fully to our a priori notions of depletion. We would expect marginal cost to be rising with each tonne extracted, thus marginal cost increases should be observed even for maintaining a given rate of production. Also, different resource pools, depending on their geological characteristics should have different rates of increase of marginal cost. In Chapter 3, we developed the idea of a cumulative cost function as an index of the geological complexity of a block, and depletion could be thought of as cost increases resulting from movement along this Block Level Cumulative Cost Function (BLCCF). The BLCCF has been developed as a long run cost function in which it is assumed that capacity can be costlessly adjusted to the amount desired to be produced during any year. One would have liked to work with more specific capital and operating costs, introducing constraints which restrict the shifting of capital across blocks overtime. This would have led to a model of considerable mathematical complexity. The assumption of costless shiftability of capital has the merit that it greatly simplifies computation and is empirically not too unrealistic. After estimating the BLCCF, our subsequent aim is to use them to identify a production washing and transport programme to meet an exogenously specified, spatially distributed, time profile of demands, at minimum discounted present value of production, washing and transport cost. The Model (1) presented in Chapter 5, is a model for project identification rather than evaluation of projects already identified. Working with seamwise geological data, rather than specific investment proposals, Model (1) provides a solution to the Chronic problem of lack of availability of a shelf of feasibility reports which is particularly severe for the coal industry. The planning process in the coal industry as envisaged in our approach then consists of two phases. First, a macro model to identify crucial blocks is solved, and then tactical drilling in each of these is initiated based on the output predicted by the model to identify the locations for which project reports may be commissioned.

7.2 Estimation of the BLCCF

The BLCCR is estimated at the level of a geological block which consists of a number of coal seams. The BLCCF is built up by aggregating seam level total cost functions, which depend upon the geological parameters of the coal seam. An attempt was made to statistically correlate seam level costs to the geological parameters of a seam mined by underground methods. This proved to be unsuccessful because the data were not sufficiently differentiated. Perhaps it is important to collect data on costs incurred in various stages of mining and geological parameters that influence costs there so that cost functions for each stage could be estimated. These can then be aggregated to arrive at a seam level total cost curve. In the absence of statistical cost functions we relied upon the engineering estimates of how costs in each stage of mining are influenced by geological variables. It was however possible to correlate costs of open cast mines to the coal overburden ratio of the coal seam. Similarly it was possible to statistically correlate* the yield in a coal washery ~~given~~ to the raw coal and clean coal ash, from washery data (Appendix 3A.2).

Both these analyses should by themselves be useful to the researchers and policy makers in this area.

The BLCCF is computed for each block by estimating cost incurred and output obtained as seam after seam is exploited in an increasing cost sequence. The set of points as obtained above is approximated by a quadratic function. This characterises the geological complexity of a block by just two parameters. The nature of geological complexity of each block can be easily understood by observing the impact on these parameters of a change in factor prices. In the course of estimation of the BLCCF we also obtained a forecast of the distribution of production between open cast and underground methods of mining and how this changes in response to a change in factor prices, (Table 3.5.10).

7.3 The programming model

In Chapter 5 we use the BLCCF's as inputs and specify an overall model to minimise the discounted present value of production, washing and transportation costs, while meeting an exogenously specified spatially distributed time profile of demands. Chapter 5 presents solutions to the model when transport costs are absent by a simple dual enumeration procedure. The marginal cost curves obtained provide~~s~~ an economic dimension to Indian coking coal reserves which hitherto had only been characterised by their geological attributes. For Prime Coking Coal, reported in Fig. 5.1 the marginal cost increases from 55 Rs./te to 155 Rs./te as the additional cumulative output builds up, to 600 M. tonnes, the present annual output being 7.37 m.t. It must be noted that these refer to an optimal programme of mining. The marginal cost functions reveal intrinsic differences in the nature of deposits of Prime and Medium Coking Coal. Thus although the marginal cost curve of the Prime Coking coal deposit lies above that of Medium coking coal deposit, marginal costs in the latter increase at a higher rate than the former. Table 5.3.6 computes the marginal cost at various points in time with respect to varying rates of growth of demand for coking coal in India. Thus at 5% per annum rate of growth, for the present scale of exploitation the marginal cost in the Prime coking coal sector increases by 40% and that in Medium coking coal by 38% over the next 50 years. The statistical approximations to the cost schedules, and their shifts in response to factor price have been reported in table 5.3.5. These estimates we believe would be valuable in evolving long run pricing policies and as inputs into models constructed for exploring choices on interfuel substitution.

7.4 Interactions between production and transportation cost

In chapter 6 we solve the model for medium coking coal by considering the transportation costs. From the results we obtain the fieldwise production and linkage programme of medium coking coal, for various time horizons. The model also suggests fieldwise washing capacity requirements as well as shadow prices of clean coking coal at all steel plants. It thus provides a ranking in terms of coking coal cost. We also study the impact of changing the wage rate and discount rate on fieldwise contribution to the total output. The importance of integrating production and transport costs is examining by solving the model with transport cost doubled and conducting a two stage optimisation. It is found that integration of production and transport costs does not lead to significant gains in relative terms. Thus if one worked out a production programme without considering transport costs and used this as capacities to minimise transport cost only, the increase in cost would only be .51% over the integrated optimisation exercise. Even then the savings in absolute terms are 44 million Rs. However, as steel plants are located away from the North Eastern region in future, the transport cost would become important. Similarly, the methodology developed would be useful, for analysing the non-coking coal industry where transport cost could be expected to play a major role as the demand centres for non-coking coal are many and are dispensed over the country.

The line of analysis that is developed in this study should be extended to the planning of extraction programmes of other minerals. The empirical results presented are expected to be useful to policy makers in coal, energy and related industries in India.

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