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DYNAMIC PROCEDURES AND INCENTIVES IN PUBLIC GOOD ECONOMIES[1]

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In this paper we consider economies involving one public good, one private good, and convex technology and propose an informationally decentralized dynamic nontatonnement procedure that converges in general from the initial endowments to an allocation in the core of the economy. We then consider a general class of procedures and show that there exists none in the class that is locally incentive compatible and individually rational. These results show that there exists a trade-off between the requirements of local incentive compatibility and equitable cost sharing.

KEYWORDS: Equitable cost sharing, decentralized procedure, local incentive compatibility, maximally Pareto improving, coalitionally noncoercive, core.

1. INTRODUCTION

AN OPTIMAL PROVISION of a public good and an equitable sharing of its cost is a central issue of welfare economics. This paper is concerned with both the normative and strategic aspects of this problem.

We propose an informationally decentralized dynamic nontatonnement procedure that converges in general from the initial endowments to an allocation in the core.[2] The procedure may be seen as enunciating a plausible method of cooperation among the agents for achieving an optimal provision of a public good and an equitable sharing of its cost.

As in the related literature (Drèze and de la Vallée Poussin (1971), Malinvaud (1972) and others), we also study the procedure from the viewpoint of noncooperative game theory, i.e. whether the agents have incentives for truthful revelation. In this regard, we obtain a result that holds for such procedures in general. We consider a general class of procedures and show that there exists none in the class that is locally incentive compatible (Laffont and Maskin (1983) and others) and converges in general to at least an "individually rational" Pareto optimal allocation.

[1] This is an extensive revision of Chander (1988) and is heavily inspired by discussions with Jacques Drèze, Henry Tulkens, Dilip Mookherjee, and Debraj Ray. The revision was carried out in part during a stay at CORE in 1988 and for another part during a visit to Instituto de Analisis Economico in 1990. I am grateful to these institutions for their hospitality and to the participants of seminars at California Institute of Technology and CORE. Two unknown referees of *Econometrica* made comments which lead to substantial improvement of the paper. Also the stimulating comments of the editor are gratefully acknowledged.

[2] The core (in public good economies) is always nonempty (Foley (1970)). It has long been seen as the guideline in the search for equitable allocations. The ratio equilibrium (Kaneko (1977), Mas-Colell and Silvestre (1989)) and the egalitarian-equivalent allocations (Moulin (1987)) thus belong to the core. Similary, Yen (1990) argues that an equitable cost sharing allocation must belong to the core.

We know from the existing literature (Roberts (1979), Fujigaki and Sato (1981), and Laffont and Maskin (1983)), however, that there exist procedures that are locally incentive compatible and converge to an allocation which is "maximally Pareto improving" but not in general individually rational or in the core.[3] This means that our results may also be viewed as an analysis of the interplay between the normative and strategic aspects of decentralized procedures for achieving an optimal provision of a public good. In particular, they show that even the weak requirement of *local* incentive compatibility imposes severe distributional constraints; and there exists a trade-off between the requirements of local incentive compatibility and equitable cost sharing as emphasized in footnote 2.

The contents of this paper are as follows. Section 2 states the basic model of an economy involving one public good, one private good, and convex technology and reports some general results that are used later in the paper. Section 3 describes the procedure and proves its convergence to an allocation in the core. Section 4 analyses the issue of local incentive compatibility. Section 5 draws the conclusion.

2. THE ECONOMIES

We consider economies consisting of one public good, whose quantity is denoted by x, one private good, whose quantity is denoted by y, and n consumers. Each consumer is characterized by his or her utility function u^i defined on R^2_+ (the nonnegative orthant of R^2) and by a positive endowment w^i of the private good. The public good can be produced from the private good, which is the only input. The production relation is described by a cost function $g: R_+ \to R_+$ which associates with every quantity x of public good the minimum cost $g(x)$. We make the following assumptions. Let $N = \{1, \ldots, n\}$ denote the set of consumers.

ASSUMPTION (A.1): (1) *Each u^i is C^2 and quasi-concave*; (2) *for at least one i, u^i is strictly quasi-concave*; (3) *for each i, $\partial u^i(x, y)/\partial x \geqslant 0$ and $\partial u^i(x, y)/\partial y > 0$ for all $(x, y) \in R^2_+$*; (4) *for each i, $u^i(0, w^i) > u^i(x, 0)$ for all $x \geqslant 0$ such that $g(x) \leqslant \Sigma w^i$.*

Assumption (A.1.4) rules out the possibility of a consumer giving up every last bit of the private good. It is not essential for the analysis below, but is made only in order to avoid certain boundary problems.

[3] The term "maximally Pareto improving" is due to Mas-Colell (1980). The more familiar term "individually rational" is a game theoretic concept and used presently as such only. Both the terms are formally defined later in the paper.

