Pseudopotential approach to nonlinear dust acoustic waves in dusty plasma

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(Received 29 April 2002; accepted 9 July 2002)

A method is outlined to derive Sagdeev's pseudopotential which can take into account the dusty charge fluctuation. Numerical solutions of Sagdeev's master equations are obtained to show the existence of solitary waves. Also, for small amplitude, explicit analytical expressions for solitary waves and double layers are obtained.

I. INTRODUCTION

Dusty plasma can be defined as an ionized gas which contains charged particles. It exists in astrophysical bodies (for example, the radial structure of Saturn's rings, narrow rings of Uranus, etc.)[1–3] The discovery of the dust accoustic wave (DAW) and the dust ion accoustic wave (DIAW)[5] gave new impetus to the study of dusty plasma. Nonlinear properties of dusty acoustic waves has been the subject of study by several authors.[4–13] Linear and nonlinear dust accoustic waves have been studied experimentally.[14–16] Ion-acoustic shocks[17] and ion-acoustic solitary waves[18] have been observed recently in the laboratory. In fact, dusty plasma physics has become one of the most rapidly growing fields in plasma physics in particular and science in general (for a survey and other references see Ref. 19). Many of the studies used multicomponent plasma models to study plasma dynamics. If one takes into account dusty charge fluctuation, analytical study becomes difficult. However, one can derive Korteweg–de Vries (KdV) type equations using the reductive perturbation technique. Neglecting dusty charge fluctuation makes the analysis an extension of a multicomponent plasma model with the dusty component having large mass and, hence, ion and electron inertia may be neglected. Using this model, large amplitude dusty accoustic waves have been studied for both magnetized and nonmagnetized dusty plasma.[20–22]

In the present work a method of obtaining the exact Sagdeev's potential for the dusty plasma has been obtained without neglecting the dust charge fluctuation. Sagdeev's pseudopotential equation has been numerically solved to obtain soliton solutions. The plan of the paper is as follows. In Sec. II basic equations of the dusty plasma dynamics are given and the Sagdeev's potential has been derived from these equations. In Sec. III some special cases are considered where successive approximations are made for the dust charge. In Sec. IV small amplitude expansion for the pseudopotential is derived. Analytic solutions for $\phi(\xi)$ in terms of ξ are also given in this section. Numerical solution of the Sagdeev's equation $d\phi/d\xi=\sqrt{-2\psi(\phi)}$ are discussed in Sec. V. Section VI is kept for the conclusion.

II. BASIC EQUATIONS AND DERIVATIONS OF PSEUDOPOTENTIAL

We consider a dusty plasma whose constituents are electrons, ions and dust grains. The electron and ions are taken as Boltzmannian so that the densities are given by

$$n_e=n_{e0}\exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

$$n_i=n_{i0}\exp\left(\frac{-e\phi}{T_i}\right), \tag{2}$$

where n_{e0}, n_{i0} are the unperturbed electron and ion number densities. T_e, T_i are the temperatures and ϕ is plasma potential.

Equations governing the dust dynamics are

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0, \tag{3}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}=-\frac{q_d}{m_d}\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi[e(n_e-n_i)-q_d n_d], \tag{5}$$

$$\frac{\partial q_d}{\partial t}+u_d\frac{\partial q_d}{\partial x}=I_e+I_i, \tag{6}$$

where u_d, n_d, q_d, m_d, respectively, are the dust particle velocity, number density, charge, and the mass. I_e, I_i are the electron and ion currents, respectively. Following Goertz[1] we take the following expressions for I_e, I_i:

$$I_e=-\pi a_d^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e\exp\left(\frac{eq_d}{a_d T_e}\right), \tag{7}$$

$$I_i=\pi a_d^2 e\left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i\exp\left(\frac{-eq_d}{a_d T_i}\right), \tag{8}$$

where m_e, m_i are electron and ion masses, a_d is the radius of dust particle with surface charge q_d. In order to study Eqs. (1)–(6) we introduce the following normalizations:

$$x=\lambda_d\bar{x}, \tag{9}$$

$$\phi=\frac{T_e}{e}\bar{\phi}, \tag{10}$$

$$t=\frac{1}{\omega_{pd}}\tau, \tag{11}$$

$$n_d=n_{d0}\bar{n}_d, \tag{12}$$

$$q_d=q_{d0}\bar{q}_d, \tag{13}$$

$$u_d=\sqrt{\frac{T_e}{m_d}}\bar{u}_d, \tag{14}$$

$$a_d=\lambda_d\bar{a}_d, \tag{15}$$

where

$$\frac{1}{\lambda_d^2}=\frac{1}{\lambda_{de}^2}+\frac{1}{\lambda_{di}^2}, \tag{16}$$

$$\lambda_{dj}=\left(\frac{k_BT_j}{4\pi q_j^2n_{j0}}\right)^{1/2} \tag{17}$$

$(j=i,e)$, λ_{dj} is the Debye length for jth species, $q_e=-e$ and $q_i=e$,

$$\omega_{pd}=\left(\frac{4\pi z_{d0}^2e^2n_{d0}}{m_d}\right)^{1/2} \tag{18}$$

is the dust frequency and z_d is the grain charge number given by $z_d=q_d/e$. With these normalizations equations (3)–(6) take the following forms:

$$\frac{\partial\bar{n}_d}{\partial\tau}+\frac{\partial}{\partial\bar{x}}(\bar{n}_d\bar{u}_d)=0, \tag{19}$$

$$\frac{\partial\bar{u}_d}{\partial\tau}+\bar{u}_d\frac{\partial\bar{u}_d}{\partial\bar{x}}=-z_{d0}\bar{q}_d\frac{\partial\bar{\phi}}{\partial\bar{x}}, \tag{20}$$

$$\frac{\partial^2\bar{\phi}}{\partial\bar{x}^2}z_{d0}^2=\bar{n}_e-\bar{n}_i-\bar{q}_dz_{d0}\bar{n}_d, \tag{21}$$

$$\frac{\partial\bar{q}_d}{\partial\tau}+\bar{u}_d\frac{\partial\bar{q}_d}{\partial\bar{x}}=\bar{I}_e+\bar{I}_i, \tag{22}$$

where

$$\bar{I}_e=-\frac{\pi\bar{a}_d^2}{z_{d0}\omega_{pd}}\lambda_d^2\left(\frac{8T_e}{\pi m_e}\right)^{1/2}\bar{n}_{e0}n_{d0}\exp\bar{\phi}\exp\left(\frac{e\bar{q}_dq_{d0}}{\bar{a}_d\lambda_dT_e}\right), \tag{23}$$

$$\bar{I}_i=\frac{\pi\bar{a}_d^2}{z_{d0}\omega_{pd}}\lambda_d^2\left(\frac{8T_i}{\pi m_i}\right)^{1/2}\bar{n}_{i0}n_{d0}\exp\left(-\bar{\phi}\frac{T_e}{T_i}\right)\times\exp\left(-\frac{e\bar{q}_dq_{d0}}{\bar{a}_d\lambda_dT_i}\right). \tag{24}$$

In order to use quasipotential analysis, the dependant variables are made to be functions of a single variable $\xi=\bar{x}-M\tau$, where M defines the Mach number. Equations (19)–(22) reduce to

$$-M\frac{d\bar{n}_d}{d\xi}+\frac{d}{d\xi}(\bar{n}_d\bar{u}_d)=0, \tag{25}$$

$$-M\frac{d\bar{u}_d}{d\xi}+\bar{u}_d\frac{d\bar{u}_d}{d\xi}=-z_{d0}\bar{q}_d\frac{d\bar{\phi}}{d\xi}, \tag{26}$$

$$\frac{d^2\bar{\phi}}{d\xi^2}z_{d0}^2=\bar{n}_e-\bar{n}_i-\bar{q}_dz_{d0}\bar{n}_d, \tag{27}$$

$$(-M+\bar{u}_d)\frac{d\bar{q}_d}{d\xi}=\bar{I}_e+\bar{I}_i. \tag{28}$$

Equation (25) gives

$$\bar{u}_d=M\left(1-\frac{1}{\bar{n}_d}\right). \tag{29}$$

From Eqs. (25) and (29), we obtain

$$\bar{u}_d=\frac{z_{d0}}{M}\int_0^{\bar{\phi}}\bar{n}_d\bar{q}_d\,d\bar{\phi}. \tag{30}$$

