Effect of ion temperature on large-amplitude ion-acoustic solitary waves in relativistic plasma

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Effect of ion temperature on the conditions for existence of solitary waves in a relativistic plasma is studied using Sagdeev's pseudopotential approach. It is shown that the ion temperature puts a restriction on the values of V, the soliton velocity. It is also shown that for small amplitude and cold ions, the present results agree with the existing published results. Numerical solutions of the equation of motion derived from the pseudopotential are obtained to see the effect of ion temperature on the width and amplitude of the ion-acoustic solitary waves.

I. INTRODUCTION

Ion-acoustic solitary waves in a collisionless plasma have been studied extensively during the last decade or so.[1–4] Small but finite amplitude solitary waves are adequately described by the Korteweg–de Vries (KdV) equation which can be obtained by reduction perturbation technique.[5–14]

Recently properties of ion-acoustic solitary waves in a collisionless plasma consisting of nondrifting ions have been studied by a few authors.[15–18] Most of these studies, however, were based on reductive perturbation technique and hence are valid for small amplitude only. But large amplitude solitary wave solution is relevant in view of recent experimental observations.[19,20] It was Roychoudhury *et al.*[21] who first used a nonperturbative approach to obtain large-amplitude solitary wave solution in a relativistic plasma. It was later investigated in detail by Ghosh and Roy.[22] However, they, like others, neglected electron inertia. It was recently[23] shown by Kuehl and Zhang that the effects of electron inertia are much more important than relativistic effects. In this paper we shall study the effect of ion temperature in a relativistic plasma using Sagdeev's pseudopotential approach[24] without neglecting the effect of electron inertia. It is shown by Kuehl and Zhang[23] that a significant effect of electron inertia is to limit the regime in which solitary wave solutions exist to that in which ion velocities are essentially nonrelativistic. It will be shown here that one very significant effect of ion temperature is to limit the range of values of V, the soliton velocity, even in the nonrelativistic region.

The plan of the paper is as follows. In Sec. II we derive the exact pseudopotential starting from the basic system of equations and compare our results with the previously published results. In Sec. III we discuss the conditions under which solitary wave solutions are possible. We also compare our results with those obtained from perturbative approach. Finally, Sec. IV is kept for discussion and conclusion.

II. BASIC EQUATIONS AND PSEUDOPOTENTIAL APPROACH

The basic system of equations governing the plasma dynamics in unidirectional propagation is given by

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=0, \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t}+u\frac{\partial u_\alpha}{\partial x}+\frac{\sigma}{n}\frac{\partial p}{\partial x}=-\frac{\partial \varphi}{\partial x}, \tag{2}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+3p\frac{\partial u}{\partial x}=0, \tag{3}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e u_e)=0, \tag{4}$$

$$\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}=\frac{1}{\mu}\left(\frac{\partial \varphi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right), \tag{5}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=n_e-n, \tag{6}$$

when

$$u_\alpha=u\left(1+\frac{u^2}{2c^2}\right), \tag{7a}$$

$$\mu=\frac{m_e}{m_i}=\frac{1}{1836}\ (\text{approx.}), \tag{7b}$$

where m_i and m_e are electron and ion masses, respectively. Here σ is the ratio of the ion temperature T_i to the electron temperature T_e, n and n_e are the ion and electron densities, respectively, normalized to the unperturbed ion density; u, u_e, and c are ion and electron fluid velocities and the velocity of light, respectively, normalized to the ion-acoustic speed $C_s=(KT_e/m_i)^{1/2}$, where K and T_e are Boltzmann's constant and the electron temperature, respectively. The distance x and time t are normalized to the Debye length and the ion-plasma period, respectively. The ion pressure p is normalized to $(n_0KT_i)^{-1}$, n_0 being the unperturbed ion density. The electrostatic potential φ is normalized to KT_e/e, where e is the electron charge.

