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THE CONTINUOUS SAMPLING PLAN THAT MINIMISES THE AMOUNT OF INSPECTION

By D. T. GHOSH
Indian Statistical Institute

SUMMARY. Several combinations of (i, f) are possible that will ensure the same AOQL under the continuous sampling inspection plan first introduced by Dodge. A procedure is developed here to find a unique (i, f) that will achieve the AOQL requirement and also minimise the amount of inspection when the process average $\bar{p}$ is known. The procedure is extended for the cases when the incoming quality may follow a two-point or three-point Binomial distribution. In fact, this procedure can be used if p follows a continuous distribution which can be approximated by discrete probabilities for some given values of p.

1. Introduction

Dodge (1943) introduced the continuous sampling plan CSP-1 applicable for continuous production. The plan provides for corrective inspection with a view to having a 'limiting average outgoing quality (AOQL)' which will not be exceeded no matter what quality is submitted. The plan visualises two phases of inspection. At the outset 100% of the units produced consecutively are inspected till i units in succession are found clear of defects. Then only a fraction f of the units, chosen at random, are inspected. If a sample unit is found to be defective, immediately 100% inspection is resorted to until again i units in succession are found clear of defects. All defective units found are replaced by good ones.

The elements of the plan i and f are so chosen that they together ensure a desired AOQL. However the choice of (i, f) is not unique and several combinations of (i, f) will ensure the same AOQL though with different amount of inspection. There is no objective procedure of determining a specific combination of (i, f).

One procedure would be to select f not too low such that the spotty quality p_t (%) is not high. p_t(%) is defined as the percent defective in a consecutive run of $N = 1000$ units for which the probability of acceptance under sampling phase is .10. However, p_t (%) is not always a suitable criterion, firstly because the incoming quality may never be as bad as p_t(%) and

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secondly because it depends on the value of N which has been arbitrarily chosen as 1000. $p_t(\%)$ may not appear numerically high if N is taken to be large as can be seen from below.

TABLE 1. p_t (%) FOR DIFFERENT N

f	N				
	1000	2000	3000	4000	5000
.0020	68.3	43.7	32.0	25.0	20.0
.0266	8.4	4.3	2.9	2.1	1.7

It is also not true that selection of a higher value of f to protect against spotty quality will necessarily amount to larger inspection. This can be seen from the amount of inspection for a few plans given below.

TABLE 2. AMOUNT OF INSPECTION (%) FOR A FEW SELECTED PLANS

Incoming quality ($\bar{p}$)	AOQL = 5.0 %		
	f = .20 i = 13	f = .10 i = 21	f = .05 i = 29
.01	23.2	12.0	6.7
.02	25.7	14.5	8.7
.03	28.7	17.4	11.5
.04	31.3	20.7	14.9
.05	34.1	24.6	19.2
.06	37.3	28.9	24.4
.07	40.6	33.7	30.5
.08	44.0	38.9	37.6
.09	47.5	44.5	45.2
.10	51.5	50.3	53.2
.12	58.3	61.9	68.6

For all other acceptance rectification sampling plans developed by Dodge it was attempted to minimise the amount of inspection if the process is controlled at a certain process average $\bar{p}$. This aspect was not considered in the case of continuous sampling plan.

It is, therefore, natural to look for a combination of (i, f) that would not only ensure a desired AOQL but would also require the minimum amount of inspection for $\bar{p}$, the process average. In this paper, we show that for certain cases it is possible to select a unique combination of (i, f) to ensure this twin objective.

2. Notations

The symbols and the formulae used here are those originally adopted by Dodge and they are listed below:

The symbol p_L denotes AOQL;

p_A average outgoing quality AOQ;

p_1 the quality level for which AOQL is reached;

$\bar{p}$ the process average;

p the incoming proportion defective;

and q $1-p$.

i denotes the number of defect free consecutive items which will direct a change from 100% to sampling inspection. i is essentially an integer. f denotes the sampling fraction. u stands for the expected number of items inspected following the finding of a defect. v stands for expected number of items that will be passed under the sampling procedure before a defect is found. This includes the sampling units actually inspected as well as the uninspected units produced between successive sampled units.

