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Grain size distribution in suspension from bed materials

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ABSTRACT

Experimental results show that the grain size distribution of suspended material is related to flow parameters and grain size distribution in the bed. A theoretical model has been developed to compute the suspension grain size distribution on the basis of diffusion equations, taking into account the effect of hindered settling due to the increased concentration in suspension. Fluid velocity closest to the bed is estimated by using the concept of migration velocities of particles in the bed layer. Comparisons of data computed by the proposed method and data from actual observations show generally good agreement.

INTRODUCTION

Sediment transport under hydrodynamic conditions has received considerable attention in terms of the basic problems related to the phenomenon of grain sorting. During the processes of grain sorting, grain sizes in a river normally decrease in a downstream direction. Our understanding of sorting processes is important for the solution of practical problems in the field of aggradation and degradation, siltation and the paleocurrent analysis of ancient river systems. It is also of interest in studies in which the grain size of suspended sediment must be predicted from the nature of the bed material.

The logarithmic transformation of particle sizes ($\varphi = -\log_2 D$, where D is in millimetres) of naturally transported sediments, both ancient and modern, often reveals a normal distribution (Krumbein, 1934; Blench, 1952; Sengupta, 1975), although in some cases, log–hyperbolic and log–skew–laplace grain size distributions have also been reported (Bagnold & Barndorff-Nielsen, 1980; Christiansen *et al.*, 1984; Fieller *et al.*, 1984, 1992; Sengupta *et al.*, 1991). Investigation of a fluvial system revealed that the suspension deposits on sand bars have a tendency to develop a lognormal distribution with increasing distance in the downstream direction (Sengupta 1975).

To gain further insights into the process of grain sorting in rivers, grain size distributions of suspended loads at different flow velocities over six sand beds of different grain size distributions were studied in laboratory flumes (Sengupta 1975, 1979; Ghosh *et al.*, 1979, 1981; Sengupta *et al.*, 1991). Controlled experiments have shown that grain size distribution of suspended sediments is related to bed materials, flow velocity and height of suspension above the sand bed. These studies have also shown that a sorting process is initiated immediately above the bed and that the grain sizes of the bed layer influence the size distribution of the suspended load above. A theoretical model has been developed by Ghosh *et al.* (1981) to estimate the grain size distribution of bed load and suspended load from bed materials and flow parameters. Following this model it is possible to calculate the suspension concentration at any height above the bed if the relative concentration of the particle at the bed is known. This method allows direct computation of suspended load from the grain size distribution of a bed without going through an intermediate stage (bed load) as required by other methods (Einstein, 1950; Gessler, 1965). The Gessler method is used to compute the bed layer grain size distributions above the sand bed and the Rouse equation is then used to obtain the grain size distribution in suspension, using the bed loads as references. Several existing methods for computation of bed load and suspended load have been discussed in detail in Ghosh *et al.* (1981). Each of these methods has been tested by comparing the computed values with actual observations of the grain size distributions of suspended load samples collected in laboratory flumes. The use of any particular method cannot be claimed to be superior to the others for the prediction of concentration at the level of suspension. Ghosh & Mazumder (1981) have also studied the conditions of symmetry, unimodality and lognormality of grain size in suspension. They have shown that the suspended load will have a tendency to be lognormally distributed for that size range of bed material for which the settling velocity is linearly distributed on a logarithmic scale.

The purpose of this article is to re-examine the mathematical model developed by Ghosh *et al.* (1981) for the direct computation of suspended grain size distribution from the bed materials, taking the following into account: (1) the flow velocity at the top of the bed layer (bed layer thickness amounts to 2·5 mm) is assumed to be the same as the migration velocity of a representative size fraction (4·5φ) in the bed layer; and (2) the fall velocity of sand particles decreases with an increase in the sediment concentration in suspension. According to Maude & Whitmore (1958), the term 'hindered settling' is used to designate the decrease in the fall velocity of sediment in suspension resulting from an increase in sediment concentration. From the equations of mass and momentum in the fluid–sediment mixture, Thacker & Lavelle (1977) have shown that hindered settling may be accounted for by a factor of $(1-c)^{\alpha}$ multiplied by the single particle fall velocity w_0, where c is the concentration of material per unit volume and α is the exponent of reduction of fall velocity; this is consistent with the data obtained by Maude & Whitmore (1958). Lavelle & Thacker (1978) and Woo *et al.* (1988) have used similar concepts to model the reduction of particle fall velocity in sediment laden flows. The efficiency of the present model has been tested by comparing the computed values with observed grain size distribution data collected in close-circuit hydraulic flumes by Sengupta (1975, 1979) and Ghosh *et al.* (1979, 1981).