ASSUMPTION (A.2): *The cost function g is C^2 and satisfies $g(0)=0$, $\partial g(x)/\partial x>0$ and $\partial^2 g(x)/\partial x^2 \geq 0$ for all $x \geq 0$, and there exists a finite x such that $g(x) > \Sigma w^i$.*

An *allocation* is a $(n+1)$-tuple $(x, y^1, \ldots, y^n) \in R_+^{n+1}$, where (x, y^i) denotes the consumption bundle of consumer i. An allocation $(x, y^1, \ldots, y^n)$ is feasible if and only if $g(x) + \Sigma y^i \leqslant \Sigma w^i$. An allocation $(x, y^1, \ldots, y^n)$ is *Pareto optimal* if it is feasible and if there does not exist a feasible allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ such that $u^i(\bar{x}, \bar{y}^i) > u^i(x, y^i)$ for all $i \in N$. A Pareto optimal allocation $(x, y^1, \ldots, y^n)$ is *maximally Pareto improving* if $u^i(x, y^i) \geqslant u^i(0, w^i)$ for all $i \in N$.

Let $Z = \{(x, y^1, \ldots, y^n) \in R_+^{n+1} | g(x) + \Sigma y^i \leqslant \Sigma w^i\}$. Then, Z is the *set of all feasible allocations*. Under (A.2), it is clearly nonempty, convex, and compact.

We shall often denote consumer i's marginal rate of substitution by $\pi^i(x, y^i)$, i.e., $\pi^i(x, y^i) \equiv (\partial u^i(x, y^i)/\partial x)/(\partial u^i(x, y^i)/\partial y^i)$ and the marginal cost by $\gamma(x)$, i.e., $\gamma(x) \equiv \partial g(x)/\partial x$.

ASSUMPTION (A.3): $\Sigma_{i \in N} \pi^i(0, w^i) > \gamma(0)$.

The economies which do not satisfy this assumption are of little interest because in that case the initial endowment itself is Pareto optimal. We now state a well-known lemma.

LEMMA 2.1: *A feasible allocation $(x, y^1, \ldots, y^n)$ is Pareto optimal if and only if $\Sigma \pi^i(x, y^i) - \gamma(x) \leqslant 0$ and $(\Sigma \pi^i(x, y^i) - \gamma(x))x = 0$.*

We shall often adopt the following notation. Let $(\pi^1, \ldots, \pi^n)$ be some n-tuple of real numbers. Then $\pi^S = \Sigma_{i \in S} \pi^i$, $S \subset N$.

An allocation $(x, y^1, \ldots, y^n)$ is *attainable* by a coalition $S \subset N$ if $g(x) + y^S \leqslant w^S$. An allocation $(x, y^1, \ldots, y^n)$ is *individually rational* if there is no consumer $i \in N$ and an allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ attainable by $\{i\}$ such that $u^i(\bar{x}, \bar{y}^i) > u^i(x, y^i)$. An allocation $(x, y^1, \ldots, y^n)$ is a *core allocation* if it is feasible and if there exists no coalition $S \subset N$ and an allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ attainable by S such that $u^i(\bar{x}, \bar{y}^i) > u^i(x, y^i)$ for all $i \in S$.

An allocation $(x, y^1, \ldots, y^n)$ is *coalitionally noncoercive* if it is feasible and if there exists no coalition $S \subset N$ such that $g(x) + y^S < w^S$, i.e., if no coalition pays more than the full cost of public good. Note that a feasible allocation $(x, y^1, \ldots, y^n)$ would be coalitionally noncoercive if and only if $g(x) + y^N = w^N$ and $y^i \leqslant w^i$ for all $i \in N$, i.e., each consumer pays some amount (possibly zero) of the public good cost—no one enjoys a gain in the private good.

We now illustrate these concepts by extending the well-known Kolm triangle diagram for the case of an economy in which there are two consumers and $g(x) = x$. The extended diagram is shown in Figure 1.

