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NOTES

A NOTE ON WEAKLY DOMINATED EXPERIMENTS

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SUMMARY. Let $\mathcal{E} = (X, \mathcal{A}, \mathcal{P})$ be an experiment. Mussmann and Luschgy (1985) showed that if the dimension of $\mathcal{E}$ is a non-measurable cardinal, then weak domination of $\mathcal{P}$ implies that every measure dominated by $\mathcal{P}$ has a support. In this note we give an example to show that the converse is not true. The example also answers another question raised by Mussmann and Luschgy.

1. Introduction

Let $\mathcal{E} = (X, \mathcal{A}, \mathcal{P})$ be an experiment, where X is a set, $\mathcal{A}$ a σ-algebra of subsets of X and $\mathcal{P}$ a family of probability measures on $(X, \mathcal{A})$. $\mathcal{E}$ is said to be weakly dominated if there is a semi-finite localizable measure μ which dominates $\mathcal{P}$ and $\mathcal{P} \subset L_1(\mu)$. See Luschgy and Mussmann (1985) for details.

Let $Ca(\mathcal{E})$ be the set of all bounded measures on $(X, \mathcal{A})$. A positive bounded measure ν on $(X, \mathcal{A})$ is said to have a $\mathcal{E}$-support if there is an S in $\mathcal{A}$ such that $\nu(S^c) = 0$ and $\nu(S \cap E) = 0$ implies $P(S \cap E) = 0$ for all P in $\mathcal{P}$.

Define $U(\mathcal{E})$, $V(\mathcal{E})$ by

$$U(\mathcal{E}) = \{\nu \in Ca(\mathcal{E}) : |\nu| \text{ has a } \mathcal{E}\text{-support}\}$$
$$V(\mathcal{E}) = \{\nu \in Ca(\mathcal{E}) : \forall P \in \mathcal{P}, P(A) = 0 \Rightarrow \nu(A) = 0\}.$$

Mussmann and Luschgy (1985) raise the following questions :

(i) Does $V(\mathcal{E}) \subset U(\mathcal{E})$ imply $\mathcal{E}$ is weakly dominated ? (Page 185)

(ii) Does $U(\mathcal{E}) = Ca(\mathcal{E})$ imply $\mathcal{E}$ is weakly dominated ? (Page 189).

Below we give an example which answers both the questions in the negative. This example first appeared in Ramamoorthi and Yamada (1983).

2. The example

Let Y be a subset of $(0, 1]$ of cardinality $\aleph_1$. Let $X = Y \cup (-Y)$

$$\mathcal{A} = \{A \subset X : \exists\, B \text{ symmetric about 0 such that } A \,\Delta\, B \text{ is countable}\}$$
$$\mathcal{P} = \{P_x : x \in X\} \text{ where } P_x \text{ is the degenerate measure at } x.$$

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If ν is any measure on $(X, \mathcal{A})$, then ν is discrete. This follows by noting that $\mathcal{A}$ restricted to Y and $-Y$ gives respectively the power set on Y and $-Y$ and then a theorem of Ulam (1930) gives that ν restricted to Y and $-Y$ is discrete. We thus have

(a) $$V(\mathcal{E}) = Ca(\mathcal{E})$$

(b) $$U(\mathcal{E}) = V(\mathcal{E}).$$

However $\mathcal{E}$ is not weakly dominated. If μ is any measure dominating $\mathcal{P}$ then $\mu\{x\} > 0$ for $x \in X$ and $\{I_{(x)}, x \in Y\}$ will not have a μ-essential supremum in $\mathcal{A}$.

References

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