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Bayes and Minimax Procedures for Finite Population Sampling under Measurement Error Models

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Abstract: We consider Bayes and minimax estimates of population mean. Bayes estimates of domain total, mean under simple random sampling from a finite population when the true values of the characteristic can not be observed, but only the values mixed with some measurement errors are observed. Bayes and minimax procedures in stratified random sampling under measurement error models have also been investigated.

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1. **Introduction** : Let U denote a finite population of a known number N of identifiable units labelled $1,\cdots,i,\cdots,N$. Associated with each i is a real quantity 'y_i', the value of a variable 'y' on unit i. Our problem is to estimate the population mean $\overline{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, domain mean $\overline{y}_D = y_D/N_D$, where $y_D = \sum_{i\in D} y_i$, N_D is the number of units in a domain (subpopulation) D by a sample survey for which a sample s is selected with probability $p(s)$ according to a sampling design p. In this paper we shall consider that s is selected by simple random sampling without replacement (srswor) or by stratified random sampling. We assume that whenever $i \in s$, the true value y_i of 'y' can not be observed but a different value Y_i, mixed with measurement

errors, is observed. A general treatment for inference problem under measurement error models has been considered in Fuller (1987). Prediction in finite population under error- in-variables superpopulation models has been earlier considered by Bolfarine (1991), Mukhopadhyay (1992).

In this paper we shall consider Bayes estimates and minimax estimates of population mean, Bayes estimates of domain mean in simple random sampling without replacement from a finite population. We shall also derive Bayes and minimax estimates of linear functions of strata means and minimax choice of sample sizes in stratified random sampling.

2. Bayes and minimax estimates:

Let $\mathcal{X}$ be the sample space of a random variable X and $f = \{f_w; w \in \Omega\}$ be a class of probability distributions f of X indexed by a parameter $\omega \in \Omega$, the parameter space and g be a numerical - valued function defined on Ω whose value $g(\omega)$ we want to estimate. f is a class of superpopulation distributions of X. An estimate δ is a non-randomised decision function which specifies for each $x \in \mathcal{X}$ the value $\delta(x)$ which is chosen to estimate $g(\omega)$. The loss involved in estimating $g(\omega)$ by $\delta(x)$ is $L(\omega, \delta)$ and is often a squared error function $(\delta(x) - g(\omega))^2$. The risk function R associated with δ is $E_x L(w, \delta)$ where E_x denotes expectation with respect to probability distribution $f_w(x)$ of X.

It is assumed that w has a prior distribution $\lambda(w)$ and Bayes estimate with respect to (wrt) prior λ is defined as the one for which the average risk $\int R(w, \delta) d\lambda(w)$ is minimum. If λ is not known, one may use minimax estimate which is defined as an estimate δ which minimises the maximum risk $\sup_{w \in \Omega} R(w, \delta)$.

The following theorems connect Bayes estimates and minimax estimates.

<u>Theorem 1</u>: If $\{\lambda n\}$ is a sequence of a priori probability distributions and $\{r_n\}$ the sequence of associated Bayes risks and if $r_n \to r$ as $n \to \alpha$ and if there exists some estimate δ for which the risk $R(w, \delta) \leq r \forall w$ then δ is a minimax estimate.

<u>Theorem 2</u>: If δ, r are a minimax procedure and minimax risk respectively, assuming that the observation X follow any probability distribution $f \in f_0$ and if $f \supset f_0$ is a space of distributions for which the risk associated with δ does not exceed r then δ is a minimax procedure and r the minimax risk for all the distributions of X in f.

3 Models for an unstratified population :

We shall assume that the unknown true values $\underset{\sim}{y} = (y_1, \cdots, y_N)$ are the realised values of N random variables. However, since both are unknown, we shall make no notatioinal difference between y_i and the random variable of which it is a realization.

