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Fuzzy set theoretic interpretation of object shape and relational properties for computer vision

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The computation of various geometrical and topological properties of objects, as well as the inter-relationships among them, is useful and important in image processing and computer vision problems. Many of these properties and relations are ill-defined and presented here are some approaches to define them using fuzzy set theoretic concepts. It is assumed that the objects are segmented into a two-tone mask from a grey tone image. The properties considered are bigness, position, convexity, circularity, elongatedness, straightness and angular orientation. The relationships considered are relative position, relative orientation, degree of surroundedness and the degree of betweenness. A man–machine interaction based on these properties is also proposed and illustrative examples of the results of executed algorithms are presented.

1. Introduction

The general approach to image understanding and description consists of three stages:

(*a*) segmentation of the image into regions or objects of interest;

(*b*) evaluation of properties of individual objects and description of relationship among various objects;

(*c*) real world indentification of scene.

In this paper we assume that stage (*a*) is faithfully completed and concern ourselves with stage (*b*) only. Our long term goal is to generate a description in a language very similar to natural language so that man–machine communication is possible. Hence, we concentrate here on properties that can be visually perceived and verbally described.

Our input is a set of object masks contained in the rectangular image frame. For simplicity, we consider that the objects are topologically simply connected. Also, we assume that the individual object properties do not affect and are not affected by the relationships among objects. Thus we avoid some complex Gestalt principles as well as the problems of geometric illusion.

The individual object properties we are interested in include size (such as bigness), position in the image frame (say, at the middle), convexity, circularity, symmetry, elongatedness, straightness and the degree of horizontal/vertical orientation. The relational properties among objects may include relative position (such as, A to the left of B), relative orientation (say, A is parallel to B), degree of surroundedness (A is partly surrounded by B) and degree of betweenness (A sits between B and C). Rosenfeld (1982) pointed out the problems of defining these properties and relationship.

One approach to solving the problems is to assign fuzzy truth values to these properties. Our work is motivated by this idea.

The fuzzy set theoretic approach has been used by Chaudhuri and Dutta Majumder (1980) in obtaining a polygonal approximation of a closed curve as well as in symmetry analysis. Thus, we have not tried to analyse these properties again here. Of the other work reported here, a related paper is due to Koczy (1988) who proposed a method of describing the relative position of two objects by a fuzzy degree. We have given an alternative and generalized approach to this problem. In fact our motivation is to give fuzzy definitions to all the properties listed above so that the machine can follow and carry out simple instructions in human-like language such as 'pick a few elongated objects' or 'place object A above object B'.

This paper is organized as follows. The concepts of fuzzy and ultrafuzzy subsets are introduced and a method of creating an ultrafuzzy set from a fuzzy set is described in § 2. The individual object properties and their fuzzy membership evaluation approaches are discussed in § 3. Section 4 deals with the problems of relational properties between objects. The problems of man–machine interaction using these membership values are approached with examples in § 5. Some scope for further work is presented in § 6.

2. Fuzzy and ultrafuzzy subsets: a note

Consider a set of objects U. A fuzzy subset (Zadeh 1965) of U may be defined in terms of some properties of objects in U. Let the property be bigness of the objects. Then the fuzzy subset called big objects can be considered as a mapping μ_{big} from U into [0, 1]. For any object $A \in U$, $\mu_{\text{big}}(A)$ is called the degree of membership of A in μ_{big}. When a functional form can be given to $\mu_{\text{big}}(A)$, it is called a membership function. As shown below in (1), μ_{big} can be expressed as a monotonic function of the area of the objects.

A crisp (i.e. ordinary, non-fuzzy) subset of U can be regarded as a special case of fuzzy subset, where the mapping μ_{big} is into {0, 1}. Suppose, we are given an object A_1 and asked the question 'Is A_1 big?'. Then we should transform the fuzzy subset into an ordinary subset by thresholding the membership of A_1. If $\mu_{\text{big}}(A_1)$ is greater than the threshold, then we call the membership of A_1 to the ordinary subset to be 1 and our answer to the above question is 'yes'; else, the answer is 'no'.

One of the reasons for using the fuzzy approach is its ability to express natural language-like properties. Thus, one may use the linguistic hedge 'very' and define a fuzzy subset as 'very big objects'. A simple way of doing it is proposed by Zadeh:

$$\mu_{\text{very big}}(A) = [\mu_{\text{big}}(A)]^2$$

He also proposed the definition of hedge 'more or less' as

$$\mu_{\text{more or less big}}(A) = [\mu_{\text{big}}(A)]^{0\cdot 5}$$

We can extend these to superlative hedges like 'extremely' and 'very approximately'. Thus,

$$\mu_{\text{extremely big}}(A) = [\mu_{\text{big}}(A)]^4$$

$$\mu_{\text{very approximately big}}(A) = [\mu_{\text{big}}(A)]^{0\cdot 25}$$

Another basic purpose of using a fuzzy subset is its ability to quantify vagueness and ambiguity. However, once the membership function is defined, it maps the set of objects precisely into [0, 1] and no ambiguity is encountered in further analysis. To make the fuzzy model more realistic, the concept of an ultrafuzzy subset (Zadeh 1983) has been introduced. The membership of a given object in an ultrafuzzy subset lies in an interval, rather than being single-valued. A typical example is given in Fig. 1 for the fuzzy subset 'big objects', where bigness is defined in terms of area. It is clear that the 'membership interval' will be less, i.e. the ambiguity in defining a big object will be less when the area is too small or too big while the ambiguity will be maximum at some intermediate value. We can construct such an ultrafuzzy subset using the following steps.