To obtain a solitary wave solution we make the dependent variables depend on a single independent variable $\xi=x-Vt$ where V is the velocity of the solitary wave. Now Eqs. (1)–(6) can be written as

$$-V\frac{dn}{d\xi}+\frac{d}{d\xi}(nu)=0, \tag{8}$$

$$-V\frac{du_\alpha}{d\xi}+u\frac{du_\alpha}{d\xi}+\frac{\sigma}{n}\frac{dp}{d\xi}=-\frac{d\varphi}{d\xi}, \tag{9}$$

$$-V\frac{dp}{d\xi}+u\frac{dp}{d\xi}+3p\frac{du}{d\xi}=0, \tag{10}$$

$$-V\frac{dn_e}{d\xi}+\frac{d}{d\xi}(n_e u_e)=0, \tag{11}$$

$$-V\frac{du_e}{d\xi}+u_e\frac{du_e}{d\xi}=\frac{1}{\mu}\left(\frac{d\varphi}{d\xi}-\frac{1}{n_e}\frac{dun_e}{d\xi}\right), \tag{12}$$

$$\frac{d^2\varphi}{d\xi}=n_e-n. \tag{13}$$

We integrate Eqs. (8)–(10) with the following boundary conditions.

When

$$\xi\rightarrow\infty,\quad \varphi\rightarrow 0,\quad u\rightarrow u_0,\quad p\rightarrow p_0,\quad n\rightarrow 1. \tag{14}$$

Equation (8) can be integrated to give

$$n=\frac{u_{01}}{(1-u/V)}, \tag{15}$$

where

$$u_{01}=1-\frac{u_0}{V}. \tag{16}$$

Equation (10) can be integrated to give

$$p=n^3p_0. \tag{17}$$

If we replace $1/n$ in (9) by $[(1-u/V)/u_{01}]$, multiply it by 2 and subtract it from σ'/V times (10) we get the following differential equation

$$-3\sigma'\frac{dp}{d\xi}+\frac{3\sigma'}{V}\frac{d}{d\xi}(up)+2V\frac{du_\alpha}{d\xi}-2u\frac{du_\alpha}{d\xi}=2\frac{d\varphi}{d\xi}, \tag{18}$$

where

$$\sigma'=\frac{\sigma}{u_{01}}, \tag{18a}$$

Integrating (18) and using the boundary conditions (14) we get

$$-3\sigma'p+\frac{3\sigma'p}{V}u+2uV\left(1+\frac{u^2}{2c^2}\right)-2u^2\left(1+\frac{u^2}{2c^2}\right)$$
$$+u^2\left(1+\frac{u^2}{4c^2}\right)=2\varphi'-3\sigma'p_0u_{01}, \tag{19}$$

where

$$\varphi'=\varphi-a_0 \tag{20}$$

where

$$a_0=-Vu_0\left(1+\frac{u_0^2}{2c^2}\right)+\frac{u_0^2}{2}\left(1+\frac{3u_0^2}{4c^2}\right). \tag{21}$$

Eliminating p and u from (15), (17), and (19), a sixth-order equation in n is obtained. The explicit form of this equation is

$$-3\sigma'p_0u_{01}n^6+(V^2-2\varphi'+3\sigma'p_0u_{01})n^4-V^2n^2u_{01}^2$$
$$+(V^4/c^2)\{(n-u_{01})^2[(n+3u_{01})/4]\}=0. \tag{22}$$

Henceforth, we will write σ in place of σ'. There is no general analytical solution of Eq. (22). However, it can be solved for n keeping terms up to $0(1/c^2)$. After some algebra we obtain

$$n=\bar{n}+\frac{v^2}{c^2}n_1 \tag{23}$$

when

$$\bar{n}=\left(\frac{(V^2-2\varphi'+2\sigma p_0u_{01})-[(V^2-2\varphi'+3\sigma p_0u_{01})^2-12\sigma p_0V^2u_{01}^3]^{1/2}}{6\sigma p_0u_{01}}\right)^{1/2}, \tag{24}$$

$$n_1=-\frac{V^2}{8}\left(\bar{n}-\frac{6u_{01}^2}{\bar{n}}+\frac{8u_{01}^3}{\bar{n}^2}-\frac{3u_{01}^4}{\bar{n}^3}\right)$$
$$\times[(V^2-2\varphi'+3\sigma p_0u_{01})^2-12\sigma p_0V^2u_{01}^3]^{-1/2}. \tag{25}$$

