# On the Incentives for Cooperative Research

Tarun Kabiraj
Indian Statistical Institute, Kolkata

**Abstract**

We portray situations to show that non-cooperative R&D can occur even if the probability of success in R&D is large. We then model synergy in cooperative R&D and show that if the innovation size is large, cooperative research will occur.

*Key words*: Cooperative R&D, non-cooperative R&D, synergy.
*JEL Classification Index*: D43, L13, O31.

---

**Correspond to**: Tarun Kabiraj, Economic Research Unit, Indian Statistical Institute, 203 B. T. Road, Kolkata 700108, India.
E-mail: tarunkabiraj@hotmail.com; Fax: 91-33-25778893.

## 1. Introduction

In a paper published in Letters, Marjit (1991) studies the incentives of firms for doing cooperative research when they compete in the product market non-cooperatively. The paper shows that `uncertainty' in the R&D outcome can alone result in a research joint venture (RJV). Under cooperative research firms are assumed to do research in a single lab and share the R&D outcome and investment. Then research cooperation occurs if the probability of success is either low or quite high.

Combs (1992) considers an otherwise Marjit model but allows the RJV to operate two research labs and share the research output, if success. So R&D cost is duplicated. The model is structured in a way that the probability of success under research cooperation is twice that of non-cooperative research.[1] The model shows that research cooperation occurs only when the probability of success is relatively high.

The present paper is an extension of Marjit and Combs. We consider the situation when firms are capable of imitating the rival's innovation costlessly, unless the innovation is protected by a patent. When both firms are successful in innovation, we assume that only one firm is awarded protection; this means, each such firm has one-half chance of wining the patent. With this assumption we show in the context of Marjit model that if R&D cost is low, cooperative research will occur if and only if the probability of success is small. This completely reverses the Combs result. However, when R&D cost is relatively high, we get back the Marjit result.

We then construct a model endogenizing the probability of success but retain their assumption regarding imitation and patenting of an innovation. As in Combs, firms do their research always in their own lab, but we structure the model in such a way that the extent of cost reduction under cooperation is doubled when both firms are successful in innovation. Then research cooperation occurs if the extent of cost reduction is relatively large. We thus draw attention to the size of the innovation in the decision of cooperative vs. non-cooperative R&D. While there is a big literature on this issue,[2] no works so far

[1] Here there are $m$ research projects, and exactly one of these is successful. Each lab has the capacity for one project and the RJV selects two projects without replacement.

[2] A subset of the literature comprises of Katz (1986), D'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Suzumura (1992), Choi (1993), Kabiraj and Mukherjee (2000), and Silipo and Weiss (2005).

have focused on this aspect. Moreover, we have explicitly incorporated synergies in R&D due to employing complementary resources. Combs result follows as a special case.

The setup of the paper is as follows. We provide two models in the following two sections. The fourth section concludes the paper.

## 2. Model 1

Both in Marjit and Combs, the market is symmetric duopoly under independent research if both firms are successful in R&D, and it is monopoly when only one of them is successful, whereas under cooperative R&D the market is always duopoly if at least one firm is successful.

The implicit assumption underlying their models is that either the firms cannot imitate rivals' innovations at all (so that patenting of an innovation is irrelevant), or all firms which are successful in research get equal patent protection. Neither of these assumptions seems realistic. Assume that the firms can costlessly imitate rivals' innovations. This calls for patent protection to give incentives to the innovators. It is then obvious that if there is only one successful firm, it will get patent protection. It is also reasonable to assume that under cooperative research, the research coalition will get protection for the innovation and the member firms will share the knowledge. But if under independent research both firms come up with the same innovation, which firm will be awarded patent protection? It is unlikely that both the firms will apply for patent at the same date and time. Under this circumstance, it is reasonable to assume that each successful firm will win the patent with probability ½. Hence under independent research the market structure will be monopoly of the patented firm. This leads to the following comparison in the context of Marjit model.

Denote the symmetric duopoly and monopoly payoffs by $\pi^d$ and $\pi^m$ respectively, and the probability of success and R&D investment by $r$ and $R$ respectively. Then cooperative R&D will occur if and only if

$$r\pi^d - \frac{R}{2} > [\frac{r^2}{2} + r(1-r)]\pi^m - R$$

$$\text{or,} \qquad \frac{R}{2} > \phi(r) \equiv \frac{r(2-r)}{2}\pi^m - r\pi^d \tag{1}$$

Note that,

$$\phi(0) = 0\,, \quad \phi(1) = \frac{\pi^m}{2} - \pi^d > 0$$

$$\phi'(r) = 0 \Leftrightarrow r = r^* \equiv \frac{\pi^m - \pi^d}{\pi^m} \text{ and } \phi''(r) < 0\,.$$

Therefore, $\phi(r)$ reaches a unique maximum at $r^*$. This leads to the following result.

