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SINGLE SAMPLING THREE-DECISION PLAN BY ATTRIBUTES PROVIDING AVERAGE QUALITY PROTECTION

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SUMMARY. Single sampling three-decision plan by attributes providing average quality protection is discussed. Two types of three-decision criteria (i) accept-screen-reject outright (ASR plans) and (ii) accept-screen-accept with penalty to supplier (ASP plans) are considered. Some illustrative numerical example for construction of such plans are given. For ASR plans a characterisation theorem is proved and alternative ways of characterisation is indicated. It is shown that the three-decision plans involve smaller average amount of inspection as compared with the 'corresponding' Dodge and Romig's plans.

1. Introduction

Single sampling plans by attributes with three-decision criteria providing lot quality protection for specified values of consumer's risk expressed in terms of probabilities of misclassification were discussed by Pandey *et al.* (1972). The plans provided minimum average amount of inspection at process average quality. Single sampling three-decision plans minimising average total cost with discrete and continuous prior distributions for the incoming lot quality were discussed by Pandey (1974a) and Pandey (1973) respectively. It was empirically shown in Pandey (1974b) that the three-decision plans involve smaller average total cost as compared with the two-decision plans. Motivated by the three-decision plans providing lot quality protection (Pandey *et al.* 1972) single sampling plans by attributes with three-decision providing average quality protection are discussed here. Two types of three-decision criteria are considered : (i) criteria with accept-screen-outright reject type of decision (ASR plans) as introduced in Section 2 and discussed in Sections 3 and 4 and (ii) criteria with accept-screen-accept with a penalty imposed on the supplier type of decision as discussed in Section 5 under ASP plans. A characterisation theorem based on point of inflexion and other related parametres is proved in Section 3 and a comparison of ASR plans and 'corresponding' Dodge and Romig's plan is attempted in Section 5.

2. The three-decision criteria

The concept of three-decision in sampling inspection was introduced by Pandey *et al.* (1972). For the purpose of developing plans in this paper the three-decision criteria D, as described by Pandey (1974a, pp. 349), i.e., classify the lots in three quality grades A, B and C and accept grade A as good lot, screen grade B lots and reject a grade C lot out right, is used. Single sampling plan by attributes with three-decision criteria D is defined by the parameters n, c_1 and c_2 and is operated as follows :

Take a random sample of n from N and let x be the number of defectives in the sample then

$$\begin{aligned} &\text{accept if} \quad 0 \leqslant x \leqslant c_1 \\ &\text{screen if} \quad c_1 < x \leqslant c_2 \\ &\text{reject if} \quad c_2 < x \leqslant n. \end{aligned} \qquad \ldots \quad (1)$$

We shall refer to such a plan as ASR plans. The plans discussed here are applicable in those situations where

(1) items are inspected on lot-by-lot basis,

(2) inspection does not involve any destructive or very costly testing, i.e., 100% inspection is permissible,

(3) any defective item if detected can be either replaced with a good item or rectified,

(4) quality protection is expressed in terms of average outgoing quality limit (AOQL).

3. The operating characteristic curve

For incoming lot quality p the probability of acceptance $P_a(p)$, probability of screening $P_s(p)$ and probability of rejection $P_r(p)$ under an ASR plan are defined as follows :

$$P_a(p) = B(c_1; n, p) \qquad \ldots \quad (2)$$

$$P_s(p) = B(c_2; n, p) - B(c_1; n, p) \qquad \ldots \quad (3)$$

$$P_r(p) = 1 - B(c_2; n, p) \qquad \ldots \quad (4)$$

where $$B(c; n, p) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}.$$

For p not exceeding 0.10 Poisson approximation to $B(c; n, p)$ is used by Dodge and Romig (1959). All the results of this article are based on Poisson approximation of the binomial distribution.

We shall use the following notations :

p_s^* : p-coordinate of the maximum ordinate of the curve of $P_s(p)$ against p.

p_a^0 : p-coordinate of the point of inflexion of $P_a(p)$. The corresponding values for $P_s(p)$ and $P_r(p)$ can similarly be defined.

