COMMUN. STATIST. -SIMULA., 19(2), 637-648 (1990)

ON TESTIMATING THE WEIBULL SHAPE PARAMETER

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Key Words and Phrases: testimator; preliminary test; shape and scale parameters; Weibull and extreme value distributions.

ABSTRACT

A well-known estimator for the shape parameter of a two-parameter Weibull distribution from a failure-censored sample involves a preliminary test of a null hypothesis concerning the parameter. Efficiency of the resulting testimator varies with the chosen level of significance of the test. We present an alternative procedure with a higher efficiency over this for each significance level under various circumstances. Appropriate choice of significance level is also discussed.

1. INTRODUCTION

We consider estimating the shape parameter β of the cumulative distribution function (cdf)

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right],\ 0 < x,\ \theta,\ \beta < \infty$$

for the random variable X said to have the Weibull (1939, 1951) distribution. For this, data are supposed to be provided as the first r ordered observations x_i, $i = 1, \ldots, r$ on X in a sample of size n obtained through a failure-censored life test. Equivalently, $y_i = \ell_n x_i$, $i = 1, \ldots, r$ (ℓ_n is natural logarithm) are the first r ordered observations on $Y = \ell_n X$ which follows the extreme-value distribution with a cdf

$$G(y) = 1 - \exp\left[-\exp\left(\frac{y-u}{b}\right)\right],$$

where $b = 1/\beta$, $u = \ell_n\theta$.

Writing $w_i = (y_i - u)/b$, $T_r = -\sum_1^r (y_i - y_r)$, $N = -E\sum_1^r (w_i - w_r)$, an unbiased estimator $\hat{b} = \frac{T_r}{N}$ for b is given by Bain (1972) who noted that $T = 2N\,\hat{b}/b$ approximately follows the chi-square distribution with 2N degrees of freedom (df). Then, $(N-1)/T_r$ and $(N-2)/T_r$ are respectively approximately unbiased and biased estimators of β with approximate variance $\beta^2/(N-2)$ and mean square error (MSE) $\beta^2/(N-1)$. Singh and Bhatkulikar (1977) considered applying the UMP (uniformly most powerful) level-α $(0 < \alpha < 1)$ test of the null hypothesis H_0: $\beta = 1$ against the one-sided alternative H: $\beta > 1$ using the statistic T_r noting that (i) X has the exponential distribution when $\beta = 1$ and that (ii) values of β below 1 are rare in practice. Based on this preliminary test they recommended the estimator $\tilde{\beta}$ for β given as

$$\tilde{\beta} = K\left(\frac{c}{T_r} - 1\right) + 1, \text{ if } \lambda_\alpha \leq 2T_r < \infty$$

i.e. if H_0: $\beta = 1$ is accepted at level of significance α

$$= \frac{c}{T_r}, \text{ otherwise.}$$

Here c is taken as either $(N-1)$ or $(N-2)$, $K(0 < K < 1)$ as a constant purported to control the value of $E(\tilde{\beta}-\beta)^2$ and λ_α is such that $\text{Prob}\left[\chi^2_{2N} < \lambda_\alpha\right] = \alpha$, i.e. λ_α is the lower $100\alpha\%$ point of the chi-square variable χ^2_{2N} with df 2N and the error in approximating the distribution of $2T_r$ under H_0 by that of χ^2_{2N} is neglected. Clearly the magnitude of the mean square error (MSE) $E(\tilde{\beta}-\beta)^2$ varies with α but Singh et al (1977) do not consider a criterion for the choice of α except illustrating numerically how the MSE varies with α. So, in what follows we consider three alternative approaches to probe into this problem of taking account of the effect of α on the MSE and thereby suggest a more efficient choice of an alternative testimator.