**Proposition 1**: Given that the *R&D cost is low, non-cooperative research will occur if and only if the probability of success is large.*

**Proof**: Suppose $(R/2) \le \phi(1)$. Then given the characterization of the $\phi(r)$ function, $\exists\ \tilde{r} < r^*$ such that the inequality (1) holds for all $r < \tilde{r}$. Hence the result. QED

The result is shown in *Figure 1*. Here Combs' result is just reversed. However, for $\phi(1) < (R/2) < \phi(r^*)$, we have Marjit's result, that is $\exists\ \{\tilde{r}_1, \tilde{r}_2\}$, $\tilde{r}_1 < \tilde{r}_2$ such that cooperative R&D occurs $\forall r \in [0, \tilde{r}_1)$ and $\forall r \in (\tilde{r}_2, 1]$.

## 3. Model 2

Consider the Marjit-Combs models, but assume that the probability of success is endogenous, and cooperative research exploits synergies due to complementarity of inputs. However, their assumption regarding imitation and patent protection is retained.

Two firms, 1 and 2, produce a homogeneous good using only two inputs, X and Y, and compete in the product market non-cooperatively.[3] Given the production technology, let the unit cost of final goods production be given by $c = c_x + c_y$, where $c_x$ and $c_y$ are respectively X-input and Y-input costs involved in one unit final production. To make the structure simple, we further assume that, given the market demand function $p(q)$, initially production is just marginally not profitable, i.e., $c = p(o)$. This means, the initial payoff of each firm is zero.

Now suppose that in the first stage a firm, through R&D, can reduce $c_x$ or $c_y$ (or both) by the extent of $\varepsilon > 0$. We call $\varepsilon$ as the size of the innovation; $\varepsilon$ is assumed exogenous. Assume that a firm under independent research can reduce the cost to either

[3] We may think X and Y as two input vectors.

$(c_x - \varepsilon)$ or $(c_y - \varepsilon)$, but not both (due to fund constraint or capacity constraint). This means, under non-cooperative R&D a successful firm has a production cost $(c - \varepsilon)$. When only one firm is successful, the market structure becomes monopoly of the successful firm. If both firms are successful, the market structure is symmetric duopoly. The corresponding payoffs under these two situations are $\pi^m(\varepsilon)$ and $\pi^d(\varepsilon)$, respectively.

Now consider cooperative research. We assume that under RJV the firms engage in non-overlapping research in their respective research lab and then combine their research outcomes to benefit from the complementarity of inputs. Thus, one firm is engaged in reducing X-cost and the other in reducing Y-cost. Therefore, under RJV each firm will have unit cost of production $(c - 2\varepsilon)$ when both firms are successful, and $(c - \varepsilon)$ when only one firm is successful. In either case, the market structure is symmetric duopoly. The corresponding payoff of each firm is either $\pi^d(2\varepsilon)$ or $\pi^d(\varepsilon)$, as the case may be.

Finally, we assume that the research outcomes are probabilistic. Let $r(R_i)$ be the probability that firm $i$ will succeed in research if it invests $R_i$ amount of resources. In particular, we assume that the R&D technology is given by the function

$$r(R_i) = \sqrt{R_i}, \quad i = 1,2 \tag{2}$$

The function is concave and increasing and the relevant domain is $[0,1]$. For the existence of an interior solution we further assume that

$$\pi^d(2\varepsilon) < 2 \quad i.e., \quad \varepsilon < \bar{\varepsilon} \tag{3}$$

It is quite obvious that

$$\pi^d(2\varepsilon) > \pi^d(\varepsilon) \quad \text{and} \quad \pi^m(\varepsilon) > 2\pi^d(\varepsilon) \tag{4}$$

Therefore, given any first stage choice of $\{R_1, R_2\}$, the expressions for the expected payoffs of the firms under cooperative and non-cooperative research are respectively,

$$E_i(C) = r(R_i)r(R_j)\pi^d(2\varepsilon) + 2r(R_i)(1 - r(R_j))\pi^d(\varepsilon) - R_i \tag{5}$$

$$E_i(N) = r(R_i)r(R_j)\pi^d(\varepsilon) + r(R_i)(1 - r(R_j))\pi^m(\varepsilon) - R_i \tag{6}$$

for $i, j = 1, 2;\ i \neq j$.

