Income Inequality and Spatial Distribution: Firm Location, Product Quality and Welfare of Poor

Namrata Gulati

Thesis submitted to the Indian Statistical Institute

in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

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Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation

The key idea explored in this thesis is the following: though being poor in itself is a huge disadvantage, the situation might be influenced considerably by the type of neighbourhood the poor lives in as private establishments like educational institutions, health care facilities or credit institutions take both the location and income mix of people into account while making strategic decisions like whether to enter into the neighbourhood at all, and, upon entry, what price and quality to choose for their products and services. Is staying with the rich a virtue for the poor or a source of resentment? Are the poor living in poor neighborhoods better-off because living in an affluent one costs too much? Or are they significantly worse-off as they do not even have access to many basic facilities? These are the kinds of questions we are interested in exploring in this thesis.

A casual walk across the city streets of any developing country might suffice to illustrate the idea. While moving across poor neighborhoods one comes across many roadside vendors selling tea, providing barber services and so on from shops requiring minimal physical investment and, understandably, the quality on offer is quite basic. As one moves into relatively richer neighbourhoods, one is bound to come across more sophisticated counterparts of the same products and services: road side vendors are replaced by air-conditioned cafes, shining beauty salons and so on. Similar products and services could become heavily capital intensive and highly specialized in nature depending on the income mix of the neighbourhood. Changes in price of course reflect these changes in quality level.

Given these differences in price and quality, which neighbourhood does an individual prefer to be in? Answer to this depends not just on the cost relative to income, but also on the ease of access of the facilities. This is because certain goods and services are required at regular intervals so that distance becomes an important factor. In the less developed countries distance from schools is an important factor leading to high drop-out rates. Simlarly distance from the nearby health care facility is a major reason resulting in higher mortality of both mother and child during child birth in rural areas of developing countries. How readily a product or service is available is thus determined by the neighborhood an individual lives in. So it is the interaction of the two, the individual's income and his postcode, that determines his welfare. This thesis attempts to capture this interaction by developing simple models that integrate consumer's income distribution with spatial distribution and explores the consequences of an increase in income inequality on the welfare of the poor in general, and their access to market in particular.

There is a substantial body of evidence showing how neighborhood poverty affects poor people's ability to access facilities such as health care and schooling. Consider health care first. Montgomery et al. (2005) find that both household and neighborhood living standards can make a significantly important difference to health. They report striking differentials in health depending on the region: poor city dwellers often face health risks that are nearly as bad as what is seen in the countryside, and sometimes the risks are decidedly worse. For instance, they find that in the slums of Nairobi, rates of child mortality substantially exceed those found elsewhere in Nairobi, and are high enough even to exceed rural rates of mortality. Wilson (1987) argues that concentrated poverty leaves neighborhoods without the middle-class capital to support strong local organizations. Thus there is less investment in the health care resources. Similarly according to the October 31, 2006 news release from the Stanford University School of Medicine, death rates are highest among people of low socioeconomic status who also lived in affluent neighborhoods. In India Das and Hammer (2005) have found that doctors located in the poorest neighborhoods are one full standard deviation worse than doctors located in the richest neighborhoods of Delhi. Also, in India the ratio of hospital beds to population in rural areas is fifteen times lower than that for urban areas, and the ratio of doctors to population in rural areas is almost six times lower than that in the urban population (Deogaonkar, 2004). As per the records of the Ministry of Health and Family Welfare, Government of India, a total of 74% of the graduate doctors live in urban areas, serving only 28% of the national population, while the rural population remains largely unserved. Although the health care facilities are overwhelmingly concentrated in urban areas, the economic distance, which includes cost of health care, prevents access for the urban poor (Deogaonkar, 2004). While the rural poor are underserved, at least they can access the limited number of government-supported medical facilities that are available to them. The urban poor fares even worse because they cannot afford to visit the private facilities that thrive in India's cities (Price Water House Coopers, 2007).

Neighborhood determines not just the quality and price of the service but also its ease of access. In a survey on rural Rajasthan, Banerjee and Duflo (2009) distinguished between three broad categories of health care facilities: public, private and traditional, and even within traditional and private practitioners they find huge variation in the level of qualification. This difference in qualification was replicated in the cost of an average visit to these facilities. The mean and median distance to the traditional healer is much lower than the distance to the closest private facility, implying convenient accessibility of relatively poor and cheap quality services.

Similarly on education a study by the Education Policy and Data Center (2008) has found that the net school attendance rate variation is higher in the rural areas, and, in about half of the countries of the world, the net attendance rate is strongly negatively correlated with the relative poverty rate. This is because higher prevalence of poor households has spillover effects on the sub-national region, leading to, for example, fewer common resources for school. For India Agrawal (2010) reports that educational facilities are distributed unequally between rural and urban areas. In rural areas, students suffer from scarcity and inadequate accessibility of schools (as well as poor quality of education) which also forces them to travel larger distances. Tilak and Sudarshan (2001) has found statistically significant, strong and inverse relationship between levels of educational attainment and levels of poverty in Indian households. Poor households are unable to get good quality education.

1.2 Summary

This thesis explores the interaction of income inequality with the neighbourhood effect in determining market outcomes like price and quality of products and services and the decision of the providers of the goods and services whether or not to enter into the neighbourhood. We develop simple and tractable models that integrates consumer's income distribution with spatial distribution of their location and investigate the effects of income inequality on market outcomes and on the welfare of the poor under diferent scenarios. For instance, we consider different spatial distributions like when the poor lives side by side with the rich in a mixed community vis-a-vis when they are spatially segregated. We examine different specifications of the product or service under consideration like whether the product is homogeneous or differentiated, and whether the product quality is given exogenously or determined endogenously as part of the market outcome. We also consider different market structures like whether the number of providers of the goods and services is fixed or there is free entry and exit. Finally we consider alternative location decisions, in particular the decisions of the providers whether to locate in the poor or rich neighbourhoods.

In the presence of neighbourhood effects the effect of income inequality can be contrasted from the following simple illustration. The poor can easily be worse-off living in a rich neighbourhood owing to the soaring prices of products and services reflecting the higher willingness to pay of his rich neighbours. On the other had, if he lives in a poor neighbourhood and the average income of the neighbourhood is low enough, the providers of the product or service might not enter into the neighbourhood at all as they will not be able to recover their fixed costs of production. In this scenario living with the rich might be welfare improving for the poor as at least some poor get to access the product or service since the providers could recover their fixed costs due to the higher willingness to pay of the rich.

The intuition for the mechanism working through all the three chapters of the thesis can be explained as follows. The idea is that the effect of any income distribution over the spatial horizon is reflected in the demand structure and this shapes the market outcome. Consumers' willingness to pay varies with their income: people with higher income level generally have a higher willingness to pay and a preference for improved quality. But firm's quality and price choice depends on the average willingness to pay of the consumers since production is costly and generally involves fixed costs. This average willingness to pay might either be in line with or be in conflict with the individual preferences. This brings in the element of externality. Because of the presence of this externality, an individual with a certain level of income might face very different trade-offs in the presence of one level of inequality as opposed to the other. For example, when the proportion of the rich is relatively high then the market outcome in terms of quality and price of the good or service will also be high reflecting preferences and willingness to pay of the rich. But high product quality requires higher fixed cost implying that only a limited number of firms can break-even. The consumers are surely better-off with the higher product quality but worse-off from the high price and fewer number of firms. The trade-off is just the opposite when the proportion of poor is relatively high. Now there will be low price and product quality but greater number of firms since the fixed cost is lower.

Chapter 2 of the thesis makes this trade-off clear by considering a homogeneous product in a competitive framework with free entry and exit. It is very interesting to investigate the role of inequality in such an ideal market structure. In chapter 2 we first lay out the basic preference and industrial structure integrating consumer's income distribution with spatial distribution of their location. This basic structure is maintained throughout the thesis. The preference structure reflects the higher willingness to pay of the richer consumers and the consumers' reluctance to travel farther to access the product or service under consideration. What is specific about chapter 2 is the spatial structure: we consider a circular city across which the consumers are uniformly distributed with rich and poor consumers living side by side. In this set-up we explore the consequences of an increase in income inequality on the market outcomes and on the market access and welfare of the poor.

We find an inverted-U shaped relationship between income inequality and the welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer. Interestingly the same inverted-U shaped relationship is also observed between income inequality and market access of the poor. There exist multiple equilibria: a bad equilibrium where all the poor are excluded exists simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group and show that poor are better-off staying in the mixed-income economy as long as the poor income is below a feasibility threshold.

The reason for this inverted-U shaped relationship can be traced to the opposing welfare impacts of income inequality working through equilibrium price and number of firms. Consumers benefit from the increase in number of firms but lose from the increase in price. In the model when product quality is exogenously given, both price and number of firms increase steadily as the neigbourhood of the poor becomes richer. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and hence the market access of the poor increases as the number of firms increases. But, beyond a point, the adverse price effect takes over. When product quality is endogenous, the number of firms also start decreasing beyond a point to accommodate the higher costs of production associated with higher product quality that results when the society becomes richer. The study by Li and Zhu (2004) lends strong empirical support to our theoretical results. They find an inverted-U association between self-reported health status and inequality using individual data from the China Health and Nutrition Survey (CHNS).

In chapter 2 we have considered product quality to be homogeneous. In chapter 3 we consider the possibility for the firms to offer different quality products. This is quite appropriate for a discussion on income inequality since a rise in income level may not altogether replace the existing facilities, but rather may lead to rise in parallel facilities of different qualities.

Richer consumers prefer higher quality products. So firms may choose quality differentiation as a way to effectively reduce price competition and reach out to various sections of consumers. Some firms will concentrate on the high quality and price and hence depend on consumers with high income. Others will offer cheaper products of lower quality to cater to low income groups. But product differentiation makes sense only if there is enough demand for differentiated products in the market. In the absence of that the firms will offer same quality product as otherwise they will not be able to break even. So the absolute values of the incomes as well as the relative size of different income groups are important in determining firm's quality choice. Another key aspect is the difference in the fixed costs of production. If the cost of providing high quality is too high relative to the perceived benefits then the firm will not be in a position to offer high quality and price to effectively exploit product differentiation to its advantage.

As in the second chapter, here too consumers differ in their locations. This imposes another constraint on the firm's choice. This is because instead of traveling all the way to buy their most preferred quality product, consumers might go for the product that is accessible relatively easily. Presence of travel costs thus inhibits firm to effectively segregate the market with respect to income. So the firm needs to carefully weigh its options before deciding the quality. These trade-offs have important bearings on the market outcome in terms of quality offered and price being charged and hence on the welfare of the consumers.

We find that for a homogenous distribution of income or when the poor's income or density is too low, both firms offer the same quality. For a homogenous income distribution firm does not perceive much benefit from product differentiation. Similarly when income and density of the poor is low, it implies a low demand for a different variety. In these scenarios the poor are either left completely unserved, or they end up buying whatever the market has to offer. Given this, for a very high difference in the fixed costs, both firms offer the low quality. But when the difference in the fixed costs is low, both firms offer the high quality.

For a more heterogeneous income distribution and an intermediate range of the difference in fixed costs, one firm offers the high quality and the other the low quality. Product differentiation on one hand allows firm to alleviate price competition and, on the other hand, serves consumers' demand better. Within this there can either be horizontal dominance – both firms serving either income groups, or vertical dominance – all the rich buying the high quality product and the poor buying the low quality product. Horizontal dominance arises when the travel cost outweighs the income and quality difference. When the income and quality difference is substantial compared to the travel cost, it makes the case for vertical dominance. We show that although in general a rise in income inequality has a spiraling negative effect on the welfare of the poor, there are situations, particularly when the poor income is very low, when an increase in the rich income could be welfare improving for the poor.

In the last two chapters we have restricted ourselves to the spatial structure where rich and poor live side by side in the same location. But any discussion on neighbourhood effects will be incomplete until it addresses the scenario where the rich and poor are geographically segregated – poor living in the poor ghettos while rich live in the rich neighbourhoods. We take up this issue of regional inequality in chapter 4. While discussing the issue of regional inequality, it is natural to focus on the question of firm location. So far the firms in our discussion are quite passive in terms of location decision: the location of the two firms are fixed in chapter 3; although location decision was allowed in chapter 2, but, because of the focus on symmetric equilibrium, the entering firms had to locate symmetrically in the circular city. In chapter 4 we take up the firms' location decision seriously.

To investigate the interaction of regional inequality with the spatial aspect we consider a circular city with two regions, rich and poor, where the potential customers have to bear a travel or transportation cost to access the facility under consideration. The interaction of regional inequality with the spatial aspect gives rise to an interesting trade-off between the consumers' willingness to pay and the firm's potential market size. Since the richer consumers have a higher willingness to pay, the firm has obvious incentive to locate in the rich region. But, in the presence of travel costs, the preference structure implies that to access the same quality product a rich customer is willing to travel further than a poor customer. Then if a firm locates in the rich region, it is almost certainly going to lose the poor customers as they will find it costlier to travel all the way to the rich region to access the product. Instead if the firm locates in the poor region, it serves all the poor who live close by, and the rich may not mind to travel to the poor region to access the product. This trade-off between willingness to pay and market size determine firms' location choice.

To illustrate this trade-off in a simple way we first discuss the scenario when there is a single firm in the city deciding whether to locate in the poor or rich region. This trade-off leads to an interesting result: when the income gap between rich and poor is relatively narrower and the travel cost and the size of the poor region is relatively lower, there exists an equilibrium where the monopoly firm locates in the poor region. By locating in the poor neighborhood the firm ensures a larger market size by exploiting the rich consumers' willingness to travel higher distance. It serves all the poor in the neighbourhood and some rich who live relatively closer to the poor neighbourhood.

Next we extend our analysis by allowing free entry and exit so that the number of firms is determined endogenously. The intuition spelt out for the monopoly firm case extends here too: the location equilibrium is such that the area of operation of terminal firms in the poor region encroaches into the neighbouring richer region, that is, some rich people commute to the terminal firm located in the poor region to access the product, and not the other way round. In equilibrium, there are quality and price ladders where consumers residing in the interior of the rich region are offered the highest quality and are charged with the highest price. Finally we find that the utilities of consumers located at comparable distances from the firm increase with the increase in income.

Chapter 2

Income Inequality, Spatial Distribution and Welfare of Poor

2.1 Introduction

There exists a substantial body of evidence demonstrating how neighbourhood effects interacting with income inequality affect poor people's ability to access basic facilities like health care services, schooling, and so on. Unfortunately there is very little analytical research to understand this interaction. This chapter makes an early attempt to model this interaction by integrating consumer's income distribution with spatial distribution, and look at the consequence of an increase in income inequality on the welfare of the poor in general, and their access to market in particular.

In this chapter we consider a homogeneous product in a competitive framework with free entry and exit. It is very interesting to investigate the role of inequality in such an ideal market structure. In this chapter we first lay out the basic preference and industrial structure integrating consumer's income distribution with spatial distribution of their location. This basic structure is maintained throughout the thesis.

The preference structure reflects the higher willingness to pay of the richer consumers and the consumers' reluctance to travel farther to access the product or service under consideration. The industrial structure is characterized by the presence of a fixed cost of production. What is specific about this chapter is the spatial structure: we consider a circular city across which the consumers are uniformly distributed with rich and poor consumers living side by side. The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms locate equidistantly around the circumference of the city. In the second stage, firms decide price and quality of the product or service. In this set-up we explore the interaction of income inequality with the neighbourhood effect in determining the market outcomes and its consequences on the market access and welfare of the poor.

We find an inverted-U shaped relationship between income inequality and the welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer. Interestingly the same inverted-U shaped relationship is also observed between income inequality and market access of the poor. The reason for this inverted-U shaped relationship can be traced to the opposing welfare impacts of income inequality working through equilibrium price and number of firms. Consumers benefit from the increase in number of firms but lose from the increase in price. In the model when product quality is exogenously given, both price and number of firms increase steadily as the neighbourhood of the poor becomes richer. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and hence the market access of the poor increases as the number of firms increases. But, beyond a point, the adverse price effect takes over. When product quality is endogenous, the number of firms also start decreasing beyond a point to accommodate the higher costs of production associated with higher product quality that results when the society becomes richer. The study by Li and Zhu (2004) lends strong empirical support to our theoretical results. They find an inverted-U association between self-reported health status and inequality using individual data from the China Health and Nutrition Survey (CHNS).

We find that the nature of equilibrium depends on two income thresholds of the poor. We identify an *upper income threshold* for the poor income such that all poor consumers get served by the market only if the poor income is above this upper income threshold. On the other hand there exists a *lower income threshold* for the poor income such that no poor consumer is served if the poor income is below this lower threshold. When the poor income is in between the upper and lower income thresholds, there are pockets of the city where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

We have also identified the possibility of multiple equilibria. There exists a whole range of parameter values such that a bad equilibrium and a good equilibrium exist side by side for the same parameter configurations. Under the good equilibrium at least some poor (if not all of them) gets served, those who are located closer to the firms. Whereas under the bad equilibrium all the poor are excluded; the firms completely ignore their presence and choose the price and quality as if there were only rich individuals residing in the city. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to the complete exclusion possibility. We have also found that poor are more likely to be completely excluded when they are a minority: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number.

Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group. We have identified a *feasibility income threshold* in a single-income economy such that it is not feasible for any firm to operate if the common income is below this feasibility threshold. Comparing mixed versus single-income economies we show that poor are better-off staying in the mixed-income economy as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income economy as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income poor economy.

When the quality choice is endogenous, we find interesting interactions between product quality and number of firms. This is because with higher income, there is an increase in the average quality on offer. With increase in quality the fixed cost of providing this higher quality also increases. With the given market this would result in the equilibrium number of firms to fall to make up for the higher fixed cost. This in turn increases the average distance that the consumer has to travel in order to access the facility. This resembles the recent development experience quite accurately. For example, traditional Indian markets have rapidly transformed themselves in the post liberalization period. There has been a transition from fragmented owner-run small retail shops to medium and big retail chains. Similarly large health care centers have registered rapid growth displacing small neighbourhood health clinics. Thus, in the process of development, although on one hand better quality is made available, but, on the other hand, consumers need to pay higher price – first in terms of higher fees which is the offer price, and then in terms of the increase in the average distance traveled to access the facilities. For instance, in a survey on rural Rajasthan, Banerjee and Duflo (2009) distinguish between three broad categories of health care facilities: public, private and traditional, and even within traditional and private practitioners they find huge variation in the level of qualification. This difference in qualification is replicated in the cost of an average visit to these facilities. Also they find that the mean and median distance to the traditional healer is much lower than the distance to the closest private facility, implying convenient accessibility of relatively poor and cheap quality services.

The idea that people with higher income generally have higher valuation for services like health, education, or credit, and so have higher willingness to spend on them, and that firms do take this into account while making strategic decisions was first developed by Gabszewicz and Thisse (1979) in the vertical differentiation models introduced by them. This strand of the literature has been developed and extended further by Shaked and Sutton (1982, 1983, 1987).[1] In our model we allow consumers to differ with respect to both their income and location. The basic horizontal product differentiation model was introduced by Hotelling (1929) and was later developed by d'Aspremont et al. (1979) and Salop (1979). The literature on industrial organization that follows these seminal works (for example, Economides, 1989, 1993; Neven and Thisse, 1990; Degryse, 1996) looks at product specifications combining both the vertical and horizontal characteristics. But the industrial organization literature typically does not allow for the possibility of exclusion. Atkinson (1995) is the only work that we are aware of that looks at the possibility of non-consumption arising out of income gap in the context of determination of poverty and capability by firm behaviour and industrial structure. But people even at the same level of income might not consume because of higher distance, and this is especially relevant for services like health, education, or credit. This is the feature we would like to highlight in our work.

Tarasov (2008) is the paper that comes closest to our work in terms of the result.

[1]Relatively recently Benassi et al. (2006) analyze the effect of income concentration on product differentiation based on a given income distribution while Yurko (2009) furthers this research by considering more general specification of income distribution function.

They have also identified an inverted-U shape relationship between welfare of the poor and the fraction of the rich in a general equilibrium model of monopolistic competition with free entry, heterogenous firms and consumers that share identical but non-homothetic preferences. While the intuition is similar, we derive the result from a much simpler framework.

The chapter is organized as follows. Section 2.2 outlines the model with the spatial structure capturing the inequality-neighbourhood interaction. In section 2.3 we characterize the equilibrium for the generic scenario where the poor has partial market access while the rich has complete access and discusses the welfare implications when product quality is exogenously given. The endogeneity of product quality is taken up in section 2.4. Section 2.5 concludes. The detailed proofs are developed in the appendix.

2.2 The Model

Our model adapts the framework of Salop (1979). There is a circular city of circumference 1 unit. Two types of consumers, rich and poor, are uniformly distributed along the circumference of the city: there are f_P proportion of poor with income Y_P and f_R proportion of rich with income Y_R . Obviously $Y_R > Y_P$. The total number of consumers is normalized to 1.[2]

There are n firms are located equidistant to each other around the circle so that the distance between adjacent firms is $\frac{1}{n}$. The number of firms is not fixed; it is determined endogenously from free entry and exit condition.[3]

[2]The model with a general income distribution is developed in Appendices A.1 and A.2.

[3]We are not modeling firms' location choice, rather our interest is to analyze the extent of entry. It is the extent of entry that determines the market access of the poor and hence their welfare. Our justification of this modeling structure is similar to Tirole (1988): " Omitting the choice of location

We use the notations x_j for location of firm j, p_j for price charged by firm j and θ_j for quality of the product offered by firm j, $j = 1, 2, ..., n$. Each consumer buys either one unit of the product from his most preferred firm, or does not buy the product at all. Let θY be the *gross* utility a consumer with income Y enjoys from consuming the product. Utility of a consumer at location z with income Y and purchasing from firm j is given by

$$U(z, Y, j) = Y\theta_j - p_j - t|x_j - z|,$$

where t denotes per unit travel cost. It is worth mentioning that even though the location choice is on a 'circle', each point on the circle is identified with a point on the real line and the distance function $|.|$ is the standard distance function on the real line. Note that the reservation utility (the utility the consumer enjoys without having the product) is Y, implying that minimum quality is $\theta_j = 1$.

Note that the particular form of the utility function is such that everywhere it satisfies the "single-crossing" condition: $\partial\left(\frac{\partial U/\partial\theta}{\partial U/\partial Y}\right)/\partial Y > 0$. Hence any indifference curve in (θ, Y)- plane of a higher income household cuts a indifference curve of lower-income household from below. Epple and Romano (1998) interprets that this condition corresponds to a positive income elasticity of demand for quality.[4] Also, as in Das and Donnenfeld (1987,1989), this implies that the individual with higher income has higher willingness to pay for the same marginal increase in quality. This signifies not just their ability to pay but their preference to pay for higher quality even if it comes at a higher price. The underlying assumption is that richer individuals are likely to be better informed about the benefits of quality education or health care system, and are willing to pay more for all these services.

allows us to study the entry issue in a simple and tractable way" (page 283).

[4]Using a variety of approaches Rubinfeld and Shapiro (1989) estimates a positive income elasticity of demand for educational quality.

Also, $Y\theta$ signifies that the welfare from consumption of all goods and services increases if he chooses to buy the product/service under consideration. For example, if θ represents the quality level of health services, then this would imply that owing to access to the better health facility an individual is able to derive higher satisfaction from the consumption of all other goods and services. Similarly an educated person is better able to appreciate the value of other things because of higher degree of awareness and understanding. This enhances his utility from his overall spending. It is interesting to note that θ can be interpreted in a more broad sense to include other aspects of value. Take for instance; one may conceptualize quality to imply lower waiting time in the case of health care services. Lower waiting time can be ensured by providing better infrastructure which comes at a higher cost.

There is a fixed cost of production that depends on the quality of the product, $F(\theta_j)$. We assume that this fixed cost of production is strictly increasing and convex in quality: $F'(\theta_j) > 0$, $F''(\theta_j) > 0$. The marginal cost of production, $c(\theta_j)$, is independent of output but depends on quality. We assume that the marginal cost of production is also strictly increasing and convex in quality: $c'(\theta_j) > 0$, $c''(\theta_j) > 0$. For example, better quality schools or hospitals need higher infrastructural investments like better equipments, laboratories, and so on which are fixed cost in nature. At the same time marginal cost is also higher since higher salaries are to be paid to attract better quality teachers, doctors or nurses.

Finally, profit of firm j charging a price p_j and offering the quality θ_j is given by

$$\pi_j = [p_j - c(\theta_j)]D_j - F(\theta_j),$$

where D_j denotes demand faced by firm j. Given the spatial structure, we elaborate in the next subsection how D_j depends on firm j's own strategic choices, (x_j, p_j, θ_j), and on the strategic choices of the two adjacent firms, $(x_{j-1}, p_{j-1}, \theta_{j-1})$ and $(x_{j+1}, p_{j+1}, \theta_{j+1})$.

The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms locate equidistantly around the circumference of the city. In the second stage, firms choose their prices and quality simultaneously.

2.2.1 Demand Structure

Consider firm j located between the two adjacent firms $j-1$ and $j+1$. Let $\delta_{j,j+1}(Y)$ denote the distance from firm j of the marginal consumer with income Y who is indifferent between firms j and $j+1$, that is, $U(x_j+\delta_{j,j+1},Y,j)=U(x_j+\delta_{j,j+1},Y,j+1)$. It follows that $\delta_{j,j+1}(Y)=\dfrac{1}{2t}\left[(Y(\theta_j-\theta_{j+1})+p_{j+1}-p_j)+\dfrac{t}{n}\right]$. Utility of this marginal consumer is $\dfrac{1}{2}\left[Y(\theta_j+\theta_{j+1})-(p_{j+1}+p_j)-\dfrac{t}{n}\right]$.

Let $\overline{Y}_{j,j+1}$ denote the income level such that the consumer with income $\overline{Y}_{j,j+1}$ who is indifferent between firms j and $j+1$ at a distance $\delta_{j,j+1}\left(\overline{Y}_{j,j+1}\right)$ is also indifferent between buying and not buying, that is, $U\left(x_j+\delta_{j,j+1}\left(\overline{Y}_{j,j+1}\right),\overline{Y}_{j,j+1},j\right)=\overline{Y}_{j,j+1}$. It follows that

$$\overline{Y}_{j,j+1}=\frac{(p_{j+1}+p_j)+t\left|x_{j+1}-x_j\right|}{(\theta_j+\theta_{j+1}-2)}.$$

Now consider the consumers with income $Y<\overline{Y}_{j,j+1}$. Let $\eta_{j,j+1}(Y)$ denote the distance from firm j of the consumer with income $Y<\overline{Y}_{j,j+1}$ who is indifferent between buying and not buying from firm j, that is, $U(x_j+\eta_{j,j+1},Y,j)=Y$. It follows that $\eta_{j,j+1}(Y)=\dfrac{1}{t}\left[Y(\theta_j-1)-p_j\right]$. But note that $\eta_{j,j+1}(Y)<0$ for $Y<\dfrac{p_j}{\theta_j-1}\equiv\underline{Y}_j$, that is, consumers with income $Y<\underline{Y}_j$ are not buying from firm j even when they are located at the same location as firm j. The implication for demand is that the measure of consumers located between x_j and x_{j+1} and buying from firm j is $\eta_{j,j+1}(Y)$ for all $\underline{Y}_j\leq Y<\overline{Y}_{j,j+1}$, and 0 for $Y<\underline{Y}_j$.

Proceeding in the same way we can define $\overline{Y}_{j,j-1}$, $\underline{Y}_j$, $\delta_{j,j-1}(Y)$ and $\eta_{j,j-1}(Y)$ symmetrically replacing $j+1$ with $j-1$ in the corresponding expressions and conclude that the measure of consumers located between x_j and x_{j-1} and buying from firm j is $\delta_{j,j-1}(Y)$ for $Y \geq \overline{Y}_{j,j-1}$, $\eta_{j,j-1}(Y)$ for $\underline{Y}_j \leq Y < \overline{Y}_{j,j-1}$, and 0 for $Y < \underline{Y}$.

To sum up, demand for firm j's product generated from the consumers located between x_j and x_{j+1} is

$$D_{j,j+1} = \begin{cases} \dfrac{1}{2t}\left(Y\left(\theta_j - \theta_{j+1}\right) + p_{j+1} - p_j + \dfrac{t}{n}\right) & \text{for } Y \geq \overline{Y}_{j,j+1} \\ \dfrac{Y\left(\theta_j - 1\right) - p_j}{t} & \text{for } \underline{Y}_j \leq Y < \overline{Y}_{j,j+1} \\ 0 & \text{for } Y < \underline{Y}_j. \end{cases}$$

Similarly, demand for firm j's product generated from the consumers located between x_j and x_{j-1} is

$$D_{j,j-1} \begin{cases} \dfrac{1}{2t}\left(Y\left(\theta_j - \theta_{j-1}\right) + p_{j-1} - p_j + \dfrac{t}{n}\right) & \text{for } Y \geq \overline{Y}_{j,j-1} \\ \dfrac{Y\left(\theta_j - 1\right) - p_j}{t} & \text{for } \underline{Y}_j \leq Y < \overline{Y}_{j,j-1} \\ 0 & \text{for } Y < \underline{Y}_j. \end{cases}$$

Clearly, $D_j = D_{j,j+1} + D_{j,j-1}$.

It is interesting to note the difference in demand patterns arising from the relatively rich and poor. For the relatively rich consumers (with $Y \geq \overline{Y}_{j,j-1}$ or $Y \geq \overline{Y}_{j,j+1}$) firm j has to compete with the two adjacent firms, and the demand reflects that: D_j does depend on the strategic choices of the two adjacent firms, $(x_{j-1}, p_{j-1}, \theta_{j-1})$ and $(x_{j+1}, p_{j+1}, \theta_{j+1})$. In contrast, firm j does not compete with its adjacent firms for the relatively poor consumers (with $\underline{Y}_j \leq Y < \overline{Y}_{j,j-1}$ and $\underline{Y}_j \leq Y < \overline{Y}_{j,j+1}$); they form a captive market for firm j over which it exercises some monopoly power.

Difference between the rich and poor gets reflected in the price and quality responses to demand also. Price response for the part of demand arising from the rich, $\dfrac{\partial \delta_{j,j+1}}{\partial p_j} = \dfrac{\partial \delta_{j,j-1}}{\partial p_j} = -\dfrac{1}{2t}$, is clearly lower than that arising from the poor, $\dfrac{\partial \eta_{j,j+1}(Y)}{\partial p_j} = \dfrac{\partial \eta_{j,j-1}(Y)}{\partial p_j} = -\dfrac{1}{t}$, because of the presence of competitive pressure.

2.2.2 Characterizing the Symmetric Equilibrium

Given the symmetric model structure, in what follows we characterize the symmetric equilibrium where each of the n entering firms chooses the same price in stage 2, that is, $p_j = p$, and $\theta_j = \theta$ for all j. Then in stage 1 entry (that is, the number of operating firms) is determined by the zero-profit condition.

In a symmetric equilibrium the income thresholds relevant to define the demand structure become

$$\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1} = \frac{2p + \dfrac{t}{n}}{2(\theta - 1)} = \frac{p}{\theta - 1} + \frac{t}{2n(\theta - 1)} \equiv \overline{Y}, \tag{2.1}$$

and

$$\underline{Y}_{j,j+1} = \underline{Y}_{j,j-1} = \frac{p_j}{\theta - 1} = \frac{p}{\theta - 1} \equiv \underline{Y}. \tag{2.2}$$

We always consider the scenario where the rich has complete market coverage, that is, $Y_R \geq \overline{Y}$. Then, depending on whether the poor has complete or partial coverages, that is, depending on the position of Y_P vis-a-vis $\overline{Y}$ and $\underline{Y}$, we have the following cases to consider:

(1) $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$;

(2) $Y_R > \overline{Y}$ and $Y_P = \overline{Y}$;

(3) $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$;

(4) $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$;

(5) $Y_R = \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$.

Both rich and poor have full market coverage under case (1) and case (2). There is complete market coverage for rich, but only partial coverage for poor under case (3) and case (5). There is complete market coverage for rich, but no coverage for poor under case (4). Case (2) and case (5) exemplify the 'kinked equilibrium' possibilities as in Salop (1979) and therefore are considered separately. To develop intuition, we first characterize the symmetric equilibrium with a fixed quality. Then we endogenize the choice of quality.

2.3 Equilibrium with Quality Fixed

In this section we assume that product quality is fixed at the level $\theta > 1$, that is, $\theta_j = \theta$, for all j. In what follows we analyze in detail case (3), the most generic case where all the rich consumers are served, whereas, for the poor, some are served while others are left out. Analysis of the other cases is similar, and we summarize and discuss the relevant results in sections 2.3.3 and 2.3.4.

2.3.1 Case (3): $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$

Let us first derive the expression for demand faced by firm j. It follows from the demand structure discussed in section 2.2.1 that

$$\begin{aligned} D_j &= f_R\left[\delta_{j,j+1}(Y_R) + \delta_{j,j-1}(Y_R)\right] + f_P\left[\eta_{j,j+1}(Y_P) + \eta_{j,j-1}(Y_P)\right] \\ &= f_R\left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + t\left|x_j - x_{j-1}\right| + t\left|x_{j+1} - x_j\right|}{2t}\right] + 2f_P\left[\frac{Y_P(\theta - 1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\left[\dfrac{f_R + 2f_P}{t}\right]$.

In stage 2, given entry and location decisions in the earlier stage, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = \left[p_j - c(\theta)\right] \cdot \left[\frac{f_R + 2f_P}{t}\right]. \tag{2.3}$$

In the symmetric equilibrium firms locate symmetrically, that is, $\left|x_j - x_{j-1}\right| =$

$|x_{j+1} - x_j| = \frac{L}{n}$, for all j. The implication for demand is

$$D_j = f_R \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + 2t\left(\frac{L}{n}\right)}{2t} \right] + 2f_P \left[\frac{Y_P(\theta - 1) - p_j}{t} \right]. \tag{2.4}$$

Finally, firms' entry decision is determined by the zero-profit condition. Using (2.3) the expression for profit becomes

$$\pi_j = [p_j - c(\theta)]D_j - F(\theta) = [p_j - c(\theta)]^2 \cdot \left[\frac{f_R + 2f_P}{t} \right] - F(\theta),$$

so that the zero-profit condition implies

$$[p_j - c(\theta)]^2 \cdot \left[\frac{f_R + 2f_P}{t} \right] - F(\theta) = 0. \tag{2.5}$$

Using (2.3), (2.4), and (2.5) we derive the equilibrium price and number of firms:

$$p = c(\theta) + \sqrt{\frac{tF(\theta)}{1 + f_P}}, \tag{2.6}$$

$$\frac{L}{n} = \frac{1}{t(1 - f_P)} \left[(1 + 3f_P)\sqrt{\frac{tF(\theta)}{1 + f_P}} - 2f_P [Y_P(\theta - 1) - c(\theta)] \right]. \tag{2.7}$$

Note that since the firms are competing for rich consumers, both price and number of firms are independent of the rich income. Price is also independent of the poor income. But, since the poor forms a captive market for the firms the size of which is restricted by their income, number of firms increases with the poor income. As poor income increases, demand size of each firm increases, and, price remaining the same, each firm makes more than normal profit. This super-normal profit attracts fresh entry of firms into the city.

It is also interesting to note that for f_P close to zero, $\frac{L}{n}$ is inversely related to t (the measure of transport cost). That is, as the transport cost increases the number of firms increase in equilibrium. The reason is that as t increases market power of each firm goes up owing to increased horizontal differentiation. So each firm is able to make higher profit which induces further entry. When f_P is strictly positive then, with increase in t, there is a counteracting effect at work: higher transportation cost imposes higher cost on the poor leaving a larger section of them unserved; this reduces firm's profit leading to a fall in the number of firms. Thus, the net effect is ambiguous when f_P is strictly positive.