First, following Ohtani (1987) we propose the following alternative to the above preliminary test estimator (PTE). Our estimator for β is

$$\hat{\beta} = K\left(\frac{c}{T_r} - 1\right) + 1, \text{ if } L < 2T_r < \infty$$

$$= \frac{c}{T_r}, \text{ if } 0 < 2T_r < L$$

taking c as in Singh et al (1977), K and L as positive constants to be suitably chosen to control the magnitude of the MSE $E(\hat{\beta}-\beta)^2$. Following Sclove, Morris, and Radhakrishnan (1972) and Adke, Waikar, and Schuurmann (1987) we call it a testimator because the estimator $\hat{\beta}$ like $\tilde{\beta}$ involves a prior test. Though $\hat{\beta}$ is not essentially different from $\tilde{\beta}$ we show below numerically that L can be chosen in particular ways so that for several choices of r, K, and α for a fixed n illustrated by Singh et al (1977) the MSE($\hat{\beta}$) turns out less than the corresponding MSE($\tilde{\beta}$). This really emphasizes the need for appropriate rather than conventional choice of α in practice in employing Singh et al's (1977) estimator.

Recently, Pandey, Malik, and Srivastava (1989), modifying Singh et al's (1977), have given another testimator involving a test of H_0: $\beta = \beta_0$ (allowing β_0 different from 1) against the two-sided alternatives H': $\beta \neq \beta_0$ and studied the right choice of α considering a range of values of $p = \frac{\beta}{\beta_0}$. A modification of Pandy et al's (1989) for a possible improvement is being examined by us as a separate piece of investigation.

2. PERFORMANCE OF THE TESTIMATOR

Respectively to $c = (N-1)$, $(N-2)$, $\tilde{\beta}(\hat{\beta})$ will be written as $\tilde{\beta}_1(\hat{\beta}_1)$, $\tilde{\beta}_2(\hat{\beta}_2)$. Writing $J(a,p) = \frac{1}{\Gamma(p)} \int_a^{\infty} \exp(-t)t^{p-1}dt$, for $0 < p$, $a < \infty$, their biases and MSE's work out as

$$\text{Bias}(\tilde{\beta}_1) = \beta(K-1)\left[J(X_\alpha, N-1) - b\ J(X_\alpha, N)\right] \tag{2.1}$$

$$\text{MSE}(\tilde{\beta}_1) = \beta^2\left(\frac{1}{N-2} - (1-K^2)\left\{\frac{N-1}{N-2} J(X_\alpha, N-2) - 2\frac{Kb+1}{K+1} J(X_\alpha, N-1) - b^2\frac{(1-K-2\beta)}{1+K} J(X_\alpha, N)\right\}\right)$$

where $X_\alpha = \lambda_\alpha/2b$. Assuming $N > 2, \ldots$ (2.2)

Bias $(\hat{\beta}_1)$ and MSE($\hat{\beta}_1$) follow from (2.1), (2.2) replacing λ_α there by L. Bias and MSE formulae for $\tilde{\beta}_2$, $\hat{\beta}_2$ follow similarly but are not shown here to save space. Writing $\beta_1 = \beta/2$, noting

$$\frac{\partial}{\partial L} J\left(\frac{L}{2b}, p\right) = -\frac{1}{\Gamma(p)}\beta_1^p \exp\left(-L\ \beta_1\right) L^{p-1}$$

one gets $\frac{\partial}{\partial L}$ MSE($\hat{\beta}_1$) $= \beta_1^N \exp(-L\ \beta_1) L^{N-3}(1-K)$.

$$(4\beta_1 + K-1)\left[L-2(N-1)\right]\left[L - \frac{2(N-1)(k+1)}{4\beta_1 + (K-1)}\right] \Big/ \Gamma(N) \ .$$

