BRIEF COMMUNICATIONS

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Arbitrary amplitude double layers in dusty plasma

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Arbitrary amplitude double layers in a dusty plasma consisting of warm dust particles and two temperature isothermal ions are studied using Sagdeev's pseudopotential technique. Regions of the existence of double layers are investigated both analytically and numerically. It is seen that when one considers two-temperature ions, both compressive and rarefactive solitons are found but only compressive double layers could be obtained. It is also shown that the result obtained in the small-amplitude limit conforms to the perturbation result obtained using the reduction perturbation technique.

The study of dusty plasmas is of considerable interest. This is because dusty plasmas occur in nature in various forms, such as planetary rings, cometary tails, interstellar clouds, etc.[1–8] Some time ago, Rao *et al.*[9] theoretically found the existence of extremely low phase velocity dust acoustic waves in an unmagnetic plasma, whose constituents are inertial charged dust fluid and Boltzmann distributed electrons and ions. A recent[10] laboratory observation of low phase velocity dust acoustic waves suggested a significant depletion of the electron number density there by supporting the theoretical findings of Rao *et al.*[9] Apart from this, some experimental studies of dusty plasmas have also been made recently.[11–13] Very recently, Mamun *et al.*[14] demonstrated that a dusty plasma with inertial dusty fluid and Boltzmann, distributed ions admits only negative amplitude solitary waves associated with nonlinear dust acoustic waves. Subsequently Roychoudhury and Mukherjee[15] showed that finite dusty temperature restricts the region for the existence of such nonlinear solitary waves. The motivation of the present paper is to extend the model of Mamun *et al.*[13] to a dusty plasma consisting of dust particles and two-temperature isothermal ions and to study the existence of both solitary waves and double layers without neglecting the dust temperature. The temperature of dust particles is important, owing to thermalization with the ions or orbital effects.[16] In fact, it is suggested that the dust in planetary rings may have a large value of dust temperature compared to the electron temperature.[17] Recently, Tagare[18] extended the model of Ref. 14. However, he did not give any numerical results. In particular, in the study of double layers, the numerical analysis is of importance as the velocity of the solitary waves and ϕ_m, the maximum value the electric potential can take, are determined simultaneously and to find them is not always a trivial exercise. Also, as mentioned before, the temperature of dust particles plays a not too insignificant role in determining the region of the existence of double layers. Again, the dust particles occur in a multicomponent plasma and, if one considers more than one ion species, both the compressive and rarefactive solitons may exist in dust acoustic waves, and so it will be interesting to see whether double layers of both types may also exist. This motivates us to consider a dusty plasma consisting of warm dust particles and two-temperature isothermal ions and derive the exact pseudopotential to study solitary waves and, in particular, to find the conditions for the existence of double layers. We consider a fluid model of dusty plasma in which the electron number density is assumed to be sufficiently depleted, i.e., $n_e \ll Z_d n_d$. The one-dimensional equations governing the dynamics of dusty plasma are

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d n_d) = 0, \tag{1}$$

$$\frac{\partial \dot{u}_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\sigma}{n_d}\frac{\partial p_d}{\partial x} = \frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d - n_{ic} - n_{ih}, \tag{3}$$

where

$$n_{ic} = \mu \exp\left(-\frac{\phi}{\mu + \nu\beta}\right), \tag{4}$$

$$n_{ih} = \nu \exp\left(-\frac{\beta\phi}{\mu + \nu\beta}\right) \tag{5}$$

are component ions with low temperature T_c and normalized initial density μ, and ions with high temperature T_h and normalized initial density ν, respectively.

Since dust mass is quite heavy compared to the ion masses, the ion mass inertia effects are neglected here.

Also, $\mu+\nu=1$ and β is given by

$$\beta=\frac{T_c}{T_h}; \tag{6}$$

also

$$T_{\text{eff}}=\frac{T_c+T_h}{\mu T_h+\nu T_c}. \tag{7}$$

Both the ion species, as evident from Eqs. (4) and (5), are assumed to follow the Boltzmann distribution. Here n_d refers to the dust particle density, while u_d refers to the dust particle velocity. All the quantities appearing in Eqs. (1)–(5) are normalized. Here n_d is normalized to n_{d0}, the unperturbed dust density, u_d is normalized to the dust acoustic speed $c_d=(T_{\text{eff}}/m_d)^{1/2}$. Time t and distance x are normalized to w_{pd}^{-1} and the Debye length $\lambda_D=[T_{\text{eff}}/4\pi Z_d n_{d0} e^2]^{1/2}$, respectively, where w_{pd}^{-1} is the inverse of dust plasma frequency given by and $w_{pd}^{-1}=(m_d/4\pi n_0 Z_d^2 e^2)^{1/2}$. Here Z_d is the dust charge. The electrical potential ϕ is normalized to T_{eff}/e. Also, $\sigma=T_d/T_{\text{eff}}$. We also take the equation of state as

