Testing for Unit Roots in Panels With a Factor Structure

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Abstract

This paper considers various tests for the unit root hypothesis in panels, where the cross section dependence is due to common dynamic factors. Three situations are studied. First, the common factors and idiosyncratic components may both be nonstationary. In this case all GLS type test statistics possess a standard normal limiting distribution, whereas the OLS based test statistics are invalid. If the common component is $I(1)$ and the idiosyncratic component is stationary (the case of cross-unit cointegration), then both the OLS and the GLS statistics fail. Finally, if the idiosyncratic components are $I(1)$ but the common factors are stationary, then the OLS based test statistics are not applicable, whereas the GLS type statistics do not have problems in this situation. A Monte Carlo study is conducted to gauge the small sample performance of these tests and a panel data set of 16 countries is used to test the hypothesis that interest rates are stationary.

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1 Introduction

Panel unit root tests are proposed to improve the power of standard univariate unit root tests. However, an important problem with the application of panel data methods to regional data is that the data usually exhibit substantial cross sectional dependence. In recent years, two different approaches have been advanced to cope with such situations. Chang (2002, 2004), Breitung and Das (2005), and Harvey and Bates (2003) assume "weak" error dependence that can be characterized by the fact that all eigenvalues of the error covariance matrix are bounded as the number of cross section units tend to infinity. On the other hand, the work of Choi (2002), Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), and Pesaran (2005) assumes a "strong" form of dependence that is due to common factors. In this case the largest eigenvalue of the covariance matrix tend to infinity as the cross section dimension (N) increases.[1]

In this paper, we consider four panel unit root tests under strong dependence. The first test statistic is the pooled OLS t-statistic that ignores a possible cross-section dependence. The simple robust OLS t-statistic suggested by Breitung and Das (2005) was found to perform well in the case of weak cross section dependence. In this paper we show that in the presence of common factors the test has a nonstandard limiting distribution under the null hypothesis. We also study test statistics based on suitable transformations of the variables. Two different approaches are considered. First, the inverse of the estimated residual covariance matrix is used to compute the GLS t-statistic. The second approach employs a transformation matrix that eliminates the common factors. Such test procedures are suggested by Phillips and Sul (2003) and Moon and Perron (2004). We focus on the latter test statistic that is based on the principal component estimator of the common factors.

These test statistics are used to test the null hypothesis that all time series in the panel data set are $I(1)$. This null hypotheses may be the result of three different situations. In *case (a)* we assume that both the common and idiosyncratic components are nonstationary. In *case (b)* it is assumed that the common factors are $I(1)$ and the idiosyncratic components are $I(0)$ (that is, there exist a cointegration relationship among the panel units), whereas in *case (c)* the common factors are $I(0)$ and the idiosyncratic components are $I(1)$. In this paper we study the asymptotic properties of alternative panel unit root tests in these

[1]For a recent review of the literature see Breitung and Pesaran (2005).

three situations. We show that no test procedure is able to cope with case (b), where the series are cross-cointegrated. In the other cases at least some of the tests are valid.

The rest of the paper is organized as follows. In Section 2, we analyze the asymptotic properties of the OLS based test statistics. In Section 3 the asymptotic properties of the GLS type test procedure are considered. Section 4 presents the results of some Monte-Carlo experiments and Section 5 provides an empirical illustration. Some concluding remarks are offered in Section 6.

2 The OLS based test statistic

Consider a collection of time series $\{y_{i0}, \ldots, y_{iT}\}_{i=1,\ldots,N}$ that is generated by the composed process

$$
\begin{aligned}
y_{it} &= \gamma_i' f_t + u_{it} \ , && (1)\\
f_t &= A f_{t-1} + v_t \ , && (2)\\
u_{it} &= \theta u_{i,t-1} + \varepsilon_{it} \ , && (3)
\end{aligned}
$$

where f_t is a $r \times 1$ vector of unobservable common factors, γ_i is a $r \times 1$ vector of non-random factor loadings, $\Gamma = [\gamma_1, \ldots, \gamma_N]'$ is the matrix of factor loadings and u_{it} is an idiosyncratic error component. The eigenvalues of the $r \times r$ matrix A are on or inside the unit circle of the complex plane. Specifically, we will focus on the special case $A = \rho I_r$ with $|\rho| \leq 1$.

To simplify the exposition we have left out any deterministic terms and short-run dynamics. The inclusion of constants, time trends and short-run dynamics is straightforward and is considered in Breitung and Das (2005). Similar versions of the factor model are considered by Phillips and Sul (2003), Moon and Perron (2004) and Pesaran (2005).

The assumptions on the error processes are summarized in

Assumption 1: The vector $\varepsilon_t = [\varepsilon_{1t}, \ldots, \varepsilon_{Nt}]'$ *is i.i.d. with* $E(\varepsilon_t) = 0$ *and* $E(\varepsilon_t \varepsilon_t') = \Sigma$*, where* Σ *is a positive definite (not necessarily diagonal) matrix with bounded eigenvalues and* $\lim_{N\to\infty} N^{-1} \sum_{i=1}^N \gamma_{ik}^2 = c_k < \infty$ *for* $k \in \{1, \ldots, r\}$ *as* $N \to \infty$*. The* $r \times 1$ *error vector* v_t *is i.i.d. with* $E(v_t) = 0$*,* $E(v_t v_t') = I_r$ *and* $E(\varepsilon_t v_t') = 0$*. Furthermore,* $E(\varepsilon_{it}^4) < \infty$ *for all* i *and* t *and* $E(v_{jt}^2 v_{kt}^2) < \infty$ *for all* j, k, t.