A suitable modification of these notations may be made in cases where more than one point of inflexion exist. However, such modifications are not relevant to p_a^0 since the point of inflexion of $P_a(p)$ is unique.

p_a^t : the point where the tangent at the point of inflexion of $P_a(p)$ cuts the p-axis ;

S_a^0 : slope of $P_a(p)$ at its point of inflexion ;

p_A^0 : p-coordinate of the point of inflexion (which is unique) of $pP_a(p)$;

S_A^0 : slope of $pP_a(p)$ at its point of inflexion.

Using these notations we shall prove the following characterisation theorem.

Theorem 1 : *The acceptance sampling three-decision plan by attributes is completely characterised by p_a^0, p_a^t and p_s^* points on its OC curve.*

Proof: Let (n, c_1, c_2) and (n', c_1', c_2') be any two plans with values of p_a^0, p_a^t and p_s^* and $p_a^{0'}$, $p_a^{t'}$ and $p_s^{*'}$ respectively. It is enough to show that $p_a^0 = p_a^{0'}$, $p_a^t = p_a^{t'}$ and $p_s^* = p_s^{*'}$ imply $n = n'$, $c_1 = c_1'$ and $c_2 = c_2'$. From $p_a^0 = p_a^{0'}$ we get

$$\frac{c_1}{n} = \frac{c_1'}{n'} \qquad \dots \quad (5)$$

and $p_a^t = p_a^{t'}$ gives

$$\frac{c_1}{n} + \frac{c_1!}{n c_1^{c_1}} \sum_{r=0}^{c_1} \frac{(c_1)^r}{r!} = \frac{c_1'}{n'} + \frac{c_1'!}{n' c_1'^{c_1'}} \sum_{r=0}^{c_1'} \frac{(c_1')^r}{r!}. \qquad \dots \quad (6)$$

For p_s^* and $p_s^{*'}$ we get the relations

$$(n\, p_s^*)^{c_2 - c_1} = \frac{c_2!}{c_1!} \qquad \dots \quad (7)$$

and
$$(n' p_s^{*'})^{c_2'-c_1'} = \frac{c_2'!}{c_1'!} \qquad \ldots \quad (8)$$

From (5) and (6) we get

$$\frac{(c_1-1)!}{c_1^{c_1}} \sum_{r=0}^{c_1} \frac{(c_1)^r}{r!} = \frac{(c_1'-1)!}{c_1'^{c_1'}} \sum_{r=0}^{c_1'} \frac{(c_1')^r}{r!} \qquad \ldots \quad (9)$$

We shall show that $f(c_1) = \frac{(c_1-1)!}{c_1^{c_1}} \sum_{r=0}^{c_1} \frac{(c_1)^r}{r!}$ is one-to-one strictly decreasing function of c_1. Write

$$f(c_1) = \sum_{r=0}^{c_1} \frac{c_1!}{r!} (c_1)^{r-c_1-1}$$

To prove that $f(c_1) > f(c_1+1)$ for any positive integer c_1.

Consider
$$f(c_1) = \frac{c_1!}{c_1^{c_1+1}} \sum_{r=0}^{c_1} \frac{c_1^r}{r!}$$

$$= \frac{c_1! e^{c_1}}{c_1^{c_1+1}} \sum_{r=0}^{c_1} e^{-c_1} \frac{c_1^r}{r!}$$

$$= \frac{e^{c_1} \; \Gamma(c_1+1)}{c_1^{c_1+1}} \int_{c_1}^{\infty} \frac{e^{-x} x^{c_1} \; dx}{\Gamma(c_1+1)}$$

$$= \frac{e^{c_1}}{c_1^{c_1+1}} \int_{c_1}^{\infty} e^{-x} x^{c_1} \; dx$$

$$= e^{c_1} \int_1^{\infty} e^{-c_1 x} x^{c_1} \; dx, \qquad \ldots \quad (10)$$

by making suitable transformation of variables.

