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SHORT COMMUNICATIONS

A GROUP SINGLE SAMPLING ATTRIBUTE PLAN TO ATTAIN A GIVEN STRENGTH

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ABSTRACT

A single sampling attribute plan cannot be designed satisfying the given strength exactly. It is suggested that a group consisting of *atmost* three single sampling plans be operated at random with certain proportions so that the group as a whole attains the given strength exactly. The problem of choosing *atmost* three plans with the proportion of times they should be operated with minimum average sample size is shown to be a linear programming problem. An example with solution by simplex method is given.

1. Introduction

We consider an industrial situation in which a series of lots of size N items arrive at the inspection station. For acceptance sampling inspection an exact single sampling plan (SSP) (n,c) cannot be designed satisfying $Q(p_1) = 1-P(p_1) = \alpha$ and $P(p_2) = \beta$ where $p_1(p_2)$ is the satisfactory (unsatisfactory) quality level, $\alpha(\beta)$ is the producer's (consumer's) risk specified by the decision maker (DM) and $P(p)$ is the operating characteristic of the SSP (for details, see Hald [4]).

For obtaining a SSP *close* to α and β, Chakraborty [2] modelled the problem as a fuzzy goal programming (FGP). The FGP model is shown to be equivalent to four sub-problems corresponding to the different combinations of the membership functions of fuzzy goals. Each of these subproblems corresponds to a different type of *close to the goals* SSP according to the different combinations of the requirements of $Q(p_1)\,(\leqslant,\geqslant)\alpha$ and $P(p_2)\,(\leqslant,\geqslant)\beta$

The present paper is a continuation of the earlier paper (Chakraborty [2]) and here we propose a system of SSP consisting of a group of *atmost* three plans from the set of plans corresponding to the four types of SSPs considered in Chakraborty [2] for the attainment of risks exactly at the specified levels. An individual SSP of the system is to be chosen at random for a particular lot in a specified proportion. The idea of such *group of SSPs* has been used in the MIL-STD 105D (see Hald [4]) where a group of three SSPs is utilized for acceptance inspection though the procedure employed differs substantially from the one dealt within the present paper.

2. Group Single Sampling Attribute Plan

A SSP from any one of the types of plans can be obtained under Poisson conditions from the following general goal programming model (for notation, explanation and solution procedure. see Chakraborty [1]).

$$\text{Min } Z = P_1(n+c) + P_2 w_1 d_1^{+2} + P_2 w_2 d_1^{-2} + P_3 w_3 d_2^{+2} + P_3 w_4 d_2^{-2},$$

Subject to

$$Q(p_1) + d_1^{+2} - d_1^{-2} = \alpha,$$

$$P(p_2) + d_2^{+2} - d_2^{-2} = \beta,$$

$$d_1^{+} d_1^{-} = 0, \; d_2^{+} d_2^{-} = 0.$$

Thus given $(p_1, \alpha, p_2, \beta)$ and k sets of P_j's and w_j's, one easily obtains $(k > 3)$ SSPs each of them is individually satisfactory to the DM.

Assume that the DM has chosen k SSPs $(n_j, c_j), j = 1, 2, \ldots, k$, having producer's and consumer's risks of α_j, β_j, respectively.

The problem is to determine x_j $(j = 1, 2, \ldots, k)$, the proportion of times the jth SSP should be operated at random so that the expected average sample size is minimised subject to attaining the given strengths α and β exactly. The mathematical model is the following:

$$\text{Minimize } x_o = n_1 x_1 + n_2 x_2 + \ldots + n_k x_k,$$

subject to

$$\begin{aligned} x_1 + x_2 + \ldots + x_k &= 1, \\ \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_k x_k &= \alpha, \\ \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k &= \beta, \\ x_1, x_2, \ldots, x_k &\geqslant 0. \end{aligned} \tag{1}$$

This is a linear programming (LP) problem and can be easily solved by the simplex algorthm.

Note: (1) If LP is feasible then it will provide a solution with *atmost* three x_j's $> 0 \Rightarrow$ Group consisting of atmost three SSPs.

(2) If LP is not feasible then this scheme cannot be utilised to obtain Group SSPs to attain the risks exactly.

Example 1. Let $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.06$, $\beta = 0.10$, (Hald [4, p.50]). Let the DM choose the following five SSPs as satisfactory plans. We are to determine a group SSP.

S. No.	n_j	c_j	α_j	β_j
1	85	2	0.055	0.116
2	90	2	0.063	0.095
3	110	3	0.026	0.105
4	120	3	0.034	0.072
5	138	3	0.052	0.035

Solving the problem by the simplex method we obtain the group SSP consisting of SSPs (85, 2), (90, 2) and (120, 3) and these are to be operated at random with proportions 0.560, 0.147 and 0.293 respectively and will have an average sample size 96.012.

Remarks : The problem (1) can be viewed as three dimensional linear *knapsack problem* (see Dantzig [3]) and can also be solved by adapting and extending the *centre of gravity* method (see Dantzig [3, pp. 160-166]) to solve the special LP of the form (1). However, the computational effort would be of the same order as that of simplex algorthm.

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