Now consider the first stage problem.[4] Under cooperative research firm $i$'s problem is:

$$\max_{R_i} \; E_i(C)$$

The first order condition yields:

$$r'(R_i)r(R_j)\pi^d(2\varepsilon) + 2r'(R_i)(1-r(R_j))\pi^d(\varepsilon) = 1$$

Assuming symmetric equilibrium and using (2) and (3), we have:

$$r_C = \sqrt{R_C} = \frac{2\pi^d(\varepsilon)}{2+2\pi^d(\varepsilon)-\pi^d(2\varepsilon)} \tag{7}$$

Similarly, the problem under non-cooperative research is:

$$\max_{R_i} \; E_i(N)$$

The corresponding symmetric equilibrium value is:

$$r_N = \sqrt{R_N} = \frac{\pi^m(\varepsilon)}{2+\pi^m(\varepsilon)-\pi^d(\varepsilon)} \tag{8}$$

One can easily show that as $\varepsilon$ goes up, both $R_C$ and $R_N$ go up.

Now plugging the values of $R_C$ and $R_N$ in (5) and (6) respectively, we get the symmetric expected payoffs under cooperative and non-cooperative research as:

$$E^*(C) = R_C \quad and \quad E^*(N) = R_N \tag{9}$$

Then, cooperative research will form if and only if $E^*(C) > E^*(N)$, that is,

$$\frac{2\pi^d(\varepsilon)}{2+2\pi^d(\varepsilon)-\pi^d(2\varepsilon)} > \frac{\pi^m(\varepsilon)}{2+\pi^m(\varepsilon)-\pi^d(\varepsilon)}$$

or,

$$LHS \equiv \frac{2\pi^d(\varepsilon)}{\pi^m(\varepsilon)} > \frac{2-\pi^d(2\varepsilon)}{2-\pi^d(\varepsilon)} \equiv RHS \tag{10}$$

We can write the following proposition.

**Proposition 2**: *The firms will necessarily go for cooperative research if the size of the innovation is relatively large.*

---

[4] If $r$ is independent of $R$ and is specified exogenously, as in Marjit or Combs, we shall get back Combs type result. Comparing (5) and (6), cooperative research will occur iff $r > [\pi^m(\varepsilon) - 2\pi^d(\varepsilon)]/[\pi^d(2\varepsilon) + \pi^m(\varepsilon) - 3\pi^d(\varepsilon)]$.

**Proof**. Consider inequality (10). As $\varepsilon \to \bar{\varepsilon}$, $\pi^d(2\varepsilon) \to 2$ but $\pi^d(\varepsilon) < 2$. So as $\varepsilon \to \bar{\varepsilon}$, $RHS \to 0$, whereas $LHS > 0$. Therefore, inequality (10) necessarily holds for relatively large $\varepsilon$. QED

The intuition of the result follows from (5) and (6). The incentive for non-cooperative R&D comes from the fact that when only one firm succeeds in R&D, it emerges as monopoly, whereas under cooperative R&D in this case the market structure becomes symmetric duopoly. On the other hand, incentives for cooperative research come from two sources. First, if either of the firms succeeds, they share the knowledge, whereas under non-cooperative case a firm gets a zero profit if it fails. Second, if both firms succeed, the market structure is symmetric duopoly in both cooperative and non-cooperative cases, but under cooperative research the firms enjoy a cost reduction twice of that under non-cooperation. The larger the size of the innovation, the greater the cost difference is between the two situations; hence larger is the incentive for cooperative research when the innovation is large. Under this situation the firms also invest a larger amount in R&D. This means, the expected payoff under cooperative research will be larger compared to non-cooperative research when the innovation is large.

**Proposition 3**: *If the demand function is linear, then* $\exists\, \varepsilon^* \in (0, \bar{\varepsilon})$ *such that cooperative research will occur if and only if* $\varepsilon \in (\varepsilon^*, \bar{\varepsilon})$*; when* $\varepsilon < \varepsilon^*$*, it will be non-cooperative research.*

**Proof:** Consider any linear demand function of the form $p(q) = a - q$. Then $\pi^m(\varepsilon) = \varepsilon^2/4$, $\pi^d(\varepsilon) = \varepsilon^2/9$, $\pi^d(2\varepsilon) = (4\varepsilon^2)/9$, $\bar{\varepsilon} = 2.12$, $LHS = \frac{8}{9}$, and $RHS = \frac{18 - 4\varepsilon^2}{18 - \varepsilon^2}$ is a decreasing function of $\varepsilon$. Hence, $\exists\, \varepsilon^* \in (0, \bar{\varepsilon}) \mid \varepsilon \gtrless \varepsilon^* \Leftrightarrow LHS \gtrless RHS$. It can be easily checked that $\varepsilon^* = 0.8$ (approx.).[5] QED

[5] Note that in the linear demand example the critical values of $\varepsilon$ are independent of the demand shift parameter.

