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ENDOGENOUS GROWTH AND THE SIZE OF THE MARKET

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Abstract: A model of endogenous growth is considered where demand plays a crucial role in determining the rate of technical progress and growth. We show that the size of the middle class is extremely important in determining the rate of growth and the latter is zero unless the economy has a sufficiently large middle class. The existence of long run positive growth is also shown to be dependent upon the initial conditions.

1. INTRODUCTION

If one looks at the world's growth experience over the last hundred years or so one comes across the fact that the levels of per capita income or standards of living have been rising continuously over time. Even casual empiricism would suggest that this rise has been made possible mainly through continuing technical progress. This technical progress, in turn, has been the result of conscious R & D efforts by firms. Thus, if one is looking for a theoretical explanation of growth of per capita income, one has to face the question: what determines the rate of technical progress in an economy?

Clearly, the issue is closely related to economic development as well. If one could satisfactorily understand the mechanics of technical progress and growth, one could also explain why growth rates have been so much different across countries. In particular, an understanding of the mechanics of growth enables one to identify the constraints on economic development. Looking at the recent literature, quite a bit of it has developed, over the past ten years or so, focusing on endogenous technical progress and growth [see, for example, Romer (1986, 1990), Lucas (1988), and Grossman and Helpman (1991b)]. In these models, the rate of growth is obviously determined by the rate of technical progress. But more

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importantly, the rate of technical progress is determined by the amount of "resources" available in the economy. A large part of the literature makes a distinction between skilled and unskilled labour and emphasizes the role of human capital formation in R & D, technical progress and growth. In other words, R & D is assumed to use human capital in the form of skilled labour and therefore its speed depends on the human capital resources available in the economy. For example, in Romer (1990), the growth rate of the economy is shown to be equal to zero if the endowment of human capital in the economy is less than some minimum level. In other models, labour is assumed homogeneous and its endowment determines technical progress and growth; the higher the endowment of labour, the lower being the cost of R & D [see, for example, Grossman and Helpman (1991b)]. In short, in the existing models, the rate of technical progress and growth is determined from the supply side.

In contrast, the present paper views endogenous growth from the demand side. It is well understood that R & D expediture by firms depends upon their profit opportunities. These profit opportunities, in turn, are to a large extent limited by the expenditure made by the consumers on the products of the firms. In the model that is developed below, we show that this expenditure by the consumers depends upon the total profits generated in the economy and, in particular, on profits generated in the mass consumption good sector. Thus, in our model, technical progress and growth are crucially determined by the size of the mass consumption good sector. It is perhaps helpful to describe the mechanics of growth in our model in somewhat more detail at the very outset.

We consider an economy producing one homogeneous good and one differentiated good. The differentiated good is produced at two different quality levels: high and low. There are three classes of income earners: the rich, the middle class and the poor. The rich consumes the high quality, the middle class the low quality and the poor are too poor to consume any differentiated good at all. Technical progress, in the form of quality improvement, takes place only in the differentiated good. It takes place first in the high quality good sector and then trickles down to the low quality good sector with a time lag. In other words, technical progress in the high quality good sector is the driving force of growth in the economy.

The rate of technical progress in the high quality good sector depends upon the level of profits in that sector and hence on the level of expenditure made on the high quality good. By assumption, the high quality good is consumed only by the profit earners (i.e. the rich). Therefore, the rate of technical progress depends upon the level of profits generated in the economy and, in particular, on the level of profits generated outside the high quality good sector. The homogeneous good is assumed to be perfectly competitive. Hence, the only profit generated outside the high quality good sector is the low quality or the mass consumption good sector. The size of the mass consumption good sector and the size of the middle class, therefore, determine the rate of technical progress in the high quality good sector as well as the over all rate of growth in the economy.

We have defined the middle class somewhat broadly as those people who are able to consume the mass consumption good and whose expenditure generates profit outside the high quality good sector. The poor, on the other hand, are defined to be those whose expenditure does not generate any profit. This does not seem to be an unrealistic characterization of the poor in developing economies where a significant part of the population lives below the poverty line and does not incur expenditure which may give rise to profits. Since in our model, growth depends on the size of the middle class, a direct implication is that promotion of a portion of the poor to the middle class increases the rate of growth. We show that this has to be done through the formation of skills among the poor. But the main difference between our work and the existing models is that in the present work, human capital formation does not affect growth directly from the supply side. Instead, it works through the channel of demand by creating more purchasing power. We also show that a transfer of purchasing power from the rich to the middle class or to the poor actually reduces the rate of growth. We therefore conclude that in a developing economy the rate of growth can be increased not through redistributive tax-transfer policies, but by increasing the productivity of the poor, and thereby creating more purchasing power in the economy. In a recent paper, Murphy, Shleifer and Vishny (1989) have emphasized the role played by the size of the market, income distribution and, in particular, by the size of the middle class in the process of industrialization of less developed countries. In their model, for increasing returns technology to break even, sales must be high enough to cover fixed set-up costs. The role of demand has also been emphasized in another recent paper by Eswaran and Kotowal (1993) which has been developed along the lines of Murphy, Shleifer and Vishny. The main difference between the Murphy, Shleifer and Vishny approach and ours is that while the former is confined to a static framework, our focus is primarily on growth. Apart from this, there are, of course, quite a few differences in the details of model building. The endogenous growth model that we develop here is, in spirit, related to the "quality-ladder" approach of Grossman and Helpman (1991a). In a growth model where growth occurs in terms of rising product qualities, we have introduced different classes of income earners and have shown that the rate at which product quality rises over time crucially depends on the size of the middle class.