Now consider $f(c_1)$ to be defined for all real $c_1 > 0$. If we can prove that $f'(c_1) < 0$ for all c_1, it will imply, in particular, that $f(c_1) > f(c_1+1)$ for integral values of c_1.

Now,

$$f'(c_1) = e^{c_1} \int_1^\infty e^{-c_1 x}\, x^{c_1}\, dx + e^{c_1}\left[\int_1^\infty \{e^{-c_1 x}\, x^{c_1} \log x - e^{-c_1 x} x^{c_1+1}\} dx\right]$$

$$= e^{c_1}\left[\int_1^\infty e^{-c_1 x}\, x^{c_1}\, \{\log x - (x-1)\} dx\right]$$

Denote by $\phi(x)$ the function $\log x - (x-1)$

$$\frac{d\phi(x)}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x} < 0 \text{ for } x > 1,$$

i.e., $\phi(x)$ is a decreasing function of x for $x > 1$. But $\phi(1) = 0$. Hence $\phi(x) < 0$ for $1 < x \leqslant \infty$. It implies that $\int_1^\infty e^{-c_1 x} x^{c_1} \{\log x - (x-1)\} < 0$, i.e., $f'(c_1) < 0$ and therefore $f(c_1) > f(c_1+1)$ which was required to be proved.

Further, $\lim_{c_1 \to 0} f(c_1) = \infty$ and $\lim_{c_1 \to \infty} f(c_1) = 0$ showing that $f(c_1)$ is strictly decreasing one-to-one function of c_1 (Figure 1).

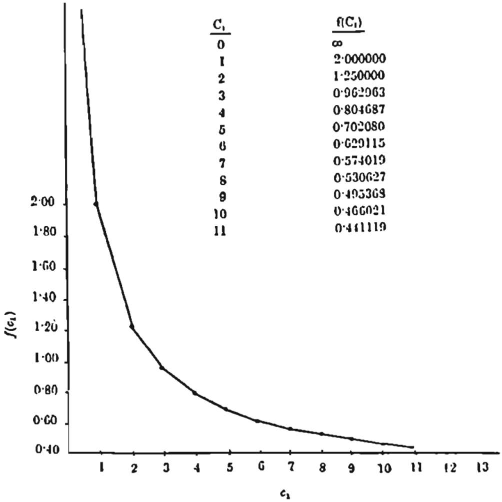

Figure 1. Curve for $f(c_1)$ against c_1.

Since

$$\frac{(c_1-1)!}{c_1^{c_1}} \sum_{r=0}^{c_1} \frac{(c_1)^r}{r!}$$

is strictly decreasing one-to-one function of c_1 there can not exist two integral values of c_1 for which (9) holds and hence we get $c_1 = c_1'$ which amounts to $n = n'$ from (5). Further from (5), (7), (8) and $p_s^* = p_s^{*'}$ we get

$$\ln k = \frac{1}{c_2'-c_1'} \ln \frac{c_2'!}{c_1'!} - \frac{1}{c_2-c_1} \ln \frac{c_2!}{c_1!} \qquad \ldots \quad (11)$$

where $k = c_1'/c_1$ and ln denotes natural logrithm. Since $c_1' = c_1$, (11) implies

$$\frac{1}{c_2'-c_1'} \ln \frac{c_2'!}{c_1'!} = \frac{1}{c_2-c_1} \ln \frac{c_2!}{c_1!} \qquad \ldots \quad (12)$$

which means $c_2' = c_2$ proving the result.

Corollary : *The acceptance sampling three-decision plan by attributes is completely characterised by any one of the following combinations of parameters*

(a) p_a^0, S_a^0 *and* p_s^*

(b) p_A^0, S_A^0 *and* p_s^*

The proof of this corollary is similar to Theorem 1 and is omitted.