To test the null hypothesis that y_{it} is a random walk process for all $i = 1, \ldots, N$, the panel data unit root test is based on the autoregression

$$y_{it} = \phi_i y_{i,t-1} + e_{it} \ .$$

Consider the test of the unit root hypothesis $\phi_1 = \cdots = \phi_N = 0$ against the homogeneous alternative $\phi_1 = \cdots = \phi_N < 0$. Following Levin, Lin and Chu (2002) the test is based on the pooled regression

$$\Delta y_t = \phi y_{t-1} + e_t \ , \tag{4}$$

where $\Delta y_t = [y_{1t}, \ldots, y_{Nt}]'$, $y_{t-1} = [y_{1,t-1}, \ldots, y_{N,t-1}]'$ and $e_t = [e_{1t}, \ldots, e_{Nt}]'$. The pooled OLS t-statistic is

$$t_{ols} = \frac{\sum\limits_{t=1}^{T} y'_{t-1}\Delta y_t}{\widehat{\sigma}\sqrt{\sum\limits_{t=1}^{T} y'_{t-1} y_{t-1}}} \ . \tag{5}$$

where $\widehat{\sigma}^2 = (NT)^{-1}\sum_{t=1}^{T} \widehat{e}'_t\widehat{e}_t$ and $\widehat{e}_t = \Delta y_t - \widehat{\phi} y_{t-1}$ denotes the residual vector.

Following Breitung and Das (2005) and Jönsson (2005) we will also consider the robust t-statistic that employs "panel corrected standard errors" (PCSE). The test statistic results as

$$t_{rob} = \frac{\sum\limits_{t=1}^{T} y'_{t-1}\Delta y_t}{\sqrt{\sum\limits_{t=1}^{T} y'_{t-1}\widehat{\Omega} y_{t-1}}}, \tag{6}$$

where

$$\widehat{\Omega} = \frac{1}{T}\sum_{t=1}^{T} \widehat{e}_t\widehat{e}'_t \ .$$

Using a sequential limit theory, Breitung and Das (2005) show that if all eigenvalues of the covariance matrix Ω are bounded, this test statistic is distributed as $\mathcal{N}(0, v_\Omega)$, where $v_\Omega = \lim_{N\to\infty} tr(\Omega^2/N)/(tr\Omega/N)^2$. However, if Ω has a factor structure, then r eigenvectors are $O(N)$ and, therefore, the limiting distribution is no longer standard normal. In what follows we derive the asymptotic properties of the test statistics t_{ols} and t_{rob} if it is assumed that the time series possess a factor structure.

Under the null hypothesis it is assumed that all components of the vector y_{it} are $I(1)$. We will consider three different situations:

$$\text{case (a)} \qquad A = I_r \text{ and } \theta = 1 \tag{7}$$

$$\text{case (b)} \qquad A = I_r \text{ and } |\theta| < 1 \tag{8}$$

$$\text{case (c)} \qquad A = \rho I_r \text{ where } |\rho| < 1 \text{ and } \theta = 1 \tag{9}$$

In case (a) it is assumed that both the common factors and the idiosyncratic components are nonstationary. For example, it may be assumed that the error components follow the same autoregressive process with $\theta = \rho$ under the null and alternative hypotheses (Phillips and Sul 2003, Moon and Perron 2004, Pesaran 2005). In case (b) it is assumed that the series share r common stochastic trends. Accordingly there exists a $N \times (N - r)$ matrix Q such that $Q'y_t$ is stationary. Following Banerjee et al. (2005) this situation is called *cross-unit cointegration*. Finally, in case (c) the nonstationarity of the elements of y_t is due to the idiosyncratic component.

In the following theorem the limiting distributions of the test statistics are given.

Theorem 1: *Assume that y_t is generated as in (1) – (3) and Assumption 1. Let $\lambda_i^* = \lim_{N\to\infty} N^{-1}\lambda_i$ denote the limit of the i'th (ordered) eigenvalue of the covariance matrix $\Omega = E(e_t e_t')$. As $T \to \infty$ is followed by $N \to \infty$ the asymptotic properties of the OLS based test statistics for the three cases (7) – (9) can be summarized as follows:*

$$\textit{case (a):} \qquad N^{-1/2}\, t_{ols} \;\Rightarrow\; \frac{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)dW_i(a)}{\sqrt{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)^2 da}}, \qquad t_{rob} \;\Rightarrow\; \frac{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)dW_i(a)}{\sqrt{\sum_{i=1}^{r} \lambda_i^{*2} \int_0^1 W_i(a)^2 da}}$$

$$\textit{cases (b):} \qquad N^{-1/2} t_{ols} \;\Rightarrow\; \frac{\left[\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)dW_i(a)\right] - \psi}{\sqrt{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)^2 da}}, \qquad t_{rob} \Rightarrow \frac{\left[\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)dW_i(a)\right] - \psi}{\sqrt{\sum_{i=1}^{r} \lambda_i^{*2} \int_0^1 W_i(a)^2 da}}$$

$$\psi = (1 - \theta) \lim_{N\to\infty} tr(\Sigma/N)$$

$$\textit{case (c)} \qquad t_{ols} \textit{ is } -O_p(\sqrt{N}), \quad t_{rob} \to -\infty \textit{ as } min(N,T) \to \infty$$

and $W_1(a), \ldots, W_r(a)$ are independent standard Brownian motions.

