Effect of finite ion temperature on large-amplitude solitary kinetic Alfvén waves

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By using the Sagdeev pseudopotential method, solitary kinetic Alfvén waves (SKAW) are studied in a low-β plasma, taking into account the electron inertia and ion temperature. Solitary wave solutions, and the electric and magnetic fields, are obtained using the knowledge of the pseudopotential analysis in plasma dynamics. It is found that both hump and dip solitons exist for SKAWs, conforming to the results of Freja scientific satellite observations in space.

I. INTRODUCTION

The existence of finite-amplitude solitary kinetic Alfvén waves (SKAW) propagating in an oblique direction to the ambient magnetic field in a plasma has been studied by several authors.[1–4] Kinetic Alfvén waves play a significant role in the coupling between the ionosphere and magnetosphere. It is known that the ideal magnetohydrodynamic Alfvén waves are not of a dispersive nature. However, if the perpendicular wavelength is comparable to the ion gyroradius, the ions will no longer follow the magnetic lines of force, whereas the electrons, due to their small Larmor radius, will still be attached to the field lines. This produces a charge separation and leads to what are called kinetic Alfvén waves. These waves have dispersion for an oblique propagation. Solitary kinetic Alfvén waves are possible due to the interplay of this dispersive characteristic and nonlinear steepening. Hasegawa and Mima[5] studied analytically the solitary Alfvén waves propagating in a direction oblique to the ambient magnetic field in a plasma with $1 \gg \beta \gg m_e/m_i$; where β is the ratio of the kinetic pressure to the magnetic one. Yu and Shukla[3] showed the existence of localized finite-amplitude Alfvén waves with density hump. Later Shukla *et al.*[6] investigated the exact nonlinear slow Alfvén wave in low-β plasma. Kalita and Kalita[7] studied exact nonlinear Alfvén waves in a low-β plasma ($\beta \ll m_e/m_i \ll 1$) and found that both super- and sub-Alfvénic rarefactive solitons exist depending upon the angle of inclination of the propagation vector to the magnetic field. Subsequently the large-amplitude solitary Alfvén waves were studied by Kalita and Bhatta.[8] However, they did not obtain the solutions for the electric and magnetic fields.

Recently the large-amplitude Alfvén wave research received an added impulse owing to the observational studies on the data from the Freja scientific satellite observations.[9,10] The data from the Freja satellite showed that the auroral low-frequency turbulence is dominated by strong electromagnetic spikes, which show solitary structure and have a possible interpretation as SKAWs. Also, these SKAWs are of both hump density and dip density solitons. Recently, Wu *et al.*[11] attempted to relate the analytical solution of SKAWs to some events observed from the Freja satellite data. Wu *et al.*[12] extended their studies to cases where they took into accounts the pressure gradient effect as well as the electron mass. However, in this case they did not calculate the finite-amplitude soliton solution either analytically or numerically.

In this paper we derived the exact Sagdeev pseudopotential in order to find the region of existence of finite-amplitude SKAWs, taking into account the pressure gradient term for the electron, electron mass, and also the finite ion temperature. Ion temperature is of significance in the study of Alfvén waves, as unlike ion acoustic waves, Alfvén waves exist even when the nonisothermal condition, viz. $T_e \gg T_i$ is violated.[13] This is because in a magnetized plasma oscillations of same frequencies can propagate in which the electric field is not irrotational and cannot be expressed in terms of a single scalar potential. These oscillations are weakly attenuated as their velocities are much greater than v_{T_i}; thermal velocity of ions.

We obtained the solitary wave solutions and the electric and magnetic fields numerically using the knowledge of the pseudopotential. We found that both hump and dip density solitons may exist, though there exists a critical ion temperature beyond which compressive solitons would not exist. The plan of the paper is as follows.

In Sec. II the basic equations governing the dynamics of a homogeneous plasma in a uniform ambient magnetic field are given. Thence, based on the quasipotential analysis, the exact pseudopotential is derived analytically. In Sec. III the solitary wave solutions are discussed. Section IV is kept for discussions and conclusions.