4. The optimal plans

Let p denote the incoming lot quality. Assuming $p \leqslant .10$ which is a reasonable assumption for most of the industrial situations, the average amount of inspection for ASR plan can be written as

$$I = n+(N-n) \sum_{c_1+1}^{c_2} e^{-np}(np)^r/r\,! \qquad \ldots \quad (13)$$

Since under ASR plan the rejected lots do not contribute to the average outgoing quality after inspection, p_A is given by

$$p_A = p\,\frac{N-I_1}{N} \qquad \ldots \quad (14)$$

when the defectives found in the sample are replaced except in the case of rejection and where I_1 is the average amount of inspection conditional that the lot is either accepted or screened, i.e.,

$$I_1 = \frac{nP_a(p)+NP_s(p)}{P_a(p)+P_s(p)}.$$

The average outgoing quality limit (p_L) is the maximum value of p_A that will result under any sampling plan, considering all possible values of p in the submitted product. The value of p for which this maximum value of p_A occurs is designated as p_m.

Differentiating (14) with respect to p and equating to zero we get

$$\frac{dp_A}{dp} = \frac{N-I_1}{N} - \frac{p}{N}\frac{dI_1}{dp} = 0. \quad \ldots \quad (15)$$

Substituting for I_1 in (15) we get

$$P_a^2(p)+P_a(p)P_s(p) = npe^{-np}\frac{(np)^{c_1}}{c_1!}\left(P_a(p)+P_s(p)\right)-e^{-np}\frac{(np)^{c_2+1}}{c_2!}P_a(p) \quad \ldots \quad (16)$$

Let the solution be $p = p_m$ and then denoting $np_m = x$

$$p_L = \left(\frac{1}{n}-\frac{1}{N}\right) y \quad \ldots \quad (17)$$

where
$$y = x\sum_{r=0}^{c_1} e^{-x}\frac{x^r}{r!}\Big/\sum_{r=0}^{c_2} e^{-x}\frac{x^r}{r!}$$

From (16) we get

$$\left(\sum_{r=0}^{c_1} e^{-x}\frac{x^r}{r!}\right)\left(\sum_{r=0}^{c_2} e^{-x}\frac{x^r}{r!}\right)$$

$$= e^{-x}\frac{x^{c_1+1}}{c_1!}\sum_{r=0}^{c_2} e^{-x}\frac{x^r}{r!} - e^{-x}\frac{x^{c_2+1}}{c_2!}\sum_{r=0}^{c_1} e^{-x}\frac{x^r}{r!} \quad \ldots \quad (18)$$

Solving (18) by Newton-Raphson method of approximation we obtain the solution x, the value of y for the values of $c_1 = 0$ to 20 as given in Table 1. The smallest feasible value of c_2 is taken corresponding to a given c_1 in (18).

TABLE 1. THE VALUES OF x SATISFYING (18) AND CORRESPONDING VALUES OF y

c_1	c_2	x	y	c_1	c_2	x	y
0	2	1.414213	0.414214	11	13	11.215362	8.158714
1	3	2.230375	0.952182	12	14	12.140244	8.953887
2	4	3.092727	1.558027	13	15	13.068499	9.750142
3	5	3.965123	2.208438	14	16	14.000797	10.564794
4	6	4.849036	2.891019	15	17	14.930630	11.379249
5	7	5.742012	3.598241	16	18	15.864714	12.199019
6	8	6.641530	4.325121	17	19	16.800401	13.023669
7	9	7.547868	5.068162	18	20	17.737896	13.852822
8	10	8.459299	5.824794	19	21	18.677091	14.686149
9	11	9.374142	6.593061	20	22	19.617890	15.523357
10	12	10.292835	7.371440				

The value of c_1 giving minimum value of I at a specified process average p is given by

$$\Delta \bar{I}(c_1-1) \leqslant 0 < \Delta \bar{I}(c_1) \qquad \dots \quad (19)$$

where $\bar{I}$ denotes the value of I when $p = \bar{p}$. For a specified values of process average ($\bar{p}$), AOQL (p_L) and the lot size (N), the plan (n, c_1, c_2) which minimises $\bar{I}$ would be the optimal plan under consideration. For determination of such plans, the relevant equations and conditions are (17), (18) and (19).