In what follows, we develop the basic model in section 2. In section 3 we characterize the equilibrium. Section 4 discusses the main results of the paper. Section 5 considers some further implications of the basic model. Section 6 concludes the paper.

2. THE BASIC MODEL

We consider an economy producing two types of goods: one homogeneous good which we call good 0 and which serves as the numeraire good and one differentiated good which is produced in two different quality levels, high and low, denoted by

h and m respectively (m for mass consumption). There is a single primary factor of production identified as labour. There are three classes of income earners: rich (h), middle class (m) and poor (l). For simplicity, we assume that all income earners within a class are alike.[1] There are N_h number of rich people and each of them is endowed with one unit of labour. In addition, each gets a share of profits. In particular, total profits are shared equally by the N_h high income people. Thus, a rich person earns $w+\pi/N_h$ where w is the wage rate and π is total profits generated in the economy. A member of the middle class, on the other hand, is endowed with one unit of labour but has no share in profits. Accordingly, she earns an income w. Finally, a person belonging to the low income group is endowed with β units of labour and earns an income βw. By assumption, $\beta<1$, i.e. a low income person is less efficient than the middle class or the rich. The sizes of the middle and low income groups are given by N_m and N_l respectively.

Next, we come to the specification of the demand side. Each individual consumes either one or zero units of the differentiated good. As consumers, all individuals are assumed to have identical preferences represented by the utility function

$$\begin{aligned} U_j &= (y_j - p_s)\alpha_s \quad \text{if consumes quality } s, \\ &= y_j \quad \text{otherwise}; \; j=l,m,h; \; s=m,h. \end{aligned} \tag{2.1}$$

In equation (2.1), U_j is the utility of a consumer belonging to jth income group, y_j is her income, p_s is the unit price of the jth quality good and α_s is a constant. We assume that other things remaining the same, higher quality goods yield higher utility. Therefore, we have $1<\alpha_m<\alpha_h$. A few observations about the utility maximization problem are now in order.

First, it is to be noted that individuals do not save and maximize only their instantaneous utilities. The underlying assumption is that there is no credit market.[2] We make this simplifying assumption to keep aside the role of savings in determining investment expenditure on R & D. As explained below, investment expenditure on R & D in our model is met out of undistributed profits. We assume away individual savings just to put emphasis on the role of demand in determining R & D.

Since there is no individual savings, consumers, in each period, spend their entire income on consumption. Individual j consuming quality s also consumes (y_j-p_s) units of the homogeneous good since the price of the homogeneous good is unity by choice of numeraire. In case the consumer does not consume any quality good, she spends her entire income on the homogeneous good getting a utility y_j. The direct utility function underlying the indirect utility function (2.1) is of the form $U=c_0\alpha_s$ when consumes quality s and $U=c_0$ otherwise where c_0 is

[1] The analysis may be extended to take into account continuous income distribution without much difficulty.

[2] Alternatively, we may assume that each individual lives for exactly one period and leaves no bequest.

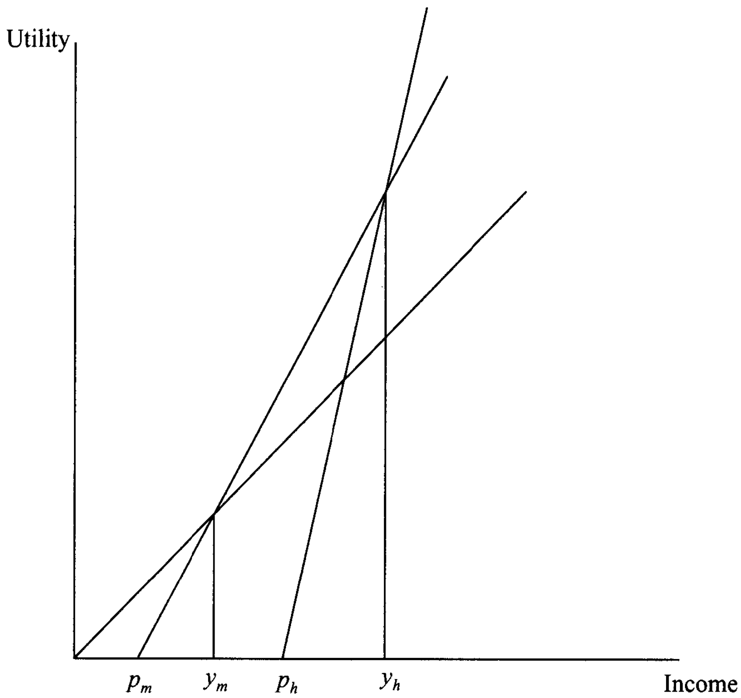

Figure 1.

consumption of the homogeneous good.