Similarly considering the boundary conditions $\xi\rightarrow\infty$, $\varphi\rightarrow 0$, $u_e\rightarrow 0$, $n_e\rightarrow 1$, and integrating (11) and (12) we get

$$n_e=\left[\frac{V}{(V-u_e)}\right], \tag{26}$$

$$\varphi=\ln[V/(V-u_e)]+1/2\mu[(V-u_e)^2-V^2]. \tag{27}$$

It is convenient to obtain the so-called Sagdeev potential, such that

$$\frac{d^2\varphi}{d\xi}=-\frac{d\psi}{d\varphi}, \tag{28}$$

where the pseudopotential is

$$\psi(\varphi)=\psi_e(\varphi)+\psi_i(\varphi), \tag{29}$$

where

$$\psi_e(\varphi) = -\int n_e\, d\varphi = -\int n_e \left(\frac{\partial \varphi}{\partial u_e}\right) du_e \tag{30}$$

and

$$\psi_i(\varphi) = \int n\, d\varphi = \int \bar{n}\, d\varphi + \frac{V^2}{c^2}\int n_1\, d\varphi. \tag{31}$$

The pseudopotential ψ given by Eq. (29) contains the terms ψ_e due to electrons and ψ_i due to ions. In order to evaluate the integral (30) we differentiate (27) and get

$$\frac{\partial \varphi}{\partial u_e} = (V-u_e)^{-1} - (V-u_e)/\mu, \tag{32}$$

where u_e is given implicitly by Eq. (27). Assuming $\psi_e=0$ at $\varphi=0$, we get

$$\psi_e = 1 - \frac{V}{V-u_e} + \frac{Vu_e}{\mu}. \tag{33}$$

This agrees with the result obtained in Ref. 23. Now the integration of the ion term can be done by the following change of variable

$$\cosh\theta = (12\sigma p_0 V^2 u_{01}^3)^{-1/2}(V^2 - 2\varphi' + 3\sigma p_0 u_{01}), \tag{34}$$

whence

$$\sinh\theta = (12\sigma p_0 V^2 u_{01}^3)^{-1/2}[(V^2-2\varphi'+3\sigma p_0 u_{01})^2 - 12\sigma p_0 V^2 u_{01}^3]^{1/2} \tag{35}$$

From (24) we get

$$\bar{n} = \left(\frac{V^2 u_{01}}{3\sigma p_0}\right)^{1/4} e^{-\theta/2}. \tag{36}$$

So

$$\begin{aligned}\int \bar{n}\, d\varphi &= \int \left(\frac{V^2 u_{01}}{3\sigma p_0}\right)^{1/4} e^{-\theta/2}\frac{d\varphi}{d\theta}\, d\theta \\ &= -(3\sigma p_0 V^6 u_{01}^7)^{1/4}\int e^{-\theta/2}\frac{(e^{\theta}-e^{-\theta})}{2}\, d\theta \\ &= -(3\sigma p_0 V^6 u_{01}^7)^{1/4}\left(e^{\theta/2} + \frac{1}{3}e^{-3\theta/2}\right) + C_1,\end{aligned} \tag{37}$$

where C_1 is an integrating constant. Using (25) we get

$$\begin{aligned}\int n_1\, d\varphi = \frac{V^2}{8}\Bigg[&-\left(\frac{V^2u_{01}}{3\sigma p_0}\right)^{1/4} e^{-\theta/2} \\ &- 6u_{01}^2\left(\frac{3\sigma p_0}{V^2u_{01}}\right)^{1/4} e^{\theta/2} + 4u_{01}^3\left(\frac{3\sigma p_0}{V^2 u_{01}}\right)^{1/2} e^{\theta} \\ &- u_{01}^4\left(\frac{3\sigma p_0}{V^2u_{01}}\right)^{3/4} e^{3/2\theta}\Bigg].\end{aligned} \tag{38}$$

