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SOME EFFECTS OF A MAGNETIC FIELD ON THE FLOW OF A NEWTONIAN FLUID THROUGH A CIRCULAR TUBE

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In this paper, we investigate some characteristics of the flow of a viscous incompressible Newtonian fluid through a circular tube under a transverse magnetic field. The problem is of high interest from the view point that externally imposed magnetic field has considerable influence on the biological system of human life. Moreover, flows in channels and tubes are often studied keeping in view their applications in physiological flow problems, for example, blood flow through artery. Assuming the magnetic field constant and the viscosity of the fluid dependent on temperature, we determine the distributions of axial velocity and temperature in terms of the Hartman number and a parameter, respectively arising from the imposed magnetic field and the associated temperature field. The results are computed numerically and discussed.

1. INTRODUCTION

A variety of fluid flows in channels and tubes are investigated owing to their applications in physiological and engineering problems. Based on Navier-Stokes equations, many researchers have developed mathematical models for transportation of blood through arteries. Shah[1] has reviewed the fully developed and developing solutions for blood flowing in a tube, channel and annular duct. Weisman and Mockros[2] solved the mass conservation equation for straight and coiled permeable tube flows utilizing an empirical relation for oxygen—hemoglobin dissociation curve, a constant effective diffusivity and a Newtonian parabolic velocity profile. Attempts have also been made to study the influence of magnetic field on physiological fluid flows. It is well known that externally imposed magnetic field plays an important role in stimulating the functions of various biological systems of the body. For example, it helps in regenerating tissues of the body (Bansal[3]). McMichael and Deutsch[4] studied the magneto-hydrodynamics of laminar flow in slowly varying tubes in an axial magnetic field. Subsequently, Desikachar and Rao[5] analysed the influence of a magnetic field on the blood oxygenation process. Krishna and Rao[6] investigated the motion of a viscous incompressible flow through a nonuniform channel under a transverse magnetic field. The results obtained by them are found useful in analysing some biomedical problems. In the cases of blood oxygenation and hemodialysis blood is removed from the body for processing and returned to the body. In situations like these, it is important to control the temperature of the blood when it is out of the body in order to prevent damage. Shah[1] studied the effects of temperature on the transport phenomena of blood.

In the present paper, we consider the problem of flow of a viscous incompressible Newtonian fluid through a circular tube under a transverse magnetic field. As such, this may serve as a simple model for physiological flow e.g., blood flow through a straight artery of large diameter. Rodkiewicz[7] discussed that blood may be considered a Newtonian fluid when flowing through conduits of a large diameter. Assuming the viscosity of the fluid to be dependent on the temperature, we determine the distributions of velocity field and associated temperature field in terms of the appropriate parameters.

2. Formation of the Problem

Let us consider the steady laminar flow of an electrically conducting Newtonian fluid through a circular tube of radius R. We use cylindrical polar coordinates (r, θ, z), with z-axis lying along the axis of the tube and the origin on the axis of the tube. A constant magnetic field of strength B_0 is originally imposed perpendicular to the axis of the tube.

The fluid is assumed to be flowing parallel to the axis of the tube with velocity v under the influence of a constant pressure gradient and the Lorenz force. Due to symmetry of the problem the flow will be independent of θ. The velocity components in the r, θ and z directions are, respectively given by

$$v_r = 0,\ v_\theta = 0,\ v_z = v_z(r). \qquad \dots (1)$$

The effects of the induced magnetic field and the electric field are assumed negligible. No external electric field is applied and the Joule heating is negligible in comparison with viscous heating. The equations for the momentum and energy can be written now in simple forms

$$\frac{1}{r}\frac{d}{dr}\left(\mu r \frac{dv_z}{dr}\right) - \frac{dp}{dz} - \sigma B_0^2 v_z = 0 \qquad \dots (2)$$

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \mu\left(\frac{dv_z}{dr}\right)^2 = 0 \qquad \dots (3)$$

where $\frac{dp}{dz}$ is the pressure gradient, σ the electrical conductivity, k the coefficient of thermal conductivity, T the fluid temperature and μ the coefficient of viscosity. r is the measure of the distance along the radial direction.