Before we investigate this case any further, it is important to identify parameter values, in particular the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$, where the income thresholds $\overline{Y}$ and $\underline{Y}$ are endogenous (as expressed in equations (2.1) and (2.2)). Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ and $\underline{Y}$ we find that $Y_P < \overline{Y}$ implies

$$Y_P\left(\theta - 1\right) - c\left(\theta\right) < \frac{3 + f_P}{2}\sqrt{\frac{tF\left(\theta\right)}{1 + f_P}},$$

whereas $Y_P > \underline{Y}$ implies

$$Y_P\left(\theta - 1\right) - c\left(\theta\right) > \sqrt{\frac{tF\left(\theta\right)}{1 + f_P}}.$$

Combining the two we get

$$\sqrt{\frac{tF\left(\theta\right)}{1 + f_P}} < Y_P\left(\theta - 1\right) - c\left(\theta\right) < \frac{3 + f_P}{2}\sqrt{\frac{tF\left(\theta\right)}{1 + f_P}}.$$

Similarly, $Y_R > \overline{Y}$ implies

$$\left[f_R Y_R + f_P Y_P\right]\left(\theta - 1\right) - c\left(\theta\right) > \frac{3 + f_P}{2}\sqrt{\frac{tF\left(\theta\right)}{1 + f_P}}.$$

Thus we conclude that case (3) arises when the poor and rich incomes are such that

$$\begin{aligned} \frac{c(\theta)+\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)} &< Y_P < \frac{c(\theta)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)} \\ &\text{and} \\ f_R Y_R + f_P Y_P &> \frac{c(\theta)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)}. \end{aligned} \tag{2.8}$$

So we have identified an upper income threshold and a lower income threshold for the poor income such that if the poor income is in between these two thresholds whereas the rich income is high enough so that the average income is higher than the upper income threshold, then the firms do not compete with the adjacent firms for the poor consumers but do so only for the rich consumers. All the rich consumers are served by the market, but some poor are left out – only those poor who are located closer to the firms get served.

2.3.2 Income Inequality and Welfare of the Poor

Now we use this generic case (3) to analyze the impact of income inequality on the welfare of the poor.

Consider the rich consumers first. Since all the rich consumers are served, the market access of the poor can be thought of as in proportion to that of the rich. To calculate the aggregate consumer surplus of the rich community as a whole we proceed as follows. Surplus to a rich consumer located at a distance x from the firm from which it is buying is $Y_R\theta - p - tx - Y_R$.[5] Since there are n firms each with a market coverage of $\frac{L}{2n}$ on either side of its location, the aggregate consumer surplus of the

[5] Recall that the reservation utility of the rich is Y_R.

rich community is

$$CS_R = 2n \int_0^{\frac{L}{2n}} \left[Y_R (\theta - 1) - p - tx \right] dx = L \left[Y_R (\theta - 1) - p - \frac{t}{4} \left(\frac{L}{n} \right) \right].$$

As expected, consumer surplus increases with income (Y_R), quality of the product (θ), and number of firms (n), and decreases with travel cost (t) and price (p). Since price and number of firms are endogenous, substituting their equilibrium values from equations (2.6) and (2.7) we derive the expression for aggregate consumer surplus of the rich community solely in terms of the parameters of the model:

$$CS_R = L \left[Y_R (\theta - 1) + \frac{1}{2} \frac{f_P}{f_R} Y_P (\theta - 1) - c(\theta) \left(1 - \frac{1}{2} \frac{f_P}{f_R} \right) - \sqrt{\frac{tF(\theta)}{1 + f_P}} \left(1 + \frac{1 + 3f_P}{4f_R} \right) \right]. \tag{2.9}$$

It is interesting to note that consumer surplus of the rich increases even when the income of the poor increases. As noted in the last section, as poor income increases price remains the same but number of firms increases. A higher number of firms implies less travel cost for the rich and hence their consumer surplus increases.

Coming to the poor consumers, consider their market access first. Not all the poor can afford to buy the product: only the poor up to the distance $\frac{Y_P (\theta - 1) - p}{t}$ from any firm are buying the product; those in between the distance $\frac{Y_P (\theta - 1) - p}{t}$ and $\frac{L}{2n}$ cannot afford it. Hence the aggregate real consumption of the poor community is

$$C_P = 2n \int_0^{\frac{Y_P(\theta - 1) - p}{t}} dx = \frac{2n}{t} \left[Y_P (\theta - 1) - p \right].$$

The tension between price and number of firms is clear: an increase in number of firms increases market access while a price increase reduces it. Substituting the equilibrium values of price and number of firms we get

$$C_P = \frac{2L(1 - f_P)}{(1 + 3f_P)\sqrt{\frac{tF(\theta)}{1 + f_P}} - 2f_P \left[Y_P (\theta - 1) - c(\theta) \right]} \left[Y_P (\theta - 1) - c(\theta) - \sqrt{\frac{tF(\theta)}{1 + f_P}} \right]. \tag{2.10}$$

Finally consider the aggregate consumer surplus of the poor. Since the poor in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{L}{2n}$ from any firm does not buy the product, their consumer surplus is zero. Hence the aggregate consumer surplus of the poor is

$$CS_P = 2n\left[\int_0^{\frac{Y_P(\theta-1)-p}{t}} \left[Y_P(\theta-1)-p-tx\right]dx\right] = \frac{n}{t}\left[Y_P(\theta-1)-p\right]^2.$$

Similar to the aggregate real consumption, an increase in number of firms increases aggregate consumer surplus of the poor while a price increase reduces it. Substituting the equilibrium values we derive

$$CS_P = \frac{L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}}-2f_P\left[Y_P(\theta-1)-c(\theta)\right]}\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right]^2. \tag{2.11}$$

Note that both the aggregate real consumption and consumer surplus of the poor increases as poor income increases. There are two effects at work. First is the direct 'valuation effect': quality of the product remaining the same as income increases value of the product to the consumers increases. Second effect is the indirect effect working through the increase in number of firms as poor income increases. Both the effects work in the same direction reinforcing each other.

Now to see the effect of income inequality on the welfare of the poor we conduct the following comparative static analysis: we vary f_P keeping Y_P and Y_R fixed. That is, we follow the poor with the same income level and compare the aggregate market access and consumer surplus of the poor community as a whole when they live in relatively richer societies (as f_P decreases from 1 to 0).

This comparative static exercise is worked out in Appendix A.3 and the results are illustrated in the following four figures depicting aggregate real consumption of the poor (Figure 2.1), aggregate expenditure of the poor (Figure 2.2), aggregate consumer surplus of the poor (Figure 2.3) and aggregate consumer surplus of the rich (Figure 2.4) for a specific set of parameter values under case (3).

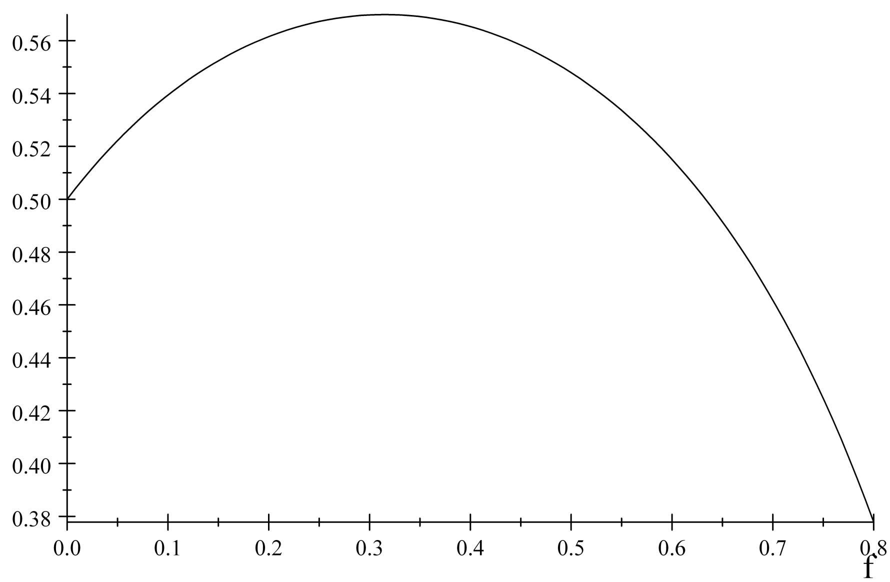

Figure 2.1: Aggregate Real Consumption of the Poor

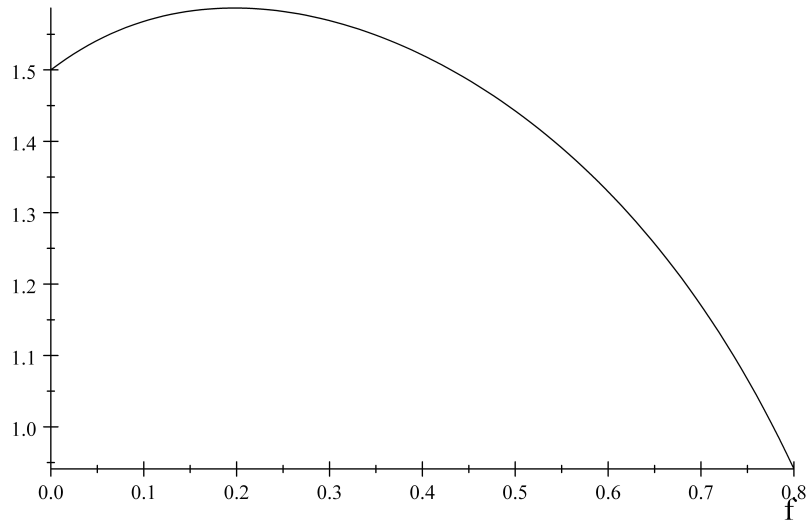

Figure 2.2: Aggregate Expenditure of the Poor

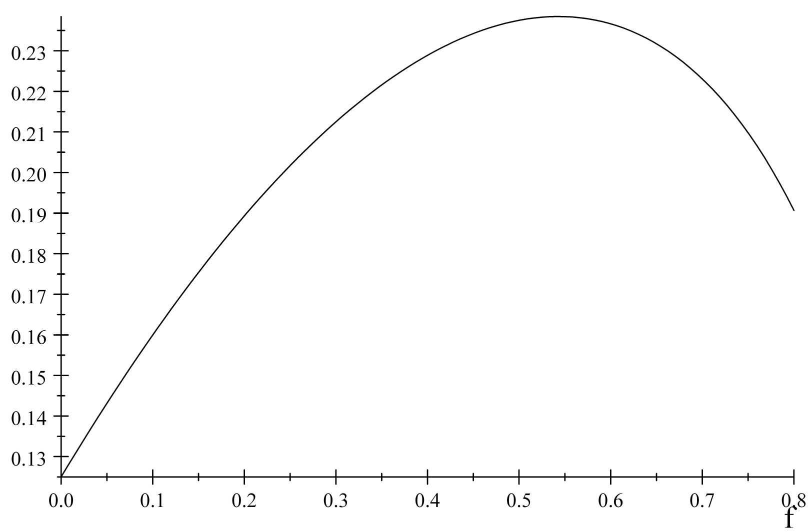

Figure 2.3: Aggregate Consumer Surplus of the Poor

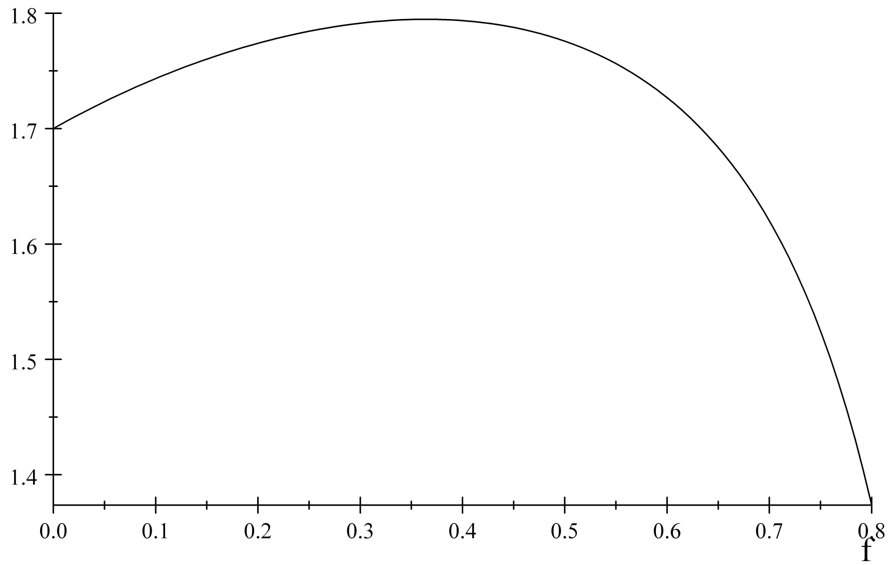

Figure 2.4: Aggregate Consumer Surplus of the Rich

It is very interesting to observe the "inverted-U" shaped relationship between f_P and aggregate real consumption of poor, f_P and aggregate expenditure of poor, and f_P and aggregate consumer surplus of poor.[6] That is, if we compare a cross-section

[6]Establishing the exact "inverted-U" shaped relationship is algebraically tedious. In Appendix A.3 we show the following:

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0} > 0 \text{ and } \left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1} < 0,$$

and

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} > 0 \text{ and } \left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1} < 0.$$

of societies, the poor community as a whole is initially better-off living in relatively richer societies (as f_P decreases from 1). But, beyond a point, the aggregate consumer surplus of poor starts declining as the society becomes richer.

The reason for this inverted-U shaped relationship can be traced to the equilibrium price and number of firms. It can be checked from their equilibrium values given in equations (2.6) and (2.7) that both price and number of firms increases steadily as f_P decreases from 1 to 0.[7] Consumers benefit from the increase in number of firms but lose from the increase in price. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and the number of poor served increases as the number of firms increases. But, beyond a point, the adverse price effect takes over.

It is important to highlight the role of the spatial structure, in particular to point out that we are getting the inverted-U shape in both real consumption and consumer surplus because the number of firms are also changing endogenously. When we conduct the same analysis with number of firms fixed, both real consumption and consumer surplus of the poor decreases steadily as f_P decreases; that is, we do not see any inverted-U shape in the relationships. The reason is that price increase steadily without any compensating increase in the number of firms.

This clearly indicates the "inverted-U" shaped relationship.

[7]While it is obvious from equation (2.6) that $\frac{\partial p}{\partial f_P} < 0$, from equation (2.7) we derive

$$\frac{\partial}{\partial f_P}\left(\frac{L}{n}\right) = \frac{\left(3f_P^2 + 6f_P + 7\right)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4\left(1+f_P\right)\left[Y_P\left(\theta-1\right) - c\left(\theta\right)\right]}{2t\left(1-f_P\right)^2\left(1+f_P\right)} < 0$$

since, under case (3), $\frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}} > Y_P\left(\theta-1\right) - c\left(\theta\right)$ implies

$$\left(3f_P^2 + 6f_P + 7\right)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4\left(1+f_P\right)\left[Y_P\left(\theta-1\right) - c\left(\theta\right)\right] > \left(1-f_P\right)^2\sqrt{\frac{tF(\theta)}{1+f_P}} \geq 0.$$

2.3.3 Summary of Equilibrium with Quality Fixed

In the last two subsections we have analyzed in detail the generic case (3) where all the rich consumers are served but only some of the poor consumers are served, others are left out of the market. Analysis of the other four cases are similar and, for the sake of brevity, we do not repeat the detailed analysis in the text and relegate it to Appendix A.4. Instead, in this section we summarize the income ranges of rich and poor under which different cases arise and discuss the implications of income inequality in the following section.

Case (1): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P > \overline{Y}$:

This case arises when

$$Y_R > Y_P > \frac{c(\theta) + \frac{3}{2}\sqrt{tF(\theta)}}{(\theta - 1)}.$$

In this case firms compete for both consumer types – rich and poor, and all consumers of each type are served.

Case (2): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P = \overline{Y}$:

This case arises when

$$\frac{c(\theta) + \dfrac{3 + f_P}{2}\sqrt{\dfrac{tF(\theta)}{1 + f_P}}}{(\theta - 1)} < Y_P < \frac{c(\theta) + \frac{3}{2}\sqrt{tF(\theta)}}{(\theta - 1)}.$$

Firms compete for the rich, but the marginal poor who is indifferent between two adjacent firms is also indifferent between buying and not buying; all consumers of each type are served though.

Case (3): $\boldsymbol{Y}_R > \overline{Y}$ and $\underline{Y} < \boldsymbol{Y}_P < \overline{Y}$:

This case arises when

$$\frac{c(\theta)+\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)} < Y_P < \frac{c(\theta)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)}$$

and

$$f_R Y_R + f_P Y_P > \frac{c(\theta)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)}.$$

Firms compete only for the rich, but are monopolist with respect to the poor. All the rich consumers are served, but some poor are left out of the reach of market.

Case (4): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P < \underline{Y}$:

This case arises when

$$Y_P < \frac{c(\theta)+\sqrt{\dfrac{tF(\theta)}{1-f_P}}}{(\theta-1)}$$

and

$$Y_R > \frac{c(\theta)+\dfrac{3}{2}\sqrt{\dfrac{tF(\theta)}{1-f_P}}}{(\theta-1)}.$$

Firms are competing only for the rich and all the rich consumers are served. But, unfortunately, all the poor consumers are left out.

Case (5): $\boldsymbol{Y}_R = \overline{Y}$ and $\underline{Y} < \boldsymbol{Y}_P < \overline{Y}$:

This case arises when

$$\frac{c(\theta)+\sqrt{\dfrac{tF(\theta)}{2}}}{(\theta-1)} < Y_P < \frac{c(\theta)+\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)}$$

and

$$\frac{c(\theta)+\sqrt{2tF(\theta)}}{(\theta-1)} < f_R Y_R + f_P Y_P < \frac{c(\theta)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)}.$$

In this case firms are monopolists with respect to the poor, but even the marginal rich is also indifferent between buying and not buying. All the rich consumers are served. Some poor consumers are served too. Figure 2.5 summarizes all these cases plotting the lower and upper bounds of incomes for different values of f_P, the proportion of poor people.

2.3.4 Implications of Income Inequality

Our analysis of the different cases summarized in the last section has a number of implications of income inequality.

Upper threshold for $\boldsymbol{Y}_P$

From cases (1), (2) and (3) it is clear that there exists an upper income threshold for Y_P, call it $\overline{Y}_P$, defined by

$$\overline{Y}_P \equiv \frac{c\left(\theta\right)+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF\left(\theta\right)}{1+f_P}}}{\left(\theta-1\right)}$$

such that all poor consumers are served only if $Y_P \geq \overline{Y}_P$.

Case (1) shows the existence of another income threshold, $\dfrac{c\left(\theta\right)+\frac{3}{2}\sqrt{tF\left(\theta\right)}}{\left(\theta-1\right)} > \overline{Y}_P$, such that if the income of the poor is above this threshold, then not only all poor consumers are served but, in addition, each firm has to compete with its adjacent firms for *both* poor and rich customers. Equilibrium price and number of firms reflect this competition.

Lower thresholds for $\boldsymbol{Y}_P$

There are two lower income thresholds for the poor, $\underline{Y}_P$, such that no poor consumer is served if $Y_P < \underline{Y}_P$. Interestingly which threshold is relevant depends on the income of the rich.

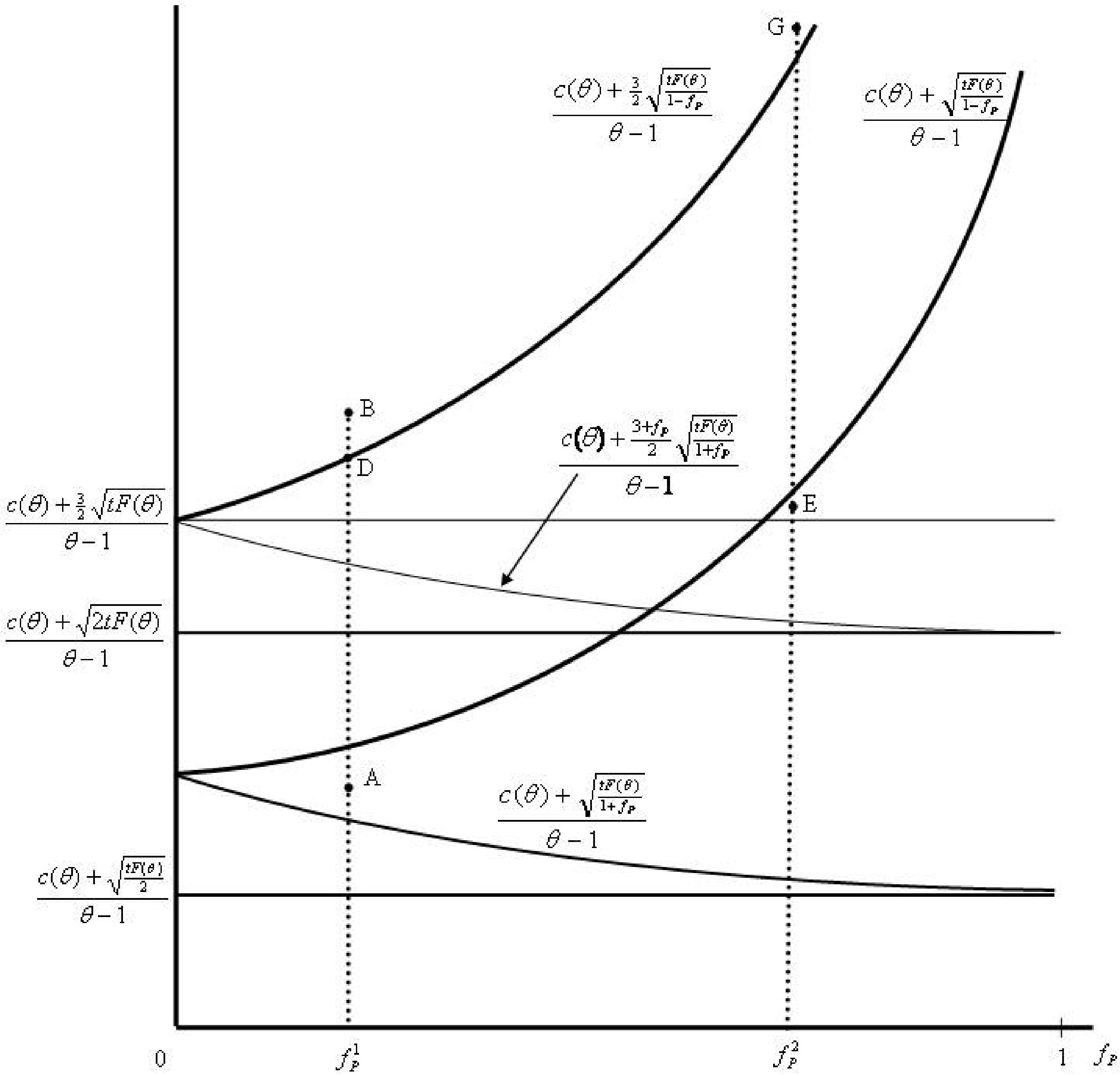

Figure 2.5: Different Equilibrium Possibilities

When the rich income is high enough so that firms are competing for the rich (cases (1), (2) and (3)), the lower income thresholds for the poor is given by

$$\underline{Y}_P^1 \equiv \frac{c(\theta) + \sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta - 1)}.$$

But when the rich income is reasonably low in the sense that the marginal rich is indifferent between buying and not buying (case (5)), then this lower income threshold becomes

$$\underline{Y}_P^2 \equiv \frac{c(\theta) + \sqrt{\dfrac{tF(\theta)}{2}}}{(\theta - 1)}.$$

Implication of Income Gap between Rich and Poor:

Notice that $\underline{Y}_P^2 < \underline{Y}_P^1$, that is, the lower income threshold of the poor is lower when the rich income is reasonably low. Thus poor are better off when the income gap between the rich and poor is low.

When the poor income is in between the upper and lower income thresholds, there are pockets of the city where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

Possibility of Multiple Equilibria

Case (4) generates the possibility of multiple equilibria. Consider, for example, the income distribution depicted by points A and B in Figure 2.5: there are f_P^1 proportion of poor with income given by the height of A and $(1 - f_P^1)$ proportion of rich with income B. The income distribution is such that parameter configurations for both cases (3) and (4) are satisfied, generating the multiple equilibria. The equilibrium under case (3) is a good equilibrium where at least some poor get served. The equilibrium outcome under case (4) is a bad outcome: all the poor are excluded; the firms completely ignore their presence and choose the price as if there were only rich individuals residing in the city.

Implication of Income Gap:

Note once again the implication of higher income gap between rich and poor. If the rich income were below the height of D, then this complete exclusion possibility of the poor would not have arisen. It is the higher income gap that exposes the poor to this vulnerable situation.

The implication of income gap could be even more damaging for a multiple equilibria situation like the one depicted by the other income distribution shown in Figure 2.5: there are f_P^2 proportion of poor with income given by the height of E and $(1 - f_P^2)$ proportion of rich with income G. Here the multiplicity occurs with cases (4) and (1). Recall that case (1) is the best possible outcome that can happen to the poor – income of the poor is high enough so that all the poor are served, and, at the same time, the firms are forced to compete for them. But even then a higher income gap exposes them to the possibility of complete exclusion.

The Case of Minority Poor:

Poor are more likely to be completely excluded when they are a minority, that is, when f_P is low: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number. For instance, in Figure 2.5, with the same income levels A for poor and B for rich, the complete exclusion possibility does not arise when the proportion of poor is f_P^2; but this possibility does arise when the proportion of poor is f_P^1.

2.3.5 Comparison with a Single-Income Economy

In section 2.3.2 we have identified scenarios where the poor could be better-off living in relatively richer societies. To see how the possibility arises in the simplest possible way it is interesting to compare our model economy with two income groups with a single-income economy. A single-income economy refers to a city inhabited by a single income group; that is, at each point of the city there is a measure 1 of consumers with the same income Y. The single-income economy model is analyzed in Appendix A.5 and the relevant comparison is highlighted below.

The Feasibility Income Threshold in a Single-Income Economy:

In a single-income economy it is not feasible for any firm to operate unless the common income is at least $\frac{c(\theta) + \sqrt{2tF(\theta)}}{(\theta - 1)}$. If the income is below this feasibility threshold, the willingness to pay is so low that it is not possible for the firms to recover the fixed cost of production. The implication for a single-income poor economy with common income Y_P is that nobody gets to enjoy the product or service when $Y_P < \frac{c(\theta) + \sqrt{2tF(\theta)}}{(\theta - 1)}$.

Comparing a Single-Income Economy with a Mixed-Income Economy:

With reference to a single-income economy, a mixed-income economy is the one that we are considering so far where at each point of the city there are f_R proportion of rich with income Y_R and f_P proportion of poor with income Y_P. Since both the lower income thresholds of the poor, $\underline{Y}_P^1$ and $\underline{Y}_P^2$, are strictly less than the feasibility threshold, $\frac{c(\theta) + \sqrt{2tF(\theta)}}{(\theta - 1)}$, it is clear that poor are better-off staying in the mixed-income community as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income economy as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income poor economy.

2.4 Equilibrium with Endogenous Quality

In this section we consider the scenario where product quality is not fixed any more, it is an endogenous choice of the firms. As in the last section we continue to focus on the symmetric equilibrium where all the entering firms have equal market share, charge the same price and offer the same quality product. In what follows we first illustrate the method of analysis by detailing the generic case (3) in the next subsection. In the two subsections that follow we discuss the implications of income inequality on the welfare of the poor focusing on the channel working through endogenous quality choice.

2.4.1 Case (3): $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$

Quality being an endogenous choice, demand faced by firm j becomes

$$\begin{aligned} D_j &= f_R \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + t\,|x_j - x_{j-1}| + t\,|x_{j+1} - x_j| + Y_R\,(2\theta_j - \theta_{j-1} - \theta_{j+1})}{2t} \right] \\ & \quad +2f_P \left[\frac{Y_P\,(\theta_j - 1) - p_j}{t} \right] \end{aligned}$$

so that price and quality responses to demand are given by $\dfrac{\partial D_j}{\partial p_j} = -\left[\dfrac{f_R + 2f_P}{t}\right]$, and $\dfrac{\partial D_j}{\partial \theta_j} = \dfrac{f_R Y_R + 2 f_P Y_P}{t}$, respectively.

In stage 2, firm j chooses price and quality to maximize profit. The first-order condition with respect to price has the same implication as in equation (2.3). The first-order condition with respect to quality implies

$$c'(\theta_j) \cdot D_j + F'(\theta_j) = [p_j - c(\theta_j)] \cdot \frac{\partial D_j}{\partial \theta_j}. \tag{2.12}$$

It is easy to see that the implication for symmetric location leads to the same condition as in equation (2.4) and the zero-profit condition in stage 1 leads to equation (2.5).

Using equations (2.3), (2.12), (2.4) and (2.5) we derive the same expressions for equilibrium price and number of firms as in equations (2.6) and (2.7); only difference is that, in addition, equilibrium quality (θ) gets determined implicitly from the following condition:

$$c'(\theta) + F'(\theta) \cdot \sqrt{\frac{t}{F(\theta)}} \cdot \frac{1}{\sqrt{f_R + 2f_P}} = \frac{f_R Y_R + 2 f_P Y_P}{f_R + 2 f_P}. \tag{2.13}$$

Notice that the right-hand side of equation (2.13) is an weighted average of the rich and poor incomes. Imposing enough convexity on the cost structure it is easy to check that equilibrium quality increases as either poor income or rich income or their weighted average increases.[8]

Comparing with the fixed quality model, one important difference to note is that *both* equilibrium price and number of firms now respond to changes in poor income, rich income, or their weighted average. The reason is that both price and number of firms respond to changes in product quality, and product quality is determined by income. From equation (2.6) it is clear that price responds positively to changes in quality since both fixed and marginal costs are increasing in quality. Since product quality increases with incomes, it is clear that price responds positively to changes in incomes too. On the other hand, it is established in Appendix A.7 that there is an inverted-U shape relationship between equilibrium number of firms and quality. As can be seen from equation (2.7), there are two forces at work. First is the cost effect, both fixed and marginal. As quality increases, costs increase, leading to a reduction in number of firms so that each existing firm can break-even. On the other hand, the effect that works in the opposite direction is the valuation effect working through the higher willingness to pay of the poor consumers. Recall that, under this case (3), the poor forms a captive market for the firms. As product quality increases, the

[8] The sufficient condition, as derived in Appendix A.6, is $c''(\theta) \geq 0$ and $F''(\theta) > \frac{[F'(\theta)]^2}{2F(\theta)}$.

poor's willingness to pay for the product increases (Y_P remaining the same), leading to an increase in the number of firms. This effect is similar to the effect of increase in poor income on equilibrium number of firms discussed in the context of fixed quality model. Quality response having an inverted-U shape, the relationship between equilibrium number of firms and income has an inverted-U shape too.

Proceeding in the same way as in the fixed quality model we derive exactly the same ranges of poor and rich incomes as in (2.8) under which case (3) occurs. The important difference is that, unlike the fixed quality model, the equilibrium quality is endogenously determined by equation (2.13). Similarly the income ranges of poor and rich incomes under which the other cases arise are exactly the same as the fixed quality model, with the added qualification that equilibrium product quality are endogenously determined by conditions similar to equation (2.13). We do not mention the income ranges here again to avoid repetition.

2.4.2 Income Gap and Market Access of the Poor

Now we explore the implications of income inequality on the welfare of the poor. As explained in the last subsection, the endogeneity of product quality makes all the endogenous variables in the model – price, product quality and number of firms – dependent on incomes – poor income (Y_P), rich income (Y_R), or their weighted average. This makes it possible for us to conduct the following comparative static analysis that was not possible in the fixed quality model: we vary Y_R keeping fixed Y_P and all other parameters of the model. We investigate the implications of this increased income gap on the market access of the poor in this subsection, and on the aggregate consumer surplus of the poor community as a whole in the next subsection.

Recall that there exists an *upper income threshold* for the poor, $\overline{Y}_P$, such that all poor consumers are served only if $Y_P \geq \overline{Y}_P$. When product quality is endogenous, $\overline{Y}_P$ is determined jointly from the following two relationships

$$\overline{Y}_P = \frac{c(\theta) + \dfrac{3+f_P}{2}\sqrt{\dfrac{tF(\theta)}{1+f_P}}}{(\theta-1)} \tag{2.14}$$

and

$$c'(\theta) + F'(\theta) \cdot \sqrt{\frac{t}{F(\theta)}} \cdot \frac{1}{\sqrt{f_R + 2f_P}} = \frac{f_R Y_R + 2 f_P Y_P}{f_R + 2 f_P}. \tag{2.15}$$

The expression for the upper income threshold is the same as in section 2.3.4, whereas the second condition is equation (2.13) determining equilibrium product quality. We have put the two together to emphasize on their joint determination. The chain of determination is as follows. Given Y_P and Y_R, equation (2.13) determines product quality (θ). Then θ determines the upper income threshold ($\overline{Y}_P$) from equation (2.14).

Now we analyze how the upper income threshold changes when the income gap between the rich and poor widens by increasing Y_R while keeping Y_P fixed. Recall that when Y_R increases, θ increases. The same convexity condition on the cost structure (see footnote 8) also guarantees a convex U-shaped relationship between $\overline{Y}_P$ and θ: as θ increases $\overline{Y}_P$ decreases in the beginning , reaches a minimum, and then starts increasing. This relationship is established in Appendix A.8. The reason for this U-shape can once again be traced back to the price effect and the number-of-firms effect that we have already encountered in the fixed quality model. In addition, we now have the valuation effect working through the higher willingness to pay by the poor. This can be seen clearly from the general expression of the upper income threshold involving price, product quality and number of firms:

$$\overline{Y}_P = \frac{p + \dfrac{tL}{2n}}{\theta - 1}.$$

Clearly the upper income threshold increases with price but decreases with number of firms. As product quality increases, price increases due to increased cost, both fixed and marginal. Recall from the last subsection that the effect on number of firms is ambiguous: cost effect tends to reduce the number of firms while the higher willingness to pay by the poor who forms the captive market for the firms, the valuation effect, leads to an increase in the number of firms. Finally the direct impact of the valuation effect (increase in θ keeping Y_P the same) is to lower the upper income threshold of the poor. In the beginning as θ increases the valuation effect – working directly, and indirectly through the number of firms – dominates. But, beyond a point, the cost effect – working through price and number of firms – takes over. This results in the U-shape in the relationship between $\overline{Y}_P$ and θ. Since θ increases with Y_R, the implication is that as the income gap between the rich and poor widens (by increasing Y_R while keeping Y_P fixed), the upper income threshold of the poor also decreases in the beginning, reaches a minimum, and then starts increasing.