MATHEMATICAL MODEL

In a steady and uniform turbulent flow, where the concentration c varies only with the verticle coordinate y throughout the depth, and the diffusion coefficients of sediment and water are assumed to be equal (i.e. $\varepsilon_s=\varepsilon_m$), the vertical distribution of sediment concentration can be described (Hunt, 1954) as

$$\varepsilon_s\frac{dc}{dy}+(1-c)c\,w=0, \qquad (1)$$

where w is the sediment fall velocity in a fluid–sediment mixture (see Appendix for a full explanation of the notation). Experiments on fluid–sediment mixtures have shown a substantial reduction of the particle fall velocity due to the presence of neighbouring particles. The fall velocity of sediment, w, varies with concentration, c, as a result of a hindered settling effect (Maude & Whitmore, 1958) and is given by

$$w=w_0(1-c)^{\alpha}, \qquad (2)$$

where w_0 is the fall velocity in clear water and α is the exponent of reduction of fall velocity, which varies from 2 to 5 depending on particle Reynolds number and the size of non-cohesive sediment particles.

For fully developed turbulent flow, the momentum diffusion coefficient for sediment ε_s is defined as

$$\tau=-\rho\,\overline{u'v'}=\rho\varepsilon_s\frac{du}{dy}=\rho u_*^2\left(1-\frac{y}{d}\right), \qquad (3)$$

where τ is the turbulent shear stress, $-\rho\,\overline{u'v'}$ is the Reynolds stress, d is the flow depth, ρ is the density of fluid and u_* $(=\sqrt{\tau_0/\rho})$ is the friction velocity. The von Kármán velocity distribution used by Hunt (1954) is given by

$$\frac{u-u_m}{u_*}=\frac{1}{\chi}\left\{\left(1-\frac{y}{d}\right)^{1/2}+B\ln\left[\frac{B-(1-y/d)^{1/2}}{B}\right]\right\}, \qquad (4)$$

where satisfies the maximum velocity (u_m) at the free surface ($y=d$). B is an integrating constant and χ is the von Kármán constant (0·4). If $B=1$ in Eq. (4), the velocity distribution (4) agrees with that of von Kármán (1930) in which the velocity gradient is infinite at the bed. The constant B was chosen by Hunt (1954) to give the best fit with the experimental data. Ghosh *et al.* (1981) determined the constant B from the condition that $u=0$ at $y=k_s$, the roughness of the bed. Since k_s is extremely small in comparison with the depth of flow d, the ratio k_s/d is neglected for higher powers of k_s/d. Under this assumption and after relatively few calculations (Ghosh *et al.*, 1979) the constant B is obtained as

$$B=1-\frac{1}{2}\frac{k_s}{d}+\exp\left(-1-\frac{\chi\,u_m}{u_*}\right). \qquad (5)$$