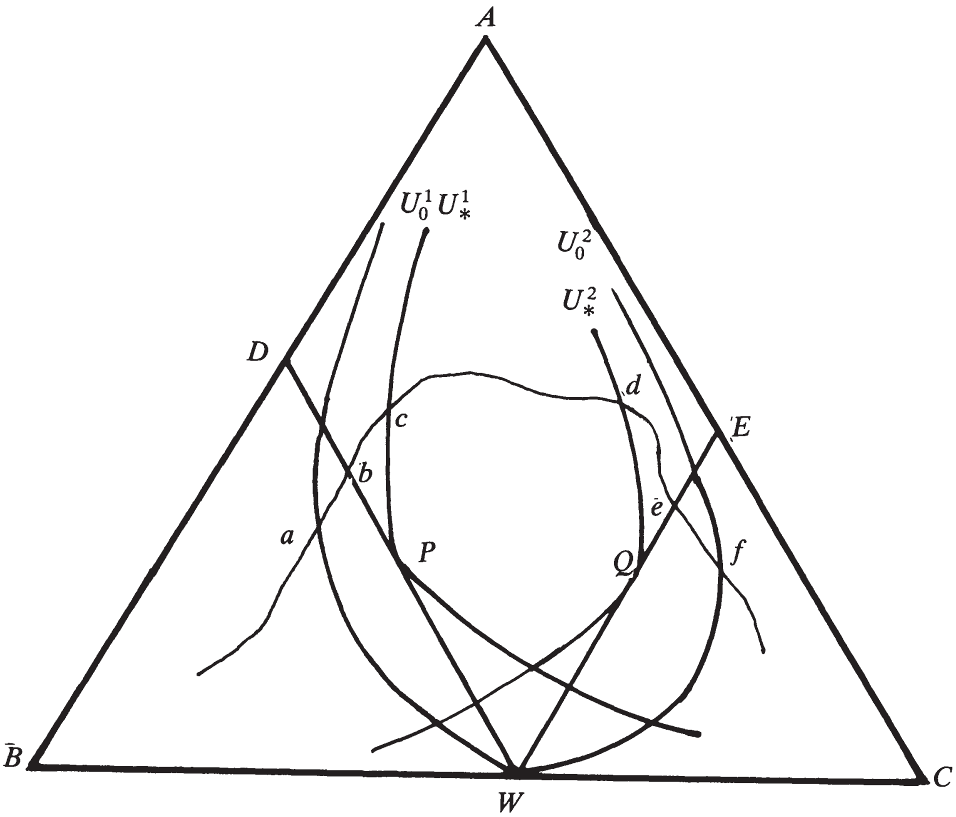

FIGURE 1

Let the point W on the horizontal side BC represent the initial endowment $(0, w^1, w^2)$. Then the points in the parallelogram WDAE represent coalitionally noncoercive allocations. The points in the triangle DBW and EWC represent allocations attainable by consumers 1 and 2, respectively, and the points P and Q their best attainable allocations.

Let the set of points where the indifference curves of the two consumers are tangent to each other be represented by the curve passing through the points a, b, c, d, e, and f. Then the points on the curves af, be, and cd represent, respectively, the maximally Pareto improving, the coalitionally noncoercive Pareto optimal, and the individually rational Pareto optimal allocations. Since the representation is in reference to two consumers only, the core coincides with the set of individually rational Pareto optimal allocations. For more than two consumers the core would in general be a smaller set.

We prove a simple but fundamental lemma, which characterizes coalitionally noncoercive Pareto optimal allocations. This lemma plays a crucial role in the proof of Theorem 3.4 below.

LEMMA 2.2: *Let $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ be a coalitionally noncoercive Pareto optimal allocation which is not a core allocation. Then for any allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ which is attainable by a coalition $S \neq N$ and such that $u^i(\bar{x}, \bar{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$ (and there exists at least one such allocation) we must have $\bar{x} < \overset{*}{x}$.*

The lemma justifies our representation in Figure 1 that the points P and Q be below the curves bc and de, respectively. It is also quite general as its validity does not depend on the convexity assumption on the cost function g.

Proof of Lemma 2.2: Since $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ is Pareto optimal but not in the core, there exists, by definition, a proper coalition $S \neq N$ and an allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ attainable by it such that $u^i(\bar{x}, \bar{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$. This means $g(\bar{x}) + \bar{y}^S \leqslant w^s$ and $u^i(\bar{x}, \bar{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$. Suppose contrary to the assertion that $\bar{x} \geqslant \overset{*}{x}$. Let $(\hat{x}, \hat{y}^1, \ldots, \hat{y}^n)$ be the allocation defined as $\hat{x} = \bar{x}$, $\hat{y}^i = \bar{y}^i$ for all $i \in S$ and $\hat{y}^i = w^i$ for all $i \in N \setminus S$. Then, clearly $g(\hat{x}) + \hat{y}^N \leqslant w^N$, i.e., $(\hat{x}, \hat{y}^1, \ldots, \hat{y}^n)$ is feasible. Since $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ is coalitionally noncoercive, $\overset{*}{y}^i \leqslant w^i$ for all $i \in N$. This means $(\hat{x}, \hat{y}^1, \ldots, \hat{y}^n)$ is a feasible allocation which is such that $u^i(\hat{x}, \hat{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$ and $u^i(\hat{x}, \hat{y}^i) \geqslant u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in N \setminus S$. Therefore, there exists an allocation $(x, y^1, \ldots, y^n)$ which is such that $u^i(x, y^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in N$, since the utility functions are strictly monotonic in terms of the private good. But this contradicts that $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ is Pareto optimal. Hence, we must have $\bar{x} < \overset{*}{x}$.

3. THE PROCEDURE AND ITS CONVERGENCE TO CORE

The procedure is described by the differential equation system:

$$\dot{x}(t) = \pi^N(t) - \gamma(t), \tag{3.1}$$

$$\dot{y}^i(t) = -(\pi^i(t)/\pi^N(t))\gamma(t)\dot{x}(t), \tag{3.2}$$

$$x(0) = 0, \quad y^i(0) = w^i, \quad i \in N, \tag{3.3}$$

where $\pi^i(t) \equiv \pi^i(x(t), y^i(t))$ and $\gamma(t) \equiv \gamma(x(t))$.[4] (Recall the notation that $\pi^N(t) = \sum_{i \in N} \pi^i(t)$.)

This procedure is very much like the Malinvaud (1972) and Drèze and de la Vallée Poussin (1971) (MDP) procedure except that the instantaneous cost is distributed proportionally to current individual valuations and not according to some exogenously given parameters. Equation (3.2) could also be interpreted as the instantaneous analog of the cost sharing rule corresponding to the ratio equilibrium.[5]

Lemma 3.3: *Under Assumptions* (A.1), (A.2), *and* (A.3), *there exists for the system of differential equations* (3.1)–(3.3) *a unique solution* $z(t)$: $[0, +\infty) \to Z$, $z(0) = (0, w^1, \ldots, w^n)$. *This solution is contained entirely in the set of coalitionally noncoercive allocations and converges to a Pareto optimal allocation in this set.*

[4] As usual a dot over a time variable denotes the operator d/dt.
[5] This latter interpretation is due to Mas-Colell and Silvestre (1989).

