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EFFECTS OF HOUSEHOLD SIZE ON HOUSEHOLD EXPENDITURE PATTERN : AN APPLICATION OF AN ADDILOG ENGEL MODEL

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SUMMARY. In this paper, is presented a system of addilog Engel functions which is more general than the system of log-linear type Engel functions and the traditional system of per capita indirect addilog Engel function. This system, referred as the addilog model, satisfies the aggregation criterion and gives rise to variable partial elasticities with respect to income and household size.

Based on the consumer expenditure data for individual households from the 17th round of National Sample Survey of India relating rural and urban areas separately in the state of Uttar Pradesh, estimates of income and households-size variables elasticities relating the addilog model, income elasticities relating the per capita addilog model and income and households-size constant elasticities are obtained at average levels of income and household size of four income groups. These estimates are contrasted and analysed.

The comparative results provide evidence to signify that (i) the use of the less general log-linear Engel model may give rise to an over-estimation of two partial elasticities and (ii) for estimating income elasticities the use of the less general per capita addilog model may involve over-estimation for necessaries and under-estimation for luxuries.

1. Introduction

The simplest hypothesis allowing for the effect of variation in household size is given by supposing that consumption per person depends only on the level of income per person. There is no doubt that this assumption is easy to handle and may explain the broad phenomena of consumption patterns, but it is apparent that it will not be exactly true in general (see Stone, 1951, 10–11). A better approach, however, will be one which introduce explicitly the effect of household size in addition to that of household income in the estimation of Engel functions. Most of the studies (See Tobin (1950), Houthakker (1957), Crocket (1960), Liviatan (1964), Iyengar, Jain and Srinivasan (1975)) in the past following these lines have employed the well-known Cobb-Douglas (or log-linear) type of Engel function which suffers from the limitation, viz., it implies constant elasticities with respect to income and household size and does not possess the property of aggregation.

In view of these shortcomings, the present study develops a system of additive Engel functions which relate every specific expenditure to two explanatory variables, viz., household income and household size. This Engel model, referred as the addilog model, differs from the log-linear Engel model as it satisfies the aggregation criterion and gives rise to variable elasticities. with respect to income and household size, and is more general than Houthakker (1960)'s per capita indirect addilog Engel functions model. The addilog model is developed in Section 2.1, whereas Section 2.2 discusses the properties of the partial elasticities with respect to income and household size. Next Section 2.3 outlines a method of estimation of the parameters which is applied to the consumer expenditures cross-section data on individual households described in Section 3.1. The results on income and household-size elasticities according to the addilog model, long-linear model and per capita addilog model are presented in Tables 2.1 and 2.2, and discussed in Section 3.2. In last Section 4 we give our concluding remarks.

2. Addilog system

2.1. *Additive Engel functions model with household size effect.* The functional form of Engel function which introduces explicitly the effect of household size along with that of household income with a view to explain household expenditure pattern, considered by the studies as mentioned in Section 1, is log-linear

$$\log e_i = \log a_i + b_i \log E + c_i \log N \quad \ldots \quad (2.1.1)$$

where e_i and E[1] are the household specifics expenditure on i-th commodity and household total outlay and N is the household size. Since this form does not add up, Houthakker's (1960), suggestion may be followed to make it add up by writing

$$e_i/E = a_i E^{b_i} N^{c_i} / \sum_{k=1}^{K} a_k E^{b_k} N^{c_k} (i = 1, \ldots, K) \quad (2.1.2)$$

This may be rewritten as

$$e_i = E / \sum_{k=1}^{K} a_{ki} E^{b_{ki}} N^{c_{ki}} \quad \ldots \quad (2.1.3)$$

where $a_{ki} = a_k/a_i$, $b_{ki} = b_k - b_i$ and $c_{ki} = c_k - c_i$.

This system of additive Engel functions reduces to the Houthakker (1960)'s system of indirect addilog Engel functions when $(b_i + c_i)$ remains unchanged for different values of 'i'. Model (2.1.2), therefore, is more general

[1]Here after named 'household income', for brevity sake.

than the traditional per capita addilog Engel functions model. We shall refer to (2.1.2), for brevity sake, the addilog model in our later discussion.