Using Dodge's result we have

$$u = \frac{1-q^i}{pq^i} \quad \dots \quad (1)$$

$$v = \frac{1}{fp} \quad \dots \quad (2)$$

$$F \text{ (amount of inspection)} = \frac{u+fv}{u+v} = \frac{f}{f+(1-f)(1-p)^i} \quad \dots \quad (3)$$

$$p_A = p(1-F) = p\left(1-\frac{u+fv}{u+v}\right) \quad \dots \quad (4)$$

$$p_L = p_1\left(1-\frac{f}{f+(1-f)(1-p_1)^i}\right) \quad \dots \quad (5)$$

$$p_1 = \frac{ip_L+1}{i+1} \quad \dots \quad (6)$$

$$f = \frac{(1-p_1)^{i+1}}{ip_L+(1-p_1)^{i+1}}. \quad \dots \quad (7)$$

3. ON SOME PROPERTIES OF p_1, i, f AND F

Here we study some properties of p_1, i, f and F.

Lemma 1 : *For a given p_L, the value of proportion defective p_1 for which the AOQL value is attained decreases as i increases.*

Proof : $$p_1 = \frac{ip_L+1}{i+1} = \frac{(i+1)p_L}{i+1} + \frac{1-p_L}{i+1} = p_L + \frac{1-p_L}{i+1}$$

Since $0 < 1-p_L < 1$, the result is proved.

Lemma 2 : *For a given p_L, the sampling fraction f decreases as i increases.*

Proof : $$f = \frac{(1-p_1)^{i+1}}{ip_L+(1-p_1)^{i+1}}$$

So $$\frac{1}{f} = \frac{ip_L}{(1-p_1)^{i+1}}+1.$$

Since $0 < 1-p_1 < 1$ and $0 < p_L < 1$, $(1-p_1)^{i+1}$ decreases and ip_L increases as i increases. Hence the result.

Lemma 3 : *For a given p_L, the amount of inspection for a given incoming quality p for a given i is given by the relation*

$$F_p(i) = \frac{1}{1+\frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{1-p_L} \cdot \left(\frac{1-p}{1-p_L}\right)^i}. \quad \ldots \quad (8)$$

Proof : Using (3) and (7) we have

$$F_p(i) = \frac{f}{f+(1-f)(1-p)^i}$$

$$= \frac{1}{1+\left(\frac{1}{f}-1\right)(1-p)^i} = \frac{1}{1+\frac{ip_L}{(1-p_1)^{i+1}} \cdot (1-p)^i}$$

Using (6) we have

$$\frac{ip_L}{(1-p_1)^{i+1}} = \frac{ip_L}{1} \times \frac{(i+1)^{i+1}}{i^{i+1}(1-p_L)^{i+1}}$$

$$= \frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{1-p_L} \cdot \frac{1}{(1-p_L)^i}.$$

Hence the result.

We now rewrite $F_p(i)$ as $F(x)$, where

$$F(x) = \frac{1}{1+cb^x \dfrac{(x+1)^{x+1}}{x^x}}. \quad \ldots \quad (9)$$

It may be noted that $c = \dfrac{p_L}{1-p_L}$ and $b = \dfrac{1-p}{1-p_L}$ are constants for a given p and p_L and $b < 1$ for $p > p_L$.