PROOF OF LEMMA 3.3: Observe that along any solution of (3.1)–(3.3), if one exists,

$$\gamma(t)\dot{x}(t)+\dot{y}^N(t)=0, \quad \dot{x}(t)\geqslant 0, \quad \dot{y}^i(t)\leqslant 0 \quad (i\in N),$$

and

$$\begin{aligned}\dot{u}^i(x(t),y^i(t)) &= \frac{\partial u^i}{\partial x(t)}\dot{x}(t)+\frac{\partial u^i}{\partial y^i(t)}\dot{y}^i(t)\\ &= \frac{\partial u^i}{\partial y^i(t)}\left(\pi^i(t)-\frac{\pi^i(t)}{\pi^N(t)}\gamma(t)\right)(\pi^N(t)-\gamma(t))\\ &= \frac{\partial u^i}{\partial y^i(t)}\left(\frac{\pi^i(t)}{\pi^N(t)}\right)(\pi^N(t)-\gamma(t))^2\geqslant 0.\end{aligned}$$

Therefore, $g(x(t))+y^N(t)=w^N$, $x(t)\geqslant 0$, $u^i(x(t),y^i(t))\geqslant u^i(x(0),y^i(0))$ and $y^i(t)\leqslant w^i$ along any solution of (3.1)–(3.3).

Consider the set

$$C=\left\{(x,y^1,\dots,y^n)|u^i(x,y^i)\geqslant u^i(0,w^i),\right.$$
$$\left. g(x)+\sum y^i=\sum w^i, 0\leqslant y^i\leqslant w^i\right\}.$$

Clearly, C is contained in the set of coalitionally noncoercive allocations. It is nonempty and compact. If $z=(x,y^1,\dots,y^n)\in C$ is such that $x=0$, then $y^i=w^i$, $i\in N$. Hence, if a solution exists it must be contained entirely in C. Moreover, it must never hit the boundary of C (in view of Assumption (A.1.4)) except at the initial point $(0,w^1,\dots,w^n)$ at which the right hand side of (3.1) is positive (Assumption (A.3)).

Existence: The differential equation system (3.1)–(3.3) is of the form $\dot{z}=f(z)$. The function f is defined and continuously differentiable on C (Assumptions (A.1.1) and (A.2)). This implies that f is locally Lipschitz on C. Hence, there exists for (3.1)–(3.3) a unique solution $z(t), z(0)=(0,w^1,\dots,w^n)$, on some interval in R_+ which cannot be continued on the right. As the solution $z(t), z(0)=(0,w^1,\dots,w^n)$, maps into the compact set C, with $x(t)>0$, $y^i(t)>0$ or $x(t)=0$, $y^i(t)=w^i$, $i\in N$, for all t, the interval for which the solution exists must be $[0,+\infty)$.

Quasi-stability, i.e., that any limit point of the solution is a stationary point of (3.1)–(3.3), follows from the fact that the function $L(z(t))\equiv\Sigma u^i(x(t),y^i(t))$ satisfies the definition of a Lyapounov function. $L(z(t))$ is indeed continuous, defined on the compact set C and monotonically increasing unless $z(t)$ is a stationary point, where $L(z(t))$ is constant. Hence $\lim_{t\to\infty}L(z(t))$ exists and is attained at a stationary point.

Stability, i.e., uniqueness of the accumulation point of the solution, follows from the fact that $L(z(t))$ is strictly quasi-concave (Assumption (A.1.2)) and that any stationary point of (3.1)–(3.3) is Pareto optimal. This means that the solution must converge to a unique limit point which is Pareto optimal. Since the solution is contained entirely in the compact set C, the Pareto optimal allocation to which it converges must be coalitionally noncoercive.

THEOREM 3.4: *Under Assumptions* $(A.1)$, $(A.2)$, *and* $(A.3)$, *there exists for the system of differential equations* (3.1)–(3.3) *a unique solution* $z(t)$: $[0, +\infty) \to Z$, $z(0) = (0, w^1, \ldots, w^n)$. *This solution converges to a core allocation.*

We have thus exhibited an informationally decentralized dynamic nontatonnement procedure with a stronger convergence property than has usually been obtained for such procedures.[6, 7, 8]

It was noted earlier that the MDP procedure and its descendants so far might converge to allocations that are not necessarily individually rational whereas the present one always converges to a core allocation. What is the crucial difference between the MDP procedure and the present procedure that leads to this stronger convergence property? It seems that it comes from the fact that in the present procedure $t \to y^i(t)$ is monotonically nonincreasing for all i whereas it is not in the MDP procedure. In fact, the MDP procedure allows $y^i(t) > w^i$ for some or all $t(\geqslant 0)$ and some i. Because of this the time path of the MDP procedure could be of type T_1 or T_5 as shown in Figure 2. The monotonicity of $t \to y^i(t)$ for all i rules out such a time path since in that case it must be contained entirely in the set of coalitionally noncoercive allocations (represented by the parallelogram WDAE in Figure 2). Simultaneously, the monotonicity of $t \to u^i(t)$ for all i implies that the time path must not intersect the indifference curve u^1_* or u^2_* from above. Therefore, the time path of the present procedure could be only of type T_2, T_3, or T_4 as shown in Figure 2. Convergence to those maximally Pareto improving allocations that are not core allocations (represented by the points on the curves ac and df in Figure 2) is thus ruled out. Since the procedure does converge to a maximally Pareto improving allocation, it must converge to one in the core (represented by the points on the curve cd).