We have omitted the integrating constant as it is already included in (37). It is convenient to choose the integrating constant in such a manner as to get $\psi(0)=0$. Thus we have

$$\begin{aligned}\psi_i = -\Bigg\{&(3\sigma p_0 V^6 u_{01}^7)^{1/4}\left((e^{\theta/2}-e^{\theta_0/2}) + \frac{1}{3}(e^{-3\theta/2} - e^{-3\theta_0/2})\right) + \frac{V^4}{8C^2}\Bigg[\left(\frac{V^2u_{01}}{3\sigma p_0}\right)^{1/4}(e^{-\theta/2}-e^{-\theta_0/2}) \\ &+ 6u_{01}^2\left(\frac{3\sigma p_0}{V^2u_{01}}\right)^{1/4}(e^{\theta/2}-e^{\theta_0/2}) - 4u_{01}^3\left(\frac{3\sigma p_0}{V^2u_{01}}\right)^{1/2} \\ &\times (e^{\theta}-e^{\theta_0}) + u_{01}^4\left(\frac{3\sigma p_0}{V^2u_{01}}\right)^{3/4}(e^{3/2\theta}-e^{3/2\theta_0})\Bigg]\Bigg\},\end{aligned} \tag{39}$$

where

$$\theta = \cosh^{-1}(12\sigma p_0 V^2 u_{01}^3)^{-1/2}(V^2 - 2\varphi' + 3\sigma p_0 u_{01}), \tag{40}$$

$$\theta_0 = \cosh^{-1}(12\sigma p_0 V^2 u_{01}^3)^{-1/2}(V^2 + 2a_0 + 3\sigma p_0 u_{01}), \tag{41}$$

$$u_{01} = 1 - \frac{u_0}{V}, \tag{42}$$

$$\varphi' = \varphi - a_0, \tag{43}$$

and

$$a_0 = -Vu_0\left(1+\frac{u_0^2}{2C^2}\right) + \frac{u_0^2}{2}\left(1+\frac{3u_0^2}{4C^2}\right). \tag{44}$$

Neglecting electron inertia and relativistic effect ($m_e=0$, $C=\infty$) we get

$$\psi_e(\varphi) = 1 - e^{\varphi}, \tag{45}$$

$$\psi_i(\varphi) = -[(3\sigma V^6)^{1/4}(e^{\theta/2}-e^{\theta_0/2}) + \tfrac{1}{3}(e^{-3\theta/2} - e^{-3\theta_0/2})], \tag{46}$$

where

$$\theta = \cosh^{-1}[(V^2+3\sigma-2\varphi)/(12\sigma V^2)^{1/2}], \tag{47}$$

$$\theta_0 = \cosh^{-1}[(V^2+3\sigma)/(12\sigma V^2)^{1/2}]. \tag{48}$$

These results agree completely with those obtained by Roychoudhury *et al.*[25] for a nonrelativistic plasma with warm ions. To consider the limit $\sigma\to 0$ we first expand the right-hand side (RHS) of (40) in powers of σ. Neglecting terms of $O(\sigma^2)$ we get

$$\begin{aligned}\psi_i = -\Bigg(&u_{01}V^2 Z + \frac{\sigma p_0 u_{01}}{2}\left(\frac{3}{Z}-\frac{u_{01}^2}{Z^3}\right) + \frac{V^4}{8C^2}\Bigg\{\frac{u_{01}}{Z} \\ &\times\left[1-\frac{3\sigma p_0 u_{01}}{2V^2Z^2}\left(1-\frac{u_{01}^2}{Z^2}\right)\right] + 6u_{01}Z\left[1+\frac{3\sigma p_0 u_{01}}{2V^2Z^2}\right. \\ &\left.\times\left(1-\frac{u_{01}^2}{Z^2}\right)\right] - 4u_{01}Z^2\left[1+\frac{3\sigma p_0 u_{01}}{V^2Z^2}\left(1-\frac{u_{01}^2}{Z^2}\right)\right] \\ &+ u_{01}Z^3\left[1+\frac{9}{2}\frac{\sigma p_0 u_{01}}{V^2Z^3}\left(1-\frac{u_{01}^2}{Z^2}\right)\right]\Bigg\}\Bigg) + K,\end{aligned} \tag{49}$$