We define
$$\psi(x) = cb^x \frac{(x+1)^{x+1}}{x^x},$$

$$\phi(x) = \log \psi(x);$$

and obtain

$$\phi'(x) = \log b + \log(x+1) - \log x \quad \ldots \quad (10)$$

and
$$\phi''(x) = \frac{1}{x+1} - \frac{1}{x}$$

$$= -\frac{1}{x(x+1)} \text{ which is less than zero for all } x > 0 \ \ldots \quad (11)$$

Lemma 4 : *$F'(x) \lesseqgtr 0$ according as $x \lesseqgtr x_0$ where $\dfrac{x_0}{x_0+1} = b$.*

Proof : We have $F(x) = \dfrac{1}{\psi(x)+1} = \dfrac{1}{e^{\phi(x)}+1}$

and
$$F'(x) = -\frac{e^{\phi(x)}.\phi'(x)}{(e^{\phi(x)}+1)^2}$$

$$= \frac{e^{\phi(x)}}{(e^{\phi(x)}+1)^2}.\left[\log \frac{x}{x+1} - \log b\right].$$

Hence the result.

Lemma 5 : *The function $h(x) = \dfrac{1}{x(x+1)} - \left\{\log \dfrac{b(x+1)}{x}\right\}^2$ is positive for all integer $x \leqslant x_0$ and for all $b < 1$.*

Proof: We have

$$h'(x) = -\frac{2x+1}{x^2(x+1)^2}+2\{\log b+\log(x+1)-\log x\}.\frac{1}{x(x+1)}$$

$$= \frac{1}{x(x+1)}\left\{-\frac{2x+1}{x(x+1)}+2\log b+2\log\left(1+\frac{1}{x}\right)\right\}$$

$$= \frac{1}{x(x+1)}\Big[-\frac{1}{x}-\frac{1}{x+1}+2\log b$$

$$+2\left\{\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}-\frac{1}{4(x+\theta)^4}\right\}\Big] \text{ where } 0<\theta<1$$

$$= \frac{1}{x(x+1)}\left[2\log b+\frac{1}{x}-\frac{1}{x+1}-\frac{1}{x^2}+\frac{2}{3x^3}-\frac{1}{2(x+\theta)^4}\right]$$

$$= \frac{1}{x(x+1)}\left[2\log b-\frac{x-2}{3x^3(x+1)}-\frac{1}{2(x+\theta)^4}\right] < 0 \text{ for } x \geqslant 2$$

At x_0, $b = \dfrac{x_0}{x_0+1}$ and hence $\log \dfrac{b(x_0+1)}{x_0} = 0$. Thus $h(x_0) = \dfrac{1}{x_0(x_0+1)} - 0 > 0$.

It can be easily verified that $h(x) > 0$ for $x = 1$ and for all $b < 1$. So, $h(x) > 0$ for $1 \leqslant x \leqslant x_0$ and hence the result follows.

Lemma 6 : *$F''(x) > 0$ for all $x \leqslant x_0$ and $b < 1$.*

Proof : We have

$$F''(x) = -\left[\frac{(e^{\phi(x)}+1)^2\{e^{\phi(x)}(\phi'(x))^2+e^{\phi(x)}\phi''(x)\}-2e^{\phi(x)}\phi'(x)(e^{\phi(x)}+1)e^{\phi(x)}\phi'(x)}{(e^{\phi(x)}+1)^4}\right]$$

$$= -\frac{(e^{\phi(x)}+1)e^{\phi(x)}\{(\phi'(x))^2+\phi''(x)\}-2e^{2\phi(x)}(\phi'(x))^2}{(e^{\phi(x)}+1)^3}$$

$$= -\frac{e^{\phi(x)}}{(e^{\phi(x)}+1)^3}\left[e^{\phi(x)}(\phi'(x))^2+e^{\phi(x)}\phi''(x)+(\phi'(x))^2+\phi''(x)-2e^{\phi(x)}(\phi'(x))^2\right]$$

$$= -\frac{e^{\phi(x)}}{(e^{\phi(x)}+1)^3}\left[\phi''(x)(1+e^{\phi(x)})+(\phi'(x))^2(1-e^{\phi(x)})\right]$$

Using (10) and (11) the sufficient condition for $F''(x) > 0$ is that

$$\frac{1+e^{\phi(x)}}{x(x+1)}-(\phi'(x))^2(1-e^{\phi(x)}) > 0$$

or

$$\frac{1}{x(x+1)} > (\phi'(x))^2$$

or

$$\frac{1}{x(x+1)} > \left\{\log\frac{b(x+1)}{x}\right\}^2$$

a condition which is true in view of Lemma 5. This completes the proof.