II. BASIC EQUATIONS AND PSEUDOPOTENTIAL

In the following we give the basic equations governing a homogeneous plasma in a uniform ambient magnetic field B_0 along the z direction [assuming the wave vector to be $(k_x,0,k_z)$],

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e v_{ez})=0, \quad (1)$$

$$\frac{\partial v_{ez}}{\partial t}+v_{ez}\frac{\partial v_{ez}}{\partial t}=-\alpha\left(E_z+\frac{1}{n_e}\frac{\partial n_e}{\partial z}\right), \quad (2)$$

$$\frac{\partial n_i}{\partial t}+\nabla_\perp\cdot(n_i\vec{v}_i)=0, \quad (3)$$

$$\frac{\partial v_{iz}}{\partial t}+(\vec{v}\cdot\nabla_\perp v_{iz})+\frac{\sigma}{n_i}\frac{\partial p_i}{\partial z}=Q\alpha E_z, \quad (4)$$

$$v_{ix}=Q\alpha\frac{\partial E_x}{\partial t}, \quad (5)$$

where $\sigma=Q\alpha\sigma_i$ and $\sigma_i=T_i/T_e$,

$$\frac{\partial B_y}{\partial z}=-n_i v_{ix}, \quad (6)$$

$$Q\alpha(\nabla\times\vec{E})_y=-\frac{\partial B_y}{\partial t}, \quad \text{where } \nabla_\perp\equiv\left(\frac{\partial}{\partial x},0,\frac{\partial}{\partial z}\right), \quad (7)$$

with the quasineutrality condition

$$n_i=n_e=n. \quad (8)$$

Equations (1)–(8) are written in normalized form. The density is normalized to n_0; the unperturbed plasma density, the velocity to the Alfvén velocity v_A. The electric and magnetic fields are normalized, respectively, to $T_e\omega_{ci}/(ev_A)$ and B_0. Here T_e and ω_{ci} are, respectively, the electron temperature and the ion gyrofrequency. Here $\alpha=\beta/2Q$, Q being the ratio of the electron mass to ion mass. To obtain the analytical pseudopotential we consider a stationary wave in the moving frame, defined by

$$\eta=k_x x+k_z z-\omega t. \quad (9)$$

Replacing $\partial/\partial t$ by $-\omega(d/d\eta)$, $\partial/\partial x$ by $k_x(d/d\eta)$, and $\partial/\partial z$ by $k_z(d/d\eta)$, Eqs. (1)–(7) reduce to a set of ordinary differential equations, the independent variable being η. The integration of these equations is not a difficult problem. However, before integrating the equations let us first calculate the dispersion relation without neglecting the finite ion temperature.

Using Eqs. (2)–(7) the dispersion relation is obtained as follows:

$$M_z^4(1+Q+Qk_x^2)-M_z^2(1+Q+Q\alpha k_x^2-\gamma\sigma)$$
$$+Q\alpha+\gamma\sigma=0, \quad (10)$$

where

$$M_z=\frac{\omega}{k_z}=\frac{M}{K_z}, \quad (11)$$

where M is the dimensionless phase velocity and $K_z=k_z/k,=\cos\theta,\theta$ being the angle of propagation made with the z direction and k the dimensionless wave number. When $\sigma=0$, the relation (10) reduces to the one obtained by Wu *et al.*[11,12] The solution of (10) is given by

$$M_z^2=\frac{(1+Q+Q\alpha k_x^2-\gamma\sigma)}{2(1+Q+Qk_x^2)}\pm\frac{\sqrt{(1+Q+Q\alpha k_x^2-\gamma\sigma)^2-4(Q\alpha+\gamma\sigma)(1+Q+Qk_x^2)}}{2(1+Q+Qk_x^2)}. \quad (12)$$

If one neglects Q, $Q\alpha$, and σ, then (12) reduces to

$$M_z^2=(1+Q\alpha k_x^2)(1+Qk_x^2)^{-1}. \quad (13a)$$

Writing $M_z=\omega/k_z$ and restoring the dimensions, one gets the dispersion relation for kinetic Alfvén waves,

$$\omega^2=v_A^2k_z^2(1+r_s^2k^2)(1+r_s^2k^2/\alpha), \quad (13b)$$

where r_s is the ion Larmor radius. This is same as obtained by Hasegawa and Mima.[5] Now integrating the above set of equations with the boundary conditions for solitary waves, viz. $n\to1$, $v_{ez}\to0$ $v_{iz}\to0$ as $|\eta|\to\infty$, the equation of motion can be written as (henceforth we write M for M_z)

$$\frac{d^2n}{d\eta^2}=-\frac{\partial\psi(n,M,k_x)}{\partial n}, \quad (14)$$

where the pseudopotential $\psi(n,M,k_x)$ is given by

$$\psi=\frac{(1+Q)}{QM^2k_x^2}\bar{\psi}(n), \quad (15)$$

$$\bar{\psi}(n)=-n^4\left(1-\frac{\alpha n^2}{M^2}\right)^{-2}\left[\phi_1+\frac{\alpha}{M^2}\phi_2\right.$$
$$\left.+\frac{Q}{1+Q}\frac{\alpha}{M^2}\left(\phi_3+\frac{\alpha}{M^2}\phi_4\right)+\phi_5\right], \quad (16)$$