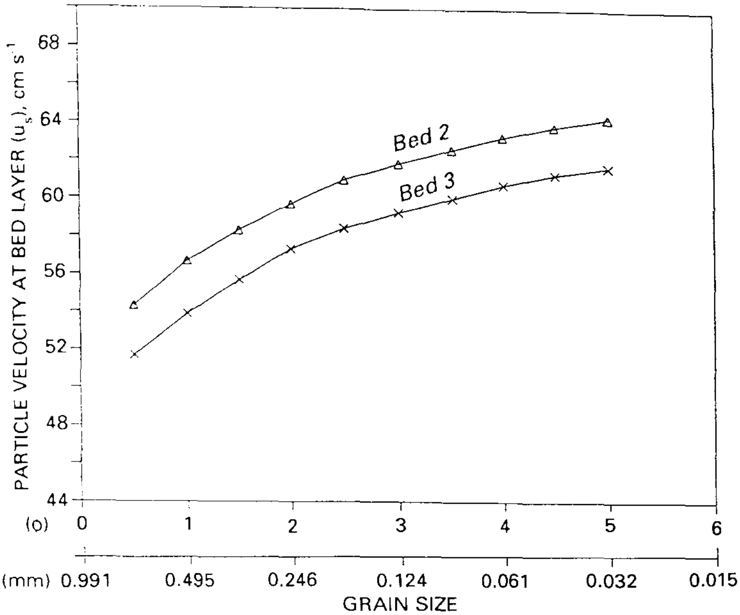

Fig. 1. Average particle velocity at the bed layer. (△) Bed 2 for $u_m=121{\cdot}3$ cm s^{-1}; (×) bed 3 for $u_m=97{\cdot}8$ cm s^{-1}.

Using Eq. (5) in Eq. (4), the velocity distribution (4) is extrapolated nearest to the bed at a height $y=0{\cdot}25$ cm$=y_1$, the interface of bed layer and suspension. A linear velocity distribution is assumed down to the bed layer zone, which is below the height $y=0{\cdot}25$ cm and down to $y=k_s$, where the velocity is assumed to be zero.

In this paper, determination of B is different from previous studies. It would be more consistent to utilize the concept of instantaneous migration velocity of sediment particles in the bed layer. The migration velocity (u_s) of a sediment particle in the bed layer is a function of shear velocity (u_*) and critical shear velocity (u_{*c}) corresponding to the condition of Shield's grain movement for different bed materials. The empirical relation for the migration velocity of a sediment particle in the bed layer is given by Engelund & Fredsoe (1976) as

$$u_s=\beta u_*[1-0{\cdot}7\sqrt{(u_{*c}/u_*)}], \qquad (6)$$

where the value of β is 9·0 for sand. As it is very difficult to measure the fluid velocity at the top of the bed load layer, it would be more reasonable to assume the migration velocity of a representative size at $y=0{\cdot}25$ cm (the top of the bed load layer) to be equal to the fluid velocity at that layer, rather than extrapolation. Therefore, the fluid velocity at the interface will be

$$u=u_s \text{ at } y=y_1. \qquad (7)$$

Using this condition at the interface, the constant B is determined from Eq. (4).

Figure 1 shows the plots of u_s using Eq. (6) at the bed layer for two maximum velocities ($u_m=97{\cdot}8$ and $121{\cdot}3$ cm s^{-1}) of water over two different sand beds (data from Ghosh *et al.*, 1981). It is clear that as the particle size decreases, the migration velocity increases and reaches asymptotically a constant velocity for the size range 0·032–0·061 mm. The fluid velocities at the top of the bed load layer ($y_1=0{\cdot}25$ cm) for two maximum velocities $u_m=121{\cdot}3$ and $97{\cdot}8$ cm s^{-1} are respectively $u=u_s=63$ and 61 cm s^{-1}. Using the determined constant B, the velocity distribution (4) is plotted against y/d in Figure 2 for two maximum velocities (u_m) above the sand beds. The agreement between the observed and computed velocities is close for the values of y shown. Since the velocity distribution (4) is valid closest to the bed, at a height of about 0·25 cm, a linear velocity profile is assumed for the bed layer from k_s to 0·25 cm as

$$u=\frac{u_s}{(y_1-k_s)}(y-k_s),\ y_1-k_s\neq 0, \qquad (8)$$

where u_s is the value read from Fig. 1.

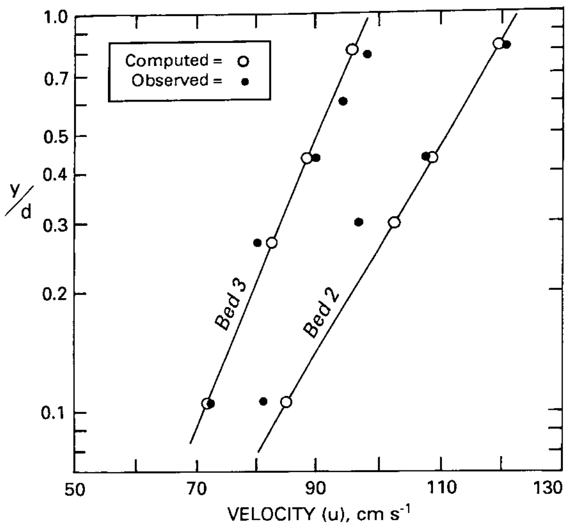

Fig. 2. Logarithmic velocity distribution, showing observed and computed velocity (see text for detailed discussion).