Next, note that the utility function exhibits strong income effects. Let us assume for the moment that $p_m < p_h$ (we shall show below that this is true in equilibrium). Then utility from the consumption of goods of different qualities as well as utility from the consumption of good 0 alone may be represented as functions of income as shown in Figure 1. Utility maximization implies that for $y < y_m$ the consumer consumes only good 0; for $y_m \leq y \leq y_h$ the consumer consumes quality m and good 0; and for $y > y_h$ the consumer consumes quality h and good 0. Therefore, there is a basis for different classes of income earners consuming different qualities in equilibrium.

We now consider technology, technical progress and market structure. We assume that in any period of time one unit of labour is required to produce one unit of quality h. Similarly, to produce one unit of quality m, it is necessary to employ c units of labour where $c < 1$. Finally, to produce one unit of the homogeneous good, one unit of labour is required. We assume that $c < \beta$.[3]

The economy is divided into a large number of sub-economies or markets [as, for example, assumed in Hart (1982)]. In each of these sub-economies, there is a monopolist firm producing the h-good and another monopolist firm producing the m-good. The homogeneous good, on the other hand, has an economy-wide market and is produced by perfectly competitive firms. Consumers are evenly

[3] Therefore, *by assumption*, the poor are unable to consume any differentiated good.

distributed between these sub-economies. Thus in each of these sub-economies, there are n_h $(=N_h/n)$ rich people, n_m $(=N_m/n)$ middle class and n_l $(=N_l/n)$ poor, where n is the number of sub-economies. These sub-economies or markets are assumed, for the time being, to be locationally separated so that consumers located in one market do not buy in others.[4] The net result is that an h or m-sector monopolist, though has a monopoly power in his local market, is none the less small with respect to the aggregate economy. On the other hand, consumers belonging to a particular market earn their profit incomes from all the markets, i.e., they have shares in all the firms operating in the economy. This, together with the assumption that labour markets are perfectly competitive, imply that each firm takes consumers' income as given.

Next we consider technical progress. Two types of technical progress takes place each period, one involving product innovation (i.e. quality improvement) and the other involving process innovation (i.e. cost reduction). The pattern of technical progress may be described in the following way. In each period, the h-sector monopolist, in each market, undertakes innovation to improve the quality of the good he sells. The extent of improvement depends upon the level of R & D expenditure and is a choice variable for the h-sector monopolist. The m-sector monopolist, in each market, on the other hand, undertakes R & D to imitate the previous period quality level of the h-sector monopolists and to reduce costs. Thus imitation and cost reduction take place with a one period lag. More specifically, in any period t, an m-sector monopolist imitates the *average* $t-1$ quality level of the h-sector monopolists and is also able to produce it at a lower cost. By assumption, therefore, the h-sector monopolists are specialized in product innovation and the m-sector monopolists in process innovation. Finally, we assume that the technology of the m-sector monopolists become common knowledge with a further one period lag. Thus in period t, any (perfectly competitive) firm can (potentially) produce the $t-1$ quality levels of the m-sector monopolists at the same cost as they were being produced by the latter.[5]

Let us assume for the time being that the h-sector monopolists are all identical in the sense that there is no difference in their expenditure on R & D in each period. Therefore they produce the same quality level. This is shown to be true in the equilibrium described in the next section particularly because each h-sector firm has the same market size. Let $\alpha_{h,t-1}$ be the quality level of the h-sector monopolists in period $t-1$. Then $\alpha_{mt}=\alpha_{h,t-1}$ because the average quality level of the h-sector monopolists is also $\alpha_{h,t-1}$. Let

$$\lambda_t=\alpha_{ht}/\alpha_{mt}=\alpha_{ht}/\alpha_{h,t-1} \tag{2.2}$$

[4] We shall indicate below how this assumption can be relaxed.

[5] Conceptually, this means that perfectly competitive firms can imitate the product innovation of $t-1$ in period t. But they can not produce the good in t because of the process innovation and the consequent cost reduction of the m-sector firms. They learn about this process innovation with a further one period lag.

Then λ_t is the *growth factor* in the economy at period t and the *growth rate* is $\lambda_t - 1$. The pattern of innovation and technology diffusion lead to the growth of α_s ($s = m, h$) over time. This leads to an increase in the utility levels which is the essence of growth in our model.

The pattern of innovation and technology diffusion imply that over time the price of a particular quality goes down. As we demonstrate in the next section, when a quality improvement takes place in the h-sector, the commodity is expensive to produce and can be consumed only by the rich. In the next period, there is a cost reducing innovation and the commodity becomes available to the masses, i.e., to the middle class. The poor, however, remain outside the market for the differentiated good; for their income βw falls short of the production costs w and cw. In other words, the poor are *assumed* to be too poor to consume any differentiated good.