The addilog model (2.1.2) can also be obtained from the indirect addilog utility function for a household

$$V[E/m_1p_1, \ldots, E/m_Kp_K]^{2} = \sum_{i=1}^{K} (\bar{a}_i/b_i)(E/m_ip_i)^{b_i}$$

if $m_i = m_i(n_1, \ldots, n_G)$, the total number of equivalent standard consumers with respect to the commodity 'i' (n_g is the number of household members in g-th age-sex group), is replaced by N^{-c_i/b_i} and $a_i = \bar{a}_i/p_i^{b_i}$. p_i's are the prices of the different commodities treated constant in a cross-sectional situation.

The addilog model (2.1.2) ignores the variation of household composition like age, sex, children versus adults etc., which has important bearing on household consumption pattern. To meet this shortcoming, the model (2.1.2) can be modified to

$$\frac{e_i}{E} = \frac{a_iE^{b_i}(1+n_1)^{c_{i1}}(1+n_2)^{c_{i2}} \ldots (1+n_G)^{c_{iG}}}{\sum_{k=1}^{K} a_kE^{b_k}(1+n_1)^{c_{k1}} \ldots (1+n_G)^{c_{kG}}} \qquad \ldots \quad (2.1.4)$$

where n_g, $g = 1, \ldots, G$ is the number of persons in g-th age-sex group. However, this improved version of our addilog model could not be taken up because of the nature of data available and the limited scope of the present study. Moreover as pointed by Muellbauer (1975), there is an identification problem in estimating the consumer unit scales if only cross-section data is available.

2.2. *Income and household-size elasticities.* The income and household-size elasticities implies by (2.1.3) are :

$$\eta_E^{e_i} = (\partial \log e_i/\partial \log E)$$

$$= 1+b_i-\bar{b} \text{ or } = 1+\sum_{j=1}^{K} w_jb_{ij} \qquad \ldots \quad (2.2.1)$$

[2]Muellbauer (1974) has shown that $V = V(E/m_1p_1, \ldots, E/m_Kp_K)$ is the household indirect utility function which corresponds to the household direct utility function $U = U(q_1/m_1, \ldots, q_K/m_K)$, where q_i is the quantity of commodity 'i' consumed by the household.

$$\eta_N^{e_i} = (\partial \log e_i/\partial \log N)$$

$$= c_i - \bar{c} \text{ or } = \sum_{j=1}^{K} w_j c_{ij} \qquad \dots \quad (2.2.2)$$

where

$$\bar{b} = \sum_{j=1}^{K} w_j b_j \text{ and } \bar{c} = \sum_{j=1}^{K} w_j c_j$$

are the means of b_j's and c_j's, respectively, weighted by the budget shares $w_j = e_j/E$. Evidently, both elasticities are variables ones, being function of both the variables E and N. Furthermore

$$\frac{\partial \eta_E^{e_i}}{\partial E} = -E^{-1} \sum_{j=1}^{K} w_j(b_j - \bar{b})^2 < 0$$

$$\frac{\partial \eta_N^{e_i}}{\partial N} = -N^{-1} \sum_{j=1}^{K} w_j(c_j - \bar{c})^2 < 0$$

It means that both partial elasticities decrease monotonically, at a pace decreasing with increasing of the corresponding variables, but otherwise (at fixed E and N) equal for all items. The income elasticities based on the addilog model, therefore, must decline with income. As there is no general theoretical presumption that they should all be falling, the addilog model is restrictive. The two elasticities seems to have lower-bound as well as upper bound (see Somermeyer and Langhout, 1972) :

$$-b_{\bar{h}} < 1 + b_i - b_{\bar{h}} < \eta_E^{e_i} < 1 + b_i - b_{\underline{h}}$$

$$c_i - c_{\bar{h}} < \eta_N^{e_i} < c_i - c_{\underline{h}}$$

with $b_{\underline{h}} = \min_h b_h$, $b_{\bar{h}} = \max_h b_h$, $c_{\underline{h}} = \min_h c_h$ and $c_{\bar{h}} = \max_h c_h$. The lower and upper bounds for income elasticities are unity at most and at least respectively, whereas for household-size elasticities the two bounds are zero at most and at least respectively. However, the lower bound for income elasticities may be zero or negative, viz., if $1 + b_i = b_{\bar{h}}$ or $< b_{\bar{h}}$ respectively, allowing for "inferior" goods to enter the budget.

Relation (2.2.1) suggests that an item i is more "necessary" or more "luxury" according as b_i is lower (especially if stronger negative) and higher (especially if positive, and nearer to 1) respectively.

Relation (2.2.2) shows that the addilog model satisfies the relation $\sum_{i=1}^{K} w_i \eta_N^{e_i} = 0$ which means that total effect of a change in household size N on all specific expenditures must be zero (see Houthakker, 1957).