The vertical distribution of sediment concentration above the bed is obtained by substituting Eq. (2) in Eq. (1) and integrating from k_s to y:

$$\int_{k_s}^{y} \frac{dc}{(1-c)^{\alpha+1}\, c} = -\int_{k_s}^{y_1} \frac{w_0}{\varepsilon_{sb}} dy - \int_{y_1}^{y} \frac{w_0}{\varepsilon_{ss}} dy, \quad (9)$$

where the sediment diffusion coefficient ε_{sb} for the bed layer zone is

$$\varepsilon_{sb} = \frac{u_*^2}{u_s}(y_1 - k_s)(1 - y/d) \quad (10)$$

and that for the suspension zone is

$$\varepsilon_{ss} = 2\chi d u_* (1 - y/d)[B - (1 - y/d)^{1/2}]. \quad (11)$$

If $\alpha=0$ in Eq. (9), the equation is exactly similar to Eq. (15) of Ghosh *et al.* (1981), except for the values of u_s and B in Eqs (10) and (11). Equation (2) shows that for $\alpha=0$, the settling velocity $w=w_0$. As α increases to 5, the particle settling velocity changes due to grain–grain interaction in suspension, but if α is very large, then $w \rightarrow 0$. To compute the concentration c_y at any height y, numerical integration of Eq. (9) has been performed for $\alpha=0$, 3, 4 and the given relative bed concentration c_{k_s} by trapezoidal rule (Conte & de Boor, 1986). The values of α used for this calculation are read from fig. 2 of Maude & Whitmore (1958). These values depend on the particle Reynolds number (R_p) ranging from 2 to 50 computed from our data. Once the concentration function c_y is known from Eq. (9) using Eq. (10) and (11), the relative suspension concentration c'_y may be obtained as

$$c'_y = c_y / \sum_{\varphi} c_y. \quad (12)$$

Equation (12) is used to calculated the relative suspension concentration c'_y of a given grain size with a settling velocity w_0 (in clear water) at any height $y>y_1$ above the bed for a given relative bed concentration $c_{k_s}(\varphi)$. If $y=y_1$, Eq. (12) corresponding to Eq. (9) gives the average relative bed layer concentration c_{bl} of sediment of different sizes.

EXPERIMENTAL STUDIES

Data used for verification of the present model are taken from the earlier publications by Sengupta (1975, 1979) and Ghosh *et al.* (1979, 1981). Two closed circuit laboratory flumes (see Ghosh *et al.*, 1986), one designed at Uppsala University and other at the Indian Statistical Institute, Calcutta, were used for conducting the controlled experiments. Bed materials of six different known grain size distributions were used. Descriptions of equipment set-up, techniques of velocity measurement, sample collection, and analysis have been given in the earlier publications. The size distributions of two different sand beds (beds 2 and 3) are used in this article for computation (Fig. 3a). The flow parameters used for verification of the model are reproduced in Table 1.

COMPARISON WITH OBSERVED VALUES

Computations based on Eqs (9) and (12) have been performed for $\alpha=0$, 3 and 4. Observed and computed values of suspended grain size distribution above two sand beds have been plotted in Fig. 3(b,c) for various values of α. The weighted relative errors between the observed and computed values have been computed by the following formula:

$$E = \sqrt{\left[\Sigma \frac{(c'_c - c'_0)^2}{c_0'^2} c_0'\right]} = \sqrt{\left[\Sigma \frac{(c'_c - c'_0)^2}{c'_0}\right]}, \quad (13)$$

where c'_c is the computed relative suspension concentration and c'_0 is the observed relative suspension concentration.

The relative errors between the observed and computed values are shown in Table 2 for the present method and the methods developed by Gessler (1965) and Ghosh *et al.* (1981).