$$p=n^{\gamma}p_0, \tag{8}$$

where $\gamma=3$ for adiabatic process and $\gamma=1$ for the isothermal process. To find the Sagdeev's pseudopotential, we make all the dependent variables depend on a single variable $\xi=x-Vt$, V being the solitary wave velocity, when Eqs. (1)–(3) reduce to ordinary nonlinear differential equations and can be solved easily. From (1) we get

$$n_d=V/(V-u_d). \tag{9}$$

Using (9) we get from (2) and (8) u_d as an implicit function of ϕ, which is given by

$$\phi=-Vu_d+u_d^2/2+\frac{\gamma\sigma}{(\gamma-1)}\left(\frac{V}{V-u_d}\right)^{\gamma-1}-\frac{\gamma\sigma}{\gamma-1}, \tag{10}$$

for $\gamma\neq 1$, and

$$\phi=-Vu_d+(u_d^2/2)+\ln\left(\frac{V}{(V-u_d)}\right), \tag{11}$$

for $\gamma=1$. For $\gamma=3$, Eq. (10) can be solved to find u_d as an explicit function of ϕ and is given by

$$u_d=V-(1/\sqrt{2})\Big[V^2+2\phi+3\sigma+\sqrt{(V^2+2\phi+3\sigma)^2-12\sigma V^2}\Big]^{1/2}. \tag{12}$$

To derive the results (10)–(12), we have used the boundary conditions $u_d\to 0$, $n_d\to 1$, $\phi\to 0$ when $\xi\to\infty$.

The Sagdeev's pseudopotential[19] is defined by

$$\frac{d^2\phi}{d\xi^2}=-\frac{\partial\psi}{\partial\phi}. \tag{13}$$

Using Eqs. (10)–(12) and Eq. (3), ψ can be easily obtained and is given by

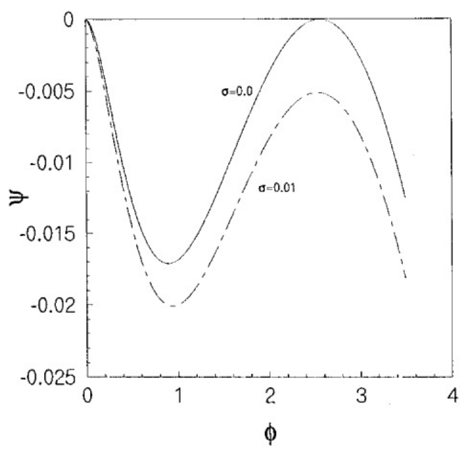

FIG. 1. The plot of $\psi(\phi)$ against ϕ with for $\mu=0.2$ and $\beta=0.05$. The solid line is for $\sigma=0$ and the broken line is for $\sigma=0.01$.

$$\psi(\phi)=-\Big[\sigma+Vu_d-\sigma\left(\frac{V}{V-u_d}\right)^3+\mu[\mu+\nu\beta](1-e^{-\phi/(\mu+\nu\beta)})+\frac{\nu(\mu+\nu\beta}{\beta}(1-e^{-\beta\phi/(\mu+\nu\beta)})\Big]. \tag{14}$$

For solitary wave solutions, the following conditions must be satisfied:

$$\psi(\phi)=0,\quad \left(\frac{\partial\psi}{\partial\phi}\right)_{\phi=0}=0 \quad\text{and}\quad \left(\frac{\partial^2\psi}{\partial\phi^2}\right)_{\phi=0}<0, \tag{15}$$

and $\psi(\phi)<0$ for ϕ lying between 0 and ϕ_m, i.e., either for $0<\phi<\phi_m$ or $\phi_m<\phi<0$. Condition $0<\phi<\phi_m$ given in Ref. 16 gives only the compressive solitons. Here, for compressive (rarefactive) solitons,

$$\left.\frac{\partial\psi}{\partial\phi_m}\right|_{\phi=\phi_m}>0(<0). \tag{16}$$

For double layers, apart from the conditions (15), one must have the following conditions [replacing the condition (16)]:

$$\psi(\phi_m)=0,\quad \left(\frac{\partial\psi}{\partial\phi}\right)_{\phi_m}=0,\quad \left(\frac{\partial^2\psi}{\partial\phi^2}\right)_{\phi=\phi_m}<0, \tag{17}$$

i.e., $\phi=\phi_m$ is a point of maximum. Here ϕ_m is the amplitude of the double layer. The condition $\partial\psi/\partial\phi=0$ at $\phi=\phi_m$ gives

$$\frac{V}{V-u_{dm}}-\mu e^{-\phi_m/(\mu+\nu\beta)}-\nu e^{-\beta\phi_m/(\mu+\nu\beta)}=0,$$

when

$$u_{dm}=V-\frac{1}{\sqrt{2}}\Big[V^2+2\phi_m+3\sigma+\sqrt{(V^2+2\phi_m+3\sigma)^2-12\sigma V^2}\Big]^{1/2}, \tag{18}$$

where we have taken $\gamma=3$. Equations (17) and (18) together will determine V and ϕ_m in terms of σ and other parameters like μ and β for the existence of double layers.