with

$$\phi_1=(n-1)^2\frac{\left(3M^2-1-\frac{2}{n}\right)}{6}, \quad (17)$$

$$\phi_2=-n(n-1)(M^2n+1)+(M^2-1)n^2\ln n, \quad (18)$$

$$\phi_3=-\frac{(n^2-1)}{2}+(M^2+1)(n-1)n-M^2n^2\ln n, \quad (19)$$

$$\phi_4=M^2n^2\frac{(n^2-1)}{2}-(1+M^2)(n-1)n^2+n^2\ln n, \quad (20)$$

$$\phi_5=\frac{\sigma n^2}{(1+Q)M^4}\left(\frac{M^2}{\gamma-2}(n^{\gamma-2}-1)-\frac{\alpha}{\gamma}(n^{\gamma}-1)-\frac{M^4}{\gamma-1}(n^{\gamma-1}+1)+\frac{M^2}{\gamma+1}(n^{\gamma+1}-1)+\alpha\ln n+\frac{M^4}{n^2}(n^2-n)-\frac{M^2}{2n^2}(n-1)-M^2\alpha(n-1)\right). \tag{21}$$

We have used the equation of state for the ion as $p_i/n_i^{\gamma}=\text{const}$, with $p_0=n_i^0=1$, $n=1$, $u_{ez}=u_{iz}=dn/d\eta=0$ as $|\eta|\to\infty$. Henceforth, for all numerical calculations, we shall take $\gamma=3$. For $\sigma=0$ the pseudopotential $\psi(\phi)$ obtained here agrees with the one obtained by Kalita and Bhatta[8] if one neglects the term $-v_A^2/c^2$. However, our result differs slightly from the one obtained by Wu *et al.*,[12] who erred in leaving out the term $1/(1+Q)$ from the coefficients of ϕ_3 and ϕ_4.

III. SOLITARY KINETIC ALFVÉN WAVES

For solitary wave solutions, the particle motion given by Eq. (14) must be confined between two points $n=1$ and $n=n_m$ and $dn/d\eta=0$ at $n=1$ and $n=n_m$. Also, in order that $\sqrt{-\psi(n)}$ is real for n lying between 1 and n_m, $\psi(n)<0$ in that region. The other two conditions are

$$(1)\quad \frac{d^2n}{d\eta^2}=0,\quad \text{at } n=1; \tag{22}$$

and

$$(2)\quad (n_m-1)\frac{d^2n}{d\eta^2}<0,\quad \text{at}\quad n=n_m; \tag{23}$$

these conditions mean that the particle is reflected back at $n=n_m$, but not at $n=1$.

From $d^2n/d\eta^2=0$ at $n=1$, it follows that $\bar{\psi}(n,M,k_z)$ reaches a maximum at $n=1$. This implies that

$$(1-M^2)(1-\alpha M^{-2})\left(1-\frac{1}{(Q+1)M^2}(Q\alpha-3\sigma)\right)<0. \tag{24}$$

Also,

$$(n_m-1)\left.\frac{d^2n}{d\eta^2}\right|_{n=n_m}<0$$

leads to

$$(1-M^2n_m)(1-\alpha M^{-2}n_m^2)\times\left(1-\frac{1}{(Q+1)M^2}(Q\alpha-\sigma(n_m^2+n_m+1))\right)>0. \tag{25}$$

Since, for the Alfvénic wave, M is of order unity, for $1\ll\alpha\ll\alpha^{-1}$ and $\alpha\ll 1$ the conditions (24) and (25) lead to (for small σ)

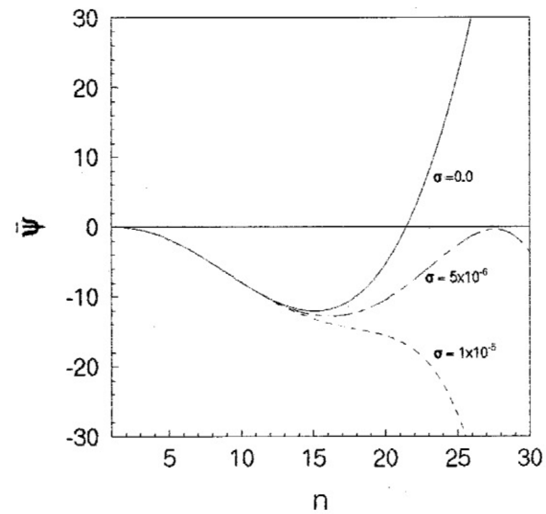

FIG. 1. A plot of $\psi(n)$ vs n for $\sigma=0.0$, 10^{-5}, and 5×10^{-6}, indicating the existence of hump solitons. Here $M=0.35$, $\alpha=0.9$, $k_x=0.5$.