Two roots of $\frac{\partial}{\partial L}$ MSE$(\hat{\beta}_1) = \mathbf{0}$ are

$$L_1 = 2(N-1)(K+1)/(4\beta_1 + K-1), \ L_2 = 2(N-1).$$

Writing

$$A = \beta_1^N (1-K)(4\beta_1 + K-1)\Big/\Gamma(N),$$

one has

$$\frac{\partial^2}{\partial L^2} \text{MSE}(\hat{\beta}_1) = - A \exp(-L\beta_1)\Big(L^{N-4}(L-L_1)(L-L_2)(L\beta_1-N+3)$$

$$- L^{N-3}(2L-L_1-L_2)\Big) .$$

To maintain parity with Singh et al (1977) we restrict to $\beta > 1$ for which $L_1 < L_2$. Noting that $\frac{\partial^2}{\partial L^2}$ MSE$(\hat{\beta}_1)\big|_{L=L_1} < 0$ and $\frac{\partial^2}{\partial L^2}$ MSE$(\hat{\beta}_1)\big|_{L=L_2} > 0$ it follows that MSE$(\hat{\beta}_1)$ has a local maximum at $L = L_1$ and a local minimum at $L = L_2$. Since it is not possible to find a global minimum of MSE$(\hat{\beta}_1)$ particularly because β is unknown we consider the following three ways of setting an appropriate L. Our findings for $\hat{\beta}_2$ are of course similar.

First we consider the minimax regret choice. The regret function for $\hat{\beta}_j$, following Ohtani (1987) is

$$\text{Regret } (\hat{\beta}_j) = \text{MSE}(\hat{\beta}_j) - \min\Big[\text{MSE}(\hat{\beta}_j)\big|_{L=0}, \ \text{MSE}(\hat{\beta}_j)\big|_{L=L_2}\Big]$$

for $j = 1, 2$. The minimax choice of L is that L which gives the smallest value among the largest of the regret values over variations in b. In practice one may calculate Regret $(\hat{\beta}_j)$ for various choices of b and L and get an approximate solution for the minimax value. Following this we find the minimax choice of L to be L_2 for $n = 20$, $r = 4, 6, 8$, $K = 2, 4, 6$ but do not show the details to save space. The value of α corresponding to this minimax is easy to obtain. Secondly, we consider the average minimum risk function approach. Following Ohtani (1987), the average risk function of $\hat{\beta}_1$ is taken as

$$\text{AR}(\hat{\beta}_1) = \int_1^\infty \left\{\text{MSE}(\hat{\beta}_1) - \min\Big[\text{MSE}(\hat{\beta}_1)\big|_{L=0}, \ \text{MSE}(\hat{\beta}_1)\big|_{L=L_2}\Big]\right\} d\beta.$$

To minimize this we solve the equation

$$0 = \frac{\partial}{\partial L} AR(\hat{\beta}_1) = \int_1^{\infty} \frac{\partial}{\partial L} MSE(\hat{\beta}_1)\, d\beta$$

$$= \left\{L-2(N-1)\right\} \int_{1/2}^{\infty} \beta_1^N \exp(-L\beta_1)\, L^{N-3}(1-K)(4\beta_1 + K-1)$$

$$\left\{L - \frac{2(N-1)(K+1)}{4\beta_1 + k-1}\right\} d\beta_1 \ .$$

So, one solution is $L = 2(N-1) = L_2$. Another root is difficult to find exactly and an approximation to that is numerically found not quite useful as it corresponds to α close to 1 indicating that the testimator is effectively the original estimator $(N-1)/T_r$, the shrinkage estimator $K\left(\frac{c}{T_r}-1\right) + 1$ hardly allowed to be used since the H_0: $\beta = 1$ is virtually rejected most of the time. Obviously the second course cannot lead to gain in efficiency. So we will take L_2 as the choice of L minimizing the average risk. It is easily verifiable likewise that $L = 2(N-2)$ gives the minimum average risk for $\hat{\beta}_2$. Thus the minimax and average risk minimization approaches coincide in this problem.