Consider next the *lower income threshold* for the poor, $\underline{Y}_P$, such that no poor consumer is served if $Y_P < \underline{Y}_P$. Like $\overline{Y}_P$, $\underline{Y}_P$ is also jointly determined from the two relationships

$$\underline{Y}_P = \frac{c(\theta) + \sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta - 1)} \tag{2.16}$$

and

$$c'(\theta) + F'(\theta) \cdot \sqrt{\frac{t}{F(\theta)}} \cdot \frac{1}{\sqrt{f_R + 2f_P}} = \frac{f_R Y_R + 2f_P Y_P}{f_R + 2f_P}, \tag{2.17}$$

with Y_P and Y_R determining θ from equation (2.13), and then θ determining the lower income threshold ($\underline{Y}_P$) from equation (2.16).[9] It is easy to check that when the cost structure is convex enough there exists a convex U-shaped relationship between $\underline{Y}_P$ and θ, similar to the relationship between $\overline{Y}_P$ and θ. The reason once again is the

[9] We consider only $\underline{Y}_P^1$ and do not repeat the same argument for $\underline{Y}_P^2$.

tension between the cost effect and the valuation effect as should be clear from the general expression of the lower income threshold involving price and product quality:

$$\underline{Y}_P = \frac{p}{(\theta - 1)}.$$

While the cost effect – working through price – tends to increase the lower income threshold, the valuation effect tends to lower it. It follows once again that, similar to the upper income threshold, as the income gap between the rich and poor widens (by increasing Y_R while keeping Y_P fixed), the lower income threshold of the poor also decreases in the beginning, reaches a minimum, and then starts increasing.

As both the upper and lower income thresholds show the same pattern, it follows that market access of the poor follow the reverse trend: increases in the beginning, reaches a maximum, and then starts declining as the income gap between the rich and poor widens. Although the upper and lower income thresholds reach their respective turfs at different levels of $\theta,$ the general trend is unmistakable.

2.4.3 Income Gap and Welfare of the Poor

In this section we will establish that, similar to the model with fixed quality, there is an inverted-U shaped relationship between income gap and welfare of the poor. Recall from section 2.3.2 that under case (3) when all the rich are served but some poor are left out of the reach of the market, the expression for aggregate consumer surplus of the poor is given by

$$CS_P = \frac{n}{t}\left[Y_P(\theta - 1) - p\right]^2.$$

Once again, two forces are at work – the cost effect and the valuation effect. The cost effect leads to a decrease in consumer surplus by increasing price and decreasing the number of firms. The valuation effect tends to increase consumer surplus directly, and, at the same time, indirectly by increasing the number of firms. As the income gap between the rich and poor widens (by increasing Y_R while keeping Y_P fixed), initially the valuation effect dominates, but the cost effect takes over at the later stage resulting in the inverted-U shaped relationship between income gap and aggregate consumer surplus of the poor.

What is interesting is that under the model with endogenous product quality we get an inverted-U shaped relationship between income gap and aggregate consumer surplus of the poor even under the most favourable case for the poor where all the poor are served by the market (case (1)).[10] Following the same methodology as outlined for case (3), we find the equilibrium price, number of firms and product quality under case (1) as follows:

$$p = c(\theta) + \sqrt{tF(\theta)}, \tag{2.18}$$

$$\frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}, \tag{2.19}$$

$$c'(\theta) + F'(\theta)\sqrt{\frac{t}{F(\theta)}} = f_R Y_R + f_P Y_P. \tag{2.20}$$

Comparing with equations (2.6), (2.7) and (2.13) it is clear that the equilibrium values of the endogenous variables under cases (1) and (3) become equivalent when $f_P = 0$, that is, when the city is inhabited by only the rich.

The right-hand side of equation (2.20) is an weighted average of the rich and poor income. Imposing the same convexity condition on the cost structure (see footnote 8) it is easy to see that, similar to case (3), equilibrium product quality increases as either poor income or rich income or their weighted average increases. Price response

[10]This was not possible in the fixed quality model as, under case (1), both the endogenous variables, price and number of firms, were independent of income or the proportion of poor.

is similar too: price increases with quality and hence with incomes. But, unlike case (3), quality response to number of firms is unambiguously negative: only the cost effect is at work, there is no counteracting valuation effect since the poor no longer forms a captive market for the firms. Quality response to number of firms being unambiguous, income response is also unambiguously negative.

Since all the poor consumers are served, the aggregate consumer surplus of the poor community is

$$CS_P = 2n \int_0^{\frac{L}{2n}} \left[Y_P (\theta - 1) - p - tx \right] dx = L \left[Y_P (\theta - 1) - p - \frac{t}{4} \left(\frac{L}{n} \right) \right],$$

and substituting the equilibrium values of price and number of firms from equations (2.18) and (2.19) we derive that the aggregate consumer surplus of the poor is given by

$$CS_P = L \left[Y_P (\theta - 1) - c(\theta) - \frac{5}{4} \sqrt{tF(\theta)} \right] \tag{2.21}$$

where θ is determined by equation (2.20).

It is shown in Appendix A.9 that the same convexity condition on the cost structure generates an inverted-U shaped relationship between CS_P and θ. Like case (3), the reason is the tension between the cost effect and the valuation effect. The difference is that the positive valuation effect is relatively weaker – only the direct effect is at work, the indirect effect working through the increase in number of firms is absent.

As θ increases as Y_R increases, it follows that there is, in general, an inverted-U shaped relationship between income gap and aggregate consumer surplus of the poor under case (1) also. But what happens to aggregate consumer surplus of the poor as the income gap increases depends on the existing income gap; that is, it is the existing income gap that determines on which side on the inverted-U we begin with. It is shown in Appendix A.9 that aggregate consumer surplus of poor is maximized at θ_{W_P} which is determined from the following equation:

$$c'(\theta_{W_P}) + \frac{5}{8}\sqrt{t} \cdot \frac{F'(\theta_{W_P})}{\sqrt{F(\theta_{W_P})}} = Y_P.$$

Interestingly, quality level in the single income economy (θ_S), where the common income level is Y_P, is determined from

$$c'(\theta_S) + \sqrt{t} \cdot \frac{F'(\theta_S)}{\sqrt{F(\theta_S)}} = Y_P.$$

On the other hand quality level in the mixed-income economy under consideration (call it θ_M) is determined from equation (2.20):

$$c'(\theta_M) + \sqrt{t} \cdot \frac{F'(\theta_M)}{\sqrt{F(\theta_M)}} = f_R Y_R + f_P Y_P.$$

Given the assumption on the cost structure it follows that $\theta_{W_P} > \theta_S$, and $\theta_M > \theta_S$. But the comparison between θ_M and θ_{W_P} depends on the extent of the existing income gap. This is illustrated in Figures 2.6, 2.7 and 2.8. Figure 2.7 illustrates the case where the starting income gap is not very high resulting in $\theta_M < \theta_{W_P}$. In this case as the income gap increases, aggregate consumer surplus of poor increases in the beginning, reaches the maximum, and then starts falling. On the other hand, Figure 2.8 illustrates the case where the starting income gap is high enough so that $\theta_M > \theta_{W_P}$. In this case aggregate consumer surplus of poor steadily declines as the income gap increases.

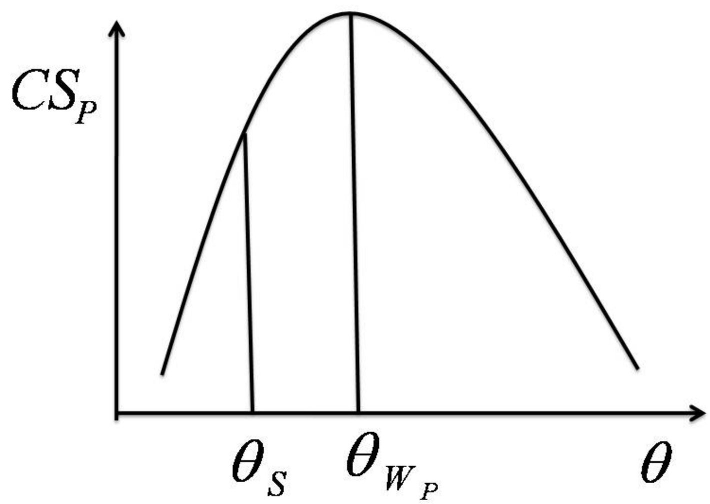

Figure 2.6: Relationship between Consumer Surplus and Quality

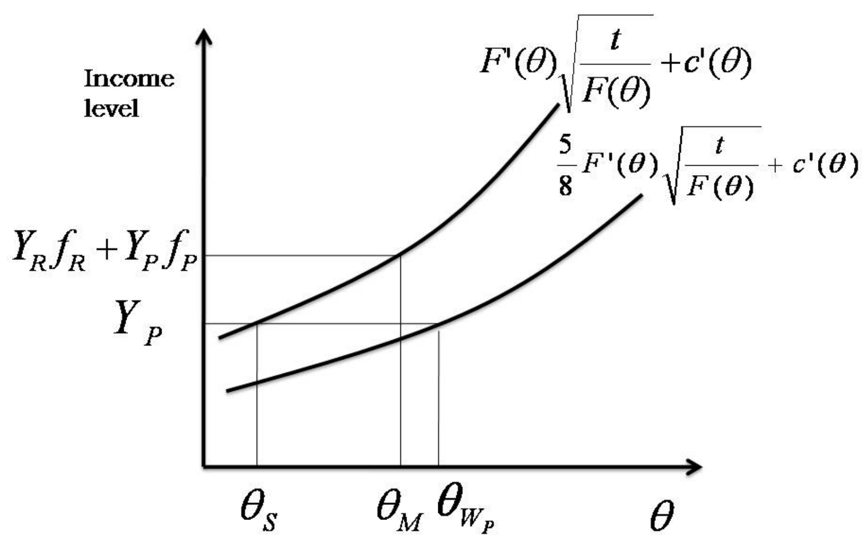

Figure 2.7: Low Income Gap: $\theta_M < \theta_{W_P}$

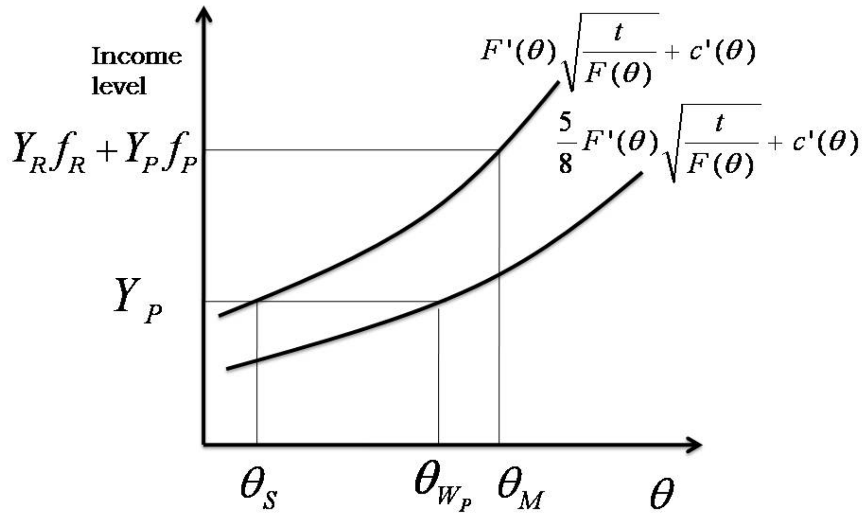

Figure 2.8: High Income Gap: $\theta_M > \theta_{W_P}$

2.5 Conclusion

In this chapter we develop a model that integrates consumer's income distribution with spatial distribution, and look at the consequence of an increase in income inequality on the welfare of the poor in general, and their access to market in particular. We find an inverted-U shaped relationship between welfare of the poor and the income gap between rich and poor. When the gap is not much, welfare of the poor initially increases and reaches its maximum. But, as the income gap widens further, the welfare gain from increase in quality is not enough to offset the corresponding rising price and travel costs, so the welfare falls. Interestingly the same inverted-U shaped relationship is also observed between income inequality and market access of the poor.

We identify an *upper income threshold* for the poor income such that all poor consumers get served by the market only if the poor income is above this upper income threshold. On the other hand there exists a *lower income threshold* for the poor income such that no poor consumer is served if the poor income is below this lower threshold. When the poor income is in between these two thresholds, there exist pockets where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

There exist multiple equilibria: a bad equilibrium where all the poor are excluded can exist simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group and show that poor are better-off staying in the mixed-income economy as long as the poor income is below a feasibility threshold.

We have considered a two-stage game where firms simultaneously decide price and quality in the second stage. The alternative game structure that can be considered is the one where choice of quality is earlier than price. Economides (1993) compares the equilibrium outcomes in these two alternative game structures. In one structure firms simultaneously take price and quality decisions in the last stage. This stage is preceded by a location stage, which itself is preceded by the entry stage. In the other game form the last stage has been split in two stages, one of quality choice to be followed by a stage of price choice. He has shown that precommitment in quality reduces the quality levels and increases the number of firms in equilibrium. The reason is that availability of precommitment in quality gives firms a strategic advantage in the quality stage and reduces competition between the firms. This increases the profit of each individual firm thus inducing more entry. So even though consumers are worse-off as the equilibrium quality is lower, they benefit because of increased entry which implies lower travel cost on average. We have considered the game structure where firms simultaneously decide price and quality because of its simplicity without affecting the main results. Since both quality and price are positively related to the income level irrespective of what game structure one chooses, the direction of the impact of income inequality remains unchanged.

The model has two important limitations. First is concerned with the endogenous product quality model: since we have restricted ourselves to the symmetric equilibrium, there exists just one quality in equilibrium; but when product quality is a strategic choice it is natural for the firms to separate the rich from the poor consumers by offering a higher quality. The current analysis can be interpreted as portraying, in some sense, the average picture. The other limitation is the assumption on the spatial structure that rich and poor live side by side in the same location. It would be interesting to investigate how the nature of the equilibrium changes when the rich and poor are geographically segregated – poor residing in the poor ghettos whereas rich live in the rich neighbourhoods. We address both these limitations in the following two chapters.

Another limitation of the model developed in this chapter is that it assumes away the element of 'externalities' that is a common feature of goods / services like health care and education. For example, an agent's demand for health care will depend (among other things) on the delay / congestion associated with the provision of this service. Consequently, an agent may prefer a facility that is located further away from his residence if such costs associated with this facility is lower. There are two ways to think about this issue. First, delay or congestion can be interpreted as a negative form of quality. A consumer may prefer a higher quality product or service (a hospital with a lower waiting time, or a school with fewer number of students per teacher) even if the facility is located further away from his residence. This precise trade-off between product quality and travel cost is taken up in chapter 3. Second, another way to incorporate delay or congestion in this model is to add that an individual consumer's utility depends negatively on the size of consumers accessing the product or service. But since the demand size is determined by the distance of the marginal consumer indifferent between two adjacent firms, it can be seen from the demand structure developed in section 2.2.1 that adding this aspect is equivalent to increasing the horizontal differentiation between firms. With increased horizontal differentiation market power of firms increases leading to higher price and higher profit attracting further entry of firms.

2.6 Appendix

A.1 The Model with a General Income Distribution

Here we set up the model with a general income distribution and establish the existence of a symmetric equilibrium. We also show that the symmetric equilibrium is unique when product quality is exogenously given. The set-up is a two-stage game. This is because instead of imposing symmetry we are proving that the symmetric equilibrium is the unique equilibrium. In the first stage, firms decide whether to enter or not and also choose their locations. In the second stage, firms decide price.

Consumers are uniformly distributed across the city: at each point of the city there is a uniform income distribution of consumers over the interval $[Y_{\min},\ Y_{\max}]$ with cumulative distribution function $G(Y)$. The probability density function is: $g(Y)$.

Based on the structure developed in the chapter demand faced by firm j, D_j, is given by

$$\begin{aligned} D_j &= \int_{\underline{Y}_j}^{\overline{Y}_{j,j-1}} \eta_{j,j-1}(Y)\, dG(Y) + \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \delta_{j,j-1}(Y)\, dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \eta_{j,j+1}(Y)\, dG(Y) \\ &\quad + \int_{\overline{Y}_{j,j+1}}^{Y_{\max}} \delta_{j,j+1}(Y)\, dG(Y). \end{aligned}$$

Substituting the expressions for η's and δ's we derive

$$\begin{aligned} D_j &= \int_{\underline{Y}_j}^{\overline{Y}_{j,j-1}} \left[\frac{Y(\theta_j - 1) - p_j}{t}\right] dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta_j - 1) - p_j}{t}\right] dG(Y) \\ &\quad + \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j-1} - p_j) + t\,|x_j - x_{j-1}| + Y(\theta_j - \theta_{j-1})}{2t}\right] dG(Y) \qquad (2.22) \\ &\quad + \int_{\overline{Y}_{j,j+1}}^{Y_{\max}} \left[\frac{(p_{j+1} - p_j) + t\,|x_{j+1} - x_j| + Y(\theta_j - \theta_{j+1})}{2t}\right] dG(Y). \end{aligned}$$

Difference between the rich and poor gets reflected in the price and quality responses to demand also. Using the Leibniz's rule (for differentiation of definite integrals) and the definitions of $\underline{Y}_j$, $\underline{Y}_j$, $\overline{Y}_{j,j-1}$ and $\overline{Y}_{j,j+1}$ we derive

$$\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}\left[\int_{\underline{Y}_j}^{\overline{Y}_{j,j-1}} dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} dG(Y)\right] - \frac{1}{2t}\left[\int_{\overline{Y}_{j,j-1}}^{Y_{\max}} dG(Y) + \int_{\overline{Y}_{j,j+1}}^{Y_{\max}} dG(Y)\right], \tag{2.23}$$

and

$$\frac{\partial D_j}{\partial \theta_j} = \frac{1}{t}\left[\int_{\underline{Y}_j}^{\overline{Y}_{j,j-1}} Y dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} Y dG(Y)\right] + \frac{1}{2t}\left[\int_{\overline{Y}_{j,j-1}}^{Y_{\max}} Y dG(Y) + \int_{\overline{Y}_{j,j+1}}^{Y_{\max}} Y dG(Y)\right]. \tag{2.24}$$

Clearly price and quality responses are lower for the part of demand arising from the rich because of the presence of competitive pressure.

Existence of the Symmetric Equilibrium

Here we establish the existence of a symmetric equilibrium where (1) n firms enter the city, firm j chooses location x_j such that $|x_j - x_{j-1}| = |x_{j+1} - x_j| = \frac{L}{n}$, for all j; in the first stage and (2) in stage 2, $p_j = p$, and $\theta_j = \theta$, for all j. We proceed in the standard backward fashion.

In stage 2, given entry and location decisions in the earlier stage, each firm decides its price and quality to maximize profit, π_j. The first-order conditions with respect to price and quality are given by

$$\frac{\partial \pi_j}{\partial p_j} = D_j + [p_j - c(\theta_j)]\frac{\partial D_j}{\partial p_j} = 0, \tag{2.25}$$

and

$$\frac{\partial \pi_j}{\partial \theta_j} = -c'(\theta_j) \cdot D_j + [p_j - c(\theta_j)]\frac{\partial D_j}{\partial \theta_j} - F'(\theta_j) = 0. \tag{2.26}$$

The n entering firms take their location decisions in stage 1. Let the locations of the n firms be $x = (x_1, x_2, ..., x_n)$. For stage 1, π_j can be expressed as a function of x as follows:

$$\pi_j(x) = \pi_j(x, p(x), \theta(x)) = [p_j(x) - c(\theta_j(x))]D_j(x, p(x), \theta(x)) - F(\theta_j(x)).$$

Note that equations (2.23) and (2.25) imply

$$\pi_j(x) = (p_j(x) - c(\theta_j(x)))^2 \times \left\{ \frac{1}{t} \left[\int_{\underline{Y}_j}^{\overline{Y}_{j,j-1}} dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} dG(Y) \right] + \frac{1}{2t} \left[\int_{\overline{Y}_{j,j-1}}^{Y_{\max}} dG(Y) + \int_{\overline{Y}_{j,j+1}}^{Y_{\max}} dG(Y) \right] \right\} - F(\theta_j(x)).$$

In order to maximize $\pi_j(x)$ with respect to x_j we do not need to take the derivatives $\frac{\partial \pi_j(x)}{\partial p_j} \cdot \frac{\partial p_j}{\partial x_j}$ and $\frac{\partial \pi_j(x)}{\partial \theta_j} \cdot \frac{\partial \theta_j}{\partial x_j}$. This is due to the envelope theorem: firm j maximizes π_j with respect to p_j and θ_j in stage 2, implying that $\frac{\partial \pi_j}{\partial p_j} = 0$ and $\frac{\partial \pi_j}{\partial \theta_j} = 0$. Using this and the Leibniz's rule we get

$$\frac{d\pi_j(x)}{dx_j} = -\frac{1}{t}[p_j(x) - c(\theta_j(x))]^2 \left[g\left(\overline{Y}_{j,j-1}\right) \frac{\partial \overline{Y}_{j,j-1}}{\partial x_j} + g\left(\overline{Y}_{j,j+1}\right) \frac{\partial \overline{Y}_{j,j+1}}{\partial x_j} \right].$$

But $\frac{\partial \overline{Y}_{j,j-1}}{\partial x_j} = \frac{t}{\theta_j + \theta_{j-1} - 2}$, and $\frac{\partial \overline{Y}_{j,j+1}}{\partial x_j} = -\frac{t}{\theta_j + \theta_{j+1} - 2}$, and, in a symmetric equilibrium, we have $\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1} = \frac{2p + \frac{tL}{n}}{2(\theta - 1)} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)} \equiv \overline{Y}$. It now follows that $\frac{d\pi_j(x)}{dx_j} = 0$ at a symmetric equilibrium, that is, a symmetric equilibrium does exist.

Finally, in stage 1, firms decide whether to enter or not. Entry (that is, the number of operating firms) is determined by the zero-profit condition:

$$\pi_j = [p_j - c(\theta_j)]D_j - F(\theta_j) = 0.$$

Uniqueness of the Symmetric Equilibrium with Quality Fixed

When product quality is exogenously given, we have the interesting result that the symmetric equilibrium defined in the last section is also unique. This result is proved below using two lemmas which are of interest on their own.

Lemma 1. *When product quality is given, each firm would locate itself in a way such that it faces equal demand from either side.*

This lemma draws upon the result that there exists a marginal consumer with an income level at which he is indifferent between the two adjacent firms, and, at the same time, is also indifferent between buying and not buying. Also, a firm does not compete with its adjacent firms for the consumers below this income level. So in case the income of this marginal consumer is not the same on both sides, the firm can do better by relocation to the extent that the marginal consumers on both sides face the same delivered price. The basic intuition for this result is drawn from Novshek (1980). The proof of this lemma is developed in Appendix A.2.

The symmetric equilibrium where each firm charges the same price and locates equidistant to each other is a simple follow-up of Lemma 1. This result is stated in Lemma 2 and proved in Appendix A.2.

Lemma 2. *When product quality is given, in equilibrium* $p_j = p$, *and* $|x_j - x_{j-1}| = |x_{j+1} - x_j| = \frac{L}{n}$, *for all* j.

The intuition for this result follows from Lemma 1. As the firm faces same demand from either side, it implies that the income of the marginal consumer from either side is the same. Observe that the firm shares its marginal consumer with the adjacent firm on either side. So the marginal consumer between firms j and $j-1$ and j and $j+1$ will have the same income. Extending this argument cyclically to all the n firms implies that the marginal consumers across all the n firms have the same income. This implies that, in the competitive range, by lowering price each firm can increase its market demand. It is the monopoly size of the demand which varies according to price for each firm. The monopoly part of the demand is independent of the interaction with the other firms. As all the firms are profit maximizers, they respond in a similar manner to the symmetric situation. Hence in the equilibrium all the firms will choose the same price, which together with lemma 1, implies that the distance between the firms is also the same.

A.2 Proofs of Lemma 1 and Lemma 2

Proof of Lemma 1

Suppose demand from either side is not the same. This implies that $\overline{Y}_{j,j+1} \neq \overline{Y}_{j,j-1}$. Without loss of generality assume that $\overline{Y}_{j,j-1} > \overline{Y}_{j,j+1}$. We argue below that firm j will be better-off by moving towards firm $j-1$.

Suppose firm j moves towards firm $j-1$ so that $\overline{Y}_{j,j-1}$ falls to $\overline{Y}'_{j,j-1}$ and $\overline{Y}_{j,j+1}$ increases to $\overline{Y}'_{j,j+1}$ in a way such that the following holds: $\overline{Y}_{j,j-1} > \overline{Y}'_{j,j-1} > \overline{Y}'_{j,j+1} > \overline{Y}_{j,j+1}$. Using these four different income levels we can break up the initial demand (that is, before the relocation) for firm j's product generated from the consumers located between x_j and x_{j-1} (denoted by $D_{j,j-1}$) into five components as

$$\begin{aligned} D_{j,j-1} &= \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j-1}-p_j)+t\left|x_j-x_{j-1}\right|}{2t}\right] dG\left(Y\right) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) \\ &+ \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) \\ &+ \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right). \end{aligned}$$

We can break up $D_{j,j+1}$ in the same way:

$$\begin{aligned} D_{j,j+1} &= \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG(Y) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG(Y) \\ &+ \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG(Y) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG(Y) \\ &+ \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y). \end{aligned}$$

With the relocation, firm j moving from x_j to x'_j, the demand changes to

$$\begin{aligned} D'_{j,j-1} &= \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j-1}-p_j)+t\left|x'_j-x_{j-1}\right|}{2t}\right] dG(Y) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j-1}-p_j)+t\left|x'_j-x_{j-1}\right|}{2t}\right] dG(Y) \\ &+ \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) \\ &+ \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y), \end{aligned}$$

and

$$\begin{aligned} D'_{j,j+1} &= \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG(Y) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG(Y) \\ &+ \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG(Y) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) \\ &+ \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y). \end{aligned}$$

Now let us compare the demand components before and after relocation.

First Component:

$$\int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j-1}-p_j)+t\left|x_j-x_{j-1}\right|}{2t}\right] dG\left(Y\right) + \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG\left(Y\right)$$

$$= \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j-1}-p_j)+t\left|x'_j-x_{j-1}\right|}{2t}\right] dG\left(Y\right) + \int_{\overline{Y}_{j,j-1}}^{Y_{\max}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG\left(Y\right).$$

Second Component:

$$\int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG\left(Y\right)$$

$$< \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j-1}-p_j)+t\left|x'_j-x_{j-1}\right|}{2t}\right] dG\left(Y\right) + \int_{\overline{Y}'_{j,j-1}}^{\overline{Y}_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG\left(Y\right).$$

Third Component:

$$\int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG\left(Y\right)$$

$$< \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}'_{j,j+1}}^{\overline{Y}'_{j,j-1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x'_j\right|}{2t}\right] dG\left(Y\right).$$

Fourth Component:

$$\int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{(p_{j+1}-p_j)+t\left|x_{j+1}-x_j\right|}{2t}\right] dG\left(Y\right)$$

$$< \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right) + \int_{\overline{Y}_{j,j+1}}^{\overline{Y}'_{j,j+1}} \left[\frac{Y\left(\theta-1\right)-p_j}{t}\right] dG\left(Y\right).$$

Fifth Component:

$$\begin{aligned} & \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) \\ = & \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y) + \int_{\underline{Y}_j}^{\overline{Y}_{j,j+1}} \left[\frac{Y(\theta-1)-p_j}{t}\right] dG(Y). \end{aligned}$$

It follows that firm j will be better-off by moving towards firm $j-1$.

Proof of Lemma 2

We have shown in Lemma 1 that firm j faces equal demand from either side implying that $\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1} \equiv \overline{Y}_j$, say. Using the expressions for $\overline{Y}_{j,j+1}$ and $\overline{Y}_{j,j-1}$ it follows from $\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1}$ that

$$\frac{(p_{j+1}+p_j)+t\,|x_{j+1}-x_j|}{2(\theta-1)} = \frac{(p_{j-1}+p_j)+t\,|x_{j-1}-x_j|}{2(\theta-1)}$$

$$\Rightarrow p_{j-1}+t\,|x_j-x_{j-1}| = p_{j+1}+t\,|x_{j+1}-x_j|. \tag{A.1}$$

Note, from their expressions derived in section 2.1, that $\overline{Y}_{j+1,j} = \overline{Y}_{j,j+1}$, and from this it follows that $\overline{Y}_j = \overline{Y}_{j+1}$. Similarly $\overline{Y}_{j-1,j} = \overline{Y}_{j,j-1}$ resulting in $\overline{Y}_j = \overline{Y}_{j-1}$. That is, we have $\overline{Y}_{j-1} = \overline{Y}_j = \overline{Y}_{j+1} \equiv \overline{Y}$, say. Now using equations (2.22), (2.23) and (A.1) the first-order condition for profit maximization with respect to price for firm j (equation (2.25)) reduces to

$$\begin{aligned} & 2\int_{\underline{Y}_j}^{\overline{Y}} [Y(\theta-1)-p_j]\,dG(Y) + \int_{\overline{Y}}^{Y_{\max}} [(p_{j+1}-p_j)+t\,|x_{j+1}-x_j|]\,dG(Y) \\ & \qquad = [p_j-c(\theta)]\left[2\int_{\underline{Y}_j}^{\overline{Y}} dG(Y) + \int_{\overline{Y}}^{Y_{\max}} dG(Y)\right]. \end{aligned}$$

It follows that

$$\begin{aligned} t\,|x_{j+1}-x_j| = & \frac{1}{\int_{\overline{Y}}^{Y_{\max}} dG(Y)} \left\{ [p_j-c(\theta)]\left[2\int_{\underline{Y}_j}^{\overline{Y}} dG(Y) + \int_{\overline{Y}}^{Y_{\max}} dG(Y)\right] \right. \\ & \left. -2\int_{\underline{Y}_j}^{\overline{Y}} [Y(\theta-1)-p_j]\,dG(Y) - \int_{\overline{Y}}^{Y_{\max}} (p_{j+1}-p_j)\,dG(Y) \right\}. \end{aligned}$$

Similarly it follows from the first-order condition for profit maximization with respect to price for firm $j-1$ that

$$\begin{aligned} t\left|x_j - x_{j-1}\right| &= \frac{1}{\int_{\overline{Y}}^{Y_{\max}} dG(Y)} \left\{ \left[p_{j-1} - c(\theta)\right] \left[2\int_{\underline{Y}_{j-1}}^{\overline{Y}} dG(Y) + \int_{\overline{Y}}^{Y_{\max}} dG(Y)\right] \right. \\ &\quad \left. -2\int_{\underline{Y}_{j-1}}^{\overline{Y}} \left[Y(\theta-1) - p_{j-1}\right] dG(Y) - \int_{\overline{Y}}^{Y_{\max}} (p_j - p_{j-1})\, dG(Y) \right\}. \end{aligned}$$

Upon substitution for $t\left|x_{j+1} - x_j\right|$ and $t\left|x_j - x_{j-1}\right|$, equation (A.1) reduces to a relationship involving only prices:

$$\begin{aligned} & p_{j-1} + \frac{1}{\int_{\overline{Y}}^{Y_{\max}} dG(Y)} \left\{ \left[p_{j-1} - c(\theta)\right] \left[2\int_{\underline{Y}_{j-1}}^{\overline{Y}} dG(Y) + \int_{\overline{Y}}^{Y_{\max}} dG(Y)\right] \right. \\ & \quad \left. -2\int_{\underline{Y}_{j-1}}^{\overline{Y}} \left[Y(\theta-1) - p_{j-1}\right] dG(Y) - \int_{\overline{Y}}^{Y_{\max}} (p_j - p_{j-1})\, dG(Y) \right\} \\ = \; & p_{j+1} + \frac{1}{\int_{\overline{Y}}^{Y_{\max}} dG(Y)} \left\{ \left[p_j - c(\theta)\right] \left[2\int_{\underline{Y}_j}^{\overline{Y}} dG(Y) + \int_{\overline{Y}}^{Y_{\max}} dG(Y)\right] \right. \\ & \quad \left. -2\int_{\underline{Y}_j}^{\overline{Y}} \left[Y(\theta-1) - p_j\right] dG(Y) - \int_{\overline{Y}}^{Y_{\max}} (p_{j+1} - p_j)\, dG(Y) \right\}. \end{aligned}$$

Similar substitutions for the n firms would imply that we will have n such relations of which $n-1$ are independent. To solve for the system of equations with n price variables we use another relationship: sum of the distances between firms is L, the length of the city. Since we have n independent equations involving n price variables appearing symmetrically, the solution is also symmetric, that is, $p_j = p$ for all j. [11] Upon substitution for equal prices in relationships like (A.1), it follows that the distances between the firms are also equal, that is, $\left|x_j - x_{j-1}\right| = \left|x_{j+1} - x_j\right| = \dfrac{L}{n}$, for all j.

[11] Uniqueness follows from the fact that it is a symmetric supermodular game (follows from Peitz, 1999, p. 34). Also see Amir(1996).

A.3 Income Inequality and Welfare of the Poor: Comparative Statics with Respect to f_P

Recall that

$$C_P = \frac{2L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c(\theta)\right]}\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right].$$

After some simplifications we derive

$$\frac{\partial C_P}{\partial f_P}\cdot\frac{1}{2L} = \frac{(1-f_P)\left[\frac{1}{2(1+f_P)}\sqrt{\frac{tF(\theta)}{1+f_P}}\right]}{(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c(\theta)\right]}$$

$$-\frac{\left[(3f_P^2+6f_P+7)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1)-c(\theta)\right]\right]\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right]}{2(1+f_P)\left[(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c(\theta)\right]\right]^2}.$$

Note that when $f_P = 1$,

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1}\cdot\frac{1}{2L} < 0, \text{ that is, } \left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1} < 0$$

since the first term vanishes and the second term is strictly negative under case (3) as $Y_P(\theta-1)-c(\theta) > \sqrt{\frac{tF(\theta)}{1+f_P}}$ and $\frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}} > Y_P(\theta-1)-c(\theta)$ implies $(3f_P^2+6f_P+7)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1)-c(\theta)\right] > (1-f_P)^2\sqrt{\frac{tF(\theta)}{1+f_P}} \geq 0.$

We argue next that when $f_P = 0$,

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0} > 0.$$

Note that

$$\begin{aligned}\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0}\cdot\frac{1}{2L} &= \frac{tF(\theta) - \left[7\sqrt{tF(\theta)} - 4\left[Y_P(\theta-1)-c(\theta)\right]\right]\left[Y_P(\theta-1)-c(\theta)-\sqrt{tF(\theta)}\right]}{2tF(\theta)} \\ &= \frac{4\left[Y_P(\theta-1)-c(\theta)\right]^2 + 8tF(\theta) - 11\left[Y_P(\theta-1)-c(\theta)\right]\sqrt{tF(\theta)}}{2tF(\theta)}.\end{aligned}$$

Denoting $A \equiv [Y_P(\theta-1)-c(\theta)]$ and $B \equiv \sqrt{tF(\theta)}$, the numerator of the above expression becomes $4A^2+8B^2-11AB$. Recall that under case (3) we have (when $f_P=0$) $B<A<\frac{3}{2}B$. Hence the problem boils down to determining the sign of $4A^2+8B^2-11AB$ when $B<A<\frac{3}{2}B$. Note that

$$\frac{\partial}{\partial A}\left[4A^2+8B^2-11AB\right]=8A-11B,$$

and

$$\frac{\partial^2}{\partial A^2}\left[4A^2+8B^2-11AB\right]=8>0.$$

That is, treating B as a parameter, $4A^2+8B^2-11AB$ is minimized at $A^*=\frac{11}{8}B$, and the minimum value is $\frac{7}{16}B^2>0$. Then we can conclude that $\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0}>0$.