Assuming

$$\int_0^{\bar{\phi}}\bar{n}_d\bar{q}_d\,d\bar{\phi}=F(\bar{\phi}), \tag{31}$$

we get

$$\bar{u}_d=\frac{z_{d0}}{M}F(\bar{\phi})=M\left(1-\frac{1}{\bar{n}_d}\right), \tag{32}$$

$$\bar{q}_d=\frac{F'(\bar{\phi})}{\bar{n}_d}=F'(\bar{\phi})\left[1-\frac{z_{d0}F(\bar{\phi})}{M^2}\right]. \tag{33}$$

If we write

$$P=\frac{d\bar{\phi}}{d\xi}, \tag{34}$$

then from Eq. (27) we have

$$\frac{P^2}{2}=\frac{1}{z_{d0}^2}\left[\bar{n}_{e0}(\exp\bar{\phi}-1)+\bar{n}_{i0}\frac{T_i}{T_e}\left\{\exp\left(-\bar{\phi}\frac{T_e}{T_i}\right)-1\right\}-z_{d0}F(\bar{\phi})\right]. \tag{35}$$

The Sagdeev potential is given by $\psi=-(P^2/2)$ so that

$$\left(\frac{d\bar{\phi}}{d\xi}\right)^2+2\psi(\bar{\phi})=0. \tag{36}$$

From Eq. (28), we obtain

$$-\frac{M}{\bar{n}_d}\frac{d\bar{q}_d}{d\bar{\phi}}P=\bar{I}_e+\bar{I}_i. \tag{37}$$

Now we have a system of differential equations:

$$\frac{d\bar{q}_d}{d\bar{\phi}}=-\frac{\bar{n}_d(\bar{I}_e+\bar{I}_i)}{PM}, \tag{38}$$

$$\frac{dF}{d\bar{\phi}}=\bar{n}_d\bar{q}_d. \tag{39}$$

From Eqs. (38) and (33),

$$F''(\bar{\phi})-\frac{Z_{d0}}{M^2}F''(\bar{\phi})F(\bar{\phi})-\frac{z_{d0}}{M^2}F'(\bar{\phi})^2=-\frac{n_d}{PM}(\bar{I}_e+\bar{I}_i). \tag{40}$$

Equations (38), (39), and (40) are the main results of this paper. To get arbitrary amplitude solitary waves without any approximation for q_d like the charge balance equation, one has to integrate these equations numerically. However, one can get analytical expressions for the pseudopotential up to any order in ϕ.

III. SOME SPECIAL CASES

A. Case (i)

We first consider the charge balance equation:

$$\bar{I}_e+\bar{I}_i=0; \tag{41}$$

this gives

$$\bar{q}_d=A+B\bar{\phi}, \tag{42}$$

where

$$A=\frac{a_dT_eT_i\ln\left[\left(\frac{T_im_e}{T_em_i}\right)^{1/2}\frac{n_{i0}}{n_{e0}}\right]}{e^2z_{d0}(T_e+T_i)}, \tag{43}$$

$$B=-\frac{a_dT_e}{e^2z_{d0}}. \tag{44}$$

Using Eq. (31), we obtain

$$F'(\bar{\phi})\left[1-\frac{z_{d0}F(\bar{\phi})}{M^2}\right]=A+B\bar{\phi}. \tag{45}$$

Integrating, we get

$$F(\bar{\phi})-\frac{z_{d0}}{2M^2}F^2(\bar{\phi})=\left(A\bar{\phi}+\frac{B}{2}\bar{\phi}^2\right). \tag{46}$$