We end this section with a couple of observations. First, technology diffusion in our model is crucial to technical progress and growth. Because of costless technology diffusion, the m-sector monopolists know that their profits are going to be zero if they do not go up in the quality ladder by imitating the h-sector monopolists. This obviously compels them to spend money on R & D and imitate. Given that the m-sector monopolists are compelled to imitate, the h-sector monopolists have no choice but to innovate in each period.

Secondly, it is to be noted that in our model decisions to innovate and the realizations of technical progress take place in the same period. At the beginning of the period, firms decide how much to invest in R & D, i.e. they decide how many workers to be hired for R & D. This leads to product improvement at the end of the period and the cost of R & D is met out of undistributed profits.[6] Thus firms are not fund constrained as far as innovations are concerned and individual savings, which are assumed to be zero in our model, do not play any role in determining R & D investment.

3. EQUILIBRIUM

We may now proceed to analyze the actual working of the model. Given perfect competition in the homogeneous good sector and given the choice of numeraire, we have

$$p_0 = w = 1 \tag{3.1}$$

Next consider the middle class. Note that a member of the middle class is unable to consume quality h. Since, to earn positive profits, an h-sector monopolist sets $p_h > w$, i.e. the marginal cost of production, and since the income of a representative middle class is w, the middle class can not afford to buy quality h. Therefore, in

[6] We may assume that workers and shareholders are paid in terms of a paper claim on the firm's product which they can exchange in the market to arrive at their desired consumption levels.

any period t, for the middle class the choice is between consuming quality α_{mt} and quality $\alpha_{m,t-1}$. The former is produced by the m-sector monopolist and the latter can be potentially produced by perfectly competitive firms through a one period lagged technology diffusion. If the $\alpha_{m,t-1}$ quality is actually produced in period t then it is sold at a price cw, the marginal cost of production. Thus, given her income w, a member of the middle class is indifferent between consuming quality α_{mt} and $\alpha_{m,t-1}$ if

$$(w-p_{mt})\alpha_{mt}=(w-cw)\alpha_{m,t-1} \tag{3.2}$$

Let $\alpha^0_{h,t-1}=(1/n)\sum_k \alpha^k_{h,t-1}$ be the average quality level in the h-sector in period $t-1$, where $\alpha^k_{h,t-1}$ is the quality level in the kth market.[7] Let λ^0_t measure the relative extent of quality improvement in the m-sectors in period t, where

$$\lambda^0_t=\alpha_{mt}/\alpha_{m,t-1}=\alpha^0_{h,t-1}/\alpha^0_{h,t-2}$$

and where the last term represents the average quality improvement in the h-sector in period $t-1$. From (3.2) we solve the price of the m-quality good as

$$p_{mt}=[c+\lambda^0_t-1]/\lambda^0_t \tag{3.3}$$

where using (3.1) we put $w=1$. Assume that c, the unit cost of production in the m-sector, is a constant.[8] Then, as of period t, the right hand side of (3.3) is given and hence p_{mt} gets completely determined. If the m-sector monopolist charges a price which is just a shade below p_{mt} as given by the right hand side of (3.3), then he gets the entire middle class market.[9]

Next consider the rich whose choice is between quality h and quality m. In the kth market, a member of the high income group is indifferent between the two qualities at period t if the price of the high quality good in the kth market satisfies

$$p^k_{ht}=[(\lambda^k_t-1)y^k_{ht}+p_{mt}]/\lambda^k_t \tag{3.4}$$

where $\lambda^k_t=\alpha^k_{ht}/\alpha_{mt}$ measures the extent of technical progress in the h-sector of the kth market in period t. Note that this improvement is over the *economy wide* average quality level in the previous period. An h-sector monopolist chooses his price just a shade below the one given by the right hand side of (3.4) and captures the entire market.

An h-sector monopolist, while choosing his price, takes p_{mt} and more importantly, y^k_{ht} as given. Along with the price, he also chooses the rate of technical progress λ^k_t. Let us assume that the cost of innovation is linearly related to the rate of growth λ^k_t-1. In other words, to achieve an improvement $\lambda^k_t>1$, $\delta(\lambda^k_t-1)$

[7] The m-sector quality levels are, of course, identical.

[8] It is possible to make c a function of the R & D expenditure of the m-sector firm. This, however, keeps the basic analysis unchanged and does not provide any additional insight.