Proof: *Case (a):* The matrix $\Omega = E(\Delta y_t \Delta y_t') = \Gamma\Gamma' + \Sigma$ is decomposed as $\Omega = V\Lambda V'$, $\Lambda =$diag$(\lambda_1, \ldots, \lambda_N)$ with the ordered eigenvalues $\lambda_1 > \ldots > \lambda_N$ and $V = [v_1, \ldots, v_N]$ is the matrix of orthonormal eigenvectors (v_i). Let $z_t = \Lambda^{-1/2}V'y_t$ denote a $N \times 1$ vector of random walk components with unit covariance matrix. We have

$$a_{NT} = N^{-1}T^{-1}\sum_{t=1}^{T} y_{t-1}'\Delta y_t = N^{-1}T^{-1}\sum_{t=1}^{T} z_{t-1}'\Lambda\Delta z_t = N^{-1}\sum_{i=1}^{N}\lambda_i\xi_{iT} \ ,$$

where, as $T \to \infty$, $\xi_{iT} = T^{-1}\sum_{t=1}^{T} z_{i,t-1}\Delta z_{it} \Rightarrow \int_0^1 W_i(a)dW_i(a)$ and $\{W_1(a), \ldots, W_N(a)\}$ represent N independent standard Brownian motions and $\Rightarrow$ indicates weak convergence with respect to the associated probability measure. Thus, since ξ_{iT} is independent of ξ_{jT} for $i \neq j$ and $E(\xi_{iT}) = 0$ for all i

$$a_{NT} = N^{-1}\sum_{i=1}^{r}\lambda_i\xi_{iT} + O_p(N^{-1/2}),$$

where we have used the fact that $\lambda_i = O(N)$ for $i = 1, \ldots, r$ and all other eigenvalues are bounded. Letting $N \to \infty$ we obtain

$$a_{NT} \ \Rightarrow \ \sum_{i=1}^{r}\lambda_i^*\int_0^1 W_i(a)dW_i(a).$$

Furthermore,

$$\begin{aligned} b_{NT} = N^{-2}T^{-2}\sum_{t=1}^{T} y_{t-1}'\widehat{\Omega}y_{t-1} &= N^{-2}T^{-2}\sum_{t=1}^{T} y_{t-1}'\Omega y_{t-1} + o_p(1) \\ &= N^{-2}T^{-2}\sum_{t=1}^{T} z_{t-1}'\Lambda^2 z_{t-1} + o_p(1) \\ &= N^{-2}\sum_{i=1}^{N}\lambda_i^2 S_{iT} + o_p(1), \end{aligned}$$

where, as $T \to \infty$, $S_{iT} = T^{-2}\sum_{t=1}^{T} z_{i,t-1}^2 \Rightarrow \int_0^1 W_i(a)^2 da$. Letting $N \to \infty$ we obtain

$$\begin{aligned} b_{NT} &= N^{-2}\sum_{i=1}^{r}\lambda_i^2 S_{iT} + o_p(1) \\ &= \sum_{i=1}^{r}\lambda_i^{*2} S_{iT} + o_p(1) \\ &\Rightarrow \sum_{i=1}^{r}\lambda_i^{*2}\int_0^1 W_i(a)^2 da. \end{aligned}$$

In a similar manner it follows that

$$\begin{aligned} c_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T} y'_{t-1}y_{t-1} \\ &= N^{-1}\sum_{i=1}^{r}\lambda_i S_{iT} + o_p(1) \\ &\Rightarrow \sum_{i=1}^{r}\lambda_i^* \int_0^1 W_i(a)^2 da. \end{aligned}$$

Thus, $t_{ols} = a_{NT}/(\widehat{\sigma}\sqrt{c_{NT}/N})$ is $O_p(N^{1/2})$ and

$$N^{-1/2}\, t_{ols} = \frac{a_{NT}}{\widehat{\sigma}\sqrt{c_{NT}}} \Rightarrow \frac{\sum\limits_{i=1}^{r}\lambda_i^* \int_0^1 W_i(a)dW_i(a)}{\sqrt{\sum\limits_{i=1}^{r}\lambda_i^* \int_0^1 W_i(a)^2 da}}$$

$$t_{rob} = \frac{a_{NT}}{\sqrt{b_{NT}}} \Rightarrow \frac{\sum\limits_{i=1}^{r}\lambda_i^* \int_0^1 W_i(a)dW_i(a)}{\sqrt{\sum\limits_{i=1}^{r}\lambda_i^{*2} \int_0^1 W_i(a)^2 da}} .$$

Case (b): Let

$$\begin{aligned} a_{NT} &= N^{-1}T^{-1}\sum_{t=1}^{T} y'_{t-1}\Delta y_t \\ &= N^{-1}T^{-1}\sum_{t=1}^{T}(\Gamma f_{t-1} + u_{t-1})'(\Gamma\Delta f_t + \Delta u_t) \\ &= \sum_{i=1}^{r}(\tilde{\lambda}_i/N)\tilde{\xi}_{iT} + N^{-1}T^{-1}\sum_{t=1}^{T} u'_{t-1}\Delta u_t + o_p(1), \end{aligned}$$

where $\tilde{\lambda}_1, \ldots, \tilde{\lambda}_r$ denote the r nonzero eigenvalues of $\Gamma\Gamma'$ and

$$\tilde{\xi}_{iT} = T^{-1}\sum_{t=1}^{T}\widetilde{V}_r' f_t,$$

where $\widetilde{V}_r = [\tilde{v}_1, \ldots, \tilde{v}_r]$ is the matrix of r eigenvectors associated with the r eigenvectors $\tilde{\lambda}_1, \ldots, \tilde{\lambda}_r$. Furthermore, $E(\Delta u_t' u_{t-1}) = E[(\theta-1)u'_{t-1} + \varepsilon'_t]u_{t-1} = (\theta-1)tr(\Sigma)$. If $T \to \infty$ is followed by $N \to \infty$, then

$$a_{NT} \Rightarrow \sum_{i=1}^{r}\lambda_i^* \int_0^1 W_i(a)dW_i(a) - (1-\theta)\lim_{N\to\infty} tr(\Sigma/N).$$