Consider next the aggregate consumer surplus of the poor:

$$CS_P=\frac{L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}}-2f_P\left[Y_P(\theta-1)-c(\theta)\right]}\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right]^2.$$

We derive

$$\frac{\partial CS_P}{\partial f_P}\cdot\frac{1}{L}=\frac{(1-f_P)\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right]\left[\frac{1}{(1+f_P)}\sqrt{\frac{tF(\theta)}{1+f_P}}\right]}{(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}}-2f_P\left[Y_P(\theta-1)-c(\theta)\right]}$$

$$-\frac{\left[(3f_P^2+6f_P+7)\sqrt{\frac{tF(\theta)}{1+f_P}}-4(1+f_P)\left[Y_P(\theta-1)-c(\theta)\right]\right]\left[Y_P(\theta-1)-c(\theta)-\sqrt{\frac{tF(\theta)}{1+f_P}}\right]^2}{2(1+f_P)\left[(1+3f_P)\sqrt{\frac{tF(\theta)}{1+f_P}}-2f_P\left[Y_P(\theta-1)-c(\theta)\right]\right]^2}.$$

Again when $f_P=1$,

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1}\cdot\frac{1}{L}<0,\ \text{that is,}\ \left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1}<0$$

since the first term vanishes and the second term is strictly negative for the same logic given above.

When $f_P = 0$,

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} \cdot \frac{1}{L} = \left[Y_P(\theta-1) - c(\theta) - \sqrt{tF(\theta)}\right] - \frac{\left[7\sqrt{tF(\theta)} - 4\left[Y_P(\theta-1) - c(\theta)\right]\right]\left[Y_P(\theta-1) - c(\theta) - \sqrt{tF(\theta)}\right]^2}{2tF(\theta)}.$$

Using the same notation as above we rewrite

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} \cdot \frac{1}{L} = [A-B] - \frac{[7B-4A][A-B]^2}{2B^2} = \frac{(A-B)}{2B^2}\left[4A^2 + 9B^2 - 11AB\right] > 0$$

by the same logic given above. Hence we conclude that $\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} > 0$.

A.4 Details of the Different Cases

Case (1): $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$:

From the demand structure discussed in section 2.1 it follows that:

$$\begin{aligned} D_j &= f_R\left[\delta_{j,j+1}(Y_R) + \delta_{j,j-1}(Y_R)\right] + f_P\left[\delta_{j,j+1}(Y_P) + \delta_{j,j-1}(Y_P)\right] \\ &= (f_R + f_P)\left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + t\left|x_j - x_{j-1}\right| + t\left|x_{j+1} - x_j\right|}{2t}\right] \\ &= \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + t\left|x_j - x_{j-1}\right| + t\left|x_{j+1} - x_j\right|}{2t}\right]. \end{aligned}$$

This implies that $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. In stage 2, given entry and location decisions in the earlier stage, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = [p_j - c(\theta)] \cdot \frac{1}{t}. \tag{A.2}$$

In the symmetric equilibrium firms locate symmetrically in location stage, that is,

$|x_j - x_{j-1}| = |x_{j+1} - x_j| = \frac{L}{n}$, for all j. The implication for demand is

$$D_j = \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + 2t\left(\frac{L}{n}\right)}{2t}\right]. \tag{A.3}$$

Finally, firms' entry decision is determined by the zero-profit condition. Using (A.2) the expression for profit becomes

$$\pi_j = [p_j - c(\theta)]D_j - F(\theta) = [p_j - c(\theta)]^2 \cdot \frac{1}{t} - F(\theta),$$

so that the zero-profit condition implies

$$[p_j - c(\theta)]^2 \cdot \frac{1}{t} - F(\theta) = 0. \tag{A.4}$$

Using (A.2), (A.3) and (A.4) we derive the equilibrium price and number of firms:

$$p = c(\theta) + \sqrt{tF(\theta)},$$

$$\frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}.$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$, where the upper income threshold $\overline{Y}$ is given by

$$\overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)}.$$

Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ implies:

$$Y_P(\theta - 1) - c(\theta) > \frac{3}{2}\sqrt{tF(\theta)}.$$

Thus we conclude that case (1) arises when

$$Y_R > Y_P > \frac{c(\theta) + \frac{3}{2}\sqrt{tF(\theta)}}{(\theta - 1)}.$$

Case (2): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P = \overline{Y}$:

This is the case of a 'kinked equilibrium' as in Salop (1979). One extreme of the kink is case (1) described above where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. The other extreme is case (3) discussed in section 2.4.1 where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left[\frac{f_R + 2f_P}{t}\right]$.

Note that since $Y_P = \overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)}$, we have

$$p = Y_P(\theta - 1) - \frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c(\theta) + \sqrt{tF(\theta)}, \text{ and } \frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}.$$

Since $p = c(\theta) + \sqrt{tF(\theta)}$ and, at the same time, $p = Y_P(\theta - 1) - \frac{tL}{2n}$, this implies

$$\frac{L}{n} = \frac{2}{t}\left[Y_P(\theta - 1) - c(\theta)\right] - 2\sqrt{\frac{F(\theta)}{t}}.$$

But we have $\frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y_P(\theta - 1) - c(\theta) = \frac{3}{2}\sqrt{tF(\theta)}. \tag{A.5}$$

For the other extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\left[\frac{f_R + 2f_P}{t}\right]$, proceeding as in case (3) we have

$$p = c(\theta) + \sqrt{\frac{tF(\theta)}{1 + f_P}}, \text{ and } \frac{L}{n} = \frac{1}{t(1 - f_P)}\left[(1 + 3f_P)\sqrt{\frac{tF(\theta)}{1 + f_P}} - 2f_P\left(Y_P(\theta - 1) - c(\theta)\right)\right].$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y_P(\theta - 1) - c(\theta) = \frac{3 + f_P}{2}\sqrt{\frac{tF(\theta)}{1 + f_P}}. \tag{A.6}$$

Combining these two extremes it follows from (A.5) and (A.6) that case (2) arises when

$$\frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}} < Y_P(\theta-1) - c(\theta) < \frac{3}{2}\sqrt{tF(\theta)},$$

that is, when

$$\frac{c(\theta)+\frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta-1)} < Y_P < \frac{c(\theta)+\frac{3}{2}\sqrt{tF(\theta)}}{(\theta-1)}.$$

Case (4): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P < \underline{Y}$:

In this case the demand for firm j is given by

$$\begin{aligned} D_j &= f_R\left[\delta_{j,j+1}(Y_R) + \delta_{j,j-1}(Y_R)\right] \\ &= f_R\left[\frac{(p_{j-1}+p_{j+1}-2p_j) + t\left|x_j - x_{j-1}\right| + t\left|x_{j+1}-x_j\right|}{2t}\right]. \end{aligned}$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{f_R}{t}$. In stage 2, given entry and location decisions in the earlier stage, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = [p_j - c(\theta)]\cdot\frac{f_R}{t}. \tag{A.7}$$

In the symmetric equilibrium firms locate symmetrically in location, that is, $|x_j - x_{j-1}| = |x_{j+1} - x_j| = \dfrac{L}{n}$, for all j. The implication for demand is

$$D_j = f_R\left[\frac{(p_{j-1}+p_{j+1}-2p_j) + 2t\left(\dfrac{L}{n}\right)}{2t}\right]. \tag{A.8}$$

Finally, firms' entry decision is determined by the zero-profit condition. Using (A.7) the expression for profit becomes

$$\pi_j = [p_j - c(\theta)]D_j - F(\theta) = [p_j - c(\theta)]^2\cdot\frac{f_R}{t} - F(\theta),$$

so that the zero-profit condition implies

$$[p_j - c(\theta)]^2 \cdot \frac{f_R}{t} - F(\theta) = 0. \tag{A.9}$$

Using (A.7), (A.8) and (A.9) we derive the equilibrium price and number of firms:

$$p = c(\theta) + \sqrt{\frac{tF(\theta)}{1 - f_P}},$$

$$\frac{L}{n} = \sqrt{\frac{F(\theta)}{t(1 - f_P)}}.$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$, where the upper and lower income thresholds are given by

$$\overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)} \text{ and } \underline{Y} = \frac{p}{\theta - 1}.$$

Substituting the equilibrium values of price and number of firms we find that $Y_P < \underline{Y}$ implies:

$$Y_P < \frac{c(\theta) + \sqrt{\frac{tF(\theta)}{1 - f_P}}}{\theta - 1},$$

whereas $Y_R > \overline{Y}$ implies

$$Y_R > \frac{c(\theta) + \frac{3}{2}\sqrt{\frac{tF(\theta)}{1 - f_P}}}{(\theta - 1)}.$$

Case (5): $\boldsymbol{Y}_R = \overline{Y}$ and $\underline{Y} < \boldsymbol{Y}_P < \overline{Y}$:

Since $Y_R = \overline{Y}$, this exemplifies another case of 'kinked equilibrium'. One extreme of the kink is case (3) described above where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left[\frac{f_R + 2f_P}{t}\right]$. For the other extreme the demand from the rich is such that total demand is given by

$$\begin{aligned} D_j &= f_R\left[\eta_{j,j+1}\left(Y_R\right)+\eta_{j,j-1}\left(Y_R\right)\right]+f_P\left[\eta_{j,j+1}\left(Y_P\right)+\eta_{j,j-1}\left(Y_P\right)\right] \\ &= 2f_R\left[\frac{Y_R\left(\theta-1\right)-p_j}{t}\right]+2f_P\left[\frac{Y_P\left(\theta-1\right)-p_j}{t}\right], \end{aligned}$$

so that the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j}=-\left[\dfrac{2f_R+2f_P}{t}\right]=-\dfrac{2}{t}$.

Note that since $Y_R=\overline{Y}=\dfrac{p}{\theta-1}+\dfrac{tL}{2n(\theta-1)}$, we have

$$p=Y_R(\theta-1)-\frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j}=-\dfrac{1}{t}$, proceeding as in case (3) we can derive the equilibrium price and number of firms as

$$p=c\left(\theta\right)+\sqrt{\frac{tF\left(\theta\right)}{1+f_P}},\ \text{and}\ \frac{L}{n}=\frac{1}{t\left(1-f_P\right)}\left[\left(1+3f_P\right)\sqrt{\frac{tF\left(\theta\right)}{1+f_P}}-2f_P\left(Y_P\left(\theta-1\right)-c\left(\theta\right)\right)\right].$$

Since $p=c\left(\theta\right)+\sqrt{\dfrac{tF\left(\theta\right)}{1+f_P}}$ and, at the same time, $p=Y_R(\theta-1)-\dfrac{tL}{2n}$, this implies

$$\frac{L}{n}=\frac{2}{t}\left[Y_R\left(\theta-1\right)-c\left(\theta\right)\right]-\frac{2}{t}\sqrt{\frac{tF\left(\theta\right)}{1+f_P}}.$$

But we have $\dfrac{L}{n}=\dfrac{1}{t\left(1-f_P\right)}\left[\left(1+3f_P\right)\sqrt{\dfrac{tF\left(\theta\right)}{1+f_P}}-2f_P\left[Y_P\left(\theta-1\right)-c\left(\theta\right)\right]\right]$. It follows that this extreme case arises under the special circumstance when

$$\left[f_RY_R+f_PY_P\right]\left(\theta-1\right)-c\left(\theta\right)=\frac{3+f_P}{2}\sqrt{\frac{tF\left(\theta\right)}{1+f_P}}. \tag{A.10}$$

For the other extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j}=-\dfrac{2}{t}$, using the first-order condition, demand symmetry and the zero-profit condition we derive

$$p=c\left(\theta\right)+\sqrt{\frac{tF\left(\theta\right)}{2}},\ \text{and}\ \frac{L}{n}=\sqrt{\frac{2F\left(\theta\right)}{t}}+\frac{2f_P\left(Y_R-Y_P\right)\left(\theta-1\right)}{t}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$\left[f_RY_R+f_PY_P\right]\left(\theta-1\right)-c\left(\theta\right)=\sqrt{2tF\left(\theta\right)}. \tag{A.11}$$

At the same time, $Y_P > \underline{Y}$ implies, for this extreme case,

$$\frac{c(\theta) + \sqrt{\frac{tF(\theta)}{2}}}{(\theta - 1)} < Y_P. \tag{A.12}$$

Combining (A.10), (A.11) and (A.12) and the fact that the lower bound for Y_P is $\frac{c(\theta) + \sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta - 1)}$ under case (3) which is just the other extreme for case (5), we conclude that case (5) arises when

$$\frac{c(\theta) + \sqrt{\frac{tF(\theta)}{2}}}{(\theta - 1)} < Y_P < \frac{c(\theta) + \sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta - 1)}$$

and

$$\frac{c(\theta) + \sqrt{2tF(\theta)}}{(\theta - 1)} < f_R Y_R + f_P Y_P < \frac{c(\theta) + \frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta - 1)}.$$

A.5 Single-Income Economy

Consider a circular city where at each point of the city there is a measure 1 of consumers with the same income Y. Since there is only one income, we have the following three cases to consider:

(1) $Y > \overline{Y}$;

(2) $Y = \overline{Y}$;

(3) $\underline{Y} < Y < \overline{Y}$.

Case (1): $\boldsymbol{Y} > \overline{Y}$:

From the demand structure discussed in section 2.2.1 it follows that:

$$\begin{aligned} D_j &= \delta_{j,j+1}(Y) + \delta_{j,j-1}(Y) \\ &= \frac{(p_{j-1} + p_{j+1} - 2p_j) + t\,|x_j - x_{j-1}| + t\,|x_{j+1} - x_j|}{2t}. \end{aligned}$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. Now, similar to the analysis of the two-income groups, using the first-order condition, demand symmetry and the zero-profit condition we derive

$$p = c(\theta) + \sqrt{tF(\theta)},$$

$$\frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}.$$

Substituting these equilibrium values of price and number of firms into the expressions for $\overline{Y}$ we conclude that case (1) arises when

$$Y > \frac{c(\theta) + \frac{3}{2}\sqrt{tF(\theta)}}{(\theta - 1)}.$$

In this case firms compete for the consumers and all consumers are served.

Case (2): $\boldsymbol{Y} = \overline{Y}$:

This, once again, is a case of a 'kinked equilibrium'. One extreme of the kink is case (1) described above where the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. For the other extreme, demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta - 1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{2}{t}$.

Note that since $Y = \overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)}$, we have

$$p = Y(\theta - 1) - \frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c(\theta) + \sqrt{tF(\theta)}, \text{ and } \frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}.$$

Since $p = c(\theta) + \sqrt{tF(\theta)}$ and, at the same time, $p = Y(\theta - 1) - \frac{tL}{2n}$, this implies

$$\frac{L}{n} = \frac{2}{t}\left[Y(\theta - 1) - c(\theta)\right] - 2\sqrt{\frac{F(\theta)}{t}}.$$

But we have $\frac{L}{n} = \sqrt{\frac{F(\theta)}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y(\theta - 1) - c(\theta) = \frac{3}{2}\sqrt{tF(\theta)}. \tag{A.13}$$

For the other extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$, using the first-order condition, demand symmetry and the zero-profit condition we derive

$$p = c(\theta) + \sqrt{\frac{tF(\theta)}{2}}, \text{ and } \frac{L}{n} = \sqrt{\frac{2F(\theta)}{t}}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y(\theta - 1) - c(\theta) = \sqrt{2tF(\theta)}. \tag{A.14}$$

Combining (A.13) and (A.14) we conclude that case (2) arises when

$$\sqrt{2tF(\theta)} < Y(\theta - 1) - c(\theta) < \frac{3}{2}\sqrt{tF(\theta)}.$$

In this case also all the consumers are served, but the marginal consumer who is indifferent between two adjacent firms is also indifferent between buying and not buying.

Case (3): $\underline{Y} < \boldsymbol{Y} < \overline{Y}$ **:**

This is the second extreme of case (2) discussed above where demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta-1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$. As above using the first-order condition, demand symmetry and the zero-profit condition we derive

$$p = c(\theta) + \sqrt{\frac{tF(\theta)}{2}}, \text{ and } \frac{L}{n} = \sqrt{\frac{2F(\theta)}{t}}.$$

In the symmetric equilibrium $D_j = \frac{L}{n}$. Then $D_j = 2\left[\frac{Y(\theta-1) - p_j}{t}\right]$ and $p = c(\theta) + \sqrt{\frac{tF(\theta)}{2}}$ give

$$\frac{L}{n} = \frac{2}{t}\left[Y(\theta-1) - c(\theta) - \sqrt{\frac{tF(\theta)}{2}}\right].$$

Since $\frac{L}{n} = \sqrt{\frac{2F(\theta)}{t}}$, and, at the same time, $\frac{L}{n} = \frac{2}{t}\left[Y(\theta-1) - c(\theta) - \sqrt{\frac{tF(\theta)}{2}}\right]$, it follows that

$$Y(\theta-1) - c(\theta) = \sqrt{2tF(\theta)}.$$

So we conclude that case (3) can occur under this limiting case where $Y(\theta-1) - c(\theta) = \sqrt{2tF(\theta)}$. Following Salop (1979) we can ignore this limiting case.

- From the analysis of the three cases under the single-income economy it is clear that the minimum income required for any firm to operate is such that $Y(\theta-1) - c(\theta) = \sqrt{2tF(\theta)}$. That is, the feasibility income threshold is $\frac{c(\theta) + \sqrt{2tF(\theta)}}{(\theta-1)}$.

A.6 Sufficient Condition for the Positive Relationship between Quality and Income

Denoting the right-hand side of equation (2.13) as Y let us rewrite the equation as

$$c'(\theta) + \frac{F'(\theta)}{\sqrt{F(\theta)}} \cdot \sqrt{\frac{t}{1+f_P}} = Y.$$

Total differentiating both sides we derive

$$\frac{d\theta}{dY} = \frac{1}{c''(\theta) + \sqrt{\frac{t}{1+f_P}} \cdot \frac{1}{\sqrt{F(\theta)}} \left[F''(\theta) - \frac{(F'(\theta))^2}{2F(\theta)} \right]}.$$

It follows that the sufficient condition for $\frac{d\theta}{dY} > 0$ is

$$c''(\theta) \geq 0 \text{ and } F''(\theta) > \frac{(F'(\theta))^2}{2F(\theta)}.$$

A.7 Inverted-U Shaped Relationship between Number of Firms and Quality

From equation (2.7) we derive

$$\frac{\partial}{\partial\theta}\left(\frac{L}{n}\right) = \frac{1}{t(1-f_P)} \left[\frac{(1+3f_P)}{2} \sqrt{\frac{t}{1+f_P}} \cdot \frac{F'(\theta)}{\sqrt{F(\theta)}} + 2f_P c'(\theta) - 2f_P Y_P \right].$$

Assuming $\lim_{\theta\to 1} F'(\theta) = 0 = \lim_{\theta\to 1} c'(\theta)$, it is clear that $\lim_{\theta\to 1} \frac{\partial}{\partial\theta}\left(\frac{L}{n}\right) < 0$.[12] Also, $\frac{\partial}{\partial\theta}\left(\frac{L}{n}\right) = 0$ for the value of θ, call it θ^*, such that

$$\frac{(1+3f_P)}{2} \sqrt{\frac{t}{1+f_P}} \cdot \frac{F'(\theta^*)}{\sqrt{F(\theta^*)}} + 2f_P c'(\theta^*) - 2f_P Y_P = 0.$$

Since

$$\begin{aligned} \frac{\partial^2}{\partial\theta^2}\left(\frac{L}{n}\right) &= \frac{1}{t(1-f_P)} \left[(1+3f_P) \sqrt{\frac{t}{1+f_P}} \cdot \frac{1}{\sqrt{F(\theta)}} \left[F''(\theta) - \frac{(F'(\theta))^2}{2F(\theta)} \right] + 2f_P c''(\theta) \right] \\ &> 0 \text{ when } c''(\theta) \geq 0 \text{ and } F''(\theta) > \frac{(F'(\theta))^2}{2F(\theta)}, \end{aligned}$$

[12] Recall that the minimum quality level is $\theta = 1$.

it is clear that $\frac{L}{n}$ reaches a minimum at θ^{*}.

Now we can conclude that there is a U-shaped relationship between $\frac{L}{n}$ and θ implying an inverted-U shaped relationship between n and θ.

A.8 U-shaped Relationship between Upper Income Threshold and Quality

The upper income threshold is given by

$$\overline{Y}_P = \frac{c(\theta) + \frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}}}{(\theta-1)}.$$

Define $y(\theta) \equiv c(\theta) + \frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}}$. Clearly $y'(\theta) = c'(\theta) + \left(\frac{3+f_P}{4}\right)\sqrt{\frac{t}{1+f_P}}\frac{F'(\theta)}{\sqrt{F(\theta)}} > 0$ and $y''(\theta) = c''(\theta) + \left(\frac{3+f_P}{4}\right)\sqrt{\frac{t}{1+f_P}}\frac{1}{\sqrt{F(\theta)}}\left[F''(\theta) - \frac{(F'(\theta))^2}{2F(\theta)}\right] > 0$ since $c''(\theta) \geq 0$ and $F''(\theta) > \frac{(F'(\theta))^2}{2F(\theta)}$. Also, $y(1) > 0$ as long as $c(1) \geq 0$ and $F(1) > 0$. Hence $y(\theta)$ is a strictly increasing and strictly convex function with a strictly positive intercept. Since $\overline{Y}_P$ is the average of this function, $\overline{Y}_P = \frac{y(\theta)}{(\theta-1)}$, it follows that there is an U-shaped relationship between $\overline{Y}_P$ and θ.

A.9 Inverted-U Shaped Relationship between Consumer Surplus and Quality under Case (1)

From equation (2.21) we derive

$$\frac{\partial(CS_P)}{\partial\theta}\cdot\frac{1}{L} = Y_P - c'(\theta) - \frac{5}{8}\sqrt{t}\cdot\frac{F'(\theta)}{\sqrt{F(\theta)}}.$$

Assuming $\lim_{\theta\to 1} F'(\theta) = 0 = \lim_{\theta\to 1} c'(\theta)$, it is clear that $\lim_{\theta\to 1}\frac{\partial(CS_P)}{\partial\theta} > 0$. Also, $\frac{\partial(CS_P)}{\partial\theta}\cdot\frac{1}{L} = 0$ for the value of θ, call it θ_{W_P}, such that

$$Y_P - c'(\theta_{W_P}) - \frac{5}{8}\sqrt{t}\cdot\frac{F'(\theta_{W_P})}{\sqrt{F(\theta_{W_P})}} = 0.$$

Since

$$\begin{aligned}\frac{\partial^2 (CS_P)}{\partial \theta^2} \cdot \frac{1}{L} &= -c''(\theta) - \frac{5}{8}\sqrt{t} \cdot \frac{1}{\sqrt{F(\theta)}} \left[F''(\theta) - \frac{(F'(\theta))^2}{2F(\theta)} \right] \\ &< 0 \text{ when } c''(\theta) \geq 0 \text{ and } F''(\theta) > \frac{(F'(\theta))^2}{2F(\theta)},\end{aligned}$$

it is clear that CS_P reaches a maximum at θ_{W_P}.

Now we can conclude that there is an inverted-U shaped relationship between CS_P and θ.

Chapter 3

Income Inequality, Neighbourhood Effects and Product Quality

3.1 Introduction

One limitation of the second chapter is that we restricted ourselves to the symmetric equilibrium so that there exists just one quality in equilibrium. The analysis in the second chapter can be interpreted as depicting, in some sense, the average picture. Typically differences in household's incomes cause variations in the quality of products and services demanded. In this chapter we explore the possibility for the firms to offer different quality products.

With increase in income consumers value of the given quality goes up. Also wealthier individuals have preference for higher quality products. So firms may choose quality differentiation as a way to effectively reduce price competition and reach out to various sections of consumers. Some firms will concentrate on the high quality and price and hence depend on consumers with high income. The idea is to exploit higher willingness to pay of the rich consumers. Others will offer cheaper products of lower quality in order to cater to low income groups.[1] But product differentiation makes sense only if there is enough demand for differentiated products in the market.[2] In the absence of that the firms will offer same quality product as otherwise they will not be able to break even. So the absolute values of the incomes as well as the relative size of different income groups are important in determining firm's quality choice. Another key aspect is the difference in the fixed costs of production.[3] If the cost of providing high quality is too high relative to the perceived benefits then the firm will not be in a position to offer high quality and charge a higher price to effectively exploit product differentiation to its advantage.

As in the second chapter, here too consumers differ in their locations. This imposes another constraint on the firm's choice. This is because instead of traveling all the way to buy their most preferred quality product, consumers might go for the product that is accessible relatively easily. Presence of travel costs thus inhibits firm to effectively segregate the market with respect to income. So the firm needs to carefully weigh its options before deciding the quality. These trade-offs have important bearing on the market outcome in terms of quality offered and price being charged and hence on the welfare of the consumers.

In order to study these trade-offs and its implications in the simplest possible manner we simplify the model set-up in this chapter in the following way. The main simplification is that the number of firms is now fixed.[4] We consider a linear city where the rich and the poor live side by side and two duopolist firms are positioned at the

[1] Classic works by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) provide motivation for vertical product differentiation.

[2] See Yurko (2009).

[3] For instance, Ronnen (1991), Fajgelbaum et al.(2009), Liao (2008) allow the fixed cost to be quality dependent, where high quality product costs more than a low quality one.

[4] Allowing for free entry without assuming symmetry would have made the model intractable.

maximal distance from each other on the two ends of the city.[5] To focus on the competition over quality choice we assume that the locations of the two firms are fixed. Competition is modeled as a two-stage game. In the first stage firms simultaneously choose between two qualities, high and low. In the second stage the firms compete in prices. We identify conditions when there is a symmetric equilibrium with each firm offering the same quality or an asymmetric equilibrium with each firm offering different quality products. For the asymmetric equilibrium we distinguish between two scenarios: vertical dominance and horizontal dominance. Vertical dominance occurs when all the rich, irrespective of the distance, buy the high quality product whereas the poor buy the low quality product. This arises when the vertical attribute, that is, the difference in the incomes and quality of the products dominate the travel cost. On the other hand horizontal dominance occurs when the travel cost is high enough to discourage consumers to buy their most preferred quality products. Instead, they end up buying the product that is available in the close neighborhood.

It turns out that the prominent factors contributing to the firms' price and quality choices are income inequality, relative proportion of rich and poor, and the cost differential between the high and low quality products relative to the perceived benefit. We find that when the income of poor is too low then both firms ignore their presence and offer the same quality. Both firms offer the low quality when difference in the fixed costs of the high and low quality products is high relative to the income level in the society. On the other hand both firms offer high quality when income of the rich is sufficiently high compared to the difference in fixed costs. For the intermediate

[5] As there are only two firms, the assumption of a linear city structure instead of a circle is innocuous. As in Degryse (1996), one could have assumed that firms are located on the circumference of a circle of length 1 unit with locations being given exogenously at 0 and $\frac{1}{2}$ respectively, without changing the results.

level of the difference in fixed costs, there is an asymmetric equilibrium with one firm offering high quality and the other low quality. It follows that the welfare of the poor initially increases and then falls as there is income growth as a result of rise in income and proportion of the rich. Given the quite low income of the poor, if the rich income is also reasonably low, the firms offer only the low quality product catering to the rich and all the poor consumers are shut out of the market. For a relatively higher level of the rich income the possibility of asymmetric equilibrium emerges where poor are better-off as at least some of them can access the product that was earlier unavailable. But for a relatively high proportion of the rich, again the symmetric equilibrium prevails with both firms offering the high quality product and ignoring all the poor consumers in the process.

Similarly, when the income of the poor is relatively high, then again both firms offer low quality if the difference in the fixed cost is high. Or else both of them offer high quality when the income of rich is relatively high but the proportion of poor is low enough that product differentiation does not make sense. For the intermediate range of income gap firms choose to differentiate the market. Market differentiation allows alleviating price competition. The firm serving the rich is now able to charge a higher price to take advantage of the higher willingness to pay of the rich. Since the poor does not have a too high preference for high quality they opt for the low quality. This allows the low quality firm to charge a higher price taking advantage of his monopoly position with the poor. Poor are definitely worse-off on two accounts. First, they are priced out from buying the high quality product. In addition, they end up paying a high price for the low quality product. Thus the rise in income inequality has a spiraling negative effect on the welfare of the poor.

Our framework is close to Degryse (1996) who discusses the interaction between horizontal and vertical differentiation in determining a bank's choice whether to offer the remote access facility or not. The key difference with our work is that in order to emphasize on the market access and welfare of the poor, the possibility of non-consumption is an important aspect in our framework which he does not consider. The other works in the industrial organization literature implicitly assume that the market is either covered or uncovered. For example, Wauthy (1996) and Liao (2008) show that covered or uncovered markets are endogenous outcomes and depend on the degree of consumer heterogeneity. However, while Wauthy (1996) assumes that the costs of improving quality are zero, both papers consider only vertical product differentiation.

The chapter is organized as follows. In section 3.2 the basic framework is laid down. Section 3.3 discusses the price equilibrium in the second stage. Section 3.4 analyzes the quality choice in the first stage and the resulting equilibrium outcomes and welfare implications. Section 3.5 concludes by summarizing the main findings. The detailed proofs are developed in the Appendix.

3.2 Basic Model

In this section we outline the assumptions on preferences, technologies, market structure and income distribution of the economy we propose to study.

3.2.1 Consumer Preferences

Consumer are of two types: rich consumers with income Y_R and poor consumers with income Y_P, with $Y_R > Y_P$. At each point of the linear city of length L units, there are f_R proportion of rich and f_P proportion of poor, such that $f_R + f_P = 1$. Hence each individual is defined by his income level $Y \in \{Y_R, Y_P\}$ and location z on the linear city.[6] There are two firms located at the either end of the linear city offering quality θ, where $\theta \in \{\theta_H, \theta_L\}$ and $\theta_H > \theta_L$. Each consumer can buy a single unit of the product. Let $Y\theta$ be the gross utility of a consumer with income Y from consumption of the good/service of quality θ. In addition, there is a disutility from travel which enters linearly in the utility function. We denote the per-unit travel cost by t. Let p_j be the price charged and θ_j be the quality offered by firm j, $j = 1, 2$. Utility of the consumer located at the distance z from firm 1, with income Y and buying quality θ_j at price p_j is then given by

$$U(z, Y, p_j, \theta_j) = \begin{cases} Y\theta_1 - p_1 - tz & \text{if he buys from firm } 1, \\ Y\theta_2 - p_2 - t \mid L - z \mid & \text{if he buys from firm } 2, \\ Y & \text{if he does not buy.} \end{cases}$$

Y is the reservation utility of the consumer implying that $\theta_H > \theta_L > 1$.

The model incorporates the attributes of both horizontal as well as vertical differentiation with horizontal differentiation featured in the distance traveled and vertical differentiation in quality choice. We take distance literally to imply the physical distance traveled by the consumer. It is apparent from the utility function that the total price paid by the consumers (which includes the transportation cost) differs from the net market price received by the producer. Because of this difference between the actual price and delivered price there might be consumers even at the same income level who are left out of the market.

3.2.2 Firms

As mentioned above, there are two firms located at the either end of the linear city. We assume a two stage game between the firms. Investment in quality is made in

[6]That is, this is not a model of location choice by the consumers.

the first stage which can be either θ_L or θ_H. In the second stage firms simultaneously decide the price. Each firm faces two types of cost: a fixed cost denoted by $F(\theta)$ and a marginal cost given by $c(\theta)$. We assume that marginal cost of production is independent of the output level, but both fixed and marginal costs increase with the improvement in quality, that is, $F(\theta_H) > F(\theta_L)$ and $c(\theta_H) > c(\theta_L)$.

We assume that firms do not price discriminate between consumers and charge them the same price irrespective of their incomes and locations. But there is implicit price discrimination arising out of the differences in locations of individuals on the linear city. This difference in the actual cost borne by an individual has an implication on the number of consumers who finally buy the product.

After having chosen the quality in the first stage firms simultaneously choose the price in the second stage. Profit of firm j charging a price p_j and offering quality θ_j is given by

$$\pi_j = [p_j - c(\theta_j)]D_j - F(\theta_j),$$

where D_j is the demand faced by firm j and depends on its own strategic choices of price and quality and also on the strategic choices of the other firm. As mentioned above, because of the spatial aspect, there might be people who are left unserved owing to the greater distance.

3.3 Price Equilibrium

Given the first stage equilibrium of quality choice, there are several possible subgames: (θ_H, θ_H) - both firms offer high quality; (θ_L, θ_L) - both firms offer low quality; (θ_H, θ_L) - one firm offers high quality and the other offers low quality. In each of this subgame, given the quality choices, firms simultaneously compete in prices.

3.3.1 Same Quality by Both Firms

In this section we analyze a subgame where both firms offer the same quality which can be either be θ_H or θ_L. Given the income level of poor, this may imply either full market coverage where all poor are served, or partial market coverage where some poor are left unserved. The following two sub-sections characterize equilibrium under the full and partial market coverage respectively.

Full Market Coverage

Throughout the paper we assume that the rich income is high enough so that all rich consumers are served. In this subsection we establish conditions for an equilibrium where there is full market coverage, that is, all poor consumers are served. So each firm competes for each income type for their demand. Demand faced by each firm is determined by the distance of the marginal consumer who is indifferent between the two firms. Let the marginal consumer with income Y who is indifferent between firm 1 and firm 2, with both firms offering the same quality θ, be located at a distance z from firm 1. Then

$$U(z, Y, p_1, \theta) = U(z, Y, p_2, \theta) \quad \Rightarrow \quad z = \frac{p_2 - p_1 + tL}{2t}.$$

As all the rich and poor are being served, this would imply that demand faced by firm 1, is $D_1(.) \;=\; (f_R + f_P)\left[\dfrac{p_2 - p_1 + tL}{2t}\right]$. Profit for firm 1 is then given by

$$\pi_1 \;=\; (f_R + f_P)[p_1 - c(\theta)]\left[\frac{p_2 - p_1 + tL}{2t}\right] - F(\theta).$$

In stage 2, given the quality decision in the earlier stage, firms choose its price to maximize profit, π. For the firm 1 the first-order condition for the profit maximization with respect to price implies $D_1(.) \;=\; \dfrac{(f_R + f_P)[p_1 - c(\theta)]}{2t}$. After substitution and simplification this reduces to $p_2 \ = \ 2p_1 - c(\theta) - tL$. Similar exercise for firm 2 would imply that $p_1 \ = \ 2p_2 - c(\theta) - tL$. From the above two equations it follows that

$$p_1 \ = \ p_2 \ = tL + c(\theta).$$

So there is a unique symmetric equilibrium where both firms charge the same price given that in the initial stage they offer the same quality. Unlike the Betrand competition[7] firms are able to charge above the marginal cost because of horizontal differentiation. Using the first-order condition with respect to price the expression for profit reduces to

$$\pi_i = (f_R + f_P)[p_i - c(\theta)]D_i(.) - F(\theta) = (f_R + f_P)[p_i - c(\theta)]^2 \cdot \frac{1}{2t} - F(\theta), \ i = 1, 2.$$

On substituting for p in equilibrium firm's profit is given by

$$\pi_i = \frac{(f_R + f_P)[tL]^2}{2t} - F(\theta), \ i = 1, 2.$$

Clearly profits in the equilibrium where both the firms produce low quality is higher than the case where both of them produce high quality. This is because even though the mark-up over the marginal cost is the same because of competition and symmetric equilibrium but there is difference in the fixed cost of quality. As both income types are served, the density of rich and poor have equal weightage in determination of firm's profit. Higher the travel cost higher is the profit level for the firm as this raises the extent of horizontal differentiation between the two firms.