Solving, we have

$$F(\bar{\phi})=\frac{M^2}{Z_{d0}}\left[1-\sqrt{1-\frac{2z_{d0}}{M^2}\left(A\bar{\phi}+\frac{B}{2}\bar{\phi}^2\right)}\right]. \tag{47}$$

B. Case (ii)

$$\bar{I}_e+\bar{I}_i\neq 0. \tag{48}$$

We shall assume a linear relation for $\bar{q}_d$

$$\bar{q}_d=1+b\bar{\phi}. \tag{49}$$

Using Eqs. (37) and (49) and equating the coefficients of $\bar{\phi}$ from both sides we have

$$b=\frac{a_dT_e}{e^2z_{d0}}\left[\frac{2z_{d0}}{M^2(r-1)}-1\right], \tag{50}$$

where $r=(T_e/T_i)$. Finally we get

$$F(\bar{\phi})=\frac{M^2}{z_{d0}}\left[1-\sqrt{1-\frac{2z_{d0}}{M^2}\left(\bar{\phi}+\frac{1}{2}b\bar{\phi}^2\right)}\right]. \tag{51}$$

C. Case (iii)

$$\bar{q}_d=1+a_1\bar{\phi}+a_2\bar{\phi}^2+\ldots, \tag{52}$$

$$F(\bar{\phi})=\frac{M^2}{z_{d0}}\left[1-\sqrt{1-\frac{2z_{d0}}{M^2}\left(\bar{\phi}+\frac{a_1}{2}\bar{\phi}^2+\frac{a_2}{3}\bar{\phi}^3+\ldots\ldots\right)}\right], \tag{53}$$

where a_1, a_2.. can be found from consistency equations using Eqs. (33), (35), and (37). However, if we neglect terms of $o(\phi^4)$ then a_1, a_2, a_3 can be found from the equations

$$-\frac{M}{z_{d0}}\left\{\bar{n}_{e0}+\frac{\bar{n}_{i0}}{T_i}T_e-z_{d0}\left(a_1+\frac{z_{d0}}{M^2}\right)\right\}^{1/2}a_1 =A\left(1+\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_e}\right)+B\left(-\frac{T_e}{T_i}-\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_i}\right), \tag{54}$$

$$\frac{1}{M}Da_1-\frac{M}{Z_{d0}}\left(2a_2D+\frac{1}{2}CDa_1\right) =A\frac{ea_2q_{d0}}{\bar{a}_d\lambda_dT_e}-B\frac{ea_2q_{d0}}{\bar{a}_d\lambda_dT_i}+\frac{A}{2}\left(1+\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_e}\right)^2 +\frac{B}{2}\left(-\frac{T_e}{T_i}-\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_i}\right)^2, \tag{55}$$

$$-\frac{M}{z_{d0}}\left(3a_3D+CDa_2-\frac{C^2a_1D}{8}+\frac{EDa_1}{2}\right) +\frac{1}{M}\left(2a_2D+\frac{CDa_1}{2}\right)+Da_1\left(\frac{a_1}{2M}+\frac{z_{d0}}{2M^3}\right)=F, \tag{56}$$

where A,B,C,D,E,F are given by

$$A=-\frac{\pi\bar{a}_d^2}{z_{d0}\omega_{pd}}\lambda_d^2\left(\frac{8T_e}{\pi m_e}\right)^{1/2}\bar{n}_{e0}n_{d0}\exp\left(\frac{eq_{d0}}{\bar{a}_d\lambda_dT_e}\right), \tag{57}$$

$$B=\frac{\pi\bar{a}_d^2}{z_{d0}\omega_{pd}}\lambda_d^2\left(\frac{8T_i}{\pi m_i}\right)^{1/2}\bar{n}_{i0}n_{d0}\exp\left(-\frac{eq_{d0}}{\bar{a}_d\lambda_dT_i}\right), \tag{58}$$