Under the null hypothesis the least-squares estimator $\widehat{\phi}$ is $O_p(T^{-1}N^{-1/2})$ and, therefore,

$$\begin{aligned}\widehat{e}_{it} &= \Delta y_{it} - \widehat{\phi} y_{i,t-1} \\ &= \Delta y_{it} + O_p(T^{-1/2}N^{-1/2}) \\ &= \gamma_i' v_t + \Delta u_{it} + O_p(T^{-1/2}N^{-1/2}).\end{aligned}$$

It follows that $\widehat{\Omega} = E(\Delta y_t \Delta y_t') + o_p(1)$. Accordingly, as $N \to \infty$, the (normalized) r largest eigenvalues of $\widehat{\Omega}$ converge to the nonzero eigenvalues of $\Gamma\Gamma'$. If $T \to \infty$ is followed by $N \to \infty$ we obtain

$$b_{NT} = N^{-2}T^{-2}\sum_{t=1}^{T} y_{t-1}'\widehat{\Omega} y_{t-1} \ \Rightarrow \ \sum_{i=1}^{r} \lambda_i^{*2} \int_0^1 W_i(a)^2 da$$

and

$$c_{NT} = N^{-1}T^{-2}\sum_{t=1}^{T} y_{t-1}' y_{t-1} \ \Rightarrow \ \sum_{i=1}^{r} \lambda_i^{*} \int_0^1 W_i(a)^2 da.$$

Thus,

$$\begin{aligned} N^{-1/2}\, t_{ols} &= \frac{a_{NT}}{\widehat{\sigma}\sqrt{c_{NT}}} \Rightarrow \frac{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a) dW_i(a) - (1-\theta) \lim\limits_{N\to\infty} tr(\Sigma/N)}{\sqrt{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a)^2 da}} \\ t_{rob} &= \frac{a_{NT}}{\sqrt{b_{NT}}} \Rightarrow \frac{\sum_{i=1}^{r} \lambda_i^* \int_0^1 W_i(a) dW_i(a) - (1-\theta) \lim\limits_{N\to\infty} tr(\Sigma/N)}{\sqrt{\sum_{i=1}^{r} \lambda_i^{*2} \int_0^1 W_i(a)^2 da}} .\end{aligned}$$

Case (c): Using the same notation as in case (a) we obtain

$$a_{NT} = N^{-1}\sum_{i=1}^{r} \lambda_i \xi_{iT} + o_p(1),$$

where under the assumption $A = \rho I_r$ with $|\rho| < 1$

$$\xi_{iT} \ \xrightarrow{p} \ \rho - 1 \quad i = 1, \ldots, r$$

and, therefore, $a_{NT} \ \xrightarrow{p} \ (\rho - 1)\left(\sum_{i=1}^{r} \lambda_i^*\right)$. Furthermore,

$$\begin{aligned} b_{NT} &= N^{-2}\sum_{i=1}^{N} \lambda_i^2 S_{iT} + o_p(1) \\ &= \sum_{i=1}^{r} (\lambda_i^2/N^2) S_{iT} + N^{-2} \sum_{j=r+1}^{N} \lambda_j^2 S_{jT} .\end{aligned}$$

Since S_{iT} is $O_p(T^{-1})$ for $i = 1, \ldots, r$ and $E(S_{jT}) = 1/2$ for $j = r+1, \ldots, N$ it follows that

$$b_{NT} = O_p(T^{-1}) + O_p(N^{-1}).$$

Similarly, if N is fixed we have

$$\begin{aligned} c_{NT} &= N^{-1} \sum_{i=1}^{N} \lambda_i S_{iT} \\ &= \sum_{i=1}^{r} (\lambda_i/N) S_{iT} + N^{-1} \sum_{j=r+1}^{N} \lambda_j^2 S_{jT} \\ &= O_p(T^{-1}) + O_p(1). \end{aligned}$$

It follows that $t_{ols} = a_{NT}/\sqrt{c_{NT}/N}$ is $-O_p(N^{1/2})$ and t_{rob} tends to $-\infty$ whenever $min(N,T) \to \infty$. ■

These results show that the OLS t-statistic is severely biased in all cases due to the fact that it tends to infinity as $N \to \infty$. According to the results of Breitung and Das (2005) this behavior is anticipated since v_Ω is $O(N)$ if some eigenvalue is of order $O(N)$.

It is interesting to note that for $r = 1$ the limiting distribution of the robust OLS statistic in case (a) is identical to the limiting distribution of the Dickey-Fuller test statistic. Indeed, in this case, the test is asymptotically equivalent to a Dickey-Fuller test applied to the first principal component $v_1' y_t$. Thus, the limiting behavior of this test statistic is dominated by the common factor and the idiosyncratic components do not affect the limiting distribution.

3 Tests based on GLS regressions

Since the GLS estimator of ϕ in (4) is more efficient than the OLS estimator, a more powerful test statistic can be constructed based on the GLS estimator of ϕ. The GLS t-statistic is given by

$$t_{gls} = \frac{\sum_{t=1}^{T} y_{t-1}' \widehat{\Omega}^{-1} \Delta y_t}{\sqrt{\sum_{t=1}^{T} y_{t-1}' \widehat{\Omega}^{-1} y_{t-1}}}.$$

It is important to note that this test statistic can only be computed if $T > N$ as otherwise the estimated covariance matrix $\widehat{\Omega}$ is singular. This is an important drawback compared to the OLS based test statistics.