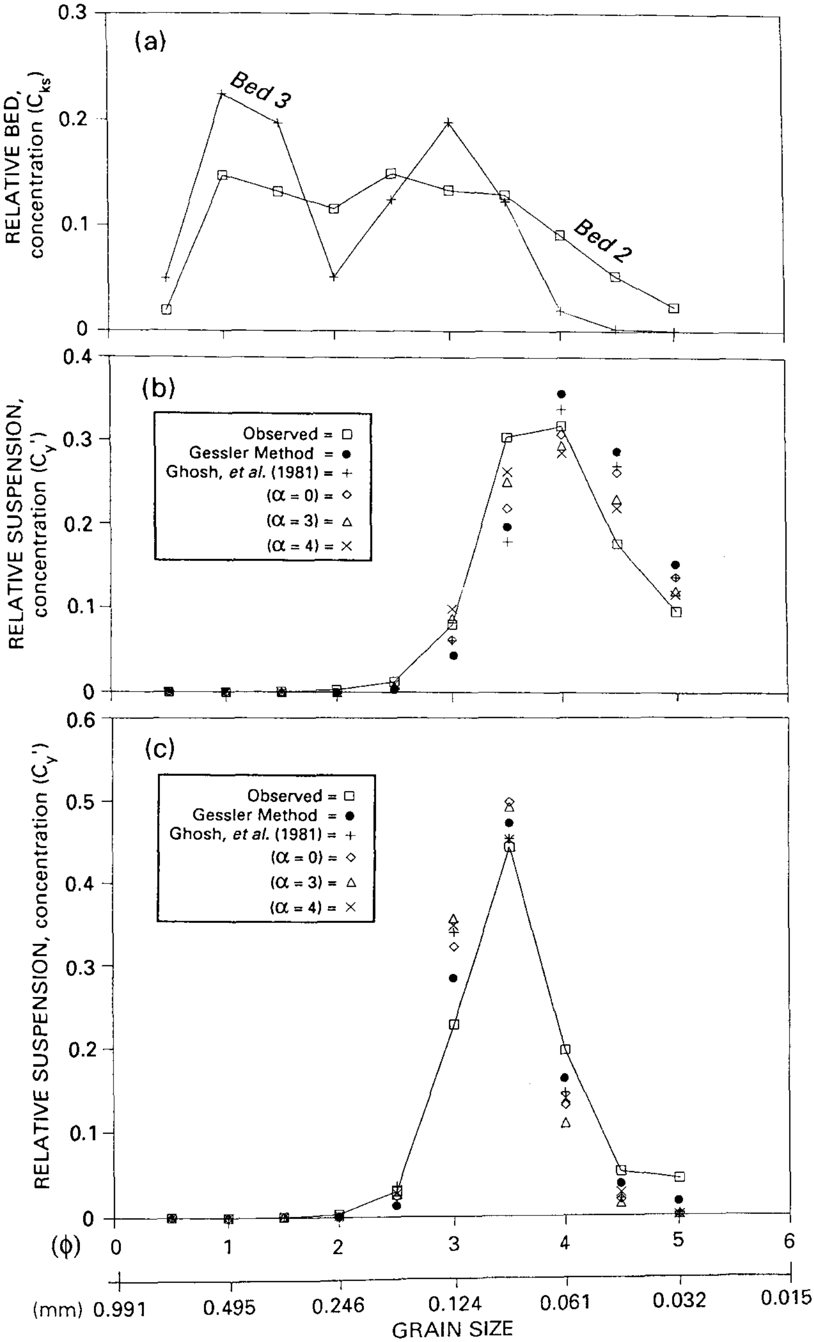

Fig. 3. (a) Grain size distributions of sand beds 2 and 3. (b) Grain size distribution in suspension (c_y' at y=23·3 cm) above sand bed 2. (c) Grain size distribution in suspension (c_y' at y=17·5 cm) above sand bed 3.

It is clear from Table 2 that for bed 2, the smallest error is obtained by the present method, whereas for bed 3, the smallest error is obtained by Gessler's method. On the whole, errors obtained by different

Table 1. Flow parameters used for computation.

