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AN INVESTIGATION ON THE EFFECT OF INSPECTION ERROR ON AOQ OF A SINGLE SAMPLING PLAN ACCEPTANCE RECTIFICATION SYSTEM

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SUMMARY. In this paper we study the effect of inspection error on average outgoing quality of a single sampling acceptance rectification plan. We define a p^u such that if the lot quality $p > p^u$ then $AOQ(e) > AOQL$. It is established that if both e_1 and e_2 are less than $AOQL$ and equal then $p^u > 0.5$.

1. Introduction

In recent times several authors have studied the nature and magnitude of inspection error and its effect on acceptance sampling plan, a precise account of which may be had from Dorris *et al.* (1978). AOQ expressions for different types of sample and lot disposition in presence of inspection error are available in Case *et al.* (1975) and Wortham *et al.* (1970).

Inspection error was found to cause a significant change in the shape of the AOQ curve. Given the inspection errors e_1 and e_2, AOQ curve in general following an initial peak decreases and then begins a monotonic increase as $p \to 1$. The conventional concept of the Average Outgoing Quality Limit (AOQL) is, thus, not meaningful in the presence of inspection errors.

In this paper we study the effect of inspection error in details for the acceptance rectification scheme S3-L3 in which the apparent defectives are replaced whenever they are found either in the sample or in the lot.

2. Notations

The inspection error may be of two types : e_1 is the probability of classifying a good item as a defective one and e_2 is the probability of classifying a defective item as a good one. The average outgoing quality is denoted by AOQ, AOQL (without inspection errors) by p_L and the probability of acceptance of a lot of quality p by $L(p)$. $P(c)$ will denote the probability of obtaining

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c defectives in a sample of fixed size n. The letter e is used within bracket or as a suffix to represent the comparable quantity in presence of inspection error.

3. Effect of inspection error (e_1, e_2) on incoming quality (p) and average outgoing quality (AOQ).

It has been shown by Lavin, (1946), and Collins *et al.* (1976) that due to error in inspection the probability of acceptance of the lot, $L(p)$ will be obtained by replacing the true fraction defective (p) by the apparent fraction defective p_e where

$$p_e = p(1-e_2)+(1-p)e_1. \qquad \ldots \quad (1)$$

It is seen that $L(p_e) \lesseqgtr L(p)$ according as $p \lesseqgtr \frac{e_1}{e_1+e_2}$. For the type of acceptance rectification plan (S3-L3) we have from Beainy *et al.* (1981)

$$\text{AOQ}(e) \simeq pL(p_e)+\frac{pe_2(1-L(p_e))}{1-p_e} \qquad \ldots \quad (2)$$

if the sample size is negligible in comparison with the lot size. Assuming Poisson approximation for $L(p)$ we have

$$\frac{\partial \text{AOQ}(e)}{\partial e_1} = -np(1-p)P(c_e)+p(1-p)e_2\Big[\frac{n(1-p_e)P(c_e)+(1-L(p_e))}{(1-p_e)^2}\Big] < 0$$

for all p if $e_2 = 0$.

Thus, if the inspection error is of Type 1 only, the AOQ(e) curve will lie wholly below the original AOQ curve for perfect inspection. This generalises the findings of Case, Bennett and Schmidt (1975) for a particular value of e_1.

It will be interesting to study the behaviour of AOQ(e) curve in presence of both kinds of inspection errors in relation to AOQ curve for different values of p. It follows from (2) that AOQ(e) $\geqslant$ AOQ if

$$L(p_e)-L(p)+\frac{e_2(1-L(p_e))}{1-p_e} \geqslant 0. \qquad \ldots \quad (3)$$

Noting that (i) $p \geqslant p_e$, (ii) $L(p) \leqslant L(p_e) \leqslant 1$ and (iii) $1-p_e \leqslant 1$ for $p \geqslant \frac{e_1}{e_1+e_2}$ we observe that AOQ(e) curve must be lying above the AOQ curve for all $p \geqslant \frac{e_1}{e_1+e_2}$. It follows, as a corollary, that if $e_1 = 0$ then for all $p > 0$ AOQ(e) will be larger than AOQ. The particular case considered in Case *et al.* (1975) is in agreement with this general result.