Another approach to deal with the cross section dependence due to common factors is the elimination of the common factors by employing a transformation matrix Q with the property $Q'\Gamma = 0$. The transformed regression is

$$\Delta\zeta_t = \phi^* \zeta_{t-1} + e_t$$

where $\zeta_t = Qy_t$ and $e_t = Q\varepsilon_t$. Phillips and Sul (2003) suggest to use an estimated version of the matrix

$$Q_{PS} = (\Gamma_\perp' \Sigma \Gamma_\perp)^{-1/2} \Gamma_\perp',$$

where $\Gamma_\perp$ is an orthogonal complement of Γ, which can be estimated by using a least-squares approach. Moon and Perron (2004) employ the (the estimated analog) of the matrix[2]

$$Q_{MP} = \Sigma^{-1/2}(I_N - V_r V_r')$$

where $V_r = [v_1, \ldots, v_r]$ is the matrix of eigenvectors associated with the largest r eigenvalues of Ω.

An important advantage of the GLS t-statistic is that it does not require an assumption about the structure of the covariance matrix. Specifically, no assumption about the number of common factors are required. On the other hand, this generality gives rise to very poor small sample properties (see section 5). To improve the small sample properties of the test it is therefore desirable to impose some structure on the covariance matrix. Assume that the innovations admit a (strict) factor structure such that $\Omega = \Gamma\Gamma' + \Sigma$. Using the well known result on the inverse of a sum of two matrices, we obtain

$$\Omega^{-1} = \Sigma^{-1} - \Sigma^{-1}\Gamma(I_r + \Gamma'\Sigma^{-1}\Gamma)^{-1}\Gamma'\Sigma^{-1} \ . \tag{10}$$

To estimate Ω^{-1} consistent estimators for Λ and Σ are required. Following Bai and Ng (2002) and Moon and Perron (2004) a principal component approach can be adopted. Let $\widehat{V}_r = [\widehat{v}_1, \ldots, \widehat{v}_r]$ denote the matrix of r eigenvectors of the matrix $\widehat{\Omega}$ associated to the r largest eigenvalues. The matrix Γ is estimated as $\widehat{\Gamma} = \widehat{V}_r$ and $\widehat{\Sigma} = (I_N - V_r V_r')\widehat{\Omega}(I_N - V_r V_r')$.

In the following theorem the null distribution of the GLS type test statistics are presented.

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[2]Moon and Perron (2004) assume that the idiosyncratic components are independent. To allow for (weakly) correlated idiosyncratic components we pre-multiply their transformation matrix by $\Sigma^{-1/2}$.

Theorem 2: *Let y_t be generated as in (1) – (3). If $T \to \infty$ is followed by $N \to \infty$, then according to the cases given in (7) – (9) it holds that*

$$\begin{array}{lll} \textit{case (a):} & t_{gls} \Rightarrow \mathcal{N}(0,1), & t_{MP} \Rightarrow \mathcal{N}(0,1) \\ \textit{case (b):} & t_{gls} \ \textit{is} \ O_p(N^{1/2}) & t_{MP} \ \textit{is} \ O_p(N^{1/2}T^{1/2}) \\ \textit{case (c):} & t_{gls} \Rightarrow \mathcal{N}(0,1), & t_{MP} \Rightarrow \mathcal{N}(0,1) \end{array}$$

Proof: *Case (a).* By using the same notations and decomposition of Ω as in Theorem 1, we write

$$\begin{aligned} d_{NT} = N^{-1/2}T^{-1}\sum_{t=1}^{T} y'_{t-1}\widehat{\Omega}^{-1}\Delta y_t &= N^{-1/2}T^{-1}\sum_{t=1}^{T} y'_{t-1}\Omega^{-1}\Delta y_t + o_p(1) \\ &= N^{-1/2}T^{-1}\sum_{t=1}^{T} z'_{t-1}\Delta z_t + o_p(1) \\ &= N^{-1/2}\sum_{i=1}^{N} T^{-1}\sum_{t=1}^{T} z_{i,t-1}\Delta z_{it} + o_p(1) \\ &\Rightarrow \mathcal{N}(0,1/2) \end{aligned}$$

and

$$\begin{aligned} e_{NT} = N^{-1}T^{-2}\sum_{t=1}^{T} y'_{t-1}\widehat{\Omega}^{-1}y_{t-1} &= N^{-1}T^{-2}\sum_{t=1}^{T} y'_{t-1}\Omega^{-1}y_{t-1} + o_p(1) \\ &= N^{-1}T^{-2}\sum_{t=1}^{T} z'_{t-1}z_{t-1} + o_p(1) \\ &\xrightarrow{p} 1/2 \end{aligned}$$

as $T \to \infty$ is followed by $N \to \infty$. It follows that $t_{gls} = d_{NT}/\sqrt{e_{NT}} \Rightarrow \mathcal{N}(0,1)$.

As shown by Bai and Ng (2004) the PC estimator of the idiosyncratic component $\Delta\widehat{u}_t = \Sigma^{1/2}Q_{MP}\Delta y_t$ is a consistent estimator of Δu_t as $min(N,T) \to \infty$. It follows that, as $T \to \infty$, $T^{-1/2}\widehat{u}_{[aT]} \Rightarrow \Sigma^{1/2}\overline{W}(a)$, where $\overline{W}(a) = [W_1(a), \ldots, W_N(a)]'$ is a vector of independent Brownian motion with unit variances and

$$\begin{aligned} \tilde{d}_{NT} &= N^{-1/2}T^{-1}\sum_{t=1}^{T} \zeta'_{t-1}\Delta\zeta_t \\ &= N^{-1/2}T^{-1}\sum_{t=1}^{T} \widehat{u}'_{t-1}\Sigma^{-1}\Delta\widehat{u}_t \\ &\Rightarrow \mathcal{N}(0,1/2). \end{aligned}$$

Furthermore

$$\begin{aligned}
\tilde{e}_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T}\zeta_{t-1}'\zeta_{t-1} \\
&= N^{-1}T^{-2}\sum_{t=1}^{T}\widehat{u}_{t-1}'\Sigma^{-1}\widehat{u}_{t-1} \\
&= N^{-1}T^{-2}\sum_{t=1}^{T}u_{t-1}'\Sigma^{-1}u_{t-1} + o_p(1) \\
&\stackrel{p}{\to} 1/2
\end{aligned}$$

as $T \to \infty$ is followed by $N \to \infty$. Therefore, $t_{MP} = \tilde{d}_{NT}/\sqrt{\tilde{e}_{NT}} \Rightarrow \mathcal{N}(0,1)$.