Consider the following general class of superpopulation model distributions ξ_1 of $\underset{\sim}{y} = (y_1, \cdots, y_N)$ such that for given μ, ξ_1 is a distribution in hyperplanes in R_N with $\bar{y}(= \frac{1}{N}\sum_{i=1}^{N} y_i) = \mu$ and

$$E\sum_{i=1}^{N}(y_i - \mu)^2 \leq (N-1)\sigma_e^2. \tag{3.1}$$

σ_e^2a constant and E deonting (here and also subsequently) expection wrt superpopulation models (ξ_1 and others relevant from the context). The distribution ξ_2 of $\underset{\sim}{Y_s} = \{Y_i, i \in s\}$ is considered to be a member of the class with the property that the conditional distribution of Y_i given y_i is independent and

$$E(Y_i|y_i) = y_i, \; V(Y_i|y_i) = \sigma_u^2 \tag{3.2}$$

Let C denote the class of distributions $\{\xi = \xi_1 \times \xi_2\}$. Consider the subclass $C_0 = \{\xi_0 = \xi_{10} \times \xi_{20}\}$ of C where ξ_{10} is such that given $\mu, \underset{\sim}{y}$ is distributed as a N-variate singular normal distribution with mean vector $\mu \underset{\sim}{1} n$ and dispersion matrix $\sum$ having constant values of

$$\sigma_{ii} = \frac{N-1}{N}\sigma_e^2(\forall i) \text{ and } \sigma_{ij} = -N\sigma_e^2(\forall i \neq j) \tag{3.3}$$

$\underset{\sim}{1}_p = (1, \cdots i)'_{p\times 1}$, ξ_{20} is a pdf on R_n such that the condition distribution of Y_i given y_i is independent normal with mean and variances as stated in (3.2).

We assume that μ is distributed a priori normally with mean 0 and variance θ^2. We shall obtain Bayes estimate with respect to this prior regarded as a member of the sequence $\{\lambda_\theta\}$ of prior distributions and obtain the limit, if any, of the corresponding sequence of Bayes risk $\{r_\theta\}$ as $\theta \to \alpha$, say, r. Then if we can find some estimate δ for which the risk $R(\xi, \delta)$ does not exceed r, without assuming the normality of distributions, δ will be a minimax estimate by theorems 1 and 2.

We assume that the sample is drawn by srswor and the sample is $(1, 2, \cdots, n)$.

4. Bayes and minimax estimates of $\bar{y}$:

It follows that $\underset{\sim}{y_s} = \{y_i, i \in s\}$ follows a n-variate normal distribution ξ'_{10} with mean $\mu \underset{\sim}{1_n}$ and dispersion matrix $\underset{\sim}{\sum_n}$ having elements $\sigma_{ii} = \frac{N-1}{N}\sigma_e^2(\forall i)$ and $\sigma_{ij} = -\frac{\sigma_e^2}{N}$ $(\forall i \neq j)$. The conditional likelihood of $\underset{\sim}{Y_s}$ given μ is therefore, a n-variate normal with mean $\mu \underset{\sim}{1_n}$ and dispersion matrix $\underset{\sim}{\overset{*}{\sum}}$ having elements $\sigma_{ii} = \sigma_u^2 + \frac{N-1}{N}\sigma_e^2(\forall i)$ and $\sigma_{ij} = -\frac{\sigma_e^2}{N}(\forall i \neq j)$. It is easily seen that $\overline{Y}_s(=$

$\frac{1}{n}\sum_{i=1}^{n} Y_i$) is sufficient for μ. The posterior distribution of μ given $\bar{Y}_s$ is normal with mean

$$E(\mu|\bar{Y}_s) = \frac{\bar{Y}_s}{1+\frac{\sigma'^2}{n\theta^2}} = \delta_\theta \text{ say} \quad (4.1)$$

$$\text{where } \sigma'^2 = \sigma_u^2 + \frac{N-n}{N}\sigma_e^2 \quad (4.2)$$

and posterior variance

$$V(\mu|\bar{Y}_s) = \frac{\sigma'^2}{n+\frac{\sigma'^2}{\theta^2}} = r_\theta \text{ (say)} \quad (4.3)$$

Since (4.1) and (4.3) are independent of $\bar{Y}_s$, these are, respectively, Bayes estimate of μ and Bayes risk of (4.1).