	Bed 2	Bed 3
Depth d (cm)	30·0	30·0
Sampling height H (cm)	25·0	20·0
Effective sand bed height h (cm)	~1·7	~2·5
Maximum velocity u_m (cm s^{-1})	121·3	97·8
Roughness k_s (cm)	0·0297	0·0450
Prediction height y (cm)	23·3	17·5
Slope J	0·0020	0·0018
Temperature (°C)	19·0	19·0

Table 2. Errors between computed and observed suspended loads above beds 2 and 3.

Bed no.	Height y (cm)	Gessler (1965) method	Ghosh *et al.* (1981) method	Present method		
				$\alpha=0$	$\alpha=3$	$\alpha=4$
2	23·3	0·39	0·36	0·30	0·20	0·18
3	17·5	0·29	0·36	0·36	0·33	0·30

methods are of the same order (except for the present method for bed 2 for $\alpha=3$ and 4) and it seems that the present method for both beds 2 and 3 gives better accuracy than the method developed by Ghosh *et al.* (1981). The present method can be claimed to be a more realistic one because it utilizes the concepts of instantaneous migration velocity of particles in the bed layer and the hindered settling effect (as discussed in the previous article). The other real advantage of this method is that it affords direct computation of the suspended load from the bed's grain size distribution without going through an intermediate stage (bed load) as required by the other methods.

The main drawbacks in using Gessler's method seem to arise from extrapolation of the critical shear stress for the grain sizes (Gessler, 1965, fig. 8), and a two stage computation: (1) bed to bed layer and (2) bed layer to suspension by the Rouse suspension equation. Extrapolation below a value of grain Reynolds number (Re_*) of 10 gives nearly the same value of critical shear stress (T_c) irrespective of grain sizes, where it appears from the Shields diagram and its modifications that T_c should increase slightly with a decrease in Re_* (Miller *et al.*, 1977).

CONCLUSIONS

A method of computing the grain size distribution of suspended sediment from the texture of the bed material has been developed with the help of the diffusion approach. The method utilizes the reduction of grain settling velocity with sediment concentration in suspension. The migration velocity of sediment particles is computed in the bed layer and is used to predict the fluid velocity immediately above the sand bed. The results obtained by the present method compare well with experimental observations.

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APPENDIX

Explanation of notation

- B Integrating constant
- c Concentration of material per unit volume
- c_{bl} Average bed layer concentration in proportion
- c_{k_s} Relative bed concentration at k_s
- c_y Concentration distribution at any predicting height
- c'_y Relative concentration distribution at any height y above the bed
- d Initial depth of water as measured from flume base (cm)
- h Average height of the bed during collection of the suspended load sample (cm)
- H Height of sample collection from the flume base (cm)
- J slope
- k_s Same height as the bed roughness as defined by Einstein (1950) (cm)
- Re_* Grain Reynolds number
- T_c Dimensionless critical shear stress
- u Time averaged velocity of water in the flow direction (cm s^{-1})
- u_* Shear velocity $\sqrt{\tau_{0/\rho}}$ (cm s^{-1})
- u_{*c} Critical shear velocity (cm s^{-1})
- u_m Maximum velocity of water (cm s^{-1})
- u' Fluctuating velocity in the x-direction (cm s^{-1})
- u_s Migration velocity of sediment (cm s^{-1})
- w_0 Settling velocity of sediment in clear water (cm s^{-1})
- w Settling velocity of particle in a fluid–sediment mixture (cm s^{-1})
- v' Fluctuating velocity in the y-direction (cm s^{-1})
- y Prediction height above the sand bed (cm)
- y_1 Height of bed layer (0·25 cm) (cm)
- α Exponent of reduction of fall velocity
- ε_m Water diffusion coefficient (cm^2 s^{-1})
- ε_s Sediment diffusion coefficient (cm^2 s^{-1})
- τ Turbulent shear stress at any point in the fluid (mass cm^{-1} s^{-2})
- τ_0 Bottom shear stress (mass cm^{-1} s^{-2})
- φ $-\log_2 D$, a measure of grain size (where D is in millimetres)
- χ von Kármán constant (0·4).