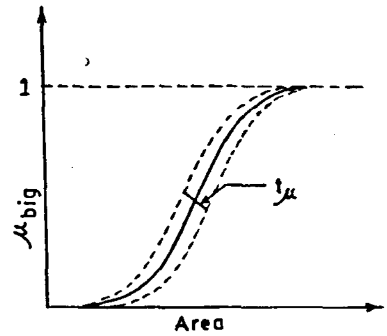

Figure 1. Membership function for an ultrafuzzy set.

(*a*) Use (1) in § 3 to find $\mu_{big}(A)$. For simplicity, let it be μ.

(*b*) Decide the maximum thickness of the membership function t_{max} and the value $\mu = \mu_0$ where the thickness is $t = t_{max}$.

(*c*) Define t as

$$) = \begin{cases} t_{max} \sin \dfrac{\mu}{\mu_0}, & \mu \leqslant \mu_0 \\ t_{max} \sin \dfrac{1-\mu}{1-\mu_0}, & \mu \geqslant \mu_0 \end{cases}$$

(*d*) Find the normal to the tangent of the membership curve at μ. Cut the normal on both sides by a distance $t/2$.

In the following sections, the methods of finding fuzzy membership are only described. Using the above procedure, ultrafuzziness can also be embedded. One way to make a hard decision under an ultrafuzzy environment is to choose randomly a membership value within the membership interval of the object under consideration. The chosen membership is then thresholded to give, say 'big' or 'not big' hard decisions.

3. Individual object properties

There exist many size, shape and topological properties of an object which can be given fuzzy interpretation. Some of these properties are described below.

3.1. *Bigness*

Bigness may be graded in terms of the area $S(A)$ of an object A. If B is an object which is certainly big, i.e. its membership to the set of big objects $\mu_{big}(B) = 1$, then a simple relation may be

$$\mu_{\text{big}}(A)=\begin{cases}[S(A)/S(B)]^{\beta} & \text{if } S(A)\leqslant S(B),\quad \beta\geqslant 1\\ 1 & \text{otherwise}\end{cases} \tag{1}$$

The term β is used for controlling the crispness of the membership function. In all the following relations we assume that $\beta \geqslant 1$ unless otherwise stated.

3.2. *Circularity and convexity*

The definition of circle given in terms of its quadratic equation classifies an object to either 'circles' or 'non-circles'. However, human perception of circularity seems to be gradual rather than abrupt. The degree of circularity may be given a fuzzy membership value $\mu_c(A)$, a simple measure of which is

$$\mu_c(A)=[4\pi S(A)/G^2(A)]^{\beta}$$

where $G(A)$ is the perimeter of A. However, it is a poor measure for a circular object with an uneven border. A better measure is as follows. Let 0 be the 'centre of mass' of the object whose coordinates are

$$x(O)=\frac{1}{S(A)}\sum_{p\in A}x(p),\quad y(O)=\frac{1}{S(A)}\sum_{p\in A}y(p)$$

and let m and δ be the mean and standard deviation of the distance of border points of A from O. Then, we may define

$$\mu_c(A)=[1-(\delta/m)^2]^{\beta} \tag{2}$$

Similar definition can also be given for fuzzy grade of ellipticity.

Convexity also has a strict definition, but one can give a gradation of convexity. Let $H(A)$ denote the convex hull of A. Then $H(A)-A$ may be called the convex deficiency of A. The fuzzy grade of convexity is defined as

$$\mu_{\text{co}}(A)=\left[1-\frac{S(H(A)-(A))}{S(H(A))}\right]^{\beta} \tag{3}$$

In general, a non-convex object can be represented as a convex set and its convex deficiency. Proceeding in this way, a concavity tree (Sklansky 1972) of the object can be created and at each concavity node, a fuzzy degree of convexity may be attributed.

3.3. *Elongatedness*

Elongatedness is another property that should not be binary truth-valued and a fuzzy gradation of elongatedness is more appropriate. This grade may be found in terms of $S(A)/G^2(A)$ as in the case of circularity but it is not a good measure if the object contains a noisy border.

An alternative definition is based on the width $W(A)$ of the object A. An object is elongated if its area is much larger than its width. The fuzzy degree of elongatedness is defined as

$$\mu_e(A)=\left[1-\frac{W^2(A)}{4\pi S(A)}\right]^{\beta} \tag{4}$$

However, evaluation of width is a difficult task. One idea due to Rosenfeld (1982) is to use shrinking of the object in a digital grid. Loosely speaking, shrinking strips the border *pels* of the object in each iteration. The width may be termed as twice the number of shrinking iterations necessary so that the object vanishes completely.