Finally,

$$\eta_E^{e_i} - \eta_E^{e_j} = b_{ij}$$

and

$$\eta_N^{e_i} - \eta_N^{e_j} = c_{ij},$$

which are independent of E and N.

2.3. *Estimation of the parameters*[3]. We make our addilog model stochastic, by incorporating into it disturbance term u_i $(i = 1, ..., K)$. As a result, we get

$$e_i = a_i E^{1+b_i} N^{c_i} e^{u_i} \Big/ \left\{ \sum_{j=1}^{K} a_j E^{b_j} N^{c_j} e^{u_j} \right\} \text{ for } i = 1, ..., K \qquad ... \quad (2.3.1)$$

This specification has the advantage that also the randomized specific expenditures satisfy the budget constraint.

From (2.3.1) we derive, by division and logarithmic transformation

$$\log (e_i/e_k) = \log a_{ik} + b_{ik} \log E + c_{ik} \log N + v_{ik} \quad (i, k = 1, ..., K; i \neq k) \qquad ... \quad (2.3.2)$$

where

$$a_{ik} = a_i/a_k,\ b_{ik} = b_i - b_k,\ c_{ik} = c_i - c_k \text{ and } v_{ik} = u_i - u_k.$$

We observe that (2.3.2) consists of $(K \frac{1}{2} (K-1))$ equations, but only $K-1$ of these are mutually independent which in turn are obtained by setting $k = 1$ and varying i. Somermeyer and Langhout (1972), and also Huyser and Semermeyer (1971) applied the method of "ordinary least squares" (O.L.S.) to the equations similar to that of the system (2.3.2) *separately*. This yields best linear unbiased and consistent estimates of the parameters under the assumption that the disturbances v_{ik}'s over the individual households are

[3] As the present study employs cross-section data on individual households, this estimation procedure refers to models applied to cross-section ungrouped data.

homoskedastic. These estimates are mutually consistent in the sense that:

$$\hat{b}_{ik} = -\hat{b}_{ki} \qquad \text{(reversability)}$$

and

$$\hat{b}_{ik} = \hat{b}_{ij}+\hat{b}_{jk} \qquad \text{(triangularity)};$$

same holds for c_{ik}'s and $\log a_{ik}$'s.

Instead of considering the equations of the system (2.3.2) separately, we can deal with them simultaneously (see e.g. Zellner, 1962). However, as the set of independent variables is the same in all the equations, the generalized least squares estimator in the present case is the vector with components same as the O.L.S. estimators obtained from the separate equations.

It can be easily seen that the above estimation remainsu neffected by setting k equal to any number and varying i in (2.3.2).

In matrix notation (in order to be precise), the BLUE of the parameters of the addilog model is

$$\beta_{ij} = (X'X)^{-1}X'Y_i-(X'X)^{-1}X'Y_j$$

where

$$Y_i = \begin{bmatrix} y_{i1} \\ \cdot \\ \cdot \\ \cdot \\ y_{iH} \end{bmatrix} \qquad X = \begin{bmatrix} 1 & x_{11} & x_{21} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x_{1H} & x_{2H} \end{bmatrix} \qquad \beta_i = \begin{bmatrix} \log a_i \\ b_i \\ c_i \end{bmatrix}$$

$\beta_{ij} = \beta_i - \beta_j$, $y_{ih} = \log e_{ih}$, $x_{1h} = \log E_h$ and $x_{2h} = \log N_h$, and H is the number of sample households. It is quite interesting to note that $\hat{\beta}_{ij} = \hat{\beta}'_i - \hat{\beta}'_j$, where

$$\hat{\beta}'_i = \begin{bmatrix} \log \hat{a}'_i \\ \hat{b}'_i \\ \hat{c}'_i \end{bmatrix}$$

are O.L.S. estimates of the parameters of the log-linear Engel relations

$$\log e_i = \log a'_i + b'_i \log E + c'_i \log N, \; i = 1, \ldots, K.$$

Therefore, the above estimation of the addilog model may be easily obtained from that of the log-linear model. We have followed this very simple procedure for estimating the addilog model.

The log-linear Engel relationships are not derived from the theory and if estimated as regressions have an omitted variable, i.e.

$$Z = \log \left(E / \sum_{i=1}^{K} a_i E^{b_i} N^{c_i} e^{u_i} \right)$$

of the addilog model

$$y_i = \log a_i + b_i x_1 + c_i x_2 + Z + u_i, \quad i = 1, \ldots, K.$$

It may, therefore, be worth citing that $\hat{\beta}_i' = (X'X)^{-1}X'Y_i$ is biased and inconsistent for β_i, but $\hat{\beta}_i' - \hat{\beta}_j'$, is BLUE for $\beta_i - \beta_j (= \beta_{ij})$.