4. Optimum combination of (i, f) that minimises the amount of inspection for a given process average $\bar{p}$ and a desired p_L

The procedure is developed on the basis of the following theorems.

Theorem 1 : *For a given p_L, the amount of inspection for a given incoming quality $\bar{p}$ decreases monotonically for all i as long as $\frac{i}{i+1} < \frac{1-\bar{p}}{1-p_L}$ and then increases monotonically for all i for which $\frac{i}{i+1} > \frac{1-\bar{p}}{1-p_L}$ i,e. attains its minimum for the value of i for which $\frac{i}{i+1} = \frac{1-\bar{p}}{1-p_L}$ for all $\bar{p} > p_L$.*

Proof : We have $F_{\bar{p}}(i) = \frac{f}{f+(1-f)(1-\bar{p})^i}$.

The result follows from Lemmas 3 and 4 once we note that $b = \frac{1-\bar{p}}{1-\bar{p}_L}$.

Theorem 2 : *For a given p_L and the process average $\bar{p} > p_L$ the amount of inspection goes on decreasing with increase in i and corresponding decrease in p_1 as long as $p_1 > \bar{p}$ and becomes minimum when $p_1 = \bar{p}$.*

Proof : Let $\bar{p}$ be less than or equal to p_1.

Then
$$\bar{p} \leqslant p_1 = \frac{ip_L+1}{i+1} = p_L + \frac{1-p_L}{i+1}$$

$$\Rightarrow \bar{p}-p_L \leqslant \frac{1-p_L}{i+1}$$

$$\Rightarrow \frac{\bar{p}-p_L}{1-p_L} \leqslant \frac{1}{i+1}$$

$$\Rightarrow 1-\frac{\bar{p}-p_L}{1-p_L} \geqslant 1-\frac{1}{i+1}$$

$$\Rightarrow \frac{i}{i+1} \leqslant \frac{1-\bar{p}}{1-p_L}$$

Hence the result.

The following simple algorithm will, therefore, determine the integer i_0, the optimum value of i which minimises the amount of inspection for a given $\bar{p}$ provided $\bar{p} > p_L$.

Algorithm :

(*a*) Solve $\frac{i}{i+1} = \frac{1-\bar{p}}{1-p_L}$

(*b*) If i is an integer take $i_0 = i$

(*c*) If i is not an integer choose $i_0 = [i]$ if $F_{\bar{p}}([i]) < F_{\bar{p}}([i+1])$. Otherwise choose $i_0 = [i+1]$

(*d*) Find f_0 corresponding to this i_0.

N.B. 1. If $\bar{p} \leqslant p_L$, there is no positive integer i for which $\frac{i}{i+1} = \frac{1-\bar{p}}{1-p_L}$ and hence we can not find any optimum plan as the amount of inspection goes on decreasing as i increases. In fact, no inspection is required in such a case. For practical consideration, however, i can not be increased indefinitely as the value of f in that case also goes on decreasing giving rise to higher value of $p_t\,(\%)$.

N.B. 2. It is interesting to note that the optimum plan is one for which the limiting value of *AOQ* i,e, p_L is attained at the given process average $\bar{p}$.

The optimum inspection plan for a wide range of *AOQL* and process average $\bar{p}$ is given in Appendix 1. The table gives $(i, f, p_t(\%))$ for a given p_L. For $\bar{p} \leqslant p_L$ it is recommended that the plan for $\bar{p}$ which is just greater than p_L in the table should be used.

Generally it is found that if the process is controlled around process average $\bar{p}$ the incoming quality will not be as high as $p_t(\%)$ as shown in some cases. However, the customer may stipulate some value of $p_t(\%)$ to protect against any sudden deterioration of quality and this chosen value of $p_t(\%)$ may differ from what is shown in the optimal plan in the appendix. Under such a situation the given procedure is to be modified. We consider the following problem which includes a new restriction.