$$C=\frac{\frac{\bar{n}_{e0}}{3}-\frac{\bar{n}_{i0}}{3}\frac{T_e^2}{T_i^2}-z_{d0}\left(\frac{2a_2}{3}+\frac{z_{d0}}{M^2}a_1+\frac{z_{d0}^2}{2M^4}\right)}{z_{d0}D^2}, \tag{59}$$

$$D=\left\{\bar{n}_{e0}+\frac{\bar{n}_{i0}T_e}{T_i}-z_{d0}\left(a_1+\frac{z_{d0}}{M^2}\right)\right\}^{1/2}, \tag{60}$$

$$E=\frac{1}{z_{d0}^2}\cdot\frac{\frac{\bar{n}_{e0}}{12}+\frac{\bar{n}_{e0}T_e^3}{12T_i^3}-z_{d0}\left(\frac{a_3}{2}+\frac{2z_{d0}}{3M^2}a_2+\frac{z_{d0}}{4M^2}a_1^2+\frac{3z_{d0}^2}{2M^4}a_1+\frac{5z_{d0}^3}{4M^6}\right)}{\bar{n}_{e0}+\bar{n}_{i0}\frac{T_e}{T_i}-z_{d0}\left(a_1+\frac{z_{d0}}{M^2}\right)}, \tag{61}$$

$$F=\frac{Aea_2q_{d0}}{\bar{a}_d\lambda_dT_e}\left(1+\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_e}\right)-\frac{Bea_2q_{d0}}{\bar{a}_d\lambda_dT_i}\times\left(-\frac{T_e}{T_i}-\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_i}\right)+\frac{A}{6}\left(1+\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_e}\right)^3+\frac{B}{6}\left(-\frac{T_e}{T_i}-\frac{ea_1q_{d0}}{\bar{a}_d\lambda_dT_i}\right)^3. \tag{62}$$

IV. SMALL AMPLITUDE EXPANSION

Let us assume that the pseudopotential can be written

$$\frac{d^2\phi}{d\xi^2}=-\frac{\partial\psi}{\partial\phi}=A_1\phi-A_2\phi^2, \tag{63}$$

where

$$A_1=\frac{\bar{n}_{e0}}{z_{d0}^2}+\frac{\bar{n}_{i0}}{z_{d0}^2}\frac{T_e}{T_i}-\frac{a_1}{z_{d0}}-\frac{1}{M^2}, \tag{64}$$

$$A_2=\frac{a_2}{z_{d0}}+\frac{3a_1}{2M^2}+\frac{3Z_{d0}}{2M^4}-\frac{\bar{n}_{e0}}{2z_{d0}^2}+\frac{\bar{n}_{i0}}{2z_{d0}^2}\frac{T_e^2}{T_i^2}. \tag{65}$$

By a transformation $\phi(\xi)=w(z)$ with $z=\operatorname{sech}\alpha\xi$, the above equation reduces to

$$\alpha^2z^2(1-z^2)\frac{d^2w}{dz^2}+\alpha^2z(1-2z^2)\frac{dw}{dz}-A_1w+A_2w^2=0. \tag{66}$$

Since $z=0$ is a regular singularity, we seek a solution in the form

$$w(z)=\sum_{r=0}^{\infty}b_rz^r. \tag{67}$$

The series truncates at $r=3$ if $\alpha=\sqrt{A_1/4}$, $b_2=3A_1/2A_2$, $b_0=0$, $b_1=0$ and we get a solution in the form

$$\phi(\xi)=\left(\frac{3A_1}{2A_2}\right)\operatorname{sech}^2\left(\frac{\bar{x}-M\tau}{\delta}\right), \tag{68}$$

where $\delta=\sqrt{4/A_1}$. If we now include ϕ^3 term in $d^2\phi/d\xi^2$ one can write

$$\frac{d^2\phi}{d\xi^2}=A_1\phi-A_2\phi^2+A_3\phi^3, \tag{69}$$

where

$$A_3=\frac{\bar{n}_{e0}}{6z_{d0}^2}+\frac{\bar{n}_{i0}T_e^3}{6z_{d0}^2T_i^3}-\frac{a_3}{z_{d0}}-\frac{4a_2}{3M^2}-\frac{a_1^2}{2M^2}-\frac{3z_{d0}a_1}{M^4}-\frac{5z_{d0}^2}{2M^6} \tag{70}$$

and A_1, A_2 are as mentioned above.