To find a minimax estimate for μ, we consider if r_θ tends to a limit as $\theta \to \alpha$. It is seen that

$$\lim_{\theta\to\alpha} r_\theta = \frac{\sigma'^2}{n} = r \text{ (say)} \quad (4.4)$$

By theorems 1 and 2 if we can find some estimate δ for which the risk does not exceed r, for all $\xi \in C$ without assuming the normality of the distributions as in ξ_{10} and ξ_{20}, then δ is a minimax estimate. Trying $\delta = \bar{Y}_s(= \lim_{\theta\to\alpha} \delta_\theta)$ we see that the risk corresponding to δ is, (denoting by E_p expectation wrt sampling design p and E_1 expectation wrt ξ_1-distribution)

$$\begin{aligned} R(\mu,\bar{Y}_s) &= E_pE(\bar{Y}_s-\mu)^2 \\ &= E_pE_1[E\{(\bar{Y}_s-\mu)^2|\underset{\sim}{y}\}] \\ &= E_pE_1[\frac{\sigma_u^2}{n} + \bar{y}_s^2 + \mu^2 - 2\mu\bar{y}_s] \\ &= E_1[\frac{\sigma_u^2}{n} + \frac{N-n}{nN}\cdot\frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^2] \\ &\le \frac{\sigma_u^2}{n} + \frac{N-n}{nN}\sigma_e^2 = r. \quad (4.5) \end{aligned}$$

Hence $\bar{Y}_s$ is a minimax estimate of $\mu(=\bar{y})$.

5. Bayes estimate of domain total, mean.

Suppose we want to estimate the total $y_D = \sum_{i\in D} y_i$, mean $\bar{y}_D = y_D/N_D$ of a domain (subpopulation) D of known size N_D. Bayes estimate of y_D is the posterior mean,

$$\underset{\theta}{\delta}(y_D) = E(y_D|\underset{\sim}{Y_s}) = E\{\sum_{i\in s\cap D} y_i + \sum_{i\in\bar{s}\cap D} y_i|\underset{\sim}{Y_s}\} \quad (5.1)$$

where $\bar{s} = U - s$. For this we have to find the posterior distribution of $y_i, i = 1, 2, \cdots, N$.

The model (3.3) implies that $\underset{\sim}{y}^* = (y_1, \cdots, Y_{N-1})$ is a $(N-1)$-variate non-singular normal distribution with mean vector $\mu \underset{\sim}{1}_{N-1}$ and $(N-1)\times(N-1)$-dispersion matrix $\sum^*$ with elements $\sigma_{ii} = \frac{N-1}{N}\sigma_e^2 (\forall i)$ and $\sigma_{ij} = -\frac{\sigma_e^2}{N}(\forall i \neq j)$. Posterior distribution of $\underset{\sim}{y}^*$ given $\underset{\sim}{Y}_s$ is, therefore,

$$f(\underset{\sim}{y}^* \mid \underset{\sim}{Y}_s)\alpha \exp[-\frac{1}{2}\{(\underset{\sim}{y}^* - \mu \underset{\sim}{1}_{N-1})' \overset{*^{-1}}{\sum}(\underset{\sim}{y}^* - \mu \underset{\sim}{1}_{N-1}) + (\underset{\sim}{Y}^* - \underset{\sim}{y}^*)D^{-1}(\underset{\sim}{Y}^* - \underset{\sim}{y}^*)\}] \tag{5.2}$$

where $\underset{\sim}{Y}^* = (Y_1, \cdots, Y_{N-1})'$ and D is a $(N-1)\times(N-1)$ -diagonal matrix $(\sigma_u^2, \cdots, \sigma_u^2)$. It is seen, (5.2)

$$\alpha \exp[-\frac{1}{2}(\underset{\sim}{y}^* - \underset{\sim}{m})' M (\underset{\sim}{y}^* - \underset{\sim}{m})] \tag{5.3}$$

where

$$\underset{\sim}{m} = \overset{-1}{\underset{\sim}{M}}(\mu \overset{*-1}{\sum} \underset{\sim}{1}_{N-1} + D^{-1}\underset{\sim}{Y}^*) \tag{5.4}$$

and $M = \sum^{*^{-1}} + D^{-1}$. Therefore, $\underset{\sim}{y}^*$ follows a $(N-1)$-variate normal distribution with posterior mean $\underset{\sim}{m} = (m_1, \cdots, m_{N-1})$ where