Let $\bar{M} = N\bar{p}$, $a = n\bar{p}$ and for a given c_1 the corresponding values of c_2, a and y be denoted by $c_2^{(c_1)}$, a_{c_1} and y_{c_1} respectively. Further let $\bar{p}/p_L$ be denoted as k. Consider the $\bar{M}$, k plane. For given value of AOQL $= p_L$, for any pair ($\bar{M}$, k), a particular pair (c_1, a_{c_1}) can be found which makes $\bar{I}$ a minimum. Since the acceptance number c_1 assumes only discrete values minimum value of $\bar{I}$ will be found for many pairs ($\bar{M}$, k) for the same value of c_1. From this it is evident that on an $\bar{M}$, k plane there exist zones in which the acceptance numbers are identical. To find the boundary lines of these zones it was noted as by Dodge and Romig (1959) that for certain pairs ($\bar{M}$, k) two pairs of (c_1, a_{c_1})

exist, giving the same minimum value of $\bar{I}$. These values of c_1 were found to differ by 1 in all such cases. The boundary point for any two adjacent zones would be given under the optimal condition (19) as

$$\bar{M} = \frac{(a_{c_1+1}-a_{c_1}) + a_{c_1}\sum_{c_1+1}^{c_2^{(c_1)}} e^{-a_{c_1}}\frac{(a_{c_1})^r}{r!} - a_{c_1+1}\sum_{c_1+2}^{c_2^{(c_1+1)}} e^{-a_{c_1+1}}\frac{(a_{c_1+1})^r}{r!}}{\sum_{c_1+1}^{c_2^{(c_1)}} e^{-a_{c_1}}\frac{(a_{c_1})^r}{r!} - \sum_{c_1+1}^{c_2^{(c_1+1)}} e^{-a_{c_1+1}}\frac{(a_{c_1+1})^r}{r!}} \quad \ldots \quad (20)$$

Thus to obtain optimal plan (n, c_1, c_2) one may consider c_1, c_2 and y satisfying (18), i.e., from Table 1 and then by a process of iteration boundary points for the adjacent zones on $\bar{M}$, k plane can be obtained, subsequently sketching all the zones each having identical value of c_1. This chart on the $\bar{M}$, k plane will give the value of c_1 minimising $\bar{I}$ for specified p/p_L and lot size (N). The corresponding value of n is to be determined from (17).

The iterative procedure to obtain boundary point on $\bar{M}$, k plane is given below :

Step 1 : Choose some arbitrary value of c_1 and hence c_1+1.

Step 2 : Assume N as infinite, obtain n_{c_1} and n_{c_1+1} and hence a_{c_1} and a_{c_1+1} from (17) using the values y_{c_1} and y_{c_1+1} from Table 1.

Step 3 : Obtain the value of $\bar{M}$ and hence N from (20) using chosen values of c_1 and c_1+1 and corresponding values of $c_2^{(c_1)}$, $c_2^{(c_1+1)}$ and the values a_{c_1} and a_{c_1+1} as obtained in Step 2.

Step 4 : From (17) using N as obtained in Step 3 and the values of y_{c_1} and y_{c_1+1}, obtain new values of a_{c_1} and a_{c_1+1}.

Step 5 : Again obtain N from (20) using now new values of a_{c_1} and a_{c_1+1} obtained in Step 4.

Step 6 : Obtain more accurate values of a_{c_1} and a_{c_1+1} from (17) using N as obtained in Step 5.

Step 7 : Terminate the iteration for a fixed $k = p/p_L$ when at two successive stages of iterations the solutions are almost identical.

For example, let $\bar{p} = 0.005$, $p_L = 0.050$. For $k = \bar{p}/p_L = 0.100$ and $c_1 = 0, 1, 2, 3$, and 4 and using the corresponding values of y from the table of x and y (Table 1) the boundary points $\bar{M}$ for the adjacent zones were obtained as given in Table 2 under above procedure.

