ON THE PROBLEM OF SCREENING DEFECTIVE ARTICLES DURING PRODUCTION

By MOTOSABURO MASUYAMA

Meteorological Research Institute, Tokyo

and

Indian Statistical Institute, Calcutta

1. Introduction

The subject of this note is the problem of screening defective articles by a destructive test during production. As far as I am aware, this problem has not been considered before and my aim in this note is to give some preliminary consideration to the problem and draw attention to certain possible approaches.

When I discussed with Mr. T. Okutu, quality control engineer of the Research Institute, Fuji Photofilm Co., about my idea of applying integral geometry to estimate the total area of defects on a sheet of film base, Okutu asked me how to screen those defective portions during production.

The problem raised by Okutu is neither the problem of estimation nor the problem of testing hypotheses. Our main concern is to remove and reject the defective portions as much as possible during production.

For the sake of simplicity, let us consider a one-dimensional discrete case, for example, a series of roll or cut films during production.

Now according to the usual method of sampling purely at random we can pick out the defective articles, (i.e. the defective rolls in our case) in proportion to the fraction of defective articles in the population times the sample size, on an average. This method, however, is not likely to be very useful for "screening" the defective units, so that we must use some biased method of sampling, so as to "condense" the defective articles in the population when we hit an effective article and "dilute" the defective articles otherwise.

We assume that the total length T of the series of products is sufficiently large to enable us to neglect the anomalies which may arise at both ends of this series.

We shall consider two different methods of condensation viz. the centred method and the bordered method, and one method of dilution.

2. THE CENTRED METHOD

The centred method of condensation consists of the following sequential procedure.

Sample at random one article during production and test it. If it is effective, remove $2n$ neighbouring articles from the series of products, the first one being the centre.

We assume that the distance between two consecutive runs of the defective articles is greater than n. This assumption is approximately satisfied in most practical cases.

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Let D, k and r be the total number of the defective articles, the number of defective articles which are contained in the removed articles of length $2n$, and the total number of the runs of defective articles respectively. Thus k is a random variable and is identically equal to zero when $n = 0$. We put $D/r = m_D$.

The expectation of k is given by

$$\mathcal{E}\{k\} = rn(n+1)/(T-D). \qquad \dots \quad (1)$$

The proportion of defective articles in the rest of the series or the probability of the second random sample being a defective article will be

$$p(n) = (D-k)/(T-2n-1). \qquad \dots \quad (2)$$

We want to maximize the expectation of the difference

$$f(n) = p(n)-p(0), \qquad \dots \quad (3)$$

which is equivalent to maximizing

$$g(n) = \{2an-rbn(n+1)\}/(b-2n), \qquad \dots \quad (4)$$

where we put

$$D(T-D) \equiv DE = a, \quad T-1 = b. \qquad \dots \quad (5)$$

The required solution is given by

$$n = \{b-\sqrt{b^2-2(2a-rb)/r}\}/2$$

$$= \{T-1-\sqrt{T^2-(1+4m_DE)}\}/2, \qquad \dots \quad (6)$$

where E is the total number of the effective articles in T.

In most cases which we might come across in a factory $E \gg D$ and $T \gg 1$, so that we may use an approximation

$$n \doteqdot m_D, \qquad \dots \quad (7)$$

where m_D cannot be smaller than 1 by definition.

We shall give an illustration, where $T = 1{,}000$ and $r = 5$.

D	m_D	n	$[n]$ or $[n]+1$	$\mathcal{E}\{f(n)\}/p(0)$
10	2	1.48	1	0.1%
50	10	9.09	9	0.9%
100	20	17.84	18	1.8%

3. The bordered method

Next we consider the bordered method. The main change lies in the method of removing certain consecutive articles from the original series of products.

Sample at random two articles simultaneously. The n articles between them are removed from T. Test the first two articles. If both of them are effective, sample two articles at random from the rest of the series of the products. If at least one of the two articles tested is defective, apply the method of dilution.

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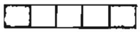

We assume here that i) the total number T of articles is sufficiently large and ii) the length of the i-th run of defective articles, say d_i, is less than or equal to n and at most one run of defective articles is contained in the removed articles. This last assumption may oversimplify the actual problem, but this model gives us the upper bound of the effectiveness of our method.

Let the total number of the runs of defective articles be r. Then the expectation of the number of defective articles which are contained between two articles tested will be

$$\mathcal{E}\{k\} = r(n-m_D+1)/\{E+r(2-m_D)-n+1\}, \qquad \dots \quad (8)$$

where E and m_D are the total number of the effective articles in the original series of length T and the mean of d_i respectively.

Thus the proportion of the defective articles in the rest is on the average

$$p(n) = \{D-\mathcal{E}\{k\}\}/(T-n-2), \quad n \geqslant d_i. \qquad \dots \quad (9)$$

Putting

$$\left.\begin{aligned} A &= m_D(E+2r-D+2), \\ B &= 1+m_D, \\ C &= E+2r-D+T-2, \end{aligned}\right\} \qquad \dots \quad (10)$$

and $$G = (E+2r-D)(T-2),$$

we have the solution $$n = \{A-\sqrt{A^2-B(AC-BG)}\}/B, \qquad \ldots \quad (11)$$

which maximizes $p(n)$.

In the case where $T \gg D$ and $T \gg 1$, we have

$$n/T \doteqdot (m_D-1)/(m_D+1), \qquad \ldots \quad (12)$$

as an approximation.

We shall show how effective our method is. We put

$$f(n) = p(n)-D/(T-2), \qquad \ldots \quad (13)$$

and $T = 1{,}000$ and $r = 5$.

D	m_D	n	$[n]$ or $[n]+1$	$f(n)/p(0)$
6	1.2	99.4	99	1.3%
10	2.0	328.2	328	12.0%

In the last case where $D = 10$ our assumptions do not hold and the efficiency is over-estimated.

4. THE METHOD OF DILUTION

Our method of dilution is as follows: We sample at random one (in the case of the centred method) or two (in the case of the bordered method). If we hit defective article or articles, we screen the defective articles on both sides of the sampled article or articles until the effective article appears. In the case of second or further sampling, if a defective article appears in the position next to the removed one, we test the removed articles until the effective article appears.

Let the length of the i-th run of defective articles be d_i and its standard deviation be σ_D; then the expectation of the number of defective articles which are destroyed in this stage is

$$v = \sum_{i=1}^{r} d_i^2/D = (\sigma_D^2+m_D^2)/m_D \geqslant m_D \geqslant 1. \qquad \ldots \quad (14)$$

There could be more efficient methods of screening, if some special information about the series of articles were available beforehand.

The sample size will be fixed by the cost of performing the destructive test. Sometimes the total number of effective articles destroyed may be fixed by the cost consideration.

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