Our aim is to find optimum (i_0, f_0) for a given process average $\bar{p}$ to achieve jointly a desired AOQL and a stipulated $p_t(\%)$.

The modified algorithm for this will be as follows :

(a) Solve $(1-p_t)^n = .10$ for n (not necessarily integer).

(b) Fix $f_1 = \frac{n}{N}$; usually N is taken as 1000 if nothing else is indicated.

(c) Using (6) and (7) find an integer i for which f is nearest to f_1.

(d) Compute (i_0, f_0) assuming that there is no restriction on $p_t(\%)$.

(e) If $f_0 < f_1$, choose (i, f) as the optimum plan.

(f) If $f_0 > f_1$ choose (i_0, f_0) as the optimum plan.

It is easy to see that the plan so selected is the minimum inspection plan under the imposed restriction.

5. Optimum (i, f) when the incoming quality p follows a two-point binomial distribution.

The incoming quality p may change over time due to various reasons and it may be proper to consider p as having a probability distribution rather than having a fixed value $\bar{p}$ as discussed earlier. Though p is usually taken to follow a Beta distribution we will assume that p follows a two point Binomial distribution. It will be quite logical and practical too to assume that the incoming quality is controlled most of the time at $p_{(1)}$ and occasionally at $p_{(2)}$.

We will work out the optimum CSP plan under the assumption that the incoming quality is $p_{(1)}$ with probability w_1 and $p_{(2)}$ with probability w_2 so that

$$0 < p_L < p_{(1)} < p_{(2)} < 1 \text{ and } w_1 + w_2 = 1$$

Under the given situation the average amount of inspection will be

$$F(i) = w_1 \, F_1(i) + w_2 \, F_2(i)$$

where

$$F_j(i) = \frac{1}{1 + \frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{1-p_L} \cdot \left(\frac{1-p_{(j)}}{1-p_L} \right)^i}; \; j = 1, 2$$

The optimum i is obtained by solving the equation

$$\frac{dF(i)}{di} = 0$$

i.e. $$w_1 A_1 \left[\log \frac{i}{i+1} - \log \frac{1-p_{(1)}}{1-p_L} \right] + w_2 A_2 \left[\log \frac{i}{i+1} - \log \frac{1-p_{(2)}}{1-p_L} \right] = 0$$

where we have

$$A_j = \frac{\frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{1-p_L} \cdot \left(\frac{1-p_{(j)}}{1-p_L} \right)^i}{\left\{ 1 + \frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{1-p_L} \cdot \left(\frac{1-p_{(j)}}{1-p_L} \right)^i \right\}^2}; \; j = 1, 2 \qquad \ldots \quad (12)$$

and $$A_j > 0.$$

The optimum plan is based on the following lemmas and theorem.

Lemma 7 : *$F_2(i)$ is greater than $F_1(i)$ for all $0 < p_{(1)} < p_{(2)} < 1$ and for all positive i.*

Proof : The result follows immediately once we compare $F_1(i)$ with $F_2(i)$ using formula (8).

Lemma 8 : *Let i_1 and i_2 be respectively the optimum values of i in relation to incoming quality $p_{(1)}$ and $p_{(2)}$. Then $i_1 > i_2$.*

Proof : We have $\frac{i_1}{i_1+1} = \frac{1-p_{(1)}}{1-p_L}$, $\frac{i_2}{i_2+1} = \frac{1-p_{(2)}}{1-p_L}$ and $p_{(1)} < p_{(2)}$.

Hence the result.

Lemma 9 : *The second derivative of $F_1(i)$ with respect to i is positive for all $i \leqslant i_1$.*

Proof : Since $p_L < p_{(1)}$, We have $b = \frac{1-p_{(1)}}{1-p_L} < 1$ and the result follows from Lemma 6.