For a particular budget allocation $w = (w_1, \ldots, w_K)$, estimates of income and household-size elasticities may be obtained as

$$\hat{\eta}_E^{e_i}(w) = 1 + \hat{b}_{i1} - \sum_{j=2}^{K} w_i \hat{b}_{i1}$$

and

$$\hat{\eta}_N^{e_i}(w) = \hat{c}_{i1} - \sum_{j=2}^{K} w_i \hat{c}_{j1}.$$

These, however, are not BLUE, since $w_j \left(= 1 / \sum_{k=1}^{K} a_{k1} E^{b_{k1}} N^{c_{k1}} \right)$'s are of the stochastic nature. However, by linearization of w_j's one may construct an approximate standard errors for the two elasticities for the purpose of statistical inference (see Jain and Tendulkar, 1973). The same is done in the present case.

3. Application

3.1. *The data used.* The above method of estimation is applied to the consumption expenditure data for individual households from the 17th round of National Sample Survey of India covering the period September 1961 to July 1962. The sampling design adopted in urban sector of a state was a stratified two-stage one where the first stage units were the urban blocks, and the second stage units were the households. In the case of rural sector of a state, villages formed the units at the first stage and households at the second. The sample was selected in the form of three independent sub-samples

For definition, concept, and sample design of the survey, see National Sample Survey (1961); Instructions to Field Workers.

Because of the limitations of the ready availability of data and the exploratory nature of this exercise, only one-subsample for rual and urban areas separately is considered in the state of 'Uttar Pradesh (U.P.) which has fairly large sample size. Number of households in the rural and the urban samples is 316 and 200. But the entire sample could not be used as the present model can be applied to data containing only those sample households which report non-zero expenditure on all the items of consumption. As large number of sample households especially with low income report zero expenditure on relatively luxury items, viz., clothing, and milk and milk products, the following five broad item-groups of consumption adding up to total expenditure could be considered :

1. Cereals and cereal substitutes (or total cereals)
2. Pulses
3. Other food items (except 1 and 2).
4. Fuel and light
5. Other non-food items (excluding 4).

This provides us with the effective sample size as 305 and 189, reducing the original sample only by 3.5 percent and 5.5 percent, for rural and urban areas of U.P. respectively.

Based on this data, two complete addilog models have been estimated for rural and urban areas. To estimate the models by least squares method, we could either give the same weight to each observation in the sample or alternatively assign a probability weight to each household, this probability being that of selection of the household according to the sampling design adopted. The N.S.S. provides these probability weights or the "multipliers". Both alternatives were tried. The weighted least squares estimates did not differ appreciably from the corresponding unweighted least squares estimates. We shall, therefore, present the results based on weighted least squares estimation..

As income and household-size elasticities based on the addilog model are decreasing functions of income and household size E and N, in order to estimate them at different levels of E and N, the entire sample in both areas is classified into four somewhat homogeneous groups with respect to household income. The distribution of the sample households according to the four income-groups alongwith average household income and average household size for each income-group is displayed in Table 1.

TABLE 1. DISTRIBUTION OF SAMPLE HOUSEHOLDS, AVERAGE HOUSEHOLD INCOME AND HOUSEHOLD SIZE BY MONTHLY HOUSEHOLD INCOME-CLASSES IN RURAL AND URBAN AREAS OF UTTAR PRADESH : 1961-62

monthly household income-class (Rs.)	number of sample households	average household income (Rs.)	average household size	average house-hold per capita income (Rs.)
		rural		
0 — 55	71	35.79	3.34	10.72
55 — 90	82	72.71	5.73	12.69
90 — 135	77	109.92	6.38	17.23
> 135	75	211.49	8.53	24.70
all classes	305	107.64	6.06	17.76
		urban		
0 — 55	37	40.79	2.51	16.25
55 — 90	62	74.08	5.00	14.82
90 — 135	47	108.04	5.89	18.34
> 135	43	264.16	7.53	35.08
all classes	189	119.25	5.30	22.50

3.2. *Results of the estimation.* Results of our calculations are shown in Table 2.1 and 2.2 which present estimates of variable income and household-size elasticities $\eta_E(\bar{E}, \bar{N})$ and $\eta_N(\bar{E}, \bar{N})$ at average income and household-size levels $\bar{E}$ and $\bar{N}$ for a household income-group and their approximate standard errors. Estimates of income elasticity $\eta_{E'}(\bar{E}')$ according to the per capita addilog model (see Jain, 1976), at average per capita income level $\bar{E}'(E' = E/N)$ of each income group are also obtained and presented in these two tables. The figures given within brackets relate to the log-linear and the per capita log-linear models and so represent estimates of constant income and family size elasticities η_E and η_N and their standard errors, and that of constant per capita income elasticity $\eta_{E'}$, respectively.