[9] For the time being, it is assumed that an m-sector monopolist does not compete with an h-sector monopolist to capture the high income market. We, however, explain below the implication of this assumption and how this behaviour might be consistent with profit maximization.

units of labour are required the cost of which is $w\delta(\lambda_t^k-1)$ where δ is a positive constant. Therefore, in the kth market, the profit of the h-sector monopolist in period t may be written as

$$\pi_{ht}^k=(p_{ht}^k-w)n_h-\delta(\lambda_t^k-1) \tag{3.5}$$

Substituting the value of p_{ht}^k from (3.4) in (3.5) we get

$$\pi_{ht}^k=[(\lambda_t^k-1)y_{ht}^k/\lambda_t^k+p_{mt}/\lambda_t^k-w]n_h-\delta(\lambda_t^k-1) \tag{3.6}$$

The h-sector monopolist in the kth market maximizes profits by choosing λ_t^k. Now, it is clear from (3.6) that $p_{m,t+1}$, which is a function of the average λ_t^k, affects the level of profit of the h-sector monopolist in period $t+1$. However, since $p_{m,t+1}$ is a function of the *average* quality improvement in the h-sectors at period t, each h-sector monopolist assumes that the effect of his own technical improvement on the economy wide average is *negligible.* This makes the intertemporal profits of the h-sector monopolists independent and the h-sector monopolist in the kth market maximizes his single period profits by choosing the rate of product improvement λ_t^k. Maximization of π_{ht}^k with respect to λ_t^k yields

$$\delta(\lambda_t^k)^2=(y_{ht}^k-p_{mt})n_h \tag{3.7}$$

Recalling our assumption that the rich are all identical, i.e. $y_{ht}^k=y_{ht}$ for all k, we conclude that the rate of technical progress must be the same in all the markets. Thus we can write

$$\delta(\lambda_t)^2=(y_{ht}-p_{mt})n_h \tag{3.8}$$

where λ_t denotes the common rate of technical progress in the h-sectors. To determine this common rate of technical progress, we need to find out the *equilibrium* value of $(y_{ht}-p_{mt})$. But before determining λ_t, note that a common λ_t across all h-sectors imply $\lambda_t^0=\lambda_{t-1}=\alpha_{h,t-1}/\alpha_{h,t-2}$.

The profit of the kth sector monopolist may be written as

$$\pi_{ht}^k=[(\lambda_t-1)y_{ht}/\lambda_t+p_{mt}/\lambda_t-w]n_h-(y_{ht}-p_{mt})n_h/\lambda_t+\delta \tag{3.9}$$

by substituting the value of $\delta\lambda$ from equation (3.8). Hence, summing over k and simplifying we can write the aggregate profit $\pi_{ht}=\sum_k \pi_{ht}^k$ as

$$\pi_{ht}=[(\lambda_t-2)y_{ht}/\lambda_t+2p_{mt}/\lambda_t-w]N_h+n\delta \tag{3.10}$$

Note that the y_{ht} term appearing so far in the equations was strictly speaking the income of a representative rich *as expected* by the h-sector firms. Clearly, given their *expectations* the h-sector firms were choosing their rates of technical progress and product price. We assume that in equilibrium these expectations turn out to be correct, i.e. $y_{ht}=(\pi_{ht}+\pi_{mt})/N_h+w$ where the values of π_{mt} and in particular π_{ht} are those that are obtained from the model. Therefore, substituting the value of y_{ht} in equation (3.10) and simplifying, we get

$$(y_{ht}-p_{mt})n_h=\lambda_t(\pi_{mt}+n\delta)/2n \tag{3.11}$$

Finally, using (3.11) to manipulate equation (3.8), we may write the equilibrium value of λ_t as

$$\begin{aligned}\lambda_t &= (\pi_{mt}+n\delta)/2n\delta \qquad \text{for} \quad \lambda_t>1\\ &=1 \quad \text{otherwise}.\end{aligned} \tag{3.12}$$

Clearly, λ_t can not be less than unity. It can not be less than unity because λ_t-1 is the *rate of quality improvement* which can at most be zero but can never be negative. In other words, it can never pay the h-sector firms to choose an inferior quality level which has been improved upon in the past. The least they can do is to undertake no quality improvement at all in which case $\lambda_t=1$. Hence the minimum value λ_t can take is unity.

Equation (3.12) shows that the rate of innovation and growth is directly related to the level of profits in the m-sector and inversely to the cost of innovation. We assume that L_m amount of labour is engaged in R & D activities in each of the m-sectors to achieve a cost reduction from w to cw. We further assume that L_m and c are fixed and not choice variables of the firms. Consequently, the level of profits in all the m sectors taken together is given by

$$\begin{aligned}\pi_{mt} &= (p_{mt}-c)N_m-nL_m\\ &=(1-c)(\lambda_{t-1}-1)N_m/\lambda_{t-1}-nL_m\end{aligned} \tag{3.13}$$

Substituting the value of π_{mt} in equation (3.12) we get

$$\begin{aligned}\lambda_t &= (2\delta)^{-1}[(1-c)(\lambda_{t-1}-1)n_m/\lambda_{t-1}-L_m+\delta] \qquad \text{for} \quad \lambda_t>1\\ &=1 \quad \text{otherwise}.\end{aligned} \tag{3.14}$$

The right hand side of (3.14) being a function of λ_{t-1} alone may be represented by $f(\lambda_{t-1})$ and consequently we can write the expression for λ_t in the following condensed form

$$\lambda_t=\max\{1,f(\lambda_{t-1})\} \tag{3.15}$$

Equation (3.15) expresses λ_t as function of λ_{t-1}. Since $f'(\lambda_{t-1})>0$ and $f''(\lambda_{t-1})<0$, the relationship between λ_t and λ_{t-1} is upward rising and concave as shown in Figure 2. Note that the relationship is valid only for $\lambda_{t-1}>1$. For $\lambda_{t-1}=1$, from (3.3), $p_{mt}=c$ and hence $\pi_{mt}=0$ (because L_m would also be equal to zero in this case). The intuition is that if there is no technical progress in the h-sector in period $t-1$, there is nothing to imitate for the m-sector firms in period t and the m-sector monopolists become perfectly competitive. Therefore, in this case, $\lambda_t=1$. We superimpose the 45° line in Figure 2 and denote the stagnation point (where $\lambda_t=\lambda_{t-1}=1$) by S.

