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SOME CONSIDERATIONS OF DODGE AND ROMIG SINGLE SAMPLING PLANS UNDER INSPECTION ERROR

By SUKALYAN SENGUPTA and ANUP MAJUMDAR
Indian Statistical Institute

SUMMARY. In industrial situations where attribute acceptance sampling plans are used, inspection error very often crops up. In this paper some results have been established that can be used to match a usual Dodge-Romig Single plan with a Dodge-Romig plan when inspection errors are operative.

1. Introduction

In industries where attribute acceptance sampling plans are used, inspection error very often crops up. This is more so when inspection is visual or subjective and a number of characteristics have to be looked for.

Inspection error can be of two types—(i) the error of judging a good item as defective, and (ii) the error of judging a defective item as good. In this paper we give some results that can be used to match a usual Dodge-Romig (1959) single sampling plan (D-R plan) with a D-R plan when inspection errors are operative. Both the LTPD and AOQL plans have been considered and numerical examples are given to illustrate the results.

2. Definition and notations

Let p be the process fraction defective and e_1, e_2 the probabilities of 1st and 2nd type of error respectively.

It is assumed that (i) e_1 and e_2 are known and remain constant and independent of working conditions, time of shift, severity of defects and process fraction defectives etc; and (ii) a single inspector or several inspectors have the same error probabilities. Then under inspection error apparent defective $p_e = p(1-e)+e_1$ where $e = e_1+e_2$ and as shown by Lavin (1946), the apparent number of defective in a sample of size n_e (say) will follow binomial distribution with parameters n_e and p_e. The lots are assumed to come from a process under Binomial control with p as the process average.

We will denote by (n, c) the D-R plan, for lot size N, lot tolerance fraction defective p_t with consumers' risk $\beta = 0{\cdot}10$ or AOQL p_L and process average $\bar{p}$. We will suffix 'e' to denote different parameters under error of misclassification.

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3. LTPD SINGLE SAMPLLING PLAN

We will assume Poisson approximation to O.C. As a first step, therefore, we need to replace the hypergeometric solution of the sample size n of a D-R plan by n_g which can easily be obtained by using c and β of the D-R plan.

Result 3.1 : Under inspection error (n_{ge}, c) is an equivalent D-R plan for $N_e = N/h$, process average $\bar{p}_e = \bar{p}h$ where $h = (1-e)+e_1/p_t$ and $n_{ge} = n_g/h$.

Proof : Denote $G(c, m) = \sum_{x=0}^{c} e^{-m}m^x/x\,!$ Thus $\beta = G(c, n_g p_t)$ and $\beta_e = G(c, n_{ge}p_{te})$ where $p_{te} = p_t(1-e)+e_1$. Now to have $\beta = \beta_e$, (Hamaker, 1950), $n_g p_t = n_{ge}p_{te}$ i.e. $n_{ge} = n_g/h$ where h is as stated. To show that ATI is minimised at $\bar{p}_e$, denote ATI by $I(N, n, \bar{p}, c)$. Then $I(N_e, n_{ge}, \bar{p}_e, c) = I(N, n, \bar{p}, c)/h$, since $n_{ge}\bar{p}_e = n_g\bar{p}$. Let (n_{1e}, c_1) be any other plan satisfying $\beta = \cdot 10$ at p_t, also let $n_{1e} = n_1/h$ for some n_1, then

$$I(N_e, n_{1e}, \bar{p}_e, c_1) = I(N, n_1, \bar{p}, c_1)/h$$

But $I(N, n_1, \bar{p}, c_1) > I(N, n, \bar{p}, c)$ since (n, c) minimised ATI.QE.D.

Numerical example : Given $N = 1500$, $p_t = 0{\cdot}10$, $\bar{p} = 0{\cdot}03$, $e_1 = 0{\cdot}01$ and $e_2 = 0{\cdot}027$. From D-R table $(n, c) = (105, 6)$ which gives $n_g = 105{\cdot}6 \simeq 106$. By calculation, $h = 1{\cdot}063$, giving $n_{ge} = 100$, $N_e = 1410$, and $\bar{p}_e = 0{\cdot}032$.

4. AOQL SAMPLING PLANS

We assume that no mis-classification occurs during screening and rectification sequence, followed by rejection of a lot. Then AOQ will be (Hill, 1962; Minton, 1972).

$$p_{ae} = \{(N_e - n_e)/N_e\}pG(c, n_c p_e)(1-e_2)+pe_2. \quad \ldots \quad (4.1)$$

As pointed out by Hald (1981) the AOQ function reaches a maxima then decreases and again increases almost linearly for large values of p. Thus to make the concept of AOQ meaningful we note the following requirements :

(i) It is clear that for $p = 1$, AOQ $\simeq e_2$. Thus if the local maxima i.e. the point on AOQ curve where its slope is 0, is to be the required AOQL, then e_2 has to be less than this maxima.

(ii) Otherwise it is possible to obtain an upper limit of the incoming quality level say P_U at which AOQ reaches the maxima and if it is feasible to restrict the incoming quality level below P_U then the local maxima can be taken as the AOQL. In practice the first part of p_{ae} will be negligible and it will suffice if incoming quality is less than AOQL/e_2.

Taking the second derivative of p_{ae} we note that it has only one point of inflexion for $1 > p > 0$ i.e. $1-e_2 > p_e > e_1$ and the second derivative is negative for $n_e e_1 \leqslant n_e p_e \leqslant A$ where

$$A = n_e e_1 + \tfrac{1}{2}\{c+2-n_e e_1+\sqrt{(c+2-n_e e_1)^2+8n_e e_1}\}.$$

Thus p_{ae} has only one maxima and the value of $n_e p_e$ maximising p_{ae} is in the range mentioned.

