MATHEMATICAL NOTE ON AREA SAMPLING

By MOTOSABURO MASUYAMA

[*Editorial Note:* Although there is nothing basically new in the following mathematical note, yet it is published to draw attention to the possibilities of the application of integral geometry to statistical problems.]

Let C_0 be a fixed closed, not necessarily convex, curve on the Euclidean plane, and C be an orientated movable curve, not necessarily closed, g be an orientated movable line, and the number of intersection-points between C and g within C_0 be n, and the sum of the lengths of arcs of C which are contained in C_0 be l. Then the integral

$$J = \int nCg \qquad \dots \quad (1)$$

where C and g mean the kinematical density of C and g, is put into the form

$$J = 2\int lC \qquad \dots \quad (2)$$

by Crofton's (1868) formula, if C be fixed.

Next, if we fix g at first, then we have from Poincaré's (1912) formula

$$\int nC = 4sL \qquad \dots \quad (3)$$

where s is the sum of chords made by g cut by C_0 and L is the length of C.

Then we have from (1) and (3)

$$J = 4L\int sg \qquad \dots \quad (4)$$

On the other hand, we have

$$\int sg = \pi F_0 \qquad \dots \quad (5)$$

where F_0 is the area of C_0. This formula was first obtained by Crofton for the convex curve C_0, but the condition "convexity" is not necessary, as was pointed out by Blaschke (1935).

Thus we have from (2), (4) and (5)

$$\int lC = 2\pi F_0 L. \qquad \dots \quad (6)$$

We owe this formula to Santaló (1935).

If we consider an area T which contains all possible unions of C_0 and C then,

$$l_E = \frac{\int lC}{\int C} = \frac{2\pi F_0 L}{2\pi T} = \frac{F_0 L}{T} \qquad \dots \quad (7)$$

is the expectation of l when the co-ordinates of the end point of C are selected at random within T, and the orientation of the tangent of C at this end point is also selected at random.

The author (*Reports of Statistical Application Research*, 1953) has subsequently shown interalia that the orientation need not be randomized for a movable rectifiable curve.

REFERENCES

[1] BLASCHKE, W. (1935): *Vorlesungen über Integralgeometrie*, Heft I. Teubner, Leipzig.

[2] CROFTON, M. W. (1868): On the theory of local probability applied to straight lines drawn at random in a plane, etc. *Phil. Trans. Roy. Soc.*, 158, 181.

[3] MASUYAMA, M. (1953): A rapid method of estimating basal area in timber survey—an application of integral geometry to areal sampling problems. *Sankhyā*, 12, 291-302.

[4] MASUYAMA, M. (1953): Rapid methods of estimating the sum of specified areas in a field of given size. *Rep. Stat. Appl. Res.*, 2, 113-110.

[5] POINCARÉ, H. (1912): *Calcul des Probabilités*, Chap. VII, Gauthier-Villars, Paris.

Paper received: February, 1953.