More generally, one can define thickness from the skeleton (Blum 1967) of the object. There exist approaches (Zhang and Suen 1986) for obtaining connected skeleton. For each skeletal pixel, the thickness may be defined as twice the smallest distance of the skeletal pixel to the border of the object. The average of these thicknesses over all skeletal pixels may be termed as the thickness of the object. However, the skeleton is very sensitive to noise and the noisy end branches should be deleted before processing. Also, an object may have different elongated parts and it may be useful to find the degree of elongatedness for each part. Then a tree structure of elongated parts may be created similar to the concavity tree stated above. The topic is being investigated in detail and useful results will be communicated in a separate paper.

3.4. *Degree of straightness*

If an object A contains only one elongated part then it may be meaningful to find its degree of straightness. Consider the centre of mass O and find a line through O so that the squared sum of distances of all points in A is minimum. This line is called the 'major axis' of A—more detail is given by Parui and Dutta Majumder (1983). Next, find the smallest rectangle R with sides parallel and perpendicular to the major axis so that $A \subseteq R$. The degree of straightness may be defined as

$$\mu_{st}(A) = \left[F \frac{S(A \cap R)}{S(R)} \right]^{\beta} \tag{5}$$

where F is a factor dependent on elongatedness of A. We can define

$$F = \begin{cases} 1 & \text{if } \mu_c(A) > t \\ \mu_e(A) & \text{otherwise} \end{cases}$$

where $0 < t \leqslant 1$ is a pre-defined threshold on elongatedness.

3.5. *Degree of (horizontal or vertical) orientation*

Again, the degree of horizontal or vertical orientation is more meaningful if the object A is elongated and straight. For an object with zero thickness, i.e., a line, the problem is simple. Find the angle θ of the line with the horizontal direction and define the degree of horizontal orientation as

$$\mu_{ho}(A) = \left[1 - \frac{2\theta}{\pi} \right]^{\beta} \tag{6 a}$$

while the degree of vertical orientation may be defined as

$$\mu_{ve}(A) = 1 - \mu_{ho}(A) \tag{6 b}$$

For an object with finite thickness, the angle θ is measured between its major axis and the horizontal direction. Equation (6) is then used for the degree of horizontal (vertical) orientation (Fig. 2).

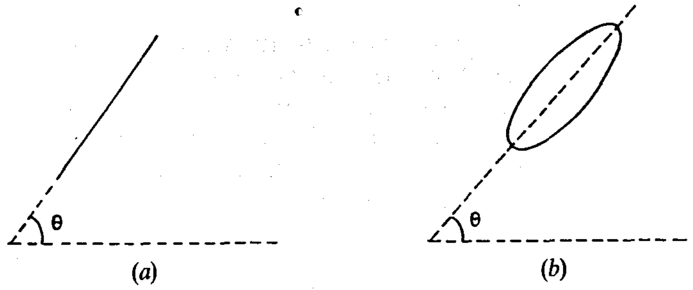

Figure 2. Evaluation of degree of horizontal/vertical orientation. (*a*) For a line, (*b*) for an extended object.

3.6. *Position of an object in an image frame*

Let A be an object in the image frame and we want to describe its position with respect to the frame. In particular we want to fill in the blank of the phrase 'A is at the ... of the image frame' by words like 'middle', 'right', 'left', with possible hedges like 'more or less', 'extreme', etc.

3.6.1. *Middleness*

Let O be the centre of the image frame and O' be the centre of mass of A. It is intuitively understood that an object (especially convex) A can be said to be at the middle if the euclidean distance $d(O, O')$ between O and O' is very small (Fig. 3(*a*)).

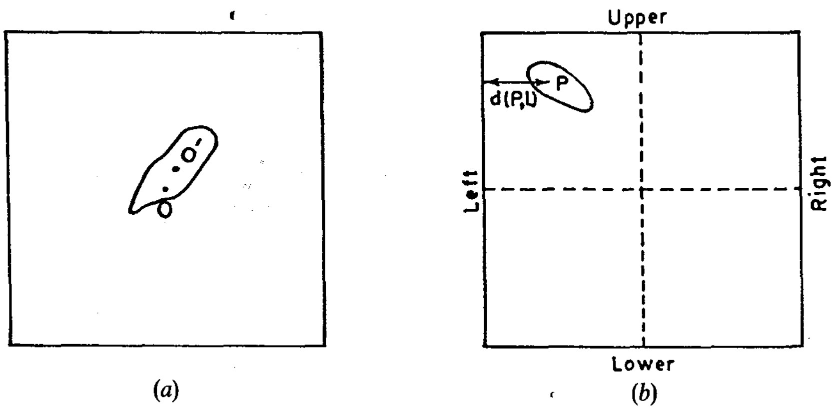

Figure 3. Position of an object in the image frame. (*a*) Degree of middleness, (*b*) degree of leftness, rightness, etc.

We consider middleness as a fuzzy membership value $\mu_{mi}(A)$ and define it in terms of $d(O, O')$ and the length of the diagonal L of the image frame

$$\mu_{mi}(A) = \left[1 - \frac{2d(O, O')}{L}\right]^{\beta} \tag{7}$$

It may be noted that this definition makes $\mu_{mi}(A)$ invariant under rotation of A with respect to the image frame, which is desirable. Another definition of $\mu_{mi}(A)$ is possible but this one is simple and it gives results consistent with human visual judgement.