$$n_m^{-1}<M^2<1, \tag{26}$$

or

$$n_m^{-1}>M^2>1, \tag{27}$$

and another inequality $(1-\alpha M^{-2}n^2)\neq 0$ is satisfied automatically.

For $\alpha\simeq 1$, $Q\ll 1$ (24) leads to

$$(1-M^2)(1-\alpha M^{-2})\left(1+\frac{3\sigma}{M^2}\right)<0, \tag{28}$$

while (25) leads to

$$(1-M^2n_m)(1-\alpha M^{-2}n_m^2)[1+\sigma M^{-2}(n_m^2+n_m+1)]>0. \tag{29}$$

It is found that for $\alpha\sim 1$ both hump and dip solitons exist. To show the region of existence of the solitary wave solution, $\bar{\psi}(n)$ is drawn against n for different σ, in Fig. 1. It is seen that for $\sigma>5\times10^{-6}$ the solitary wave solution does not exist. This implies that the upper limit for the ratio of ion temperature to the electron temperature, viz. σ_i, is of the order of 10^{-2}. Here we have taken $M=0.35$, $\alpha=0.9$, $k_x=0.5$. Since $\alpha\sim 1$, β is of order $\beta\sim10^{-3}$. In Fig. 2, $\bar{\psi}(n)$ is plotted against n to show the existence of the dip soliton solution. Here $M=1.5$, $\alpha=0.5$, $k_x=0.5$.

The soliton solution can be obtained by integrating Eq. (14). In Fig. 3 the solitary wave solution is shown for $\sigma=0.0$ and $\sigma=5\times10^{-6}$. The electric fields and magnetic fields could be obtained from the set of Eqs. (2)–(6).

Here E_x can be obtained by integrating the equation

$$\frac{dE_x}{d\eta}=\frac{1}{\alpha\sqrt{-\psi(n)}}\left[\frac{1+Q}{Q}\left(1-\frac{1}{n}\right)-\frac{k_z^2\alpha}{w^2}(n-1)\right],$$

whereas E_z and B_y are given by

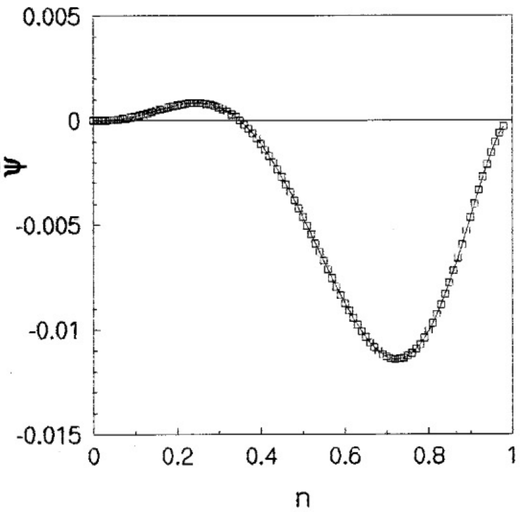

FIG. 2. A plot of $\psi(n)$ vs n for $\sigma=0.0$, $\sigma=0.10^{-4}$, indicating the existence of dip solitons. The unbroken line is for $\sigma=0.0$, and the line with circles is for $\sigma=10^{-4}$. As can be seen, these two lines are indistinguishable. Here $M=1.5$, $\alpha=0.5$, $k_x=0.5$.

$$E_z=\left(\frac{M^2k_z}{n^3\alpha}-\frac{\alpha k_z}{n}\right)\sqrt{-\psi(n)},$$

$$B_y=-\left(\frac{M}{k_z}(n-1)(1+Q)-\frac{Q\,\dfrac{(Q+1)\alpha}{k_xM}\,n(n-1)}{\sqrt{-2\psi(n)}}\right).$$

In Fig. 4, E_x, B_y (normalized) are plotted against η for two values of σ, viz. 0, and 5×10^{-6}.