Finally, note that the consumers are required to reveal the same type of information in both the procedures. The above therefore means that the MDP procedure does not make full use of the information revealed whereas the present one does.

[6] This has some game theoretic interest in itself. Very few dynamic theories for solution concepts are available in the game theory literature (Stearns (1968), Billera (1972), and Kalai, Maschler, and Owen (1975)). As far as the core concept is concerned we are aware of only one contribution so far, by Shiao and Wang (1974), for games with transferable utilities only.

[7] Chander and Tulkens (1992) extend this procedure to a model of transfrontier pollution and give an additional justification for this procedure.

[8] From a normative point of view, since the procedure selects a core allocation, it can be viewed as a solution concept for equitable cost sharing of a public good. Yen (1990) shows that when so interpreted it satisfies all the desiderata for a solution concept to be reasonable: namely, group rationality, symmetry, continuity, uniqueness, non-core-face allocation and also cost monotonicity (Moulin (1987)) at least when the cost function is linear.

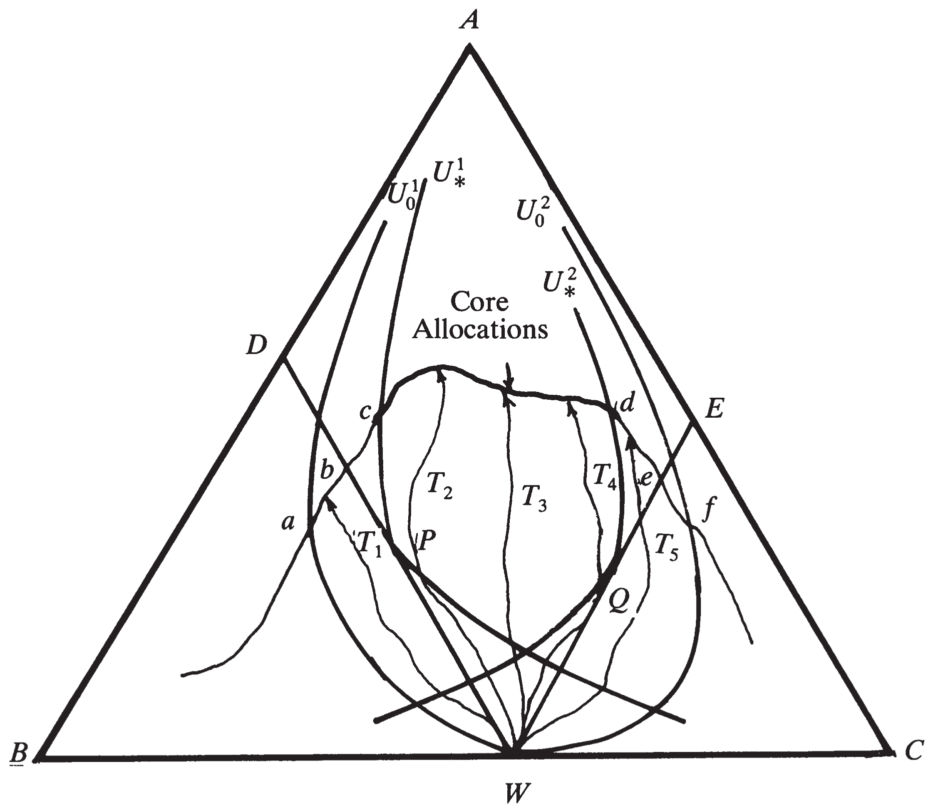

FIGURE 2

PROOF OF THEOREM 3.4: Lemma 3.3 shows that the solution converges to a coalitionally noncoercive Pareto optimal allocation, say $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$. We must prove that $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ must in fact be a core allocation.

Suppose contrary to the assertion that $(\overset{*}{x}, \overset{*}{y}^1, \ldots, \overset{*}{y}^n)$ is not a core allocation. Then, by Lemma 2.2 there exists a proper coalition $S \neq N$ and an allocation $(\bar{x}, \bar{y}^1, \ldots, \bar{y}^n)$ attainable by it, i.e. $g(\bar{x}) + \bar{y}^S \leq w^S$, such that $\bar{x} < \overset{*}{x}$ and $u^i(\bar{x}, \bar{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$.

Since $z(t) \equiv (x(t), y^1(t), \ldots, y^n(t))$: $[0, +\infty) \to C$ is continuous and $x(0) = 0$, $x(t)$: $[0, +\infty) \to [0, \overset{*}{x}]$ is also continuous. Therefore, there exists a $t \in [0, +\infty)$ such that $x(t) = \bar{x}$. Moreover, since the solution is contained in the set C, we must have $g(x(t)) + y^N(t) = w^N$ and $y^i(t) \leq w^i$ for all $i \in N$. Therefore, $x(t) = \bar{x}$ and $g(x(t)) + y^S(t) \geq w^S$, i.e., $x(t) = \bar{x}$ and $y^S(t) \geq \bar{y}^S$ (implying that $y^i(t) \geq \bar{y}^i$ for at least one $i \in S$). It follows that $u^i(\bar{x}, \bar{y}^i) \leq u^i(x(t), y^i(t))$ for at least one $i \in S$. Since $t \to u^i(t)$ is monotonically nondecreasing along the trajectory of the procedure (as shown in the proof of Lemma 3.3), $u^i(x(t), y^i(t)) \leq u^i(\overset{*}{x}, \overset{*}{y}^i)$. This means $u^i(\overset{*}{x}, \overset{*}{y}^i) \geq u^i(\bar{x}, \bar{y}^i)$ for at least one $i \in S$. But this contradicts that $u^i(\bar{x}, \bar{y}^i) > u^i(\overset{*}{x}, \overset{*}{y}^i)$ for all $i \in S$. Hence our supposition is wrong. This proves the theorem.