3.6.2. *Leftness, rightness, etc.*

Suppose that $\mu_{mi}(A)$ is very low. Then it is necessary to test if A is to the 'left', 'right', etc. of the image frame. A simple approach is as follows.

Partition the image frame into four quadrants as shown. If A is in one quadrant as in Fig. 3(*b*) then it has non-zero degree of membership to the 'left' and 'upper' directions. To find fuzzy membership to the 'left' direction $\mu_{le}(A)$, find the normal distance of each point P of A from the left side image frame $d(P, l)$ and average them as

$$D(A, l) = \frac{1}{S(A)} \sum_{p \in A} d(P, l) \tag{8}$$

If M is the length of each side of the image frame then

$$\mu_{le}(A) = \begin{cases} \left[1 - \dfrac{2D(A, l)}{M}\right]^{\beta} & \text{if } D(A, l) \leqslant M/2 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Similar definitions can be given to memberships in other directions. Let $\mu_{up}(A)$ denote the membership to upper direction. The membership to upper-left direction $\mu_{up-le}(A)$ can be found from $\mu_{up}(A)$ and $\mu_{le}(A)$. It is clear that $\mu_{up-le}(A)$ should be high if

(*a*) both $\mu_{le}(A)$ and $\mu_{up}(A)$ are high and
(*b*) $|\mu_{le}(A) - \mu_{up}(A)|$ is low.

A function that may be used is

$$\begin{aligned} \mu_{up-le}(A) &= \frac{\mu_{le}(A) + \mu_{up}(A)}{2}\left[1 - \frac{|\mu_{le}(A) - \mu_{up}(A)|}{\max\{\mu_{le}(A), \mu_{up}(A)\}}\right] \\ &= 0 \quad \text{if } \mu_{le}(A) = \mu_{up}(A) = 0 \end{aligned} \tag{10}$$

The second relation is used to avoid a 0/0 problem of first relation.

These definitions do not make μ's rotation invariant of A. In fact, it is not desirable. In practice, an object may have parts in all the four quadrants of the image partition leading to a non-zero contribution of membership to different sides.

3.6.3. *Filling the phase*

To fill the gap in the phrase 'A is at the ... of the image frame' the word X should be chosen for which

$$\mu_X(A) = \max_i \{\mu_i(A)\} \tag{11}$$

where i denotes le, up, up–le, mi, etc. Suppose $X = \text{le}$, then we accept that A is at the 'left' of the frame. Next, we may test if we can use the hedge 'very' before 'left' by testing if $\mu_{le}^2(A) \geqslant T_{le}$ where T_{le} is a pre-defined threshold. Embedding ultrafuzziness is also possible by the approach described in § 2.

Relative properties

4.1. *Relative positions of two objects*

For simplicity, consider two point objects A and B. We try to quantify the property 'B to the X of A' where X denotes 'left', 'right', 'above' and 'below', by fuzzy membership value denoted by $\mu_X(B, A)$. A simple solution is to draw a horizontal or vertical line through A to denote the X-direction and measure the angle θ that AB makes with the X-direction at A. If $\theta = 0$ then we expect that $\mu_X(B, A) = 1$. If $|\theta| \geqslant \pi/2$ then $\mu_X(B, A) = 0$. A function that monotonically maps $0 \leqslant |\theta| \leqslant \pi/2$ into [0, 1] can be used for $\mu_X(B, A)$. It may be ensured that $\mu_{\text{left}}(B, A) = \mu_{\text{right}}(A, B)$.

When A and B are not point objects and have finite area, then the problem is more difficult. An attempt to extend the above method may be as follows. Find $\mu_X(P, Q)$ for all points $P \in A$ and $Q \in B$ and make an aggregate over all $\mu_X(P, Q)$s. One possibility is

$$\mu_X(B, A) = \frac{1}{S(A)S(B)} \sum_{p \in A} \sum_{Q \in B} \mu_X(P, Q) \tag{12}$$

where $S(A)$ and $S(B)$ are areas of A and B, respectively. It may be seen that $0 \leqslant \mu_X(B, A) < 1$. However, the definition has the disadvantage that $\mu_X(B, A) \neq 1$ even when B is 'visually' to the left of A. In addition, it is computationally expensive. An alternative and simple method is described below.

Enclose the object A by the smallest rectangle with horizontal and vertical sides. Extend the sides in both directions. As shown in Fig. 4, B may be situated (*a*) entirely outside the rectangle (as B_1 and B_2), (*b*) entirely within the rectangle (as B_3). Draw diagonals to the rectangle and hatch regions as shown in Fig. 4. Note that parts inside the rectangle are also hatched. The cross-hatching refers to regions where memberships to two neighbouring positions e.g. 'left' and 'below' are non-zero. Find the area of B that falls under the X-region. Let this be $S_x(B)$. Find the centre of mass $O_x(B)$ for each x for which $S_x(B)$ is non-zero. Let X denote 'left'. Find the distance of $O_x(B)$ from the horizontal bisector of the rectangle. Let this be $d_x(B)$. Then the degree of membership of 'B to the left of A' may be denoted as

$$\mu_x(B, A) = \left[\frac{1}{1 + d_x(B)} \frac{S_x(B)}{S(B)}\right]^{\beta} \tag{13}$$

We can define membership to a composite direction such as 'left-below' using (5).