Thirdly, we consider a thumb rule keeping in view that (a) L_1 ($< L_2$) is a local maximal and L_2 is a local minimal point of $MSE(\hat{\beta}_1)$ and that (b) our purpose in this paper is to show that our testimator competes favorably with Singh et al's (1977) and hence we need to evaluate their performances under comparable circumstances keeping our procedure as simple as possible. So, we prescribe the following thumb rule for the choice of L especially because with this we achieve an appreciable gain in efficiency over Singh et al's (1977) in numerous situations illustrated in table II below.

Our thumb rule, to compare $\hat{\beta}_1$ against $\tilde{\beta}_1$ when α is the chosen level of significance for the latter, is

i) $L = \frac{1}{2}\lambda_\alpha$ if $\lambda_\alpha < L_1$

ii) $L = (\lambda_\alpha + L_2)/2$ if $L_1 < \lambda_\alpha < L_2$

and iii) $L = L_2$ if $\lambda_\alpha > L_2$.

Here the multiplier $\frac{1}{2}$ is rather arbitrary but we use it as it gives good gain in efficiency in the cases illustrated in Table II below. Some other positive

TABLE I

Showing efficiency of $\hat{\beta}_j$ (j = 1, 2) for the minimax regret and minimum average risk approaches.

n = 20										
			Efficiency $e(\hat{\beta}_1)$					Efficiency $e(\hat{\beta}_2)$		
			K					K		
r	L_2	b	.2	.4	.6	L_2	b	.2	.4	.6
	4.33 (α=.33)	.2	100.01	100.00	100.00	2.33 (α=.09)	.2	187.26	186.89	186.52
		.4	100.99	100.75	100.51		.4	217.17	208.97	200.99
4		.6	105.43	104.19	102.87		.6	296.26	263.32	234.02
		.8	111.89	109.47	106.63		.8	435.13	351.67	282.52
		1.0	116.66	114.28	110.52		1.0	605.06	471.91	345.28
	8.88 (α=.38)	.2	100.00	100.00	100.00	6.88 (α=.20)	.2	129.06	129.06	129.05
		.4	100.33	100.25	100.17		.4	132.52	131.67	130.81
6		.6	105.43	104.15	102.81		.6	160.76	152.45	144.36
		.8	118.75	114.57	109.96		.8	233.22	202.41	174.37
		1.0	131.76	126.73	119.15		1.0	333.18	278.18	218.16
	13.78 (α=.40)	.2	100.00	100.00	100.00	11.78 (α=.25)	.2	116.98	116.98	116.98
		.4	100.08	100.06	100.04		.4	117.58	117.43	117.29
8		.6	103.84	102.93	101.98		.6	132.11	128.33	124.53
		.8	120.03	115.40	110.43		.8	188.25	168.18	149.33
		1.0	140.28	133.56	123.68		1.0	280.52	238.79	191.35

multiplier may as well be employed if gain in efficiency is achieved. Similar are our choices to study $\hat{\beta}_2$ versus $\tilde{\beta}_2$. By efficiency of an estimator for β with MSE($\cdot$) we mean $e = \frac{100\,\beta^2}{(N-2)\text{MSE}(\cdot)}$. Efficiency for the minimax regret and equivalently for the minimum average risk choice of L are shown in Table I.

For comparison we take n = 20 and several r, K, α as illustrated in Singh et al (1977) and following the rules (i) - (iii) work out the MSE($\hat{\beta}_j$), and present

TABLE II

Efficiency of $\hat{\beta}_j$ and $\tilde{\beta}_j$ the latter within parentheses in various situations. n = 20.