4. INCENTIVE COMPATIBILITY

As mentioned in the beginning we now consider the question whether the agents have incentives to truthfully reveal their marginal rates of substitution.

Could an agent gain (in utilities) by announcing false marginal rates of substitution?

We know from the incentive literature (Hurwicz (1972) and others) that there exists no procedure in which the agents have dominant strategies whose equilibrium outcomes are always Pareto optimal. In order to obtain positive results therefore Fujigaki and Sato (1981), Champsaur and Rochet (1983), and Laffont and Maskin (1983) consider a weaker incentive requirement by assuming that the agents are myopic, i.e., they maximize the instantaneous payoff—the rate of change of utility $\dot{u}^i(t)$. We show below that weakening the incentive requirement in this way can help matters only to a limited extent.

For the sake of a more transparent analysis we shall restrict ourselves to economies that satisfy Assumptions (A.1)–(A.3) but have only two consumers and a linear cost function, i.e., $N=\{1,2\}$ and $g(x)=x$. Extension of the results to the more general case is straightforward but notationally more cumbersome.

A *procedure* is a 3-tuple (F,G^1,G^2) where $F(\cdot)$, $G^1(\cdot)$, and $G^2(\cdot)$ are some arbitrary real valued and continuously differentiable functions on R^2 such that the following differential equation system has a unique (continuous) solution:

$$\dot{x}(t)=F(\pi(t)), \tag{4.1}$$

$$\dot{y}^i(t)=G^i(\pi(t)), \tag{4.2}$$

$$x(0)=0, \quad y^i(0)=w^i, \quad i=1,2, \tag{4.3}$$

where $\pi(t)\equiv(\pi^1(t),\pi^2(t))$ and $\pi^i(t)\equiv\pi^i(x(t),y^i(t))$. It will be often convenient to drop the time subscript when it is not essential for the argument.

A procedure (F,G^1,G^2) is *balanced* if $F(\pi)+G^1(\pi)+G^2(\pi)=0$ for all $\pi^1,\pi^2\in R$, i.e., if $\dot{x}(t)+\dot{y}^1(t)+\dot{y}^2(t)=0$ for all $t\geq 0$.

A procedure (F,G^1,G^2) is *Pareto satisfactory* if $\pi^1+\pi^2=1\Leftrightarrow F(\pi)=G^1(\pi)=G^2(\pi)=0$ for all $\pi^1,\pi^2\in R$, i.e., if the stationary points of the differential equation system correspond to Pareto optimal allocations. In what follows, it will be convenient to rewrite (4.2) as

$$\begin{aligned}\dot{y}^i(t)&=-\pi^i(t)F(\pi(t))+T^i(\pi(t))\\&=-\pi^i(t)\dot{x}(t)+T^i(\pi(t)),\quad i=1,2.\end{aligned} \tag{4.4}$$

A procedure (F,G^1,G^2) is then defined equivalently as the 3-tuple (F,T^1,T^2).

For the remaining definitions we need to note that $du^i(t)/dt=(\partial u^i(t)/\partial y^i(t))(\pi^i(t)\dot{x}(t)+\dot{y}^i(t))$.

A procedure (F,T^1,T^2) is *monotonic* in terms of utilities if $T^i(\pi^1,\pi^2)\geq 0$ for all $\pi^1,\pi^2\in R$, equivalently, if $\pi^i\dot{x}+\dot{y}^i\geq 0$, $i=1,2$.

A procedure (F,T^1,T^2) is *locally incentive compatible* (in dominant strategies) if $T^1(\pi^1,\pi^2)\geq(\pi^1-s^1)F(s^1,\pi^2)+T^1(s^1,\pi^2)$ for all $\pi^1,\pi^2,s^1\in R$, and analogously for consumer 2.

Local incentive compatibility means that no consumer can increase $\dot{u}^i(t)$ (the rate of change of utility) by reporting $s^i(t)\neq\pi^i(t)$. Since by assumption the functions F, T^1, and T^2 are differentiable, a procedure (F,T^1,T^2) is locally incentive compatible only if

$$\frac{\partial}{\partial s^1}\left((\pi^1 - s^1)F(s^1,\pi^2) + T^1(s^1,\pi^2)\right)\Big|_{s^1=\pi^1} = 0$$

for all $\pi^1, \pi^2, s^1 \in R$.