Now we investigate for what values of p, AOQ(e) will be smaller than AOQ for a given choice of e_1 and e_2.

It follows from (3) that AOQ(e) < AOQ for $0 \leqslant p \leqslant \frac{e_1}{e_1+e_2}$ if

$$L(p_e)\left[1-\frac{e_2}{1-p_e}\right] < L(p)-\frac{e_2}{1-p_e} \qquad \ldots \quad (4)$$

e_1 and e_2 will be usually small and we assume that $e_1+e_2 \leqslant 1$. In this case it is seen that necessary, though not sufficient condition for AOQ(e) to be less than AOQ is that $L(p) > \frac{e_2}{1-p_e}$ since $e_1+e_2 \leqslant 1 \Longrightarrow \frac{e_2}{1-p_e} \leqslant 1$. This is illustrated in Fig. 1. Thus there exists a p for which $L(p) = \frac{e_2}{1-p_e}$ and beyond this

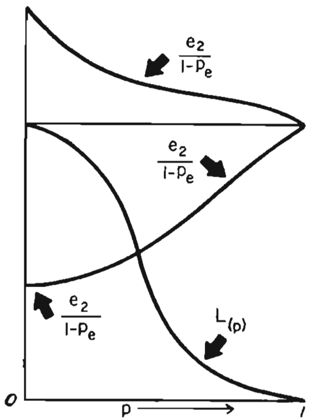

Figure 1

p AOQ(e) will be always greater than AOQ. The exact value of p for which AOQ(e) is equal to AOQ can be obtained from (4) with ' < ' replaced by ' = ' provided e_1, e_2 and the elements of the sampling plan are known.

4. Incoming quality for which AOQ (e) is more than AOQL

As $p \to 1$, AOQ(e) $\to 1$ and as pointed out earlier the sampling plan will not ensure an AOQL as visualised by Dodge-Romig. However, from a study of AOQ(e) curve it appears that if the incoming quality lies within some particular range the AOQ(e) will still not exceed the AOQL of the sampling plan. Since in practical situation, the incoming quality from a controlled process will usually be within some known interval for p an attempt is made to find

(i) set of values of incoming quality for which the AOQL is exceeded given a pair of (e_1, e_2), and

(ii) the allowable range of (e_1, e_2) which can be absorbed by the sampling plan without increasing the AOQL, given that incoming quality in practice will not exceed a specified amount.

(a) It is difficult to determine p^0 the value of p for which AOQ(e) is same as AOQL without knowing the elements of the sampling plan and its OC. However, some useful approximation can be made. We note that

$$\text{AOQ}(e) \leqslant \text{AOQL if } L(p_e)\left[1-\frac{e_2}{1-p_e}\right]+\frac{e_2}{1-p_e} \leqslant \frac{p_L}{p}. \qquad \ldots \quad (5)$$

The r.h.s. in (5) is a decreasing function of p lying between p_L and ∞. The quantity $\frac{e_2}{1-p_e}$ is an increasing function lying between $\frac{e_2}{1-e_1}$ and 1. $L(p_e)\left[1-\frac{e_2}{1-p_e}\right]$ lies between 1 and 0. Suppose for some p^u the curve $\frac{e_2}{1-p_e}$ crosses the curve $\frac{p_L}{p}$. Then $p^0 \leqslant p^u$ when $\frac{e_2}{1-p^u_e} = \frac{p_L}{p^u}$. It then follows that

$$p^u = \frac{p_L(1-e_1)}{e_2+p_L(1-e_1-e_2)}. \qquad \ldots \quad (6)$$

If the incoming quality exceeds p^u (which does not depend on the particular sampling plan) then with the inspection errors AOQL stipulation can never be met. The exact value of the incoming quality upto which AOQ(e) will not exceed AOQL may be obtained by solving (5) numerically and p^u can be taken as the initial value for starting the iteration. The graph of right hand side of (5) may cross that of l.h.s. (both plotted against p) at more than one values of p. In such cases the AOQL stipulation is ensured as long as incoming quality is less than or equal to minimum of these values of p. This is shown in Fig. 2.