The transformation $F=\phi-(A_2/3A_3)$ changes the above equation to

$$\frac{d^2F}{d\xi^2}-B_1F+B_2F^3=0. \tag{71}$$

At $B_2=0$ the Duffing equation yields a stable or unstable soliton solution depending on whether $B_1>0$ or $B_1<0$. If $B_2\neq0$, we put $z=\tanh k\xi$ and Eq. (71) reduces to

$$k^2(1-z^2)^2\frac{d^2F}{dz^2}-2k^2z(1-z^2)\frac{dF}{dz}-B_1F+B_2F^3=0, \tag{72}$$

where $B_1=A_1-2A_2\mu+3A_3\mu^2$, $B_2=-A_3$ with $A_1-A_2\mu+A_3\mu^2=0$, $\mu=A_2/3A_3$. Here the Frobenius method requires an infinite series, but whose sum can be calculated and turns out to be

$$F(z)=a_0(1-z^2)^{1/2}, \tag{73}$$

which gives

$$\phi(\xi)=\frac{A_2}{3A_3}\pm\left(\frac{2B_1}{B_2}\right)^{1/2}\operatorname{sech}k\xi\quad\text{where}\quad k=\sqrt{B_1}. \tag{74}$$

From Eq. (69), we obtain

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2=\frac{A_1}{2}\phi^2-\frac{A_2}{3}\phi^3+\frac{A_3}{4}\phi^4. \tag{75}$$

The boundary conditions for the formation of a double layer are

$$\left(\frac{d\phi}{d\xi}\right)^2=0\quad\text{at}\quad\phi=0\quad\text{and}\quad\phi=\phi_m. \tag{76}$$

The above conditions imply that $d\phi/d\xi$ can be written in the form

$$\frac{d\phi}{d\xi}=k\phi(\phi_m-\phi), \tag{77}$$

where $k=\sqrt{A_3/2}$, $\phi_m=2A_2/3A_3$ and the conditions for the existence of the double layer are

$$9A_1A_3=2A_2^2,\quad A_3>0. \tag{78}$$

Applying the tanh method we obtain the double-layer solution as

$$\phi(\xi)=\frac{1}{2}\phi_m\left[1\pm\tanh\left(\frac{\xi}{\delta}\right)\right], \tag{79}$$

where

$$\delta=\frac{3\sqrt{4A_3}}{A_2}. \tag{80}$$

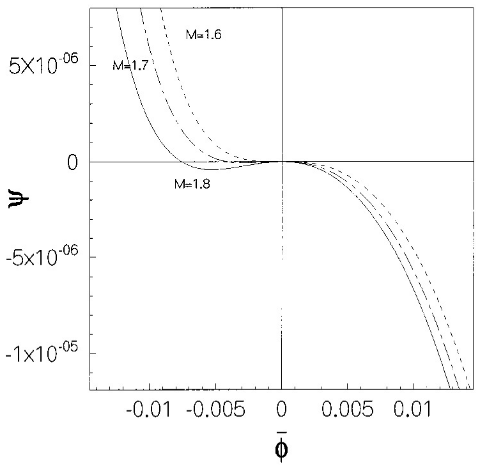

FIG. 1. Sagdeev's potential $\psi(\bar{\phi})$ vs $\bar{\phi}$ is plotted for different values of M where the dust-plasma parameters are $n_{i0}=1.1\times10^{10}\ \mathrm{cm}^{-3}$, $n_{e0}=10^{10}\ \mathrm{cm}^{-3}$, $n_{d0}=10^{7}\ \mathrm{cm}^{-3}$, $T_e=10$ eV, $a_d=10^{-6}$ cm, $q_{d0}=-93.96e$.