Theorem 3 : *There exists a unique i_0 $(i_2 \leqslant i_0 \leqslant i_1)$ for which the amount of inspection $F(i)$ is minimum provided $0 < p_L < p_{(1)} < p_{(2)} < 1$.*

Proof : We have $F'(i) = w_1F_1'(i)+w_2F_2'(i)$

We note from Lemmas 4 and 9

(a) For $i < i_2$, $F_1'(i) < 0$, $F_2'(i) < 0$

(b) For $i > i_1$, $F_1'(i) > 0$, $F_2'(i) > 0$

(c) For $i_2 < i < i_1$, $F_1'(i) < 0$, $F_1''(i) > 0$, $F_2'(i) > 0$ and $w_1 > 0$ and $w_2 > 0$.

Hence there exists a unique i_0 in the range (i_2, i_1) for which $F'(i_0) = 0$.

Hence the result.

The optimum integer i_0 can be obtained by evaluating $F(i)$ successively in the range (i_2, i_1) for integer i only till $F(i) \geqslant F(i-1)$ and fixing $i_0 = i-1$. f_0 can now be obtained from equations (6) and (7).

6. Optimum (i, f) when the incoming quality p follows a three-point binomial distribution

This is a logical extension of the previous case and we make use of the following theorem.

Theorem 4 : *Let the underlying distribution for incoming quality p be three-point Binomial $p_{(j)}$, with probability w_j such that $0 < p_L < p_{(1)} < p_{(2)}$*

$< p_{(3)} < 1$ *and* $w_1+w_2+w_3 = 1$. *Then there exists a unique* i_0 *for which* $F(i) = \sum_{j=1}^{3} w_j F_j(i)$ *is minimum provided each* $p_{(j)}$ *is greater than* p_L.

Proof: We have $F(i) = w_1F_1(i)+w_2F_2(i)+w_3F_3(i)$

$$= w_1F_1(i)+G(i) \text{ (say)}$$

Let i_1, i_2 and i_3 be the optimum values of i for incoming qualities $p_{(1)}$, $p_{(2)}$ and $p_{(3)}$ respectively. In view of what has been proved earlier, there exists a unique $i_m (i_3 < i_m < i_2)$ for which $G(i)$ attains its minimum value.

We further note that

(a) For $i < i_m$, $F_1'(i) < 0$, $G'(i) < 0$

(b) For $i > i_1$, $F_1'(i) > 0$, $G'(i) > 0$

(c) For $i_m < i < i_1$, $F_1'(i) < 0$, $F_1'(i) > 0$, $G'(i) > 0$

and (d) $w_1 > 0$.

Hence there exists a unique i_0 in the range (i_m, i_1) such that $F'(i_0) = 0$. Hence the result.

The optimum i_0 can be easily obtained by evaluating $F(i)$ for integer i in the range (i_3, i_1) till $F(i) \geqslant F(i-1)$ and taking i_0 as $i-1$.

The graphs showing the amount of inspection for a given p_L and for successive values of i are traced in Appendix 2 for some selected cases where incoming quality p follows a one-point, two point or a three-point Binomial distribution.

7. Optimum (i, f) when the incoming quality p follows a continuous probability distribution

This is possible as the result in the previous section can be again extended. We approximate the continuous distribution by a discrete one so that $p_{(j)}$ occurs with probability w_j such that $\sum_{j=1}^{k} w_j = 1$ and $0 < p_L < p_{(1)} < p_{(2)} \ldots < p_{(k)} < 1$. We obtain $i_1, i_2, \ldots i_k$ which are optimal for the respective one point Binomial cases so that

$$\frac{i_1}{i_1+1} = \frac{1-p_{(1)}}{1-p_L}, \quad \frac{i_2}{i_2+1} = \frac{1-p_{(2)}}{1-p_L}, \ldots \frac{i_k}{i_k+1} = \frac{1-p_{(k)}}{1-p_L}$$

Then the optimal i_0 in the range (i_k, i_1) is obtained by evaluating $F(i)$ successively till $F(i) \geqslant F(i-1)$.