We now introduce the following nondimensional quantities

$$v_z^* = \frac{v_z}{v_m}, \; v_m = -\frac{R^2}{\mu_0}\frac{dp}{dz}$$

$$\xi = \frac{r}{R}, \; \mu^* = \frac{\mu}{\mu_0}, \; T^* = \frac{T_0 - T}{T_0}$$

$$P_r = \frac{\mu_0 C_p}{k} \quad \text{(Prandtl number)}$$

$$E_c = \frac{v_m^2}{C_p T_0} \quad \text{(Eckert number)}$$

where μ_0 is the coefficient viscosity at temperature $T = T_0$ (on the axis) and C_p is the specific heat at constant pressure.

Equations (2) and (3) are reducible now to dimensionless forms

$$\frac{d}{d\xi}\left(\mu^* \xi \frac{dv_z^*}{d\xi}\right) + 4\xi - 4 v_z^* \xi L = 0 \qquad \text{... (4)}$$

$$\frac{d^2 T^*}{d\xi^2} + \frac{1}{\xi}\left(\frac{dT^*}{d\xi}\right) - \mu^* E_c P_r \left(\frac{dv_z^*}{d\xi}\right)^2 = 0 \qquad \text{... (5)}$$

where $L = \dfrac{\sigma B_0^2 R^2}{4\mu_0}$ is the Hartman number

The boundary conditions are given by

(i) $v_z^* = 0$ at $\xi = 1$,

(ii) $\dfrac{dv_z^*}{d\xi} = 0$ at $\xi = 0$,

(iii) $T^* = 0$ at $\xi = 0$,

(iv) $\dfrac{dT^*}{d\xi} = 0$ at $\xi = 0$. ... (6)

We now put

$$T^* = \frac{1}{\beta}\theta^* \qquad \text{... (7)}$$

where β is the measure of variation of viscosity with temperature, and

$$\mu^* = e^{-\theta^*}. \qquad \text{... (8)}$$

Taking (7) and (8) into account in (4) and (5) we obtain, respectively

$$\frac{d^2 v_z^*}{d\xi^2} + \frac{1}{\xi}\frac{dv_z^*}{d\xi^2} + 4(1 - L v_z^*)\, e^{\theta^*} = 0 \qquad \text{... (9)}$$

and

$$\frac{d^2\theta^*}{d\xi^2} + \frac{1}{\xi}\frac{d\theta^*}{d\xi} - Ne^{-\theta^*}\left(\frac{dv_z^*}{d\xi}\right)^2 = 0 \qquad \text{... (10)}$$

where $N = \beta E_c P_r$.

The boundary conditions (6) are transformed to

(i) $v_z^* = 0$ at $\xi = 1$,

(ii) $\frac{dv_z^*}{d\xi} = 0$ at $\xi = 0$,

(iii) $\theta^* = 0$ at $\xi = 0$,

(iv) $\frac{d\theta^*}{d\xi} = 0$ at $\xi = 0$. ... (11)

Treating L and N as parameters we seek solutions of eqns. (9) and (10), subject to boundary conditions (11), in the next section.

3. Numerical Solutions and Discussions

The system of eqns. (9) and (10) with the condition (11) are treated here as two-point boundary value problem. To solve these equations, we apply a shooting method (Hall and Watt[8]). First we guess an arbitrary value for v_z^* at $\xi = 0$ and integrate the equations from $\xi = 0$ to $\xi = 1$, utilizing the conditions (ii), (iii) and (iv) of (11), by Runge-Kutta method. The value of v_z^* obtained at $\xi = 1$ either overshoots or undershoots the prescribed value $v_z^* = 0$ at $\xi = 1$. The process is repeated every time reguessing the value of v_z^* at $\xi = 0$ until v_z^* obtained at $\xi = 1$ matches with its prescribed value within admissible tolerance. The solutions for v_z^* and θ^* thus obtained are plotted, respectively in Figs. 1-2 and Figs. 3-4.

These figures demonstrate the variations of the velocity and temperature profiles, respectively due to choices of the Hartman number L = 0, 0.1, 0.2, 0.3, 0.4 (N = 0.2, 0.4) and the parameter N (corresponding to temperature field) = 0.1, 0.2, 0.3, 0.4, 0.5 (L = 0.1, 0.3). After examining them we reach the following conclusions :

(i) v_z^* is maximum at the centre of the tube and decreases with the increase of ξ and finally becomes zero at the wall ($\xi = 1$) in all the individual cases.

(ii) For a fixed value of N, v_z^* is found to decrease with the increase of L. Core-like character (Gold[9]) of the velocity distribution is less apparent in the cases under consideration.