TABLE 2. THE VALUES OF $\bar{M}$ ON THE BOUNDARIES FOR ASR PLANS

c_1	c_1+1	$\bar{M}$	y
0	1	1.404712	0.414214
1	2	16.389320	0.952182
2	3	136.381060	1.558027
3	4	978.750790	2.208438
4	5	6430.144500	2.891019

Example 1 : For illustration the optimal plan for $c_1 = 0$; $\bar{p} = 0.005$, $\bar{p}_L = 0.05$ and $y = 0.414214$ is obtained below :

The value of $\bar{M}$ corresponding to $k = 0.100$ from Table 2 is 1.404712 for $c_1 = 0$ and $c_1+1 = 1$. The value of the lot size at the boundary of the zone for $c_1 = 0$ is obtained by dividing the value of $\bar{M}$ by the value of $\bar{p}$, i.e.,

$$N = \frac{1.404712}{.005} = 280.94.$$

Using (17) we got the value of sample size as 8.05. The average lot size for the $c_1 = 0$ zone is taken as 144 and the lot range as 8–280.

Similarly, proceeding successively for $c_1 = 1, 2, 3$ and 4 and using the values from Table 2 we obtain the optimal plans as given in Table 3.

TABLE 3. SOME ILLUSTRATIVE OPTIMAL ASR PLANS FOR $\bar{p} = 0.005$ AND $p_L = 0.05$

lot	$\bar{N}$	c_1	c_2	n
8 – 280	144	0	2	8
281 – 3277	1770	1	3	18
3278 – 27276	15277	2	4	31
27277 – 195750	111514	3	5	44
195751 – 1286028	740890	4	6	57

5. Special plan (ASP plans)

In the previous section we have considered ASR plans. From the applicability conditions (Section 2) it is apparent that the items are assumed to be in good supply. During a receiving inspection it may be possible to afford outright rejection of a lot under this three-decision criteria. But if suddenly demand increases manifold due to emergency like war, epidemic or famine it may not be possible to afford outright rejection of a lot. However, it may be desirable to discourage poor quality material by imposing some penalty to the supplier on the basis of a sampling inspection. In such cases the three-decision criteria under ASR plan may be modified as follows :

Take a random sample of n from N and if x denotes the number of defectives in n then

$$\begin{array}{ll} \text{accept if} & 0 \leqslant x \leqslant c_1 \\ \text{screen if} & c_1 < x \leqslant c_2 \\ \text{accept with penalty if} & c_2 < x \leqslant n \end{array} \qquad \ldots \quad (21)$$

We shall refer to the plan with this three-decision criteria as ASP plan. Some aspects of utilisation of the lots accepted with penalty to the supplier under an ASP plan need clarification at this stage. A lot so accepted may be either screened or used as such depending on whether a defective item is rendered completely unusable or it satisfies the functional requirements without hampering it considerably. Sometimes such a lot may be kept aside and may be used when actual non-availability occurs due to short supply. If supply is of stochastic nature and the distribution is known reasonably well, it may be worth investigating an optimal decision minimising regret or loss in respect of instantaneous or deferred use of such lots.

Assuming that the lots accepted with penalty under ASP plans are used as such without being screened the average amount of inspection is given by

$$I = n+(N-n) \sum_{c_1+1}^{c_2} e^{-np} (np)^r/r! \qquad \ldots \quad (22)$$

which is the same expression as (13).