We can approximate p_{ae} in this region by

$$\frac{(N_e-n_e)}{N_e}\{pG(c, n_e p_e)(1-e_2)+pe_2\}. \quad \dots \quad (4.1a)$$

Result 4.1 : Let $c \neq 0$. Under inspection error (n_e, c_e) is the equivalent D-R plan for $N_e = N/\theta$, $\bar{p}_e = \bar{p}/(1-e)$ and p_L, where $n_e = n/\theta$, $c_e = c+n_e e_1$ (c_e is rounded off to the narest integer), $\theta = 1/\left\{b+\dfrac{xe_2'}{y-xe_2'/b(c+2-x)}\right\}$, $b = (1-e_2)/(1-e)$ and $e_2' = e_2/(1-e)$.

Furthermore $p_{Le} = p_L$ occurs at $(x+k)/n_e$ where $k = \left(\dfrac{1}{\theta}-b\right) x/b(c+2-x)$ and x is the x value of usual D-R plan. (Dodge and Romig, Sampling inspection table, 1959, pp. 37–39) i.e. the value of np at which AOQ reaches the maximum and $y = p_L/\left(\dfrac{1}{n}-\dfrac{1}{N}\right)$ under the assumption of no inspection error.

Proof : Let the AOQL occur at p_2. We first show that

$$G(c_e, n_e\{p_2(1-e)+e_1\}) \simeq G(c, n_e p_2(1-e)) \text{ if } c_e = c+n_e e_1.$$

Let these probabilities be equal, α being the common value. Using Cornish-Fisher formula we get

$$n_e\{p_2(1-e)+e_2\} \simeq c_e+1-u_\alpha\sqrt{c_e+1}+\frac{1}{3}(u_\alpha^2-1) \quad \dots \quad (4.1.1a)$$

$$n_e p_2(1-e) \simeq c+1-u_\alpha\sqrt{c+1}+\frac{1}{3}(u_\alpha^2-1) \quad \dots \quad (4.1.1b)$$

where u_α is the α-th fractile of Normal distribution. From 4.1.1(a and b), we get

$$n_e e_1(c_e-c) = 1-u_\alpha/(\sqrt{c_e+1}+\sqrt{c+1}). \quad \dots \quad (4.1.2)$$

Note that $u_a/(\sqrt{c_e+1}+\sqrt{c+1}) \leqslant u_a/2\sqrt{c+1}$. From numerical investigation with different values of c, e_1, e_2, we obtain

$$0{\cdot}19 \leqslant u_a/(\sqrt{c_e+1}+\sqrt{c+1})$$
$$\leqslant u_a/2\sqrt{(c+1)} \leqslant {\cdot}133$$

for $c \neq 0$. We can thus consider $c_e = c+n_e e_1$. Hence,

$$p_{Le} = \frac{N_e-n_e}{N_e}\{p_2G(c_e, n_ep_2(1-e))+p_ee_2\}$$

$$= \left(\frac{1}{n_e}-\frac{1}{N_e}\right)\{x_ebG(c, x_e)+x_ee_2'\} \qquad \dots \quad (4.1.3)$$

where $b = (1-e_2)/(1-e)$, $e_2' = e_2(1-e)$ and $x_e = n_ep_2(1-e)$.

Writing $y_e = \{x_ebG(c, x_e)+x_ee_2'\}$ we get

$$p_{Le} = \left(\frac{1}{n_e}-\frac{1}{N_e}\right) y_e. \qquad \dots \quad (4.1.4)$$

Let $p_{Le} = p_L$, $n/n_e = N/N_e = \theta$. Then $y_e = y/\theta$.

When there is no error, $p_L = \left(\frac{1}{n}-\frac{1}{N}\right) y$ and $y = xG(c, x) = x^2g(c, x)$, where $g(c, x) = e^{-x}x^c/c\,!$

To satisfy (4.1.3) x_e should be such that

$$y_e = x_eG(c, x_e)+x_ee_2' = x_e^2g(c, x_e) = y/\theta. \qquad \dots \quad (4.1.5)$$

For values of e_2 as discussed earlier, and for $e_1 > e_3$ (which is usually true), numerical investigation showed that x_e occurs in the neighbourhood of x, say $x_e = x+k$. Then by Newton's formula $k = x\left(\frac{1}{\theta}-b\right)/b(c+2-x)$. Thus by using Taylor's expansion and neglecting higher order forms of k, $x_eG(c, x_e) \simeq xG(c, x)$. Now, using all these results and putting the value of k in (4.1.4) we get the required value of θ. It is seen that

$$I(N_e, n_e, \bar{p}_e, c_e) = I(N, n, \bar{p}, c)/\theta.$$

The proof that ATI is minimum for this plan is analogous to the proof in Result 3.1.

Note : When $e_1 = 0$, $c_e = c$ and $b = 1$. This will also be true when $c = 0$.

Numerical example : Given $N = 2500$, $\bar{p} = {\cdot}042$, $p_L = {\cdot}05$, $e_1 = {\cdot}009$, $e_2 = 0{\cdot}1$. From D-R table $(n, c) = (125, 10)$ and from table (2.3) of D-R table $x = 8{\cdot}05$, $y = 6{\cdot}528$ and $p_L = {\cdot}04961$.

By calculation, we get $N_s = 2918$, $n_s = 146$, $c_s = 11{\cdot}772 \simeq 12$, $\bar{p}_s = {\cdot}0404$. Actual calculation gives $p_{Ls} = {\cdot}04968$ at $p_2 = {\cdot}059$ whereas using result (4.1), $p_{Ls} = {\cdot}04956$ at $p_2 = {\cdot}0573$.

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