		$\hat{\beta}_1$		α = .01	$\hat{\beta}_2$		
			K			K	
r	b	.2	.4	.6	.2	.4	.6
	.2	99.51 (82.32)	99.64 (87.12)	99.76 (91.77)	183.92 (161.92)	184.40 (167.72)	184.87 (173.64)
	.4	108.80 (90.08)	116.52 (95.33)	117.45 (98.99)	210.12 (191.90)	204.12 (192.00)	198.03 (190.98)
4	.6	162.04 (115.55)	158.30 (116.55)	142.77 (113.90)	311.37 (282.92)	280.80 (257.53)	247.54 (232.16)
	.8	233.58 (143.71)	203.48 (141.24)	164.51 (129.12)	567.23 (454.98)	426.66 (365.44)	318.40 (290.12)
	1.0	302.23 (179.56)	241.24 (163.32)	180.53 (141.92)	1039.27 (705.63)	660.14 (522.76)	410.53 (365.07)
	.2	99.95 (85.08)	99.96 (88.90)	99.97 (92.70)	128.71 (111.65)	128.80 (115.90)	128.88 (120.23)
	.4	95.29 (68.28)	96.53 (77.35)	112.10 (86.34)	123.35 (91.24)	124.89 (100.43)	126.36 (110.04)
6	.6	164.19 (99.21)	180.93 (107.25)	166.93 (110.38)	168.06 (135.37)	174.52 (138.88)	167.89 (138.92)
	.8	416.71 (176.14)	324.35 (166.08)	218.46 (145.85)	436.49 (265.64)	331.75 (229.78)	240.09 (191.87)
	1.0	973.11 (293.88)	465.29 (236.55)	248.85 (178.51)	1527.41 (554.27)	648.74 (392.58)	331.19 (264.15)
	.2	100.00 (96.34)	100.00 (97.33)	100.00 (98.27)	116.97 (113.08)	116.98 (114.11)	116.98 (115.10)
	.4	96.37 (60.54)	97.32 (69.65)	98.24 (79.69)	110.99 (72.73)	112.52 (82.10)	114.04 (92.68)
8	.6	128.37 (80.65)	160.50 (91.80)	164.50 (100.09)	117.12 (96.43)	138.80 (105.54)	145.74 (112.71)
	.8	420.06 (169.09)	348.74 (164.68)	232.58 (146.92)	368.66 (211.22)	301.44 (193.24)	222.64 (167.86)
	1.0	1708.19 (372.98)	567.46 (278.09)	268.56 (195.28)	2136.27 (571.31)	676.54 (384.60)	316.31 (248.98)

(continued)

TABLE II (Continued)

Efficiency of $\hat{\beta}_j$ and $\tilde{\beta}_j$ the latter within parentheses in various situations. n = 20.

		$\hat{\beta}_1$		α = .05	$\hat{\beta}_2$		
			K			K	
r	b	.2	.4	.6	.2	.4	.6
	.2	99.86 (95.83)	99.90 (96.93)	99.93 (97.99)	186.96 (185.57)	186.66 (185.63)	186.37 (185.69)
	.4	99.44 (91.85)	99.61 (94.35)	99.77 (96.57)	216.19 (213.00)	208.29 (206.09)	200.57 (199.23)
4	.6	119.71 (101.83)	120.49 (102.61)	116.88 (102.56)	295.07 (291.81)	262.62 (260.76)	233.66 (232.71)
	.8	156.57 (117.50)	147.50 (115.01)	133.11 (111.05)	434.26 (432.56)	351.26 (350.58)	282.36 (282.12)
	1.0	190.95 (131.91)	171.46 (126.85)	146.53 (119.23)	734.84 (609.44)	536.58 (474.24)	370.15 (346.22)
	.2	99.99 (98.85)	99.99 (99.15)	99.99 (99.45)	129.01 (128.07)	129.02 (128.32)	129.03 (128.57)
	.4	98.34 (83.02)	98.78 (87.43)	99.20 (91.79)	129.82 (117.35)	129.68 (120.40)	129.51 (123.38)
6	.6	101.07 (91.27)	130.66 (95.29)	130.19 (98.23)	155.74 (143.19)	149.11 (141.00)	142.39 (137.82)
	.8	250.04 (125.60)	221.81 (122.81)	176.12 (117.06)	326.29 (227.26)	269.58 (200.70)	212.53 (174.38)
	1.0	478.17 (172.26)	324.69 (157.99)	211.53 (138.82)	835.18 (369.80)	495.96 (299.87)	295.76 (228.01)
	.2	100.00 (99.99)	100.00 (100.00)	100.00 (100.00)	116.98 (116.98)	116.98 (116.98)	116.98 (116.98)
	.4	98.87 (93.80)	99.16 (95.42)	99.45 (96.99)	115.75 (113.36)	116.07 (114.29)	116.39 (115.21)
8	.6	98.19 (89.27)	98.84 (92.52)	99.36 (95.45)	125.75 (120.78)	123.87 (120.39)	121.77 (119.61)
	.8	295.18 (115.28)	265.32 (113.72)	200.08 (110.45)	299.17 (179.29)	257.40 (163.41)	202.70 (147.16)
	1.0	935.96 (163.22)	457.69 (151.27)	247.18 (134.81)	1278.48 (296.08)	570.47 (248.52)	296.66 (196.07)