As all these estimates are worked out from those of the parameters of the log-linear and the per capita log-linear models, the latter are presented in Appendix—Table A.1.

All estimates of income elasticity based on the per capita addilog or the addilog model, as expected, confirm to the theoretical property of the addilog model that income elasticity is a monotonic decreasing function of income.

TABLE 2.1[4] ITEMWISE ESTIMATES OF INCOME AND FAMILY-SIZE ELASTICITIES $\eta_E(\overline{E},\overline{N})$ AND $\eta_N(\overline{E},\overline{N})$ AND THEIR APPROXIMATE STANDARD ERRORS BASED ON THE ADDILOG MODEL, AND INCOME ELASTICITY $\eta_{\hat{E}}(\hat{E}')$ BASED ON THE PER CAPITA ADDILOG MODEL, FOR FOUR HOUSEHOLD INCOME-GROUPS, RURAL AREAS OF UTTAR PRADESH: 1961-62

item	monthly household income class (Rs.)	$\eta_E(\overline{E},\overline{N})$	approximate s.e. of $\eta_E(\overline{E},\overline{N})$	$\eta_N(\overline{E},\overline{N})$	approximate s.e. of $\eta_N(\overline{E},\overline{N})$	$\eta_{\hat{E}}(\hat{E}')$
total cereals	0—55	.710	.030	.270	.034	.005
	55—90	.599	.034	.345	.038	.617
	90—135	.450	.041	.444	.045	.491
	>135	.196	.054	.610	.060	.321
	all classes	.444 (.468)	.045 (.035)	.449 (.459)	.050 (.039)	.478 (.508)
pulses	0—55	.957	.063	−.107*	.071	.862
	55—90	.846	.064	−.039*	.072	.814
	90—135	.697	.067	.061*	.076	.688
	>135	.443	.075	.226	.083	.518
	all classes	.690 (.714)	.069 (.069)	.066* (.076*)	.077 (.077)	.675 (.705)
other food items	0—55	1.615	.062	−.476	.069	1.400
	55—90	1.504	.060	−.408	.067	1.413
	90—135	1.355	.060	−.308	.067	1.287
	>135	1.100	.064	−.142	.071	1.117
	all classes	1.348 (1.372)	.061 (.063)	−.303 (−.293)	.068 (.071)	1.274 (1.304)
fuel and light	0—55	.708	.043	−.0871*	.048	.663
	55—90	.597	.045	−.081*	.050	.616
	90—135	.448	.048	.018*	.053	.490
	>135	.194	.056	.247	.062	.320
	all classes	.442 (.466)	.050 (.047)	.086* (.096)*	.055 (.052)	.477 (.507)
other non-food items	0—55	2.632	.096	−.971	.107	2.361
	55—90	2.422	.092	−.902	.103	2.313
	90—135	2.273	.084	−.803	.093	2.187
	>135	2.018	.069	−.637	.077	2.017
	all classes	2.266 (2.290)	.070 (.095)	−.708 (−.788)	.088 (.107)	2.174 (2.204)

[4] See notes of Table 2.2.

As expected, rural income elasticities are higher than urban income elasticities. The positive difference could be attributed to the fact that for an income-group rural households have relatively low average level of income compared to urban household. Item-groups 'cereals and cereal substitutes', 'pulses', and 'fuel and light' turn out 'necessities' with income elasticities less than unity for all households except for rural households belonging to the lowest income-group, for whom item-group 'pulses' seems a semi-luxury