Case (b). As in case (a) we have

$$\begin{aligned}
d_{NT} = N^{-1/2}T^{-1}\sum_{t=1}^{T}y_{t-1}'\widehat{\Omega}^{-1}\Delta y_t &= N^{-1/2}T^{-1}\sum_{t=1}^{T}y_{t-1}'\Omega^{-1}\Delta y_t + o_p(1) \\
&= \sum_{i=1}^{N}N^{-1/2}T^{-1}\sum_{t=1}^{T}z_{i,t-1}\Delta z_{it} + o_p(1)
\end{aligned}$$

The sets $\{z_{1t},\ldots,z_{rt}\}$ and $\{z_{r+1,t},\ldots,z_{Nt}\}$ correspond to the r nonstationary factors and $N-r$ linear transformations of the stationary idiosyncratic components. As $T \to \infty$ we obtain

$$d_{NT} = -(1-\theta) + O_p(N^{-1/2}).$$

Furthermore, as $T \to \infty$

$$\begin{aligned}
e_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T}y_{t-1}'\widehat{\Omega}^{-1}y_{t-1} \\
&= N^{-1}T^{-2}\sum_{t=1}^{T}y_{t-1}'\Omega^{-1}y_{t-1} + o_p(1) \\
&= N^{-1}T^{-2}\sum_{t=1}^{T}z_{t-1}'z_{t-1} + o_p(1) \\
&\Rightarrow N^{-1}\sum_{i=1}^{r}W_i(a)^2 da.
\end{aligned}$$

It follows that $t_{gls} = d_{NT}/\sqrt{e_{NT}}$ is $O_p(N^{1/2})$.

Since $\widehat{u}_t$ is consistent for u_t (cf. Bai 2003), it follows

$$\begin{aligned}\tilde{d}_{NT} &= N^{-1/2}T^{-1}\sum_{t=1}^{T}\zeta_{t-1}'\Delta\zeta_t \\ &= N^{-1/2}T^{-1}\sum_{t=1}^{T}\widehat{u}_{t-1}'\Sigma^{-1}\Delta\widehat{u}_t \\ &= O_p(N^{1/2})\end{aligned}$$

and

$$\begin{aligned}\tilde{e}_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T}\zeta_{t-1}'\zeta_{t-1} \\ &= N^{-1}T^{-2}\sum_{t=1}^{T}\widehat{u}_{t-1}'\Sigma^{-1}\widehat{u}_{t-1} \\ &= O_p(T^{-1}).\end{aligned}$$

It follows that $t_{MP} = \tilde{d}_{NT}/\sqrt{\tilde{e}_{NT}}$ is $O_p(N^{1/2}T^{1/2})$.

Case (c). Using

$$\begin{aligned}d_{NT} = N^{-1/2}T^{-1}\sum_{t=1}^{T}y_{t-1}'\widehat{\Omega}^{-1}\Delta y_t &= N^{-1/2}T^{-1}\sum_{t=1}^{T}y_{t-1}'\Omega^{-1}\Delta y_t + o_p(1) \\ &= \sum_{i=1}^{N}N^{-1/2}T^{-1}\sum_{t=1}^{T}z_{i,t-1}\Delta z_{it} + o_p(1),\end{aligned}$$

where $z_{1t}, \ldots, z_{rt}$ are $I(0)$ and $z_{r+1,t}, \ldots, z_{Nt}$ are $I(1)$, we obtain as $T \to \infty$ is followed by $N \to \infty$

$$d_{NT} \Rightarrow \mathcal{N}(0, 1/2)$$

and

$$\begin{aligned}e_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T}y_{t-1}'\widehat{\Omega}^{-1}y_{t-1} \\ &= N^{-1}T^{-2}\sum_{t=1}^{T}y_{t-1}'\Omega^{-1}y_{t-1} + o_p(1) \\ &= N^{-1}T^{-2}\sum_{t=1}^{T}z_{t-1}'z_{t-1} + o_p(1) \\ &\xrightarrow{p} 1/2\end{aligned}$$

It follows that t_{gls} has a standard normal limiting distribution.