In Fig. 5, E_z is plotted against η for the two values of σ mentioned above. It is seen that both a hump and dip structure exists for E_z, while E_x and B_y have shock wave, like structures. For Figs. 3, 4, 5 the parameters M, α, k_n are taken to be the same as in Fig. 1.

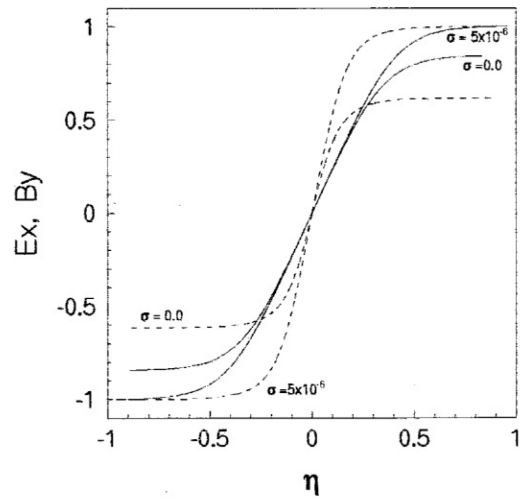

FIG. 4. A plot of E_x and B_y vs η for $\sigma=0.0$, $\sigma=5\times10^{-6}$. Other parameters are the same as in Fig. 1.

IV. DISCUSSIONS AND CONCLUSIONS

Since the Alfvén waves can exist for values of T_i/T_e not satisfying $T_i/T_e\ll1$, it is worthwhile to see the effect of ion temperature on the existence of SKAWs. We have also taken into account the electron inertia. An exact analytical solution for the pseudopotential is obtained from which the solitary wave solution and the electric and magnetic fields can be obtained. It is found that for $M<1$ only hump solitons exist, whereas for $M>1$ only dip solitons may exist. The effect of the ion temperature and the electron inertia is to restrict the region of existence of both types of solitary wave solutions.

Figure 5 shows that E_z has a structure containing both hump and dip structures and is similar to the experimental data of the F4 experiment of the Freja satellite on 3 March 1993. A small but finite ion temperature also changes the shape of the solitary wave solution, as will be clear from Fig. 3 and Fig. 5. It is very likely that SKAWs may play a significant role in the study of auroral plasma. In that case study

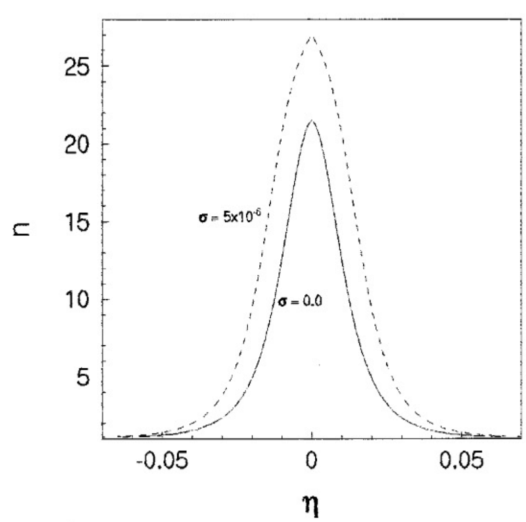

FIG. 3. A plot of n vs η for $\sigma=0.0$ and $\sigma=5\times10^{-6}$, showing the soliton solutions. Other parameters are the same as in Fig. 1.

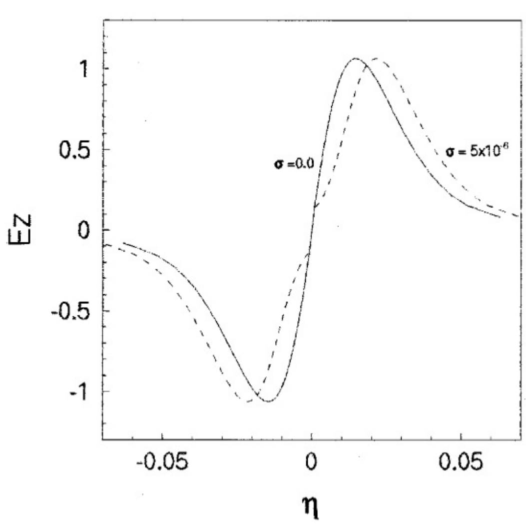

FIG. 5. A plot of E_z vs η for $\sigma=0.0$ and $\sigma=5\times10^{-6}$. For other parameters see Fig. 1.

of large-amplitude solitary Alfvén waves is of considerable importance, both from the theoretical and experimental points of view.

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