Proceeding on the line of (15)—(17) of Section 4 we get

$$\frac{dp_A}{dp} = \frac{N-n}{N} \left[1 - \sum_{c_1+1}^{c_2} e^{-np} \frac{(np)^r}{r!} + e^{-np} \frac{(np)^{c_2+1}}{c_2!} - e^{-np} \frac{(np)^{c_1+1}}{c_1!} \right] = 0. \qquad \ldots \quad (23)$$

Solving (23) for p suppose we get the solution $p = p'$. Let $p'n = x$ then we get (24) and (25) analogous to expressions (17) and (18) as follows.

$$p_L = y\left(\frac{1}{n} - \frac{1}{N}\right), \text{ i.e., } n = \frac{Ny}{Np_L + y} \qquad \dots \quad (24)$$

where

$$y = x\left(1 - \sum_{c_1+1}^{c_2} e^{-x}\frac{x^r}{r!}\right)$$

$$1 - \sum_{c_1+1}^{c_2} e^{-x}\frac{x^r}{r!} = e^{-x}\,\frac{x^{c_1+1}}{c_1!} - e^{-x}\,\frac{x^{c_2+1}}{c_2!} \qquad \dots \quad (25)$$

Solving (25) by Newton-Raphson method for the values of $c_1 = 0$ to 21 we obtain the solution x, and the value of y as given in Table 4.

TABLE 4. THE VALUES OF x SATISFYING (25) AND THE CORRESPONDING VALUE OF y

c_1	c_2	x	y	c_1	c_2	x	y
0	3	1.500000	0.433158	11	15	9.646886	7.463453
1	4	1.000000	0.907877	12	16	10.494637	8.206485
2	6	2.444306	1.395127	13	17	11.348001	8.057475
3	7	3.183143	1.981920	14	18	12.206446	9.716324
4	8	3.944987	2.600902	15	19	13.069517	10.481834
5	9	4.724959	3.152097	16	20	13.936821	11.634342
6	10	5.510603	3.912070	17	21	14.808015	12.031571
7	11	6.320748	4.595737	18	22	15.682820	12.569263
8	12	7.144284	5.294427	19	23	16.534527	13.602765
9	13	7.970872	6.006078	20	24	17.442188	14.313414
10	14	8.805380	6.729958	21	25	18.326320	15.027411

Argueing as in Section 4 and proceeding on the identical lines we get the boundary points $\bar{M}$ for the adjacent zones on the $\bar{M}$, k plane under ASP plan for $c_1 = 0, 1, 2, 3, 4, 5$ as given in Table 5 when the same values of $\bar{p}$ and p_L as earlier are used.

Proceeding on the lines as in the previous section some illustrative optimal ASP plans are obtained and are given in Table 6.

TABLE 5. THE VALUES OF $\overline{M}$ ON THE BOUNDARIES FOR ASP PLANS

c_1	c_1+1	$\overline{M}$	y
0	1	1.1779	0.4332
1	2	14.0262	0.9079
2	3	166.5458	1.3951
3	4	1320.0289	1.9819
4	5	7926.4795	2.6009
5	6	90196.0000	3.1521

TABLE 6. SOME ILLUSTRATIVE OPTIMAL ASP PLANS

lot	$\overline{N}$	c_1	c_2	n
8 – 236	144	0	3	8
237 – 2805	1621	1	4	18
2806 – 33310	18058	2	6	28
33311 – 264007	148659	3	7	40
264008 – 1585296	924652	4	8	52
1585297 – 18039320	9812308	5	9	63

As mentioned these plans are only illustratives. An exhaustive table for such AOQL plans with three-decision criteria can be worked out following the steps given here.

6. Comparison with Dodge and Romig's plan

Given a Dodge and Romig's single sampling plan for a fixed value of the lot size (N), process average ($\bar{p}$) and AOQL (p_L) with sample size n and acceptance number c we shall define a 'corresponding' plan (n, c_1, c_2) with three-decision criteria where $c_1 = c$ and $c_2 > c_1$. Such 'corresponding' plans may be neither unique nor optimal in the sense of Dodge and Romig (DR). However, for a given plan with three-decision criteria a corresponding unique Dodge and Romig's optimal plan can be defined.