LEMMA 4.5: *Let (F, T^1, T^2) be a locally incentive compatible procedure. Then, $F(\pi^1,\pi^2)$ is nondecreasing in π^1 and π^2 for all $\pi^1,\pi^2 \in R$.*

PROOF OF LEMMA 4.5: Since the procedure is locally incentive compatible, $T^1(\pi^1,\pi^2) \geq (\pi^1 - s^1)F(s^1,\pi^2) + T^1(s^1,\pi^2)$ for all $\pi^1,\pi^2, s^1 \in R$. Take $s^1 = \pi^1 + \Delta\pi^1$. Then, $T^1(\pi^1,\pi^2) \geq (-\Delta\pi^1)F(\pi^1+\Delta\pi^1,\pi^2) + T^1(\pi^1+\Delta\pi^1,\pi^2)$. Similarly, $T^1(\pi^1+\Delta\pi^1,\pi^2) \geq (\Delta\pi^1)F(\pi^1\ \pi^2) + T^1(\pi^1,\pi^2)$. Adding these two inequalities, we get $\Delta\pi^1(F(\pi^1+\Delta\pi^1,\pi^2) - F(\pi^1,\pi^2)) \geq 0$ for all π^1,π^2 and $\Delta\pi^1 \in R$. This proves that $F(\cdot)$ is nondecreasing in π^1, and similarly in π^2.

LEMMA 4.6: *Let (F,T^1,T^2) be a Pareto satisfactory and locally incentive compatible procedure. Then*

$$T^1(\pi^1,\pi^2) = \int_{1-\pi^2}^{\pi^1} F(\theta,\pi^2)\,d\theta \quad \textit{for all} \quad \pi^1,\pi^2 \in R.$$

PROOF OF LEMMA 4.6: Since the procedure is locally incentive compatible,

$$\frac{\partial}{\partial s^1}\left((\pi^1 - s^1)F(s^1,\pi^2) + T^1(s^1,\pi^2)\right)\Big|_{s^1=\pi^1} = 0 \quad \text{for all} \quad \pi^1,\pi^2 \in R,$$

that is

$$-F(\pi^1,\pi^2) + \frac{\partial T^1(\pi^1,\pi^2)}{\partial \pi^1} = 0 \quad \text{for all} \quad \pi^1,\pi^2 \in R.$$

This means

$$T^1(\pi^1,\pi^2) = \int_{1-\pi^2}^{\pi^1} F(\theta,\pi^2)\,d\theta + K \quad \text{for all} \quad \pi^1,\pi^2 \in R.$$

Since (F,T^1,T^2) is Pareto satisfactory, $F(1-\pi^2,\pi^2) = T^i(1-\pi^2,\pi^2) = 0$, $i = 1,2$, for all $\pi^2 \in R$. Therefore, $K = 0$. This proves the lemma.

LEMMA 4.7: *Let (F,T^1,T^2) be a balanced, Pareto satisfactory, and locally incentive compatible procedure. Then it must be monotonic in terms of utilities and converge to a maximally Pareto improving allocation.*

PROOF OF LEMMA 4.7: Since (F,T^1,T^2) is locally incentive compatible, $T^1(\pi^1(t),\pi^2(t)) \geq (\pi^1(t) - s^1(t))F(s^1(t),\pi^2(t)) + T^1(s^1(t),\pi^2(t))$ for all $s^1(t) \in R$. In particular, this inequality holds for $s^1(t) = 1 - \pi^2(t)$. Thus,

$$\begin{aligned} T^1(\pi^1(t),\pi^2(t)) \geq{}& (\pi^1(t) + \pi^2(t) - 1)F(1-\pi^2(t),\pi^2(t)) \\ &+ T^1(1-\pi^2(t),\pi^2(t)) = 0, \end{aligned}$$

since (F, T^1, T^2) is Pareto satisfactory. Similarly, for consumer 2. Thus, $T^i(\pi(t)) \geq 0$ for all $t \geq 0$ and $i = 1, 2$, i.e. (F, T^1, T^2) is monotonic in terms of utilities.

By definition, the differential equation system (4.1)–(4.3) or (4.4) has a unique solution. Since the procedure is balanced (by hypothesis) and monotonic (as shown), the solution must be contained in a compact set. Therefore, $L(z(t))$ is a suitable Lyapounov function. The proof then follows by similar arguments as in the proof of Lemma 3.3.

Let E denote the class of economies satisfying Assumptions (A.1)–(A.3) and with at least two consumers, i.e., $n \geq 2$.

THEOREM 4.8: *There exists no procedure which is balanced, Pareto satisfactory, locally incentive compatible, and which converges to an individually rational allocation for each economy in E.*

PROOF OF THEOREM 4.8: Let (F, T^1, T^2) be some procedure which is balanced, Pareto satisfactory, and locally incentive compatible. Then, since the procedure is balanced,

$$\begin{aligned}\dot{y}^2(t) &= \pi^1(t) F(\pi^1(t), \pi^2(t)) - T^1(\pi^1(t), \pi^2(t)) \\ &\quad - F(\pi^1(t), \pi^2(t)) \\ &= (\pi^1(t) - 1) F(\pi^1(t), \pi^2(t)) - \int_{1-\pi^2(t)}^{\pi^1(t)} F(\theta, \pi^2(t))\, d\theta\end{aligned}$$

by Lemma 4.6. This implies that

$$\dot{y}^2(t) \begin{cases} > 0 & \text{if } \pi^1(t) > 1 \text{ and } \pi^2(t) = 0, \\ = 0 & \text{if } \pi^1(t) = 1 \text{ and } \pi^2(t) = 0, \end{cases}$$

since $F(\cdot)$ is a nondecreasing (Lemma 4.5) and continuous function and $F(\pi^1, \pi^2) > 0$ if $\pi^1 + \pi^2 > 1$ (from the fact that the procedure is Pareto satisfactory and $F(\cdot)$ is nondecreasing).