Similarly, we obtain

$$\begin{aligned}\tilde{d}_{NT} &= N^{-1/2}T^{-1}\sum_{t=1}^{T}\zeta'_{t-1}\Delta\zeta_t \\ &= N^{-1/2}T^{-1}\sum_{t=1}^{T}\widehat{u}'_{t-1}\Sigma^{-1}\Delta\widehat{u}_t \\ &\Rightarrow \mathcal{N}(0,1/2)\end{aligned}$$

and

$$\begin{aligned}\tilde{e}_{NT} &= N^{-1}T^{-2}\sum_{t=1}^{T}\zeta'_{t-1}\zeta_{t-1} \\ &= N^{-1}T^{-2}\sum_{t=1}^{T}\widehat{u}'_{t-1}\Sigma^{-1}\widehat{u}_{t-1} \\ &= N^{-1}T^{-2}\sum_{t=1}^{T}u'_{t-1}\Sigma^{-1}u_{t-1} + o_p(1) \\ &\xrightarrow{p} 1/2\end{aligned}$$

as $T \to \infty$ is followed by $N \to \infty$. Thus, $t_{MP} = \tilde{d}_{NT}/\sqrt{\tilde{e}_{NT}} \Rightarrow \mathcal{N}(0,1)$. ■

4 Small sample performance

In this section, we present the results of some Monte Carlo experiments performed to investigate the finite sample performance of the four tests analyzed in Sections 2 and 3, i.e., the pooled OLS t-statistic, the robust test (t_{rob}), the GLS t-statistic and the test based on the transformation suggested by Moon and Perron [MP] (2004). In addition we have included the GLS statistic that imposes the factor structure by using the (estimated) inverse covariance matrix given in (10). This test statistic is indicated by t^*_{gls}.

In our Monte Carlo simulations, the data are generated as in (1) – (3) with $r = 1$ (the single factor model). All starting values are set equal to zero and v_t is drawn from a $\mathcal{N}(0,1)$ distribution. The factor loadings γ_i are drawn from a U(0,2) distribution and the error vector $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \dots \varepsilon_{Nt}]'$ is independently drawn from a $\mathcal{N}(0,\Sigma)$ distribution. The covariance matrix of the idiosyncratic components is set to $\Sigma = I_N$.

Table 1 reports the actual sizes of the tests, where both components are generated as random walks, that is, $\theta = 1$ and $\rho = 1$. This is equivalent to case (a) in sections 2 and 3. Our theoretical findings suggest that in this situation the OLS based test statistics are biased, whereas the GLS type test statistics are asymptotically valid. These results are confirmed by the simulation results presented in Table 1. Theorem 1 implies that $N^{-1/2}t_{ols}$ is asymptotically distributed as the Dickey-Fuller t-statistic. Thus, the actual size tends to the probability

$$\begin{aligned}\lim_{T,N\to\infty} P(t_{ols} < -1.645) &= \lim_{N\to\infty} P(t_{ols}/\sqrt{N} < -1.645/\sqrt{N}) \\ &= P(\tau < 0),\end{aligned}$$

where τ is distributed as $\int W(a)da(a)/\sqrt{\int W(a)^2 da}$. Since the distribution of the Dickey-Fuller t-statistic is skewed, this limiting probability is somewhat larger than 0.5. Indeed the results of our Monte Carlo simulations suggest that the actual size of the OLS t-statistic tends to a value (slightly) larger than 0.5.

From Theorem 1 it also turns out that in a single-factor model the robust t-statistic is distributed as the Dickey-Fuller t-statistic. The 5% critical value of this distribution is -1.95 compared to to the critical value of -1.645 for the standard normal distribution. Therefore, we expect a moderate positive size bias of the robust OLS t-statistic. Indeed, our Monte Carlo simulations suggest that this test statistic slightly over-rejects as T becomes large.

As suggested by our Monte Carlo simulations, the actual size of the GLS type test statistics that impose the factor structure (MP and t^*_{gls}) are close to the nominal size. In contrast, the original GLS t-statistic is severely biased if T is of a similar magnitude as N. The results suggest that the nominal size is attained when T is at least 10 times larger than N.

Table 2 presents the actual sizes of the test if the panel units are cross-cointegrated (case b). As stated in Theorem 1 and 2, all tests are severely biased in this case. The robust OLS test (t_{ols}) has the smallest size bias among all tests. From Theorem 2 it turns out that the GLS type statistics tend to $-\infty$ and, therefore, the actual size tends to one. Indeed, the results of the Monte Carlo study suggest that as N and T becomes large, the actual size of the GLS type statistics are close to one.

The theoretical considerations of sections 2 and 3 suggest that in case (c) the actual sizes of the OLS based statistics tend to one, whereas the actual sizes of the GLS statistics tend to the nominal ones as N and T tend to infinity. The results of our Monte Carlo simulation presented in Table 3 confirm this theoretical findings. However, the size bias of the t_{GLS} statistic converges rather slowly to zero as T tends to infinity.

5 Empirical illustration

In this section we examine whether nominal interest rates are stationary or have unit roots. Whether shocks to the interest rates are permanent or transitory is not only essential in understanding the nature of shocks to interest rates but might have important implications for the monetary authority especially with regard to a stabilization policy. Several macroeconomic models suggest presence of unit roots in nominal interest rate. A long run (cointegration) relationship among various interest rates of different maturities also requires that interest rates are unit root process. Employing a univariate framework most studies found that unit root hypothesis for interest rates cannot be rejected. See, for example, Perron (1989), Rose (1988), and Stock and Watson (1988). However, as we have also mentioned in the introduction, such non-rejection of unit root hypothesis might be due to lack of power of standard univariate unit root tests. As multivariate tests based on cross-section data provides a substantial power gain, it might be worth examining the hypothesis of unit roots in interest rates in a panel unit root framework. As shown by Wu and Zhang (1996), there is a significant contemporaneous correlation among OECD country's interest rates. Therefore, it is plausible to consider panel unit root tests which are robust to such cross sectional dependence.

We have used a panel data set of annual short term (3 months) interest rates of 16 developed countries. The sample ranges from 1980 to 2003. Data has been collected from the Datastream database. Table 6 provides the various tests considered in this paper along with Levin-Lin-Chu (LLC) and Im et al. (IPS) tests. All tests strongly suggest that interest rates are stationary.[3] As expected, the LLC and IPS test show stronger rejections than the tests which are robust to cross-sectional dependence. Our conclusion that interest rates are stationary is consistent with the findings of Wu and Zhang (1996). We thus cast doubts on previous studies relying on the non-stationarity of interest rates. In particular, the evidences of a long run relationship among various interest rates with different maturities and long run Fisher relationship may not be considered as appropriate.