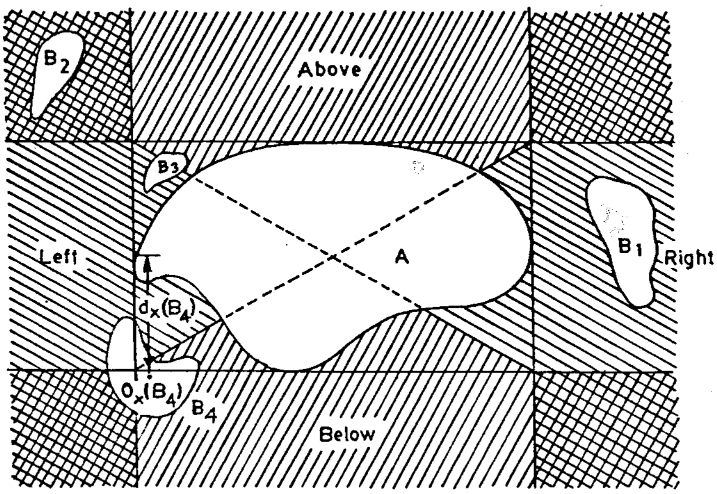

Figure 4. Relative position of two objects.

4.2. *Degree of betweenness*

We want to decide to which degree an object C is in between two objects A and B. To do so, let us first define the zone of betweenness. Consider two lines P_1Q_1 and P_2Q_2 each of which touches both A and B at one point only. The resultant enclosed region excluding A and B is called the zone of betweenness (JB) described in Fig. 5(a). Several situations that may occur while testing betweenness are shown in Figs 5(a)–5(d). Consider Fig. 5(a) where the object C is entirely within the JB while D is partly within the JB. So we expect that the degree with which C is between A and B is unity while the degree of D should be less than 1. However, in Fig. 5(b) we would expect that the degree of betweenness (DB) for C is 1 although it has parts outside JB. Note that the region C_1 is on the outer side of P_1Q_1 while C_2 is on the outer side of P_2Q_2.

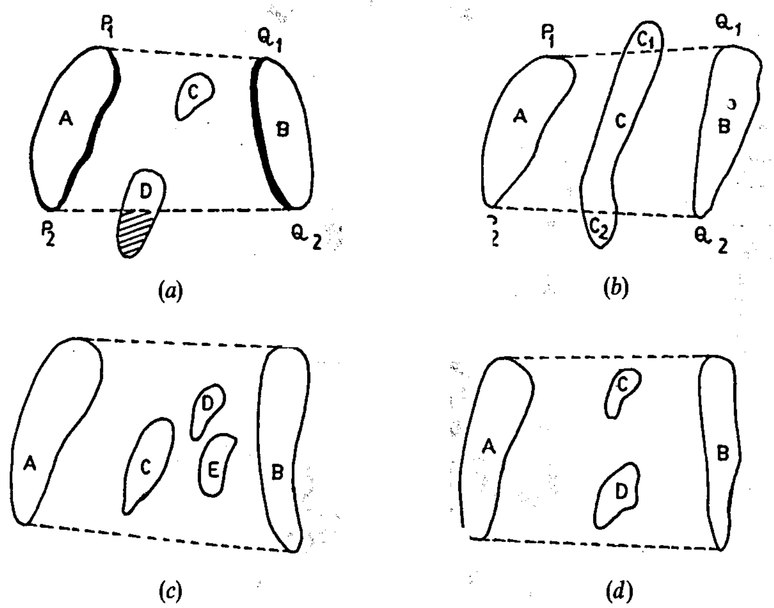

Figure 5. Different situations for the degree of betweenness. In (a) the zone of betweenness is enclosed by the thick and dotted lines.

There may be more than one object sitting within the JB of A and B. In Figs 5(a) and 5(d), the DB of object C is not affected by that of object D. But in a situation like Fig. 5(c), object E is obscured by object C and hence E is not between A and B. Similarly, C is partly obscured by D and E and hence its DB should be less than 1.

A measure of fuzzy DB should be able to capture the ideas behind all such situations and yet it should be as simple as possible. However, one should take care of two special cases.

Special case 1

Consider Fig. 6. In this case we would expect that $\mu_{db}(C, A, B) = 0$. Rather $\mu_{db}(A, C, B)$ should have a non-zero value. However, $\mu_{db}(A, C, B)$ will be less meaningful than the degree of surroundedness of A by C. So, before calling the routine to compute $\mu_{db}(A, B, C)$ it is wise to call the routine for finding the degree of surroundedness for each pair from $\{A, B, C\}$. If in each case, the degree is low, then the routine for $\mu_{db}(A, C, B)$ may be called. The problem of surroundedness will be discussed later.

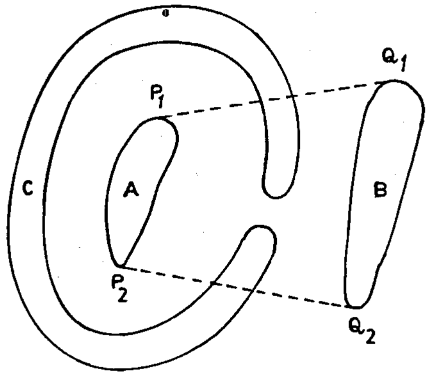

Figure 6. Special case of degree of betweenness.