The cut-off level of Y_P which ensures full market coverage is determined as follows. As all the poor are buying, it implies that the marginal poor indifferent between the two firms is better off buying the product. Let $\overline{Y}$ be the income level at which the consumer who is indifferent between the two firms, is also indifferent between buying and not buying. We have derived above that the distance from firm 1 at which the

[7] See Tirole (1988), p .209 and Vives (1999), p .117.

consumer with income $\overline{Y}$ is indifferent between the two firms is $\frac{p_2 - p_1 + tL}{2t}$. Since at this distance the consumer with income $\overline{Y}$ is also indifferent in buying and not buying, it follows that $\overline{Y}\theta - p_1 - t\left(\frac{p_2 - p_1 + tL}{2t}\right) = \overline{Y}$, that is , $\overline{Y} = \frac{p_1 + p_2 + tL}{2(\theta - 1)}$. As all poor are buying, it follows that $Y_R > Y_P > \overline{Y}$. On substituting the equilibrium value of p the above inequality implies

$$Y_R \; > \; Y_P \; > \; \overline{Y} \; = \; \frac{1}{\theta - 1}\left[c(\theta) + \frac{3tL}{2}\right]$$

As is apparent from above, the cut-off is lower, lower is the level of the marginal cost as well as the travel cost. Marginal and travel costs are the prices an individual has to pay to buy the product. Higher the price, less will be the market coverage. Also observe that an increase in θ without any increase in marginal cost unambiguously reduces the cut-off. Intuitive way to understand this is to think of θ as the individuals valuation of the product. A rise in just individual's valuation without any corresponding rise in the marginal cost induces an individual to participate.[8] This is the direct artifact of the particular form of utility function that we have assumed. It is interesting to see how the cut-off level varies with a change in θ. With the increase in quality, consumers gross utility increases; but now he also has to pay a higher price. Which affect dominates depends on curvature of the marginal cost curve. This is clear from the following

$$\frac{\partial \overline{Y}}{\partial \theta} = \frac{1}{(\theta - 1)}\left[c^{'}(\theta) - \frac{c(\theta)}{(\theta - 1)} - \frac{3tL}{2(\theta - 1)}\right]$$

[8] WaterAid-India's rural sanitation program was making slow progress in 1995-96. A lack of demand from households meant that partner NGOs had constructed only 460 out of 1,100 latrines planned for the 12-month period. WaterAid-India decided that it was time to reformulate its strategy and focus on marketing sanitation. As a result of this change in approach, by the first six months of 1997-98, partner NGOs had achieved a dramatic turnaround in demand and constructed 5,000 latrines. For more on the role on information see Jalan and Somanathan (2008) and Banerjee et al. (2008).

When the marginal cost is linear in θ, $\dfrac{\partial Y}{\partial \theta}$ is strictly negative, implying that the valuation effect outweighs the cost effect. For a convex cost, it is initially negative, but for higher level of θ it might be positive. So depending on the parameter values, there might be an inverted U-shape relation between the cut-off level of income and quality. It is pertinent to observe that both the cut-off level of income and the equilibrium price level are insensitive to the income distribution. This is because once the income level of poor is high enough, the firms do not care for the income gap owing to the competitive pressure. Thus both rich and poor are treated symmetrically and their relative disparity does not matter. Above results can be summarized in the following proposition.

Proposition 1: *In a subgame where both firms offer the same quality there exists a unique equilibrium with full market coverage iff* $Y_P(\theta - 1) \geq \left[c(\theta) + \dfrac{3tL}{2}\right]$. *The equilibrium is characterized by the following properties.*

1. *All poor and rich are served.*

2. *Both firms charge the same price,* $p_1 = p_2 = tL + c(\theta) \equiv p_C$.

3. *Market price increases with increase in* t *and* $c(\theta)$*, but does not depend on* Y_P, Y_R, f_R *or* f_P.

4. *In equilibrium each, firms profit is given by* $\pi = \dfrac{(f_R + f_P)[tL]^2}{2t} - F(\theta)$.

We would like to look at the impact of income gap on consumer surplus. But as observed above, equilibrium price is independent of the income gap or the relative proportion of poor and rich, implying that, in the case of full market coverage, relative income gap is immaterial. The net surplus to a consumer with income Y_i and located at the distance x from the firm from which he is buying is $Y_i\theta - p - tx - Y_i$, where

$Y_i \in \{Y_R, Y_P\}$. Recall that, the reservation utility is given by his income level, Y_i. Since there are 2 firms each with a market coverage of $\frac{L}{2}$, the aggregate consumer surplus CS_i of the individuals with income Y_i and proportion δ_i, where $\delta_i \in \{f_R, f_P\}$ is

$$CS_i \;=\; 2\delta_i \int_0^{\frac{L}{2}} \left[Y_i\left(\theta-1\right)-p-tx\right]dx = \delta_i L\left[Y_i\left(\theta-1\right)-p-\frac{tL}{4}\right].$$

As expected, consumer surplus increases with income Y_i, and individual's valuation for the product given by θ, and decreases with travel cost and price. Since price is endogenous, substituting the equilibrium value of p this reduces to

$$CS \;=\; \delta_i L\left[Y_i\left(\theta-1\right)-\frac{5tL}{4}-c(\theta)\right].$$

It is apparent that the welfare of rich is higher than poor by virtue of their higher income and the relative gap in the income level does not affect welfare. Again how does the welfare of the consumer changes with change in θ depends on the curvature of the cost curve and the parameter values. This leads us to the following proposition.

Proposition 2: *Let* $Y_P(\theta-1) \;\geq\; \left[c(\theta)+\frac{3tL}{2}\right]$, *aggregate consumer surplus of consumers falls in* t *and* $c(\theta)$, *increases in their own income and proportion but is independent of the income gap.*

This highlights the case when income gap is not substantial, and the competitive force undermines firm's market power.

Partial Market Coverage

Next we consider the case where not the entire market is served: some poor consumers are left unserved owing to the greater distance from the firms. In this case the marginal poor consumer indifferent between the two firms prefers to go without buying the product. So each firm has some monopoly power over the poor since it does not compete with the other firm for the poor. In what follows we consider the situation where all rich consumers are served, some poor located closer to the firms are also served while other poor consumers are left out. It is here where the distinction between the travel cost faced by each individual becomes pronounced.

Demand faced by each firm from the rich is derived exactly the same way as above, that is, demand from the rich is given by the distance of the marginal rich indifferent between the two firms: $\frac{p_2 - p_1 + tL}{2t}$. But now each firm's demand from the poor is different: it is given by the distance from the firm where a poor becomes indifferent between buying and not buying. The distance of this indifferent poor consumer, d_p, is determined from $Y_P\theta - p_1 - td_p = Y_P$, that is, $d_p = \frac{Y_P(\theta - 1) - p_1}{t}$. So the total demand faced by firm 1 is $D_1(.) = \frac{f_R[p_2 - p_1 + tL]}{2t} + \frac{f_P[Y_P(\theta - 1) - p_1]}{t}$. Given this demand, firm $1's$ profit is

$$\pi_1 = [p_1 - c(\theta)]\left[\frac{f_R[p_2 - p_1 + tL]}{2t} + \frac{f_P[Y_P(\theta - 1) - p_1]}{t}\right] - F(\theta).$$

The first-order condition for the profit maximization with respect to price implies

$$D_1(.) = \frac{f_R + 2f_P}{2t}[p_1 - c(\theta)].$$

After substitution and simplification this reduces to

$$p_1 = \frac{1}{2(f_R + 2f_P)}[f_R(p_2 + tL) + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)].$$

Similar condition for firm 2 would imply a symmetric equilibrium with both firms charging the same price given by

$$p = \frac{1}{f_R + 4f_P}[f_R tL + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)].$$

Price is independent of the income of rich, as firms are competing for them. But, since the firms have some monopoly power over the poor, equilibrium price increases with poor's income as the firms exploit their higher willingness to pay. This implies, that there could be different prices depending on income of the poor, inspite of the fact that the quality being offered is the same. Also price increases with increase in f_R but falls with rise in f_P. As f_R increases firm's demand goes up causing price level to rise, whereas price level falls with increase in f_P. With the rise in the proportion of poor, there are two opposing forces at work. Even though demand increases but, at the same time, population of poor being left out of the market also rises. It is the latter effect which prevails over the former and hence brings down the price level. This provides an interesting insight arising from the spatial nature of the model.

Using the first-order condition with respect to price the expression for profit reduces to

$$\pi_i = (f_R + 2f_P)[p_i - c(\theta)]D_i(.) - F(\theta) = (f_R + 2f_P)[p_i - c(\theta)]^2 \cdot \frac{1}{2t} - F(\theta),\ i = 1, 2.$$

On substituting for p in equilibrium firm's profit is given by

$$\pi_i = \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL\ -\ 2f_P c(\theta) + 2f_P Y_P(\theta - 1)}{f_R + 4f_P}\right]^2 - F(\theta),\ i = 1, 2.$$

Observe that, unlike the case of full market coverage, proportions of rich and poor does not enter symmetrically in the firm's profit expression. Profit is more sensitive to the proportion of poor and also income of poor reflecting the fact that it is the poor whose coverage is partial.

In what follows we investigate the parameter values for which the above case arises. Recall that this case arises when all rich are being served, but some poor, depending on their distances from the firm, are left unserved. As above $\overline{Y} = \dfrac{p_1 + p_2 + tL}{2(\theta - 1)}$. Similarly for $j = 1, 2,$ define $\underline{Y_j}$ to be the level of income such that the consumer even at the location of firm j is indifferent between buying and not buying, that is, $\underline{Y_j}(\theta - 1) = p_j$, implying $\underline{Y_j} = \dfrac{p_j}{\theta - 1}$. Clearly this above case occurs when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$. Substituting the equilibrium values of price into the expression for $\overline{Y}$

and $\underline{Y}$ we find that $\underline{Y} < Y_P < \overline{Y}$ implies

$$\frac{f_R tL}{f_R + 2f_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3f_R + 4f_P)tL}{2f_R + 4f_P} + c(\theta).$$

It is easy to check that both the cut-offs are increasing in f_R and decreasing in f_P. Increase in the lower bound with increase in f_R simply implies that as the proportion of rich increases, some poor will be served only if Y_P is high enough. On the other end increase in the upper bound with increase in f_R signifies that for all poor to be served Y_P should increase. The two together imply that increase in the proportion of the rich makes it less likely for all poor to be served. The intuition for this is straight forward. With increase in f_R, firms demand increases implying that price level increases which raises the cost of consumption for poor. The opposite holds for the increase in f_P.

Similarly, $Y_R > \overline{Y}$ implies

$$\frac{[2f_R Y_R + 4f_P(2Y_R - Y_P)](\theta - 1)}{2f_R + 4f_P} > \frac{(3f_R + 4f_P)tL}{2f_R + 4f_P} + c(\theta).$$

This implies that the partial market coverage is a possibility when the income gap is relatively higher. The following proposition, summarizes the above discussion.

Proposition 3: *A unique equilibrium in a subgame where both firms offer same quality, with the partial market coverage exists iff*

$$\frac{f_R tL}{f_R + 2f_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3f_R + 4f_P)tL}{2f_R + 4f_P} + c(\theta)$$

and

$$\frac{[2f_R Y_R + 4f_P(2Y_R - Y_P)](\theta - 1)}{2f_R + 4f_P} > \frac{(3f_R + 4f_P)tL}{2f_R + 4f_P} + c(\theta)$$

and is characterized by the following properties.

1. *Poor at relatively higher distance from the firm are left unserved.*

2. *Both firms charge the same price,* $p_1 = p_2 = \dfrac{1}{f_R + 4f_P}[f_R tL + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)] \equiv p_M$.

3. p_M *increases with increase in* Y_P *and* f_R *but falls with increase in* f_P.

4. *Profit for each firm is given by* $\pi = \dfrac{f_R + 2f_P}{2t}\left[\dfrac{f_R tL \ - \ 2f_P c(\theta) + 2f_P Y_P(\theta - 1)}{f_R + 4f_P}\right]^2 - F(\theta)$.

5. $p_C \ > \ p_M$, *that is price in the case of full market coverage is higher than the case of partial market coverage.*

As discussed above, this case arises when some poor are left unserved. It has been proved in Appendix A.1.1 and A.1.2 that p_M increases with increase in f_R but falls with rise in f_P. This has a direct implication on the welfare of rich. Rich are better-off staying in relatively poor neighborhood. Also, it warrants a mention that the price level under the partial market coverage is lower than the price under full market coverage. This is because price is sensitive to income of the poor, which is relatively low in the case of partial market coverage. This has been formally proved in Appendix A.1.3.

As above, the net surplus of the rich consumer located at a distance x from the firm from which it buys is given by $Y_R\theta - p - tx - Y_R$. As all rich are being served, the consumer surplus of rich is given by

$$CS_R \ = \ 2f_R \int_0^{\frac{L}{2}} \left[Y_R\left(\theta - 1\right) - p - tx\right] dx = f_R L \left[Y_R\left(\theta - 1\right) - p - \frac{tL}{4}\right].$$

As expected, consumer surplus increases with income Y_R and falls with the rise in price and travel cost. Because p falls with rise in f_P and fall in Y_P, this implies that rich are better-off in a relatively poor neighborhood when quality level is fixed. Rise in the welfare with increase in Y_R and f_R is obvious.

Finally consider the aggregate consumer surplus of the poor. Since the poor in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{L}{2}$ does not buy the product from any firm, their consumer surplus is zero. Hence the aggregate consumer surplus of the poor is

$$CS_P = 2f_P \left[\int_0^{\frac{Y_P(\theta-1)-p}{t}} [Y_P(\theta-1)-p-tx]\,dx\right] = \frac{f_P\,[Y_P(\theta-1)-p]^2}{t}.$$

It is apparent from above that welfare of the poor is negatively related to the price level; which falls with the increase in the proportion of poor and rises with the increase in the proportion of rich. On substituting the equilibrium value of p welfare of the poor is given by

$$CS_P = \frac{f_P\,[(f_R+2f_P)[Y_P(\theta-1)-c(\theta)]-f_R tL]^2}{t(f_R+4f_P)^2}.$$

Note that even though p also rises with the rise in Y_P but the income effect dominates the price effect. This results in the increase in the welfare of the poor with rise in Y_P. This leads us to the following proposition.

Proposition 4: *For the case of partial market coverage, with both firms offering the same quality, welfare of rich consumers falls with increase in Y_P but rises with increase in f_P, whereas the welfare of poor consumers rises with increase in both the income level Y_P and f_P but falls with rise in f_R.*

No Poor Being Served

At the other extreme is the scenario where no poor is served. This holds when Y_P is so low that firm does not find it worthwhile to serve them. On the other hand, all the rich consumers are served and firms are competing for them. In this case, firm $1's$ demand is given by

$$D_1(.) = \frac{f_R[p_2-p_1+tL]}{2t}.$$

Working exactly the same way as in the case of full market coverage, this implies that in equilibrium price is

$$p = tL + c(\theta).$$

This case arises when $Y_R > \overline{Y} = \frac{1}{\theta - 1}\left[c(\theta) + \frac{3tL}{2}\right]$, and $\underline{Y} = \frac{p}{\theta - 1} > Y_P$ implying that $Y_P < \frac{tL + c(\theta)}{\theta - 1}$. Above result can be summarized in the following proposition.

Proposition 5: *In a subgame where both firms offer the same quality, there exists a unique equilibrium with no poor being served iff* $Y_P(\theta - 1) < tL + c(\theta)$ *and* $Y_R > \overline{Y} = \frac{1}{\theta - 1}\left[c(\theta) + \frac{3tL}{2}\right]$. *The equilibrium characterized by the following properties.*

1. *No poor is served.*

2. *Both firms charge the same price,* $p_1 = p_2 = tL + c(\theta) \equiv p_C$.

3. *Market price increases with increase in* t *and* $c(\theta)$, *but does not depend on* Y_P, Y_R, f_R *or* f_P.

4. *Each firm's profit is given by* $\pi = \frac{f_R[tL]^2}{2t} - F(\theta)$.

It merits a mention that in this case the equilibrium price is the same as the first case of full market coverage. As income of poor is low, the firms completely ignore their presence and cater only to the rich. As a result the market size of each firm is smaller, affecting firm's profit adversely.

3.3.2 High quality by One Firm and Low by the Other

In this section we consider a subgame where the duopolistic firms operate with different quality levels so that one firm offers high quality, θ_H, and the other low quality,

θ_L. Depending on the relative dominance of either travel cost (horizontal attribute) or income gap (vertical attribute) this may lead to the following subcases.

Vertical Dominance

Vertical Dominance arises when there is complete market segregation. A rich consumer, even at the location of the firm producing low quality, has a preference for high quality over low quality, that is, $Y_R\theta_H - p_H - tL \; > \; Y_R\theta_L - p_L$. Similarly, a poor consumer at the location of the firm offering high quality prefers low quality over high quality, that is, $Y_P\theta_L - p_L - tL \; > \; Y_P\theta_H - p_H$. Combining these two inequalities we get

$$(Y_R - Y_P)(\theta_H - \theta_L) \; > \; 2tL.$$

As is apparent from the equation, this case arises when the income and quality difference, that is, the vertical attribute, outweighs the travel cost, the horizontal attribute. We call this the *vertical dominance*. So the two forces, quality and income differences reinforce each other leading to this outcome. The rationale is that income and quality gaps are so high that rich are willing to travel all the way to access the high quality product, whereas poor, even at the location of high quality producing firm, find it beyond their means. Similarly, given the income and the quality gaps, each firm finds it more profitable to serve either type exclusively.

To investigate further, we determine the demand faced by each firm. As above, depending on the income of the poor, there can be either full market coverage or partial market coverage where some poor are left out. The two cases are discussed below.

Partial Market Coverage

We first analyze the case when some poor consumers are left unserved. Recall that the poor even at the location of the firm offering high quality prefers low quality over high quality. What follows is that only the poor located closer to the firm offering low quality are served, others are left out. Demand from the poor is given by the distance at which the marginal poor is indifferent between buying and not buying. As derived in the context of partial market coverage when both firms were offering same quality, this distance is $\frac{Y_P(\theta_L - 1) - p_L}{t}$, where p_L denotes the price charged by the firm offering low quality. Thus the firm has some monopoly power over the poor. Since no rich buys the low quality product, total demand faced by the firm offering low quality is $D_L(.) = \frac{f_P[Y_P(\theta_L - 1) - p_L]}{t}$. Hence the profit of the firm offering low quality is

$$\pi_L = [p_L - c(\theta_L)]\frac{f_P[Y_P(\theta_L - 1) - p_L]}{t} - F(\theta_L).$$

The first-order condition for profit maximization with respect to price implies that $D_L(.) = \frac{f_P[p_L - c(\theta_L)]}{t}$. After simplification it follows that in equilibrium price charged by the firm offering low quality is

$$p_L = \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2}.$$

As the firm has some monopoly power over the poor, price charged increases in the income of the poor. Price also increases with increase in θ_L. On substituting for p_L, implied profit of the firm is

$$\pi_L = \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L).$$

There is partial market coverage when $L > \frac{Y_P(\theta_L - 1) - p_L}{t}$, that is, when the poor at the other end of the city is not willing to buy. Substituting for the equilibrium price, it follows that not all poor consumers will be served when $Y_P(\theta_L - 1) < c(\theta_L) + 2tL$. But there are some poor, located relatively closer to the firm offering low quality, who are willing to buy. Specifically, the poor at the location of the firm is better-off buying, which implies that $Y_P\theta_L - p_L > Y_P$. On substituting for p_L, the condition reduces to $Y_P > \dfrac{c(\theta_L)}{\theta_L - 1}$. Putting the two inequalities together, we get that there is partial market coverage when

$$\frac{2tL + c(\theta_L)}{\theta_L - 1} \; > \; Y_P \; > \; \frac{c(\theta_L)}{\theta_L - 1}.$$

Similarly we can evaluate the demand for the firm offering high quality. We assume that Y_R is high enough that all rich consumers are buying. For the case of vertical dominance rich buys only the high quality product. So the demand faced by the firm offering high quality is $f_R L$. It follows that the profit of the firm is

$$\pi_H = [p_H - c(\theta_H)]f_R L - F(\theta_H).$$

Pricing strategy of the firm is the following: the firm sets its price such that the marginal rich consumer, that is, the rich consumer at the location of the firm offering low quality, is indifferent between buying high quality and low quality products, that is $Y_R\theta_H - p_H - tL = Y_R\theta_L - p_L$. This implies that $p_H \; = \; Y_R(\theta_H - \theta_L) + p_L - tL$. This allows the firm to extract the maximum possible surplus from the consumers. The pricing strategy above is plausible as the competition between the firms has been relaxed for two reasons. First, the two firms now offer two distinct qualities. Secondly, as the income gap is substantial, there is a market segregation. Price and profit level for the firm offering high quality is obtained by a simple substitution for p_L. So the price level of the firm offering high quality is given by

$$p_H = Y_R(\theta_H - \theta_L) \; + \; \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL.$$

It is easy to check that p_H increases with increase in Y_R and θ_H. With the rise in income and quality consumer's willingness to pay increases and this results in the higher price. With rise in θ_L, there are three forces at work. The first is the valuation effect which increases p_L. The second is the cost effect. These two together put an upward pressure on p_H. Finally, there is the competition effect: as the quality differentiation between the firms falls, competition increases. This lowers p_H. Which effect dominates depends of the curvature of the cost curve. Similarly, an increase in Y_P has positive spill over effect on p_H. As poor constitutes captive market of the firm offering low quality so an increase in Y_P results in rise in p_L. Because of the higher p_L, the high quality firm can charge a higher price that makes the marginal rich indifferent between the two quality products. The fact that p_H falls with increase in the travel cost may appear counterintuitive. But higher the travel cost lower the surplus that the firm can take away from the rich consumer to make him indifferent between the two firms. Therefore the price falls. Finally the high quality firm's profit is given by

$$\pi_H = \left[Y_R(\theta_H - \theta_L) \ + \ \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H) \right] f_R L - F(\theta_H).$$

The equilibrium above has been worked out assuming that all rich are served and at least some poor are buying. This implies that Y_R has to be high enough so that the rich even at the location of the firm offering low quality is willing to buy, that is, $Y_R\theta_H - p_H - tL \ > \ Y_R$. After substituting for p_H, the condition reduces to: $[2Y_R - Y_P](\theta_L - 1) \ > \ c(\theta_L)$. Above results are summarized in the following proposition.

Proposition 6: *An equilibrium with vertical dominance exists when* $(Y_R - Y_P)(\theta_H - \theta_L) \ > \ 2tL$. *When* $\dfrac{2tL + c(\theta_L)}{\theta_L - 1} \ > \ Y_P \ > \ \dfrac{c(\theta_L)}{\theta_L - 1}$ *and* $[2Y_R - Y_P](\theta_L - 1) \ > \ c(\theta_L)$ *there is partial market coverage. For this equilibrium we have*

1. $p_H = Y_R(\theta_H - \theta_L) \ + \ \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL,$

2. $p_L = \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2},$

3. $\pi_L(.) = \dfrac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L),$

4. $\pi_H(.) = \left[Y_R(\theta_H - \theta_L) \ + \ \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - F(\theta_H).$

5. *p_H increase with the increase in Y_R, Y_P and θ_H. p_L increases with increase in θ_L and Y_P and is independent of Y_R. Both prices are unaffected by the change in the relative proportions.*

For this to qualify as an equilibrium, neither of the firms should have any incentive to deviate. Conditions for this have been laid down in Appendix A.2. It is shown that the proposed price strategy will qualify to be an equilibrium only when the income gap is substantial. This is intuitive: only for the relatively larger income gap each producer can benefit by serving the segregated market.

To calculate the net consumer surplus, we proceed as in the previous sections. In the case of vertical dominance there is complete market segregation. So the net consumer surplus of a rich consumer located at a distance x from the firm offering high quality is $Y_R\theta_H - p_H - tx - Y_R$. Since all the rich consumers buy from the firm offering high quality, the aggregate consumer surplus for the rich is given by

$$CS_R \ = \ f_R \int_0^L \left[Y_R\left(\theta_H - 1\right) - p_H - tx\right] dx \ = \ f_R L \left[Y_R(\theta_H - 1) - p_H - \frac{tL}{2}\right].$$

Surplus increases with Y_R and falls with p_H. After substituting for p_H consumer surplus for the rich reduces to

$$f_R L \left[\frac{(2Y_R - Y_P)(\theta_L - 1)}{2} - \frac{c(\theta_L)}{2} + \frac{tL}{2}\right].$$

Following observations warrant a mention. First, surplus for the rich is independent of θ_H. With increase in θ_H, utility of rich goes up because of the valuation effect. But increase in θ_H further raises the intensity of vertical dominance. This allows the firm to extract the entire surplus from the rich consumers. For the reason outlined above, surplus increases with increase in tL. Change in surplus of the rich consumers with increase in θ_L depends on how price changes with change in θ_L.

Next we evaluate the consumer surplus of poor. Surplus of poor when not all them are served is given by

$$CS_P = f_P \int_0^{\frac{Y_P(\theta_L - 1) - p_L}{t}} \left[Y_P\left(\theta_L - 1\right) - p_L - tx\right] dx = \frac{f_P \left[Y_P\left(\theta_L - 1\right) - p_L\right]^2}{t}.$$

On substituting for p_L this reduces to

$$CS_P = \frac{f_P[Y_P\left(\theta_L - 1\right) - c(\theta_L)]^2}{4t}.$$

As expected, consumer surplus increases with the increase in the income level of poor. Although increase in Y_P allows the firm to increase price, the valuation effect dominates this price effect, leading to an increase in the net consumer surplus. As discussed earlier, change in the consumer surplus with a change in quality depends on the assumption on the marginal cost curve. When the marginal cost is linear in θ, consumer surplus goes up with increase in quality. Observe that the consumer surplus is independent of Y_R or θ_H. The following proposition summarizes the results on consumer surplus.

Proposition 7. *Suppose that, under vertical dominance, there exists an equilibrium with partial market coverage. Then*

1. *the consumer surplus of rich increases with increase in Y_R and t but falls with increase in Y_P, and*

2. *the consumer surplus of poor is unaffected with the changes in Y_R or θ_H, increasing in Y_P, but decreasing in transportation cost.*

Full Market Coverage

There is full market coverage when the poor at the other end of the city is willing to travel all the way to buy low quality product. Demand for the firm offering low quality is $f_P L$. Given that the firm is a monopolist with respect to the poor, it is able to extract the maximum possible surplus from them. It charges the price such that the marginal poor is indifferent between buying and not buying that is, $p_L = Y_P(\theta_L - 1) - tL$. Following the similar analysis as above it follows that there is full market coverage when $Y_P > \dfrac{2tL + c(\theta)}{\theta_L - 1}$. Demand and the pricing strategy for the firm offering high quality remains the same as under partial market coverage. Results are summarized in the following proposition.

Proposition 8: *An equilibrium with Vertical Dominance exists when* $(Y_R - Y_P)(\theta_H - \theta_L) > 2tL$. *There will be full market coverage when* $\dfrac{2tL + c(\theta_L)}{\theta_L - 1} < Y_P$. *For the case of full market coverage we have*

1. $p_H = Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL,$

2. $p_L = Y_P(\theta_L - 1) - tL,$

3. $\pi_L(.) = f_P L[Y_P(\theta_L - 1) - tL - c(\theta_L)] - F(\theta_L),$

4. $\pi_H(.) = \left[Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL - c(\theta_H) \right] f_R L - F(\theta_H).$

It has been shown in the Appendix A.3 that the equilibrium with the full market coverage will not exists as the firm offering low quality will have an incentive to deviate. This is because income of both poor and rich is high enough that increase profit due to increase in the market coverage outweighs the loss coming from the reduced price.

Horizontal Dominance

As opposed to vertical dominance, horizontal dominance arises when both the firms serve both income groups. The rich consumer at the location of the firm producing low quality prefers low quality rather than traveling all the way to the other end of the city for high quality implying that $Y_R\theta_L - p_L > Y_R\theta_H - p_H - tL$. Given their income, the rich consumers do not perceive quality difference to be high enough to refrain from buying the low quality product. Similarly, the poor consumer at the location of the firm producing high quality prefers high quality over low quality available at the other end implying that $Y_P\theta_H - p_H > Y_P\theta_L - p_L - tL$. Together the two inequalities imply

$$2tL > (Y_R - Y_P)(\theta_H - \theta_L).$$

This inequality lends important insight to understand the mechanism. This case arises when travel cost is significantly large as compared to the income differences. Because of this consumers prefer to settle for the quality available close by to avoid the high travel cost. When the relative income gap is not substantial and services are available at relatively larger distance individuals prefer to buy whatever quality is easily accessible. To characterize the nature of equilibrium, we again look at two situations: full market coverage and partial market coverage.

Full Market Coverage

We first determine demand for the high quality product arising from the poor consumers. Let the distance of the marginal poor consumer from the firm offering high quality be x. x is determined from $Y_P\theta_H - p_H - tx = Y_P\theta_L - p_L - t(L - x)$. Thus the demand from poor consumers for the high quality product is

$$D_P = \frac{f_P\{Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}.$$

Demand from rich consumers is determined in a similar way. So total demand faced by the firm offering high quality is

$$D_H(.) = \frac{(Y_R f_R + Y_P f_P)(\theta_H - \theta_L) - (f_R + f_P)[p_H - p_L + tL]}{2t}.$$

Observe that the firm offering high quality serves relatively larger proportion of rich than poor. This is because the rich, by the virtue of their higher income, have relatively higher preference for better quality. This makes them more willing to travel greater distance for the better quality product. Profit for the firm is

$$\pi_H(.) = [p_H - c(\theta_H)]\frac{(Y_R f_R + Y_P f_P)(\theta_H - \theta_L) - (f_R + f_P)[p_H - p_L + tL]}{2t} - F(\theta_H).$$

The first-order condition for profit maximization with respect to price implies $D_H(.) = \frac{(f_R + f_P)[p_H - c(\theta_H)]}{2t}$. On substitution and simplification it follows that $2(f_R + f_P)p_H = (f_R Y_R + f_P Y_P)(\theta_H - \theta_L) + (f_R + f_P)[p_L + tL + c(\theta_H)]$. Similar exercise for the firm offering low quality implies that in equilibrium

$$p_L = \frac{1}{3(f_R + f_P)}\left\{ (f_R Y_R + f_P Y_P)(\theta_L - \theta_H) + (f_R + f_P)[3tL + 2c(\theta_L) + c(\theta_H)]\right\}$$

and

$$p_H = \frac{1}{3(f_R + f_P)}\left\{ (f_R Y_R + f_P Y_P)(\theta_H - \theta_L) + (f_R + f_P)[3tL + 2c(\theta_H) + c(\theta_L)]\right\}.$$

It is worth observing that p_L falls whereas p_H increases with the rise in general income level and the relative proportion of the rich. The intuition for this comes from the preference structure. With the rise in the general income level, firm offering high quality attracts more consumers at the expense of the firm offering low quality. An increase in the market demand leads to higher price for the high quality product.

Opposite is true for the firm offering low quality. Profit for each firm is obtained by simple substitution of the price level.

The cut-off level of Y_P that ensures full market coverage under horizontal dominance is determined as follows. Utility of the marginal poor individual buying from firm offering high quality is

$$U(Y_P, p_H, \theta_H) = Y_P\theta_H - p_H - \frac{t[Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t}.$$

As the marginal poor consumer is better-off buying, his utility from consumption is greater than his reservation utility Y_P. This implies that there will be full market coverage if

$$Y_P \geq \frac{3tL + c(\theta_H) + c(\theta_L)}{(\theta_H - 1) + (\theta_L - 1)}.$$

Above results can be summarized in the following proposition.

Proposition 9: *An equilibrium with horizontal dominance exists iff* $(Y_R - Y_P)(\theta_H - \theta_L) < 2tL$. *There will be full market coverage iff* $Y_P \geq \frac{3tL + c(\theta_H) + c(\theta_L)}{(\theta_H - 1) + (\theta_L - 1)}$. *The equilibrium is characterized by the following properties.*

1. *All poor are served.*

2. $p_H = \frac{1}{3(f_R + f_P)}\Big\{ (f_RY_R + f_PY_P)(\theta_H - \theta_L) + (f_R + f_P)[3tL + 2c(\theta_H) + c(\theta_L)]\Big\}.$

3. $p_L = \frac{1}{3(f_R + f_P)}\Big\{ (f_RY_R + f_PY_P)(\theta_L - \theta_H) + (f_R + f_P)[3tL + 2c(\theta_L) + c(\theta_H)]\Big\}.$

4. p_H *increases with the increase in* Y_R, Y_P, f_R *and* f_P *where as* p_L *falls with the increase in* Y_R, Y_P, f_R *and* f_P.

5. $\pi_L(.) = \frac{\Big[\{f_RY_R + f_PY_P\}(\theta_L - \theta_H) + (f_R + f_P)[3tL + c(\theta_H) - c(\theta_L)]\Big]^2}{18t(f_R + f_P)^2} - F(\theta_L).$

6. $\pi_H(.) = \frac{\Big[\{f_RY_R + f_PY_P\}(\theta_H - \theta_L) + (f_R + f_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(f_R + f_P)^2} - F(\theta_H).$

To calculate the aggregate consumer surplus for the case of horizontal dominance we proceed as follows. Observe that both firms serve both rich and poor. So the surplus of the consumers also depends on, which firm they buy from. The net consumer surplus to the rich buying quality θ_i, where $\theta_i \in \{\theta_H, \theta_L\}$, located at the distance x from the firm from which he is buying is $Y_R\theta_i - p_i - tx - Y_R$. The rich consumers upto the distance $\frac{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL}{2t}$ buy from the firm offering high quality, whereas the remaining rich consumers buy the low quality product. The aggregate consumer surplus for rich is given by

$$CS_R = f_R \int_0^{X_1} \left[Y_R\left(\theta_H - 1\right) - p_H - tx\right] dx + f_R \int_0^{X_2} \left[Y_R\left(\theta_L - 1\right) - p_L - tx\right] dx,$$

where $X_1 = \frac{\{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}$ and $X_2 = \frac{\{Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL\}}{2t}$.

As there are many opposing forces at work, it is difficult to say anything conclusive about the change in aggregate consumer surplus with changes in the general income level or relative proportions. To get some idea let us first look at the consumer surplus of the rich already buying from the firm offering high quality. With the rise in Y_R there are two opposite effects that influence this surplus. First, utility of the rich increases because of the valuation effect. This raises the surplus. But, with increase in Y_R, p_H also increases leading to a fall in the surplus. For $\theta_H - 1 > \frac{f_R(\theta_H - \theta_L)}{3}$, the former effect dominates, resulting in the rise in the surplus. We also need to check how the fraction of rich consumers served by the firm offering θ_H changes with the rise in Y_R. It is easy to calculate that the ratio of the rich served increases if $\frac{3}{2} > f_R$. So it implies that the surplus of the rich being served by the firm offering high quality increases with increase in Y_R.[9] Surplus of the rich buying low quality increases unambiguously. This is because p_L falls with the increase in Y_R. Thus the two effects,

[9]Note $f_R < \frac{3}{2}$ implies $\theta_H - 1 > \frac{f_R(\theta_H - \theta_L)}{3}$.

income and price effects, reinforce each other. But there is fall in the fraction of rich served by the firm offering low quality as $\frac{3}{2} > f_R$.

Similarly one can evaluate change in the surplus of rich with increase in Y_P. Intuition spelt out in the last paragraph continues to help. With an increase in Y_P there only price affect that determines the surplus of the rich continuing to buy from the same firm. Clearly p_H increases with increase in Y_P leading to a fall in the surplus of the consumers continuing to buy from the firm offering θ_H. Also the number of rich consumers buying from the firm offering high quality falls. On the other hand both the surplus and fraction of rich buying from the firm offering low quality increases. Because of the opposing forces at work it is difficult to conclude about the overall impact on the surplus of the rich with a rise in Y_P.

With an increase in f_R and f_P, p_H increases while p_L falls implying a corresponding change in the surplus of the consumers continuing to buy from the respective firms. Also, there is redistribution of consumers between firms with change in price.