$$m_i = \frac{1}{N\sigma'^2}[N\mu\sigma_u^2 + (N-n)Y_i\sigma_e^2 - (n\bar{Y}_s - Y_i)\frac{\sigma_e^2\sigma_u^2}{\sigma_e^2+\sigma_u^2}], i = 1, \cdots, n \tag{5.5}$$

$$m_j = \frac{1}{N\sigma'^2}[N\mu(\sigma_e^2 + \sigma_u^2) - n\bar{Y}_s\sigma_e^2], j = n+1, \cdots, N-1 \tag{5.6}$$

and dispersion matrix $M^{-1} \left(= ((\sigma_{ij}^*))\right)$ with elements

$$\sigma_{ii}^* = A\{(N-1)\sigma_u^2 + (N-n)\sigma_e^2\}, i = 1, \cdots, n \tag{5.7}$$

$$\sigma_{ij}^* = -A\sigma_e^2\sigma_u^2, i \neq j = 1, \cdots, n \tag{5.8}$$

$$\sigma_{kk}^* = \frac{\sigma_e^2\{(N-1)\sigma_u^2 + (N-n-1)\sigma_e^2\}}{N\sigma'^2}, k = n+1, \cdots, N-1 \tag{5.9}$$

$$\sigma_{hl}^* = -\frac{(\sigma_u^2+\sigma_e^2)\sigma_e^2}{N\sigma'^2}, k \neq l = n+1, \cdots, N-1 \tag{5.10}$$

$$\sigma_{il}^* = -\frac{\sigma_e^2\sigma_u^2}{N\sigma'^2}, i = 1, \cdots, n\ ; l = n+1, \cdots, N-1 \tag{5.11}$$

where

$$A = \frac{\sigma_u^2}{(\sigma_e^2 + \sigma_u^2)N\sigma'^2} \tag{5.12}$$

and σ'^2 is defined in (4.2). The posterior distribution of μ given the sufficient statistic $\bar{Y}_s$ remains the same as stated in section 4. Therefore,

$$\delta_\theta(\bar{y}_D) = E[E\{\sum_{i\in s\cap D} y_i + \sum_{i\in \bar{s}\cap D} y_i|\mu, \bar{Y}_s\}|\bar{Y}_s]$$

$$= E\left[\frac{1}{N\sigma'^2}\left\{NN_D\mu\sigma_u^2 + N(N_D - n_{1D})\mu\sigma_e^2 + n_{1D}\sigma_e^2 Y_1 \left(\frac{(N-n+1)\sigma_u^2 + (N-n)\sigma_e^2}{\sigma_e^2+\sigma_u^2}\right) + n\sigma_e^2\bar{Y}_s\left(\frac{n_{1D}\sigma_e^2}{\sigma_e^2+\sigma_u^2} - N_D\right)\right\}|\underset{\sim}{Y_s}\right] \tag{5.13}$$

where n_{1D} denotes the number of units in $s\cap D$. Bayes estimate of y_D under $N(0,\theta^2)$- prior of μ and superpopulation models ξ_0 is obtained by using (4.1) in (5.13).

6. Bayes and Minimax procedures in Stratified Random Sampling:

Suppose that the population is divided into L strata and the h-th stratum consists of a known number $N_h(>0)$ of units with true values $y_{hi}(i = 1,\cdots,N_h)$ of the characteristic 'y' $(h = 1,\cdots,L)$. A sample s_h of predetermined size $n_h(>0)$ is selected independently from the h-th stratum by srswor. When the analysis is made for a fixed sample we shall assume without loss of generality that $s_h = (h_1,\cdots,h_{n_h})$. We assume that y_{h_i}, when $i \in s_h$, can not be observed but some other value Y_{hi}, mixed with measurement error, is observed. The sampled data is $\underset{\sim}{Y_s} = (\underset{\sim}{Y_{s1}},\cdots,\underset{\sim}{Y_{sL}}$ where $\underset{\sim}{Y_{sh}} = (Y_{hi}, i = 1,\cdots,n_h)$. Let c_h be the cost of sampling a unit in the hth stratum. Our object is to find Bayes and minimax estimates of a linear function $F = \sum_{h=1}^{L} a_h\bar{y}_h$ where $\bar{y}_h$ the population mean for the hth stratum, a_h are known constants (which without loss of generality can be assumed to satisfy $\sum_{n=1}^{L} a_h = 1$) and the loss function in estimating F by δ is