As is clear from Figure 2, there could be two steady state equilibria with positive growth. However, E^* is locally stable and equilibrium E' is not. To see why E^* is locally stable, consider an arbitrary starting point $\lambda^0_{t-1}>\lambda'$. Then the consecutive

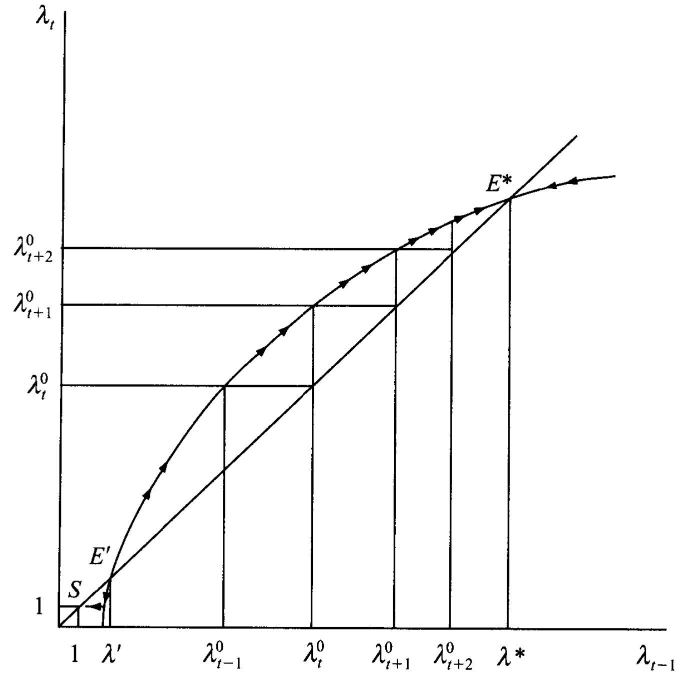

Figure 2.

growth factors in the future periods are λ_t^0, λ_{t+1}^0 and so on as shown in the Figure. Eventually the growth factor converges to the steady state level λ^*. On the other hand, if the initial value of λ_{t-1} is less than λ', then the economy eventually goes to the stagnation point S (where the growth rate is zero) and stays there permanently. Thus the unstable equilibrium E' is surrounded by two stable equilibria E^* and S on its two sides.

4. DISCUSSION

A few comments on the nature of equilibrium are now in order. Firstly, it is to be noted that in this model since all the residual income is spent on the homogeneous good, by Walras Law the labour market automatically clears. This can, of course, be verified by calculating the demand for labour and by checking that it is always equal to the supply of labour.

Secondly, it is clear from Figure 2 that the long run steady state growth rate depends on the *initial conditions.* In particular, if the initial growth rate is less than λ', then in the long run the economy will reach the stagnation point S. Similarly, if the *initial quality level* in the m-sector is less than $(1-c)^{-1}$ then the middle class will not consume any differentiated good. This can be easily checked from the utility function and the fact that the price of the differentiated good can not be less than cw. In this case, growth will never take off. Therefore, not only the initial growth rate but also the initial absolute quality level would have to be greater than a critical value in order to have positive long run growth. This has

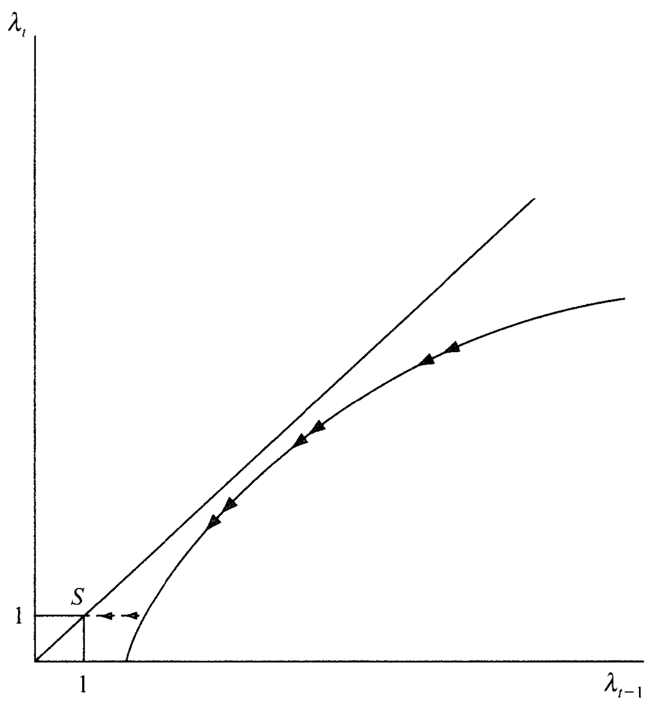

Figure 3.

a straight forward implication for growth in underdeveloped countries. In most of these countries the existing quality levels are quite low. Our model implies that these countries are confined to a *low-level stagnation trap.* Unless there is a *big push* in terms of technology from outside, growth can never take off in these economies. The same comments can be made about countries with initially low rates of growth.