TABLE 2.2 ITEMWISE ESTIMATES OF INCOME AND FAMILY-SIZE ELASTICITIES $\eta_E(\overline{E}, \overline{N})$ AND $\eta_N(\overline{E}, \overline{N})$ AND THEIR APPROXIMATE STANDARD ERRORS BASED ON THE ADDILOG MODEL, AND INCOME ELASTICITY $\eta'_E(\overline{E}')$ BASED ON THE PER CAPITA ADDILOG MODEL, FOR FOUR HOUSEHOLD INCOME-GROUPS, URBAN AREAS OF UTTAR PRADESH : 1961-62

item	monthly household income class (Rs.)	$\eta_E(\overline{E}, \overline{N})$	approximate s.e. of $\eta_E(\overline{E}, \overline{N})$	$\eta_N(\overline{E}, \overline{N})$	approximate s.e. of $\eta_N(\overline{E}, \overline{N})$	$\eta'_E(\overline{E}')$
total cereals	0—55	.487	.039	.310	.036	.417
	55—90	.404	.046	.353	.043	.443
	90—135	.287	.051	.416	.047	.382
	>135	−.033*	.063	.587	.050	.190
	all classes	.221	.055	.452	.051	.312
		(.243)	(.045)	(.455)	(.042)	(.356)
pulses	0—55	.668	.073	.024*	.069	.617
	55—90	.583	.076	.008*	.071	.643
	90—135	.465	.079	.120*	.074	.582
	>135	.120*	.086	.302	.080	.390
	all classes	.400	.081	.166	.076	.621
		(.422)	(.078)	(.170)	(.073)	(.555)
other food items	0—55	1.452	.059	−.227	.056	1.544
	55—90	1.308	.057	−.183	.054	1.270
	90—135	1.251	.054	−.121	.050	1.208
	>135	.931	.062	.051*	.058	1.017
	all classes	1.185	.058	−.085*	.054	1.148
		(1.207)	(.064)	(−.081)*	(.060)	(1.182)
fuel and light	0—55	.826	.048	−.009*	.045	.797
	55—90	.742	.049	.035*	.046	.823
	90—135	.625	.050	.067*	.047	.762
	>135	.306	.057	.209	.054	.570
	all classes	.560	.053	.133	.049	.701
		(.582)	(.050)	(.137)	(.047)	(.735)
other non-food items	0—55	2.155	.080	−.621	.075	1.819
	55—90	2.071	.073	−.578	.068	1.845
	90—135	1.954	.072	−.515	.067	1.784
	>135	1.635	.055	−.344	.051	1.592
	all classes	1.888	.064	−.479	.060	1.723
		(1.910)	(.077)	(−.470)	(.072)	(1.757)

Notes: 1. Figures within brackets relate to general log-linear model.
2. Figures marked as '*' are not significant even at 10% level under two sided test.
3. Unmarked elasticity estimates are significant at 5% level.

with income elasticity close to unity. Remaining item-groups like 'food items other than cereals and pulses', and non food items except fuel and light' turn out 'luxury' with income elasticities more than unity for all households except few. Exception is comprised of urban households lying in the highest income-group for whom income elasticity for 'other food items-group' is less than but close to unity. All income elasticity estimates are significant except those marked with asterike '*'. This indicates that for urban households in the highest income-group demand for items like 'cereals' and 'pulses' is not elastic.

Estimates of income elasticities $\eta_{E'}(\bar{E}')$ and $\eta_E(\bar{E}, \bar{N})$ relating the per capita addilog and the addilog models, respectively, are appreciably different from one another for each household income-group. In fact, $\eta_{E'}(\bar{E}')$ is smaller than $\eta_E(\bar{E}, \bar{N})$ for households with low level of income and vice versa for households with high level of income. However, for an overall average household, $\eta_{E'}(\bar{E}')$ compared to $\eta_E(\bar{E}, \bar{N})$ is on the higher side for necessities and on the lower side for luxuries. As the addilog model is more general than the per capita addilog model, this in a way suggests that for income elasticities the use of the per capita addilog model may involve over-estimation in respect of essential items of consumption and under-estimation for luxury items.

Let us now examine estimates of $\eta_N(\bar{E}, \bar{N})$, the elasticities of specific expenditures with respect to household size, calculated at average levels of household income and household size $\bar{E}$ and $\bar{N}$ for each income-group. Estimates are significant in almost all cases, except for item-groups 'pulses' and 'fuel and light' consumed by a household with income less than Rs. 135 and for 'other food items' consumed by an urban household with income more than Rs. 135. At overall average levels of E and N based on the entire sample of households, estimate turns out insignificant for 'pulses' and 'fuel and light' in rural areas and 'other food items' in urban areas. The significant household-size elasticities are all positive for necessary item-groups and negative for luxury item-groups.