Theorem 2 : *For a given lot size (N), process average ($\bar{p}$) and average outgoing quality limit (AOQL) as p_L if there exists a DR plan (optimal) with AOI as I_1^0 and an optimal three-decision ASR plan with AOI as I_2^0 then $I_2^0 \leqslant I_1^0$.*

To prove this result the following lemma will be used.

Lemma : *For given lot size* (N), *process average* ($\bar{p}$) *and AOQL* (p_L) *if there exists an optimal Dodge and Romig (DR) plan* (n, c_1) *with AOI as* I_1^0 *and a corresponding three-decision ASR plan* (n, c_1, c_2), $c_1 < c_2$ *with AOI as* I_2 *then* $I_2 \leqslant I_1^0$.

Proof: Since

$$I_1^0 = n+(N-n) \sum_{c_1+1}^{n} e^{-n\bar{p}} (n\bar{p})^r/r!$$

and

$$I_2 = n+(N-n) \sum_{c_1+1}^{c_2} e^{-n\bar{p}}(n\bar{p})^r/r!$$

the result of the lemma follows immediately from

$$I_1^0 = I_2+(N-n) \sum_{c_2+1}^{n} e^{-n\bar{p}} (n\bar{p})^r/r!$$

as

$$(N-n) \sum_{c_2+1}^{n} e^{-n\bar{p}}(n\bar{p})^r/r! \geqslant 0.$$

By definition $I_2^0 \leqslant I_2$ which implies the result of Theorem 2.

Corollary : *For given lot size* (N), *process average* ($\bar{p}$) *and AOQL* (p_L) *if there exist Dodge and Romig (optimal) single sampling and three-decision optimal ASP plan with AOI equal to* I_1^0 *and* I_2^0 *respectively then* $I_2^0 \leqslant I_1^0$.

7. General remarks

The use of plans providing average quality protection in terms of AOQL has been questioned. AOQL has been criticised as an inadequate measure as it does not provide a sharp upper bound for average outgoing quality. Anscombe (1958) viewed AOQL as only statistician's guarantee and remarked that in practice it is not consumer's requirement. He emphasised the need for an alternative measure. So far no adequate alternative measure has been found. Hillier (1964) proposed average extra defective limit (AEDL) as an alternative measure to obtain unique plan in case of continuous sampling plan (CSP-1) of Dodge (1943). But AEDL is based on AOQL and is mainly a way out to obtain a unique CSP-1 plan (i.e., value of parameters i and f for given value of AOQL and incoming lot quality p) rather than an alternative measure. It would be of interest to examine inflexion average quality, i.e., value of PL(p) for $p = p_A^0$ as a sharper bound alternative to AOQL for a given plan with three-decision criteria.

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References

Anscombe, F. J. (1958): Rectifying inspection of a continuous output. *Jour. Amer. Stat. Assn.*, 53, 702-719.

Dodge, H. F. (1943): Sampling inspection plan for continuous production. *Ann. Math. Stat.*, 14, 264-279.

Dodge, H. F. and Romig, H. G. (1959): *Sampling Inspection Tables*, second edition, John Wiley and Sons, Inc. New York.

Golub, A. (1953): Designing single sampling inspection plans when the sample size is fixed. *Jour. Amer. Stat. Assn.*, 48, 278-288.

Hillier, F. S. (1964): New criteria for selecting continuous sampling plans. *Tech.*, 6, 161-178.

Pandey, R. J. and Chawdhary, A. K. (1972): Single sampling plan by attributes with three-decision criteria. *Sankhyā*, B, 34, 265-278.

Pandey, R. J. (1973): Bayesian single sampling plan by attributes with three-decision criteria for continuous prior distribution. *Tech. Report* No. Math.-Stat. 42/73, Indian Statistical Institute.

——— (1974a): Bayesian single sampling plan by attributes with three-decision criteria for discrete prior distribution. *Sankhyā*, B, 36, 348-390.

——— (1974b): A note on sampling plan by attributes with three-decision criteria and 'corresponding' Dodge and Romig's plan. *Tech. Report* No. Math.-Stat. 17/74, Indian Statistical Institute.

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