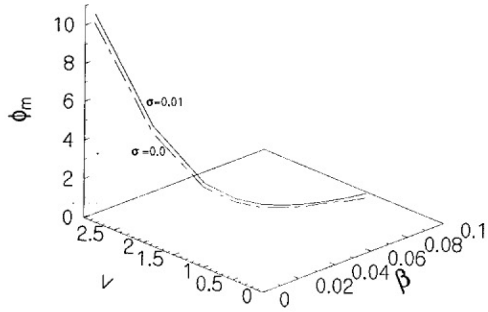

FIG. 2. A three-dimensional plot of ϕ_m against V and β showing the region of existence for double layers for different values of β. Here μ is kept fixed at $\mu=0.1$.

We have numerically solved the Eqs. (17) and (18) for ϕ_m and V for a certain range of the parameters σ, μ, and β.

In Fig. 1, $\psi(\phi)$ is plotted against ϕ with $\mu=0.2$ and $\beta=0.05$. The solid line is for $\sigma=0.0$ and the broken line is for $\sigma=0.01$. It is seen that for $\sigma=0.0$, a double layer exists and V and ϕ_m are found to be 1.20 and 2.38, respectively. However, it is found that for $\sigma=0.01$, double layers cease to exist. Figure 2 shows the domain of the existence of double layers in a three-dimensional plot, where ϕ_m is plotted against V and β. Here μ is kept fixed at $\mu=0.1$. Here V and ϕ_m were calculated for numerous values of the parameters μ and β, but rarefactive double layers (negative ϕ_m) could not be obtained.

Finally, the small-amplitude expansion of $\psi(\phi)$ is given to obtain an analytical solution of double layers that conforms to the perturbation results of Ref. 18

If one neglects terms of $0(\phi^5)$, then $\psi(\phi)$ can be written as

$$\psi(\phi)=A_1\left(\frac{\phi^2}{2}\right)-A_2\left(\frac{\phi^3}{6}\right)-A_3\left(\frac{\phi^4}{24}\right), \tag{19}$$

where

$$A_1=\frac{1}{V^2-\gamma\sigma}-1, \tag{20}$$

$$A_2=\left(\frac{(3V^2+\gamma^2\sigma)}{(V^2-\gamma\sigma)^3}\right)-\frac{\mu+\nu\beta^2}{(\mu+\nu\beta)^2}, \tag{21}$$

$$A_3=-\frac{(12V^2-6\gamma\sigma(\gamma-1)+\gamma^2\sigma(\gamma+1))}{(V^2-\gamma\sigma)^4}-3\frac{(V^2+\gamma^2\sigma)^2}{(V^2-\gamma\sigma)^5}+\frac{\mu+\nu\beta^3}{(\mu+\nu\beta)^3}. \tag{22}$$

The soliton solution for the pseudopotential (19) is given by

$$\phi(\xi)=\frac{2a_1}{\{a_2\pm\sqrt{(a_2^2-4a_1a_3)}[\cosh^2(\xi)-1]\}}. \tag{23}$$

Here $a_1=-A_1/2$, $a_2=A_2/6$, $a_3=A_3/24$.

The peak amplitude ϕ_m in this case can be both negative or positive. For example, [taking the negative sign in the denominator of (23)], if one takes $\mu=\nu=\beta=0.5$, $V=1.1$, and $\sigma=0.01$, one gets $\phi_m=-0.5344$. Again, if one takes $\mu=0.2$, $\nu=0.8$, $\beta=0.1$, $V=1.1$, and $\sigma=0.1$, one would get $\phi_m=0.1354$.

However, for the double layer solution, the conditions $\psi(\phi_m)=0$ and $(\partial\psi/\partial\phi)_{\phi=\phi_m}=0$ imply that $\psi(\phi)$ has a double root at $\phi=\phi_m$. The condition for double roots is

$$A_2^2+3A_1A_3=0. \tag{24}$$

Condition (24) will determine V. To give a numerical example, we take $\mu=0.2$, $\nu=0.8$, $\beta=0.1$ and $\sigma=0.01$. Here V and ϕ_m are found to be 0.94 and 0.28, respectively. However, numerically we could not find rarefactive double layers. When condition (24) is satisfied, (19) can be written as

$$\psi(\phi)=-\frac{A_3\phi^2}{24}(\phi-\phi_m)^2, \tag{25}$$

where $\phi_m=-2A_2/A_3$.

Solving the differential equation

$$\frac{d^2\phi}{d\xi^2}=-\frac{\partial\psi}{\partial\phi},$$

we get

$$\phi(\xi)=\frac{\phi_m}{2}\left[1-\tanh\left(\frac{\phi_m}{2}\sqrt{\frac{A_3}{12}}\,\xi\right)\right].$$

This result is in conformity with the perturbation result.

To conclude, we have shown that the finite dust temperature plays a significant role in the study of double layers in dusty plasma and changes the values of V and ϕ_m considerably for the existence of double layers. Also, while in the presence of two ion species both compressive and rarefactive solitons may exist, numerical analysis yielded only compressive double layers. Our result is valid for both large- and small-amplitude double layers.

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