(iii) For a fixed value of L, v_z^* is found to increase with the increase of N at any $\xi(\neq 1)$.

(iv) In all the individual cases, θ^* is found to increase from its zero value at the centre with the increase of ξ and becomes maximum at the wall.

(v) For a fixed value of L, θ^* is found to increase with the increase of N at any $\xi(\neq 0)$.

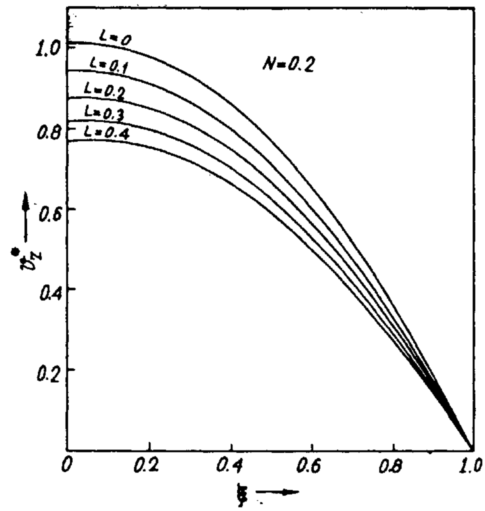

FIG. 1. Axial velocity component v_z^* for values of the Hartman number L = 0, 0.1, 0.2, 0.3, 0.4 (N = 0.2).

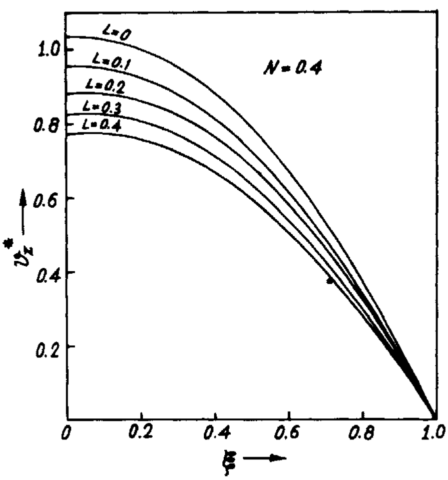

FIG. 2. Axial velocity component v_z^* for values of the Hartman number L = 0, 0.1, 0.2, 0.3, 0.4 (N = 0.4).

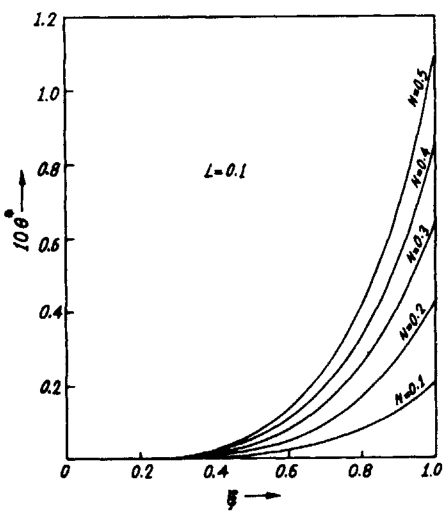

FIG. 3. Distribution of temperature θ^* for values of the parameter N = 0.1, 0.2, 0.3, 0.4, 0.5 (L = 0.1).

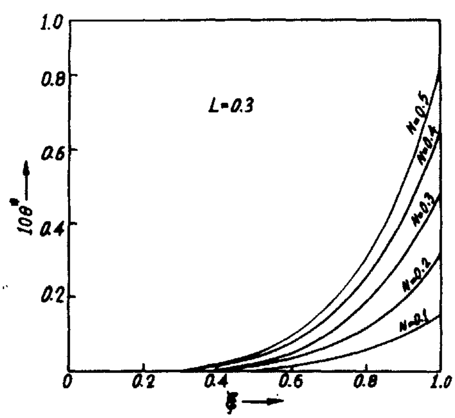

FIG. 4. Distribution of temperature θ^* for values of the parameter N = 0.1, 0.2, 0.3, 0.4, 0.5 (L = 0.3).

(vi) For a fixed value of N, θ^* is found to decrease with the increase of L at any $\xi(\neq 0)$.

The results obtained here are not only of interest in hydrodynamics but also useful for better understanding of the problem of controlling blood flow through arteries and veins, by the application of external magnetic field.

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