The result is shown in *Figure 2*. It is now clear hat the choice of cooperative vs. non-cooperative R&D depends on the amount of cost reduction. The larger the amount of cost reduction, the larger the possibility that cooperative research will occur.

## 4. Conclusion

In this paper we have shown that if at a time only one rival gets a patent of the same innovation, then there are situations when non-cooperative R&D is to be preferred to cooperative R&D for high probabilities. This reverses the result of Combs. We have also shown that Combs type result can occur if there is synergistic gain due to complementary inputs. We have provided a model incorporating such synergies. In our case the cost reduction under cooperative research is twice of that under independent research when both firms are successful in R&D. The larger the extent of cost reduction, the larger the incentive for cooperative research vis-à-vis the non-cooperative research is. We thus focus on the importance of the size of the innovation.

## References


Choi, J.P. (1993), “Cooperative R&D with product market competition”, *International Journal of Industrial Organization* 11, 553-571.

Combs, K.L. (1992), “Cost sharing vs. multiple research projects in cooperative R&D”, *Economics Letters* 39, 353-357.

D’Aspremont, C. and A. Jacquemin (1988), “Cooperative and non-cooperative R&D in duopoly with spillovers”, *American Economic Review* 5, 1133-37.

Kabiraj, T. and A. Mukherjee (2000), “Cooperation in R&D and production --- A three-firm analysis”, *Journal of Economics* 71, 281-304.

Kamien, I.M., E. Muller and I. Zang (1992), “Research joint ventures and R&D cartels”, *American Economic Review* 82, 1293-1306.

Katz, M.L. (1986), “An analysis of cooperative research and development”, *Rand Journal of Economics* 4, 527-543.

Marjit, S. (1991), “Incentives for cooperative and non-cooperative R and D in duopoly”, *Economics Letters* 37, 187-191.

Silipo, D.B. and A. Weiss (2005), “Cooperation and competition in a duopoly R&D market”, *Research in Economics* (forthcoming)

Suzumura, K. (1992), “Cooperative and non-cooperative R&D in an oligopoly with spillovers”, *American Economic Review* 82, 1307-20.

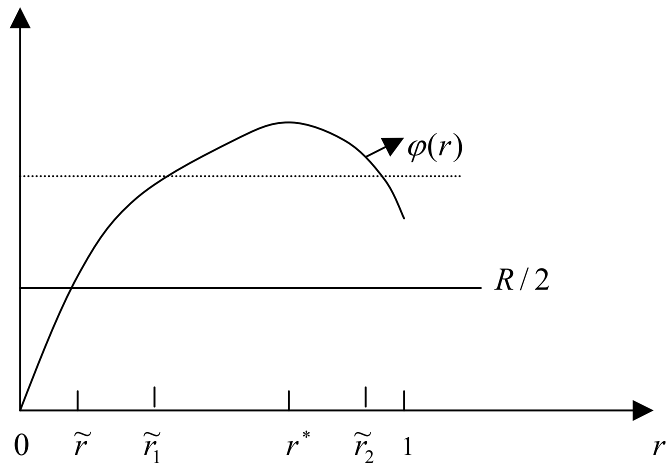


Figure 1

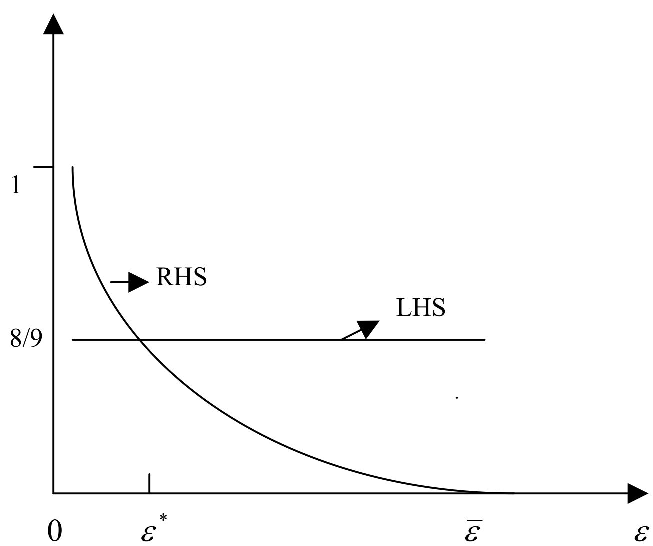


Figure 2