where

$$Z = (1 - 2\varphi'/V^2)^{1/2} \tag{50}$$

and

$$K=u_{01}\left[V^2Z_0+\frac{\sigma p_0 u_{01}}{2}\left(\frac{3}{Z_0}-\frac{u_{01}^2}{Z_0^3}\right)\right]+\frac{V^2}{8C^2}\left\{\frac{u_{01}}{Z_0}\right.$$
$$\times\left[1-\frac{3\sigma p_0 u_{01}}{2V^2Z_0^2}\left(1-\frac{u_{01}^2}{Z_0^2}\right)\right]+6u_{01}Z_0\left[1+\frac{3\sigma p_0 u_{01}}{2V^2Z_0^2}\right.$$
$$\left.\times\left(1-\frac{u_{01}^2}{Z_0^2}\right)\right]-4u_{01}Z_0^2\left[1+\frac{3\sigma p_0 u_{01}}{V^2Z_0^2}\left(1-\frac{u_{01}^2}{Z_0^2}\right)\right]$$
$$\left.+u_{01}Z_0^3\left[1+\frac{9}{2}\frac{\sigma p_0 u_{01}}{V^2Z_0^2}\left(1-\frac{u_{01}^2}{Z_0^2}\right)\right]\right\} \tag{51}$$

when

$$Z_0=(1+2a_0/V^2)^{1/2}. \tag{52}$$

Now considering $\sigma\rightarrow 0$ and neglecting electron inertia (i.e., assuming $m_e/m_i=0$) we get

$$\psi(\varphi)=-e^{\varphi}-V^2\left(1-\frac{u_0}{V}\right)\left[Z\left(1+\frac{3V^2}{4C^2}\right)+\frac{V^2}{8C^2Z}-\frac{V^2Z^2}{2C^2}\right.$$
$$\left.+\frac{V^2Z^3}{8C^2}\right]+C_0, \tag{53}$$

where

$$Z=(1-2\varphi'/V^2)^{1/2}, \tag{54a}$$

C_0 is such that $\psi(0)=0$. (54b)

This is identical with the result obtained by Roychoudhury *et al.*[21] Also in the limit $\sigma\rightarrow 0$, our result agrees with those of Kuehl and Zhang[23] where they obtained the pseudopotential for a relativistic plasma with cold ions.

III. SOLITARY WAVE SOLUTION

The form of pseudopotential would determine whether a soliton-like solution of Eq. (28) will exist or not. The conditions for existence of solition is[26]

$$\left.\frac{\partial^2\psi}{\partial\varphi^2}\right|_{\varphi=0}<0. \tag{55}$$

This is the condition for the existence of a potential well. Another condition is

$$\psi(\varphi_c)\geqslant 0, \tag{56}$$

where φ_c is given by

$$\varphi_c=(V^2+3\sigma p_0 u_{01}-\sqrt{12\sigma p_0 V^2 u_{01}^3})/2+a_0\ . \tag{57}$$

This condition is obtained in the following way: The pseudopotential ψ becomes complex if the density of the ion is complex. This occurs if

$$\varphi>(V^2+3\sigma p_0 u_{01}-\sqrt{12\sigma p_0 V^2 u_{01}^3})/2+a_0.$$

Hence, the particle moving in a pseudopotential well will be reflected back at a certain point $\varphi=\varphi_m$, φ_m being the value of φ when $\psi(\varphi)$ cuts the φ axis from below.

Let us first consider the simple case when $\sigma=0$, $m_e=0$, and $u_0=0$. As $c\rightarrow\infty$, it can be shown that the soliton solution exists for $1\leqslant V\leqslant 1.6$ (Ref. 22). If we neglect the electron inertia and ion temperature, the condition for existence of solitary waves reduces to[21] $1<\bar{V}<1.6$, where

$$\bar{V}=\frac{V}{1+2a_0/V}\ .$$

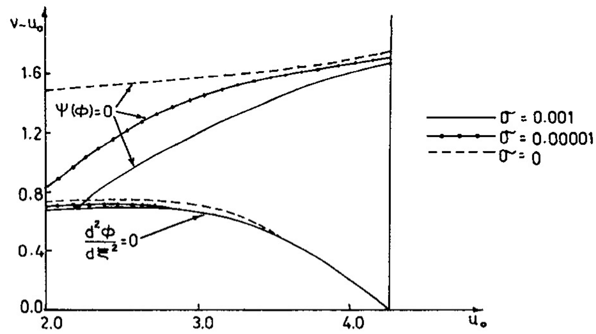

FIG. 1. Plot of $V-u_0$ against $u_0/10$ for $m_i/m_e=1836$, $u_0/c=0.02$. The solid lines show the boundaries for $\sigma=0.001$, the broken lines for $\sigma=0$ and the lines with circles for $\sigma=0.000\,01$. We have taken only the positive values of $V-u_0$.