Special case 2

Consider Fig. 7. In these situations the lines P_1Q_1 or P_2Q_2 intersect, or they cannot be drawn. Again the degree of surroundedness plays a more prominent role than DB in such cases. Although DB can be found by some complex means, we leave it for discussion in a separate paper.

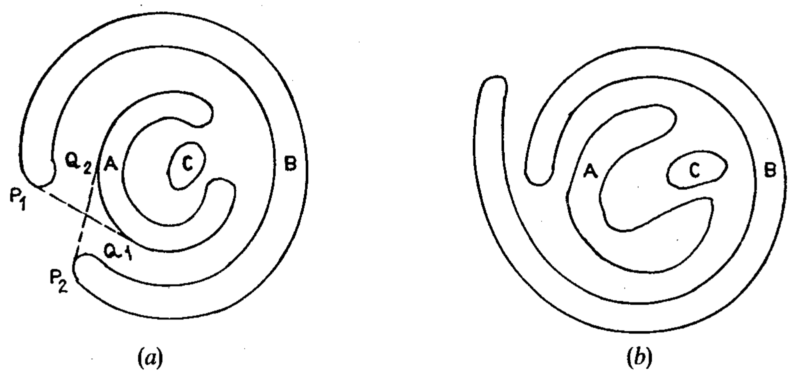

Figure 7. Other special cases for degree of betweenness. In (*b*) the *JB* cannot be drawn properly.

A simple algorithm for finding DB may be as follows.

Step 1

Find the JB of the objects A and B by defining lines P_1Q_1 and P_2Q_2. If the lines intersect or if they cannot be drawn (as in Special case 2), stop.

Step 2

Test whether C is the only object lying partly or totally within the JB. If C is the only object, continue. Else, go to Step 4.

Step 3

If C is the only object, then: if C lies entirely within JB, let $\mu_{db}(C, A, B) = 1$; if C extends beyond the JB on one side as in the case of object D in Fig. 5(*a*) then find the area $S(C - C_1)$ of the portion that is inside the JB. Define $\mu_{db}(C, A, B) = [S(C - C_1)/S(C)]^{\beta}$. If C extends on both sides beyond the JB as in the case of Fig. 5(*b*) then find the areas $S(C_1)$ and $S(C_2)$ of the portions C_1 and C_2 outside the JB. Let $S_o = \min\{S(C_1), S(C_2)\}$ and $S_{in} = S(C) - S(C_1) - S(C_2)$. Then define

$$\mu_{db}(C, A, B) = [(S_{in} + 2S_o)/S(C)]^\beta \tag{14}$$

Stop.

Step 4

(If there exist objects other than C within the JB). Find the lines that touch the objects at one point and cut the lines P_1P_2 and Q_1Q_2 in equal proportions. As shown in Fig. 8

$$\frac{\overline{P_1V_1}}{\overline{P_1P_2}} = \frac{\overline{Q_1V_2}}{\overline{Q_1Q_2}}$$

Find parts $C_1^o, C_2^o, \ldots, C_n^o$ of C that are segmented by the lines of extreme extent of other objects within the JB. Find S_{in} as in Step 3 and make $S_{in} \Leftarrow S_{in} - \sum_{i=1}^{n} S(C_i^o)$. Use (14) to find $\mu_{db}(C, A, B)$. Stop.

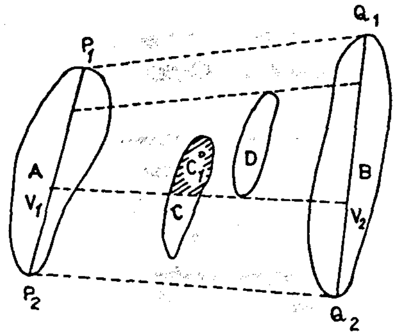

Figure 8. Situation when more than one object lies in JB.

4.3. *Degree of surroundedness*

Classically, we consider (complete) surroundedness as follows. Let A and B be two objects and let F be the border of the image frame in which they are situated. If A surrounds B, then we cannot go from any point of B to any point of F without crossing A.

According to this crisp definition, A_1 surrounds B_1 while A_i does not surround B_i, $i = 2, \ldots, 5$ in Fig. 9. But a person who is not familiar with this definition, will perhaps describe the objects as follows

A_1 completely surrounds B_1

A_2 does not surround B_2

A_3 partly surrounds B_3

A_4 completely surrounds B_4

A_5 and B_5 partly surround each other

If a linguistic description as above is necessary, the surroundedness should be given a fuzzy grade of membership. Let us see how it can be done.