TABLE II (Continued)

Efficiency of $\hat{\beta}_i$ and $\tilde{\beta}_i$ the latter within parentheses in various situations. n = 20.

		$\hat{\beta}_1$			$\alpha = .10$	$\hat{\beta}_2$	
			K			K	
r	b	.2	.4	.6	.2	.4	.6
	.2	99.95 (99.03)	99.97 (99.28)	99.98 (99.53)	187.26 (187.24)	186.89 (186.87)	186.52 (186.51)
	.4	100.22 (96.28)	100.18 (97.34)	100.14 (98.32)	217.17 (217.09)	208.97 (208.91)	200.99 (200.95)
4	.6	104.47 (101.97)	103.51 (101.96)	102.44 (101.62)	296.26 (296.14)	263.32 (263.25)	234.02 (233.98)
	.8	135.13 (112.37)	130.12 (110.30)	121.74 (107.46)	435.17 (435.05)	351.69 (351.63)	282.53 (282.50)
	1.0	159.59 (121.73)	148.52 (118.51)	133.14 (113.51)	605.27 (605.24)	472.03 (472.01)	345.32 (345.32)
	.2	100.00 (99.86)	100.00 (99.89)	100.00 (99.93)	129.05 (128.98)	129.05 (129.00)	129.05 (129.02)
	.4	99.40 (92.41)	99.56 (94.70)	99.72 (96.34)	131.77 (128.58)	131.12 (128.76)	130.45 (128.91)
6	.6	102.94 (95.14)	102.36 (97.09)	101.68 (98.58)	159.08 (153.98)	151.32 (147.95)	143.69 (141.71)
	.8	194.63 (117.50)	180.75 (114.95)	154.46 (110.98)	231.55 (228.16)	201.49 (199.82)	173.93 (173.23)
	1.0	338.39 (146.91)	260.70 (138.77)	188.56 (127.04)	623.14 (339.63)	421.45 (282.09)	273.76 (219.98)
	.2	100.00 (99.99)	100.00 (100.00)	100.00 (100.00)	116.98 (116.98)	116.98 (116.98)	116.98 (116.98)
	.4	99.57 (93.80)	99.68 (95.42)	99.79 (96.99)	116.98 (113.36)	116.99 (114.29)	116.99 (115.21)
8	.6	100.69 (89.27)	100.64 (92.52)	100.51 (95.45)	129.44 (120.78)	126.45 (120.39)	123.37 (119.61)
	.8	233.55 (115.28)	218.49 (113.72)	177.68 (110.45)	185.10 (179.29)	166.36 (163.41)	148.41 (147.16)
	1.0	636.68 (163.22)	381.05 (151.27)	228.29 (134.81)	912.77 (296.08)	493.30 (248.52)	279.35 (196.04)

(continued)

TABLE II (Continued)

Efficiency of $\hat{\beta}_j$ and $\tilde{\beta}_j$ the latter within parentheses in various situations. n = 20.