$$L(F,\delta) = (\delta - F)^2 + \sum_{h=1}^{L} c_h n_h \tag{6.1}$$

We shall regard n_h's as fixed for the purpose of finding the estimates. If δ^* be a minimax estimate for given n_h, we shall choose the n_h so as to minimise the risk

$$R(F,\delta^*) = E_p E(\delta^* - F)^2 + \sum_{h=1}^{L} c_h n_h, \tag{6.2}$$

as a function of the n_h. As before we do not make any notational difference between y_{hi} and the random variable of which it is a realised value.

Consider the following general class of superpopulation model distributions $\xi_1 = \prod_{h=1}^{L} \xi_{1h}$ of $\underset{\sim}{y} = (\underset{\sim}{y}_1, \cdots, \underset{\sim}{y}_L)$ where $\underset{\sim}{y}_h = (y_{h1}, \cdots, y_{hN_h})$ such that for given μ_h, ξ_{1h} is a distribution in hyperplanes of R_{N_h} with $\bar{y}_h (= \sum_{i=1}^{N_h} y_{hi}/N_h) = \mu_h$ and $E\sum_{i=1}^{N_h}(y_{hi} - \mu_h)^2 \leq \sigma_{\epsilon h}^2$. Consider also the class of distributions $\xi_2 = \prod_{h=1}^{L} \xi_{2h}$ of $\underset{\sim}{Y}_s$ such that the conditional distribution of Y_{hi} given y_{hi} are independent with

$$E(Y_{hi}|y_{hi}) = y_{hi}, \quad V(Y_{hi}|y_{hi}) = \sigma_{hu}^2 \tag{6.3}$$

Let $C = \{\xi = \xi_1 \times \xi_2\}$. Consider the subclass $C_0 = \{\xi_0 = \xi_{10} \times \xi_{20}\}$ of C where $\xi_{10} = \prod_{h=1}^{L} \xi_{1h_0}, \xi_{1h_0}$ such that given μ_h, $\underset{\sim}{y}_h$ is distributed as a N_h- variate singular normal distribution with mean vector $\mu_h \underset{\sim}{1}'_{N_h}$ and dispersion matrix $\sum_h = ((\sigma_{hij}))$ where

$$\sigma_{hii} = \frac{N_h - 1}{N_h}\sigma_{\epsilon h}^2 (\forall i), \sigma_{hij} = -\frac{\sigma_{\epsilon h}^2}{N_h}(\forall i \neq j) \tag{6.4}$$

ξ_{2h_0} is a pdf on R_{N_h} such that the conditional distribution of Y_{hi} given y_{hi} are independent normal with mean and variance as in (6.3). Suppose that the prior distribution of μ_h is independent $N(0, \theta^2)$. It follows that ξ_{10} implies that the distribution of $\underset{\sim}{y}_s = (\underset{\sim}{y}_{s1}, \cdots, \underset{\sim}{y}_{sL})$, $\underset{\sim}{y}_{sh} = (y_{h1}, \cdots, y_{hn_h})$ is the product of L distributions, the hth one being n_h-variate normal with mean μ_h and variance $\frac{N_h - 1}{N_h}\sigma_{\epsilon h}^2$ for each component and covariance $-\frac{\sigma_{\epsilon h}^2}{N_h}$ for each pair of components. Again, the set $\underset{\sim}{\bar{Y}}_s = (\bar{Y}_{s1}, \cdots, \bar{Y}_{sL})$ of sample means is a sufficient statistic for $(\mu_1, \cdots, \mu_L)$ and hence for F.