Consider any point P in E. Draw lines through P. If there exist at least one line

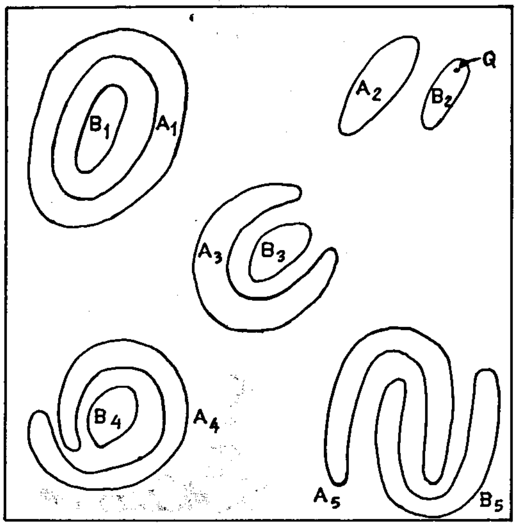

Figure 9. Situations for degree of surroundedness.

through P that meets A at two border points in opposite directions without crossing the interior of A then P is said to be surrounded by A. Notice from Fig. 9 that the point Q does not satisfy this condition and hence Q is not surrounded by A. Consider the case of a digital image where a point is replaced by a pel. Find the pels of B that are surrounded by A. If n is the number of pels out of a total N, these are surrounded by A, then we define the degree of surroundedness $\mu_{ds}(B, A)$ as

$$\mu_{ds}(B, A) = \left(\frac{n}{N}\right)^{\beta} \tag{15}$$

A computationally more complex definition may be as follows. For each P find the angle $\alpha(p)$ over which any line drawn through P meets A at two points without crossing its interior. Then the degree of surroundedness of P is $\alpha(P)/\pi$. Then $\mu_{ds}(B, A)$ may be modified as

$$\mu_{ds}(B, A) = \left[\sum_{p \in B} \alpha(P)/(N\pi)\right]^{\beta} \tag{16}$$

where $0 \leqslant \alpha(P) \leqslant \pi$.

The following properties can be readily observed

(*a*) A does not surround B or vice versa if A and B are convex, but the converse is not true.

(*b*) If A and B partly surround each other then both A and B are non-convex.

(*c*) If A completely surrounds B then A is non-convex.

It may be useful to distinguish the surroundedness relation of the pair A_1, B_1 from that of A_4, B_4. To do so, we say that B_1 lies in the 'hole' of A_1.

4.4. *Degree of (parallel) alignment*

For two straight lines in space, it is easy to define the degree of (parallel) alignment between them. Let θ be the angle (smaller of the angles) between the lines. If $\theta = 0$

then the lines are perfectly parallel. If $\theta = \pi/2$ then the lines are perpendicular to each other. The fuzzy degree of parallelism may be defined as

$$\mu_{pa}(A, B) = [1 - 2\theta/\pi]^{\beta} \tag{17}$$

Note that $\mu_{pa}(A, B) = \mu_{pa}(B, A)$.

For objects of finite area, especially if the objects are compact or if the objects are non-convex and have several elongated branches, the degree of parallelism is more difficult to define. One solution to the problem is to define the major axis through the centre of mass as in § 3.4 for both the objects and find the angle between them. The above equation then gives the degree of parallel alignment.

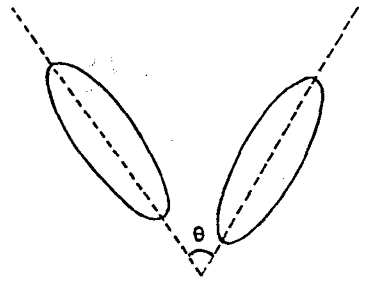

Figure 10. Determination of angle for parallel/perpendicular alignment.

5. Man–machine interaction

In the preceding sections we saw how an object shape property and an inter-relation among objects can be quantified. Let us now try to find some application in man–machine communication. Suppose there are several objects $A_1, A_2, \ldots, A_n$ in an image frame and the machine is asked to execute one or more of the following instructions.

(*a*) Sort all the big/convex/elongated/straight objects.
(*b*) Pick the objects from the middle/left/right ... etc. of the image frame.
(*c*) Place an object A_i to the left/right/ ... of another object A_j.
(*d*) Place A_i to the left/right/ ... and in parallel/perpendicular to A_j.
(*e*) Place A_i between A_j and A_k.
(*f*) Place A_i so that it is maximally surrounded by A_j.
(*g*) Pick the object to the left of A_i.
(*h*) Pick the object between A_i and A_j.
(*i*) Pick the object surrounded by B_j.

Other combinations of instructions may also be generated for execution. Depending on the situation, an instruction may not be followed. For example, in (*f*) above, if A_j is convex, it cannot surround A_i. Similarly, in (*e*) A_j and A_k may be so close that A_i cannot be placed in between them. Let us now see how the above instructions can be executed.

To execute (*a*), the fuzzy degrees of bigness/elongatedness/ straightness are found and the objects are sorted according to the decreasing order of magnitude of membership values. If an ultrafuzzy set is used, a random number generator is used to choose a value within the membership interval of each object and then the sorting is done.

To execute (*b*), the non-zero membership valued objects are chosen and ordered in decreasing order of magnitude of membership. The ultrafuzzy model is tackled as above.

To execute (*c*)–(*f*), the placement is made to a place where the fuzzy grade of 'to the xxx of' relation has maximum possible membership. If placement is not possible, a message is returned. For (*c*) and (*e*) only translation is allowed while for (*d*) and (*f*) rotation and translation are allowed.

To execute (*g*)–(*i*), the membership of the relation of all objects are found and the object with maximum membership is picked.