			$\hat{\beta}_1$		$\alpha = .50$	$\hat{\beta}_2$	
			K			K	
r	b	.2	.4	.6	.2	.4	.6
	.2	100.00 (100.00)	100.00 (100.00)	100.00 (100.00)	187.26 (185.77)	186.89 (185.77)	186.52 (185.77)
	.4	100.99 (100.59)	100.75 (100.45)	100.51 (100.31)	217.17 (190.06)	208.97 (189.08)	200.99 (188.03)
4	.6	105.43 (104.33)	104.19 (103.37)	102.87 (102.32)	296.26 (219.69)	263.32 (211.85)	234.02 (203.46)
	.8	111.89 (110.71)	109.47 (108.58)	106.63 (106.04)	435.17 (290.43)	351.69 (264.84)	282.53 (237.52)
	1.0	116.66 (116.12)	114.28 (113.82)	110.52 (110.20)	605.27 (406.93)	472.03 (354.21)	345.32 (291.32)
	.2	100.00 (100.00)	100.00 (100.00)	100.00 (100.00)	129.06 (129.05)	129.06 (129.05)	129.06 (129.05)
	.4	100.33 (100.22)	100.25 (100.16)	100.17 (100.11)	132.52 (129.76)	131.67 (129.59)	130.81 (129.42)
6	.6	105.43 (104.61)	104.15 (103.54)	102.81 (102.41)	160.76 (143.94)	152.45 (140.46)	144.36 (136.79)
	.8	118.75 (117.51)	114.56 (113.67)	109.96 (109.39)	233.22 (195.21)	202.40 (178.46)	174.37 (161.26)
	1.0	131.76 (131.33)	126.73 (126.38)	119.15 (118.91)	333.18 (291.24)	278.18 (251.70)	218.16 (205.26)
	.2	100.00 (100.00)	100.00 (100.00)	100.00 (100.00)	116.98 (116.98)	116.98 (116.98)	116.98 (116.98)
	.4	100.08 (100.35)	100.06 (100.04)	100.04 (100.03)	117.58 (117.12)	117.43 (117.09)	117.29 (117.05)
8	.6	103.84 (103.34)	102.93 (102.56)	101.98 (101.74)	132.11 (125.18)	128.33 (123.26)	124.53 (121.25)
	.8	120.03 (119.00)	115.40 (114.66)	110.43 (109.97)	188.25 (168.62)	168.18 (155.52)	149.33 (142.19)
	1.0	140.28 (139.98)	133.56 (133.32)	123.68 (123.52)	280.52 (261.66)	238.79 (226.62)	191.35 (185.28)

the efficiencies $e(\hat{\beta}_j)$, $e(\tilde{\beta}_j)$, j = 1, 2, showing the latter values within the parentheses below the former ones in Table II.

Calculating $MSE(\hat{\beta}_j)$, Regret $(\hat{\beta}_j)$, j = 1, 2, for n = 20, r = 4, 6, 8, K = 0.2, 0.4, 0.6, trying several pairs of (b, L)-values we work out the minimax values of L which equal L_2 and which agree with the minimum average risk values of L also and note the corresponding α-values and then present the corresponding efficiency values in Table I below.

3. CONCLUSIONS

Interestingly, when $\alpha = 0.50$, Singh et al's (1977) procedure is almost as efficient as the ones based on minimax and equivalently the minimum average risk procedure but the latter is also much less efficient for lower α-values especially when b is close to 1 but not so if b is away from unity and more importantly the latter's efficiency never falls below 100% whereas that on the former does so when α is low and b is away from unity. But our thumb rule produces gain in efficiency right through for every α and every b and especially so when α is low and b is close to unity and the main purpose of this paper is just to demonstrate this. It is easy to report the value of α corresponding to each choice of L according to our thumb rule but we do not show this here to save space.

ACKNOWLEDGMENT

The authors are grateful to the referee and to Professor S.R. Adke, an Editorial Board Member, for their helpful suggestions for improvement over two earlier drafts.

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Received October 1988; Revised February 1990.

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