Consumer surplus of the poor is given by

$$CS_P = \left[\int_0^{Z_1} \left[Y_P\left(\theta_L - 1\right) - p_H - tx\right] dx\right] + \left[\int_0^{Z_2} \left[Y_P\left(\theta_L - 1\right) - p_L - tx\right] dx\right],$$

where $Z_1 = \dfrac{\{Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}$ and $Z_2 = \dfrac{\{Y_P(\theta_L - \theta_H) - (p_L - p_H) + tL\}}{2t}$.

Similar logic works if one looks at the changes in consumer surplus of poor owing to the changes in Y_P, Y_R, f_P and f_R. With the rise in Y_P market share of the firm offering high quality increases at the expense of the market share of the firm offering low quality. Also, with an increase in Y_P price of θ_H rises but that of θ_L falls. So the overall result is ambiguous. What one can infer conclusively from the above analysis is that with the increase in income there is a redistribution of market shares in favor of the firm offering high quality.

Partial Market Coverage

When the income level of the poor is low then some poor, those who are farther away from either firm, might be left unserved. The firms compete for the rich but have some monopoly power over the poor. Demand faced by each firm from the rich is derived exactly the same way as above, that is, the demand from the rich consumer for the firm offering high quality is given by the distance of the marginal consumer indifferent between the two firms. So the demand from the rich is given by $\frac{f_R[Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t}$. Firm's demand from the poor is determined by the distance such that poor becomes indifferent between buying and not buying, that is, $\frac{f_P[Y_P(\theta_H - 1) - p_H]}{t}$. Total demand for the firm producing high quality is then given by

$$D_H(.) = \frac{f_R\{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL\} + 2f_P\{Y_P(\theta_H - 1) - p_H\}}{2t}.$$

The first order condition with respect to price implies $D_H(.) = \frac{(f_R + 2f_P)[p_H - c(\theta_H)]}{2t}$. Similarly we derive the price response for the firm offering low quality.

This case is worked out in the similar way as the case of partial market coverage with both firms offering the same quality. Once the equilibrium price level is solved, we investigate the parameter values for which the above case arises. As above this possibility arises when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$. As the algebra is quite involved, details of the analysis are given in the Appendix A.4. The main results are summarized in the following proposition.

Proposition 10: *An equilibrium with the partial market coverage exists if*

$$\frac{2tLf_R + \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P)}{[f_R + 2f_P][(\theta_L - 1) + (\theta_H - 1)]} < Y_P < \frac{[3f_R + 4f_P]tL + (f_R + 2f_P)[(c(\theta_L) + c(\theta_H)]}{[f_R + 2f_P][(\theta_L - 1) + (\theta_H - 1)]}$$

and

$$\{Y_R[f_R + 4f_P] - 2f_PY_P\}[(\theta_H - 1) + (\theta_L - 1)] > \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P) + tL[3f_R + 4f_P],$$

and is characterized by the following properties.

1. *All rich are served. Poor at relatively higher distance from either firm are left unserved.*

2. *Both p_H and p_L increase with the increase in Y_P. But, when Y_R increases, p_H increases while p_L falls.*

As poor constitutes the captive market for the firms both the prices charged increase with their income. As income of the rich goes up, market demand for the firm offering high quality goes up at the expense of the firm offering low quality. As a result it is optimal for the firm offering high quality to raise its price, whereas for the firm offering low quality to reduce its price.

It is difficult to calculate the change in the consumer surplus with change in relative proportions of rich and poor as the algebra is quite involved. What is unambiguous though is that price charged by both the firms increase with the increase in income of the poor. This implies that welfare of the poor already buying high or low quality products decreases due to the price effect.

The two cases, vertical dominance and horizontal dominance, illustrate how income disparity interacts with travel cost to determine the pattern of equilibrium. Even with apparently the same outcome with one firm offering high quality and the other low, there are differences with respect to the size and the type of customers each firm serves depending on the level and extent of income inequality. This has its bearing on the welfare of the consumers.

Intermediate Case

An intermediate case arises when one firm serves both the income types whereas the other firm serves just one income group. The first possibility we consider is where the firm producing low quality θ_L serves both the income types, but the firm producing θ_H serves only the rich. This implies $Y_R\theta_L - p_L > Y_R\theta_H - p_H - tL$ and $Y_P\theta_L - p_L - tL > Y_P\theta_H - p_H$. The first inequality says that the firm offering low quality serves some rich consumers as well. The second inequality says that poor even at the extreme prefers low quality to high quality. Together these two conditions imply

$$p_H - p_L > \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2}.$$

It has been shown in the Appendix A.5 that in equilibrium when there is full market coverage this condition implies

$$\frac{c(\theta_H) - c(\theta_L)}{3} > \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} - \frac{2Y_R(\theta_H - \theta_L)}{3} + \frac{2tLf_P}{3f_R}$$

$$\Rightarrow \frac{3f_R}{f_P}\left[\frac{c(\theta_H) - c(\theta_L)}{3} - \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} + \frac{2Y_R(\theta_H - \theta_L)}{3}\right] > 2tL.$$

This highlights the scenario when the high marginal cost limits the access of the poor to the high quality product. For example in the case of the medical services, high-tech assistance might only exasperate the cost of services, making it unviable for the poor. This case may coincide with either vertical or horizontal dominance thus giving rise to the possibility of multiple equilibria. This is clear from the following inequalities. Define $\Omega = \left[\frac{c(\theta_H) - c(\theta_L)}{3} - \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} + \frac{2Y_R(\theta_H - \theta_L)}{3}\right]$. It will be the case of vertical dominance if either

$$[Y_R - Y_P](\theta_H - \theta_L) \geq \frac{3f_R\Omega}{f_P} > 2tL$$

or

$$\frac{3f_R\Omega}{f_P} > [Y_R - Y_P](\theta_H - \theta_L) \geq 2tL$$

holds, whereas horizontal dominance occurs if we have

$$\frac{3f_R\Omega}{f_P} \;>\; 2tL \;>\; [Y_R - Y_P](\theta_H - \theta_L).$$

Given horizontal dominance, an increase in the proportion of rich raises the possibility of the intermediate case: firm offering high quality serves only the rich, whereas firm offering low quality serves both income groups. Given horizontal dominance, as proportion of rich increases, price charged by the firm producing high quality also rises. Higher prices forces the poor to go away without consuming, raising the possibility of intermediate equilibrium. The poor are worse-off as they now have access to only the low quality product. Similarly given vertical dominance, an increases in the proportion of rich again raises the possibility of intermediate case. This is because with the increase in the price of the high quality product, rich may now be forced to buy the low quality product as well.

The condition under which this equilibrium can be sustained is stated in Appendix A.5. The opposite case, that is, when firm producing high quality serves both income types but the firm producing low quality serves only poor, cannot be sustained in an equilibrium. The fact that the high quality firm serves both income groups implies that income difference is not substantial. In this case the low quality firm can strategically deviate and be better-off.

3.4 Quality Stage and Equilibrium Outcomes

In the last section we have discussed the various possibilities that will arise in the second stage of the game, the price stage. In this section we consider the first stage of the game, the quality stage where each firm decides which quality to offer, θ_H or θ_L. The two stages combined together gives us all the possible equilibrium outcomes that may arise in this game under consideration.

To convey the main message of the analysis in a cleaner way we introduce two simplifications in this section. The first one is a simplifying assumption on the structure of the marginal cost, $c(\theta)$. The simplest possibility is to assume that the marginal cost is linear in quality. Even if the marginal cost is convex in quality, our presentation of the quality stage of the game will be simplified if we assume that the increase in the gross utility owing to an improvement in quality more than outweighs the loss due to a higher marginal cost. For example, consider the lower bound of the poor income in Proposition 5 below which no poor is served when both firms offer the same quality, $\frac{tL + c(\theta)}{\theta - 1}$. Our simplifying assumption would imply that $\frac{tL + c(\theta_H)}{\theta_H - 1} < \frac{tL + c(\theta_L)}{\theta_L - 1}$, that is, if no poor is served when both firms offer the high quality, then same will happen to the poor when both firms offer the low quality also. The second simplification is just for the purpose of exposition: we discuss only the case of vertical dominance for the case of the partial market coverage. As will be clear from the following analysis, the nature of equilibrium outcomes is very similar under the case of horizontal dominance but involves a lot of tedious algebraic expressions without adding much to our understanding.

From the analysis in the last section let us first summarize the relevant thresholds for Y_P under which different equilibrium possibilities arise. Proposition 5 defines the lower bound for Y_P below which no poor is served when both firms offer the same quality, $\frac{tL + c(\theta)}{\theta - 1}$. Define

$$Y_P^1 \equiv \frac{tL + c(\theta_H)}{\theta_H - 1}.$$

Since our simplifying assumption above implies that $\frac{tL + c(\theta_H)}{\theta_H - 1} < \frac{tL + c(\theta_L)}{\theta_L - 1}$, it follows that if $Y_P < Y_P^1$ then poor consumers are completely excluded when the firms offer the same quality, no matter whether that common quality is high or low.

Similarly Proposition 1 identifies the upper bound for Y_P such that when both firms offer the same quality all the poor consumers are served only if $Y_P > \frac{1}{\theta - 1}\left[c(\theta) + \frac{3tL}{2}\right]$. Define

$$Y_P^2 \equiv \frac{1}{\theta_H - 1}\left[\frac{3tL}{2} + c(\theta_H)\right].$$

Since our simplifying assumption implies that $\frac{1}{\theta_H - 1}\left[c(\theta_H) + \frac{3tL}{2}\right] < \frac{1}{\theta_L - 1}\left[c(\theta_L) + \frac{3tL}{2}\right]$, it follows that if $Y_P^1 < Y_P < Y_P^2$ then there could only be partial market coverage of the poor consumers when the firms offer the same quality.

Finally, Proposition 6 defines the upper cut-off level for Y_P above which all poor consumers are served when there is vertical dominance. Let us define this upper cut-off as

$$Y_P^3 \equiv \frac{3tL + c(\theta_L) + c(\theta_H)}{(\theta_H - 1) + (\theta_L - 1)}.$$

Observe that our assumption on the marginal cost structure guarantees that $Y_P^2 < Y_P^3$.

These income thresholds and the fact that fixed cost depends on product quality ($F(\theta_H) > F(\theta_L)$) impose restrictions on the quality that the firms can offer. There are various forces that influence firms' decisions. In the price stage we have observed the trade-off in terms of market coverage: a firm can charge a higher price to take advantage of the higher willingness to pay of the rich consumers, but, in the process, it will lose its market size by losing the poor consumers. In the quality stage, the opportunity to offer different quality products, can relax the price competition. But here too it has to weigh its choices given the existence of different pressures. First, market share of the firm offering low quality might shrink since individuals prefer high quality over low quality. Second, the high-quality firm also may lose some market share since a poor consumer with not too intense preference over quality (due to relatively lower income) can now opt for the lower quality product. Third, even though higher quality induces higher willingness to pay, to provide the higher quality firm has to bear a higher fixed cost also.[10] In what follows we use the thresholds for Y_P, the trade-off over market shares in the price stage and the trade-off over market shares and fixed costs in the quality stage to characterize the possible equilibrium outcomes.

3.4.1 $Y_P < Y_P^1$

When $Y_P < Y_P^1$, if both firms offer the same quality, the poor consumer even at the location of the firm cannot afford to buy the product, no matter what the common quality is. On the other hand since $\frac{c(\theta_L)}{\theta_L - 1} < Y_P^1$, it follows from Proposition 6 that when the two firms offer two different quality products, some of the poor consumers residing closer to the firm offering the low quality product can buy the low quality product. So the question is to investigate what quality profile will prevail in equilibrium.

Consider first whether both firms offering low quality, that is, the quality profile (θ_L, θ_L), is an equilibrium outcome. As stated in Proposition 5, profit of each firm is $\pi(\theta_L, \theta_L) = \frac{f_R[tL]^2}{2t} - F(\theta_L)$. Suppose one firm deviates and offers the high quality, θ_H. To find out the profit under this deviation we have to consider the price equilibrium followed by the subgame (θ_H, θ_L) under vertical dominance. Since $Y_P < Y_P^1 < \frac{2tL + c(\theta_L)}{\theta_L - 1}$, there can only be partial market coverage of the poor as long as $\frac{c(\theta_L)}{\theta_L - 1} < Y_P$. Assuming this it follows from Proposition 6 that the profit of the deviating firm will be $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) = \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - F(\theta_H)$. Clearly the firm will deviate if $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) > \pi(\theta_L, \theta_L)$. So we can conclude that when $Y_P < Y_P^1$, (θ_L, θ_L) is an equilibrium outcome if $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) \leq$

[10]See Coibon and Hallack (2007) for the theoretical determinants of substantial differences in demand elasticities and associated markups among products of heterogeneous quality. He also refers to empirical studies like Bresnahan (1987), which have reported substantial variation in estimated price elasticities of demand and associated markups across products for oligopolistic industries.

$\pi\left(\theta_L, \theta_L\right)$, that is, if

$$\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - \frac{f_R[tL]^2}{2t} \leq F(\theta_H) - F(\theta_L). \tag{3.1}$$

Consider next whether (θ_H, θ_H) could be an equilibrium outcome. Proposition 5 says that each firm's profit is $\pi\left(\theta_H, \theta_H\right) = \dfrac{f_R[tL]^2}{2t} - F(\theta_H)$. Suppose one firm deviates and offers the low quality, θ_L, so that the relevant subgame is once again (θ_L, θ_H) under vertical dominance. By the same logic given above, there could only be partial market coverage of the poor so that, following Proposition 6, the profit of the deviating firm is $\pi_L|_{\text{deviation}}\left(\theta_L, \theta_H\right) = \dfrac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L)$. Hence we conclude that (θ_H, θ_H) is an equilibrium outcome if $\pi_L|_{\text{deviation}}\left(\theta_L, \theta_H\right) \leq \pi\left(\theta_H, \theta_H\right)$, that is, if

$$F(\theta_H) - F(\theta_L) \leq \frac{f_R[tL]^2}{2t} - \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t}. \tag{3.2}$$

Finally consider whether firms could offer different qualities in equilibrium, that is, whether (θ_H, θ_L) or (θ_L, θ_H) could be equilibrium outcomes. We consider only (θ_H, θ_L) since the other case is just symmetric. From the above analysis it is clear that the high quality producing firm will have no incentive to deviate if $\pi_H\left(\theta_H, \theta_L\right) \geq \pi\left(\theta_L, \theta_L\right)$, that is, if $F(\theta_H) - F(\theta_L) \leq \left[Y_R(\theta_H - \theta_L) + \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - \dfrac{f_R[tL]^2}{2t}$, whereas the low quality producing firm will have no incentive to deviate if $\pi_L\left(\theta_H, \theta_L\right) \geq \pi\left(\theta_H, \theta_H\right)$, that is, if $\dfrac{f_R[tL]^2}{2t} - \dfrac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} \leq F(\theta_H) - F(\theta_L)$. Combining the two inequalities we conclude that (θ_H, θ_L) is an equilibrium outcome if

$$\begin{aligned} \frac{f_R[tL]^2}{2t} - \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} \quad &\leq \quad F(\theta_H) - F(\theta_L) \\ &\leq \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - \frac{f_R[tL]^2}{2t}. \end{aligned} \tag{3.3}$$

We would like to highlight two points that that emerge from inequalities (3.1), (3.2) and (3.3) above. The first observation relates to the difference in the fixed costs of producing the two quality products, $F(\theta_H) - F(\theta_L)$. Given all the other parameter values, both the firms offer the low quality if the difference in fixed costs is too high (inequality (3.1)) and offer the high quality if the difference in fixed costs is low enough (inequality (3.2)), whereas, for an intermediate difference in fixed costs, one firm offers high quality while the other goes for low quality.

The second observation is on the income of the rich, Y_R, and their relative proportion in the city population, f_R, given, of course, all other parameter values, in particular, given the poor income and the difference in fixed costs. Given the quite low income of the poor, if either the rich income is reasonably low (inequality (3.1)) or the proportion of rich is very high (inequality (3.2)), the poor consumers are completely shut out of the market. Both the firms completely ignore their presence and offer the low quality product if the rich income is low and the high quality product if the density of the rich is high. Interestingly, for a moderate level of rich income and the proportion of rich when (inequality 3.3) is satisfied, poor are better off as at least some poor consumers residing closer to the firm producing low quality product gets access to it.

The purpose of our analysis is to investigate how the level of income and the inequality in its distribution affect the equilibrium outcome in terms of the quality offered and price charged by each firm and then to see its implications on the welfare of the consumers. Of particular interest is the welfare of the people in the lowest income category fallen deeper into poverty with long-term consequences on their health and education and thus on their future earning potential. When the poor income is so low, the poor consumer even at the location of the firm is not able to afford the product. Only the rich constitutes the market for the firms. Utility of the poor consumers is their reservation utility given by their income level Y_P. For a relatively higher level of Y_R, poor are better-off as few of them can access the product that was earlier unavailable. For a relatively high proportion of the rich, there is again a paradigm shift with both firms offering the high quality product and ignoring the poor consumers in the process. This analysis implies that in case of extreme deprivation the poor might be better-off being with the rich as at least few of them can access the product or service which was earlier beyond their reach. But as the income gap widens further, their welfare reduces to the same level as again all of them are priced out of the market. It is the combination of both the factors, low proportion and low income level of poor, that leads to this kind of outcome. From the above analysis it follows that the welfare of the poor initially increases and then falls as there is income growth (arising due to rise in income and density of rich). It seems that the welfare of the poor shows an inverted U-shaped pattern in the income of the rich.

This pattern of growth initially helps but later penalizes the poor, especially if the products or services under consideration are the merit goods like health or education, as the accessibility and quality of these services determines individual's earning capacity in future as well. So the already marginalized section of the society finds itself trapped into the vicious circle of poverty. The extreme case that one may consider is to look at the mortality rate of poor. By one estimate, in India, the infant mortality rate is 2.5 times higher among the poorest 20% of the society than among the richest 20% (Deogaonkar, 2004).

3.4.2 $Y_P^1 < Y_P < Y_P^2$

When $Y_P^1 < Y_P < Y_P^2$, it follows from Proposition 1 that there could only be partial market coverage of the poor consumers when the firms offer the same quality. Similarly since $Y_P^2 < Y_P^3$, Proposition 6 implies that there will be partial market coverage of the poor consumers when the firms offer different qualities too.

Consider first whether (θ_L, θ_L) is an equilibrium outcome. Since there is partial market coverage of the poor, it follows from Proposition 3 that profit of each firm is $\pi(\theta_L, \theta_L) = \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_L) + 2f_P Y_P(\theta_L - 1)}{f_R + 4f_P}\right]^2 - F(\theta_L)$. If any firm deviates to offer quality θ_H, Proposition 6 implies that profit of the deviating firm is $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) = \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - F(\theta_H)$. It follows that (θ_L, θ_L) is an equilibrium outcome if

$$\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_L) + 2f_P Y_P(\theta_L - 1)}{f_R + 4f_P}\right]^2 \leq F(\theta_H) - F(\theta_L). \tag{3.4}$$

Next consider whether (θ_H, θ_H) is an equilibrium outcome. When each firm offers high quality it follows from Proposition 3 that profit of each firm is $\pi(\theta_H, \theta_H) = \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_H) + 2f_P Y_P(\theta_H - 1)}{f_R + 4f_P}\right]^2 - F(\theta_H)$. If any firm deviates to offer quality θ_L, Proposition 6 implies that profit of the deviating firm is $\pi_L|_{\text{deviation}}(\theta_L, \theta_H) = \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L)$. Hence (θ_H, θ_H) is an equilibrium outcome if

$$F(\theta_H) - F(\theta_L) \leq \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_H) + 2f_P Y_P(\theta_H - 1)}{f_R + 4f_P}\right]^2 - \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t}. \tag{3.5}$$

Finally consider whether (θ_H, θ_L) could be an equilibrium outcome. From the above analysis it is clear that the high quality producing firm will have no incentive to deviate if $\pi_H(\theta_H, \theta_L) \geq \pi(\theta_L, \theta_L)$, that is, if $\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L - \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_L) + 2f_P Y_P(\theta_L - 1)}{f_R + 4f_P}\right]^2 \geq F(\theta_H) - F(\theta_L)$, whereas the low quality producing firm will have no incentive to deviate if $\pi_L(\theta_H, \theta_L) \geq \pi(\theta_H, \theta_H)$, that is, if $F(\theta_H) - F(\theta_L) \geq \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_H) + 2f_P Y_P(\theta_H - 1)}{f_R + 4f_P}\right]^2 - \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t}$.

Combining the two inequalities we conclude that (θ_H, θ_L) is an equilibrium outcome if

$$
\begin{aligned}
&\frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_H) + 2f_P Y_P(\theta_H - 1)}{f_R + 4f_P}\right]^2 - \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} \\
&\leq F(\theta_H) - F(\theta_L) \\
&\leq \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] f_R L \\
&\qquad - \frac{f_R + 2f_P}{2t}\left[\frac{f_R tL - 2f_P c(\theta_L) + 2f_P Y_P(\theta_L - 1)}{f_R + 4f_P}\right]^2.
\end{aligned}
\tag{3.6}
$$

Inequalities (3.4), (3.5) and (3.6) reiterate a similar observation made in the last subsection: given all the other parameter values, both the firms offer the low quality if the difference in fixed costs is too high (inequality (3.4)) and offer the high quality if the difference in fixed costs is low enough (inequality (3.5)), whereas, for an intermediate difference in fixed costs, one firm offers high quality while the other goes for low quality.

Now consider varying the rich income, Y_R. For this intermediate level of Y_P, when income level in the economy in general is not so high (relative to the difference in fixed costs), then both firms offer low quality. This might be more relevant for rural India where although facilities are available but the products and services are of substandard quality. Owing to low income some poor are left out.

As Y_R increases there is a transition in the market to favor the rich with the outcome being an asymmetric one where one firm offers high quality and the other low quality. Richer is an individual higher is his preference for better quality and more is his willingness to pay for it. This gives an incentive to the firm to offer high quality. Earlier when both firms were offering the low quality, it follows from Proposition 3 that the poor with market access were paying a price $\frac{1}{f_R + 4f_P}[f_R tL + 2f_P Y_P(\theta_L - 1) + (f_R + 2f_P)c(\theta_L)]$. With the rise in Y_R there is a shift in equilibrium with the price charged by the firm producing low quality being given by $\frac{Y_P(\theta_L - 1) + c(\theta_L)}{2}$ (see Proposition 6). Parameter values for which this holds is such that the price for the latter case is higher than the former. That is, the price the firm charges as a monopolist is higher than the case when it is competing for the rich. This has cascading effects on the welfare of poor. The price rise pushes more poor people out of the reach of the market. In addition since one firm switches to offering high quality which is beyond the reach of the poor, the poor consumers residing closer to this firm lose market access too. For services like education or health care, this leads to a poverty spiral. This is especially true for the developing countries where the absolute poverty levels are relatively very high. So these countries must make investment in social safety nets a development priority.

3.4.3 $Y_P^3 < Y_P$

Since $Y_P^2 < Y_P^3$, it follows from Propositions 1 and 9 that when $Y_P^3 < Y_P$ all the poor consumers are served no matter whether the firms offer the same quality or different qualities. Following the same methodology discussed in the last two subsections we can come to the following conclusion under this case when the poor are rich enough then in equilibrium both the firms will serve both income groups:

(θ_L, θ_L) is an equilibrium outcome if

$$\frac{\Big[\{f_R Y_R + f_P Y_P\}(\theta_H - \theta_L) + (f_R + f_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(f_R + f_P)^2} - \frac{(f_R + f_P)\,[tL]^2}{2t} \leq F(\theta_H) - F(\theta_L); \tag{3.7}$$

(θ_H, θ_H) is an equilibrium outcome if

$$F(\theta_H) - F(\theta_L) \leq \frac{\left[\{f_R Y_R + f_P Y_P\}(\theta_H - \theta_L) + (f_R + f_P)[3tL + c(\theta_L) - c(\theta_H)]\right]^2}{18t(f_R + \delta_P)^2}; \quad (3.8)$$

and (θ_H, θ_L) is an equilibrium outcome if

$$\frac{\left[\{f_R Y_R + f_P Y_P\}(\theta_H - \theta_L) + (f_R + f_P)[3tL + c(\theta_L) - c(\theta_H)]\right]^2}{18t(f_R + f_P)^2} \leq F(\theta_H) - F(\theta_L) \leq$$

$$\frac{\left[\{f_R Y_R + f_P Y_P\}(\theta_H - \theta_L) + (f_R + f_P)[3tL + c(\theta_L) - c(\theta_H)]\right]^2}{18t(f_R + f_P)^2} - \frac{(f_R + f_P)\,[tL]^2}{2t} \quad (3.9)$$

Once again, which quality will be offered in equilibrium depends on the difference in fixed costs relative to the general income level.

Implications of an increase in rich income on the welfare of the poor is similar to the last subsection. Other parameters remaining the same as Y_R increases equilibrium outcome switches from (θ_L, θ_L) to (θ_H, θ_L) and the low quality firm makes the poor worse off taking advantage of the relatively higher monopoly power over the poor. Still further increase in Y_R makes it more attractive for the low quality firm to reap the benefit of the higher willingness of the rich leading to the outcome where both firms produce the high quality product. The high quality comes at the higher price. But, at the same time, competition between the firms may mellow down the price level.

To sum up the following proposition characterizes the equilibrium outcomes for the overall game.

Proposition 11. *Suppose that Vertical Dominance prevails in the quality subgame.*

(1) *If* $Y_P < Y_P^1$,

(a) *both the firms offer the low quality if the difference in fixed costs is too high relative to the utility gain from the quality difference* (*if inequality* (3.1) *holds*). *The rich has complete market access while no poor is served.*

(b) *Both the firms offer the high quality if the difference in fixed costs is low enough (if inequality* (3.2) *holds). There is complete market access for rich while no poor is served.*

(c) *For an intermediate difference in fixed costs, (if inequality* (3.3) *holds) the equilibrium is* (θ_H, θ_L)*. There is partial coverage for the poor – some poor residing closer to the facilities have access to them, others get excluded.*

(2) *If* $Y_P^1 < Y_P < Y_P^2$,

(a) *both the firms offer the low quality if the difference in fixed costs is too high (if inequality* (3.4) *holds). The rich has complete market access while the poor has partial access.*

(b) *Both the firms offer the high quality if the difference in fixed costs is low enough (if inequality* (3.5) *holds). The rich has complete market access while the poor has partial access.*

(c) *For an intermediate difference in fixed costs (if inequality* (3.6) *holds), one firm offers high quality while the other offers low quality. In this case some poor buy the low quality product but no poor buys high quality product.*

(3) *If* $Y_P^3 < Y_P$*, then all the poor consumers are served no matter whether the firms offer the same quality or different qualities. The equilibrium outcome is*

(a) (θ_L, θ_L) *if inequality* (3.7) *holds,*

(b) (θ_H, θ_H) *if inequality* (3.8) *holds, and*

(c) (θ_H, θ_L) *if inequality* (3.9) *holds.*

3.5 Conclusion

In this chapter we examine how the interaction of income inequality with the neighbourhood effects determines firms' quality differentiation and the resulting welfare implications. We demonstrate that for a homogenous distribution of income or when the poor's income or density is too low, both firms offer the same quality. For a homogenous income distribution firm does not perceive much benefit from product differentiation. Similarly when income and density of the poor is low, it implies a low demand for a different variety. In these scenarios the poor are either left completely unserved, or they end up buying whatever the market has to offer. Given this, for a very high difference in the fixed costs, both firms offer the low quality. But when the difference in the fixed costs is low, both firms offer the high quality.

For a more heterogeneous income distribution and intermediate range of the difference in fixed costs, one firm offers the high quality and the other the low quality. Product differentiation on one hand allows firm to alleviate price competition and, on the other hand, serves consumers' demand better. Within this there can either be horizontal dominance - both firms serving either income groups, or vertical dominance - all the rich buying the high quality product and the poor buying the low quality product. Horizontal dominance arises when the travel cost outweighs the income and quality difference. When the income and quality difference is substantial compared to the travel cost, it makes the case for vertical dominance. We show that although in general a rise in income inequality has a spiraling negative effect on the welfare of the poor, there are situations, particularly when the poor income is very low, when an increase in the rich income could be welfare improving for the poor.

3.6 Appendix

A.1 Proof of Proposition 3

A.1.1 p_M increases in f_R

From Proposition 3, it follows that for the case of partial market coverage with both firms offering same quality, equilibrium price is given by

$$p_M = \frac{1}{f_R + 4f_P}[f_R tL + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)].$$

This implies that

$$\frac{\partial p_M}{\partial f_R} = \frac{[tL + c(\theta)](f_R + 4f_P) - [f_R tL + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)]}{(f_R + 4f_P)^2}$$
$$= \frac{2f_P[2tL - [Y_P(\theta - 1) - c(\theta)]]}{(f_R + 4f_P)^2}. \quad \text{(A.1.1.a)}$$

From Proposition 3, it follows that there is a partial market coverage with both firms offering same quality, when

$$\frac{f_R tL}{f_R + 2f_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3f_R + 4f_P)tL}{2f_R + 4f_P} + c(\theta). \quad \text{(A.1.1.b)}$$

Therefore, in this case the maximum value that $Y_P(\theta-1)-c(\theta)$ can take is $\frac{[3f_R + 4f_P]tL}{2f_R + 4f_P}$. Substituting for this in equation (A.1.1.a) we get that the corresponding minimum value that $\frac{\partial p_M}{\partial f_R}$ can take is

$$\frac{2f_P\left[2tL - \frac{[3f_R + 4f_P]tL}{2f_R + 4f_P}\right]}{(f_R + 4f_P)^2} = \frac{f_P[f_R + 4f_P]tL}{(f_R + 4f_P)^2(f_R + 2f_P)} \; > \; 0.$$

It follows that p_M increases as the proportion of the rich increases.

A.1.2 p_M falls in f_P

Proof of this has been worked out in the similar lines as above. Again we have

$$p_M = \frac{1}{f_R + 4f_P}[f_R tL + 2f_P Y_P(\theta - 1) + (f_R + 2f_P)c(\theta)].$$

This implies that

$$\frac{\partial p_M}{\partial f_P} = \frac{2[Y_P(\theta-1)+c(\theta)](f_R+4f_P) - 4[f_R tL + 2f_P Y_P(\theta-1) + (f_R+2f_P)c(\theta)]}{(f_R+4f_P)^2}$$

$$= \frac{2f_R[Y_P(\theta-1)-c(\theta)-2tL]}{(f_R+4f_P)^2}.$$

From the inequality (A.1.1.b) it follows that $Y_P(\theta-1)-c(\theta)$ has the minimum value $\frac{f_R tL}{f_R+2f_P}$; consequently the maximum value $\frac{\partial p_M}{\partial f_P}$ can take is

$$\frac{2f_R\left[\frac{f_R tL}{f_R+2f_P} - 2tL\right]}{(f_R+4f_P)^2} = -\frac{2f_R tL}{(f_R+4f_P)(f_R+2f_P)} \ < \ 0.$$

Hence p_M falls as the proportion of the poor increases.

A.1.3 $p_C > p_M$

From Proposition 1, it follows that the equilibrium price p_C when there is full market coverage, is $tL + c(\theta)$. From Proposition 3 it follows that the equilibrium price when there is partial market coverage, is $p_M = \frac{1}{f_R+4f_P}[f_R tL + 2f_P Y_P(\theta-1) + (f_R + 2f_P)c(\theta)]$. In order to prove that $p_C > p_M$, we need to show the following:

$$tL + c(\theta) \ > \ \frac{1}{f_R+4f_P}[f_R tL + 2f_P Y_P(\theta-1) + (f_R+2f_P)c(\theta)].$$

That is,

$$[tL + c(\theta)][f_R+4f_P] \ > \ [f_R tL + 2f_P Y_P(\theta-1) + (f_R+2f_P)c(\theta)].$$

This simplifies to

$$2tL + c(\theta) \ > \ Y_P(\theta-1).$$

If the above inequality is true for the maximum value that $Y_P(\theta-1)$ can take, then it will hold for all values of $Y_P(\theta-1)$ for the case implied. From the inequality (A.1.1.b) it follows that the upper threshold for $Y_P(\theta-1)$ is $\dfrac{(3f_R+4f_P)tL}{2f_R+4f_P}+c(\theta)$. Substituting for this value of $Y_P(\theta-1)$, this condition becomes

$$2tL+c(\theta) \;>\; \frac{(3f_R+4f_P)tL+2(f_R+2f_P)c(\theta)}{2f_R+4f_P}.$$

Or equivalently

$$(f_R+4f_P)tL \;>\; 0,$$

which is always true. Thus it follows that $p_C > p_M$.

A.2 Existence of Equilibrium with Partial Market Coverage under Vertical Dominance

Here we will derive the conditions for an equilibrium where no firm has any incentive for deviation. First, let us consider the firm producing high quality product. It can consider lowering its price. This will make sense only if it is able to attract poor consumers as well. At a lower price it will retain its market for the rich. So the possible deviation is to lower price such that it starts competing for the poor. Let μ denote the distance from the firm offering high quality product at which a poor consumer is indifferent between buying high quality product at new price p_H^* and low quality product at p_L. This implies

$$Y_P\theta_H - p_H^* - t\mu = Y_P\theta_L - p_L - t(L-\mu).$$

So the demand from the poor is given by

$$D_P = \frac{f_P[Y_P(\theta_H-\theta_L)-(p_{H^*}-p_L)+tL]}{2t}.$$

Market demand from rich is again given by f_RL. The expression for the profit of the firm is

$$\pi_H = [p_H^* - c(\theta_H)]\left\{f_RL + f_P\left[\frac{Y_P(\theta_H-\theta_L)-(p_{H^*}-p_L)+tL}{2t}\right]\right\} - F(\theta_H),$$

where from Proposition 6, $p_L = \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2}$. The first-order condition of profit maximization with respect to price implies

$$D_H = \frac{[p_H^* - c(\theta_H)]f_P}{2t} \text{ which implies } \pi_H = \frac{f_P[p_H^* - c(\theta_H)]^2}{2t} - F(\theta_H).$$

It follows that in equilibrium

$$p_H* = \frac{4tLf_R + 2tLf_P + 2f_PY_P(\theta_H - \theta_L) + f_PY_P(\theta_L - 1) + 2f_Pc(\theta_H) + f_Pc(\theta_L)}{4f_P}$$

and

$$\pi_H* = \frac{\left[4tLf_R + 2tLf_P + 2f_PY_P(\theta_H - \theta_L) + f_PY_P(\theta_L - 1) - 2f_Pc(\theta_H) + f_Pc(\theta_L)\right]^2}{32tf_P} - F(\theta_H).$$

We can compare this to the corresponding expression given in Proposition 6:

$$\pi_H = \left[Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL - c(\theta_H)\right]f_RL - F(\theta_H).$$

Firm will not deviate if $\pi_H > \pi_H*$, that is, if

$$\left[4tLf_R + 2tLf_P + 2f_PY_P(\theta_H - \theta_L) + f_PY_P(\theta_L - 1) - 2f_Pc(\theta_H) + f_Pc(\theta_L)\right]^2 < 32tf_Rf_PL\left[Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL - c(\theta_H)\right].$$

It is clear that above inequality is likely to hold when Y_R is high enough, implying that there will be Vertical Dominance only when the income gap is substantial.