Since the L pairs $(\mu_h, \bar{Y}_{sh})$ are independently distributed, it follows that the conditional distribution of μ_h given $\bar{Y}_{sh}$ is normal with mean

$$x_h = \frac{\bar{Y}_{sh}}{1 + \frac{\sigma_h'^2}{n_h \theta^2}} \tag{6.5}$$

and variance

$$v_h = \frac{\sigma_h'^2}{n_h + \frac{\sigma_h'^2}{\theta^2}} \tag{6.6}$$

where

$$\sigma_h'^2 = \sigma_{uh}^2 + \frac{N_h - n_h}{N_h}\sigma_{ch}^2 \tag{6.7}$$

Hence for given $\underset{\sim}{Y}_s$, the posterior distribution of F is normal with mean

$$\delta_\theta(\underset{\sim}{Y}_s) = \sum_{h=1}^{L} a_h x_h \tag{6.8}$$

and variance $\sum_{h=1}^{L} a_h^2 v_h$. $\delta_\theta(\underset{\sim}{Y}_s)$ is Bayes estimate of F and since the conditional variance is independent of $\underset{\sim}{Y}_s$, Bayes risk of (6.8) is

$$r_\theta = \sum_{h=1}^{L} a_h^2 v_h + \sum_{h=1}^{L} c_h n_h.$$

To find a minimax estimate for F, we note that

$$\begin{aligned} \lim_{\theta\to\alpha} r_\theta &= \sum_{h=1}^{L} a_h^2 \frac{\sigma_h'^2}{n_h} + \sum_{h=1}^{L} c_h n_h \\ &= \sum_{h=1}^{L} a_h^2 \{\frac{\sigma_{uh}^2}{n_h} + \frac{N_h - n_h}{n_h N_h}\sigma_{eu}^2\} + \sum_{h=1}^{L} c_h n_h, \\ &= r \text{ (say)} \end{aligned} \tag{6.9}$$

If we can find an estimator δ^* for which the risk, whatever be $\xi \in C$, does not exceed r then by theorems 1 and 2, δ^* a minimax estimate.

Trying with $\delta^*(\underset{\sim}{Y}_s) = \sum_{h=1}^{L} a_h \bar{Y}_{sh} (= \lim_{\theta\to\alpha} \delta_\theta(\underset{\sim}{Y}_s))$ we see that the risk corresponding to δ^* is

$$\begin{aligned} R(F,\delta^*) &= E_p E(\delta^* - F)^2 + \sum_{h=1}^{L} c_h n_h \\ &= E_p E[\sum_{h=1}^{L} a_h(\bar{Y}_{sh} - \mu_h)]^2 + \sum_{h=1}^{L} c_h n_h \\ &\leq \sum_{h=1}^{L} a_h^2 \{\frac{\sigma_{uh}^2}{n_h} + \frac{N_h - n_h}{n_h N_h}\sigma_{eh}^2\} + \sum_{h=1}^{L} c_h n_h \\ &= r \end{aligned} \tag{6.10}$$

since the strata are independent and by the result (4.5), using the models ξ_1. Hence $\sum_{h=1}^{L} a_h \bar{Y}_{sh}$ is a minimax estimate of F for given n_h.

7 Minimax strategy for choosing n_h:

We now choose the n_h so that the minimax risk for given n_h and the largest allowed variance (as in (6.10)) is minimised subject to the conditions that n_h are positive integers ($\leq N_h$). Such choice of n_h, $\underset{\sim}{n}^*$ (say) is a minimax choice of n_h as it satisfies

$$\max_{\xi_1 \in c} R(F, \delta^*(\underset{\sim}{n}^*)) = \min_{\underset{\sim}{n}} \max_{\xi_1 \in c} R(F, \delta^*(\underset{\sim}{n}))$$

when $\underset{\sim}{n}^* = (n_1^*, \cdots, n_L^*), \underset{\sim}{n} = (n_1, \cdots, n_L)$ and $\delta^*(\underset{\sim}{n})$ means δ is based on $\underset{\sim}{n}$. It follows that the minimax choice of n_h is given by Neyman's optimum allocation.

$$n_h^* = \sqrt{\frac{a_h^2(\sigma_{uh}^2 + \sigma_{eh}^2)}{c_h}}.$$

In particular if $F = y^= = \sum_{h=1}^{L} W_h \bar{y}_h$, $W_h = \frac{N_h}{N}$, $n_h^* = \sqrt{\frac{N_h^2(\sigma_{uh}^2 + \sigma_{eh}^2)}{c_h}}$.

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