We shall construct a class of economies belonging to E such that $t \to y^2(t)$ is monotonically increasing. A typical member of this class has two consumers. Consumer 1 has a strictly quasi-concave utility function u^1 and an initial endowment $w^1 > 0$ such that $\pi^1(0, w^1) > 1$. Consumer 2 has a utility function of the form $u^2(x, y^2) = y^2 + v(x)$, where v is a concave function. And the cost function is linear, i.e., $g(x) = x$. Suppose that $(\partial v(x)/\partial x)|_{x=0} = 0$. Then, it follows from the preceding inequalities that $t \to y^2(t)$ is monotonically increasing, since in that case $\pi^2(t) = \pi^2(0) = \pi^2(0, w^2) = 0$ and $\pi^1(t) \geq 1$ for all t. This means that in that case $y^2(t) > w^2$ for all t. Since the functions F, T^1, T^2 are continuous, there exists an $\alpha > 0$ such that if $(\partial v(x)/\partial x)|_{x=0} \leq \alpha$ (which means $\pi^2(t) \leq \alpha$ for all t), then $y^2(t) > w^2$ for all $t \geq 0$. Since the procedure is balanced, $x(t) + y^1(t) < w^1$ for all t. This clearly means that the procedure would converge to a maximally Pareto improving allocation (Lemma 4.7) which is not individually rational for consumer 1. This completes the proof.

Recall our discussion of the MDP procedure and the procedure described by (3.1)–(3.3). It was noted that the core convergence property comes from the fact that $t \to y^i(t)$ is nonincreasing for all i. We have shown above that local incentive compatibility might require $t \to y^i(t)$ to be increasing for some i. This should intuitively explain why local incentive compatibility of a procedure and the requirement of convergence to an individually rational allocation might be contradictory.

We could construct a class of economies such that $y^i(t) > w^i$ for some i. A necessary condition for this is that $\dot{y}^i(t) > 0$ for some $t \geq 0$. In the two consumer case with linear cost function it means that $\dot{y}^j(t) < -\dot{x}(t)$ for $j \neq i$ and some $t \geq 0$, i.e., at some t consumer j not only pays the full marginal cost of the public good but also transfers a positive amount of the private good to the other consumer. Thus, *even myopic* agents may not have incentives to participate.

5. CONCLUSION

We have shown that there exists an informationally decentralized dynamic nontatonnement procedure that has nice normative aspects but that in general such a procedure cannot be locally incentive compatible. Since we know from the existing literature that there do exist locally incentive compatible procedures that do not generally converge to a core allocation, it follows that there exists a trade-off between the requirements of local incentive compatibility and equitable cost sharing.

We have defined the procedure in continuous time. However, a discrete time version that also converges to the core and that might be of interest from the point of view of applications is possible.

Our analysis is restricted to the case of one public good—one private good. Though this is a standard assumption in the incentive literature, a generalization might be of interest nevertheless.[9] The procedure is well-defined in that case also and can be shown to converge to at least a maximally Pareto improving allocation. The results of Section 4 carry over in a straightforward manner. For convergence to the core the monotonicity of $t \to y^i(t)$ is critical and additional assumptions that ensure this are needed. A straightforward one is the case of quasi-linear utility functions with the assumption of separability in all public goods on the production as well as the consumption side.

[9] Chander and Tulkens (1992) study the case which is technically equivalent to that of many public goods that are additive and in one-to-one correspondence with the consumers.

Using a result of Jordan (1987), Chander and Tulkens (1990) clarify that convergence to core obtains more easily in public good economies as compared to pure exchange economies. Even in the simplest case of two-good pure exchange economies information would be needed not only about the first but also the second derivatives of utility functions. As shown in this paper, however, information about the first derivatives of utility functions alone is sufficient in the case of economies with two goods of which one is public and the other private.

We have not considered incentive compatibility in terms of maximin, Nash, expected utility maximization, and Bayes strategies. The latter two have been explored by Roberts (1987) but with essentially negative results. The second one (Roberts (1979), Truchon (1984), and Von dem Hagen (1991)) involves the assumption of complete information which is at odds with our fundamental assumption of dispersed private information.[10] We are thus left with incentive compatibility in maximin strategies. Results in this direction are positive. In particular, it can be shown that the procedure described by (3.1)–(3.3) is coalitionally (globally) incentive compatible in maximin strategies, i.e., there is some assurance that the agents will report truthfully and convergence to core will not be affected. Technicalities and limited writing space however force us not to enter into these.

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[10] By this we do not mean that this approach is without any interest. In fact, some preliminary results in this respect are reported in Chander (1987) where it is shown that Nash equilibrium strategies exist (uniquely) for each myopic agent in the procedure described by (3.1)–(3.3) and that if the agents adopt these Nash strategies then the so-defined Nash strategically stable procedure also converges to a core allocation. It can also be shown that the results of Truchon (1984), who proves the existence of perfect Nash equilibrium strategies for nonmyopic agents in the MDP procedure, carry over to our procedure and additionally that the perfect Nash equilibrium strategies are so restricted that convergence to core is not affected. Results along these lines are however to be reported in a separate paper.

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