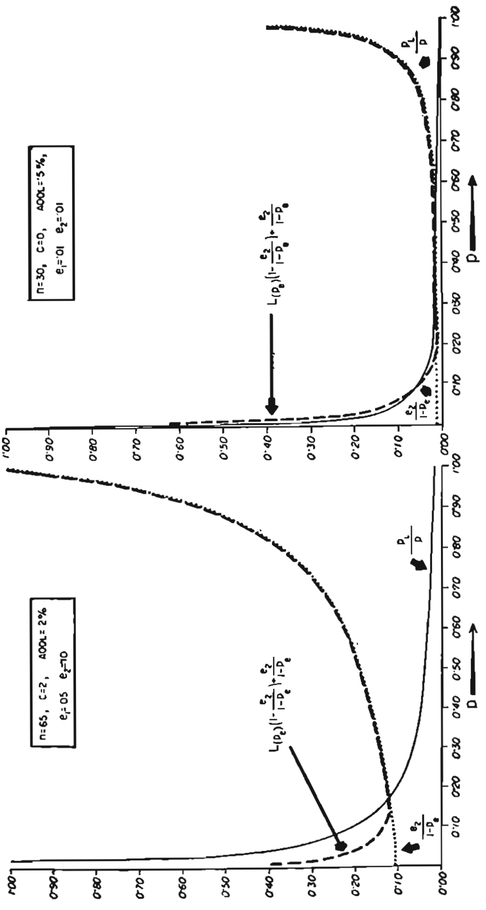

Figure 2 $\frac{p_L}{p}$, $L(p_e)\left(1-\frac{e_2}{1-p_e}\right)+\frac{e_2}{1-p_e}$, $\frac{e_2}{1-p_e}$

However, we would assume that if the incoming quality does not exceed p^u, the AOQL stipulation will be met or at least not be disturbed violently. By rearranging (6) we get

$$\frac{1}{p^u} = 1+ \frac{e_2}{1-e_1} \cdot \frac{1-p_L}{p_L} = K\frac{1-p_L}{p_L}+1. \qquad \ldots \quad (7)$$

Thus p^u depends only on $K = \frac{e_2}{(1-e_1)}$ (the ratio of e_2 and $1-e_1$) and p_L. Values of p^u for different values of K and p_L is given in Table 1 to illustrate how p^u is changed with different values of K for a given AOQL.

(b) We assume that the incoming quality of a given process does not exceed p^*. The AOQL stipulation will be met if $p^* \leqslant p^u$ or in other words if

$$K = \frac{e_2}{1-e_1} \leqslant \frac{p_L(1-p^*)}{(1-p_L)p^*} \qquad \ldots \quad (8)$$

5. Comparison of AOQL with maximum of AOQ (e) for $p \leqslant p^u$.

In order to study the extent of departure from stipulated AOQL if the incoming quality is controlled within p^u, AOQ(e) is computed for a few sampling plans as given by Dodge-Romig for some selected AOQL. Two groups of sampling plans are considered. Four error levels for each e_1 and e_2 are considered. The result of the study is shown in Table 2. The sampling plans are referred to in the Table as PL_1 and PL_2. Details of the sampling plans under PL_1 and PL_2 are as follows :

group	lot size	n	c	$\bar{p}$ (%)	AOQL (%)
	31–50	30	0	0·00–0·01	0·5
	26–50	22	0	0·00–0·02	1·0
PL_1	16–50	14	0	0·00–0·04	2·0
	6–50	6	0	0·00–0·10	5·0
	4–50	3	0	0·00–0·20	10·0
	801–1000	145	1	0·21–0·30	0·5
	801–1000	80	1	0·21–0·40	1·0
PL_2	801–1000	65	2	0·81–1·20	2·0
	801–1000	37	3	2·01–3·00	5·0
	801–1000	25	4	4·01–6·00	10·0

It can be seen from Table 2 that for PL_2 plans, the maximum of AOQ in presence of inspection error is almost same as respective AOQL for $p \leqslant p^u$.