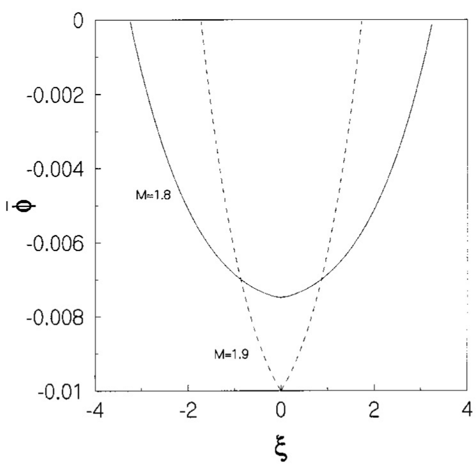

FIG. 2. Plot of $\bar{\phi}$ vs ξ for $M=1.8$, $M=1.9$, other parameters remain same as in Fig. 1.

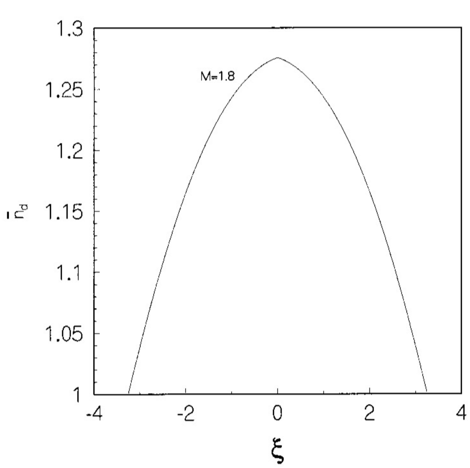

FIG. 3. Plot of $\bar{n}_d$ vs ξ for $M=1.8$, others parameters remain same as in Fig. 1.

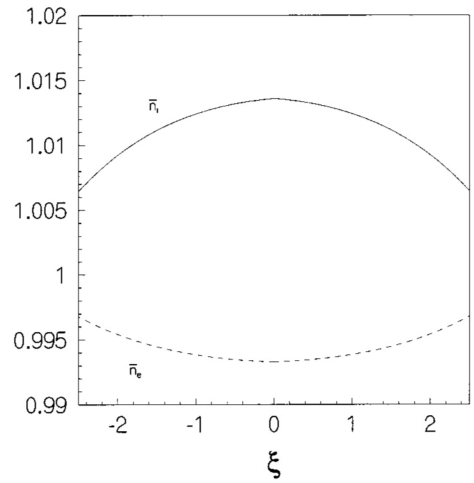

FIG. 4. Plot of $\bar{n}_e$, $\bar{n}_i$ vs ξ for $M=1.8$, other parameters remaining same as in Fig. 1.

V. RESULTS AND DISCUSSIONS

In Fig. 1, the Sagdeev potential versus ϕ for different values of M using Eqs. (35) and (51) is plotted. The values of the other parameters are given in the figure captions. It is seen that a potential well exists on the side $\phi\leqslant0$ for $1.6\leqslant M\leqslant3.1$. This indicates that solitary wave solution exists for $1.6\leqslant M\leqslant3.1$. Integrating numerically Eq. (34), the dip soliton profile shown in Fig. 2 is obtained. It is seen that while the solitary wave amplitude increases with the increase of the Mach number, the width decreases. Compressive ion and the dust density profiles shown in Fig. 3 and Fig. 4, respectively. Figure 4 shows also the electron density profile thereby showing depletion of electrons, which is to be expected as the dust density increases because of electron depletion.

VI. CONCLUSION

A nonlinear wave equation in a plasma comprising electron, ions and dust charge grains is derived. Sagdeev's potential has been derived for stationary solitary wave solution without neglecting dust charge fluctuation. Small amplitude expansion is obtained and the so-called tanh method has been applied to obtain soliton solution analytically. Numerically, the solitary wave solution is obtained, which is a dip soliton whose amplitude increases, but the width decreases, with the increase of Mach number.

ACKNOWLEDGMENT

One of the authors (R.R.) is grateful to the Council of Scientific and Industrial Research (CSIR) for a grant.

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