Also, since in the limit $\sigma\rightarrow 0$ our results completely agree with those of Ref. 23, the conditions under which the solitary wave solutions exist will be same as those given in Ref. 23. However for $\sigma\neq 0$ these conditions can not be expressed in simple analytical form and one has to have recourse to numerical analysis. We have numerically analyzed the pseudopotential for some particular values of the relevant parameters. Figure 1 shows the region where solitary wave solutions exist for $m_i/m_e=1836$ and $u_{01}/c=0.02$, and for three values of σ, viz., $\sigma=0$, $\sigma=0.0001$, and $\sigma=0.001$. The boundary curves labeled

$$\left(\frac{\partial^2\psi}{\partial\varphi^2}\right)_{\varphi=0}=0$$

correspond to linear ion-acoustic waves. It is seen from this figure that the effect of finite ion temperature is to shrink further the region where solitary wave solutions exist. However, u_0/c has very little effect. In fact, for $\sigma=0$, the boundary $\psi(\varphi)=0$ almost coincides with that obtained by Kuehl and Zhang.

In Fig. 2, $\psi(\varphi)$ is plotted against φ for different values of V(30.75, 31, 31.25, and 31.55). Other parameters involved are $u_0=30$, $u_0/c=0.02$, $\sigma=0.0001$, $p_0=1$, $\mu=1/1836$. The amplitude of the solitary wave, if it exists, would be the value of φ at which $\psi(\varphi)$ crosses the φ axis from below. The end point of the graph shows the value of φ behind which $\psi(\varphi)$ becomes complex. As can be seen from Fig. 2, solitary wave solutions exist for $V=30.75$, 31, and 31.25, but for $V=31.55$, solitary wave solutions do not exist as $\psi(\varphi)$ is negative through out the region. This clearly shows how the finite ion temperature restricts the region where solitary wave solutions exist. It may be noted that for $\sigma=0$, solitary wave solutions exist even for $V=31.55$ (see Ref. 23). [To obtain the solitary wave solutions we have numerically integrated

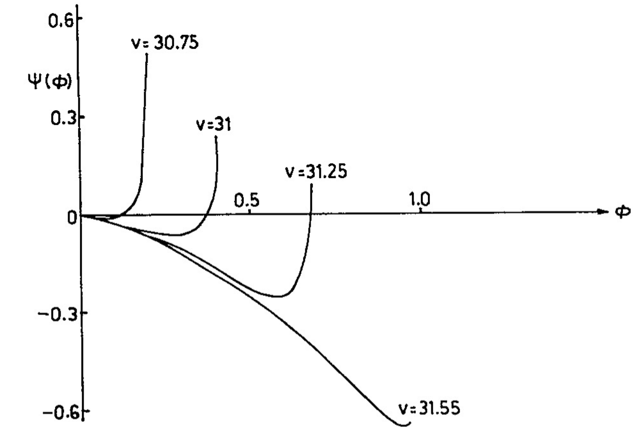

FIG. 2. Plot of $\psi(\varphi)$ against φ for several values of V, viz., V=30.75, 31, 31.25, 31.55, and $u_0/c=0.02$.

Eq. (18) for different values of σ viz., σ=0.0, 0.0001, and 0.01. Other parameters are $u_0=30$, $u_0/c=0.025$, $V=30.75$, $p_0=1$.