While interpreting these estimates it is worth noting (see Houthakker, 1957) that family-size elasticity parameter $\eta_N^{e_i}$ represents a combination of two effects, viz., (i) *specific effect* which results from the increase in the need for various goods when family size N increases. The increase in need is usually less than in proportion to increase in family size because of economies of scale in large households, (ii) *Income-effect* : since partial elasticity $\eta_N^{e_i}$ refers to influence of family size when family income is held constant, an increase in family size makes the family relatively poor, and this may induce the household to spend relatively less on various goods. For example, although an increase in family size may result in increase of a family's need for an item like clothing, the simultaneously arising need for more of the necessary item like cereals may force the household to spend less for clothing on balance. Therefore, we note that if the specific effect is stronger than the income effect, $\eta_N^{e_i}$ will be positive otherwise it will be negative. Secondly, due to the relation $\sum_{i=1}^{K} e_i \eta_N^{e_i} = 0$, $\eta_N^{e_i}$ is not all positive or negative.

In the light of these two points, the estimates of different household-size elasticities are not difficult to explain. For instance, the negative significant elasticities relating luxury items, viz., 'other food items' and 'other non-food items', may be attributed to the fact that for luxury goods the negative income-effect dominates the positive specific effect. Positive elasticities occuring in case of "necessities" have a similar interpretation.

Another interesting observation regarding family-size elasticity is that it moves along the income level. Reasons for this seems to be that (i) the negative income effect is less stronger in case of relatively better-off households; (ii) in formula

$$\eta_N^{e_i} = c_i - \sum_{j=1}^{K} c_j w_j,$$

firstly, with rise in the income level the budget shares w_j's are expected to go up for luxuries and go down for necessaries, and secondly values of the parameters c_j's for luxiries are likely to be less than those for necessaries.

Estimates of constant partial elasticities given within brackets and their counterparts related to the addilog model and calculated for an overall average household are similar in terms of sign and significance. However, in magnitude estimates of constant elasticities are always on the higher side. This shows that the less general log-linear type model containing family size and family income as the explanatory variables may result in over-estimation of income and family-size elasticities.

4. CONCLUDING REMARKS

This study presents a system of addilog Engel functions which is more general than the log-linear Engel functions model and the traditional per capita indirect addilog Engel function model. It provides for variable partial elasticities with respect to household income and household size. Secondly, at average levels of household income and household size of four income-groups, estimates of income and household-size variable elasticities relating the addilog model, income elasticities relating the per capita addilog model, and income and household size constant elasticities are contrasted and analysed.

This study provides evidence to show that based on the addilog model, (i) with rise in the level of household income, household-size elasticities go up whereas income elasticities go down; (ii) household-size elasticities are positive for necessaries and negative for luxuries implying that specific effect is stronger than income effect for necessaries and vice versa for luxiries; (iii) variable partial elasticities calculated for an overall average household are smaller than corresponding constant partial elasticities. This suggests that the use of the less general log-linear Engel model may give rise to an over-estimation of partial elasticities; (iv) income elasticities based on the addilog model are greater than those based on the per capita addilog model for households with low income and vice versa for households with high income. However, for an overall average household which corresponds to the middle income-group, income elasticities according to the addilog model are higher than those according to the per capita addilog model for luxuries and vice versa for necessaries. This seems to suggest that for estimating income elasticities the use of the less general per capita addilog model may involve over-estimation for necessaries and under-estimation for luxuries.

The present formulation of the addilog model has ignored the variation of the socio-economic factors like age and sex composition, educational status of members of the household, occupational status etc., which have important bearing on household consumption behaviour. Reason for it is the limited scope of the present study. Nevertheless, we hope to investigate this aspect in a future study. Another theoretical limitation of our model is that its estimation does not permit 'zero' item-specific consumption. As such, the use of individual data is not always possible. However, this limitation may be met by forming broad item-groups, as is done in the present study, or by using aggregate (grouped) data. To test the significance of the various elasticity estimators, we have used the sample size (less the number of estimated unknown present) as the degree of freedom. This may not be a valid procedure as our estimates are based on 'probability' (not random) sample data.

"In omitting the sample household observations with zero value of specific expenditures which are dependent variables, we are using non-randomly selected samples which, as pointed out by Heckman (1979) will give rise to biased estimation of the parameters. This problem has not been taken up as it is a very difficult one and is beyond the scope of this paper".

Our findings may not be valid universally as they are based on a particular body of data. Moreover, our results are valid only to the extent that the error specifications we have assumed are appropriate. The assumption of homoscedasticity made in estimating the model may not be realistic one. This suggests the need for further work not only with different stochastic assumptions but also with data from different countries and at different times.