Thirdly and perhaps most importantly, the long run rate of growth depends crucially on the size of the mass consumption good sector and on the size of the middle class. If N_m, the size of the middle class, goes up then the curve in Figure 2 shifts upwards and the long run growth λ^* increases. Alternatively, if the size of the middle class is less than a critical level, then the long run growth rate goes to zero as shown in Figure 3. Thus, in order to have positive growth in the long run, the size of the middle class market has to be greater than a certain minimum level. This result may be compared with Romer (1990) where the growth rate becomes zero unless the size of human capital is greater than a critical value. In other words, while in Romer, as well as in the existing literature, the rate of growth is constrained from the supply side, in our model it is constrained by demand and the size of the market.

We may get some further insights into the model by looking at the conditions under which a steady state equilibrium with positive growth exists. From Figure 2 it is clear that the $f(\lambda_{t-1})$ curve intersects the 45° line provided $\lambda_t > \lambda_{t-1}$ at the point where $f'(\lambda_{t-1}) = 1$. After some manipulation, the condition reduces to

$$[\{(1-c)n_m/2\delta\}^{1/2}-1]^2>(L_m+\delta)/2\delta \tag{4.1}$$

which is obviously satisfied for large enough N_m.

The role played by the middle class in sustaining growth may be explained heuristically in some detail. Note that each member of the high income class is just able to produce the high quality good with her one unit of labour endowment. But she does not demand any h-good unless she has a sufficiently high profit income besides her wage income. This profit income is provided by the middle class who produces a *surplus*. In particular, a middle class uses a fraction c of her labour endowment to produce the m-good she consumes. Of the remaining $(1-c)$ only a part goes to the production of the homogeneous good which she herself consumes. This can be easily seen by noting that $(w-p_m)$, i.e. the number of units of the homogeneous good she consumes, is less than $(1-c)$, since $p_m>c$. This *surplus* labour is used partly in the production of the homogeneous good that is consumed by the rich and partly in the R & D sectors of the differentiated goods. Unless this surplus is sufficiently high, there is no growth in the long run; for, unless this surplus is high, i.e. profits in the m-sectors are high, there is no *demand* for the high quality good.

Finally, in order to have a positive long run growth, profit in the h-sector must be positive. Using the value of λ_t from (3.11) and the value of p_{mt} from (3.3) we may write

$$\pi_{ht}=n\delta(\lambda_t-1)^2-N_h(1-c)/\lambda_{t-1} \tag{4.2}$$

Thus, h-sector profits are positive only if N_h is *sufficiently small.* This might seem apparantly paradoxical because it implies that a rise in sales in the h-sector reduces profits in that sector. Why this happens in our model may be explained intuitively. Note that a rise in N_h reduces the profit income of each member of the rich and hence the demand price p_{ht} goes down. This reduces total profits in the h-sector. We may, therefore, conclude that a large middle class as well as a small number of rich are necessary for long run growth. The long run growth λ^* is, of course, independent of N_h but increases with N_m once positive profit in the h-sector is guaranteed.

5. SOME FURTHER IMPLICATIONS OF THE MODEL

In the above analysis, we implicitly assumed that the m-sector firms do not find it profitable to sell their products to the rich. We may, at this point, look into the implication of this assumption. Suppose an m-sector firm enters into price competition with the corresponding h-sector firm. The minimum price it can charge for its product is given by p^*_{mt}, where p^*_{mt} must satisfy

$$(p^*_{mt}-cw)(N_h+N_m)=(p_{mt}-cw)N_m \tag{5.1}$$

p_{mt} being the price the m-sector monopolist charges when he does not enter into a price competition with the h-sector firm. The latter price is given, as before, by equation (3.3). In other words, the m-sector monopolist can reduce his price to capture the h-class market only up to the point where his profits in the enlarged market is equal to the profits he would have earned if he were selling only to the middle class. Suppose he charges this price. Then the h-sector firm can still remain in the market if

$$\pi_{ht}^{*k} = \delta(\lambda_t - 1)^2 - (w - p_{mt})n_h > 0 \tag{5.2}$$

If the above condition is satisfied, the h-sector monopolist can and will undercut the m-sector monopolist and drive her out of the h-class market. Of course, knowing this, the m-sector monopolist will not try to capture the h-class market at all by undercutting price. Since $p_{mt}^{*} < p_{mt}$, we have to put a more stringent upper bound on N_h or require a much higher value of N_m so that the h-sector monopolist remains in business and undertakes R & D.