It is seen from Fig. 3 that both the amplitude and the width of the soliton decrease when temperature increases.] Now to compare our results with those obtained in Ref. 16 using reduction perturbation technique, we write [taking $\mu=0$ and neglecting terms of order $O(\varphi^3)$]

$$\frac{d^2\varphi}{d\xi^2}=a\varphi-b\varphi^2. \tag{58}$$

From RHS of Eq. (49), we have (neglecting electron inertia)

$$a=-1+\frac{1}{(v-u_0)^2}\left(1-\frac{3u_0^2}{2c^2}\right)-\frac{3\sigma p_0}{v(v-u_0)^3} -\frac{\sigma p_0}{8c^2}\left(\frac{30v^3}{(v-u_0)^5}-\frac{108v}{(v-u_0)^3}+\frac{96}{(v-u_0)^2} -\frac{18}{v(v-u_0)}-\frac{15(3u_0^4-4vu_0^3)}{v(v-u_0)^5}\right), \tag{59a}$$

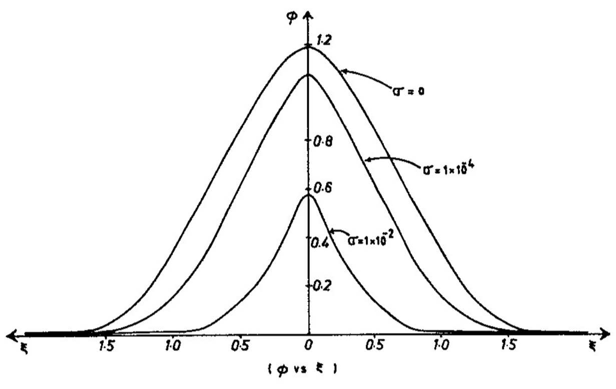

FIG. 3. Plot of $\varphi(\xi)$ against ξ for $u_0/c=0.025$ and $V=30.75$ for several values of σ, viz., σ=0.0, 0.0001, and 0.01.

$$b=-\frac{1}{2}+\frac{3}{2(v-u_0)^4}\left(1-\frac{u_0(v-u_0)}{c^2}\right)+\frac{15\sigma p_0}{v(v-u_0)^5} -\frac{v^2\sigma p_0}{8c^2}\left(\frac{315}{v(v-u_0)^7}+\frac{576}{v^4(v-u_0)^4}-\frac{810}{v^3(v-u_0)^5} -\frac{81}{v^5(v-u_0)^3}-\frac{315(3u_0^4-4vu_0^3)}{2v^5(v-u_0)^7}\right), \tag{59b}$$

and the soliton solution is

$$\varphi=\frac{3a}{2b}\,\mathrm{sech}^2\left(\frac{\xi}{\delta}\right), \tag{60}$$

where $\delta=2/\sqrt{a}$ is the width of the soliton and $3a/2b$ is its amplitude. To compare (60) with the result of Ref. 16 we first rewrite their result in the form

$$\varphi=\frac{3M}{\alpha}\,\mathrm{sech}^2\left(\frac{\Delta}{l}\right). \tag{61}$$

Also, we take $n_0=1$. Here the soliton amplitude is $3M/\alpha$ and its width is $l=(4\beta/M)^{1/2}$. Now, if we keep only first-order terms in p_0 and u_0 and u_0/c^2, we get

$$\frac{3M}{\alpha}\simeq 3M\left(1+\frac{3p_0}{2}+\frac{3}{2}\frac{u_0}{c^2}\right). \tag{62}$$

Again from (59a) and (59b) we get (after putting $v=\lambda_0+M$ and $\sigma=p_0$)

$$\frac{3a}{2b}\simeq 3M\left(1+\frac{3}{2}p_0+\frac{3}{2}\frac{u_0}{c^2}\right) \tag{63}$$

which is identical with (62). Similarly, the equivalence of δ and l, the respective widths, can be shown in a straightforward manner. Thus Nejoh's[17] result is but a particular case of our exact result.

IV. CONCLUSION AND DISCUSSION

In this paper we have studied the effect of ion temperature in a relativistic plasma using Sagdeev's pseudopotential approach. We have taken into account the effect of electron inertia in deriving the exact pseudopotential. It has been shown by Kuehl and Zhang[23] that the effect of electron inertia is to limit the regime in which solitary wave solutions exist. In the present work we have shown that the effect of finite ion temperature is to further restrict this region. To check our result with the previously published one, we have calculated the pseudopotential in the limit $\sigma\to 0$ and found that our results completely agree with those of Kuehl and Zhang. For small amplitude solitons, the pseudopotential obtained by us reproduces the results obtained by Nejoh.[17]

Though we have condidered here one electron plasma the result can be easily extended to two electron plasma and also plasma with negative ions.

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