Next we consider the possibility of deviation by the firm offering low quality. This firm might consider to charge a low price so that some rich are also willing to buy. Firm will continue to be a monopolist with respect to the poor. Demand by the poor at the new price p_L^* is given by D_p where

$$D_p = \frac{f_P[Y_P(\theta_L - 1) - p_L^*]}{t}.$$

Since this firm competes for the rich, so demand from the rich is given by μf_R where μ is determined from

$$Y_R\theta_L - p_L^* - t\mu = Y_R\theta_H - p_H - t[L-\mu]$$
$$\Rightarrow \mu = \frac{Y_R(\theta_L-\theta_H) - (p_{L^*} - p_H) + tL}{2t}.$$

Firm's profit is given by

$$\pi_L{*}(.) = [p_L^* - c(\theta_L)]\left\{ f_P \times \frac{Y_P(\theta_L - 1) - p_L^*}{t} + f_R\left[\frac{Y_R(\theta_L-\theta_H) - (p_{L^*} - p_H) + tL}{2t}\right]\right\} - F(\theta_L),$$

where from Proposition 6 it follows that $p_H = Y_R(\theta_H - \theta_L) + \dfrac{Y_P(\theta_L-1)+c(\theta_L)}{2} - tL.$ The first-order condition for profit maximization with respect to price implies

$$D_L(.) \;=\; \frac{[p_L^* - c(\theta_L)][f_R + 2f_P]}{2t}$$

that is,

$$\left\{ f_P \times \frac{Y_P(\theta_L - 1) - p_L^*}{t} + f_R\left[\frac{Y_R(\theta_L-\theta_H) - (p_{L^*} - p_H) + tL}{2t}\right]\right\} \;=\; \frac{[p_L^* - c(\theta_L)][f_R + 2f_P]}{2t},$$

which implies

$$p_L{*} = \frac{(4f_P + f_R)Y_P(\theta_L - 1) + c(\theta_L)[3f_R + 4f_P]}{2(f_R + 2f_P)}.$$

Also from the first-order condition expression for profit simplifies to

$$\pi_L*(.) \;=\; \frac{[f_R + 2f_P][p_L^* - c(\theta_L)]^2}{2t} \;-\; F(\theta_L).$$

On substitution for price it becomes

$$\pi_L*(.) \;=\; \frac{\Big[(4f_P + f_R)Y_P(\theta_L - 1) + f_R c(\theta_L)\Big]^2}{8t[f_R + 2f_P]} \;-\; F(\theta_L).$$

The corresponding expression for the profit of the firm from Proposition 6 is

$$\pi_L(.) = \frac{f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L).$$

Firm will have no incentive to deviate if $\pi_L(.) > \pi_L * (.)$, that is, if

$$2(f_R + 4f_P)f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2 \ > \ \bigg[(4f_P + f_R)Y_P(\theta_L - 1) + f_R c(\theta_L)\bigg]^2.$$

Or if

$$2(f_R + 4f_P)f_P[Y_P(\theta_L - 1) - c(\theta_L)]^2 \ > \ (4f_P + f_R)^2\bigg[Y_P(\theta_L - 1) + \frac{f_R}{4f_P + f_R}c(\theta_L)\bigg]^2.$$

Above inequality reduces to

$$2[Y_P(\theta_L - 1) - c(\theta_L)]^2 \ > \ \bigg(4 + \frac{f_R}{f_P}\bigg)\bigg[Y_P(\theta_L - 1) + \frac{f_R}{4f_P + f_R}c(\theta_L)\bigg]^2.$$

Clearly above inequality is more likely to hold when relative density of poor is high.

A.3 Existence of Equilibrium with Full Market Coverage under Vertical Dominance

Here again, we will derive the conditions for an equilibrium where no firm has any incentive to deviate. As before, we first consider the firm offering high quality product. The possible deviation that firm can consider is to lower its price. At a lower price it will retain its share of rich consumers. But the deviation will be profitable only if the firm is able to serve poor consumers as well. So we assume that at a lower price this firm starts competing for poor consumers with the firm offering low quality product. Hence the demand from the poor at lower price p_H^* is given by

$$D_P = f_P\bigg[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - p_L) + tL}{2t}\bigg].$$

So profit of the firm at the reduced price p_H^* is given by

$$\pi_H^*(.) = [p_H^* - c(\theta_H)]\bigg\{f_R L + f_P\bigg[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - p_L) + tL}{2t}\bigg]\bigg\} - F(\theta_H).$$

Where, it follows from Proposition 8, that $p_L = Y_P(\theta_L - 1) - tL$. This implies

$$\pi_H^*(.) \ = \ [p_H^* - c(\theta_H)]\bigg\{f_R L + f_P\bigg[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - [Y_P(\theta_L - 1) - tL]) + tL}{2t}\bigg]\bigg\} - F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H \;=\; \frac{[p_H^* - c(\theta_H)]f_P}{2t}.$$

This implies

$$p_H* = \frac{2tLf_R + f_P Y_P(\theta_H - 1) + f_P c(\theta_H)}{2f_P}.$$

and the profit of the firm is given by

$$\pi_H^*(.) \;=\; \frac{f_P[p_H^* - c(\theta_H)]^2}{2t} \;-\; F(\theta_H).$$

Substituting for price, expression for profit becomes

$$\pi_H^*(.) \;=\; \frac{\left[2tLf_R + f_P Y_P(\theta_H - 1) - f_P c(\theta_H)\right]^2}{8tf_P} \;-\; F(\theta_H).$$

We can compare this to the corresponding expression given in Proposition 8:

$$\pi_H(.) = [Y_R(\theta_H - \theta_L) \;+\; Y_P(\theta_L - 1) - 2tL - c(\theta_H)]f_R L - F(\theta_H).$$

Firm will not deviate if $\pi_H^*(.) < \pi_H(.)$, that is, if

$$\left[2tLf_R + f_P Y_P(\theta_H - 1) - f_P c(\theta_H)\right]^2 < \; 8tf_R f_P L\left[Y_R(\theta_H - \theta_L) \;+ Y_P(\theta_L - 1) - 2tL - c(\theta_H)\right].$$

It is clear from above that above inequality will hold when Y_R is high enough. It is intuitive that, it is only for the high difference in the income level that the equilibrium with *Vertical Dominance* will qualify.

Next we consider a similar possibility for the firm producing low quality. Deviation will be profitable only if it implies that the firm reduces its price to the extent that some rich are also willing to buy the low quality product. With the reduced price, firm will continue to serve all poor. Demand from the poor at this new price p_L^* is given by

$$D_L \;= f_P L.$$

Demand from rich is given by μf_R, such that

$$Y_R\theta_L - p_L^* - t\mu = Y_R\theta_H - p_H - t[L-\mu]$$

$$\Rightarrow \mu = \frac{Y_R(\theta_L-\theta_H)-(p_{L^*}-p_H)+tL}{2t}.$$

Firm's profit at the new price is

$$\pi_L(.) = [p_L^* - c(\theta_L)]\left\{f_P L + f_R\left[\frac{Y_R(\theta_L-\theta_H)-(p_L^*-p_H)+tL}{2t}\right]\right\} - F(\theta_L),$$

where, from Proposition 8, it follows that,

$$p_H \;=\; Y_R(\theta_H-\theta_L)+Y_P(\theta_L-1)-2tL.$$

The first-order condition of profit maximization with respect to price implies

$$D_L(.) \;=\; \frac{[p_L^*-c(\theta_L)]f_R}{2t}.$$

$$\Rightarrow\; p_L* = \frac{2f_PtL + f_RY_R(\theta_L-\theta_H)+f_Rp_H+f_RtL+f_Rc(\theta_L)}{2f_R}.$$

Substituting for p_H

$$p_L^* = \frac{2f_PtL+f_RY_P(\theta_L-1)+f_Rc(\theta_L)-tLf_R}{2f_R}.$$

Also, from the first-order condition of profit maximization, it follows that

$$\pi_L^*(.) \;=\; \frac{f_R[p_L^*-c(\theta_L)]^2}{2t} \;-\; F(\theta_L).$$

On substituting for p_L^*, it reduces to

$$\pi_L^*(.) \;=\; \frac{\left[f_R[Y_P(\theta_L-1)-c(\theta_L)]-tLf_R+2f_PtL\right]^2}{8tf_R} \;-\; F(\theta_L).$$

Whereas corresponding expression for profit given in Proposition 8 is

$$\pi_L(.) = f_PL[Y_P(\theta_L-1)-tL-c(\theta_L)]-F(\theta_L).$$

Firm will have no incentive to deviate only if

$$f_P L[Y_P(\theta_L - 1) - c(\theta_L) - tL] > \frac{\left[f_R[Y_P(\theta_L - 1) - c(\theta_L)] - tLf_R + 2f_P tL\right]^2}{8tf_R}.$$

That is, if

$$8f_P f_R tL[Y_P(\theta_L - 1) - c(\theta_L) - tL] > \left[f_R[Y_P(\theta_L - 1) - c(\theta_L)] - tLf_R + 2f_P tL\right]^2.$$

Again, above inequality can be written as

$$\frac{8f_P tL}{f_R}[Y_P(\theta_L - 1) - c(\theta_L) - tL] > \left[[Y_P(\theta_L - 1) - c(\theta_L)] - tL + \frac{2f_P tL}{f_R}\right]^2.$$

Which will never be true, implying that the firm will always have an incentive to deviate.

A.4 Partial Market Coverage with Horizontal Dominance

Here we derive the equilibrium when there is partial market coverage under the case of horizontal dominance. We first consider the case of the firm offering high quality product. As firm is competing for the rich and some poor are left unserved, it follows that the firm's demand is given by

$$D_H(.) = \frac{f_R\{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL\} + 2f_P\{Y_P(\theta_H - 1) - p_H\}}{2t}.$$

Profit of the firm producing high quality is given by

$$\pi_H(.) = [p_H - c(\theta_H)]D_H(.) - F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H(.) = \left[\frac{[p_H - c(\theta_H)](f_R + 2f_P)}{2t}\right].$$

On substitution and simplification this reduces to

$$2p_H(f_R+2f_P) = f_R\{Y_R(\theta_H-\theta_L)+p_L)+tL\}+2f_PY_P(\theta_H-1) + c(\theta_H)(f_R+2f_P). \quad \text{(A.4.1)}$$

$$\Rightarrow p_H = \frac{f_R\{Y_R(\theta_H-\theta_L)+p_L)+tL\}+2f_PY_P(\theta_H-1)\ +\ c(\theta_H)(f_R+2f_P)}{2(f_R+2f_P)}.$$

A similar exercise for the firm offering low quality results in the following

$$2p_L(f_R+2f_P) = f_R\{Y_R(\theta_L-\theta_H)+p_H)+tL\}+2f_PY_P(\theta_L-1)\ +\ c(\theta_L)(f_R+2f_P). \quad \text{(A.4.2)}$$

From equations (A.4.1) and (A.4.2) it follows that in equilibrium

$$\begin{aligned}&[4(f_R+2f_P)^2-f_R^2]p_L\\ &=\Big[f_R(f_R+4f_P)Y_R(\theta_L-\theta_H)+f_R(3f_R+4f_P)tL+2f_Rf_PY_P(\theta_H-1)\ +\ f_R(f_R+2f_P)c(\theta_H)\\ &\quad +4f_P(f_R+2f_P)Y_P(\theta_L-1)\ +\ 2(f_R+2f_P)^2c(\theta_L)\Big]\end{aligned}$$

and

$$\begin{aligned}&[4(f_R+2f_P)^2-f_R^2]p_H\\ &=\Big[f_R(f_R+4f_P)Y_R(\theta_H-\theta_L)+f_R(3f_R+4f_P)tL+2f_Pf_RY_P(\theta_L-1)+\ f_R(f_R+2f_P)c(\theta_L)\\ &\quad +4f_P(f_R+2f_P)Y_P(\theta_H-1)\ +\ 2(f_R+2f_P)^2c(\theta_H)\Big].\end{aligned}$$

This case arises when the poor consumer who is indifferent between the two adjacent firms is better-off by not buying. This implies that the distance at which a poor consumer is indifferent between buying and not buying high quality product is lower than that corresponding to the poor consumer, that is

$$\frac{Y_P(\theta_H-\theta_L)-(p_H-p_L)+tL}{2t} > \frac{Y_P(\theta_H-1)-p_H}{t}.$$

$$\Rightarrow tL > Y_P(\theta_H-1)+Y_P(\theta_L-1)-p_H-p_L. \quad \text{(A.4.3)}$$

From equations (A.4.1) and (A.4.2) it follows

$$p_L+p_H = \frac{2tLf_R+2f_PY_P(\theta_L-1)+2f_PY_P(\theta_H-1)+\{c(\theta_L)+c(\theta_H)\}(f_R+2f_P)}{f_R+4f_P} \quad \text{(A.4.4)}$$

Substituting for $p_L + p_H$ from equation (A.4.4) in equation (A.4.3) it follows that

$$tL > Y_P(\theta_H - 1) + Y_P(\theta_L - 1) \\ -\left\{\frac{2tLf_R + 2f_PY_P(\theta_L - 1) + 2f_PY_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P)}{f_R + 4f_P}\right\}.$$

It follows that there will be partial market coverage if

$$\frac{[3f_R + 4f_P]tL + (f_R + 2f_P)[(c(\theta_L) + c(\theta_H)]}{[f_R + 2f_P][(\theta_L - 1) + (\theta_H - 1)]} > Y_P.$$

Given that few poor are buying from either firm it follows

$$Y_P(\theta_H - 1) - p_H > 0 \text{ and } Y_P(\theta_L - 1) - p_L > 0$$

The two inequalities imply

$$Y_P[(\theta_H - 1) + (\theta_L - 1)] > p_L + p_H.$$

Substituting for $p_L + p_H$ from equation (A.4.4) above inequality reduces to

$$Y_P[(\theta_H - 1) + (\theta_L - 1)] > \frac{2tLf_R + 2f_PY_P(\theta_L - 1) + 2f_PY_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P)}{f_R + 4f_P}$$

$$\Rightarrow Y_P > \frac{2tLf_R + \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P)}{[f_R + 2f_P][(\theta_H - 1) + (\theta_L - 1)]}.$$

Also, we need a condition on Y_R to ensure that all rich are served. Therefore, the distance at which the rich is indifferent in buying and not buying, say the high quality product, is higher than the distance at which he indifferent between the two adjacent firms.

$$\Rightarrow \frac{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL}{2t} < \frac{Y_R(\theta_H - 1) - p_H}{t}.$$

Above equation together with equation (A.4.4) implies

$$Y_R[(\theta_H - 1) + (\theta_L - 1)] \\ > \frac{2tLf_R + 2f_PY_P(\theta_L - 1) + 2f_PY_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P)}{f_R + 4f_P} + tL.$$

That is,

$$[Y_R(f_R + 4f_P) - 2f_P Y_P][(\theta_H - 1) + (\theta_L - 1)] \; > \; \{c(\theta_L) + c(\theta_H)\}(f_R + 2f_P) + tL(3f_R + 4f_P).$$

A.5 Full Market Coverage for Intermediate Case

We initially look at the equilibrium when firm producing θ_L serves both rich and poor, whereas firm producing θ_H serves only rich. The firm offering low quality has to compete for rich, implying that the demand from rich is given by

$$D_{L_R}(.) \; = \; \frac{f_R[Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL]}{2t}. \tag{A.5.1}$$

As there is full market coverage, and poor buy only the low quality product, it implies that demand from poor is $f_P \times L$. So the profit of the firm offering low quality product is given by the following expression

$$\pi_L(.) = [p_L - c(\theta_L)]\left\{ f_P L + \frac{f_R[Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL]}{2t} \right\} - F(\theta_L).$$

The first-order condition of profit maximization with respect to price implies

$$D_L(.) \; = \; \frac{f_R[p_L - c(\theta_L)]}{2t} \tag{A.5.2}$$

hence

$$\pi_L(.) = \frac{f_R}{2t}[p_L - c(\theta_L)]^2 - F(\theta_L).$$

From equation (A.5.1) and (A.5.2) it follows that

$$p_L \; = \; \frac{2tLf_P + f_R[Y_R(\theta_L - \theta_H) + p_H + tL + c(\theta_L)]}{2f_R}.$$

Next we consider the firm offering high quality. Given the case that we consider here, firm producing high quality serves only rich. So profit of the firm offering high quality product is given by

$$\pi_H(.) = [p_H - c(\theta_H)]\left[\frac{f_R[Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t} \right] - F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H(.) \;=\; \frac{f_R[p_H - c(\theta_H)]}{2t} \tag{A.5.3}$$

and

$$\pi_H(.) = \frac{f_R}{2t}[p_H - c(\theta_H)]^2 - F(\theta_H)$$

From equation (A.5.3) it follows that

$$p_H \;=\; \frac{Y_R(\theta_H - \theta_L) + p_L + tL + c(\theta_H)}{2}.$$

Substituting for p_H in the expression for profit implies that firm's profit is given by

$$\pi_H(.) \;=\; \frac{f_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 \;-\; F(\theta_H). \tag{A.5.4}$$

The two first-order conditions imply that in equilibrium

$$p_L = tL + \frac{c(\theta_H)}{3} + \frac{2c(\theta_L)}{3} + \frac{Y_R(\theta_L - \theta_H)}{3} + \frac{4tLf_P}{3f_R}.$$

and

$$p_H = tL + \frac{Y_R(\theta_H - \theta_L)}{3} + \frac{2c(\theta_H)}{3} + \frac{c(\theta_L)}{3} + \frac{2tLf_P}{3f_R}.$$

From the above two equations it follows that

$$p_H - p_L \;=\frac{2Y_R(\theta_H - \theta_L)}{3} + \frac{c(\theta_H) - c(\theta_L)}{3} - \frac{2tLf_P}{3f_R}.$$

Let us now consider the condition under which the above can be sustained as an equilibrium. Firm producing low quality already serves both the income types. So the possible deviation can be from the firm producing high quality. Firm may consider reducing its price to the extent so that it is able to serve some consumers with low income as well. In which case profit of the firm at the new price p_H^* is given by

$$\pi_H^* = [p_H^* - c(\theta_H)]\left\{f_P\left[\frac{Y_P(\theta_H - \theta_L) - (p_H^* - p_L) + tL}{2t}\right] + f_R\left[\frac{Y_R(\theta_H - \theta_L) - (p_H^* - p_L) + tL}{2t}\right]\right\} - F(\theta_H).$$

The first-order condition with respect to price implies

$$D_H(.) = \frac{(f_P + f_R)[p_H^* - c(\theta_H)]}{2t}.$$

It follows that

$$p_H^* = \frac{(f_P Y_P + f_R Y_R)(\theta_H - \theta_L)}{2(f_P + f_R)} + \frac{p_L + tL + c(\theta_H)}{2}.$$

So in equilibrium, profit of the firm offering high quality is given by

$$\pi_H^*(.) = \frac{1}{2t}\left[\frac{(f_P Y_P + f_R Y_R)(\theta_H - \theta_L)}{2(f_P + f_R)} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 - F(\theta_H).$$

From equation (A.5.4) it follows that the corresponding expression for profit of the firm offering high quality is

$$\pi_H(.) = \frac{f_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 - F(\theta_H).$$

So the firm will not deviate if $\pi_H(.) > \pi_H^*(.)$, that is, if

$$\frac{f_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 > \frac{1}{2t}\left[\frac{(f_P Y_P + f_R Y_R)(\theta_H - \theta_L)}{2(f_P + f_R)} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2.$$

Above inequality is more likely to hold for high income of the rich consumers. This is because high income of rich consumers allows the firm offering high quality to charge high price, that more than offsets for the lost demand from the poor section of the society. Similarly we can work out the case, when firm producing θ_H serves both income types, but firm producing θ_L serves only poor. For this case firm offering low quality has an incentive to deviate. Similar calculations as above imply that firm will not deviate if

$$\begin{aligned}&\frac{f_P}{2t}\left[\frac{Y_P(\theta_L - \theta_H)}{2} + \frac{p_H + tL - c(\theta_L)}{2}\right]^2 \\ &> \frac{(f_P + f_R)}{2t}\left[\frac{(f_P Y_P + f_R Y_R)(\theta_L - \theta_H)}{2(f_P + f_R)} + \frac{p_H + tL - c(\theta_L)}{2}\right]^2.\end{aligned}$$

Clearly, this will never be true. So there can never be an equilibrium when firm offering high quality product serves both rich and poor, whereas firm offering low quality product serves only the poor.

Chapter 4

Regional Inequality, Location Choice and Quality Ladder

4.1 Introduction

In the last two chapters we have restricted ourselves to the spatial structure where rich and poor live side by side in the same location. But any discussion on neighbourhood effects will be incomplete until it addresses the scenario where the rich and poor are geographically segregated – poor living in the poor ghettos while rich live in the rich neighbourhoods. We take up this issue of regional inequality in this chapter. The moment we start discussing the issue of regional inequality, the focus naturally shifts to the question of firm location. Will a firm always find it profitable to locate in the rich neighbourhood? Or are there some advantages in locating in the poor region also?[1] So far the firms in our discussion are quite passive in terms of location decision: the location of the two firms are fixed in chapter 3; although location decision

[1]For a selective survey of firms' location choice under spatial structure, see Kilkenny and Thisse (1999).

was allowed in chapter 2, but, because of the focus on symmetric equilibrium, the entering firms had to locate symmetrically in the circular city. In this chapter we take up the firms' location decision seriously.

To investigate the interaction of regional inequality with the spatial aspect we consider a circular city with two regions, rich and poor, where the potential customers have to bear a travel or transportation cost to access the facility under consideration. The interaction of regional inequality with the spatial aspect gives rise to an interesting trade-off in terms of firms' location decision. The trade-off is between the consumers' willingness to pay and the firm's potential market size. Since the richer consumers have a higher willingness to pay, the firm has obvious incentive to locate in the rich region. But, in the presence of travel costs, the preference structure implies that to access the same quality product a rich customer is willing to travel further than a poor customer. Then if a firm locates in the rich region, it is almost certainly going to lose the poor customers as they will find it costlier to travel all the way to the rich region to access the product. Instead if the firm locates in the poor region, it serves all the poor who live close by, and the rich may not mind to travel to the poor region to access the product. This trade-off between willingness to pay and market size determine firms' location choice.

To illustrate this trade-off in a clean and simple way we first discuss the scenario when there is a single firm in the city deciding whether to locate in the poor or rich region. This trade-off leads to an interesting result: when the income gap between rich and poor is relatively narrower and the travel cost and the size of the poor region is relatively lower, there exists an equilibrium where the monopoly firm locates in the poor region. By locating in the poor neighborhood the firm ensures a larger market size by exploiting the rich consumers' willingness to travel higher distance. It serves all the poor in the neighbourhood and some rich who live relatively closer to the poor neighbourhood. Firm's location choice has important implications for the welfare of the consumers. If the firm locates in the rich region (possibly at the center), no poor consumer gets to access the product, and some of the rich, living closer to the poor region, are also excluded. Just the opposite happens when the firm locates in the poor region.

Next we extend our analysis by allowing free entry and exit so that the number of firms is determined endogenously. The intuition spelt out for the monopoly firm case extends here too: the location equilibrium is such that the area of operation of terminal firms in the poor region encroaches into the neighbouring richer region, that is, some rich people commute to the terminal firm located in the poor region to access the product, and not the other way round. In equilibrium, there are quality and price ladders where consumers residing in the interior of the rich region are offered the highest quality and are charged with the highest price. Finally we find that the utilities of consumers located at comparable distances from the firm increase with the increase in income.

Similar in spirit to our work is the paper by Bacchiega and Minniti (2009). They analyze a location choice model with two vertically differentiated firms and two regions with different incomes. Goods can be shipped across the regions at a constant unit transportation cost borne by the consumers. They show that when goods are sufficiently differentiated and income disparities are significant, high quality producers are less keen on settling in a rich region than low quality ones since the demand for their products is less distorted by transport costs. Instead, by choosing a poorer region high-quality producers better exploit this location's lower willingness to pay, while the opposite holds for low-quality firms. Since the markets are segmented, each firm practices price discrimination between markets. In our framework, firms are not allowed to price discriminate. Also in our spatial structure, even with the same income consumers vary with respect to their location. So while choosing price and quality firm faces an inverse relation between price and demand even from the consumers of the same income group. The fact that the firm cannot price discriminate between the consumers and each individual faces different transportation cost contributes to our result on the firm's location choice.

The chapter is organized as follows. Section 4.2 lays out the basic framework while section 4.3 discusses the location choice of a monopoly firm. In section 4.4 we analyze the general case with free entry and exit. Finally we conclude in section 4.5. The Appendix contains the proofs.

4.2 Basic Model

We consider a circular city with circumference L units. Consumers are uniformly distributed across the city. Each consumer is defined by location z on the circular city and income level Y, where $Y \in \{Y_R, Y_P\}$, and $Y_R > Y_P$. In order to capture the idea of regional segregation, we assume that the city has two regions: North and South of size ϕL and $(1-\phi)L$ respectively. At any stretch of the city income of the consumers is the same. Income in the North, Y_R, is greater than the level of income in the South, Y_P.

There are n firms in the city and we use the notations x_j for location of firm j, p_j for price charged by firm j and θ_j for quality of the product offered by firm j, $j = 1, 2, ..., n$. Utility of a consumer at location z with income Y and purchasing from firm j is given by:

$$U(z, Y, j) = Y\theta_j - p_j - t|x_j - z|,$$

where t denotes per unit travel cost. Note that the reservation utility (the utility the consumer enjoys without having the product) is Y, implying that minimum quality is

$\theta_j = 1$. Note that the particular form of the utility function captures the idea that for the same increase in quality people with higher income are also willing to pay more. This is reasonable as the richer individual is more likely to be better informed about the benefits of higher quality, and hence would value it more. It is also evident is that disutility from traveling is linear in distance.

There is a fixed cost of production that depends on the quality of the product, $F(\theta_j)$. We assume that this fixed cost of production is strictly increasing and convex in quality: $F'(\theta_j) > 0$, $F''(\theta_j) > 0$. The marginal cost of production, $c(\theta_j)$, is independent of output but depends on quality. We assume that the marginal cost of production is also strictly increasing and convex in quality: $c'(\theta_j) > 0$, $c''(\theta_j) > 0$. Profit of firm j charging a price p_j and offering the quality θ_j is given by

$$\pi_j = [p_j - c(\theta_j)]D_j - F(\theta_j),$$

where D_j denotes demand faced by firm j.

The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms decide their location. In the second stage, firms decide price and quality simultaneously.

4.3 Location Choice and Product Quality under Monopoly

In this chapter we are interested in examining the location choice of firms when there is regional inequality. To develop the basic intuition for the trade-off firms face in locating at the rich or poor region, in this section we begin with the location choice when there is only a single firm in the city. The general case where the number of firms is determined endogenously is taken up in the following section.

When there is a single firm, the basic idea is that the firm will choose to locate in the most profitable location taking into account the consumers' willingness to pay.

The monopolist's problem is to decide its location and price-quality combination to offer so as to maximize

$$\pi = [p - c(\theta)]D(\cdot) - F(\theta).$$

Given the spatial nature, the monopolist's demand, $D(\cdot)$, is determined by the distance of the marginal consumer who is indifferent in buying and not buying from him.

In general, where consumers are identified only by their incomes, price chosen by the monopolist is such that the consumers are pushed to their reservation utility. But in spatial model, even with the same income, consumers vary with respect to their location. So while choosing price and quality the monopolist faces an inverse relation between price and demand even from consumers of the same income group. In order to avoid existence issues, we assume that the income level of the consumers in poor region is high enough that the monopolist would make strictly positive profits by operating from there.

Consider first the structure of demand faced by the monopolist. Since there is only one firm, demand is determined by the distance, on either side, of the consumer who is indifferent between buying and not buying. Suppose, in general, this indifferent consumer is a rich consumer (with income Y_R) on one side and a poor consumer (with income Y_P) on the other side. The distance of the indifferent rich consumer, d_R, is determined from $Y_R\theta - p - td_R = Y_R$, that is, $d_R = \dfrac{Y_R(\theta-1)-p}{t}$. Similarly the distance of the indifferent poor consumer, d_P, is given by $d_P = \dfrac{Y_P(\theta-1)-p}{t}$. Putting together, the demand faced by the monopolist is $D(\cdot) = d_R + d_P = \dfrac{2\left[\frac{Y_R+Y_P}{2}(\theta-1)-p\right]}{t}$. Let us denote by Y the average of the incomes of the two indifferent consumers the monopolist faces on two sides. Clearly $Y = Y_R$ when the indifferent consumer on either side is rich. Similarly when the marginal consumer on either side is poor, $Y = Y_P$.

Finally, when the indifferent consumer on one side is rich and on the other side is poor, $Y = \dfrac{Y_R + Y_P}{2}$. It is easy to see that when the monopolist charges a price p, offers quality θ and the average of the incomes of the two indifferent consumers it faces is Y, its demand is given by

$$D(p, \theta; Y) = \frac{2[Y(\theta - 1) - p]}{t}.$$

Hence its profit is

$$\pi(p, \theta; Y) = 2[p - c(\theta)]\left[\frac{Y(\theta - 1) - p}{t}\right] - F(\theta).$$

In the price-and-quality stage, given entry and location decisions in the earlier stage, the monopolist decides its price and quality to maximize profit, $\pi(\cdot)$. The first-order conditions with respect to price and quality are given by

$$\frac{\partial \pi(\cdot)}{\partial p} = \frac{Y(\theta - 1) - p}{t} - \frac{p - c(\theta)}{t} = 0,$$

and

$$\frac{\partial \pi(\cdot)}{\partial \theta} = -2c'(\theta)\left[\frac{Y(\theta - 1) - p}{t}\right] + 2[p - c(\theta)]\frac{Y}{t} - F'(\theta) = 0.$$

These two first-order conditions imply that price of the monopolist is given by

$$p = \frac{Y(\theta - 1) + c(\theta)}{2}, \tag{4.1}$$

and the quality is determined from

$$Y = c'(\theta) + \frac{tF'(\theta)}{Y(\theta - 1) - c(\theta)}. \tag{4.2}$$

It can be checked that there exists a positive relationship between quality and the average of incomes of the two indifferent consumers. Simple differentiation of (4.2) results in

$$\frac{d\theta}{dY} = \frac{[Y(\theta - 1) - c(\theta)] + [Y - c'(\theta)](\theta - 1)}{c''(\theta)[Y(\theta - 1) - c(\theta)] + tF''(\theta) - [Y - c'(\theta)]^2}. \tag{4.3}$$

To establish the positive relationship between θ and Y, we invoke the sufficient condition for profit maximization. The second order sufficient condition for profit maximization requires $\frac{\partial \pi(\cdot)}{\partial \theta^2} \times \frac{\partial \pi}{\partial p^2} > \left[\frac{\partial \pi}{\partial \theta \partial p}\right]^2$, which, upon substitution, simplifies to $c''(\theta)[Y(\theta-1)-c(\theta)] + tF''(\theta) - [Y - c'(\theta)]^2 > 0$. Hence as long as the sufficient condition of profit maximization is satisfied, there exists a positive relation between Y and θ. It turns out that the restriction for the positive relation (as well as for the sufficient condition for profit maximization) is an assumption on the convexity of the cost structure. As consumers trade-off money with quality at constant marginal rate of substitution and the two income types are spatially segregated, enough convexity on the cost structure is important to avoid a situation in which better quality is offered in the poorer region. It follows from (4.1) that there is a positive relationship between p and Y too. There are two effects at work. Given quality, consumers' willingness to pay increases directly as his valuation for the product increases as Y increases. This effect is reinforced as quality itself increases when Y increases. The monopolist exploits this higher willingness to pay to charge a higher price.

But what happens to the monopolist's profit when Y increases? There is a clear trade-off: the higher willingness to pay allows him to charge a higher price, but, at the same time, the better quality provision has a cost implication also. Using conditions (4.1) and (4.2) the expression for realized profit when equilibrium strategies are used in the price-and-quality subgame is given by

$$\pi = \frac{1}{2t}\left[\frac{tF'(\theta)}{Y - c'(\theta)}\right]^2 - F(\theta). \tag{4.4}$$

It is shown in Appendix A.1 that $\frac{d\pi}{dY} > 0$, that is, the monopolist's profit increases as the average of incomes of the two indifferent consumers it serves goes up. Thus, in the trade-off between price and cost mentioned above, it turns out that the increase in cost is dominated by the increase in price, resulting in a higher profit for the monopolist. There are two properties that are important for this to hold. First is the single crossing property, that is, for the same marginal increase in quality, people with higher income are willing to pay more. Because of this a consumer with higher income are willing to travel more. The second property is the convexity in the cost structure which guarantees that higher quality will be produced in the richer region. These two properties together ensure that the monopolist's profit increases in the income of the region.

In the location stage, this relationship between realized profit in the price-and-quality stage and the average of incomes of the two indifferent consumers guides the monopolist to choose its location: it should choose its location so that the average of incomes of the two indifferent consumers is such that its profit is maximized. Recall that Y, the average of incomes of the two indifferent consumers that the monopolist serves, takes 3 possible values: $Y_R > \dfrac{Y_R + Y_P}{2} > Y_P$. Since $\dfrac{d\pi}{dY} > 0$, the monopolist's profit is maximized when $Y = Y_R$. That is, the monopolist locates such that the indifferent consumer it serves on either side is a rich consumer.

There are two alternative location possibilities in which the monopolist serves a rich indifferent consumer on either side. The first possibility is that the monopolist serves a rich indifferent consumer on either side by locating anywhere in the North. The second possibility is to locate in the South pole, serve all the poor consumers residing in the South and then reach further by serving some rich consumers in the North. In what follows we identify parameter values under which each possibility becomes an equilibrium location.

Since $Y = Y_R$, it follows that for either of these two location choices the price (p_R) and quality (θ_R) of the monopolist are determined by using $Y = Y_R$ in (4.1) and (4.2), that is,

$$p_R = \frac{Y_R(\theta_R - 1) + c(\theta_R)}{2},$$

and

$$Y_R = c'(\theta_R) + \frac{tF'(\theta_R)}{Y_R(\theta_R - 1) - c(\theta_R)}.$$

Hence the size of demand faced by the monopolist is given by

$$D_R = \frac{2[Y_R(\theta_R - 1) - p_R]}{t} = \frac{Y_R(\theta_R - 1) - c(\theta_R)}{t}.$$

Recall that the size of North is ϕL. Clearly the first possibility (monopolist locates in the North) qualifies to be an equilibrium location if and only if the optimal demand size, D_R, is less than the size of North, ϕL, that is,

$$Y_R(\theta_R - 1) - c(\theta_R) \leq t\phi L. \tag{4.5}$$

It is easy to check that the left hand side of condition (4.5) is an increasing function of Y_R.[2] Hence, given ϕ, L and t, condition (4.5) defines an upper bound for Y_R, call it Y_R^ϕ, such that the monopolist locating in the North is an equilibrium if and only if the rich income is below this upper bound, that is, $Y_R \leq Y_R^\phi$. Clearly this upper bound increases with ϕ, L and t. Thus, in equilibrium, the monopolist is more likely to locate in the North when the size of North is relatively larger and the travel cost is relatively higher.

