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A NOTE ON AN INVENTORY MODEL WITH STOCK-DEPENDENT CONSUMPTION RATE

B. N. Mandal

Department of Applied Mathematics, Calcutta University, 92 A.P.C. Road Calcutta 700009, India.

and

S. Phaujdar

Department of Applied Physics, Calcutta University, 92 A.P.C. Road, Calcutta 700009, India

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ABSTRACT

This note is concerned with an inventory model with instantaneous stock replenishment and stock-dependent consumption rate. The model is illustrated with a numerical example.

1. Introduction

Conventional inventory models assume demand rate and inventory level as uncorrelated quntities. A company dealing with the marketing of an item may go in for advertisement with certain per unit expenditure per unit item. This cost would naturally be proportional to the instantaneous stock level of the item and can be absorbed in the holding cost. As a return to this investment in advertisement the demand rate for the item may be boosted up in proportion to the stock level. In case of many items, e.g. consumer goods, some customers may be motivated to procure with the ease of availability or abundance of supply of the item. This behaviour may be approximated by a stock dependent demand pattern.

Gupta and Vrat [1] suggested an EOQ model through cost minimization technique to take care of stock-dependent consumption rate. They substituted the expression for variable demand rate in the total cost per unit time derived under the assumption of constant demand. Naturally, this could not take care of stock-dependent demand rate except where the demand rate is dependent on replenishment size.

In the EOQ model proposed in this note, the replenishment of stock is instantaneous, shortages are not allowed and the demand rate depends upon the current stock level. The actual form on the functional relationship between the demand rate and the stock status can be obtained from a close observation of the actual system. A demand rate linearly increasing with the stock status is assumed here. In this particular situation, the criteria of minimization of total cost per unit time and maximization of total profit per unit time would produce different results for the EOQ. The choice of profit maximization as the objective function seems to be more realistic and is followed here.

2. The EOQ Model

If q denotes the inventory level at time t, then in this case we have

$$\frac{dq}{dt} = -R(q), \tag{1}$$

where R is the demand rate at time t. The length T of each cycle is given by

$$T = \int_0^S \frac{dq}{R(q)} = F(S), \text{ say}, \tag{2}$$

where S is the highest stock level. The holding cost during T is $h\,G(S)$ where h is the unit holding cost per unit time and

$$G(S) = \int_0^S \frac{q\,dq}{R(q)}\,. \tag{3}$$

The total profit per unit time during T is thus

$$Z(S) = \frac{pS - \{A + cS + h\,G(S)\}}{F(S)} \tag{4}$$

where A is the setup cost for each cycle, c is the unit cost of the item and p is the unit selling price of the item. The optimum value of S for maximum total profit per unit time is a solution of $Z'(S) = 0$ provided $Z''(S) < 0$ for that value of S. This yields by virtue of (4)

$$F(S)\,\{(p-c)\,S - h\,G'(S)\} = F'(S)\,\{(p-c)\,S - A - h\,G(S)\} \tag{5}$$

where dashes denote derivatives of functions with respect to their arguments. Equation (5) is in general nonlinear and can be solved numerically by Newton Raphson method, say, once the explicit form of $R(q)$ is known, The optimum cycle length is then $F(S^*)$ where S^* is the solution of (5).

For a linear dependence of the demand rate on the stock level we put

$$R(q) = \alpha + \beta q \tag{6}$$

where α and β are nonnegative constants. Equation (5) then gives

$$A-\{h-\beta(p-c)\}\,\frac{(\alpha+\beta S)\ln\left(1+\frac{\beta}{\alpha}S\right)-\beta S}{\beta^{2}}=0. \tag{7}$$

It may be noted that if the criterion of minimization of total average cost per unit time is utilised, then we would obtain, instead of (7), the nonlinear equation

$$A-(h+\beta c)\,\frac{(\alpha+\beta S)\ln\left(1+\frac{\beta}{\alpha}S\right)-\beta S}{\beta^{2}}=0. \tag{8}$$

However, the classical EOQ formula [2] with uniform demand rate α and no shortage allowed, can be deduced from both (7) aud (8) by letting $\beta\to 0$ resulting in the same equation

$$\frac{1}{2\alpha}h\,S^{2}-A=0,$$

so that

$$S^{*}=\left(\frac{2A\alpha}{h}\right)^{1/2}, \tag{9}$$

which does not depend on c and p, as it should be.

The nonlinear equation (7) can be solved numerically by the Newton-Raphson method to yield the optimal value S^{*} of the order quantity. It is verified that $Z''(S) < 0$ for this value of S.

For numerical illustration we take the numerical values of the different parameters as A = Rs 100.00 per order, α = 100 units, h = Re 1.00 per unit per year, c = Rs 4.00 per unit, p = Rs 6.00 per unit and $\beta = 0.2$. For this model we obtain S^{*} = 193 units and T^{*} = 1.635 years.

3. Discussion

An EOQ model for stock-dependent consumption rate has been considered in this note. A linear relationship between the demand rate and the stock level has been assumed. The criterion of maximization of total profit per unit time has been chosen to yield the optimal solution. However, the criterion of minimization of total cost per unit time would yield a different result. Equations (7) and (8) corroborates this fact. It may be interesting to note that while in the classical EOQ formula for fixed demand rate with no shortage allowed the result does not depend on the unit cost and selling price of the item, it does depend on them for the stock-dependent demand rate.

This model can be further generalised to include the case of finite rate of replenishment of the stock as well as other plausible functional relationships which may exist between the demand rate and the stock level.

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