TABLE 1. VALUES OF $100p^u$, FOR DIFFERENT VALUES OF p_L AND K

AOQL (%) / *K*	.10	.25	.50	.75	1.00	1.50	2.00	2.50	3.00	4.00	5.00	7.00	10.00
.01	9.10	20.04	33.44	43.04	50.25	60.36	67.11	71.94	75.57	80.65	84.03	88.27	91.70
.02	4.77	11.14	20.08	27.42	33.56	43.22	50.50	56.18	60.73	67.57	72.46	79.01	84.75
.03	3.23	7.71	14.35	20.12	25.19	33.67	34.01	46.09	50.70	58.14	63.69	71.50	78.74
.04	2.44	5.90	11.16	15.89	20.16	27.57	33.78	39.06	43.60	51.02	56.82	65.30	73.53
.05	1.96	4.77	9.13	13.13	16.81	23.35	28.99	33.90	38.22	45.45	51.28	60.09	68.96
.06	1.64	4.01	7.73	11.19	14.41	20.24	25.38	29.94	34.01	40.98	46.73	55.64	64.94
.07	1.41	3.46	6.70	9.74	12.61	17.87	22.57	26.81	30.04	37.31	42.91	51.81	61.35
.08	1.24	3.04	5.91	8.63	11.21	15.99	20.32	24.27	27.88	34.25	39.68	48.48	58.14
.09	1.10	2.71	5.29	7.75	10.09	14.47	18.48	22.17	25.56	31.65	36.90	45.54	55.25
.10	0.99	2.45	4.78	7.03	9.17	13.22	16.95	20.41	23.62	29.41	34.48	42.94	52.63
.11	0.90	2.23	4.37	6.43	8.41	12.16	15.56	18.90	21.95	27.47	32.36	40.63	50.25
.12	0.83	2.05	4.02	5.92	7.70	11.26	14.53	17.61	20.49	25.77	30.49	38.49	48.08
.13	0.76	1.89	3.72	5.49	7.21	10.49	13.57	16.47	19.22	24.27	28.82	36.67	46.08
.14	0.71	1.76	3.46	5.12	6.73	9.81	12.72	15.48	18.00	22.94	27.32	34.07	44.25
.15	0.66	1.64	3.24	4.80	6.31	9.22	11.98	14.50	17.09	21.74	25.97	33.41	42.55
.16	0.62	1.54	3.05	4.51	5.94	8.69	11.31	13.81	16.29	20.66	24.75	31.99	40.98
.17	0.59	1.44	2.87	4.26	5.61	8.22	10.72	13.11	15.39	19.68	23.64	30.69	39.53
.18	0.55	1.37	2.72	4.03	5.31	7.80	10.18	12.47	14.66	18.80	22.62	29.49	38.17
.19	0.52	1.30	2.58	3.83	5.05	7.42	9.70	11.89	14.00	17.99	21.69	28.37	36.90
.20	0.50	1.23	2.46	3.64	4.81	7.08	9.26	11.36	13.39	17.24	20.83	27.34	35.71
.21	0.47	1.18	2.34	3.47	4.59	6.76	8.86	10.88	12.84	16.56	20.04	26.38	34.00
.22	0.45	1.13	2.23	3.32	4.39	6.47	8.49	10.44	12.32	15.92	19.30	25.49	33.56
.23	0.43	1.08	2.14	3.18	4.21	6.21	8.15	10.03	11.85	15.34	18.62	24.65	32.57
.24	0.42	1.03	2.05	3.05	4.04	5.97	7.84	9.65	11.41	14.70	17.98	23.87	31.05
.25	0.40	0.99	1.97	2.93	3.88	5.74	7.55	9.30	11.00	14.29	17.30	23.14	30.77

TABLE 2. COMPARISON OF MAXIMUM OF AOQ WITH AOQL FOR $p < p^u$ FOR SOME PLANS FOR DIFFERENT ERROR LEVELS:

AOQL (%)	e_1	e_2 = .01 max. of AOQ (%) PL_1	PL_2	$100p^u$	.05 max. of AOQ (%) PL_1	PL_2	$100p^u$	.10 max. of AOQ (%) PL_1	PL_2	$100p^u$	.20 max. of AOQ (%) PL_1	PL_2	$100p^u$
0.5	.01	0.95	0.50	33.22	1.12	0.50	9.05	1.39	0.51	4.74	1.28	0.63	2.43
1.0	.01	1.40	1.00	50.00	1.62	1.00	16.67	2.10	1.04	9.09	2.36	1.41	4.76
2.0	.01	2.38	2.00	66.89	2.70	2.00	28.18	3.60	2.05	16.81	4.33	2.71	9.18
5.0	.01	6.03	5.00	83.90	7.40	5.00	51.03	9.50	5.08	34.25	10.50	6.90	20.76
10.0	.01	12.50	10.00	91.66	18.00	10.00	68.75	20.20	10.55	52.38	20.47	13.80	35.48
0.5	.05	0.50	0.50	32.31	0.67	0.50	8.72	0.77	0.50	4.56	0.80	0.50	2.33
1.0	.05	1.00	1.00	48.97	1.20	1.00	16.10	1.51	1.00	8.76	1.54	1.04	4.58
2.0	.05	2.00	2.00	65.98	2.36	2.00	27.94	3.03	2.00	16.24	3.33	2.14	8.84
5.0	.05	5.00	5.00	83.33	7.24	5.00	50.00	8.70	5.40	33.33	9.52	5.00	20.00
10.0	.05	15.00	10.00	91.35	18.50	10.00	67.85	19.90	10.00	51.35	20.00	13.00	34.55
0.5	.10	0.50	0.50	31.14	0.52	0.50	8.29	0.56	0.50	4.33	0.56	0.50	2.21
1.0	.10	1.00	1.00	47.62	1.00	1.00	15.38	1.15	1.00	8.33	1.30	1.00	4.35
2.0	.10	2.00	2.00	64.75	2.15	2.00	26.87	2.61	2.00	15.52	2.71	2.00	8.41
5.0	.10	5.00	5.00	82.75	7.00	5.00	48.65	8.20	5.00	32.14	8.75	5.50	19.15
10.0	.10	16.00	10.00	90.91	18.75	10.00	66.67	18.92	10.20	50.00	18.85	12.00	33.33
0.5	.20	0.50	0.50	28.67	0.50	0.50	7.44	0.50	0.50	3.86	0.50	0.50	1.97
1.0	.20	1.00	1.00	44.69	1.00	1.00	13.91	1.00	1.00	7.47	1.02	1.00	3.88
2.0	.20	2.00	2.00	62.02	2.09	2.00	24.62	2.18	2.00	14.04	2.02	2.00	7.55
5.0	.20	6.00	5.00	80.81	6.50	5.00	45.71	7.20	5.00	29.63	7.00	5.00	17.39
10.0	.20	15.90	10.00	89.89	17.30	10.00	64.00	17.60	10.00	47.06	17.10	11.00	30.77

This is, however, not true for PL_1 plans which have less discriminating OC. However, since in most practical cases plans having sharper discriminating power are used and lot size is larger or equal to that of PL_2 plans, it can be said that if p remains smaller than p^u the AOQL stipulation will not be violently disturbed.

6. Relationship between p^u and $\frac{e_1}{e_1+e_2}$

It can be shown from (6) that $p^u > \frac{e_1}{e_1+e_2} \Longleftrightarrow e_1 < p_L$. The applicability of Dodge-Romig plan in presence of inspection errors is recommended without causing any serious departure from stipulated AOQL if p^u is large. For a given e_1, $\frac{e_1}{e_1+e_2}$ can be made larger by keeping e_2 smaller and it is noted that if e_1 and e_2 are of the same order and less than p_L, the value of p^u will exceed ·5.

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References

Beainy, I., and Case, K. E. (1981) : A wide variety of AOQ and ATI performance measures with and without inspection error. *Journal of Quality Technology*, **13**, No. 1, 1-9.

Case, K. E., Bennett, G. K., and Schmidt, J. W. (1975) : The effect of inspection error on average outgoing quality. *Journal of Quality Technology*, **7**, No. 1, 28-33.

Collins, R. D. Jr., and Case, K. E. (1976) : The distribution of observed defectives in attribute acceptance sampling plans under inspection error. *AIIE Transactions*, **8**, No. 3, 375-378.

Dorris, A. L., and Foote, B. L. (1978) : Inspection errors and statistical quality control : A survey. *AIIE Transactions*, **10**, No. 2, 184-192.

Lavin, M. (1946) : Inspection efficiency and sampling inspection plans. *Jour. Amer. Statist. Assoc.*, **41**, 432-438.

Wortham, A. W., and Mogg, J. W. (1970) : A technical note on average outgoing quality. *Journal of Quality Technology*, **2**, No. 1, 30-31.

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