Next we consider our assumption regarding the division of the economy into strictly separated sub-economies. Instead of assuming locational separation, we can make the somewhat weaker assumption that within the h and m sectors, goods are horizontally differentiated and each horizontal quality is produced by a single firm. Thus n qualities are produced within each sector. Also the population is divided into n equal groups with the jth group having the jth horizontal quality as its ideal choice. This means that the jth group will be willing to pay a lower price for the ith horizontal quality than it is willing to pay for the jth quality. Now, starting with the equilibrium described above, a firm producing the ith quality has no incentive to enter into horizontal price competition in the jth market because the relevant jth firm can always successfully drive it out of the market by undercutting its own price.

We end this section by considering the effects of a transfer from the rich to the middle class or to the poor on the long run rate of growth. Suppose each member of the rich is taxed by a lump sum amount T and the tax proceeds are equally distributed among the middle class. Thus the income of a representative rich falls by T and that of the middle class increases by TN_h/N_m. As a result, the representative h-sector profit changes to

$$\pi_{ht}^{k} = (n)^{-1}[(\lambda_t - 1)(\pi_{mt} - T) + (p_{mt} - w)N_h - \delta\lambda_t(\lambda_t - 1)] \tag{5.3}$$

Thus the equilibrium value of λ_t is given by

$$\lambda_t = (\pi_{mt} - TN_h + n\delta)/2n\delta \tag{5.4}$$

The corresponding value of π_{mt} is given by

$$\pi_{mt} = N_m[w + TN_h/N_m - cw]\{\lambda_{t-1} - 1\}/\lambda_{t-1} - L_m \tag{5.5}$$

An increase in T directly depresses the value of λ_t in (5.4) but indirectly stimulates it through an increase in π_{mt}. The net effect, however, is depressing, i.e.

$$d\lambda_t/dT = -N_h(2n\delta\lambda_{t-1})^{-1} < 0 \qquad (5.6)$$

In other words, an increase in transfer from the rich to the middle class decreases the value of λ_t for each λ_{t-1} and therefore shifts the $f(\lambda_{t-1})$ curve downwards and reduces long run growth.

Similarly we may consider the effect of a transfer from the rich to the poor. Suppose each member of the rich pays a lump sum tax T which is used to give some members of the poor exactly the middle class income w. The number of poor that can be promoted to the middle class is given by $TN_h/(1-\beta)$ and

$$d\pi_{mt}/dT = \{(1-\beta)\lambda_{t-1}\}^{-1}[(1-c)(\lambda_{t-1}-1)N_h] \qquad (5.7)$$

where w is taken to be equal to unity by choice of numeraire. The net effect of an increase in T is consequently given by

$$d\lambda_t/dT = [\{\lambda_{t-1}-1)(1-c)\}\{(1-\beta)\lambda_{t-1}\}^{-1}-1]N_h \qquad (5.8)$$

Since, by assumption, $c > \beta$, the right hand side of (5.8) is negative. Thus a transfer from the rich to the poor reduces the long run growth rate.

A possible way to increase the long run growth is to tax the homogeneous good and subsidize either the m-sector or the h-sector or both. This, however, does not seem to be an attractive policy option because it increases the well-being of the rich and the middle class at the cost of the poor who consumes only the homogeneous good.

The only feasible way to increase the growth rate, therefore, is to promote the poor to the middle class by permanently increasing their productivity. This, of course, has to be achieved through education and human capital formation. The cost of educating the poor may be financed by taxing the rich. As is clear from equation (5.8), this will temporarily reduce the growth rate. But the growth rate will pick up in future when the size of the *surplus producing* middle class is permanently expanded. The exact trade-off between the present and the future growth rates will depend upon the cost of education, discount rates and so on. But the basic message of the paper is that to increase the rate of growth the market size can be expanded by generating more income earning capacity through the formation of human capital.

6. CONCLUDING REMARKS

In this paper, we considered a model of endogenous growth where demand and in particular the size of the middle class plays an important role in determining the rate of growth. This is in contrast with the existing literature on endogenous growth where growth is explained from the supply side. The model may be extended in several ways. First, in our model, competition, working implicitly through cost less technology diffusion, is crucial to innovation, technical progress and growth. The model may be extended to situations where technology diffusion is not automatic and firms compete in R & D. Under such circumstances, one may be able to relate R & D and growth to the market structure.

Secondly and perhaps more importantly, the model may be extended to take into account international trade. Since we are trying to relate growth with the size of the market and since international trade is one of the more important channels through which markets can be expanded, this particular extension of the model might give important insights. In particular, one can imagine an advanced country producing and selling the h-quality good with an imitating backward country manufacturing and exporting the m-quality good. Mutual gains from trade can be demonstrated in this case because the advanced country would depend on the backward country for its market and the backward on the advanced for its technological progress. These possible extensions are included in our future research agenda.

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