8. NEAR-OPTIMAL CSP PLAN WHEN ANY OF $p_{(j)}$ IS LESS THAN OR EQUAL TO p_L ($p_{(j)} \leqslant p_L$)

In case the incoming quality is lower than or equal to p_L in one or more cases for the distribution of p the search for i is to be widened between i and ∞ as some $i_j \to \infty$. It may also happen that no optimum i exists as $F(i)$ may be an ever decreasing function of i.

In such a situation the value of the relevant $p_{(j)}$ may be taken as a p (among those values of p considered in Appendix 1) just greater than p_L and the optimum value i_j can be obtained. Since all i_j, $j = 1, \ldots k$ are now finite, there will exist a unique i_0 which minimises the amount of inspection for the modified plan. The value of i_0 can be easily worked out to give, so to say, the near optimal plan.

9. CONCLUDING REMARKS

All along, the emphasis has been to obtain a plan that minimises the amount of inspection for a given probability distribution of the incoming quality. If, however, there is a stipulation on $p_t(\%)$ in addition to AOQL, it should be examined whether the optimal plan (i_0, f_0) for the given AOQL also satisfies the stipulation on p_t (%). If not, (i, f) is to be worked out by trial as suggested earlier in the 'modified algorithm'.

It is obvious that the amount of inspection in such a case will be more than that of the optimum plan.

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Appendix—1

VALUES OF i, f AND p_t(%) FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGES ($100\bar{p}$)

Incoming process average	AOQL (%) 0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.5	198 .1203 1.9										
1.0	198 .1203 1.9	98 .1213 1.8									
2.0	65 .4406 0.5	98 .1213 1.8	97 .0254 8.8								
3.0	39 .6039 0.4	48 .3167 0.7	97 .0254 8.8	96 .0066 31.9							
4.0	27 .6993 0.3	32 .4483 0.5	48 .1235 1.9	96 .0066 31.9	95 .0019 90.0						
5.0	21 .7540 0.3	24 .5387 0.4	32 .2252 1.0	48 .0538 4.3	95 .0019 90.0	94 .0006 100.0					
6.0	17 .7935 0.3	19 .6068 0.4	24 .3118 0.7	31 .1261 1.8	47 .0266 8.4	94 .0006 100.0	93 .0002 100.0				
7.0	14 .8248 0.3	16 .6527 0.4	19 .3864 0.6	23 .2008 1.1	31 .0733 3.1	46 .0140 10.2	93 .0002 100.0	92 66×10^{-6} 100.0			
8.0	12 .8465 0.3	13 .7031 0.3	15 .4621 0.5	18 .2709 0.8	23 .1281 1.8	31 .0433 5.2	46 .0071 28.0	92 66×10^{-6} 100.0	91 23×10^{-6} 100.0		
9.0	11 .8577 0.3	11 .7303 0.3	13 .5067 0.5	15 .3271 0.7	18 .1863 1.2	23 .0838 2.7	30 .0287 7.9	46 .0037 53.5	91 23×10^{-6} 100.0	91 8×10^{-6} 100.0	
10.0	9 .8805 0.3	10 .7583 0.3	11 .5566 0.4	13 .3723 0.6	15 .2300 1.0	18 .1305 1.7	23 .0558 4.1	30 .0178 12.7	45 .0022 68.4	90 8×10^{-6} 100.0	98 3×10^{-6} 100.0

Appendix—1 (contd.)