Since the size of South is $(1-\phi)\,L$, for the second possibility (monopolist locates at the South pole) to be an equilibrium outcome it must be the case that $D_R > (1-\phi)\,L$, that is,

$$Y_R(\theta_R - 1) - c(\theta_R) > t\,(1-\phi)\,L.$$

At the same time, the poor residing at the border of North and South, that is, residing at the distance $\dfrac{(1-\phi)\,L}{2}$ from the South pole, must strictly prefer to buy the product

[2] $\dfrac{d\,[Y_R(\theta_R - 1) - c(\theta_R)]}{dY_R} = (\theta_R - 1) + [Y_R - c'\,(\theta_R)]\,\dfrac{d\theta_R}{dY_R} > 0$ since $\theta_R > 1$, $Y_R > c'\,(\theta_R)$, and $\dfrac{d\theta_R}{dY_R} > 0$.

than go without it, that is,

$$Y_P\theta_R - p_R - \frac{t\,(1-\phi)\,L}{2} > Y_P,$$

that is,

$$Y_P\,(\theta_R - 1) - \frac{Y_R(\theta_R - 1) + c(\theta_R)}{2} > \frac{t\,(1-\phi)\,L}{2}.$$

When this condition holds, we have

$$2Y_P\,(\theta_R - 1) > [Y_R(\theta_R - 1) - c(\theta_R)] + 2c(\theta_R) + t\,(1-\phi)\,L$$

$$\Rightarrow 2\,[Y_P\,(\theta_R - 1) - c(\theta_R)] > 2t\,(1-\phi)\,L,$$

using the condition $Y_R(\theta_R - 1) - c(\theta_R) > t\,(1-\phi)\,L$ mentioned above. Hence we conclude that locating at the South pole is an equilibrium outcome if and only if the following condition holds:

$$Y_P\,(\theta_R - 1) - c(\theta_R) > t\,(1-\phi)\,L. \tag{4.6}$$

Similar to condition (4.5), the left hand side of condition (4.6) is also an increasing function of Y_R.[3] Thus, given ϕ, L, t and Y_P, condition (4.6) defines a lower bound for Y_R, call it $Y_R^{1-\phi}$, such that the monopolist locating at the South pole is an equilibrium if and only if the rich income is above this lower bound, that is, $Y_R > Y_R^{1-\phi}$. Clearly this lower bound increases with $(1-\phi)$, L and t. Thus, in equilibrium, the monopolist is more likely to locate at the South pole when the size of South is relatively smaller and the travel cost is relatively lower. Interestingly the lower bound decreases with Y_P, implying that the monopolist is more likely to locate at the South pole when the poor income is reasonably high, that is, the income gap between the rich and poor is not substantial.

[3] $\frac{d\,[Y_P(\theta_R - 1) - c(\theta_R)]}{dY_R} = [Y_P - c'\,(\theta_R)]\,\frac{d\theta_R}{dY_R} > 0$ as long as $Y_P > c'\,(\theta_R)$.

Although locating at the South pole seems somewhat surprising, it is quite intuitive to see when this possibility occurs and it brings out clearly the trade-off between willingness to pay and market size in choosing firm location. Since the rich consumer has a higher willingness to pay, he is willing to travel further too. When the size of South is relatively lower so that the rich residing closer to the border does not have to travel too far and the travel cost is lower, the rich residing closer to the border does not mind traveling all the way to the South pole to buy the product. The higher rich income strengthens this effect further. At the same time the poor income has to be high enough so that the monopolist serves all the poor consumers before reaching out to the rich.

Note from conditions (4.5) and (4.6) that when $\phi > \frac{1}{2}$ it is possible to have $Y_R^{1-\phi} < Y_R^{\phi}$ as long as Y_P is not substantially lower than Y_R. In this case it is possible to have multiple equilibria when $Y_R^{1-\phi} < Y_R < Y_R^{\phi}$: the monopolist locating in the North is an equilibrium outcome, and, at the same time, locating at the South pole is also an equilibrium outcome. Otherwise, that is, when $\phi \leq \frac{1}{2}$, we have $Y_R^{\phi} < Y_R^{1-\phi}$, and the equilibrium is unique: the monopolist locates in the North and serves only rich consumers when $Y_R \leq Y_R^{\phi}$; it locates at the South pole and all the poor consumers are served when $Y_R > Y_R^{1-\phi}$.[4] It is quite intriguing to note that locating at the South pole is the unique equilibrium outcome when the rich income is very high. In this case the effect of higher willingness to pay by the rich clearly dominates the market size effect. Of course, the poor income also has to be high enough.

[4]Of course when $\phi \leq \frac{1}{2}$ and $Y_R^{\phi} < Y_R < Y_R^{1-\phi}$, neither of these two location possibilities occur in equilibrium. In this case the monopolist cannot serve a rich indifferent consumer on either side in equilibrium. That is, it is not possible for the monopolist to earn the maximum possible potential profit; it has to settle down for the next best alternative, that is, serve a rich indifferent consumer on one side and a poor indiferent consumer on the other side.

4.4 Location Choice and Product Quality under Free Entry

We introduce free entry and exit in this section. We are interested in finding out how the presence of regional inequality affects the industrial structure – differences in the number of firms, their location choices and quality of their products in the poor and rich regions, and the impact of this industrial structure on the welfare of the rich and poor living in these two segregated regions.

The model structure – geographically segregated habitats of rich and poor, consumer's preference and firm's cost structure – is the same as in the last section. The difference is that instead of a single monopoly firm the number of firms is now endogenously determined. The set-up is a two-stage game. In the first stage, firms decide whether to enter or not and they also choose their locations. In the second stage, firms decide price and quality simultaneously. In the following analysis strategies will be restricted not to involve 'undercutting' where one of the firms will be left with zero demand through the actions of its opponents (see Novshek (1980) for details). Also, unlike the earlier part of the thesis, in this section we analyze only the situation where all the consumers, rich and poor, are served by the market.

4.4.1 Demand Structure

Consider firm j located between the two adjacent firms $j-1$ and $j+1$. The locations of these three firms are denoted by x_j, x_{j-1} and x_{j+1} respectively. Similarly, product qualities of these three firms are denoted by θ_j, θ_{j-1} and θ_{j+1} respectively. Since all the consumers are served, demand faced by firm j is the sum of the distances from firm j of the marginal consumers indifferent between firms j and $j-1$ (denoted by

$\delta_{j,j-1}$) and between firms j and $j+1$ (denoted by $\delta_{j,j+1}$). Let $Y_{j,j-1}$ denote the income of the marginal consumer who is indifferent between firms $j-1$ and j. Similarly, let $Y_{j,j+1}$ denote the income of the marginal consumer who is indifferent between firms j and $j+1$.

Clearly we have

$$\delta_{j,j-1} = \frac{(p_{j-1} - p_j) + t\,|x_j - x_{j-1}| + Y_{j,j-1}\,(\theta_j - \theta_{j-1})}{2t}$$

and

$$\delta_{j,j+1} = \frac{(p_{j+1} - p_j) + \frac{t}{n} + Y_{j,j+1}\,(\theta_j - \theta_{j+1})}{2t}$$

so that demand faced by firm j, D_j, is given by

$$\begin{aligned} D_j &= \delta_{j,j-1} + \delta_{j,j+1} \\ &= \frac{(p_{j+1} + p_{j-1} - 2p_j) + \frac{t}{n} + t\,|x_j - x_{j-1}| + Y_{j,j+1}\,(\theta_j - \theta_{j+1}) + Y_{j,j-1}\,(\theta_j - \theta_{j-1})}{2t}. \end{aligned} \tag{4.7}$$

Since $|x_{j+1} - x_j| + |x_j - x_{j-1}| = |x_{j+1} - x_{j-1}|$, it is interesting to note that given the locations of the two adjacent firms, x_{j-1} and x_{j+1}, the precise location of firm j does not matter for its own demand. This observation has an important implication that we will exploit in the analysis of the location stage below. Note also that $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$; that is, since all the consumers are served the price response to demand is independent of income. But, as expected, the quality response to demand does depend on income: $\frac{\partial D_j}{\partial \theta_j} = \frac{Y_{j,j-1} + Y_{j,j+1}}{2t}$; it depends on the average of the incomes of the two marginal consumers firm j faces on either side.

4.4.2 Price and Quality Stage

In stage 2, given entry and location decisions in the earlier stage, each firm decides its price and quality to maximize profit, π_j. The first-order conditions with respect to price and quality are given by

$$\frac{\partial \pi_j}{\partial p_j} = D_j + [p_j - c(\theta_j)] \frac{\partial D_j}{\partial p_j} = 0,$$

and

$$\frac{\partial \pi_j}{\partial \theta_j} = -c'(\theta_j) \cdot D_j + [p_j - c(\theta_j)] \frac{\partial D_j}{\partial \theta_j} - F'(\theta_j) = 0.$$

Substituting the price and quality responses to demand, $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$ and $\frac{\partial D_j}{\partial \theta_j} = \frac{Y_{j,j-1} + Y_{j,j+1}}{2t}$, these two first-order conditions reduce to

$$\frac{Y_{j,j-1} + Y_{j,j+1}}{2t} = c'(\theta_j) + \frac{tF'(\theta_j)}{p_j - c(\theta_j)}. \tag{4.8}$$

The first-order condition with respect to price implies

$$\pi_j = \frac{1}{t}[p_j - c(\theta_j)]^2 - F(\theta_j).$$

Now using condition (4.8) the expression for realized profit when the equilibrium strategies are used in this price-and-quality subgame is given by

$$\pi_j(x,n) \equiv \pi_j(x,n,p_j(x,n),\theta_j(x,n)) = \frac{1}{t}\left[\frac{tF'(\theta_j(x,n))}{\frac{Y_{j,j-1}+Y_{j,j+1}}{2t} - c'(\theta_j(x,n))}\right]^2 - F(\theta_j(x,n)). \tag{4.9}$$

Here $x = (x_1, x_2, ..., x_n)$ stands for the locations of the n entering firms and, given entry and location decisions in stage 1, $p_j(x,n)$ and $\theta_j(x,n)$ denote the equilibrium price and quality strategies of firm j in the price-and-quality subgame.

4.4.3 Location and Entry Stage

In stage 1 firms also choose locations. Since there are two segregated regions with two distinct income groups, for the analysis of the location stage we need to distinguish between two possible types of firms: *interior firms* and *terminal firms*. Interior firms face the marginal consumers with the same income on either side, that is, $Y_{j,j-1} = Y_{j,j+1}$ for interior firm j. Terminal firms, on the other hand, face marginal consumers with two different incomes, that is, $Y_{j,j-1} \neq Y_{j,j+1}$ for terminal firm j. Since the city is circular, there are potentially two terminal firms. Each terminal firm could be located either in the poor region, or in the rich region, or at the border. Where are they located in equilibrium is an interesting question that we explore at this location stage.

Firms choose locations expecting to receive the profits of the implied equilibrium of the price-and-quality subgame. Thus the objective function of firm j in the location stage is $\pi_j(x,n) \equiv \pi_j(x,n,p_j(x,n),\theta_j(x,n))$ as expressed in equation (4.9) above. In this expression for profit product quality $\theta_j(x,n)$ is determined from (4.8). Note that the first-order condition with respect to price implies $p_j - c(\theta_j) = tD_j$. Since, as argued above, D_j is independent of x_j, it is clear from equation (4.8) that $\theta_j(x,n)$ is independent of x_j. Then it follows from (4.9) that $\pi_j(x,n)$ is independent of x_j too. That is, the realized profit of firm j in the price-and-quality subgame is independent of its precise location, given, of course, the locations of the two adjacent firms, $j-1$ and $j+1$.

But note that $\pi_j(x,n)$ depends on the incomes of the two marginal consumers, $Y_{j,j-1}$ and $Y_{j,j+1}$, and a change in location may affect profit by changing the identity of either of the marginal consumers. Since an interior firm faces the same type of marginal consumer from either side, its profit remains unaffected by small changes in its location.[5] A shift of the location of a firm to one side results in a gain of as many consumers as a loss of consumers on the other side. Therefore, for the interior firms, a symmetric equilibrium of location choice exists with each firm facing the same demand from either side.

For a terminal firm j, since $Y_{j,j-1} \neq Y_{j,j+1}$, a change in location may affect profit

[5]This is an artifact of the linear model with inelastic demand, first noticed by Novshek (1980).

by changing the identity of either of the marginal consumers. We explore below how this affects the choice of a terminal firm to locate in the poor region, or in the rich region, or at the border. As long as there is no room for confusion we denote the terminal firm under consideration as firm T and its adjacent firm in the poor region as firm P and in the region as firm $R.$

Consider first the possibility whether a terminal firm would be located at the border of the poor and rich regions. Let us examine its incentives to relocate. Note that the marginal movements of firm T will not make any difference unless the identity of the marginal consumer changes. Being located at the border, the marginal consumer it faces is a rich consumer while competing with firm $R,$ whereas it is a poor consumer when it competes with firm $P.$ And, following equation (4.7), the demand it faces is given by

$$D_T = \frac{(p_R + p_P - 2p_T) + t\,|x_R - x_P| + Y_R\,(\theta_T - \theta_R) + Y_P\,(\theta_T - \theta_P)}{2t}.$$

Suppose that firm T moves towards firm R to the extent that the marginal consumer it gets when it competes with firm P is a rich consumer instead of a poor. With this relocation firm T's demand becomes

$$D_T' = \frac{(p_R + p_P - 2p_T) + t\,|x_R - x_P| + Y_R\,(\theta_T - \theta_R) + Y_R\,(\theta_T - \theta_P)}{2t}.$$

Hence the gain in demand to firm T from this relocation is

$$D_T' - D_T = \frac{1}{2t}\,(Y_R - Y_P)\,(\theta_T - \theta_P)\,.$$

But, for this potential gain to be effective, this move of firm T towards firm R has to be feasible. That is, the distance between firms T and R has to be large enough so that firm T gets enough room to move towards firm R to change the identity of the marginal consumer from poor to rich when it competes with firm $P.$ To define this feasibility condition recall our notation δ_{TP} that stands for the distance from firm T

of the marginal consumer indifferent between firms T and P. Since with any movement of the position of a firm the position of the marginal consumer changes only by half of the distance moved, the minimum distance that firm T needs to move before the identity of the marginal consumer changes from poor to rich is $2\delta_{TP}$. Hence the effective move of firm T towards firm R will be feasible only if $|x_R - x_T| > 2\delta_{TP}$.

On the other hand if firm T moves towards firm P to the extent that the marginal consumer it faces while competing with firm R is a poor consumer instead of a rich one, then its demand becomes

$$D''_T = \frac{(p_R + p_P - 2p_T) + t\,|x_R - x_P| + Y_P\,(\theta_T - \theta_R) + Y_P\,(\theta_T - \theta_P)}{2t}.$$

Now the gain in demand to firm T from this relocation is

$$D''_T - D_T = \frac{1}{2t}\,(Y_R - Y_P)\,(\theta_R - \theta_T)\,.$$

As above, this effective move of firm T towards firm P will be feasible only if $|x_P - x_T| > 2\delta_{TR}$ where δ_{TR} denotes the distance from firm T of the marginal consumer indifferent between firms T and R.

Consider next the possibility whether a terminal firm would be located in the rich region. In the same way as above firm T gains if it relocates, though there are differences in the minimum distances that it needs to move in either direction before it can gain. The minimum distance firm T needs to move now is *less* than $2\delta_{TP}$ towards firm R and *more* than $2\delta_{TR}$ towards firm T.

Finally consider the possibility whether a terminal firm would be located in the poor region. Once again firm T can gain if it relocates. The minimum distance firm T needs to move now is *more* than $2\delta_{TP}$ towards firm R and *less* than $2\delta_{TR}$ towards firm T.

These feasibility conditions discussed here point out the necessary condition for the locations of the two terminal firms to be an equilibrium. We will use this condition below to pin down the location of firms in equilibrium.

In stage 1, firms also decide whether to enter or not. Entry (that is, the number of operating firms) is determined by the zero-profit condition:

$$\pi_j = \frac{1}{t}[p_j - c(\theta_j)]^2 - F(\theta_j) = 0. \tag{4.10}$$

4.4.4 Structure of Equilibrium

Now we proceed to characterize the structure of the equilibrium. Consider first the *interior firms located in the poor region*. Since for any such firm j, $Y_{j,j-1} = Y_P = Y_{j,j+1}$, it follows from equations (4.8) and (4.10) that p_j and θ_j are jointly determined from the following two equations:

$$c'(\theta_j) + \frac{tF'(\theta_j)}{p_j - c(\theta_j)} = Y_P$$

and

$$\frac{1}{t}[p_j - c(\theta_j)]^2 - F(\theta_j) = 0.$$

Since the conditions are the same for each interior firm located in the poor region and depend on the common parameters t and Y_P, it follows that each such firm charges the same price (call it p_P) and produces the same quality product (call it θ_P). Upon some simple substitutions we find that the sequence of determination is as follows. Given t and Y_P, first θ_P is determined from

$$c'(\theta_P) + \sqrt{t} \cdot \frac{F'(\theta_P)}{\sqrt{F(\theta_P)}} = Y_P. \tag{4.11}$$

Then p_P is determined from

$$p_P = c(\theta_P) + \sqrt{tF(\theta_P)}. \tag{4.12}$$

Similarly each *interior firm located in the rich region* produces the same quality product (θ_R) and charges the same price (p_R) that are determined, respectively, from the following two conditions:

$$c'(\theta_R) + \sqrt{t} \cdot \frac{F'(\theta_R)}{\sqrt{F(\theta_R)}} = Y_R \tag{4.13}$$

and

$$p_R = c(\theta_R) + \sqrt{tF(\theta_R)}. \tag{4.14}$$

Finally, each of the two *terminal firms* produces the same quality product (θ_T) and charges the same price (p_T) that are determined, respectively, from the following two conditions:

$$c'(\theta_T) + \sqrt{t} \cdot \frac{F'(\theta_T)}{\sqrt{F(\theta_T)}} = \frac{Y_P + Y_R}{2} \tag{4.15}$$

and

$$p_T = c(\theta_T) + \sqrt{tF(\theta_T)}. \tag{4.16}$$

We are obviously interested in comparing the prices and product qualities of the three types of firms: interior poor, interior rich and terminal firms. The three equations, (4.11), (4.13), and (4.15), make it clear that the general relationship between product quality (Q) and the average income of the two marginal consumers (Y) is given by

$$c'(\theta) + \sqrt{t} \cdot \frac{F'(\theta)}{\sqrt{F(\theta)}} = Y.$$

Imposing enough convexity on the cost structure it is easy to check that there is a positive relationship between product quality and income.[6] Since $Y_P < \dfrac{Y_P + Y_R}{2} < Y_R$, it follows that, in equilibrium, we have

$$\theta_P < \theta_T < \theta_R.$$

Also, it follows from the three equations, (4.12), (4.14), and (4.16), that the general relationship between price (p) and product quality (Q) is

$$p = c(\theta) + \sqrt{tF(\theta)}.$$

[6]The sufficient condition is $c''(\theta) \geq 0$ and $F''(\theta) > \dfrac{[F'(\theta)]^2}{2F(\theta)}$.

Since $c'(\theta) > 0$ and $F'(\theta) > 0$, and $\theta_P < \theta_T < \theta_R$, it follows that

$$p_P < p_T < p_R.$$

Thus, in equilibrium, there exists *price and quality ladders* with the highest quality offered and the highest price charged by the firms located in the interior rich region, followed by the two terminal firms and then by the firms located in the interior poor region.

Price and quality of the interior and terminal firms being determined as above let us now get back to the question of locations of firms in equilibrium. We have already argued that, for the interior firms, a symmetric equilibrium of location choice exists with each firm facing the same demand from either side. Without loss of generality (and to maintain the analogy), we characterize the locations in equilibrium where the two terminal firms also face the same demand from either side.

Note that the first-order condition with respect to profit implies that total demand faced by any firm j is

$$D_j = \frac{1}{t}[p_j - c(\theta_j)].$$

It follows that the distances between firms in the interior poor and rich regions are $\frac{1}{t}[p_P - c(\theta_P)]$ and $\frac{1}{t}[p_R - c(\theta_R)]$ respectively. Since the firms face the same demand from either side, the distance between a terminal firm and its adjacent firm in the rich region is

$$|x_R - x_T| = \frac{1}{2t}[p_R - c(\theta_R)] + \frac{1}{2t}[p_T - c(\theta_T)] = \frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_R)} + \sqrt{F(\theta_T)}\right],$$

and the distance between a terminal firm and its adjacent firm in the poor region is

$$|x_P - x_T| = \frac{1}{2t}[p_P - c(\theta_P)] + \frac{1}{2t}[p_T - c(\theta_T)] = \frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_P)} + \sqrt{F(\theta_T)}\right].$$

Also, since a terminal firm faces the same demand from either side, we have

$$\delta_{TP} = \delta_{TR} = \frac{1}{2t}[p_T - c(\theta_T)] = \frac{1}{2\sqrt{t}}\sqrt{F(\theta_T)}.$$

Now let us get back to the possibility whether a terminal firm would be located at the border. Recall that the feasibility condition for a profitable relocation by the terminal firm towards its adjacent firm in the rich region is $|x_R - x_T| > 2\delta_{TP}$. Substituting the expressions for $|x_R - x_T|$ and δ_{TP} we find

$$|x_R - x_T| - 2\delta_{TP} = \frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_R)} - \sqrt{F(\theta_T)}\right] > 0$$

since $\theta_T < \theta_R$ and $F'(\theta) > 0$. The feasibility condition for a profitable relocation being satisfied, a terminal firm cannot locate at the border in equilibrium.

Consider next the possibility whether a terminal firm would be located in the rich region. Recall that a profitable relocation requires that the minimum distance a terminal firm now needs to move towards its adjacent firm in the rich region is even less than $2\delta_{TP}$. Hence a terminal firm can always relocate and be better-off. Thus, in equilibrium, a terminal firm cannot locate in the rich region.

We are now left with the only possibility whether a terminal firm would be located in the poor region. We argue below that under reasonable conditions this possibility does qualify to hold in equilibrium. That is, the only location structure that is possible in equilibrium is where the two terminal firms are located in the poor region (diametrically opposite to each other, of course). The argument rests upon showing that the feasibility conditions for profitable relocations by the terminal firm does not hold. Suppose that firm T is located in the poor region at a distance d from the border. The feasibility condition for a profitable relocation by firm T towards its adjacent firm R is $|x_R - x_T| > 2(\delta_{TP} + d)$, that is, $\frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_R)} + \sqrt{F(\theta_T)}\right] > 2\left[\frac{1}{2\sqrt{t}}\sqrt{F(\theta_T)} + d\right]$. On the other hand the feasibility condition for a profitable relocation by firm T towards its adjacent firm P is $|x_P - x_T| > 2(\delta_{TR} - d)$, that is, $\frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_P)} + \sqrt{F(\theta_T)}\right] > 2\left[\frac{1}{2\sqrt{t}}\sqrt{F(\theta_T)} - d\right]$. Reverting the argument we can say that it will not be feasible for firm T to relocate only if we have

$$\frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_R)}+\sqrt{F(\theta_T)}\right]<2\left[\frac{1}{2\sqrt{t}}\sqrt{F(\theta_T)}+d\right]$$

and

$$\frac{1}{2\sqrt{t}}\left[\sqrt{F(\theta_P)}+\sqrt{F(\theta_T)}\right]<2\left[\frac{1}{2\sqrt{t}}\sqrt{F(\theta_T)}-d\right].$$

Upon adding the two inequalities we get

$$\frac{1}{2}\left[\sqrt{F(\theta_R)}+\sqrt{F(\theta_P)}\right]<\sqrt{F(\theta_T)}.$$

Note that the sufficient condition that guarantees a positive relationship between θ and Y, $F''(\theta)>\frac{[F'(\theta)]^2}{2F(\theta)}$, also guarantees that $\sqrt{F(\theta)}$ is also a convex function. Then the above inequality holds only if θ_T is relatively closer to θ_R as compared with θ_P, that is, as Y increases θ increases, but less than proportionately. This is a reasonable condition and we assume that this condition holds.[7] Thus we conclude that the only possible equilibrium occurs when the terminal firms are located in the poor region and the necessary condition for this equilibrium to occur is an increasing and concave relationship between Y and θ. Note that although there is only one possible equilibrium location structure, there are actually a multiple of equilibria depending on the distance of the terminal firm from the border.

Finally, let us consider the number of firms in equilibrium, both in the poor and rich regions. Suppose that the two terminal firms are located in the poor region at a distance d from the border diametrically opposite to each other on the circular city. Recall that the length of the rich region is ϕL and that of the poor region is $(1-\phi)L$.

Consider first the number of firms in the poor region. Since the total demand faced by a terminal firm is $\frac{1}{t}[p_T-c(\theta_T)]=\sqrt{\frac{F(\theta_T)}{t}}$, and it faces equal demand

[7]This condition can be guaranteed by imposing appropriate restrictions on the third order derivatives of the cost functions, $c(\theta)$ and $F(\theta)$.

on either side, it is clear that $d < \frac{1}{2}\sqrt{\frac{F(\theta_T)}{t}}$. Of the available length of the poor region, $(1-\phi)L$, each terminal firm covers a distance d on the side of the border and $\frac{1}{2}\sqrt{\frac{F(\theta_T)}{t}}$ on the other side. Since there are two such terminal firms, the total length of the poor region that remains available to the interior firms is $(1-\phi)L - 2\left[d + \frac{1}{2}\sqrt{\frac{F(\theta_T)}{t}}\right]$. Suppose there are n_P firms located in the poor region. That is, excluding the two terminal firms, there are $n_P - 2$ interior firms in the poor region each covering a total market of size $\sqrt{\frac{F(\theta_P)}{t}}$. It follows that the number of firms in the poor region, n_P, is determined from the following condition:

$$(n_P - 2)\sqrt{\frac{F(\theta_P)}{t}} = (1-\phi)L - 2\left[d + \frac{1}{2}\sqrt{\frac{F(\theta_T)}{t}}\right]. \tag{4.17}$$

Similarly, the number of firms in the rich region, n_R, is determined from the following condition:

$$n_R\sqrt{\frac{F(\theta_R)}{t}} = \phi L - 2\left[\frac{1}{2}\sqrt{\frac{F(\theta_T)}{t}} - d\right]. \tag{4.18}$$

It is interesting to note that, fixing the location of the terminal firm (that is, fixing d), as the income gap between rich and poor increases (say by increasing Y_R while Y_P remains the same) number of firms in both the poor and rich regions decreases. As the average income of the marginal consumers increases each terminal firm increases the quality of their products. But this increases its fixed cost and to cover the increased fixed cost it needs a larger market. The location remaining fixed, each terminal firm encroaches into the markets of both the rich and poor interior firms. For the interior poor region this is the only effect that leads to the decrease in the number of firms. For the rich region the number of firms decreases further since increase in rich income results in an increase in quality leading to a higher fixed cost requiring a larger market for each surviving firm. For the rich consumers buying from the interior rich firms the adverse effect of the decrease in number of firms is compensated by the increase in product quality. Unfortunately, no such compensation works for the poor consumers buying from the interior poor firms.

4.4.5 Welfare Comparison

In this subsection we examine how the spatial structure and income inequality translate into the welfare of the consumers. As illustrated in the last section, quality offered improves as the income of the individual rises. But this implies that the individual also has to pay a higher price: both in terms of the offer price and the distance which he has to travel. What happens to the the overall welfare of the consumer, given these two opposing forces, is an important question to investigate. Also it is of particular interest to see how the welfare of the poor buying a higher quality product from the terminal firm, (recall that $\theta_T > \theta_P$) compares with the welfare of the rich and the poor residing in the interior. This comparison illustrates the neighborhood effects: how forces operating outside the control of households exert influences on household choices and welfare.

Following equalities derived from the demand structure become useful for the purpose of welfare comparison. In equilibrium, utility of the marginal rich who is indifferent between buying from firm T and firm R is given by

$$Y_R\theta_R - p_R - \frac{p_R - c(\theta_R)}{2} = Y_R\theta_T - p_T - \frac{p_T - c(\theta_T)}{2}. \tag{4.19}$$

Similarly utility of the marginal poor who is indifferent between buying from firm T and firm P is given by

$$Y_P\theta_T - p_T - \frac{p_T - c(\theta_T)}{2} = Y_P\theta_P - p_P - \frac{p_P - c(\theta_P)}{2}. \tag{4.20}$$

First compare the utilities of consumers with different incomes but located furthest away from the firm. In general this utility is given by is given by $Y\theta - p - \frac{p - c(\theta)}{2}$, where price and quality levels correspond to different income levels. Since

$Y_R > Y_P$, it follows that

$$Y_R\theta_T - p_T - \frac{p_T - c(\theta_T)}{2} > Y_P\theta_T - p_T - \frac{p_T - c(\theta_T)}{2}.$$

That is, utility of the rich consumer buying from the terminal firm at the maximal distance is higher than the utility of the corresponding poor consumer. Combining with equations 4.19 and 4.20 the above inequality implies

$$Y_R\theta_R - p_R - \frac{p_R - c(\theta_R)}{2} \; > \; Y_P\theta_P - p_P - \frac{p_P - c(\theta_P)}{2}.$$

That is, rich at the maximum distance is better-off than the poor at the maximum distance.

$Y\theta - p$, whereas the utility of the consumer located at the maximum distance from the firm is given by where price and quality levels, correspond to income level Y. Clearly, as $Y_R > Y_P$. It follows, which implies, $Y_R\theta_R - p_R - [Y_P\theta_P - p_P] > \frac{p_R - c(\theta_R)}{2} - \frac{p_P - c(\theta_P)}{2} > 0$. [8] So the welfare of the consumer, irrespective of the distance, improves with rise in his income. Thus the valuation effect dominates the cost effect, leading to rise in individuals welfare with increase in the income level. Result is consistent with intuition. As income rises, firm offers higher quality, to exploit individuals higher willingness to pay for higher quality. But the increase in price should not dissuade the individual from buying, so the overall impact on the individual is positive. But how do poor consumers buying from the terminal firm compare with those buying from the firm located in the interior poor region. As, $Y_P\theta_T - p_T - \frac{p_T - c(\theta_T)}{2} = Y_P\theta_P - p_P - \frac{p_P - c(\theta_P)}{2}$, it follows that, $Y_P\theta_T - p_T - Y_P\theta_P - p_P = \frac{p_T - c(\theta_T)}{2} - \frac{p_P - c(\theta_P)}{2}$, is strictly positive. So the poor in the close vicinity of rich are better-off. The gain from the improved quality more than offsets the loss because of the higher price that they need to pay.

[8] Refer to equations (4.12) and (4.14).

4.5 Conclusion

In this chapter we analyze the role of the economic segregation in determining firm's location choice and its implication on the welfare of the consumers. We looked at the spatial distribution of the consumers as the distance is an important factor influencing individual's choice. To analyze regional inequality we assume that there are different stretches of the city circumference, with households on one stretch have the same income, whereas households on different stretches have different incomes. We assume that the consumers have distaste for distance, and the individuals with higher income have higher preference for quality.

We first characterize the location equilibrium for the monopoly firm and find the interesting result that when the income gap between the rich and poor is relatively narrower and size of the poor region is relatively smaller then firm will locate in the poor region. Extending the analysis to incorporate free entry and exit we find the similar location equilibrium: the area of operation of terminal firms in the poor region encroaches into the neighbouring richer region, that is, some rich people commute to the terminal firm located in the poor region to access the product, and not the other way round. In equilibrium, there are quality and price ladders where consumers residing in the interior of the rich region are offered the highest quality and are charged with the highest price.

Finally we demonstrate how the spatial structure affects the welfare of the consumers. We show that the utilities of consumers located at comparable distances from the firm – at the maximum distance from the firm, at the same location of the firm, or at the same distance from the firm – increase with the increase in income. The intuitive reason is that quality choice is endogenous. As the consumer's income increases, his willingness to pay increases. This implies that now the firm can offer a higher quality at a higher price. But the increase in price should not dissuade the consumer from buying, so the overall impact on the consumer is positive. We also show the that poor in the neighborhood of the rich have advantages over the poor in the interior of the poor region. This is because they now have access to the superior quality. Even though they need to pay more but the valuation affect dominates the price affect. So the similar idea expressed in the earlier two chapters resonates here also: utility of the individual is linked both to his neighbourhood and income.

4.6 Appendix

A.1 Proof of Positive Relation Between Income and Quality for a Monopoly Firm

Profit of the firm is given by

$$\pi(p,\theta;Y) = 2[p - c(\theta)]\left[\frac{Y(\theta-1)-p}{t}\right] - F(\theta).$$

Substituting for equilibrium price in the firm's profit expression, we get

$$\pi(.) = \frac{2}{t}\left[\frac{Y(\theta-1)-c(\theta)}{2}\right]^2 - F(\theta).$$

Using equation (4.2) firm's profit can be rewritten as

$$\pi = \frac{1}{2t}\left[\frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\right]^2 - F(\theta).$$

Totally differentiating the above equation we get

$$d\pi = \frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\left[\frac{F''(\theta)(Y-c^{'}(\theta)) + c''(\theta)F^{'}(\theta)}{[Y-c^{'}(\theta)]^2}\right]d\theta - F^{'}(\theta)d\theta - \left[\frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\right]\times\left[\frac{F^{'}(\theta)}{[Y-c^{'}(\theta)]^2}\right]dY$$

$$\Rightarrow \frac{d\pi}{dY} = \frac{F^{'}(\theta)}{Y-c^{'}(\theta)}\left[\frac{tF''(\theta)(Y-c^{'}(\theta)) + c''(\theta)tF^{'}(\theta) - [Y-c^{'}(\theta)]^3}{[Y-c^{'}(\theta)]^2}\right]\frac{d\theta}{dY} - \left[\frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\right]\left[\frac{F^{'}(\theta)}{[Y-c^{'}(\theta)]^2}\right].$$

Substituting for $tF^{'}(\theta)$ from equation (4.2) we obtain

$$\frac{d\pi}{dY} = \frac{F^{'}(\theta)}{Y-c^{'}(\theta)}\times\left[\frac{tF''(\theta)(Y-c^{'}(\theta)) + c''(\theta)[Y-c^{'}(\theta)][Y(\theta-1)-c(\theta)] - [Y-c^{'}(\theta)]^3}{[Y-c^{'}(\theta)]^2}\right]\frac{d\theta}{dY} - \left[\frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\right]\times\left[\frac{F^{'}(\theta)}{[Y-c^{'}(\theta)]^2}\right]$$

$$\Rightarrow \frac{d\pi}{dY} = tF^{'}(\theta)\left[\frac{F''(\theta) + \frac{c''(\theta)}{t}[Y(\theta-1)-c(\theta)] - \frac{[Y-c^{'}(\theta)]^2}{t}}{[Y-c^{'}(\theta)]^2}\right]\frac{d\theta}{dY} - \left[\frac{tF^{'}(\theta)}{Y-c^{'}(\theta)}\right]\times\left[\frac{F^{'}(\theta)}{[Y-c^{'}(\theta)]^2}\right].$$

Substituting for $\frac{d\theta}{dY}$ from equation (4.3) it becomes

$$\frac{d\pi}{dY} = \frac{tF^{'}(\theta)}{[Y - c^{'}(\theta)]^2} \times \frac{[Y(\theta - 1) - c(\theta)] + [Y - c^{'}(\theta)](\theta - 1)}{t} - \left[\frac{tF^{'}(\theta)}{Y - c^{'}(\theta)}\right] \times \left[\frac{F^{'}(\theta)}{[Y - c^{'}(\theta)]^2}\right].$$

Using equation (4.2)

$$\Rightarrow \quad \frac{d\pi}{dY} = \frac{F^{'}(\theta)}{[Y - c^{'}(\theta)]^2} \times \left[[Y(\theta - 1) - c(\theta)] + [Y - c^{'}(\theta)](\theta - 1)\right] - \left[\frac{F^{'}(\theta)}{[Y - c^{'}(\theta)]^2}\right] \times [Y(\theta - 1) - c(\theta)]$$

$$\Rightarrow \quad \frac{d\pi}{dY} = \frac{F^{'}(\theta)}{[Y - c^{'}(\theta)]^2} \times \left[[Y - c^{'}(\theta)](\theta - 1)\right],$$

which is clearly positive, given that $F^{'}(\theta) > 0$. So the profit of the firm increases in income.

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