[3] We do not report results of the GLS t-statistic as in our application T is not sufficiently large relative to N.

6 Conclusions

In this paper we analyze the size properties of various panel unit root statistics in three possible situations. First, the common factors and idiosyncratic components may both be nonstationary. In this case all GLS type test statistics possess a standard normal limiting distribution, whereas the OLS based test statistics are invalid. If the common component is $I(1)$ and the idiosyncratic component is nonstationary (the case of cross-unit cointegration), then both the OLS and the GLS statistics fail. Finally, if the idiosyncratic components are $I(1)$ but the common factors are stationary, then the OLS based test statistics are not applicable, whereas the GLS type statistics are at least asymptotically valid. These findings are confirmed by our Monte Carlo simulations.

As an important result our analysis suggest that the OLS and GLS based test statistics may give misleading results if the panel units are cross-cointegrated. Whereas Banerjee et al. (2005) demonstrate that the first generation panel unit root tests that ignore a possible cross-section dependence may be severely biased we have shown that this also the case if second generation panel unit root tests are applied that account for a possible cross-section dependence. The only approach that is able to deal with this situation is the PANIC approach suggested by Bai and Ng (2004). Their test procedure is based on a separate analysis of the common and idiosyncratic components that are estimated by using principal components.

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Table 1: Size: both components are I(1)

N	T	OLS	t_{rob}	MP	t_{gls}	t^*_{gls}
10	20	24.3	6.32	6.01	17.4	6.44
	50	23.6	7.23	6.85	10.4	6.80
	100	23.4	7.77	6.75	8.04	6.46
20	20	34.6	4.99	5.58	—	5.46
	50	35.9	6.82	5.55	18.6	5.49
	100	34.6	7.64	6.07	11.3	5.94
50	20	48.8	4.07	4.88	—	5.10
	50	50.1	6.53	5.37	—	5.59
	100	50.1	7.83	5.22	33.7	5.22

Note: t_{ols} is the pooled OLS t-statistic that ignores cross-section dependence, t_{rob} is the robust OLS t-statistic as defined in (6), MP indicates the test statistic based on the approach suggested by Moon and Perron (2004), t_{gls} is the GLS t-statistic and t^*_{gls} is the GLS statistic which impose the factor structure on the covariance matrix. The nominal size for all tests is 5%.

Table 2: Size: $f_t \sim I(1)$ and $u_t \sim I(0)$

N	T	OLS	t_{rob}	MP	t_{gls}	t^*_{gls}
10	20	35.7	11.2	16.7	31.8	16.5
	50	50.0	20.4	50.2	49.9	46.8
	100	56.5	26.9	93.5	81.9	83.6
20	20	49.1	12.1	26.0	—	26.0
	50	59.2	22.0	80.3	72.0	76.6
	100	63.9	28.6	99.9	96.6	98.3
50	20	59.8	12.3	46.9	—	47.1
	50	66.1	24.0	98.9	—	97.9
	100	70.4	29.5	1.00	98.2	1.00

Note: The idiosyncratic components are generated by an AR(1) process with autoregressive coefficient $\theta = 0.98$. The nominal size for all tests is 5%. See Table 1 for further details.

Table 3: Size: $f_t \sim I(0)$ and $u_t \sim I(1)$

N	T	OLS	t_{rob}	MP	t_{gls}	t^*_{gls}
10	20	54.7	21.4	6.01	39.5	9.15
	50	64.2	43.5	7.22	24.9	11.0
	100	67.3	57.1	6.56	18.0	10.4
20	20	74.9	21.7	5.11	—	7.23
	50	85.5	54.9	5.89	54.7	8.19
	100	88.0	73.7	5.60	32.2	8.12
50	20	91.5	20.7	4.39	—	5.35
	50	98.6	67.2	5.27	—	6.37
	100	99.5	92.2	5.06	92.0	6.27

Note: The common factor is generated by an AR(1) process with autoregressive coefficient $\rho = 0.8$. See Table 1 for further details.

Table 4: Power: $f_t \sim I(0)$ and $u_t \sim I(0)$

N	T	OLS	t_{rob}	MP	t_{gls}	t^*_{gls}
10	20	41.9	13.3	18.0	37.2	19.3
	50	67.7	30.7	52.3	62.9	56.4
	100	89.4	53.8	94.1	96.0	96.0
20	20	56.6	13.9	26.2	—	27.7
	50	78.9	34.1	81.6	85.4	83.5
	100	94.1	57.3	99.9	99.8	100
50	20	68.4	14.6	48.0	—	49.4
	50	85.7	35.8	99.5	—	99.5
	100	96.7	59.6	100	100	100

Note: The common and idiosyncratic components are generated by AR(1) processes with autoregressive coefficients $\theta = 0.98$ and $\rho = 0.98$. See Table 1 for further details.

Table 5: Panel Unit Root Test Statistics for Interest Rates

	LL	IPS	t_{rob}	MP	t^*_{gls}
Tests	–5.87*	–5.23*	–3.08*	–5.67*	–5.42*

Note: LL, IPS, t_{rob}, DPC, MP, denote the t-statistics corresponding to Levin et al (2002), Im et al (2003), the robust OLS t-statistic, the Moon-Perron test with $r = 1$ and the GLS t test imposing a single-factor structure (t^*_{gls}), respectively. The GLS t-statistic has not been considered as T is not sufficiently large relative to N.