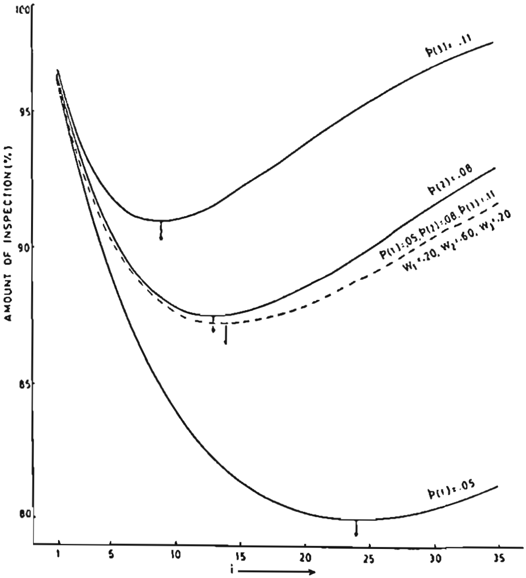
VALUES OF i, f AND p_t(%) FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGES ($100\bar{p}$)

Incoming process average	AOQL (%)										
	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
11.0	8 .8922 0.3	9 .7780 0.3	10 .5839 0.4	11 .4253 0.5	13 .2770 0.8	15 .1728 1.3	18 .0928 2.5	22 .0422 5.3	30 .0112 18.9	45 .0012 90.0	89 $3\times10^{-}$[illegible] 100.0
12.0	8 .8922 0.3	8 .7082 0.3	9 .6128 0.4	10 .4552 0.5	11 .3289 0.7	13 .2100 1.1	15 .1282 1.8	18 .0668 3.5	22 .0292 7.6	29 .0081 25.0	44 .0007 100.0
13.0	7 .9041 0.3	7 .8192 0.3	8 .6435 0.4	9 .4878 0.5	10 .3587 .0.6	11 .2570 0.9	12 .1600 1.3	15 .0960 2.3	17 .0553 4.1	22 .0203 10.9	29 .0053 36.9
14.0	6 .9162 0.3	7 .8192 0.3	7 .6762 0.3	8 .5232 0.4	9 .3917 0.6	10 .2851 0.8	11 .2024 1.1	12 .1407 1.6	14 .0833 2.7	17 .0410 5.6	21 .0166 13.4
15.0	6 .9162 0.3	6 .8408 0.3	7 .6762 0.3	7 .5619 0.4	8 .4285 0.5	9 .3160 0.7	9 .2580 0.9	11 .1606 1.4	12 .1107 2.1	14 .0641 3.5	17 .0306 7.4
16.0	5 .9286 0.2	6 .8408 0.3	6 .7100 0.3	6 .6041 0.4	7 .4696 0.5	8 .3531 0.7	8 .2926 0.8	9 .2113 1.1	11 .1282 1.8	12 .0876 2.6	14 .0497 4.0
17.0	5 .9286 0.2	5 .8631 0.3	6 .7100 0.3	6 .6041 0.4	6 .5156 0.4	7 .3944 0.6	8 .2926 0.8	8 .2435 0.9	9 .1738 1.3	10 .1213 1.9	12 .0696 3.2
18.0	5 .9286 0.2	5 .8631 0.3	5 .7470 0.3	5 .6503 0.3	6 .5156 0.4	6 .4417 0.5	7 .3328 0.7	7 .2818 0.8	8 .2036 1.1	9 .1436 1.6	10 .0991 2.2
19.0	4 .9411 0.2	5 .8631 0.3	5 .7470 0.3	5 .6503 0.3	5 .5072 0.4	6 .4417 0.5	6 .3708 0.6	7 .2818 0.8	7 .2305 1.0	8 .1707 1.3	9 .1190 1.9
20.0	4 .9411 0.2	4 .8802 0.3	4 .7873 0.3	5 .6503 0.3	5 .5072 0.4	5 .4962 0.5	6 .3708 0.6	6 .3275 0.7	7 .2395 1.0	7 .2041 1.1	8 .1437 1.6

Appendix 2

AMOUNT OF INSPECTION (%) FOR A GIVEN p_L FOR DIFFERENT VALUES OF i WHEN INCOMING QUALITY FOLLOWS A DISTRIBUTION

$p_L = .01$



Appendix 2 (*Continued*)

AMOUNT OF INSPECTION (%) FOR A GIVEN p_L FOR DIFFERENT VALUES OF i WHEN INCOMING QUALITY FOLLOWS A DISTRIBUTION

$p_L = .01$