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Appendix A

TABLE A.1. LEAST SQUARES ESTIMATES OF THE PARAMETERS OF THE LOG-LINEAR MODEL AND THE PER CAPITA LOG-LINEAR MODEL FOR FIVE ITEMS OF CONSUMPTION, RURAL AND URBAN AREAS OF UTTAR PRADESH : 1961-62

name of the items	log-linear model			per capita log-linear model	
	$\log \hat{a}'$	$\hat{b}'$	$\hat{a}'$	$\log \hat{\alpha}'$	$\hat{\beta}'$
		rural			
total cereals	0.7123	0.4677	0.4595	0.4611	0.5078
pulses	−1.7009	0.7143	0.0761	−2.0855	0.7047
other food items	−2.8816	1.3720	−0.2029	−2.5455	1.3038
fuel and light	−0.5416	0.4055	0.0963	−1.5115	0.5065
other non-food items	−6.4709	2.2900	−0.7878	−5.2625	2.2039
		urban			
total cereals	1.5082	0.2430	0.4554	0.6431	0.3549
pulses	−0.9213	0.4218	0.1606	−2.0365	0.5548
other food items	−2.1205	1.2073	−0.0814	−1.8183	1.1819
fuel and light	−0.9206	0.5817	0.1366	−1.8818	0.7349
other non-food items	−5.0542	1.9104	−0.4757	−3.8260	1.7609

Note: The per capita log-linear model: $\log(e_i/N) = \log \alpha_i' + \beta_i' \log(E/N)$ is estimated by weighted least squares with household sizes as the weights.

REFERENCES

BLOCKLAND, J. and SOMERMEYER, W. H. (1970): Effects of family size and composition on family expenditure according to an allocation model. *Report* 7020 of the Econometric Institute, Netherlands School of Economics, Rotterdam.

CROCKETT, J. (1960): Demand relationship for food. *Consumption and Savings*, Vol. 1 (ed. Irwin Friend and Robert Jones), University of Pennsylvania, 293–310.

HECKMAN, J. J. (1979): Sample selection bias as a specification error. *Econometrica*, 47, No. 1, 157–161.

HOUTHAKKER, H. S. (1957): An international comparison of household expenditure patterns. *Econometrica*, 25, 532–551.

——— (1960): Additive preferences. *Econometrica*, 28, 248–257.

HUYSER, A. P. and SOMERMEYER, W. H. (1971): Income and quality elasticities in Mexico: An application of the expenditure allocation model. *Report* 7108 of the Econometric Institute, Netherlands School of Economics, Rotterdam.

IYENGAR, N. S., JAIN, L. R. and SRINIVASAN, T. N. (1975): Economics of scale in household consumption: A case study. *Economic Theory and Practice in the Asian Setting*, Vol. 2 (Microeconomics) (ed.) Committee on Economic Teaching Material for Asian Universities), Wiley Eastern Ltd., New Delhi, India 23–36.

JAIN, L. R. and TENDULKAR, S. D. (1973): Analysis of occupational differences in consumer expenditure pattern in India. *Sankhyā*, Series B, 35, Part 2, 239–267.

JAIN, L. R. (1976): An empirical evaluation of the system of indirect addilog Engel functions. *Sankhyā*, Series C, Part 4, 107–126.

LIVIATON, N. (1964): *Consumption Pattern of Israel*, Jerusalem, Academic Press Ltd., Jerusalem, Israel.

MUELLBAUER, J. (1974): Household composition, Engel curves and welfare comparisons between households: A duality approach. *European Economic Review*, 5, 103–122.

——— (1975): Identification and consumer unit scales. *Econometrica*, 43, No. 4, 807–809.

NATIONAL SAMPLE SURVEY (1961): *Instructions to Field Workers, Design, Concept, Definition and Procedures*, Seventeenth round 1961-62, Indian Statistical Institute, Calcutta.

SOMERMEYER, W. H. and LANGHOUT, A. (1972): Shapes of Engel curves and demand curves: Implications of the expenditure allocation model applied to Dutch data. *European Economic Review*, 3.

STONE, R. (1951): The demand for food in the United Kingdom before the war. *Metroeconomica*, 13, 8.

TOBIN, J. (1950): A statistical demand function for food in the U.S.A. *J. Roy. Statist Soc.* Part II, 134.

ZELLNER, A. (1962): An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias. *Jour. Amer. Statist. Assoc.*, 57, 348–368.

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