While execution of (*a*)–(*b*) as well as (*g*)–(*i*) are rather straightforward, execution of (*c*)–(*f*) requires some placement planning similar to the path planning of a robot. In our experiment we assume that there exists sufficient space for placement and no complicated planning is required.

To examine how well the above definitions work, a set of sixteen objects are taken in an image frame. The objects are numbered 1, 2, ..., 16 as shown in Fig. 11. Most of the objects are arbitrarily shaped except those numbered 5, 15 and 16 which resemble the outline of English numerals 7, 5 and 2, respectively. The image frame is sampled into 256 × 256 cells and the objects are digitized accordingly. Ordering of these objects according to fuzzy grade of several individual object properties are presented in the Table.

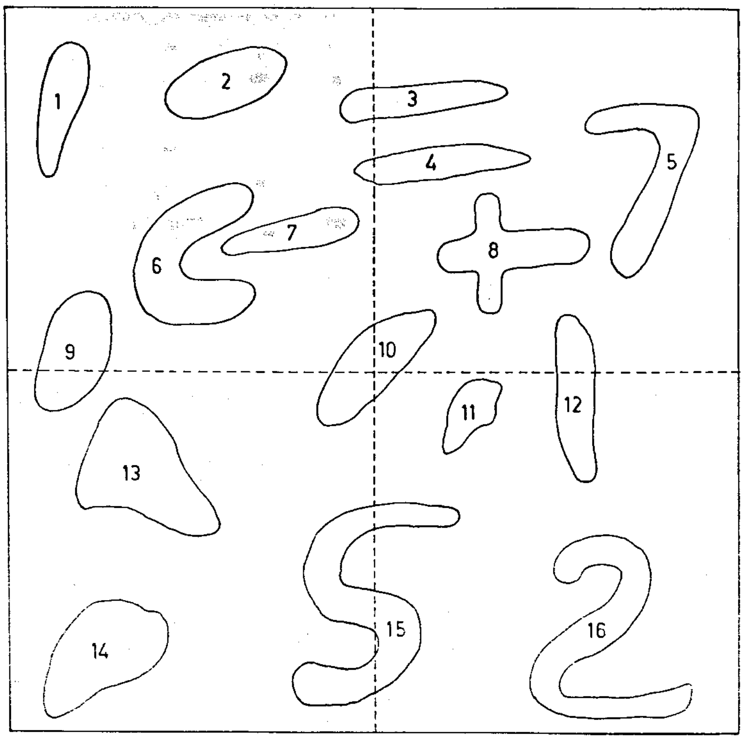

Figure 11. Some test objects for man–machine interaction.

The properties such as leftness, etc. refer to the image frame. Here simple properties such as bigness are not shown. Also, objects with rightness, upper-rightness are not shown in the Table.

Property	Rank in decreasing order
Middleness	10, 11, 7, 8, 12, 6, 4, 13, 9, 3, 2, 5, 15, 16, 14, 1
Leftness	1, 9, 14, 13, 6, 2, 7, 15, 10, 3, 4 (others need not be counted)
Upper-leftness	1, 6, 2, 7, 9, 10, 3, 4 (others need not be counted)
Elongatedness	16, 15, 5, 6, 12, 4, 3, 1, 8, 7, 10, 2, 9, 14, 13, 11
Straightness	7, 3, 4, 12, 1, 2, 10, 9, 8, 5, 6, 11, 14, 13, 15, 16
Convexity	2, 9, 10, 7, 1, 3, 4, 12, 14, 13, 11, 8, 6, 5, 16, 15

Let us now investigate the relational properties among objects. Consider the case of the relation 'to the left of'. The pairs (1,2), (2,3), (1,5), (2,5), (3,5), (4,5), etc. satisfy the exact 'to the left of' relation. Suppose it is asked to find all the objects to the left of object number 7. The answer is 9, 13, 1, 14, 6 (partly), 2 (partly). The answer 'partly' refers to the fact that fuzzy membership in the relation is less than 1. Similarly, among all triplet of objects, the 'degree of betweenness' is maximum for 1 sitting between 10 and 12. For parallelism, 3 and 4 are most parallel followed by 3 and 7, while for surroundness 6 partly surrounds 7. In fact, the degree of surroundedness for any other pair is zero.

It may be observed that these results agree well with the visual perception. Some experiments were made for placement of objects by invoking instructions such as (*c*), (*d*), (*e*) and (*f*). For example, when asked to pick an object to the left of object 7 in Fig. 11, object 6 was picked. Thus, the results were satisfactory for objects of simple shape as in Fig. 11. However, it is necessary to examine the algorithms for more complex shapes and objects with noisy borders.

6. Conclusion

Recognition and description of shape and relational properties of two-dimensional simply closed outlines using fuzzy set theoretic concepts are proposed and implemented on some artificially generated figures. The expressions are simple and yet capture the basic notion of the shape properties. Here, the relations between two or among three objects have been defined but can be extended for more than three objects, e.g. 'A sits among B_1, B_2, ..., B_n'.

The work may be useful in computer vision and scene analysis problems. It is interesting to note how these properties and relations may be extended to grey tone objects. In addition, it may be useful to extend the work for three-dimensional objects.

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